INVESTMENT IRREVERSIBILITY IN A GRANULAR WORLD

2025

BANCO DE ESPAÑA
Eurosistema

Documentos de Trabajo
N.º 2546

Tatsuro Senga and Iacopo Varotto

INVESTMENT IRREVERSIBILITY IN A GRANULAR WORLD (*)

Tatsuro Senga (**)

KEIO UNIVERSITY AND QUEEN MARY UNIVERSITY OF LONDON

Iacopo Varotto (***)

BANCO DE ESPAÑA

(*) This paper was previously circulated under the title ”Idiosyncratic Shocks and Investment Irreversibility: Capital Misallocation over the Business Cycle”. We thank participants at various seminars and conferences for helpful comments. The opinions in this paper are those of the authors and do not necessarily reflect the views of the Banco de España or the Eurosystem.
(**) t.senga@keio.jp
(***) iacopo.varotto@bde.es

Documentos de Trabajo. N.º 2546

November 2025

https://doi.org/10.53479/41857

ISSN: 1579-8666 (online edition)

Abstract

Does micro-level investment irreversibility amplify or dampen business cycles? We show this depends on the source of aggregate risk. Investment irreversibility reduces fluctuations in both aggregate output and investment when firm-level idiosyncratic shocks aggregate up to economy-wide effects. This contrasts with models driven by aggregate productivity shocks, where irreversibility has little effect on volatility. The key is whether idiosyncratic shocks are sufficiently volatile to cause the irreversibility constraint to bind cyclically for a significant mass of firms. If so, investment irreversibility hampers productivity-enhancing capital reallocation and reduces business cycle volatility. Moreover, household consumption smoothing is impeded when firms cannot adjust capital optimally, increasing real wage volatility. This labor market effect, combined with capital misallocation, reduces aggregate output volatility by 22 percent and investment volatility by 60 percent. These results highlight the importance of considering the source of economic volatility when assessing investment frictions. We provide empirical support for these predictions using firm-level investment data from Compustat.

Keywords: investment irreversibility, business cycles, idiosyncratic shocks, capital misallocation.

JEL classification: D25, E22, E23, E32.

Resumen

¿La irreversibilidad de la inversión microeconómica amplifica o atenúa los ciclos económicos? Mostramos que depende de la fuente del riesgo agregado. La irreversibilidad de la inversión reduce las fluctuaciones tanto en la producción agregada como en la inversión cuando los *shocks* idiosincrásicos de las empresas se agregan hasta producir efectos macroeconómicos. Esto contrasta con los modelos impulsados por *shocks* de productividad agregada, donde la irreversibilidad tiene poco efecto sobre la volatilidad. La clave es que los *shocks* idiosincrásicos son lo suficientemente volátiles como para hacer que la restricción de irreversibilidad se active cíclicamente para una masa significativa de empresas. Si esto ocurre, la irreversibilidad de la inversión dificulta la reasignación de capital que mejora la productividad y reduce la volatilidad de los ciclos económicos. Además, el alisamiento del consumo de los hogares se ve obstaculizado cuando las empresas no pueden ajustar el capital de manera óptima, lo que incrementa la volatilidad del salario real. Este efecto en el mercado laboral, combinado con la mala asignación de capital, reduce la volatilidad de la producción agregada en un 22 % y la volatilidad de la inversión en un 60 %. Estos resultados subrayan la importancia de considerar la fuente de la volatilidad económica al evaluar las fricciones en la inversión. Proporcionamos evidencia empírica que respalda estas predicciones utilizando datos de inversión a escala empresarial de Compustat.

Palabras clave: irreversibilidad de la inversión, ciclos económicos, *shocks* idiosincráticos, mala asignación de capital.

Códigos JEL: D25, E22, E23, E32.

1 Introduction

Investment irreversibility represents a fundamental friction in capital allocation; the substantial losses firms incur when liquidating equipment and machinery hamper efficient capital reallocation across firms.[1] This creates a policy challenge, for example, by spurring bankruptcy law reforms aimed at facilitating smoother capital transitions from distressed firms to more productive firms while preserving asset values; the 2009 restructurings of General Motors and Chrysler are a prime example of this.[2] It is well understood that investment irreversibility reduces allocative efficiency in the economy; though, its cyclical implications vary across studies, depending on the structure of the economy and the nature of shocks examined.[3]

Among papers that study the cyclical implications of micro-level investment irreversibility, this paper poses a new question: does investment irreversibility affect business cycle fluctuations when the source of aggregate fluctuations is idiosyncratic shocks rather than economy-wide aggregate shocks? If so, how and how much? We answer these questions in a model of a "granular" economy, drawing on the recent argument that the economy can experience significant fluctuations originating from firm-level idiosyncratic shocks to large firms that do not average out.[4] We find that investment irreversibility substantially dampens the volatility of aggregate output, investment, and hours worked, while amplifying consumption volatility. This occurs through two distinct channels: a direct constraint on capital allocation

[1]Investment irreversibility can arise from multiple sources including asset specificity (Ramey and Shapiro, 2001; Lanteri, 2018), technological obsolescence (Caunedo and Keller, 2021), and thin resale markets (Ottonello, 2024). Evidence from systematic analysis of corporate liquidations reveals that even general-purpose equipment and machinery incur substantial losses upon disposal, typically recovering only 50-70% of replacement cost, while specialized assets face even higher losses of 10-30%. See Kermani and Ma (2023) for detailed analysis.

[2]Section 363 sales in bankruptcy law allow expedited asset transfers to preserve going-concern value and avoid piecemeal liquidation, which enabled rapid transfer of viable assets to "New GM" and the Fiat consortium, respectively, preserving operational continuity that would have been lost in traditional liquidation proceedings (Brubaker and Tabb, 2010; Lubben, 2009; Roe and Skeel, 2010).

[3]Seminal works on investment irreversibility include Arrow (1968), Nickell (1977), Pindyck (1988, 1991), Dixit and Pindyck (1994), Demers (1991), Leahy (1993), Abel and Eberly (1994, 1996), Bertola and Caballero (1994), Bertola (1998), Bloom (2009).

[4]See Gabaix (2011) for the original hypothesis.

that leads to marginal products of capital being unequalized across firms, and an indirect general equilibrium effect where impeded consumption smoothing leads to greater real wage flexibility, which acts as a shock absorber for firms.

A direct channel stems from the impact of irreversibility on capital allocation, a mechanism we initially illustrate using a simplified partial equilibrium model and then demonstrate within our fully fledged general equilibrium model. The partial equilibrium analysis reveals that aggregate volatility is tied to the covariance between firm-level productivity and capital; a lower covariance dampens economic fluctuations. Our quantitative general equilibrium model confirms that this is a key channel. In that setting, irreversibility causes firms' capital stocks to adjust more sluggishly to idiosyncratic shocks to productivity at the firm level. This sluggishness reduces the dynamic alignment between capital and productivity, and as a consequence, aggregate investment becomes less volatile and less responsive to shocks.

An indirect general equilibrium channel works through wage flexibility, an effect absent from our partial equilibrium analysis. In the full general equilibrium model, when capital adjustment is restricted by irreversibility, households have a diminished ability to smooth consumption over time. This results in greater volatility in consumption, which in turn translates into more volatile movements in the real wage. This acts as a shock absorber for firms; a larger wage reduction during downturns helps mitigate the fall in employment and production. This effect contributes to lower volatility in aggregate hours worked and further dampens overall output fluctuations.

This result stands in sharp contrast to the "irrelevance" result found in standard business cycle models that rely exclusively on aggregate shocks.[5] The reason for this lies in the micro-level investment dynamics. We show that granular shocks, unlike their aggregate

[5] In particular, investment irreversibility is often found to play a limited role for aggregate fluctuations in standard models driven by common, economy-wide shocks. One reason is that aggregate fluctuations driven by economy-wide shocks may not be sufficiently volatile or varied to cause the irreversibility constraint to bind cyclically for a significant mass of firms (Sargent, 1980; Dow and Olson, 1992; Veracierto, 2002). Other research also suggests that lumpy investment arising from nonconvex adjustment costs plays a limited or insignificant role for aggregate dynamics. In such models, general equilibrium price adjustments, such as changes in wages and interest rates, can smooth out potential aggregate effects, leading to the same aggregate dynamics as in frictionless models (Thomas, 2002; Khan and Thomas, 2003, 2008).

counterparts, are sufficiently varied and volatile to cause the irreversibility constraint to bind *cyclically* for a significant mass of economically important firms. Specifically, in our granular economy, the share of large firms wanting to downsize but being prevented by the constraint is not only substantial but also strongly counter-cyclical. This is a critical distinction from existing models with aggregate shocks, where the fraction of constrained large firms shows virtually no cyclical variation. This active, time-varying engagement of the constraint among large firms is what elevates investment irreversibility to a prominent propagation mechanism in a granular world.

The paper takes three steps to demonstrate the aforementioned result. We first sketch a simple model to study factors that determine the volatility of aggregate output when the economy is populated by a finite number of firms: that is, the economy is granular and idiosyncratic shocks may not cancel out in the cross-section due to the fact that the law of large numbers does not hold. We then build the above-mentioned fully fledged equilibrium business cycle model to quantitatively evaluate the relative contribution of capital misallocation in driving aggregate fluctuations. Finally, to validate our model predictions, we build a firm-level panel dataset by merging firm-level accounting information from Compustat with TFP estimations of İmrohoroğlu and Tüzel (2014). We show that the model performs well in replicating the aggregate dynamics following a shock to the largest firms of the economy and the dynamics of capital misallocation among the largest firms of the economy.

Our analysis reveals that investment irreversibility in granular economies presents policymakers with a fundamental trade-off: while it dampens aggregate volatility, it also impairs capital allocation and consumption smoothing. To understand the welfare implications of this trade-off, we examine two policy interventions designed to mitigate irreversibility frictions. Our findings show stark differences in outcomes: policies that merely subsidize liquidation losses yield modest gains (0.34% in consumption-equivalent terms), while those that fundamentally enhance capital market liquidity generate substantial welfare improvements (2.52%). These results, motivated by real-world interventions such as the Troubled Asset

Relief Program, suggest that in granular economies, policymakers should prioritize reducing fundamental market frictions rather than simply compensating firms for the symptoms of irreversibility.

A key challenge in computing the equilibrium of the model under aggregate uncertainty (e.g. Khan and Thomas (2008) and Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018)) is constructing a feasible set of transition probabilities and their probability space that are consistent with the granularity of the underlying idiosyncratic productivity process. To address this, we employ the discretization method for multivariate stochastic processes developed by Terry and Knotek II (2011). This method allows us to simulate changes in transition probabilities that replicate the fluctuations of the moments of firm distribution over present and past productivities. This new approach enables the study of other real or financial frictions in economies characterized by granularity.

Related literature

There are two major areas of macroeconomics that are related to this paper. One is the literature that explores how micro-level frictions can affect aggregate dynamics in heterogeneous firm models.[6] The other is the literature on granular fluctuations, which has established that a large portion of aggregate fluctuations originates from idiosyncratic shocks to a small number of large firms.

Particularly with irreversibility on the former, Lanteri (2018) is close to this paper in that it models a secondary market for used capital explicitly, whereby investment irreversibility is endogenous and becomes prominent in dampening aggregate fluctuations with general equilibrium forces. Blanco and Baley (2024) also studies the role of partial irreversibility in generating persistent aggregate dynamics. While these papers focus on aggregate productivity as a source of fluctuations, we provide new insights into how micro-level frictions interact

[6] See early work of Bertola and Caballero (1994); Veracierto (2002) on investment irreversibility and Caballero and Engel (1999); Thomas (2002); Khan and Thomas (2003); Gourio and Kashyap (2007); Khan and Thomas (2008); Bachmann, Caballero, and Engel (2013); Bachmann and Bayer (2014) on fixed adjustment costs.

with firm-specific shocks to shape macroeconomic outcomes, highlighting the importance of considering the source of economic volatility when assessing the impact of investment frictions (Bloom, 2009; Bloom et al., 2018; Jaimovich, Terry, and Vincent, 2023). Cui (2022) and Khan and Thomas (2013) are close on this front in studying financial shocks in a model with irreversibility of business liquidation or investment.

Regarding granular fluctuations, this paper extends research on how firms' idiosyncratic shocks generate aggregate fluctuations (Jovanovic, 1987; Gabaix, 2011; Gabaix and Koijen, 2024; Carvalho and Grassi, 2019; Di Giovanni and Levchenko, 2012; Di Giovanni, Levchenko, and Méjean, 2014, 2024; Yeh, 2023; Ifergane, 2024; Burstein, Carvalho, and Grassi, 2025). This paper is also related to recent empirical work documenting fat-tailed and skewed firm dynamics (Salgado, Guvenen, and Bloom, 2019; Melcangi and Sarpietro, 2025; Ehouarne, Kuehn, and Schreindorfer, 2022). A distinct feature of our study is the inclusion of real frictions and capital misallocation—elements typically abstracted from in granular models—to understand how these documented micro-level shock patterns translate into aggregate fluctuations through the lens of investment irreversibility. As such, this paper is also connected to the misallocation literature (Restuccia and Rogerson, 2008; Hsieh and Klenow, 2010). For instance, David and Venkateswaran (2019) study how micro-level frictions contribute to allocative efficiency of the economy, while Ifergane (2024) studies the cyclical implications of misallocation in the context of granular economies like us.

