# A Merton-model approach to assessing the default risk of UK public companies

*Merxe Tudela*[*]

*and*

*Garry Young*[**]

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* Bank of England.
E-mail: merxe.tudela@bankofengland.co.uk

** Bank of England.
E-mail: garry.young@bankofengland.co.uk

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**Abstract**

This paper shows how a Merton-model approach can be used to develop measures of the probability of failure of individual quoted UK companies. Probability estimates are then constructed for a group of failed companies and their properties as leading indicators of failure assessed. Probability estimates of failure for a control group of surviving companies are also constructed. These are used in probit regressions to evaluate the information content of the Merton-based estimates relative to information available in company accounts and in assessing Type I and Type II errors. We also look at power curves and accuracy ratios. The paper shows that there is much useful information in the Merton-style estimates.

Key words: Merton models, corporate failure, implied default probabilities

JEL classification: G12, G13

**Summary**

The quantitative modelling of credit risk initiated by Merton (1974) shows how the probability of company default can be inferred from the market valuation of companies under specific assumptions on how assets and liabilities evolve. This paper employs a Merton-style approach to estimate default risk for public non-financial UK companies and assesses the reliability of these estimates using a range of different techniques.

The original Merton model is based on some simplifying assumptions about the structure of the typical firm's finances. The event of default is determined by the market value of the firm's assets in conjunction with the liability structure of the firm. When the value of the assets falls below a certain threshold (the default point), the firm is considered to be in default.

To draw conclusions on financial stability and implement the right policy measures, the estimated probabilities of failure need to be both reliable and efficient. This paper assesses the reliability of the estimates by examining their success in predicting the failure or survival of both failed companies and survivors. The efficiency of the estimates is assessed by testing the extent to which the predictive power of the estimates could be improved by incorporating other information publicly available in company accounts. Models that combine a Merton approach with additional financial information are referred to in the literature as 'hybrid models'.

The probability of default derived from our Merton-model implementation provides a strong signal of failure one year in advance of its occurrence. For example, the mean value of the estimated one-year probabilities of default for our entire sample is 47.3% for those companies that went bankrupt, and 5.4% for those that did not.

Calculation of Type I and II errors (Type I errors are defined as the percentage of actual failures classified as non-failures, Type II errors are the percentage of non-failures classified as failures) suggests that the estimated probabilities of default are successful in discriminating between failing and non-failing firms. Classifying defaults as those firms with an estimated probability of default greater than or equal to 10%, the Type I error is relatively modest at 9.2% (with a Type II error of 15.0%).

Our implementation of the Merton approach clearly outperforms a reduced-form model based solely on company account data. But our analysis also shows that the type of hybrid

models implemented here, ie those combining company account information and the Merton approach, outperform our implementation of the Merton approach, if only marginally.

## 1 Introduction

The quantitative modelling of credit risk initiated by Merton (1974) shows how the probability of company default can be inferred from the market valuation of companies under specific assumptions on how assets and liabilities evolve. This paper employs a Merton-style approach to estimate default risk for public non-financial UK companies and assesses the reliability of these estimates using a range of different techniques.

The original Merton model is based on some simplifying assumptions about the structure of the typical firm's finances. The event of default is determined by the market value of the firm's assets in conjunction with the liability structure of the firm. When the value of the assets falls below a certain threshold (the default point), the firm is considered to be in default. A critical assumption is that the event of default can only take place at the maturity of the debt when the repayment is due. In order to implement the model in practical situations, this paper shows how this assumption can be modified to allow for default to occur at any point in time and not necessarily at maturity.

KMV Corporation(1) also uses the broad Merton approach to estimate the probability of firm failure in a number of different countries over a range of different forecast horizons. But the KMV approach does not rely solely on an analytical Merton model. Instead, it uses the Merton framework to estimate the 'distance-to-default' of an individual company and then uses a proprietary database of US company histories to map this into an 'expected default frequency' (EDF), estimated by the proportion of companies with a given distance to default that have failed in practice.(2) By contrast, the calculations reported here use only publicly available information on market prices and time series estimates of parameters to measure the probability of default.

To draw conclusions on financial stability and implement the right policy measures, the estimated probabilities of failure need to be both reliable and efficient. This paper assesses the reliability of the estimates by examining their success in predicting the failure or survival of both failed companies and survivors. The efficiency of the estimates is assessed by testing the extent to which the predictive power of the estimates could be improved by incorporating other information publicly available in company accounts. Models that combine a Merton approach with additional financial information are referred in the

(1) Moody's has recently acquired KMV Corporation. The combined business of Moody's Risk Management Services and KMV is called Moody's KMV. Throughout this paper we use the terms KMV and Moody's to refer to the KMV Corporation and Moody's, respectively, before this acquisition took place.
(2) For a review of the KMV approach see Kealhofer and Kurbat (2002).

literature as 'hybrid models'. Sobehart and Keenan (2001b) provide an excellent summary of this class of models.

There are other market based indicators of default probability. Nevertheless, though dependent upon certain modelling assumptions, structural models provide a cleaner and more direct measure of implied default probability than do market prices and other indicators from the bond market. In a frictionless, risk-neutral world, the excess return generated by investment in a corporate bond relative to a government bond would simply reflect compensation for the expected loss from default. In a recent empirical study by Elton, Gruber, Agrawal and Mann (2001), however, it is shown that such compensation constitutes less than a fifth of the observed spread, with taxation and risk premia making up the lion's share. Other studies, such as Collin-Dufresne, Goldstein and Martin (2001) obtain similar findings. Thus, extracting information on expected default probability from observed credit spreads is a non-trivial spread-decomposition exercise. Credit ratings are also imperfect measures. First, credit ratings are ordinal, rather than cardinal, measures of credit quality, and take into account not only default probability but also the severity of loss given default. Second, they are designed to judge credit quality over the long term, and may therefore be inappropriate measures of the probability of default over a relatively short horizon.

The paper is organised as follows. Section 2 briefly reviews the literature on equity-based models of firm default. Section 3 shows how the original Merton model may be extended so that it can be implemented in practice. Section 4 outlines how the model may be tested. Section 5 describes the data on UK quoted companies and the sample that is used in constructing estimates of failure. Section 6 sets out the results. Section 7 concludes.

## 2 Literature review

There is a wide range of papers studying aggregate company defaults. Here we concentrate on those papers that adopt a structural or hybrid approach. The analyses by the KMV Corporation and Moody's are the most well known. For a more extensive discussion of the strengths and drawbacks of various models for valuing financial instruments that are subject to default risk,[3] we refer the reader to Nandi (1998).

Crosbie and Bohn (2002) summarise KMV's default probability model. KMV's default probability model is based on a modified version of the Black-Scholes-Merton framework in the sense that KMV allows default to occur at any point in time and not necessarily at

(3) Default risk and default probability are interchangeable terms in this paper.

the maturity of the debt. In this model multiple classes of liabilities are modelled. There are essentially three steps in the determination of the default probability. The first step is to estimate the asset value and volatility from the market value and volatility of equity and the book value of liabilities using their Merton approach. Second, the distance-to-default is calculated using the asset value and asset volatility. And finally, a default database of over 250,000 company-year data and over 4,700 incidents of default is used to derive an empirical distribution relating the distance-to-default to a default probability.

Sobehart, Stein, Mikityanskaya and Li (2000) (Moody's model) construct a hybrid default risk model for US non-financial public firms. The sample consists of 14,447 public firms with multiple observations for each firm (about 100,000 firm-year observations) and 923 default events. The aim of the model is its use as an early warning system to monitor changes in the credit quality of corporate obligors. Moody's model provides a one-year estimated default probability using a variant of Merton's option theoretic model, Moody's rating (when available), company financial statement information,[4] additional equity market information[5] and macroeconomic variables.

As with the KMV model, the variant of the Merton model applied by Sobehart *et al* (2000) is not used directly to calculate default probabilities but rather to calculate the market value and volatility of the firm's assets from equity prices. These inputs are used to derive the 'distance to default', the number of standard deviations that the value of the firm's assets must drop in order to reach the default point. Moody's combine this information into a logistic regression to obtain some default probabilities that are further adjusted to correct for the fact that their in-sample data set had a slightly different proportion of defaulting-to-non-defaulting obligors from that observed in reality.[6] According to the authors there appears to be a significant jump in performance as one moves from pure statistical models to those that include structural information. Interestingly, there is also a large gap between the pure structural model (Merton model) and Moody's hybrid model. The gap would represent the gain in accuracy derived from financial statement and rating data.