Organization

The rest of the paper is organized as follows. Section 2 introduces the granular shock mechanism in a partial equilibrium model. Section 3 develops the general equilibrium model and its solution method. Section 4 presents the model's calibration and quantitative results. Section 5 provides empirical support using firm-level data. Section 6 discusses policy implications and concludes.

2 Analytical Insights into Capital Allocation and Aggregate Volatility

This section presents analytical results derived from a simple, partial equilibrium model to illustrate key mechanisms behind aggregate fluctuations in an economy with a finite number of firms. The primary aim is to understand the relationship between capital allocation and aggregate fluctuations when idiosyncratic shocks do not average out. While this particular model itself does not incorporate investment irreversibility to render analytical solutions feasible, we will use its framework to discuss how investment irreversibility, by potentially affecting capital allocation and the covariance between productivity and capital, *could* influence aggregate fluctuations. The purpose is to build intuition for these mechanisms, which are explored in a general equilibrium settings later.

We consider an economy populated by $M \in (0, \infty)$ firms. Each firm is indexed by i. Firms produce a homogeneous good in a perfectly competitive environment. Each firm uses capital as the sole input to produce, employing a decreasing returns to scale Cobb-Douglas production function:

$$y_i = \varepsilon_i k_i^{\alpha} \tag{1}$$

where k_i is the capital stock of firm i, and ε_i is its idiosyncratic productivity. We assume that idiosyncratic productivity follows the stochastic process:

$$\ln(\varepsilon_{i,t+1}) = \xi_{i,t} + \epsilon_{i,t+1} \tag{2}$$

where $\xi_{i,t} \sim \mathcal{N}(0, \sigma_\xi)$ is a idiosyncratic component known at time t, and $\epsilon_{i,t+1} \sim \mathcal{N}(0, \sigma_\epsilon)$ is a component that is revealed at the beginning of period $t + 1$.

The firm's objective at time t is to maximize its present discounted expected future profits by choosing an investment plan $\{i_{i,t+j}\}_{j=0}^{\infty}$:

$$\mathbb{E}_t\left(\sum_{j=0}^{\infty}\frac{1}{(1+r)^j}(\varepsilon_{i,t+j}k_{i,t+j}^{\alpha}-i_{i,t+j})\right) \quad \text{s.t.} \quad k_{i,t+j+1}=(1-\delta)k_{i,j+t}+i_{i,t+j}, \quad \text{given} \quad k_{i,t}. \tag{3}$$

where r is the interest rate faced by the firm (assumed to be constant for simplicity in this analytical section) and δ is the depreciation rate of capital.

Lemma 1. If firms choose the stock of capital one period ahead, then the idiosyncratic output y_i is log-normally distributed, $\ln(y_i) \sim \mathcal{N}(\mu_y, \sigma_y)$. The parameters of this distribution are:

$$\mu_y = \mathbb{E}(\ln(\varepsilon)) + \alpha\mathbb{E}(\ln(k)) \tag{4}$$

$$\sigma_y^2 = \mathbb{V}(\ln(\varepsilon)) + \alpha\left(\frac{2-\alpha}{1-\alpha}\right)\mathbb{C}\mathrm{ov}(\ln(\varepsilon), \ln(k)) \tag{5}$$

where μ_y represents the expected value of the log of the product εk^{α}, while σ_y^2 represents its variance. The distribution of $\ln(k)$ and the covariance between $\ln(k)$ and $\ln(\varepsilon)$ are endogenous and determined by the model's parameters, including the firm's optimization problem and the stochastic process for productivity.

Proof. See Appendix A.1. ■

Many models of heterogeneous firms assume a continuum of agents ($M \to \infty$). In the absence of aggregate shocks, this implies that idiosyncratic shocks average out due to the law of large numbers. However, with a finite number of firms, this is not the case.

Proposition 1. Let aggregate output be defined as $Y_t = \sum_{i=1}^{M} y_{i,t}$. Then, aggregate output Y_t can be approximated by a log-normal distribution, $\ln(Y_t) \sim \mathcal{N}(\mu_Y, \sigma_Y)$, where:

$$\mu_Y = \ln M + \mu_y + \frac{\sigma_y^2}{2} - \frac{\sigma_Y^2}{2} \tag{6}$$

$$\sigma_Y^2 = \ln\left[\frac{e^{\sigma_y^2}-1}{M}+1\right] \tag{7}$$

with μ_y and σ_y^2 being the parameters from Lemma 1. The coefficient of variation of aggregate output is:

$$\mathbb{CV}(Y) = \frac{\mathbb{V}(Y)^{\frac{1}{2}}}{\mathbb{E}(Y)} = \sqrt{\frac{e^{\sigma_y^2} - 1}{M}}. \tag{8}$$

As with Lemma 1, the distribution of $\ln(k)$ and the covariance between $\ln(k)$ and $\ln(\varepsilon)$ are endogenous and determined by the model's parameters.[7]

Proof. See Appendix A.2. ■

With a finite number of firms ($M < \infty$), the coefficient of variation of aggregate output, given by Equation (8), is strictly positive. This non-zero variation implies that aggregate variables become random, as idiosyncratic shocks do not cancel out in the cross-section. With a finite number of firms, the Law of Large Numbers does not apply, and the period t aggregate output is the sum of the realized output of each firm, not a theoretical mean.

The coefficient of variation, $\mathbb{CV}(Y)$, is influenced by several factors. First, the number of firms (M): $\mathbb{CV}(Y)$ is inversely related to M. As M increases, the coefficient of variation decreases, reflecting the (incomplete) averaging out of idiosyncratic shocks. Second, the variance of idiosyncratic output (σ_y^2): A higher σ_y^2, reflecting greater heterogeneity in firm-level productivity and capital, positively correlates with $\mathbb{CV}(Y)$. This can be interpreted as reflecting a more skewed firm size distribution, where a few large firms have a disproportionate impact on the aggregate, as originally argued by Gabaix (2011). Finally, the covariance between capital and productivity (within σ_y^2): As shown in Lemma 1, σ_y^2 incorporates the covariance between capital and productivity. Higher covariance amplifies economic fluctuations and lower covariance dampens economic fluctuations.

The key takeaway is that aggregate output becomes a random variable, even in the absence of aggregate shocks, due to the incomplete averaging out of idiosyncratic shocks. This partial equilibrium analysis highlights the critical role of capital allocation, as reflected in

[7]Note that the results are not dependent on the specific investment decision rule. We could, in principle, assume an arbitrary joint log-normal distribution between productivity and capital, and the core results regarding the randomness of aggregate output with a finite number of firms would still hold. However, by starting with a production function and a stochastic process for productivity, we provide a more micro-founded basis for the analysis.

the covariance between productivity and capital, in determining aggregate output volatility.

However, the model takes key prices as given. To move towards a more comprehensive understanding, we now transition to a dynamic general equilibrium framework, which will allow us to explicitly model firms' responses to irreversibilities, endogenize the capital allocation process, and quantify the crucial productivity-capital alignment—as well as its indirect general equilibrium consequences, thereby offering a quantitative assessment of its impact on aggregate fluctuations.

3 General Equilibrium Model

This section develops a quantitative heterogeneous firm business cycle model to assess the impact of idiosyncratic shocks and investment irreversibility on capital misallocation and aggregate fluctuations. We build upon the standard framework where firms use capital and labor to produce a homogeneous good, and where capital is chosen one period in advance. The key departure from the standard model is that we allow the distribution of firm-level productivities to fluctuate stochastically over time, even though the underlying ergodic productivity process for individual firms is a standard Markov chain. This allows us to study the effects of idiosyncratic shocks that do not average out in the aggregate. Importantly, we maintain a rational expectations framework: agents understand the stochastic process governing the evolution of the productivity distribution and form expectations accordingly. This allows us to use recursive methods for the analysis.

3.1 Production and Investment

There is a continuum of firms in the model economy. Each firm undertakes production using a predetermined capital stock k, and labor n. Firms are heterogeneous in their idiosyncratic productivity, ε. The ergodic process for idiosyncratic productivity, ε, is a Markov chain with a finite number of states: $\varepsilon \in \{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_{N_\varepsilon - 1}, \varepsilon_{N_\varepsilon}\}$. The transition probabilities for this individual firm productivity process are given by $Pr(\varepsilon' = \varepsilon_i|\varepsilon = \varepsilon_j) \equiv \Pi^\varepsilon(\varepsilon_j, \varepsilon_i) \geqslant 0$, with the standard property that $\sum_{i=1}^{N_\varepsilon} \Pi^\varepsilon(\varepsilon_j, \varepsilon_i) = 1$ for all $j = 1, 2, \ldots, N_\varepsilon$.

At the beginning of each period, firms observe the current aggregate state of the economy and choose their current level of employment, n. Production then takes place. After production, firms make their investment decisions, subject to a partial irreversibility friction, represented by the parameter ψ.

The aggregate state of the economy is characterized by the distribution of firms across idiosyncratic productivity and capital, denoted by $\mu(\varepsilon, k)$. This distribution is defined over the space $\mathbf{S} = \mathbb{R}_+ \times \mathbb{R}_+$. Unlike standard models where this distribution is stationary, we allow $\mu(\varepsilon, k)$ to vary stochastically over time. This captures the idea that idiosyncratic shocks do not perfectly cancel out in the cross-section due to the finite number of firms. The evolution of μ is the sole source of aggregate uncertainty in the model. We model this evolution as a Markov process, with $\mu' \sim G(\mu)$, meaning that the distribution in the next period, μ', is drawn from a distribution G that depends on the current distribution, μ.

Crucially, we define a conditional transition probability for the idiosyncratic productivity process, conditional on both the current and future aggregate state: $Pr(\varepsilon' = \varepsilon_i|\varepsilon = \varepsilon_j, \mu') \equiv \Pi^\varepsilon_{\mu'|\mu}(\varepsilon_j, \varepsilon_i) \geqslant 0$. This transition probability also satisfies $\sum_{i=1}^{N_\varepsilon} \Pi^\varepsilon_{\mu'|\mu}(\varepsilon_j, \varepsilon_i) = 1$ for all $j = 1, 2, \ldots, N_\varepsilon$. The transition probability $\Pi^\varepsilon_{\mu'|\mu}(\varepsilon_j, \varepsilon_i)$ depends on both μ and μ' because, while the underlying ergodic process for ε is exogenous, the realized distribution of firms across productivity levels in the next period depends on the evolution of the aggregate state.[8]

We can now state the firm's dynamic optimization problem. Let $V(\varepsilon, k; \mu)$ be the value of a firm at the beginning of a period with idiosyncratic productivity ε and predetermined capital stock k, when the aggregate state of the economy is μ. Firms discount future values using a state-contingent discount factor $d(\mu, \mu')$. The firm's problem is:

[8] Imagine a simple example: if μ' represents a state with a much higher concentration of firms at high productivity levels than μ, then the probability of a firm transitioning from a low ε to a high ε' will be higher under $\Pi^\varepsilon_{\mu'|\mu}$ than under the ergodic transition matrix Π^ε. The probability $\Pi^\varepsilon_{\mu'|\mu}$ captures how the aggregate shock (the change from μ to μ') affects the realized transitions of firms between productivity levels. This is distinct from the time-invariant, ergodic transition probabilities Π^ε, which govern the underlying stochastic process for an individual firm's productivity.

$$V(\varepsilon, k; \mu) \;= \max_{n,k'} \;\; \varepsilon F(k,n) - \omega(\mu)n - i + \sum_{\mu'} \Pi^{\mu}(\mu'|\mu) d(\mu,\mu') \sum_{\varepsilon'} \Pi^{\varepsilon}_{\mu'|\mu}(\varepsilon'|\varepsilon) V(\varepsilon', k'; \mu') \quad (9)$$

subject to

$$\hat{i} = k' - (1-\delta)k \quad (10)$$

$$i = \hat{i}(1 - \psi \mathbb{1}_{\hat{i}<0}) \quad (11)$$

$$k', n \;\in \mathcal{R}_{+} \quad (12)$$

Equation (9) is the Bellman equation. The firm chooses labor, n, and next period's capital stock, k', to maximize the sum of current profits $(\varepsilon F(k,n) - \omega(\mu)n - i)$ and the discounted expected future value. $\omega(\mu)$ is the wage rate, which depends on the aggregate state μ. Equation (10) defines gross investment, $\hat{i}$, as the difference between next period's capital and depreciated current capital. Equation (11) defines investment, i, incorporating the irreversibility friction: if gross investment is negative (disinvestment), the firm receives only a fraction $(1-\psi)$ of the value of the capital it sells. Equation (12) imposes non-negativity constraints on capital and labor. The term $\Pi^{\mu}(\mu'|\mu)$ represents the probability of transitioning from the current aggregate state μ to the future aggregate state μ'.

As in Khan and Thomas (2013), the optimal capital decision rule takes an (S, s) form:

$$K(\varepsilon, k; \mu) = \begin{cases} k^{\star}_{u}(\varepsilon, k; \mu), & \text{if } k \leqslant \frac{k^{\star}_{u}(\varepsilon,k;\mu)}{(1-\delta)} \\ k(1-\delta), & \text{if } k \in \left[\frac{k^{\star}_{u}(\varepsilon,k;\mu)}{(1-\delta)}, \frac{k^{\star}_{d}(\varepsilon,k;\mu)}{(1-\delta)}\right] \\ k^{\star}_{d}(\varepsilon, k; \mu), & k \geqslant \frac{k^{\star}_{d}(\varepsilon,k;\mu)}{(1-\delta)} \end{cases} \quad (13)$$

Here, $K(\varepsilon, k; \mu)$ represents the optimal choice of next period's capital stock, given current productivity ε, current capital k, and the aggregate state μ. The firm has two target capital levels: an upward target, $k^{\star}_{u}$, and a downward target, $k^{\star}_{d}$. If the firm's current capital is below the inaction region (determined by the targets and the depreciation rate), it invests to reach the upward target. If current capital is above the inaction region, it disinvests (subject to the irreversibility constraint) to reach the downward target. If current capital is within the inaction region, the firm simply lets its capital depreciate.

The target capital levels are defined as:

$$k_u^\star = \arg\max_{k'} \quad -k' + \sum_{\mu'} \Pi^\mu(\mu'|\mu) d(\mu,\mu') \sum_{\varepsilon'} \Pi^\varepsilon_{\mu'|\mu}(\varepsilon'|\varepsilon) V(\varepsilon', k'; \mu') \tag{14}$$

$$k_d^\star = \arg\max_{k'} \quad -(1-\psi)k' + \sum_{\mu'} \Pi^\mu(\mu'|\mu) d(\mu,\mu') \sum_{\varepsilon'} \Pi^\varepsilon_{\mu'|\mu}(\varepsilon'|\varepsilon) V(\varepsilon', k'; \mu') \tag{15}$$

Equations (14) and (15) show that the target capital levels are chosen to maximize the expected discounted future value of the firm, taking into account the cost of investment (or disinvestment) and the transition probabilities for both the aggregate state and the firm's idiosyncratic productivity.

3.2 Households

Our model economy is populated by a unit measure of identical households. Their lifetime expected utility maximization problem is:

$$W(\lambda;\mu) = \max_{c,n^h,\lambda'} U\left(c, 1-n^h\right) + \beta \mathbb{E}\left[W(\lambda';\mu')\,|\mu\right] \tag{16}$$

$$\begin{aligned} \text{subject to:} \qquad & c + \int_{\mathbf{S}} \rho_1(\varepsilon', k'; \mu)\, \lambda'(d[\varepsilon \times k']) \\ & \leqslant w(\mu)\, n^h + \int_{\mathbf{S}} \rho_0(\varepsilon, k; \mu)\, \lambda(d[\varepsilon \times k]) \,. \end{aligned} \tag{17}$$

Households hold one-period shares in firms, denoted by λ. Given the prices—the real wage, $w(\mu)$, and the prices of shares, $\rho_0(\varepsilon, k; \mu)$ and $\rho_1(\varepsilon', k'; \mu)$—households choose their current consumption, c, hours worked, n^h, and the number of new shares, λ'. Let $C(\lambda;\mu)$ and $N(\lambda;\mu)$ represent the household decision rules for consumption and hours worked, respectively, and let $\Lambda(\varepsilon, k', \lambda; \mu)$ be the household decision rule for shares purchased in firms that will begin the next period with k' capital and current productivity ε.

3.3 Recursive Equilibrium

A recursive competitive equilibrium is a set of functions:

$$\begin{aligned} \text{prices} &: (\omega, d, \rho_0, \rho_1) \\ \text{quantities} &: (N, K, C, N^h, \Lambda^h) \\ \text{values} &: (V, W) \end{aligned}$$

that solve firm and household problems and clear the markets for assets, labor, and output as follows:

1. V satisfies equation (9), and (N, K) are the associated policy functions for firms.

2. W satisfies equations (16) and (17), and (C, N^h, Λ^h) are the associated policy functions for households.

3. $\Lambda^h(\varepsilon, k; \mu) = \mu(\varepsilon, k)$ for each $(\varepsilon, k) \in \mathcal{S}$. (Asset Market Clearing: Households' shareholdings equal the distribution of firms.)

4. The labor and goods markets clear:

$$\begin{aligned} N^h(\mu) &= \int_{\mathbf{S}} N(\varepsilon, k; \mu) \cdot \mu(d[\varepsilon \times k]) \\ C(\mu) &= \int_{\mathbf{S}} [\varepsilon F(k, N(\varepsilon, k; \mu)) - (K(\varepsilon, k; \mu) - (1-\delta)k)] \cdot Q[K(\varepsilon, k; \mu), k] \cdot \mu(d[\varepsilon \times k \end{aligned}$$

where $Q[K(\varepsilon, k; \mu), k]$ is an indicator function that takes value 1 if $K(\varepsilon, k; \mu) > (1-\delta)k$, and value $(1-\psi)$ otherwise.

5. The resulting individual decision rules for firms and households are consistent with the conditional distribution of the aggregate state, Π^μ.

3.4 Solution Method

The computational algorithm builds upon the solution method of Khan and Thomas (2008), itself based on Krusell and Smith (1998). It involves approximating the aggregate state space and solving the firm's problem recursively. A key element is the construction of a Markov chain to approximate the stochastic process for the distribution of firms across productivity transitions.

The main challenge to solving traditional heterogeneous agent model is that the aggregate state vector $\mu(\varepsilon, k)$ contains the cross-sectional distribution of firms, which is an infinite-dimensional object. Krusell and Smith (1998) and Khan and Thomas (2008) approximate the intractable cross-sectional distribution μ with a finite dimensional aggregate state variable, such as some moments of the distribution of capital, to make the model solution computable. However, as previously demonstrated in Section 2, the solution of granular economy models must confront an additional key challenge: the cross-sectional distribution μ is not only an infinite-dimensional entity but also inherently stochastic. Indeed, as shown in Section 2, the failure of the law of large numbers induces stochastic fluctuations in the marginal distribution of firms over productivity, $\mu_\varepsilon(\varepsilon)$. For this reason, we must characterize the aggregate endogenous state not only by moments of the capital distribution but also by the distribution of firms across productivity transitions, h, $N_\varepsilon \times N_\varepsilon$ arrays, where each element $h(\varepsilon_i, \varepsilon_j)$ represents the mass of firms that transitioned from productivity state ε_j in the previous period (-1) to productivity state ε_i in the current period.[9] Importantly, distribution of firms across productivity transitions embodies the information of the current and previous period marginal distribution of firm over productivity, as well as the information of the conditional transitional probability for idiosyncratic productivity, $\Pi_h^\varepsilon(\varepsilon_i|\varepsilon_j)$.[10] Accordingly, the main problem to solving granular heterogeneous firm model is the construction a feasible

[9] In Appendix B, we show that we can exactly recover μ with (h, K) in a frictionless economy.

[10] We can derive the current marginal distribution of productivity, denoted by $\mu_\varepsilon(\varepsilon)$, by summing across the previous period's states: $\mu_\varepsilon(\varepsilon_i) = \sum_j h(\varepsilon_i, \varepsilon_j)$. Similarly, the previous period's marginal distribution is $\mu_{\varepsilon,-1}(\varepsilon_j) = \sum_i h(\varepsilon_i, \varepsilon_j)$. Furthermore, from the distribution of firms across productivity transitions h, we

set of distribution of firms across productivity transitions, H, and its transition probability, Π^h. To make the problem computationally tractable, we approximate this process with a finite-state Markov chain.[11] In particular, to be consistent with the definition of granularity, we construct the set of possible aggregate productivity distributions, H, and their transition matrix, Π^h, to be consistent with the underlying ergodic productivity process, Π^ε. The key idea is to first approximate the time series of the marginal productivity distribution with a Markov chain by employing the Terry and Knotek II (2011)'s methodology, then use this to construct the transition probabilities for the joint distribution, h. The detailed description of the implementation of our numerical method to build the set of transition probability and its probability space is provided in Appendix C.

4 Quantitative Analysis

4.1 Parameters

This section describes our calibration approach. We first specify the functional forms for preferences and technology, then divide parameters into two groups: predetermined parameters set based on standard values in the literature, and calibrated parameters chosen to match key moments from both micro and macro data. We conclude by discussing how well the calibrated model fits both targeted and untargeted moments.

can also derive the conditional transition probability:

$$\Pi_h^\varepsilon(\varepsilon_i|\varepsilon_j) = \frac{h(\varepsilon_i, \varepsilon_j)}{\sum_f h(\varepsilon_f, \varepsilon_j)}, \quad \forall j, i = 1, 2, \ldots, N_\varepsilon. \tag{18}$$

[11] Let $H = \{h_1, \ldots, h_{N_h}\}$ be a set of N_h possible realizations of the h array. The transitions between these realizations are governed by a transition matrix Π^h, where $\Pi^h(h_l, h_m) = \Pr(h' = h_m \mid h = h_l) \geqslant 0$ and $\sum_{m=1}^{N_h} \Pi^h(h_l, h_m) = 1$, $\forall l = 1, 2, \ldots, N_h$.

4.1.1 Model Specification

We assume that the representative household's period utility follows the indivisible labor specification of Hansen (1985) and Rogerson (1988): $u(c, n^h) = \ln(c) + \eta(1 - n^h)$. The firm-level production function takes a Cobb-Douglas form with decreasing returns to scale: $\varepsilon F(k, n) = \varepsilon k^\alpha n^\nu$, where $\alpha + \nu < 1$. Productivity is the product of two components: $\varepsilon = \xi \cdot \epsilon$, where ξ is a persistent base component and ϵ is a transitory component. In each period, the base component ξ is redrawn with probability χ from a Pareto distribution $\mathcal{P}(e_m, e)$.[12] With probability $(1 - \chi)$, the base component remains unchanged. The transitory part follows a log-normal process: $\ln(\epsilon) \sim \mathcal{N}(0, \sigma_\epsilon^2)$.

The assumption that ξ is redrawn from a Pareto distribution with probability χ serves a dual purpose in our model. First, it generates the fat-tailed firm size distribution observed empirically, similar to Ehouarne et al. (2022).[13] Second, it captures the pervasive micro-level rank reversals among firms as even major corporations can experience dramatic sales declines, "micro disasters," that fundamentally alter their market position (Autor, Dorn, Katz, Patterson, and Van Reenen, 2020). This modelling approach is also consistent with recent evidence by Melcangi and Sarpietro (2025), who document that firm-level sales and productivity are hit by heavy-tailed shocks.

4.1.2 Calibration Strategy

We set the length of a period in the model to be one year. The parameters fall into two categories:

Predetermined parameters: We set five parameters based on standard values in the literature. The household discount factor, $\beta = 0.96$, implies an average real interest rate of four percent (Gomme, Ravikumar, and Rupert, 2011). The labor share, $\nu = 0.60$, matches

[12] The Pareto distribution $\mathcal{P}(e_m, e)$ has probability density function $f(\xi) = \frac{e \cdot e_m^e}{\xi^{e+1}}$ for $\xi \geqslant e_m$, where $e_m > 0$ is the scale parameter (minimum value) and $e > 0$ is the shape parameter determining the tail thickness.

[13] Ehouarne et al. (2022) shows that permanent idiosyncratic shocks combined with a power law distribution in firm size are crucial for generating time-variation in common idiosyncratic skewness (see also Salgado et al. (2019)).

the average labor share of income (Cooley, 1995). The depreciation rate of capital, $\delta = 0.07$, targets the average investment-to-capital ratio. The standard deviation of the transitory component, $\sigma_\epsilon = 0.022$, follows (Khan and Thomas, 2008). Finally, we set the number of firms $M = 4.5$ million to match the approximate number of firms in the US economy.

Calibrated parameters: The remaining six parameters are jointly calibrated to match key moments from the data. These parameters are: (1) the capital share, α, (2) the preference parameter, η, (3) the shape parameter of the Pareto distribution, e, (4) the scale parameter of the Pareto distribution, e_m, (5) the probability of resetting the base component, χ, and (6) the degree of investment irreversibility ψ.

We target the following six data moments: (1) the aggregate capital-output ratio of 2.3 as the average private capital-to-output ratio between 1954 and 2002 reported by Khan and Thomas (2013), (2) aggregate total hours worked of one-third, (3) standard deviation of firm-level TFP growth among the 100 largest firms of 12% (Gabaix, 2011), (4) the sales share of largest 500 firms of 40% (Gabaix, 2011), (5) the largest firm's employment share of 1% (Walmart's domestic employment size of 1.4 million as in (Carvalho and Grassi, 2019)), and (6) inaction rate defined as the mass of firms among the 100 largest firms of the previous period whose $|i/k| \leqslant 0.01$.[14]

4.1.3 Results

Tables 1 and 2 present our calibration results. Table 1 shows both targeted and untargeted moments, while Table 2 reports the calibrated parameter values.

[14]We target an intermediate value between 0.080 of Khan and Thomas (2008) and 0.111 obtained from Compustat. The investment rate is given by the ratio of capital expenditures (CAPX) to lagged PPEGT after controlling for SECTOR × YEAR. See Appendix D for further details on variable construction.

Table 1: Moments

Targeted Moments			
Description	Parameter Sensitivity	Data	Model
Capital-Output Ratio ($\frac{K}{Y}$)	α, e_m, χ	2.300	2.23
Total Hours Worked (N)	η	0.333	0.336
TOP 100 Std. Dev. $\Delta\varepsilon$ (perc.)	e, e_m	12.00	11.90
Sales share TOP 500 firms (perc.)	e_m	40.00	39.50
Walmart Employment Share (perc.)	e, e_m	1.000	1.087
$Pr(\|\frac{i}{k}\| \leqslant 0.01)$	ψ, α, e	$0.08 - 0.111$	0.097
Untargeted Moments			
Tail index of Firm size dist.		1.097	1.078
Sales share TOP 50 Firms (perc.)		24.00	22.40
Sales share TOP 100 Firms (perc.)		29.00	27.55
$Pr(\frac{\Delta k'}{k} \leqslant -0.2)$ previous TOP 500 firms		0.024	0.022

Note: The "Parameter Sensitivity" column indicates which parameters primarily affect each targeted moment, not a one-to-one mapping.