Kealhofer and Kurbat (2002) (KMV) try to replicate Moody's empirical results (Sobehart *et al* (2000)) on the Merton approach. They obtain contrary results. They claim the Merton approach outperforms Moody's ratings and various accounting ratios in predicting default.

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(4) Specifically, net income-assets ratio, assets, working capital-assets ratio, liability-assets ratio and net income-equity ratio.
(5) Stock price volatility and equity growth rate.
(6) Since the authors do not have information on all public firms that default, the adjustment is made using a subset of Moody's default database that included over 1,400 non-financial US defaults between 1980 and 1999.

Kealhofer and Kurbat (2002) explain this discrepancy by the fact that their implementation of the Merton model is more accurate than Moody's approach. This greater accuracy, according to the authors, come from the special approaches developed to estimate asset volatility.[7]

Leland (2002) examines the differences in the probabilities of default generated by two alternative structural models. The first group is termed the 'exogenous default boundary' approach, meaning models of the type of Merton (1974) and Black and Scholes (1973). A default boundary is a sufficiently low level of asset value so that the firm decides to default on its debt whenever the asset value falls below this level. The second group of models introduces an 'endogenous default boundary'. In these models the decision to default is an optimal decision by managers acting to maximise the value of equity. The default boundary depends on the expected return and volatility of assets, the risk free interest rate, leverage, debt maturity and default costs. According to the authors, it fits actual default frequencies for longer time horizons quite well, although the predicted default frequency is too low for short maturities. Moreover, the endogenous models predict that default probabilities rise with default costs and fall with bond maturity, whereas default probabilities derived from an exogenous model are invariant to these parameters. Exogenous models are also more sensitive to asset volatility. The authors cannot test the relative accuracy of these predictions due to the lack of publicly available data.

Huang and Huang (2002) using a range of structural models try to solve the question of how much of the historically observed corporate-Treasury yield spread is due to credit risk, that is, if that spread can be explained using probabilities of default derived from those structural models. To do this they calibrate the probabilities derived from the structural models to be consistent with data on historical default experience. For investment grade bonds of all maturities, credit risk accounts for only a small fraction of the spread (and even smaller for shorter maturities). For junk bonds credit risk accounts for a larger fraction.

## 3 Implementation of the Merton model

The basic insight of the Merton (1974) model is that the pay offs to the shareholders of a firm are very similar to the pay offs they would have received had they purchased a call option on the value of the firm with a strike price given by the amount of debt outstanding. As such, the option pricing techniques of Black and Scholes (1973) may be used to

(7) For further insights in this discussion see Keenan and Sobehart (1999), Stein (2000) and Sobehart and Keenan (2001a).

estimate the value of the option and the underlying probability of default.

The Merton model and the variation of the Merton model adopted in this paper assume a simple capital structure for the firm: debt plus equity. We denote the notional amount of debt by $B$, with $(T-t)$ being the time to maturity, the value of the firm is $A_t$, and $F(A,T,t)$ is the value of the debt at time $t$. The equity value at $t$ is denoted by $f(A,t)$. Then, we can write the value of the firm $A_t$ as:

$$A_t = F(A,T,t) + f(A,t) \tag{1}$$

The original Merton model assumes that the firm promises to pay $B$ to the bondholders at maturity $T$. If this payment is not met, that is, if the value of the firm is less than $B$, the bondholders take over the company and the shareholders receive nothing. Furthermore, the firm is assumed not to issue any new senior claims nor pay cash dividends nor repurchase shares prior to the maturity of the debt. Under these assumptions the shareholders effectively hold a European call option on the value of the firm.

This paper relaxes the assumption that default (or insolvency) can only occur at the maturity of the debt. The model developed here assumes that insolvency occurs the first time that assets falls short of the redemption value of debt. In other words, insolvency occurs the first time that the value of the firm falls below the default point.

To model this we use the concept of a barrier option.[8] Barrier options are options where the pay off depends on whether the underlying asset price reaches a certain level during a certain period of time. Barrier options can be classified as either knock-out options or knock-in options. A knock-out option ceases to exist when the underlying asset price reaches a certain barrier. This is the type of barrier option we are interested in here. A down-and-out call option is one type of knock-out option. It is a regular call option that ceases to exist if the asset price reaches a certain level, the barrier. The barrier level is below the initial asset price.

To derive the probability of default using a barrier option we suppose that the value of the firm's underlying assets follows the following stochastic process:

$$dA = \mu_A A dt + \sigma_A A dz \tag{2}$$

where $dz = \varepsilon\sqrt{dt}$ and $\varepsilon \sim N[0,1]$

[8] Other equity-based models of credit risk that use the concept of barrier options are Black and Cox (1976), Longstaff and Schwartz (1995) and Briys and de Varenne (1997).

and we assume a deterministic process for the liabilities:

$$dL = \mu_L L dt \qquad \textbf{(3)}$$

Let us denote the asset-liability ratio by $k$:

$$k = A/L \qquad \textbf{(4)}$$

Default occurs when $k$ falls below the default trigger or default point called $\underline{k}$ at any time within a given period. To estimate this probability of default we need to model how $k$ changes over time. Differentiate **(4)** and use **(2)** and **(3)** to obtain

$$dk = (\mu_A - \mu_L)kdt + \sigma_A kdz \qquad \textbf{(5)}$$

and define:

$$\mu_A - \mu_L = \mu_k \quad and \quad \sigma_A = \sigma_k$$

The values for $\mu_k$ and $\sigma_k$ are needed to calculate the probabilities of default. Maximum likelihood techniques are used to obtain estimates of those two parameters. In order to construct the maximum likelihood function, we first need to derive an expression for the probability density function (PDF) of $k$. Given equation **(5)** we can derive the PDF for $\ln \frac{k_T}{k_t}$ (we call this PDF 'defective density'). It can be shown that[9] the defective density function is given by $h\left(\ln \frac{k_T}{k_t}\right)$ according to the following expression:

$$\begin{aligned} h\left(\ln \frac{k_T}{k_t}\right) = \frac{1}{\sqrt{2\pi\sigma_k^2(T-t)}} \Bigg\{ & \exp\left[ - \frac{\left(\ln \frac{k_T}{k_t} - \left(\mu_k - \frac{\sigma_k^2}{2}\right)(T-t)\right)^2}{2\sigma_k^2(T-t)} \right] \\ & - \exp\left[ \frac{2\ln \frac{\underline{k}}{k_t}\left(\mu_k - \frac{\sigma_k^2}{2}\right)}{\sigma_k^2} - \frac{\left(\ln \frac{k_T}{k_t} - 2\ln \frac{\underline{k}}{k_t} - \left(\mu_k - \frac{\sigma_k^2}{2}\right)(T-t)\right)^2}{2\sigma_k^2(T-t)} \right] \Bigg\} \end{aligned} \qquad \textbf{(6)}$$

Equation **(6)** represents the probability density of not crossing the barrier and being at the point $\ln(\frac{k_T}{k_t})$ at time $T$. This expression is used to construct the likelihood function that is maximised[10] in order to obtain estimates of $\mu_k$ and $\sigma_k$. These estimates are used to calculate the probabilities of default as shown below.

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(9) Rich (1994) offers a very complete mathematical approach to barrier options.

(10) This density function must be multiplied by a Jacobian adjustment term to correct for the fact that it is the equity-liability ratio ($y$) that is observed rather than the market value of the firm-liability ratio ($k$). At the end of this section we derive an expression to map $y$ and $k$.