The calibration yields several important findings. First, the calibrated degree of capital irreversibility ($\psi = 0.35$) aligns closely with the empirical findings of Bloom (2009). Second, the reset probability $\chi = 0.02$ generates realistic firm dynamics. As Autor et al. (2020) document, the probability that a firm remains in the top 500 after one year is 0.95, which our model matches precisely with $1 - \chi = 0.98$ for annual transitions. Third, the model successfully replicates the firm size distribution. The estimated tail index (1.078) closely matches the empirical value of 1.097 reported by Carvalho and Grassi (2019). Furthermore, the model accurately reproduces the right tail of the distribution: the output shares of the top 50 and top 100 firms are very close to the empirical estimates provided by

Gabaix (2011). The model's ability to replicate the skewness of the firm size distribution is particularly important, as this feature is essential for generating granular volatility—an insight originally emphasized by Gabaix (2011). The strong performance on untargeted moments provides validation of our modelling approach. These moments—particularly the tail index and sales concentration measures—were not directly targeted in our calibration but emerge naturally from the model's structure. This suggests that our productivity process with Pareto-distributed resets captures fundamental features of firm dynamics. Finally, the model replicates the negative net investment spikes observed among the largest firms in the economy, indicating that the calibrated stochastic process generates realistic micro-level investment dynamics.

Table 2: Parameter values

Pre-determined parameters		
Description	Parameter	Value
Discount factor	β	0.96
Labor share	ν	0.60
Depreciation rate	δ	0.07
Std. dev. transitory component	σ_ϵ	0.022
Number of firms	M	4.5 mil.
Calibrated parameters		
Capital share	α	0.26
Preference parameter	η	2.15
Probability of reset	χ	0.02
Pareto scale parameter	e_m	6.89
Curvature of Pareto distribution	e	6.90
Investment irreversibility	ψ	0.35

4.2 Business Cycles

We now examine how capital irreversibility shapes key business cycle dynamics in our granular economy, distinguishing between its direct impacts on firm behavior and its broader general equilibrium consequences.

Table 3: Business Cycle Moments Benchmark Model

	$\psi = 0.35$			$\psi = 0.00$			DATA		
	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, \ln Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, \ln Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, \ln Y)$
$\ln Y$	0.294	1.000	1.000	0.377	1.000	1.000	1.299	1.000	1.000
$\ln C$	0.171	0.582	0.786	0.119	0.316	0.716	1.039	0.800	0.899
$\ln I$	1.200	4.088	0.901	1.962	5.209	0.975	5.913	4.552	0.916
$\ln H$	0.191	0.651	0.834	0.303	0.805	0.962	1.700	1.309	0.881
$\ln(1+r)$	0.025	0.086	-0.496	0.018	0.049	-0.022	1.310	1.007	-0.286

Notes: The table shows the data, benchmark ($\psi = 0.35$), and frictionless ($\psi = 0.00$) model equilibrium business cycle moments of output Y, consumption C, investment I, and hours worked H. $\sigma(x)$ is the standard deviation of x, $\sigma(x)/\sigma(Y)$ is the relative standard deviation to that of Y, and $\rho(x, \ln Y)$ is the contemporaneous correlation of x with Y. The model moments are obtained from a 15,000-period unconditional simulation using the solution of the model. All series are HP-filtered in logs with a smoothing parameter of 6.25, following Ravn and Uhlig (2002). Data sources for the reported moments are: (1) real gross domestic product (GDPCA taken from FRED), (2) real gross private domestic investment (GPDICA taken from FRED), (3) real personal consumption expenditures (PCECCA taken from FRED), (4) total nonfarm business sector hours (HOANBS taken from FRED but annualized), and (5) nominal return on 1-year Treasury bills (DGS1 taken from FRED) adjusted for realized CPI inflation (CPIAUCSL_PC1 taken from FRED). The sample refers to the period spanning from 1964 to 2019.

Before examining the specific channels through which capital irreversibility operates, it is worth noting that our granular economy generates business cycle dynamics similar to those of a typical real business cycle model with aggregate TFP shocks. As shown in Table 3, the model captures key empirical regularities: investment is substantially more volatile than output ($\sigma(\ln I)/\sigma(\ln Y) = 4.088$), consumption is less volatile than output (0.582 versus 0.800), and there are strong positive contemporaneous correlations with output in consumption, investment, and total hours worked. These patterns emerge from granular shocks—idiosyncratic productivity fluctuations at large firms—rather than aggregate TFP shocks, consistent with the seminal findings of Carvalho and Grassi (2019), who show that granular shocks can generate realistic business cycle dynamics.[15]

Another thing to note is that the model generates real interest rate dynamics different from a typical real business cycle model with aggregate TFP shocks. Also, the correlation of the real interest rates with output depends on the degree of capital irreversibility.[16] Without irreversibility ($\psi = 0.00$), the real interest rate is essentially acyclical ($\rho(\ln(1+r), \ln Y) = -0.022$), while with irreversibility ($\psi = 0.35$), it becomes moderately counter-cyclical ($\rho(\ln(1+r), \ln Y) = -0.496$). This sensitivity to capital market frictions reflects how irreversibility constrains both firms' capital adjustments and households' consumption smoothing, affecting equilibrium interest rate determination through precautionary savings motives.[17]

4.2.1 Direct Effects on Investment and Capital Allocation

To understand this result, we first decompose the mechanisms at play, beginning with the direct effects of irreversibility on firm-level behavior. Capital irreversibility (ψ) plays a significant role in shaping aggregate volatility in our granular economy, primarily through its *direct effects* on firm-level investment and capital allocation. As evidenced in Table 3, overall output volatility, $\sigma(\ln Y)$, decreases as the degree of capital irreversibility (ψ) increases,

[15]This similarity is noteworthy given several key differences in the modeling frameworks. For instance, a crucial distinction is our model's general equilibrium nature, where the wage endogenously adjusts to clear the labor market. This GE channel, which we analyze in detail in Section 4.2.2, acts as a stabilizer affecting aggregate volatility. That both models generate similar business cycle dynamics, despite these foundational differences, underscores the robustness of the granular shock mechanism. To illustrate the quantitative importance of this GE wage channel, Table 7 in Appendix E presents business cycle moments for our benchmark, frictionless, and partial equilibrium specifications side-by-side.

[16]Given the aggregate state μ , the real interest rate is computed as the expected consumption growth: $r = \frac{\sum_{\mu'} \Pi^{\mu}(\mu'|\mu) C'(\mu')}{C(\mu)} - 1$. As shown in Appendix C.2, by approximating the endogenous aggregate state with moments of the endogenous marginal distribution of k and h, we derive C' using a log-linear forecasting rule.

[17]Appendix F, Table 8, shows that the real interest rate is countercyclical when considering the 1964–2019 sample. Although it appears procyclical in the 1980–2019 sample, the degree of procyclicality remains substantially lower than that predicted by standard real business cycle models, wherein persistent improvements in technology typically induce simultaneous increases in consumption and investment demand, which, in the short run, outpace supply and generate procyclical movements in the real interest rate (Beaudry and Guay, 1996). See also Winberry (2021) for the recent contribution on the cyclicality of the real interest rates in a real business cycle model.

falling from 0.377 in the frictionless model ($\psi = 0.00$) to 0.294 in the benchmark model with irreversibility ($\psi = 0.35$).[18] This can be understood through key *direct mechanisms*. Firstly, higher irreversibility *directly* causes firm-level capital stocks to adjust more sluggishly to idiosyncratic productivity shocks. This reduced responsiveness is a key channel, leading to a lower covariance between capital and current productivity. As stated in Proposition 1, in a granular economy, such a dampened alignment of capital with productivity contributes to reducing aggregate output volatility. This dampened alignment is evident in our simulation results: the average log dispersion of the marginal product of capital across large firms ($\ln \sigma^{MPK,200}$) increases from 0.019 in the frictionless model to 0.022 in our benchmark model with irreversibility. Investment fluctuations are also dampened under irreversibility. Table 3 shows that aggregate investment volatility, $\sigma(\ln I)$, drops from 1.962 when $\psi = 0.00$ to 1.210 when $\psi = 0.35$, making it far less responsive to shocks. This finding, that irreversibilities dampen changes in aggregate investment, is similar to the work of Lanteri (2018), Blanco and Baley (2024) and Khan and Thomas (2013), while it is different from the irrelevance result from standard real business cycle models driven solely by aggregate TFP shocks. In such settings, capital irreversibility often has a much more limited effect on aggregate output volatility. A set of business cycle statistics for a comparable economy driven by aggregate TFP shocks are in Appendix H, with key moments in Table 10 therein.

4.2.2 Indirect General Equilibrium Effects: The Wage Channel

Beyond these direct impacts on firm behavior and capital allocation, irreversibility also triggers *indirect general equilibrium effects* that further shape aggregate dynamics, particularly through wage adjustments. When capital adjustment is restricted, the representative

[18] Eliminating partial investment irreversibility can have two countervailing effects on the shape coefficient of the firm size distribution. On the one hand, irreversibility limits firms' ability to shrink, creating persistence and path dependence in firm size; removing this friction allows less productive firms to contract more readily, thinning the upper tail and increasing the shape coefficient. On the other hand, greater responsiveness to shocks in the frictionless economy may increase firm-level volatility, which tends to fatten the tail and reduce the shape coefficient. In our model, the first effect dominates: the shape coefficient in the frictionless economy is 1.139, indicating a thinner-tailed, more equal firm size distribution.

household's ability to smooth consumption over time is diminished. Indeed, Table 3 shows that consumption volatility, $\sigma(\ln C)$, increases from 0.119 in the frictionless model ($\psi = 0.00$) to 0.170 with irreversibility ($\psi = 0.35$). This suggests more pronounced movements in wages, as seen in the intra-temporal optimal condition for the household's labor decision:

$$\omega(\mu) = \eta C(\mu), \tag{19}$$

where ω represents the real wage, C is aggregate consumption, and η is a preference parameter. When households cannot effectively smooth consumption through capital adjustments, they become more responsive to shocks in their labor supply decisions. Following negative shocks, the sharper decline in aggregate consumption (C) directly translates into a more substantial decrease in the real wage (ω) through the above equation. Households facing this consumption drop are more willing to accept lower wages to maintain employment, creating more elastic labor supply. This greater wage flexibility, in turn, tempers the reduction in aggregate output. Firms facing negative shocks benefit from a larger reduction in wage costs, which can mitigate the decline in employment and production. This is consistent with the observed decrease in the volatility of hours worked, $\sigma(\ln H)$, from 0.303 to 0.193 (Table 3), and contributes to the overall reduction in output volatility to 0.294 in the benchmark model.

The overall reduction in output volatility due to capital irreversibility in our granular economy stems from both the direct channel (detailed in Section 4.2.1) and the indirect general equilibrium channel detailed here. A quantitative decomposition of the relative contributions of these direct and indirect effects (GE wage) is provided in Appendix G (see Table 9 there).

4.3 Dynamics of Micro-level Investment Moments

The preceding analysis established that capital irreversibility dampens aggregate output volatility in our granular economy. To understand more deeply why and how irreversibility exerts such a significant influence when interacted with granular shocks—a contrast to its often muted role in canonical real business cycle models driven solely by aggregate TFP shocks—we now examine the micro-level investment behavior. Table 4 presents key statistics on firms' investment dynamics, comparing our benchmark granular economy with a canonical real business cycle economy featuring only aggregate TFP shocks.

Table 4: Micro Investment Behavior (Values for x and $\sigma(x)$ in percent)

	Granular Economy			RBC Economy		
Micro-moment	x	$\sigma(x)$	$\rho(x, \ln Y)$	x	$\sigma(x)$	$\rho(x, \ln Y)$
$Pr(\lvert\frac{i}{k}\rvert \leqslant 0.01)$	9.7	0.2	0.060	8.9	0.2	0.318
$Pr(\lvert\frac{i}{k}\rvert \leqslant 0.01)$ TOP 500	5.4	1.2	-0.260	5.4	0.2	-0.018
$Pr(\frac{\Delta k'}{k} \leqslant -0.20)$	0.4	0.0	0.184	0.4	0.0	-0.295
$Pr(\frac{\Delta k'}{k} \leqslant -0.20)$ TOP 500	2.2	0.4	-0.525	2.2	0.0	-0.518
$Pr(i \leqslant 0.000)$	0.7	0.0	0.049	1.6	0.0	-0.696
$Pr(i \leqslant 0.000)$ TOP 500	3.5	1.7	-0.123	3.3	0.6	0.021
$Pr((1-\delta)k \geqslant k_u^\star)$	10.3	0.0	0.106	10.3	0.0	-0.183
$Pr((1-\delta)k \geqslant k_u^\star)$ TOP 500	7.7	0.8	-0.357	7.7	0.0	-0.063

Notes: The table compares the ergodic value and cyclicality of micro investment behavior in the benchmark model (Granular Economy) with a traditional model economy whose volatility is driven by aggregate TFP shocks. For each specified micro-moment (investment behavior): The column headed 'x' reports the mean probability, expressed in percent. The column headed '$\sigma(x)$' reports the standard deviation of this probability, expressed in percentage points. The column headed '$\rho(x, \ln Y)$' reports the contemporaneous correlation of the micro-moment with the logarithm of aggregate output Y. Investment behaviors considered are: inaction rate ($Pr(\lvert\frac{i}{k}\rvert \leqslant 0.01)$), negative net investment spike rate ($Pr(\frac{\Delta k'}{k} \leqslant -0.20)$), negative investment rate ($Pr(i \leqslant 0.000)$), and the probability of the irreversibility constraint binding ($Pr((1-\delta)k \geqslant k_u^\star)$). These are calculated for both the entire economy and for the largest 500 firms from the previous period.

A critical distinction emerges from Table 4 when observing the cyclical behavior of the irreversibility constraint itself, especially for large firms. In the Granular Economy, the share of the TOP 500 firms that are constrained by irreversibility ($Pr((1-\delta)k \geqslant k_u^\star)$)

– meaning they wish to downsize their capital stock but cannot due to the constraint – exhibits notable volatility ($\sigma(x) = 0.8$ percentage points) and is strongly counter-cyclical ($\rho(x, \ln Y) = -0.357$). This indicates that during downturns, which can be initiated or amplified by adverse granular shocks hitting these large firms, a significant and cyclically varying number of these economically important firms are actively impeded by irreversibility.

In stark contrast, the Real Business Cycle Economy driven by aggregate TFP shocks shows a different picture. For the TOP 500 firms, the share constrained by irreversibility has virtually no volatility ($\sigma(x) = 0.0$) and a much weaker correlation with the aggregate cycle. While some large firms might be constrained on average, this fraction does not fluctuate meaningfully over the business cycle in response to aggregate shocks alone. This lack of cyclical variation in the mass of constrained large firms is a key reason why irreversibility often appears less consequential in such models.

This difference in the cyclical engagement of the irreversibility constraint for large firms is fundamental. When substantial granular shocks occur, capital irreversibility actively shapes the investment response of a cyclically fluctuating segment of the economy's largest firms. Their inability to disinvest optimally during periods of idiosyncratic distress (or when hit by a large negative component of a granular shock) means that capital may remain temporarily misallocated. However, it also implies that aggregate investment does not contract as sharply as it might otherwise, and productive resources are not shed as precipitously. This micro-level dynamic, particularly the time-varying incidence of binding irreversibility constraints among large firms, allows irreversibility to significantly alter aggregate business cycle dynamics. Canonical real business cycle models with only aggregate shocks typically do not generate sufficient firm-specific shock variance to cause such cyclically sensitive binding of irreversibility for a large, influential group of firms. Granular shocks, by their very nature, provide these large, idiosyncratic disturbances that make the irreversibility constraint a more active margin for key economic players.[19]

[19] Further evidence from Table 4 supports this. For instance, the inaction rate ($Pr(|\frac{i}{k}| \leqslant 0.01)$) among the TOP 500 firms is more volatile ($\sigma(x) = 1.2$ percentage points vs. 0.2 percentage points) and more strongly

5 Granular Shocks and Capital Misallocation in the data and model

Our theoretical framework argues for a significant role for granular shocks in driving capital misallocation, particularly when investment irreversibility is present. These dynamics, in turn, are argued to be important contributors to aggregate business cycle fluctuations. This section now presents empirical evidence from U.S. firm-level data to substantiate this narrative. We investigate the empirical linkage between identified granular shocks and ensuing changes in capital misallocation, and assess whether these observed dynamics align with the predictions and mechanisms of our structural model.

We first construct an annual panel dataset by merging the firm-level TFP estimates from İmrohoroğlu and Tüzel (2014) with Compustat databases from 1964 to 2019.[20] Similar to Gabaix (2011), we construct the series of granular residual as the sales-weighted idiosyncratic shocks among the 200 largest firms:[21]

$$\Theta_t^{Gabaix} = \sum_{i=1}^{100} \frac{\text{Sale}_{i,t-1}}{\text{Sale}_{t-1}^{100}} \Delta\varepsilon_{i,t} - \frac{\sum_{i=1}^{100} \Delta\varepsilon_{i,t}}{100} \tag{20}$$

where the proxy for idiosyncratic productivity shocks is the productivity growth rate $\Delta\varepsilon_{i,t} = \frac{\varepsilon_{i,t}-\varepsilon_{i,t-1}}{\varepsilon_{i,t-1}}$. To build a measure of capital misallocation, we follow Kilic and Tuzel (2025) and we approximate the marginal product of capital by the logarithm of the ratio of sales (SALE) to physical capital (PPEGT).[22] Accordingly, we measure capital misallocation using the dispersion of the marginal product of capital among the largest 200 firms, $\sigma^{MPK,200}$:

$$\sigma_t^{MPK,200} = \sqrt{\frac{\sum_{i=1}^{200}(MPK_{i,t} - MPK_t^{mean})^2}{200}} \tag{21}$$

counter-cyclical ($\rho(x, \ln Y) = -0.260$ vs. -0.018) in the granular economy. Similarly, large disinvestments ($Pr(\frac{\Delta k'}{k} \leqslant -0.20)$ by TOP 500 firms are also more volatile and counter-cyclical in the presence of granular shocks.

[20] See Appendix D for the details of the variable constructions.

[21] Note that we follow Gabaix and Koijen (2024) and we subtract the equal-weighted shocks from the weighted shocks to ensure the elimination of any common observed and unobserved aggregate factors.

[22] Idiosyncratic shocks and marginal product of capital are cleaned for 2-digit sector times year fixed effects.

Finally, we test whether the model can qualitatively and quantitatively replicate how granular misallocation responds to granular shocks. To this end, we employ Jordà (2005)'s local projection method to estimate impulse responses of granular misallocation to granular shocks. This method is based on the estimation of a series of regressions for each horizon τ for each variable. Accordingly, the linear model is as follows:

$$\ln(\sigma_{t+\tau}^{MPK,200}) = \alpha_\tau + \phi_\tau(L)z_{t-1} + \beta_\tau \Theta_t^{Gabaix} + v_{t+\tau} \quad \text{for} \quad \tau = 0, 1, 2, \ldots, \tag{22}$$

where z is a vector of control variables, $\phi_\tau(L)$ is a polynomial in the lag operator, and the shock is the identified granular shock. The shock is the granular residual. Our vector of baseline control variables, z, contains the logarithm of real GDP, 10-year T-bill, term spread (the government 5-year bond yield minus the 1-year yield), consumer price inflation, and the logarithm of the dispersion of the marginal product of capital computed excluding the largest 200.[23] Furthermore, z includes lags of the granular shock variable to control for any serial correlation in the shock variable. The term $\phi_\tau(L)$ is a first-order polynomial.

Figure 1 compares the impulse responses of the level and the first difference in the logarithm of the misallocation of granular capital to a negative granular residual of one standard deviation between the model and the data. The model performs well in quantitatively replicating the response of capital misallocation among large firms to granular shocks, as the model IRFs lie within the 95 percent confidence interval of the empirical IRFs. A negative granular shock is associated with negative productivity shocks to larger firms, increasing capital misallocation. Importantly, the discrepancy between the IRFs generated by the model and those observed in the data arises from the model's exclusion of additional frictions that are present in the real economy. These omitted frictions contribute to a higher ergodic dispersion of the marginal product of capital, while simultaneously dampening its respon-

[23] In particular, we use the annual real GDP (GDPCA from FRED). We use yield treasury securities at 10, 5, and 1-year constant maturity (DGS10, DGS5 and DGS1 from FRED). We use consumer price inflation (FPCPITOTLZGUSA from FRED). The dispersion of marginal product of capital excluding the largest 100 firms is obtained from the COMPUSTAT dataset. All series are HP-filtered using a smoothing parameter of 6.25, following Ravn and Uhlig (2002).

siveness to granular shocks. As shown in Appendix I, this result is robust across alternative sample periods and industry-level controls used in the construction of firm-level MPK.

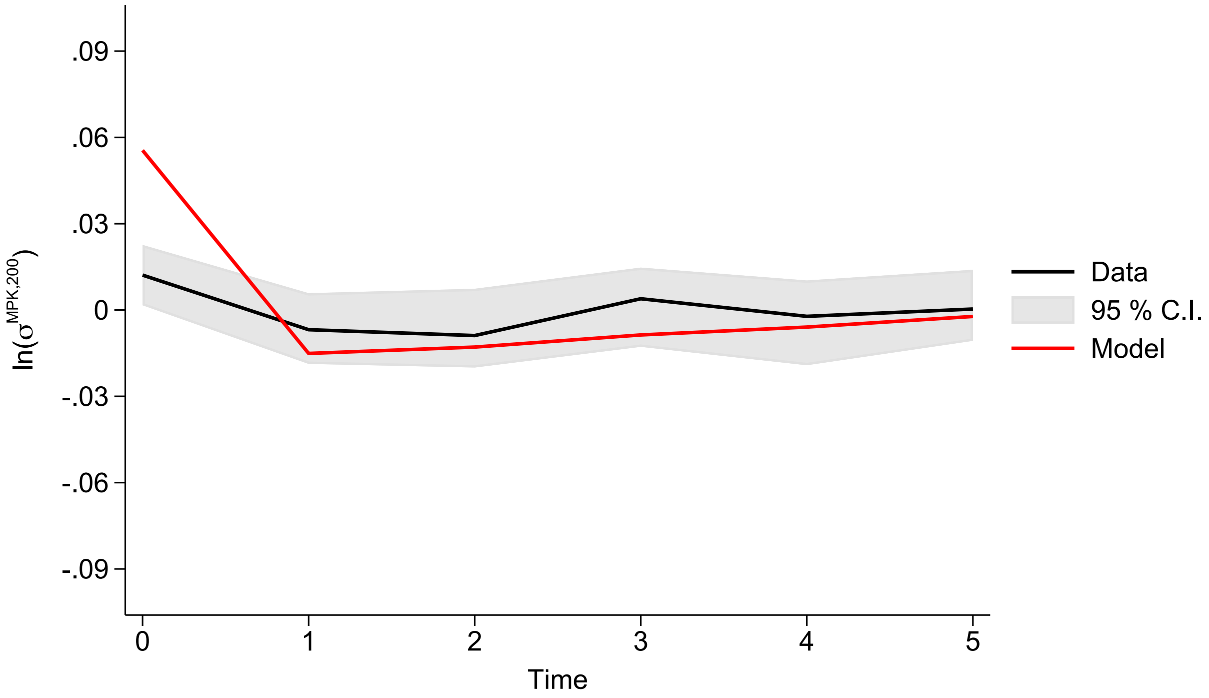


Figure 1: Effect of granular residual shock on granular misallocation.

Notes: Responses of the logarithm of the granular dispersion of the marginal product of capital to a negative one standard deviation granular residual shock. The black line represents the response estimated from the data. The red line represents the response obtained from the simulated data. The 95 percent confidence intervals of the empirical estimations are computed with a wild bootstrap of 1000 repetitions. All series are HP-filtered using a smoothing parameter of 6.25, following Ravn and Uhlig (2002). Granular misallocation is estimated conditional on 2-digit sector-by-year fixed effects. For further details on data construction, see Appendix D.

6 Policy Implications

Our preceding analysis has highlighted that investment irreversibility influences business cycle dynamics in a granular economy. While investment irreversibility may dampen fluctuations in aggregate output and investment, leading to seemingly desirable reduced volatility in those aggregate measures, these benefits are counteracted by the costs arising from suboptimal capital allocation and a deterioration in households' ability to smooth consumption, leading to greater consumption volatility. The overall welfare outcome then becomes ambiguous, necessitating careful policy evaluation that weighs these contrasting effects against each other.

This issue carries significant policy relevance. Historical interventions, such as the Troubled Asset Relief Program (TARP) in the United States, exemplify attempts to facilitate capital reallocation, particularly from distressed firms.[24] As our model demonstrates, policies that foster a better alignment between productivity and the capital stock across firms can inadvertently increase aggregate volatility. Therefore, whether such interventions are welfare-improving remains a critical question. To explore these welfare implications, we evaluate two distinct policy options designed to promote capital reallocation by mitigating the costs associated with investment irreversibility.

Our approach to calculating welfare follows the methodology established by Lucas (1987) and further developed by Krusell and Smith (1999). This involves converting differences in expected lifetime utility into "consumption equivalents" (CEV), representing the permanent percentage increase in consumption that would make a representative household indifferent between the pre-policy and post-policy economies. It is important to note that, unlike the existing studies, our exercise does not aim to eliminate business cycles entirely. Instead, we assess the welfare impact of specific policy interventions within an ongoing granular economy.

6.1 Policy Options

Government Subsidy: In this scenario, we consider a policy where the government fully absorbs the price gap between the cost of purchasing capital and the value recovered from selling it. This subsidy is financed through lump-sum taxation on the representative household. This policy aims to reduce the financial friction associated with asset liquidation, encouraging firms to adjust their capital stock more flexibly.

Eliminate Price Gap: This policy represents a more direct intervention, where the price gap is entirely removed, allowing firms to recover the full value of their capital upon sale.

[24] See U.S. Congressional Budget Office (2009) and Rattner (2010) for further details.

This effectively simulates an environment where capital is perfectly reversible, removing a pure friction from the economy.

The second policy option—eliminating the price gap entirely—may appear extreme in our stylized model, but it closely reflects actual policy objectives during financial crises. The implementation of TARP, for instance, demonstrated how governments can effectively reduce capital market frictions. By purchasing distressed assets at above-market prices, TARP eliminated the gap between book values and severely depressed market prices, creating a buyer of last resort that restored liquidity to frozen markets. The Treasury's stated goal was precisely to "restore liquidity and stability to the financial system" by ensuring that "the prices of assets would better reflect their underlying value."[25] This real-world precedent suggests that policies approximating perfect reversibility can be both feasible and necessary during periods of severe market dysfunction, particularly when large, systemically important firms face binding irreversibility constraints.