The probability[11] of the firm not defaulting until date $T$ is given by the probability of $k_T > \underline{k}$ conditional on $k_\tau > \underline{k} \quad \forall \tau \quad t \leq \tau < T \Rightarrow$

$$PD = 1 - \{[1 - N(u_1)] - \varpi\,[1 - N(u_2)]\} \tag{7}$$

where:

$$u1 = \frac{\underline{K} - \left(\mu_K - \frac{\sigma_k^2}{2}\right)(T-t)}{\sigma_k\sqrt{T-t}} \tag{8}$$

$$u2 = \frac{-\underline{K} - \left(\mu_K - \frac{\sigma_k^2}{2}\right)(T-t)}{\sigma_k\sqrt{T-t}} \tag{9}$$

$$\varpi = \exp\left[\frac{2\underline{K}\left(\mu_k - \frac{\sigma_k^2}{2}\right)}{\sigma_k^2}\right] \tag{10}$$

$$\ln\frac{\underline{k}}{k_t} = \underline{K} \tag{11}$$

Equation **(7)**, that is, the probability of default, depends on the maximum likelihood estimates, $\hat{\mu_k}$ and $\hat{\sigma_k}$ , the default point, $\underline{k}$, here set up to equal one,[12] and the assets to liability ratio via $N(u_1)$ and $N(u_2)$.

The $N(u_1)$ term in **(7)** is equivalent to the probability of default obtained using a European call option. In the case of a barrier option we have to correct that probability of default for the fact that default occurs the first time the assets to liability ratio crosses the barrier and not just at $T$. The term $\varpi\,[1 - N(u_2)]$ corrects the probability of default derived using a European call option (path independent) to take into account that the asset-liability ratio can hit the barrier before $T$ (path dependent).

A further observation is needed here. The value of the firm's assets, $A$, is unobservable and hence so is the $k$ ratio. What we can observe is the equity-liability ratio, $y = \frac{X}{L}$, $X$ being the market capitalisation of the firm. Nickell and Perraudin (1999) derive a mapping between the equity-liability ratio and the value of the firm's assets-liability ratio that this paper borrows.

Following Nickell and Perraudin (1999) we assume that the earnings flow of a firm is defined as $\delta(A - L)$, with $\delta$ a constant dividend pay-out rate. We also assume a constant

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(11) See Appendix A for a derivation of this probability.

(12) Sensitivity tests to the choice of the default point have been carried out but not reported here for the sake of brevity.

short-term interest rate of $r$. Under these assumptions the risk adjusted drift terms for assets and liabilities are $\mu_A^* = \mu_L = r - \delta$.

The risk neutral rate of return of a particular firm's equity must be equal to the dividend income plus capital gains received by equity holders, that is:

$$rX = \delta(A - L) + \frac{dE_t(X)}{dt} \quad \textbf{(12)}$$

with $X$ depending on $L$ and $A$.

We now derive an expression for $dE_t(X)$:

$$dE_t(X) = \frac{\partial X}{\partial A}dA + \frac{\partial X}{\partial L}dL + \frac{1}{2}\frac{\partial^2 X}{\partial A^2}dA^2 + \frac{1}{2}\frac{\partial^2 X}{\partial L^2}dL^2 + \frac{\partial^2 X}{\partial A \partial L}dAdL \quad \textbf{(13)}$$

Using the expressions for $dA$, $dA^2$, $dL$ and noting that $dt^2 = dtdz = 0$ and $dz^2 = dt$, the above expression reduces to:

$$dE_t(X) = \frac{\partial X}{\partial A}\mu_A^* A dt + \frac{\partial X}{\partial A}\sigma_A A dz + \frac{\partial X}{\partial L}\mu_L L dt + \frac{1}{2}\frac{\partial^2 X}{\partial A^2}\sigma_A^2 A^2 dt \quad \textbf{(14)}$$

Dividing the above expression by $dt$ and substituting into equation **(12)** we obtain:

$$rX = \delta(A - L) + \frac{\partial X}{\partial A}\mu_A^* A + \frac{\partial X}{\partial L}\mu_L L + \frac{1}{2}\frac{\partial^2 X}{\partial A^2}\sigma_A^2 A^2 \quad \textbf{(15)}$$

The above expression is a homogeneous partial differential equation in two variables, $A$ and $L$. It can be proved[(13)] that the solution to this differential equation is:

$$y(k) = k - 1 - (\underline{k} - 1)\left(\frac{k}{\underline{k}}\right)^{\lambda} \quad \textbf{(16)}$$

where

$$\lambda = \frac{1}{\sigma_A^2}\left(\frac{\sigma_A^2}{2} - \sqrt{\frac{\sigma_A^4}{4} + 2\sigma_A^2\delta}\right) \quad \textbf{(17)}$$

Using some initial values for $k$, $\mu_k$ and $\sigma_k$ we apply the Newton-Rapshon scheme to solve expression **(16)** for $k$. We then use this estimate to maximise **(6)** and get the estimates for $\mu_k$ and $\sigma_k$. Using the estimate for $\sigma_k$ we invert **(16)** to obtain the final $k$ series.

## 4 Testing the model

To test the performance of the Merton approach adopted in this paper, we calculate the probabilities of default (PDs) implied by our model for a sample of UK non-financial firms that includes a number of defaulters.[(14)] We then perform three types of tests: (1) we

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(13) A proof that expression **(16)** is a solution for equation **(15)** can be found in Appendix B.

(14) We describe the composition of the sample in the section below.

evaluate our model against the actual default experience; (2) we compare our model with other default models; and (3) we use measures of statistical power based on power curves and accuracy ratios.

For the first type of test, we compare the PD profiles for a subsample of defaulters with the timing of actual defaults to assess the accuracy of the model in predicting those failures. We also calculate the Type I and II errors. Type I errors are defined as the percentage of actual failures that the model classifies as non-failures. Type II errors are the percentage of non-failures that the model classifies as failures. Ideally we want both type of errors to be small, but clearly there is a trade-off between the two.

For the second type of test, we compare our model with other approaches. To compare the performance of our Merton approach with the information content of company account data only, we estimate a probit model. The dependent variable is a dummy that takes on the value of unity if the company went bankrupt, and zero otherwise, and regressors are company account indicators. To select the company account variables included in the probit estimations we follow Geroski and Gregg (1997), one of the most comprehensive empirical studies of the determinants of company default in the United Kingdom. To compare the accuracy of both models we calculate Type I and II errors.

The power of the PDs in explaining company default is assessed formally against other models by testing for their significance when added to the estimated probit model above. If the coefficient of the PD variable is significantly different from zero, we can conclude that the Merton approach here implemented adds value to the company account variables.

For the third type of test, following Kocagil, Escott, Glormann, Malzkom and Scott (2002), we use power curves and accuracy ratios to assess the statistical power of the models. Both testing tools evaluate the accuracy of a model in ranking defaulters and non-defaulters using the estimated probabilities of default. To plot a power curve, for a given model we rank the companies in our sample by risk score (PD) from the riskiest to the safest (horizontal axis). For a given percentage of this sample we calculate the number of defaulters included in that percentage as a proportion of the total number of defaulters in our sample (vertical axis). Thus for a sample in which 1% of companies default, a perfect model would include all the defaulters within the riskiest percentile. By contrast, in a random model the first percentile would tend to include only 1% of the defaulters and its power curve would be represented by a 45 degree line. The better the model at ranking companies the more bowed towards the upper-left corner its power curve would be. The power curve is sample dependent in that its shape is dependent on the proportion of companies in the sample that default.

The accuracy ratio, even if less visual, gives a single statistic that summarises the information content of the power curve. The accuracy ratio has values that rank from 0% (random model) to 100% (perfect model) and it is defined as the ratio of the area between the power curves of the actual and random models to the area between the power curves of the perfect and random models.

## 5 Sample and data description

The model presented in Section 3 is estimated for a sample of UK non-financial quoted[(15)] companies. Specifically, we collect 7,459 financial statements from 1990 to 2001, 65 of which correspond to firm defaults.[(16)] The sample of failed companies was constructed collecting news from FT.com about companies that went into receivership.[(17)] The sample constructed in this manner was checked against the 'deaduk' dataset in Thompson Financial Datastream and the 'Companies House' web site. The default date was selected as being the last day in which an equity price movement was observed. This may be not the exact date of default but it is a good approximation given the discrepancy and/or inaccuracy observed in the different sources consulted and the difficulty of defining a default date.

Table A disaggregates the number of failures and non-failures by year for the sample we use in the estimations (and for the sample we initially gathered for illustration purposes). It is immediately apparent that defaults are concentrated in 1990–92 ie the recession years.

All our data are downloaded from Thompson Financial Datastream. To estimate the PDs we use market capitalisation and liability data (current liabilities). The PDs are estimated on a weekly basis using a five-year rolling window. That is, we estimate equation **(6)** using five years[(18)] of weekly data to obtain the maximum likelihood estimates $\hat{\mu_k}$ and $\hat{\sigma_k}$ that are

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(15) Clearly, we cannot directly apply the model to private companies given the data requirements: we need equity market capitalisation series to estimate equation **(6)**.

(16) Initially, we identify 76 firm defaults but due to the lack of some company account data needed for our econometric specifications we use 65 to present comparable results across estimations and be able to compare the power curves for the different models. Non-defaulters are all public companies alive in 2001 and from which we have the data needed for the estimations undertaken here.

(17) Note that definition of failure does not include companies that were taken over by other companies or that went into an insolvency procedure other than receivership.

(18) Other estimation periods were used but not reported here for the sake of brevity. A five-year window is a trade-off between too short an estimation period that might include too much noise in the estimation of the drift parameter ($\hat{\mu_k}$) and too long an estimation period that might include information too far back to be relevant for the calculations of current PDs.

used to calculate the PDs. Moreover, we do not include in the maximisation procedure those observations when a dividend pay-out was made. This is to avoid any uninformative jump in equity prices. The dividend payments dates are also obtained from Datastream. The equity data (market capitalisation) are weekly data,[19] but the liability data are annual. In order to generate the necessary weekly liability data we use cubic spline interpolation routines.[20] The PDs are calculated for different time horizons, from one year to five years, but here we concentrate on 1-year and 2-year PDs.

In order to estimate the competing probit models we need company account data, in particular, profit margins, the ratio of debt to total assets, the ratio of cash to liabilities, the number of employees and sales growth. Profit margins are defined as EBITDA relative to sales and we further construct three binary (0,1) dummy variables[21] for negative profit margins, profit margins between 0% and 3%, and profit margins between 3% and 6% (therefore, profit margins greater than 6% is our reference category). The debt to assets ratio is defined as gross debt (borrowing of maturity less than a year plus capital loans with maturity greater than one year) relative to total assets. The cash to liabilities ratio is the 'total cash and equivalent' variable from Datastream relative to liabilities.

Apart from the company account data, we have also included some year dummies and/or a macroeconomic indicator[22] (GDP) in our probit estimates to account for the general economic situation. The macroeconomic data are obtained from the electronic version of the International Financial Statistics published by the International Monetary Fund.

## 6 Results

### *6.1 Implied probabilities of default*

As an initial way of measuring the accuracy of our Merton approach, we first compare the PDs of defaulting and non-defaulting companies. For defaulting companies, we calculate the 1-year ahead PD in each month of the twelve months prior to default and take the simple average of these PDs as a measure of the default probability. This is what we call *1-year PD annual average*. For non-defaulting companies, we take a simple average of the 1-year ahead PDs in each month of the preceding calendar year. We investigate the

(19) Actually, it is daily data but we use Wednesdays only to avoid any day-of-the-week effect.
(20) The goal of cubic interpolation is to get an interpolation formula that is smooth in the first derivative, and continuous in the second derivative. A linear interpolation routine is faster but the resulting curve is not very smooth. We use a cubic spline method as a way to smoothly incorporate the liabilities built by a company progressively along the accounting year.
(21) Following Geroski and Gregg (1997).
(22) Several macroeconomic variables were tested and finally we decided on GDP (see Section 6).

sensitivity of our results to these definitions below. By choosing a failure threshold for the PDs, it is possible to sort companies into those that are classified as defaulters (for whom the PD is greater than the set threshold) and those that are classified as survivors (for whom the PD is less than the threshold).

The usefulness of the estimated default probabilities generated by the model can be assessed by examining the Type I and II errors for different failure thresholds (see results in Table B). The lower the failure threshold, the smaller the Type I error (ie the proportion of companies classified as survivors that failed), but at the expense of a greater Type II error.

The success of the model depends on what threshold is appropriate for the user. If the user is a small investor wishing at all costs to avoid investing in failing companies, then the threshold would be set at a low level so as to avoid Type I errors. Conversely if the user had limited resources but wished to investigate more thoroughly the most risky companies, the threshold would be set high to avoid Type II errors. As shown in Table B, for the entire sample, choosing a failure threshold of 5%, we fail to classify as defaulters 4.6% of companies that went bankrupt. At this level, the Type I error is zero for eight of the years considered. The corresponding Type II error for the whole sample is 19.9% of non-defaulting companies. If we increase the failure threshold to a PD greater or equal than 10%, then 9.2% of our population of defaulters had not been classified as defaulters. But in this case, the Type II error is lower at 15%.

We perform a test for the equality of 1-year PD means between the defaulters group and the non-defaulters group. The 1-year PD average for the defaulter group is 47.33, for the non-defaulter group it is 5.44. The test is undertaken without assuming equality of variances between the two groups. The null hypothesis is that the difference of the two means equals zero. Under the alternative of this difference being different from zero, we reject the null at the 1% level of significance. Under the alternative of the mean for the non-defaulter group being smaller than the mean for the defaulter group, we also reject the null at the same level of significance.

To check further the accuracy of the model we calculate the Type I and II errors for the 2-year PD annual average (defined as the average of the 2-year PDs, —the probability of default in two years time from now— from the twelfth month before the default month to the $24^{th}$ before the default month) and for the 1-year PD for the twelfth month before the default month. This last measure is very strict in the sense that it gathers information for one month only, whereas the other measures compile the information content of twelve months. The results are presented in Table C.

As expected, for the same thresholds, the Type I errors are bigger for the latter measures. For a 5% threshold the Type I error for the 2-year PD annual average is 6.1%, and for the 1-year PD as defined above is 24.6%. To check if the latter is a spurious result, we calculate the Type I error for a 5% cut-off for the 1-year PD for the eleventh month before the default month, the tenth, and so on until the seventh month before the default month. The Type I error for these measures are 16.9%, 18.5%, 16.9%, 13.8% and 10.8%, respectively. These figures are more in line with the results obtained for the 1-year and 2-year PD annual averages, indicating that the figure of 24.6% is spurious. One, therefore, should always look at PDs for more than one month and relative to recent history.

We also conducted the test for the equality of means for the 1-year PD twelve months before the default. The results are similar to the ones obtained when we use the 1-year PD annual average measure. The mean value of the 1-year PD for twelve months before the default date is 32.0% for defaulters and 5.2% for non-defaulters.

By way of illustration and to assess the model's ability to reflect credit risk at the individual firm, Chart 1 represents 1-year and 2-year PDs (monthly averages) for those companies that failed in 1992.[23] The black line is the 1-year PD, that is, the probability of default in one year time in a given month. The grey curve is the 2-year PD, the probability of default in two years time in a given month. The dashed(dotted) vertical line cuts the time axis exactly one(two) year(s) before the failure date.

To correctly classify defaults, 1-year PDs should be above the chosen threshold to define failure when crossing the dashed vertical line. Similarly, 2-year PDs should be greater than the threshold when crossing the dotted vertical line. All the PD curves show rising profiles before the companies went bankrupt, and are very high in the months before the failure. That is, we observe increasing levels of risk as the date of failure draws closer. From twelve to six months before failure the 1-year PD is always greater than 50.8%, whereas from 24 to twelve months before failure the average PD is 29.1%.

Charts 2 and 3 illustrate annual averages of the one and two-year probabilities of default at the $90^{th}$, $80^{th}$, $70^{th}$, $60^{th}$, $50^{th}$ (median) and $40^{th}$ percentiles.[24] For all the percentiles the highest values of the 1-year PD are concentrated between 1990 and 1992. This indicates a greater risk of default forecasted for 1991–93. The probability of default decreases from 1993 onwards but increasing again at the end of the 1990s and the year 2000. This indicates that UK non-financial companies have become riskier during that period. It is

(23) We choose this year because it is the year with the highest number of defaults (see Table A).
(24) Smaller percentiles show very low and very stable PDs for the time period considered here.

worth noting that the PD at the $90^{th}$ percentile has worsened more than at the other percentiles, that is, the riskiest companies have become even riskier in both absolute and relative terms. The 2-year PD profiles at the different percentiles exhibit a similar pattern, with the highest PDs for 1989–92, therefore, predicting the highest risk of default for the years 1991–94. The increase in the PD profile at the end of the last decade is stronger than the one observed for the 1-year PD, and again is more acute for higher percentiles.