6.2 Welfare Implications

The long-run welfare gains from these policies, measured in consumption equivalent variation (CEV), are summarized in Table 5.

Table 5: Welfare Implication of Capital Irreversibility

	$\mathbb{E}(U)$	$\mathbb{CEV}$ (perc.)
Benchmark	24.746	
Subsidy	24.831	0.341
No-Price Gap	25.368	2.520

Under the Government Subsidy policy, the policy intervention results in a consumption-equivalent variation of approximately 0.34%. This implies that the representative household

[25]See U.S. Department of the Treasury, Office of Financial Stability (2010).

would be indifferent between residing in the initial economy with a permanently higher consumption level of 0.34% and living in the economy with the subsidy policy implemented. As shown in Table 11 of Appendix J, this policy enhances welfare primarily by reducing consumption volatility and, more significantly, by increasing average consumption. While it substantially raises investment volatility, thereby intensifying capital reallocation, its impact on consumption volatility remains limited. This outcome can be attributed to the fact that firms do not internalise the social costs associated with capital reallocation, which may lead to excessive capital destruction through divestment and potentially amplify volatility due to over-reallocation.

Under the Eliminate Price Gap policy, where capital is perfectly reversible (i.e., irreversibility costs are zero), the welfare gain is substantially larger, reaching approximately 2.52% in consumption equivalent variation. In this scenario, the policy is unequivocally efficiency-enhancing, as it removes a pure friction without introducing behavioural distortions. This policy also yields a dramatic reduction in consumption volatility, contributing significantly to the overall welfare improvement.

Our analysis reveals a fundamental insight for policymakers: in granular economies, the welfare implications of investment irreversibility are inherently ambiguous. While irreversibility dampens aggregate volatility—an apparent benefit—it simultaneously impairs allocative efficiency and consumption smoothing. The stark contrast in welfare gains between our two policy scenarios (0.34% versus 2.52%) underscores that the design of interventions matters crucially. Policies that merely subsidize the symptoms of irreversibility without addressing its root causes may inadvertently encourage excessive capital destruction, as firms fail to internalize the social costs of reallocation. In contrast, policies that enhance the fundamental liquidity and efficiency of capital markets can yield substantial welfare improvements. These findings suggest that in economies where large firms drive aggregate fluctuations, policymakers should prioritize reducing frictions in capital reallocation markets—through improved bankruptcy procedures, standardization of assets, or development of secondary markets—rather than simply compensating firms for liquidation losses. The granular nature of modern economies thus calls for a reconsideration of traditional stabilization policies, with greater emphasis on maintaining efficient capital allocation even during periods of significant idiosyncratic disturbances.

7 Conclusion

When aggregate fluctuations originate from firm-level shocks, irreversibility dampens the response of large firms to negative shocks, reducing the covariance between productivity and capital and, in turn, lowering output and investment volatility. These effects are amplified in general equilibrium: limited reallocation impairs households' ability to smooth consumption, increasing real wage flexibility and further absorbing shocks. Our analysis suggests that while investment irreversibility creates ambiguous welfare effects—dampening volatility but impairing allocative efficiency—the design of interventions matters crucially. Policymakers should prioritize enhancing the fundamental liquidity and efficiency of capital reallocation markets, enhancing allocative efficiency without encouraging excessive capital destruction.

References

Abel, Andrew B., and Janice C. Eberly. (1994). "A unified model of investment under uncertainty". *American Economic Review*, 84(5), pp. 1369-1384. https://econpapers.repec.org/article/aeaaecrev/v_3a84_3ay_3a1994_3ai_3a5_3ap_3a1369-84.htm

Abel, Andrew B., and Janice C. Eberly. (1996). "Optimal investment with costly reversibility". *The Review of Economic Studies*, 63(4), pp. 581-593. https://doi.org/10.2307/2297794

Arrow, Kenneth J. (1968). "Chapter 1. Optimal capital policy with irreversible investment". In J. N. Wolfe (ed.), *Value, Capital, and Growth*. Edinburgh University Press, pp. 1-19.

Autor, David, David Dorn, Lawrence F. Katz, Christina Patterson and John Van Reenen. (2020). "The Fall of the Labor Share and the Rise of Superstar Firms". *The Quarterly Journal of Economics*, 135, pp. 645-709. https://doi.org/10.1093/qje/qjaa004

Bachmann, Rüdiger, and Christian Bayer. (2014). "Investment dispersion and the business cycle". *American Economic Review*, 104(4), pp. 1392-1416. https://doi.org/10.1257/aer.104.4.1392

Bachmann, Rüdiger, Ricardo J. Caballero and Eduardo M. R. A. Engel. (2013). "Aggregate Implications of Lumpy Investment: New Evidence and a DSGE Model". *American Economic Journal: Macroeconomics*, 5(4), pp. 29-67. https://doi.org/10.1257/mac.5.4.29

Beaudry, Paul, and Alain Guay. (1996). "What do interest rates reveal about the functioning of real business cycle models?". *Journal of Economic Dynamics and Control*, 20(9-10), pp. 1661-1682. https://doi.org/10.1016/0165-1889(95)00916-7

Bertola, Giuseppe. (1998). "Irreversible investment". *Research in Economics*, 52(1), pp. 3-37. https://doi.org/10.1006/reec.1997.0153

Bertola, Giuseppe, and Ricardo J. Caballero. (1994). "Irreversibility and aggregate investment". *The Review of Economic Studies*, 61(2), pp. 223-246. https://doi.org/10.2307/2297979

Blanco, Andrés, and Isaac Baley. (2024). "The Macroeconomics of Irreversibility". Working Paper Series, 17, Federal Reserve Bank of Atlanta. https://doi.org/10.29338/wp2024-17

Bloom, Nicholas. (2009). "The impact of uncertainty shocks". *Econometrica*, 77(3), pp. 623-685. https://doi.org/10.3982/ECTA6248

Bloom, Nicholas, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten and Stephen J. Terry. (2018). "Really Uncertain Business Cycles". *Econometrica*, 86(3), pp. 1031-1065. https://doi.org/10.3982/ECTA10927

Brubaker, Ralph, and Charles J. Tabb. (2010). "Bankruptcy Reorganizations and the Troubling Legacy of Chrysler and GM". *University of Illinois Law Review*, 2010, pp. 1375-1410. https://doi.org/10.2139/ssrn.1549650

Burstein, Ariel, Vasco M. Carvalho and Basile Grassi. (2025). "Bottom-Up Markup Fluctuations". *The Quarterly Journal of Economics*, qjaf029. https://doi.org/10.1093/qje/qjaf029

Caballero, Ricardo J., and Eduardo M. R. A. Engel. (1999). "Explaining investment dynamics in US manufacturing: A generalized (S, s) approach". *Econometrica*, 67(4), pp. 783-826. https://doi.org/10.1111/1468-0262.00053

Carvalho, Vasco M., and Basile Grassi. (2019). "Large firm dynamics and the business cycle". *American Economic Review*, 109(4), pp. 1375-1425. https://doi.org/10.1257/aer.20151317

Caunedo, Julieta, and Elisa Keller. (2021). "Capital Obsolescence and Agricultural Productivity". *The Quarterly Journal of Economics*, 136(1), pp. 577-625. https://doi.org/10.1093/qje/qjaa028

Congressional Budget Office. (2009*). The Troubled Asset Relief Program: Report on Transactions Through June 17, 2009*. CBO Report, 3219. https://www.cbo.gov/sites/default/files/111th-congress-2009-2010/reports/06-29-tarp.pdf

Cooley, Thomas F. (ed.). (1995). *Frontiers of Business Cycle Research*. Princeton University Press. https://doi.org/10.1515/9780691218052

Cui, Wei. (2022). "Macroeconomic effects of delayed capital liquidation". *Journal of the European Economic Association*, 20(4), pp. 1683-1742. https://doi.org/10.1093/jeea/jvac023

David, Joel M., and Venky Venkateswaran. (2019). "The sources of capital misallocation". *American Economic Review*, 109(7), pp. 2531-2567. https://doi.org/10.1257/aer.20180336

Demers, Michel. (1991). "Investment under uncertainty, irreversibility, and the arrival of information over time". *The Review of Economic Studies*, 58(2), pp. 333-350. https://doi.org/10.2307/2297971

Di Giovanni, Julian, and Andrei A. Levchenko. (2012). "Country Size, International Trade, and Aggregate Fluctuations in Granular Economies". *Journal of Political Economy*, 120(6), pp. 1083-1132. https://doi.org/10.1086/669161

Di Giovanni, Julian, Andrei A. Levchenko and Isabelle Mejean. (2014). "Firms, Destinations, and Aggregate Fluctuations". *Econometrica*, 82(4), pp. 1303-1340. https://doi.org/10.3982/ECTA11041

Di Giovanni, Julian, Andrei A. Levchenko and Isabelle Mejean. (2024). "Foreign Shocks as Granular Fluctuations". *Journal of Political Economy*, 132(2), pp. 391-433. https://doi.org/10.1086/726235

Dixit, Robert K., and Robert S. Pindyck. (1994). *Investment under Uncertainty*. Princeton University Press. https://doi.org/10.1515/9781400830176

Dow, James P. J., and Lars J. Olson. (1992). «Irreversibility and the Behavior of Aggregate Stochastic Growth Models». *Journal of Economic Dynamics and Control*, 16(2), pp. 207-223. https://doi.org/10.1016/0165-1889(92)90031-9

Ehouarne, Cedric, Lars-Alexander Kuehn and David Schreindorfer. (2016). "Misallocation Cycles". Meeting Papers, 1482, Society for Economic Dynamics. https://ideas.repec.org/p/red/sed016/1482.html

Gabaix, Xavier. (2011). "The Granular Origins of Aggregate Fluctuations". *Econometrica*, 79(3), pp. 733-772. https://doi.org/10.3982/ECTA8769

Gabaix, Xavier, and Ralph S. J. Koijen. (2024). "Granular instrumental variables". *Journal of Political Economy*, 132(7), pp. 2231-2280. https://doi.org/10.1086/728743

Gomme, Paul, B. Ravikumar, and Peter Rupert. (2011,). "The Return to Capital and the Business Cycle". *Review of Economic Dynamics*, 14(2), pp. 262-278. https://doi.org/10.1016/j.red.2010.11.004

Gourio, François, and Anil K. Kashyap. (2007). "Investment spikes: New facts and a general equilibrium exploration". *Journal of Monetary Economics*, 54(1), pp. 1-22. https://doi.org/10.1016/j.jmoneco.2007.06.004

Hansen, Gary D. (1985). "Indivisible labor and the business cycle". *Journal of Monetary Economics*, 16(3), pp. 309-327. https://doi.org/10.1016/0304-3932(85)90039-X

Hsieh, Chang-Tai, and Peter J. Klenow. (2010). "Development accounting". American Economic Journal: Macroeconomics, 2(1), pp. 207-223. https://doi.org/10.1257/mac.2.1.207

Ifergane, Tomer. (2025). "Concentrated Risk: Misallocation and Granular Business Cycles". Working Paper, SSRN. http://dx.doi.org/10.2139/ssrn.4945298

İmrohoroğlu, Ayşe, and Şelale Tüzel. (2014). "Firm-level productivity, risk, and return". *Management Science*, 60(8), pp. 2073-2090. https://doi.org/10.1287/mnsc.2013.1852

Jaimovich, Nir, Stephen J. Terry, and Nicolas Vincent. (2023). "The Empirical Distribution of Firm Dynamics and Its Macro Implications". NBER Working Paper Series, 31337, National Bureau of Economic Research. https://doi.org/10.3386/w31337

Jordá, Óscar. (2005). "Estimation and inference of impulse responses by local projections". *American Economic Review*, 95(1), pp. 161-182. https://doi.org/10.1257/0002828053828518

Jovanovic, Boyan. (1987). "Micro Shocks and Aggregate Risk". *The Quarterly Journal of Economics*, 102(2), pp. 395-409. https://doi.org/10.2307/1885069

Kermani, Amir, and Yueran Ma. (2023). "Asset specificity of nonfinancial firms". *The Quarterly Journal of Economics*, 138(1), pp. 205-264. https://doi.org/10.1093/qje/qjac030

Khan, Aubhik, and Julia K. Thomas. (2003). "Nonconvex factor adjustments in equilibrium business cycle models: do nonlinearities matter?". *Journal of Monetary Economics*, 50(2), pp. 331-360. https://doi.org/10.1016/S0304-3932(03)00003-5

Khan, Aubhik, and Julia K. Thomas. (2008). "Idiosyncratic Shocks and the Role of Nonconvexities in Plant and Aggregate Investment Dynamics". *Econometrica*, 76(2), pp. 395-436. https://doi.org/10.1111/j.1468-0262.2008.00837.x

Khan, Aubhik, and Julia K. Thomas. (2013). "Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity". *Journal of Political Economy*, 121(6), pp. 1055-1107. https://doi.org/10.1086/674142

Kiliç, Mete, and Şelale Tüzel. (2025). "Investing in Misallocation". SSRN, 4464542. http://dx.doi.org/10.2139/ssrn.4464542

Krusell, Per, and Anthony A. Smith Jr. (1998). "Income and wealth heterogeneity in the macroeconomy". *Journal of Political Economy*, 106(5), pp. 867-896. https://doi.org/10.1086/250034