### *6.2 Adding other company account information*

As stated in Section 4, to compare the performance of our Merton approach with the information content of company account data only, we estimate a probit model using company account data as regressors. Here the dependent variable is a dummy that takes on the value of unity if the company went bankrupt, and zero otherwise. Given the concentration of defaulters in the recession period, we also include in this probit model a macroeconomic indicator, GDP, as an additional regressor. We test the power of PDs to explain company failure by adding them to the probit model. If the coefficient of the PD variable is significantly different from zero after controlling for company account data, we can conclude that the Merton approach adds value to the company account variables. This result is probably due to the forward-looking nature of market value data used as one of the inputs in calculating the PDs under the Merton approach. Accounting data are by definition backward looking and summarise the state of a firm at a given point in time. Market value data, on the other hand, summarise all the relevant and available information at a precise point in time and the future expectations on a firm, therefore adding value to the company account information.

In Table D we collect the results from these models. We use different measures of PDs for robustness tests. When using 1-year PDs the company account data is lagged one year, that is, it corresponds to the year before the default year —columns (1), (2) and (5). If we include 2-year PDs in the probit estimation, we lag the company account data 2 years, ie the values are those of two years prior to default —columns (3) and (4).[25] We always use the GDP of the year of default.

In column (1) of Table D we use the 1-year PD annual average. The results show that the PD variable is significant at the 1% level and that only one company account variable, the debt to assets ratio, is significant and at a lower level (5%).[26] The number of employees

[25] That is, models (1), (3) and (5) in Table D are hybrid models.

[26] The fact that the debt to asset ratio is significant even controlling for PDs, which use a similar ratio in their calculation, reveals a highly non-linear relationship between likelihood of default and the debt to asset ratio.

(included to account for size) is marginally significant. The variable accounting for the macroeconomic environment is also significant at the 1% level.[27] The conclusion is that the PD variable contains information over and above that included in publicly available company accounts.

In column (2) we re-estimate the model of column (1) excluding the PD variable. Interestingly, the profitability measures are now significant. Having negative profit margins instead of profit margins greater than 6% significantly (at the 1% level of significance) increase the likelihood of failure. Profit margins between 0% and 3% (instead of profit margins greater than 6%) also increases the probability of failure (at the 1% level). The coefficient for this last measure is, as expected, smaller than the coefficient for negative profit margins. The coefficient of profit margins between 3% and 6% is smaller than the two previous coefficients but it is not significant. If we compare these three coefficients with the ones in column (1) we clearly see the effect of omitting the PD variable. In column (1) these coefficients were not significant and did not have the correct signs or the expected increasing-in-value pattern.

Moreover, the exclusion of the PD variable increases the significance level of the debt to assets ratio (from the 5% to the 1% level). The size variable is still significant at the 10% level and the macroeconomic factor at the 1% level. It is interesting to note that the constant is not significant in the model of column (1), but becomes significant at the 5% level once we exclude the PD variable, signalling the better fit of the model in column (1).

In the final rows of Table D we report the average log-likelihood and the pseudo-$R^2$ to compare models. We include two measures of pseudo-$R^2$ based on McFadden (1974) and Cragg and Uhler (1970).[28] The pseudo-$R^2$ is between zero and one and is the analogue to the $R^2$ coefficient of determination that we calculate in the linear regression models. These measures are constructed using a likelihood ratio statistic.

Comparing the values for the average log-likelihood we see that this is bigger for the model of column (1), that is the model that includes PDs as regressor. Moreover, the

(27) We have also included yearly dummy variables instead of the macroeconomic variable with 2001 as the reference year. The dummies for the years 1990–92 and 1995 were significant. If we include the yearly dummies plus GDP, the yearly dummies are no longer significant. We also tried GDP growth, GDP deviation from its long-run trend, Industrial Production Index and its deviation from trend. All these variables were significant, but when included with the yearly dummies some of them were still significant. For this reason we report the results for the model that includes GDP (GDP=100 for 1995). Different measures of interest rates and prices were also included but failed to be significantly different from zero.

(28) For a discussion of these measures we refer the reader to Maddala (1983), pages 37–41.

pseudo-$R^2$ of the model of column (1) is more than twice[29] the pseudo-$R^2$ for the model that excludes the PDs (independently of the pseudo-$R^2$ measure chosen). Both statistics indicate, therefore, the superiority of the first model.

We run the model of column (1) by alternatively eliminating one defaulter at a time. The aim of this exercise was to check if the results were driven by a possible outlier. Since the results did not change substantially we can discard this possibility.

We estimate the models of columns (1) and (2) in Table D for the years 1990–93[30] one year at a time. The general result[31] is that in the model of column (2) the debt to assets ratio is still significant (except for 1993), but the other company account variables are not significant (except negative profit margins in 1992). The same results hold true once we include PD as a regressor, with the additional result that no accounting variable is significant for the regression of 1992 (the year with the highest number of defaults).

Columns (3) and (4) use information on PDs and company account data two years prior to the year when the default occurred. We do this as a robustness check and to evaluate the statistical power of 2-year PDs. The results are very similar to the ones of columns (1) and (2). The exception is the term sales growth, whose coefficient is now significant, and with the omission of the PD regressor only the coefficient for negative profit margins is significant. The statistical power of these two models is lower if we use average log-likelihood and pseudo-$R^2$ measures and compare with their equivalent in columns (1) and (2).

The model of column (5) is as model (1) but with a different PD variable. Here we only take the information of the 1-year PD of the twelfth month prior to the default month. Even if the coefficient for the PD measure is still significant at the 1% level, the coefficients for the accounting variables (that collect information for twelve months instead of one month only as the PD) are significant: negative profit margins and profit margins between 0% and 3%. Please note that the 1-year PD twelve months before failure is a measure twelve months prior to the default month, whereas the accounting variables are simply those of

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(29) Strictly speaking one cannot compare $R^2$'s across models with different number of regressors since the higher the number of regressors the higher the $R^2$. Notwithstanding, we have excluded one variable from our model (1) to check if the pseudo-$R^2$ was still of the same order of magnitude. Even excluding the GDP variable that has been proved to be highly significant the MacFadden pseudo-$R^2$ is 0.2785 (and higher if we exclude one of the non-significant variables). This exercise was undertaken for the rest of the models presented in Table D and the same result applies. Therefore we are confident in the comparison of pseudo-$R^2$'s across specifications.

(30) The number of defaulters is too small for the individual years from 1994 to 2001 to obtain reliable results.

(31) We do not report the results here for brevity, but they are available upon request from the authors.

the year before failure. If a company went bankrupt say, in January 2000, the accounting variables are those of the fiscal year 1999, whereas this particular PD measure is the 1-year PD for January 1999. For negative profit margins not being significant we have to include information on 1-year PD from twelve to five months before failure.

### *6.3 Power curves and accuracy ratios*

We now evaluate the ability of the different models to rank defaulters and non-defaulters using power curves and accuracy ratios as described in Section 4.

Chart 4 plots the power curve for some of the models estimated in this paper. The hybrid model is the model of column (1) in Table D. Company account data is the model of Table D, column (2). The other three curves correspond to different PD measures as stated in the graph. The power curves of Chart 4 have been constructed for the same proportion of defaulters in each model, which means that we can compare each curve with the other. But it is not possible to compare power curves produced by other models that use different data sets (the same applies for the AR index).

Observing the different curves we see that the hybrid model as here designed outperforms all other models. The 1-year PD annual average is almost identical to our hybrid model at small proportions of sample excluded. Of the models here presented, the model that uses only company account information is clearly inferior to our hybrid models or implementation of the Merton approach.

In Table E we report the accuracy ratios for the same models of Chart 4. Sobehart and Keenan (2001b) report the accuracy ratios for KMV's implementation of the Merton model (using 1-year probabilities of default) and for a hybrid model as described in Sobehart *et al* (2000). These ratios are 69.0% and 72.7%, respectively. We can use these figures as an approximate benchmark to evaluate the accuracy ratios reported in Table E. The closest models to compare with those figures are the ones for the 1-year PD annual average and the hybrid model, that is, 76.7% and 77.09%, respectively.

For a simple comparison across the models estimated in this paper, we represent the contents of Table E in the form of a graph (see Chart 5). A reduced form model of the type of Geroski and Gregg (1997) is easily outperformed by our implementation of the Merton approach, reflecting the information incorporated into market prices. The jump in performance from the pure structural Merton-based approach to our hybrid model is not as acute. One can always argue that this gap may be enhanced by the inclusion of more accounting variables. But what is important here is the existence of some information that is not captured by the Merton approach this paper uses.

## 7 Conclusions

This paper describes the derivation of default probabilities from an extended version of the Merton model and applies this to a number of UK non-financial quoted companies over the period 1990–2001.

The probability of default derived from our Merton-model implementation provides a strong signal of failure one year in advance of its occurrence. The mean value of the 1-year PD annual average measure for our entire sample is 47.3% for those companies that went bankrupt, and 5.4% for those that did not default. A more restrictive probability of default measure shows a similar pattern. The mean value of the 1-year PD for twelve months before the default date is 32.0% for defaulters and 5.2% for non-defaulters.

Calculation of Type I and II errors suggests that PDs are successful in discriminating between failing and non-failing firms. Using a threshold of 10%, that is, classifying defaults as those firms with a 1-year PD greater or equal to 10%, the Type I error is relatively modest at 9.2% (with a Type II error of 15.0%). For a 2-year PD measure the Type I and II errors for the same threshold are 12.3% and 29.9%, respectively.

If we compare our model with a reduced-form model of the type of Geroski and Gregg (1997), we can state that our implementation of the Merton approach clearly outperforms the Geroski and Gregg (1997) reduced-form model. This is independent of the specific PD measure, including the comparison of 2-year PDs and a statistical model that uses one-year lagged accounting ratios. But it also shows that the type of hybrid models implemented here, ie those combining company account information and the PDs derived from a Merton model, outperform our implementation of the Merton approach, if only marginally.

## Tables and charts

### Table A: Distribution of defaults over time

| Year | Whole sample | | Estimation sample | |
|---|---|---|---|---|
| | Non-defaults | Defaults | Non-defaults | Defaults |
| 1990 | 412 | 13 | 410 | 9 |
| 1991 | 447 | 15 | 443 | 10 |
| 1992 | 474 | 13 | 471 | 13 |
| 1993 | 484 | 8 | 482 | 7 |
| 1994 | 498 | 3 | 495 | 3 |
| 1995 | 510 | 6 | 508 | 6 |
| 1996 | 554 | 3 | 552 | 3 |
| 1997 | 597 | 5 | 595 | 5 |
| 1998 | 667 | 4 | 664 | 3 |
| 1999 | 917 | 0 | 907 | 0 |
| 2000 | 996 | 2 | 816 | 2 |
| 2001 | 1078 | 4 | 1051 | 4 |
| Total | 7634 | 76 | 7394 | 65 |

**Table B: Type I & II errors: Merton model, 1-year PDs annual average**[a]

| Sample | Type error | Threshold 5% | 10% | 15% | 20% | 30% |
|---|---|---|---|---|---|---|
| Whole sample | I | 4.61 | 9.23 | 13.85 | 20.00 | 36.92 |
| | II | 19.95 | 14.97 | 11.79 | 9.43 | 6.32 |
| 1990 | I | 0.00 | 22.22 | 22.22 | 33.33 | 33.33 |
| | II | 20.24 | 14.15 | 10.73 | 8.05 | 5.37 |
| 1991 | I | 0.00 | 0.00 | 10.00 | 20.00 | 30.00 |
| | II | 31.83 | 26.86 | 22.12 | 18.51 | 13.54 |
| 1992 | I | 0.00 | 0.00 | 0.00 | 7.69 | 23.08 |
| | II | 25.90 | 19.11 | 15.29 | 12.31 | 14.29 |
| 1993 | I | 0.00 | 0.00 | 0.00 | 0.00 | 14.29 |
| | II | 30.50 | 23.44 | 19.50 | 16.18 | 11.83 |
| 1994 | I | 0.00 | 0.00 | 0.00 | 0.00 | 100.00 |
| | II | 17.17 | 11.92 | 9.09 | 6.87 | 4.44 |
| 1995 | I | 16.67 | 16.67 | 33.33 | 33.33 | 50.00 |
| | II | 13.98 | 10.04 | 7.09 | 5.12 | 3.54 |
| 1996 | I | 0.00 | 0.00 | 0.00 | 33.33 | 33.33 |
| | II | 14.13 | 10.51 | 9.06 | 7.25 | 5.80 |
| 1997 | I | 20.00 | 20.00 | 20.00 | 20.00 | 60.00 |
| | II | 14.45 | 11.43 | 8.57 | 6.55 | 3.70 |
| 1998 | I | 33.33 | 66.67 | 66.67 | 66.67 | 100.00 |
| | II | 15.21 | 10.54 | 7.98 | 6.32 | 4.07 |
| 1999 | I | | | | | |
| | II | 19.07 | 14.55 | 11.47 | 9.26 | 6.06 |
| 2000 | I | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | II | 19.73 | 15.20 | 11.89 | 9.31 | 5.64 |
| 2001 | I | 0.00 | 0.00 | 25.00 | 25.00 | 25.00 |
| | II | 21.60 | 15.70 | 12.18 | 9.99 | 6.47 |

[a] 1-year PD annual average is the average of the 1-year PD —probability of default in one year's time from now— for the twelve months preceding the default month.

**Table C: Type I & II errors: Merton model, 2-year PDs annual average and 1-year PD twelve months before failure**

| Sample | Type error | Threshold | | | | |
|---|---|---|---|---|---|---|
| | | 5% | 10% | 15% | 20% | 30% |
| 2-year PD | I | 6.15 | 12.31 | 21.54 | 27.69 | 41.54 |
| annual average [(a)] | II | 3.68 | 29.92 | 25.18 | 21.32 | 15.90 |
| 1-year PD 12 months | I | 24.61 | 35.38 | 41.54 | 50.77 | 61.54 |
| before failure [(b)] | II | 13.22 | 10.51 | 9.06 | 7.75 | 6.20 |

[(a)] 2-year PD annual average is the average of the 2-year PD —probability of default in two years time from now— from the twelfth month before the default month to the $24^{th}$ before the default month.
[(b)] 1-year PD twelve months before failure is the 1-year PD for the twelfth month before the default month.

**Table D: Using company account data** [(a)]

| Variable | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| Constant | 0.43 | 1.36** | 0.89 | 1.53** | 0.93 |
| | (0.64) | (2.17) | (1.29) | (2.28) | (1.46) |
| 1-year PD annual average [(b)] | 0.02*** | | | | |
| | (11.03) | | | | |
| 2-year PD annual average [(c)] | | | 0.01*** | | |
| | | | (5.90) | | |
| 1-year PD 12 months before failure [(d)] | | | | | 0.01*** |
| | | | | | (5.90) |
| Profitability$< 0\%$ | 0.17 | 0.68*** | 0.25 | 0.51*** | 0.49*** |
| | (1.11) | (5.00) | (1.63) | (3.51) | (3.40) |
| $0\%$ <Profitability$< 3\%$ | 0.17 | 0.42*** | −0.09 | 0.11 | 0.30* |
| | (0.97) | (2.81) | (−0.47) | (0.59) | (1.91) |
| $3\%$ <Profitability$< 6\%$ | −0.01 | 0.14 | −0.04 | 0.07 | 0.07 |
| | (−0.03) | (0.92) | (−0.23) | (0.48) | (0.45) |
| Debt over assets | 0.31** | 0.48*** | 0.25** | 0.33*** | 0.39*** |
| | (2.52) | (4.61) | (2.37) | (3.52) | (2.99) |
| Cash over liabilities | 0.01 | −0.12 | −0.04 | −0.18 | −0.07 |
| | (0.13) | (−1.18) | (−0.37) | (−1.38) | (−0.71) |
| log number of employees | −0.6* | −0.05* | −0.04 | −0.04 | −0.05* |
| | (−1.75) | (−1.66) | (−1.11) | (−1.45) | (−1.76) |
| Sales growth | −0.11 | −0.06 | −0.16* | −0.26*** | 0.00 |
| | (−0.91) | (−0.44) | (−1.73) | (−2.98) | (2.02) |
| GDP | −0.03*** | −0.04*** | −0.04*** | −0.04*** | −0.03*** |
| | (−4.60) | (−6.18) | (−5.10) | (−5.81) | (−5.45) |
| Avg. Log-likelihood | −0.034 | −0.042 | −0.040 | −0.040 | −0.041 |
| McFadden Pseudo-$R^2$ | 0.3105 | 0.1501 | 0.1787 | 0.1296 | 0.1878 |
| Cragg & Uhler Pseudo-$R^2$ | 0.2999 | 0.1438 | 0.1717 | 0.1246 | 0.1801 |

(a) Company Account Data is for the year before the default year for models using 1-year PDs, columns (1), (2) and (5) and two years before the default year for models using 2-year PDs, columns (3) and (4). In this table we present the estimated coefficients and the z-statistics in parenthesis. $^{***}$,$^{**}$ and $^{*}$ mean that the coefficient is significant at the 1%, 5% and 10% level, respectively.