Krusell, Per, and Anthony A. Smith Jr. (1999). "On the welfare effects of eliminating business cycles". *Review of Economic Dynamics*, 2(1), pp. 245-272. https://doi.org/10.1006/redy.1998.0043

Lanteri, Andrea. (2018). "The market for used capital: Endogenous irreversibility and reallocation over the business cycle". *American Economic Review*, 108(9), pp. 2383-2419. https://doi.org/10.1257/aer.20160131

Leahy, John V. (1993). "Investment in competitive equilibrium: The optimality of myopic behavior". *The Quarterly Journal of Economics*, 108(4), pp. 1105-1133. https://doi.org/10.2307/2118461

Lubben, Stephen J. (2009). "No Big Deal: The GM and Chrysler Cases in Context". *American Bankruptcy Law Journal*, 83, p. 531. https://elischolar.library.yale.edu/ypfs-documents/9283

Lucas, Robert E. (1987). *Models of Business Cycles*. Basil Blackwell. https://archive.org/details/modelsofbusiness0000luca_z7w8

Melcangi, Davide, and Silvia Sarpietro. (2024). *Nonlinear Firm Dynamics*. Staff Report, 1088, Federal Reserve Bank of New York. https://doi.org/10.59576/sr.1088

Nickell, Stephen J. (1977). "Uncertainty and lags in the investment decisions of firms". *The Review of Economic Studies*, 44(2), pp. 249-263. https://doi.org/10.2307/2297065

Ottonello, Pablo. (2024). "Capital Unemployment". *The Review of Economic Studies*. Forthcoming. drive.google.com/file/d/1yF7yz6awOjrjoBdc27jyspvgehG59pck/view

Pindyck, Robert S. (1988). "Irreversible Investment, Capacity Choice, and the Value of the Firm". *The American Economic Review*, 78(5), pp. 969-985. https://web.mit.edu/rpindyck/www/Papers/IrreversibleInvestmentCapacityAER1988.pdf

Pindyck, Robert S. (1991). "Irreversibility, uncertainty, and investment". *Journal of Economic Literature*, 29, pp. 1110-1148. https://www.jstor.org/stable/2727613

Ramey, Valerie A., and Matthew D. Shapiro. (2001). "Displaced Capital: A Study of Aerospace Plant Closings". *Journal of Political Economy*, 109(5), pp. 958-992. https://doi.org/10.1086/322828

Rattner, Steven. (2010). *Overhaul: An Insider's Account of the Obama Administration's Emergency Rescue of the Auto Industry*. Houghton Mifflin Harcourt. https://stevenrattner.com/overhaul/

Ravn, Morten O., and Harald Uhlig. (2002). "On adjusting the Hodrick-Prescott filter for the frequency of observations". *The Review of Economics and Statistics*, 84(2), pp. 371-375. https://doi.org/10.1162/003465302317411604

Restuccia, Diego, and Richard Rogerson. (2008). "Policy distortions and aggregate productivity with heterogeneous establishments". *Review of Economic Dynamics*, 11(4), pp. 707-720. https://doi.org/10.1016/j.red.2008.05.002

Roe, Mark J., and David A. Skeel Jr. (2010). "Assessing the Chrysler Bankruptcy". *Michigan Law Review*, 108, pp. 727-772. https://dx.doi.org/10.2139/ssrn.1426530

Rogerson, Richard. (1988). "Indivisible labor, lotteries and equilibrium". *Journal of Monetary Economics*, 21(1), pp. 3-16. https://doi.org/10.1016/0304-3932(88)90042-6

Salgado, Sergio, Fatih Guvenen, and Nicholas Bloom. (2019). "Skewed Business Cycles". NBER Working Paper Series, 26565, National Bureau of Economic Research. https://doi.org/10.3386/w26565

Sargent, Thomas J. (1980). "'Tobin's *q*' and the Rate of Investment in General Equilibrium". *Carnegie-Rochester Conference Series on Public Policy*, 12, pp. 107-154. https://doi.org/10.1016/0167-2231(80)90024-X

Terry, Stephen J., and Edward S. Knotek II. (2011). "Markov-chain approximations of vector autoregressions: Application of general multivariate-normal integration techniques". *Economics Letters*, 110(1), pp. 4-6. https://doi.org/10.1016/j.econlet.2010.09.008

Thomas, Julia K. (2002). “Is lumpy investment relevant for the business cycle?”. *Journal of Political Economy*, 110(3), pp. 508-534. https://doi.org/10.1086/339746

United States Department of the Treasury. (2010). *Troubled Asset Relief Program: Two-Year Retrospective.* Reports, Office of Financial Stability. https://home.treasury.gov/system/files/initiatives/financial-stability/reports/Documents/TARP%20Two%20Year%20Retrospective_10%2005%2010_transmittal%20letter.pdf

Veracierto, Marcelo L. (2002). “Plant-level irreversible investment and equilibrium business cycles”. *American Economic Review*, 92(1), pp. 181-197. https://doi.org/10.1257/000282802760015667

Winberry, Thomas. (2021). “Lumpy investment, business cycles, and stimulus policy”. *American Economic Review*, 111(1), pp. 364-396. https://doi.org/10.1257/aer.20161723

Yeh, C.-A. (2025). “Revisiting the origins of business cycles with the size-variance relationship”. *Review of Economics and Statistics*, 107(3), pp. 864-871. https://doi.org/10.1162/rest_a_01374

Online Appendix

A Analytical Results

A.1 Proof Lemma 1

Each firm i chooses next period capital $k_{i,t+1}$ one period in advance. Idiosyncratic productivity follows

$$\ln \varepsilon_{i,t+1} = \xi_{i,t} + \epsilon_{i,t+1}, \tag{23}$$

where $\xi_{i,t} \sim N(0, \sigma_\xi^2)$ is observed at time t; $\epsilon_{i,t+1} \sim N(0, \sigma_\epsilon^2)$ is realized at the start of $t+1$; and $\xi_{i,t}, \epsilon_{i,t}$ are independent across both time and firms and mutually independent. There is no aggregate shock.

The firm's objective at time t is to maximize its present discounted expected future profits by choosing an investment plan $\{i_{t+j}\}_{j=0}^{\infty}$:

$$\mathbb{E}_t\left(\sum_{j=0}^{\infty} \frac{1}{(1+r)^j}(\varepsilon_{i,t+j}k_{i,t+j}^{\alpha} - i_{i,t+j})\right) \quad \text{s.t.} \quad k_{i,t+j+1} = (1-\delta)k_{i,j+t} + i_{i,t+j}, \quad \text{given} \quad k_{i,t}, \tag{24}$$

where r is the interest rate faced by the firm and δ is the depreciation rate of capital.

The first-order condition for $k_{i,t+1}$ is

$$\alpha E_t(\varepsilon_{i,t+1})k_{i,t+1}^{\alpha-1} = r + \delta. \tag{25}$$

Solving (25) for $k_{i,t+1}$ and taking logs yields

$$\ln k_{i,t+1} = \frac{1}{1-\alpha}\ln E_t(\varepsilon_{i,t+1}) - \frac{1}{1-\alpha}\ln\left(\frac{r+\delta}{\alpha}\right). \tag{26}$$

Given the log-normal structure,

$$E_t(\varepsilon_{i,t+1}) = \exp\left(\xi_{i,t} + \tfrac{1}{2}\sigma_\epsilon^2\right). \tag{27}$$

Substituting into (26) gives

$$\ln k_{i,t+1} = A + B\xi_{i,t}, \qquad A := \frac{1}{1-\alpha}\left(\tfrac{1}{2}\sigma_\epsilon^2 - \ln\frac{r+\delta}{\alpha}\right), \quad B := \frac{1}{1-\alpha}. \tag{28}$$

Hence $\ln k_{i,t+1} \sim N(A, B^2\sigma_\xi^2)$ with

$$E(\ln k) = A, \qquad \mathrm{Var}(\ln k) = \frac{\sigma_\xi^2}{(1-\alpha)^2}. \tag{29}$$

We have $\ln\varepsilon = \xi + \epsilon$ and $\ln k = A + B\xi$. Because (ξ, ϵ) is jointly normal and $\ln k$ is affine in ξ, $(\ln\varepsilon, \ln k)$ is bivariate normal. Their covariance is

$$\mathrm{Cov}(\ln\varepsilon, \ln k) = B\mathrm{Var}(\xi) = \frac{\sigma_\xi^2}{1-\alpha}. \tag{30}$$

Output $y = \varepsilon k^\alpha$ implies

$$\ln y = \ln\varepsilon + \alpha\ln k. \tag{31}$$

Thus $\ln y$ is normal with mean

$$\mu_y := E(\ln y) = E(\ln\varepsilon) + \alpha E(\ln k) = \tfrac{1}{2}\sigma_\epsilon^2 + \alpha A, \tag{32}$$

and variance

$$\sigma_y^2 := \mathrm{Var}(\ln y) = \mathrm{Var}(\ln\varepsilon) + \alpha^2\mathrm{Var}(\ln k) + 2\alpha\mathrm{Cov}(\ln\varepsilon, \ln k) \tag{33}$$

$$= (\sigma_\xi^2 + \sigma_\epsilon^2) + \alpha^2\frac{\sigma_\xi^2}{(1-\alpha)^2} + 2\alpha\frac{\sigma_\xi^2}{1-\alpha} \tag{34}$$

$$= \sigma_\epsilon^2 + \frac{\sigma_\xi^2}{(1-\alpha)^2}. \tag{35}$$

Equations (32)–(35) complete the proof of Lemma 1. ■

A.2 Proof of Proposition 1

Let $Y = \sum_{i=1}^M y_i$ with y_i i.i.d. $\sim LN(\mu_y, \sigma_y^2)$ as in Lemma 1. Because the first two moments of a log-normal variable are available in closed form and the $\{y_i\}_{i=1}^M$ are independent across firms, we obtain $E(Y)$ and $\mathrm{Var}(Y)$ exactly by (i) linearity of expectation and (ii) the fact that all cross-firm covariances vanish.[26] We now report these exact moments.

For $y_i \sim LN(\mu_y, \sigma_y^2)$,

$$E(y_i) = e^{\mu_y + \sigma_y^2/2}, \qquad \mathrm{Var}(y_i) = (e^{\sigma_y^2} - 1)e^{2\mu_y + \sigma_y^2}. \tag{36}$$

Independence implies

$$E(Y) = Me^{\mu_y + \sigma_y^2/2}, \tag{37}$$

$$\mathrm{Var}(Y) = M(e^{\sigma_y^2} - 1)\, e^{2\mu_y + \sigma_y^2}. \tag{38}$$

Hence the (exact) coefficient of variation is

$$CV(Y) = \frac{\sqrt{\mathrm{Var}(Y)}}{E(Y)} = \sqrt{\frac{e^{\sigma_y^2} - 1}{M}}. \tag{39}$$

B Frictionless Economy

In this Appendix, we show that the endogenous aggregate μ can be exactly characterized by the first moment of the marginal distribution of capital K and the dynamic productivity distribution h with $\psi = 0$.

In equation (9), the choice of the current level of employment can be derived from a static problem as:

$$N(\varepsilon, k; \mu) = \arg\max_n [\varepsilon k^\alpha n^\nu - \omega(\mu) n] \tag{40}$$

which yields

$$N(\varepsilon, k; \mu) = [\nu \varepsilon k^\alpha / \omega(\mu)]^{1/(1-\nu)} \tag{41}$$

Using this decision rule for employment, we can replace the first and second terms in equation

[26] The sum of i.i.d. log-normal variables is not log-normal in general. A widely used approximation (Fenton–Wilkinson; see Marlow, 1967) replaces Y with $\widetilde{Y} \sim LN(\mu_Y, \sigma_Y^2)$ chosen so that $E(\widetilde{Y}) = E(Y)$ and $\mathrm{Var}(\widetilde{Y}) = \mathrm{Var}(Y)$. Solving gives $\sigma_Y^2 = \ln\big[(e^{\sigma_y^2} - 1)/M + 1\big]$ and $\mu_Y = \ln M + \mu_y + \sigma_y^2/2 - \sigma_Y^2/2$. Because the approximation matches moments by construction, (37)–(38) remain the exact values. The approximation is only needed if one wants a closed-form pdf/cdf for Y (e.g., tail probabilities).

(9) as:

$$\varepsilon k^{\alpha} n^{\nu} - \omega(\mu) n = (1-\nu)\varepsilon^{1/(1-\nu)} k^{\alpha/(1-\nu)} \left(\frac{\nu}{\omega(\mu)}\right)^{\nu/(1-\nu)}, \tag{42}$$

and we can rewrite the problem as follows:

$$\begin{aligned} v(\varepsilon, k; \mu) = \max_{k'} \Big[& (1-\nu)\varepsilon^{1/(1-\nu)} k^{\alpha/(1-\nu)} \left(\frac{\nu}{\omega(\mu)}\right)^{\nu/(1-\nu)} + (1-\delta)k - k' \\ & + E\big[d(\mu, \mu') v(\varepsilon', k'; \mu') \mid \varepsilon, \mu\big]\Big]. \end{aligned} \tag{43}$$

This problem yields the optimal investment decision $G(\varepsilon; \mu)$ as follows:

$$G(\varepsilon; \mu) = L_0(\varepsilon) L_1(\mu) \tag{44}$$

$$L_0(\varepsilon; \mu) = \left(\sum_{\mu'} \Pi^{\mu}(\mu'|\mu) \sum_{\varepsilon'} \Pi^{\varepsilon}_{\mu'|\mu}(\varepsilon'|\varepsilon) \varepsilon'^{1/(1-\nu)}\right)^{(1-\nu)/(1-(\alpha+\nu))} \tag{45}$$

$$L_1(\mu) = \left(\frac{1 - (1-\delta)\sum_{\mu'} \Pi^{\mu}(\mu'|\mu) d(\mu, \mu')}{\alpha \sum_{\mu'} \Pi^{\mu}(\mu'|\mu) d(\mu, \mu') \left(\frac{\nu}{\omega(\mu')}\right)^{\nu/(1-\nu)}}\right)^{\frac{1-\nu}{\alpha+\nu-1}}. \tag{46}$$

This shows that the investment decision is independent of the current capital stock k, which depends only on the idiosyncratic productivity of the previous period. This implies that (1) it is sufficient to track the idiosyncratic productivity both in the current and previous periods for each firm and (2) the distribution of the current and previous idiosyncratic productivity h is a N_ε by N_ε grid point object.