(b) 1-year PD annual average is the average of the 1-year PD —probability of default in one year's time from now— for the twelve months preceding the default month.

(c) 2-year PD annual average is the average of the 2-year PD —probability of default in two years time from now— from the twelfth month before the default month to the $24^{th}$ before the default month.

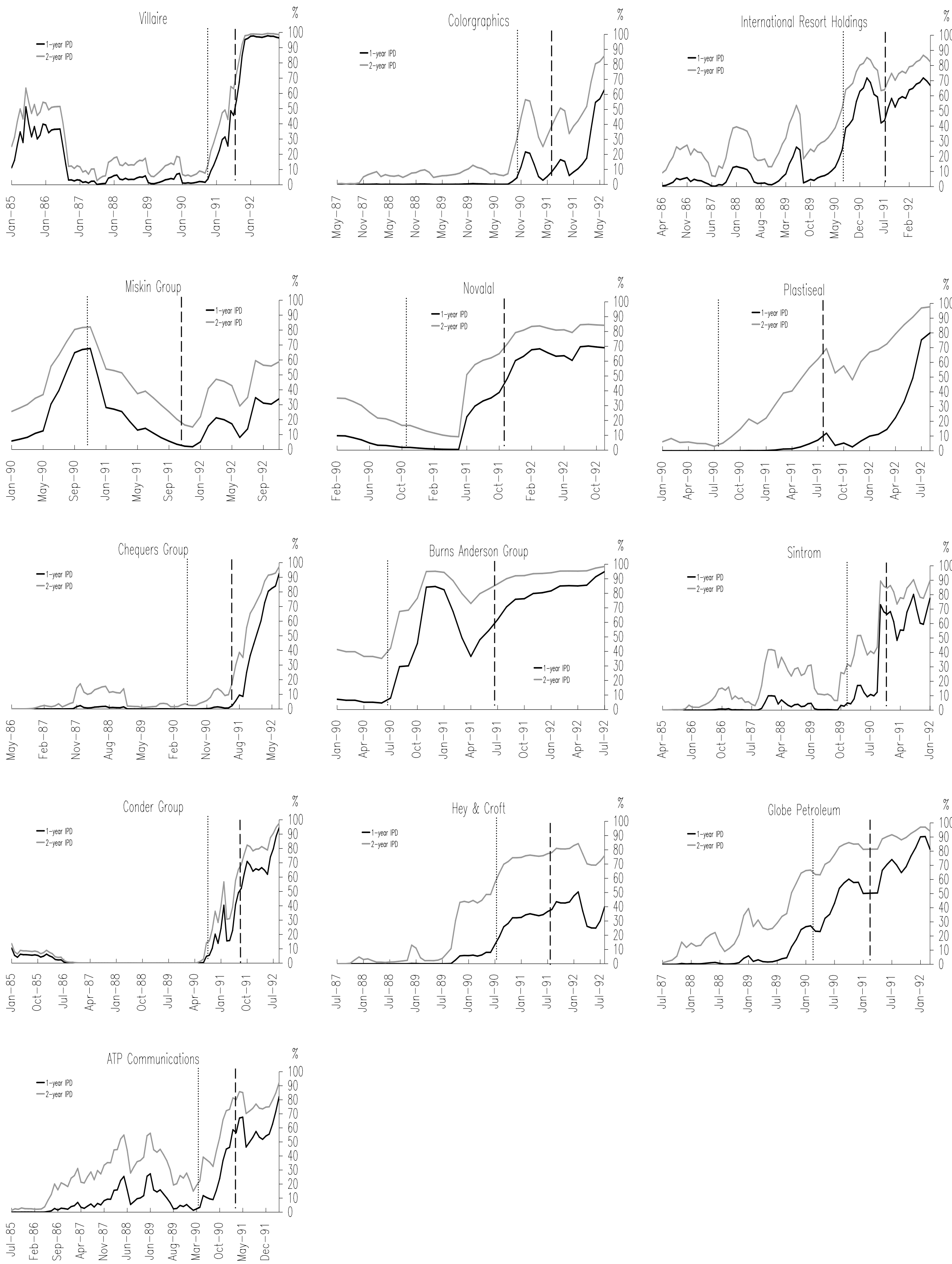
(d) 1-year PD twelve months before failure is the 1-year PD for the twelfth month before the default month.

**Table E: Accuracy ratios** [a]

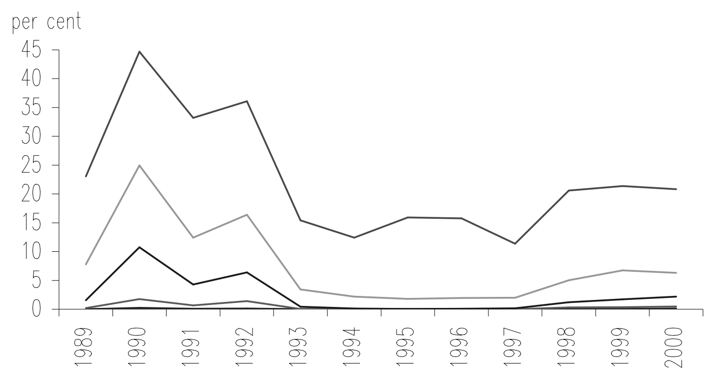
| Model | Accuracy ratio |
|---|---|
| Hybrid model | 77.09% |
| Company account data | 42.37% |
| 1-year PD annual average | 76.75% |
| 1-year PD 12 months before failure | 66.13% |
| 2-year PD annual average | 53.39% |

[a] The hybrid model is the model of column (1), Table D. Company account data is model (2), Table D.

## Chart 1: Implied probabilities of default (for defaulting firms)

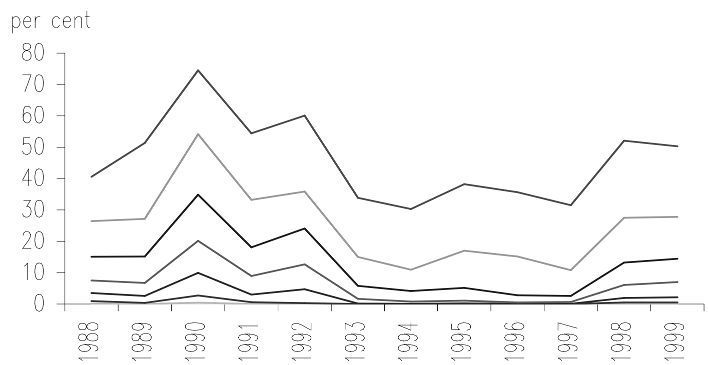


## Chart 2: Annual averages 1-year PDs



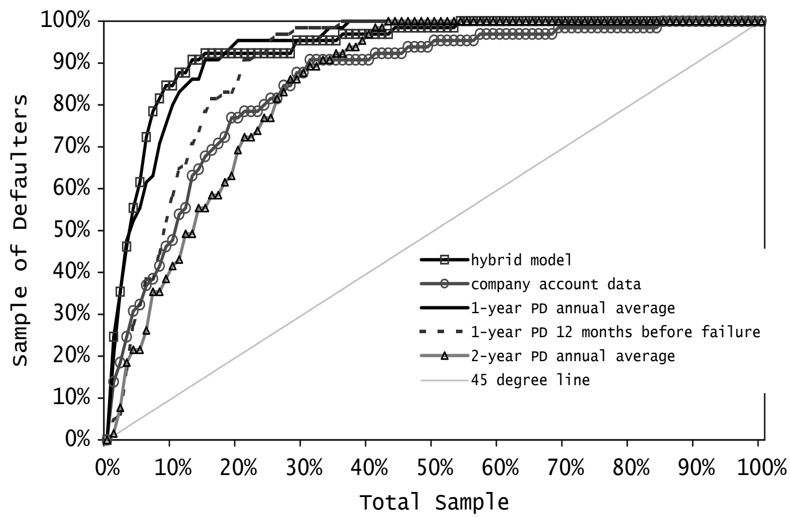

Percentiles are, from top to bottom, 90th, 80th, 70th, 60th, 50th (median) and 40th.

## Chart 3: Annual averages 2-year PDs



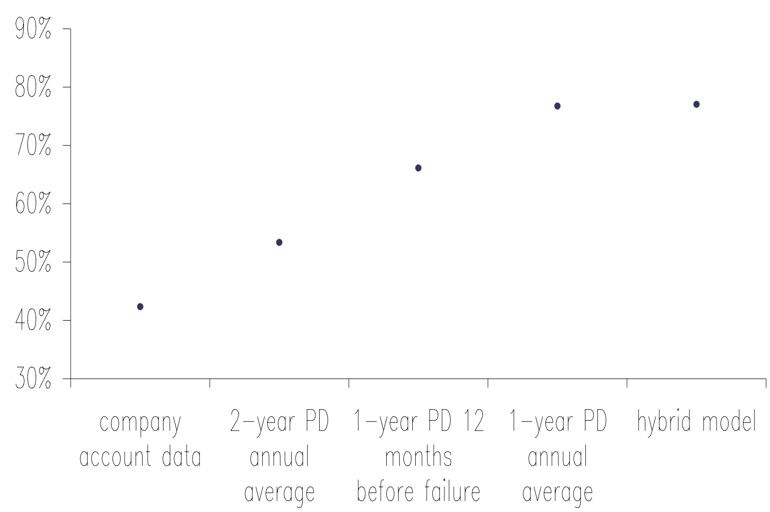

Percentiles are, from top to bottom, 90th, 80th, 70th, 60th, 50th (median) and 40th.

## Chart 4: Power curve




Note: To plot the power curve, for a given model we rank the companies in our sample by risk score (PD) from the riskiest to the safest (horizontal axis). For a given percentage of this sample we calculate the number of defaulters included in that percentage as a proportion of the total number of defaulters in our sample (vertical axis).

## Chart 5: Accuracy ratio



## Appendix A: Probability of default

Derivation of equation **(7)**: In the following lines we derive equation **(7)** of the main text. Note that

$$\int_{\ln\frac{k}{k_t}}^{\infty} h\left(\ln\frac{k_T}{k_t}\right) d\left(\ln\frac{k_T}{k_t}\right) = \int_{\underline{K}}^{\infty} h(K_T)dK_T \tag{A-1}$$

where

$$\ln\frac{k}{k_t} = \underline{K} \quad \ln\frac{k_T}{k_t} = K_T \tag{A-2}$$

Therefore, the probability of default is one minus the probability of not defaulting:

$$PD = 1 - \int_{\underline{K}}^{\infty} h(K_T)dK_T \tag{A-3}$$

We can show that:

$$1 - \int_{\underline{K}}^{\infty} h(K_T)dK_T = 1 - \left[\int_{\underline{K}}^{\infty} f(K_T)dK_T - \varpi \int_{\underline{K}}^{\infty} g_2(K_T)dK_T\right] = PD \tag{A-4}$$

where

$$\int_{-\infty}^{\underline{K}} f(K_T)dK_T = N\left[\frac{\underline{K} - \left(\mu_K - \frac{\sigma_k^2}{2}\right)(T-t)}{\sigma_k\sqrt{T-t}}\right] = N(u_1) \tag{A-5}$$

$$\varpi = \exp\left[\frac{2\underline{K}\left(\mu_k - \frac{\sigma_k^2}{2}\right)}{\sigma_k^2}\right] \tag{A-6}$$

and

$$\int_{-\infty}^{\underline{K}} g_2(K_T)dK_T = N\left[\frac{-\underline{K} - \left(\mu_k - \frac{\sigma_k^2}{2}\right)(T-t)}{\sigma_k\sqrt{T-t}}\right] = N(u_2) \tag{A-7}$$

Remember that the standard normal distribution is defined by:

$$N(u) = \int_{-\infty}^{u} f(u)du \tag{A-8}$$

We can define $N^*(u)$ as the compliment of $N(u)$:

$$N^*(u) = 1 - N(u) = 1 - \int_{-\infty}^{u} f(u)du = \int_{u}^{+\infty} f(u)du \tag{A-9}$$

So we can write the PD as:

$$PD = 1 - \{[1 - N(u_1)] - \varpi\,[1 - N(u_2)]\}$$

$$or$$

$$\boxed{PD = 1 - \{[N^*(u_1)] - \varpi\,[N^*(u_2)]\}} \qquad \textbf{(A-10)}$$

Note that the $N(u_1)$ term is equivalent to the probability of default obtained using a European call option. In the case of a barrier option we have to correct that probability of default for the fact that default occurs the first time the assets to liability ratio crosses the barrier and not just at $T$. The term $\varpi\,[1 - N(u_2)]$ corrects the probability of default derived using a European call option (path independent) to take into account that the asset-liability ratio can hit the barrier before $T$ (path dependent).

## Appendix B: Solution of the differential equation

Proof that expression **(16)** is a solution for equation **(15)**: We first need to express **(15)** in terms of $k$ and $y$. To do so we derive expressions for $\frac{\partial X}{\partial A}$, $\frac{\partial X}{\partial L}$, and $\frac{\partial^2 X}{\partial A^2}$ in terms of $y$ and $k$.

We know that

$$X = y(k)L = y(k)\frac{A}{k} \qquad \textbf{(B-1)}$$

Then

$$\frac{\partial X}{\partial A} = \frac{y(k)}{k} \qquad \textbf{(B-2)}$$

Also

$$y = \frac{X}{L} = \frac{Xk}{A} \qquad \textbf{(B-3)}$$

Then

$$\frac{\partial y}{\partial k} = \frac{X}{A} = \frac{y(k)}{k} \qquad \textbf{(B-4)}$$

Combining equations **(B-2)** and **(B-4)** we obtain:

$$\frac{\partial X}{\partial A} = \frac{\partial y}{\partial k} \qquad \textbf{(B-5)}$$

In a similar way we derive

$$\frac{\partial^2 X}{\partial A^2} = \frac{\partial^2 y}{\partial k^2}\frac{1}{L} \qquad \textbf{(B-6)}$$

and

$$\frac{\partial X}{\partial L} = y(k) - \frac{\partial y}{\partial k}k \qquad \textbf{(B-7)}$$

Dividing equation **(15)** by $L$ and substituting **(B-5)**, **(B-6)**, and **(B-7)** into **(15)** we obtain

$$ry = \delta(k-1) + \mu_A^* k\frac{\partial y}{\partial k} + \mu_L\left(y - k\frac{\partial y}{\partial k}\right) + \frac{\sigma_A^2}{2}k^2\frac{\partial^2 y}{\partial k^2} \qquad \textbf{(B-8)}$$

Using equations **(16)** and **(17)** we derive the following expressions

$$\frac{\partial y}{\partial k} = 1 - (\underline{k}-1)\lambda\left(\frac{k}{\underline{k}}\right)^\lambda\frac{1}{k} \qquad \textbf{(B-9)}$$

$$\frac{\partial^2 y}{\partial k^2} = -(\underline{k}-1)\lambda\left(\frac{k}{\underline{k}}\right)^\lambda\frac{1}{k^2} \qquad \textbf{(B-10)}$$

$$\lambda(\lambda-1) = \frac{2\delta}{\sigma_A^2} \qquad \textbf{(B-11)}$$

Substituting expressions **(B-9)** and **(B-10)** into (**(B-8)**) and knowing that $\mu_A^* = \mu_L = r - \delta$ we obtain

$$ry = \delta(k-1) + (r-\delta)y + \frac{\sigma_A^2}{2}k^2\left[-(\underline{k}-1)\lambda(\lambda-1)\left(\frac{k}{\underline{k}}\right)^{\lambda}\frac{1}{k^2}\right] \qquad \textbf{(B-12)}$$

Substituting expression **(B-11)** into the above expression and cancelling out terms we obtain

$$ry = ry \qquad \textbf{(B-13)}$$

Therefore, **(16)** is a solution for equation **(15)**.

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