It follows that the distribution of firms over idiosyncratic productivity and capital stock can be recovered, $\mu(\varepsilon_i, k_j)$, from h in each period as follows. First, we can construct $\mu_{\varepsilon,-1}(\varepsilon_j)$, the marginal distribution of firms over ε_j for $j = 1, \ldots, N_\varepsilon$ in the previous period -1, and $\mu_\varepsilon(\varepsilon_i)$, the marginal distribution of firms over ε_i for $i = 1, \ldots, N_\varepsilon$ in the current period. We can also construct $\Pi^{\varepsilon}_{h}(\varepsilon_j, \varepsilon_i)$, the transition probability $Pr(\varepsilon = \varepsilon_i \mid \varepsilon_{-1} = \varepsilon_j)$. Therefore, we can construct $\mu(\varepsilon, k)$, the distribution of firms over productivity and stock of capital in each period as

$$\mu(\varepsilon_i, k_j) = \mu_{\varepsilon,-1}(\varepsilon_j)\Pi_h^{\varepsilon}(\varepsilon_j, \varepsilon_i) \tag{47}$$

$$k_j = \frac{L_0(\varepsilon_j)\mu_{\varepsilon,-1}(\varepsilon_j)}{\sum_m L_0(\varepsilon_m)\mu_{\varepsilon,-1}(\varepsilon_m)} K \tag{48}$$

C Model Solution

In this section, we first outline the methodology used to construct the set of firm distributions across transitions and the corresponding probability space. We then describe the approach employed to solve the heterogeneous firm model.

C.1 Firm Distribution Across Transitions and its Probability Space

We construct the set of firm distributions across transitions, denoted by H, along with the associated transition probability $\Pi(h' \mid h)$, to replicate the fluctuations in the marginal distribution of firms over productivity, $\mu_\varepsilon(\varepsilon)$. To construct the transition matrix $\Pi(h' \mid h)$, we approximate the multivariate stochastic process governing the set of moments m^ε of the distribution $\mu_\varepsilon(\varepsilon)$ using a finite-state Markov chain. We then use this discretized process, together with the ergodic idiosyncratic transition probability Π^ε, to construct the firm distribution across transitions, H.

C.1.1 Moments Selection and Construction of the transition probability $\Pi(h'|h)$

To make the problem computationally feasible, we construct the set of firm distributions across transitions, denoted by H, along with the associated transition probability $\Pi(h' \mid h)$, to replicate the fluctuations in the most volatile moments of the marginal productivity distribution, $\mu_\varepsilon(\varepsilon)$. To this end, after simulating $\mu_\varepsilon(\varepsilon)$, we implement the following steps:

1. Identify the granular segment of the productivity distribution—specifically, the region of $\mu_\varepsilon(\varepsilon)$ where the Law of Large Numbers fails.

2. Select the set of most volatile moments that allow us to replicate the fluctuations in the firm distribution within this granular segment.

3. Approximate the stochastic process governing the selected moments m^{ε} of $\mu_{\varepsilon}(\varepsilon)$ using a multivariate first-order Markov chain, following the discretization method of Terry and Knotek II (2011). This yields a transition matrix $\Pi^{m^{\varepsilon}}(m^{\varepsilon\prime} \mid m^{\varepsilon})$.

Note that since ε follows a first-order Markov process, we have $\Pr(h' \mid h) \equiv \Pr(m^{\varepsilon\prime} \mid m^{\varepsilon})$. Therefore, we set $\Pi^{h} = \Pi^{m^{\varepsilon}}$.

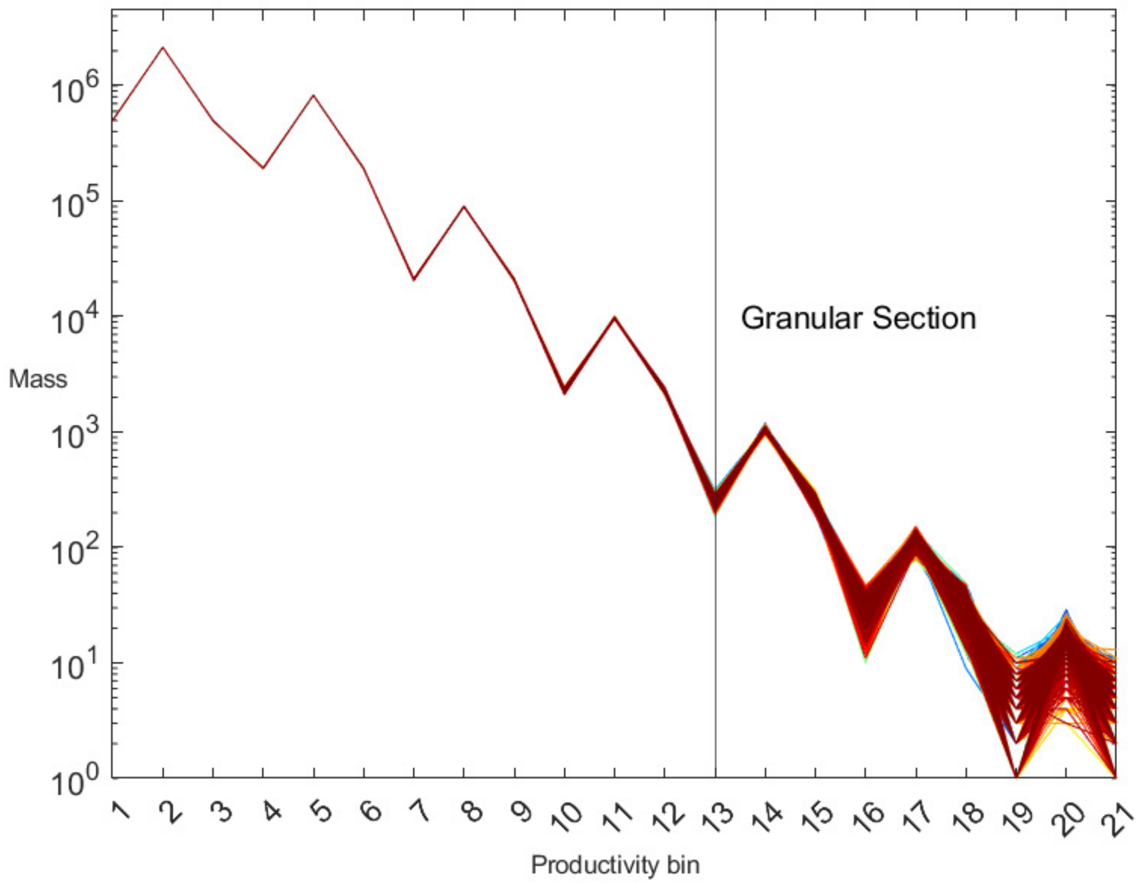
Figure 2 presents the simulated productivity distributions, $\mu_{\varepsilon}(\varepsilon)$, on a logarithmic scale over 10,000 periods. The simulation indicates that the Law of Large Numbers does not hold in the right tail of the productivity distribution. Consequently, we define the granular section of the productivity distribution as the portion of $\mu_{\varepsilon}(\varepsilon)$ associated with $\varepsilon \geqslant \varepsilon_{13}$.

Next, Figure 2 reports the distributions of the percentage deviations from their ergodic values for the moments that characterize fluctuations in the productivity distribution $\mu_{\varepsilon}(\varepsilon)$ within the granular section. Specifically, Figure 3 displays the percentage deviations of: (A) mean productivity, (B) the standard deviation of productivity, (C) Pearson's moment coefficient of skewness, and (D) firm mass—each calculated among firms with $\varepsilon \geqslant \varepsilon_{13}$.

Interestingly, the simulation reveals that the dispersion of the percentage deviation in the mean (0.398) is relatively low compared to those of the standard deviation (5.655), skewness (8.632), and firm mass (2.999). Based on this, we simulate a granular economy that replicates the cyclicality of: (1) the standard deviation, (2) Pearson's moment coefficient of skewness, and (3) firm mass within the granular section of the productivity distribution.

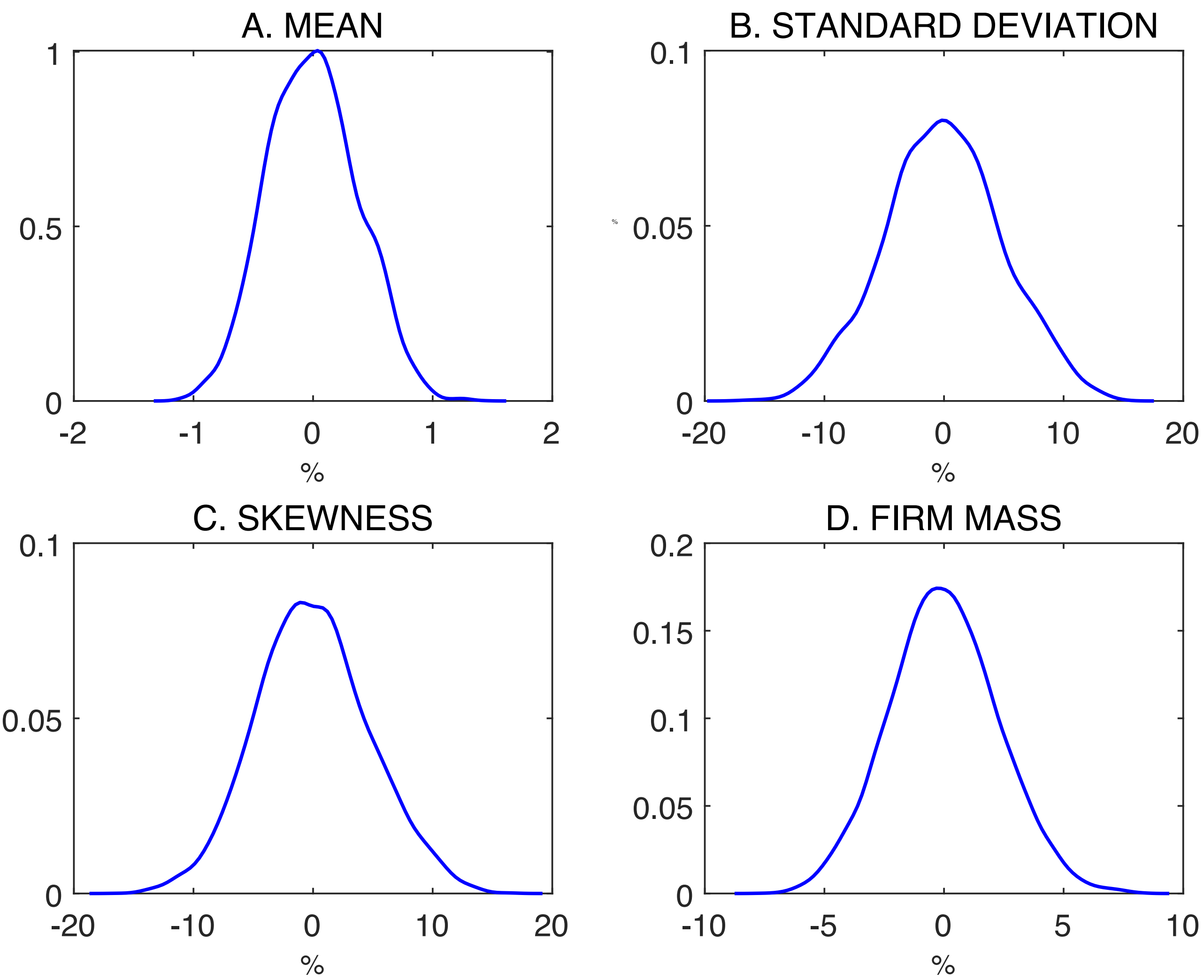
To discretize the multivariate stochastic process of the moment m^{ε}, we use a grid with three points for each moment, i.e., $d_{m^{\varepsilon}} = 3$.

Figure 2: Simulated Productivity Distribution $\mu_\varepsilon(\varepsilon)$



Notes: This figure illustrates the simulated productivity distribution, $\mu_\varepsilon(\varepsilon)$, on a logarithmic scale over 10,000 periods. Specifically, it depicts the mass of firms corresponding to each productivity level.

Figure 3: Moments of the Tail



Notes: The figure reports the distributions of percentage deviations from their ergodic values for: (A) mean productivity, (B) standard deviation of productivity, (C) Pearson's moment coefficient of skewness, and (D) firm mass—each calculated among firms with $\varepsilon \geqslant \varepsilon_{13}$. These distributions are derived from a 10,000-period simulation of the productivity distribution, $\mu_\varepsilon(\varepsilon)$.

C.1.2 Constructing H

We construct the set of firm distributions across transitions,

$$H = \{h_1, h_2, \ldots, h_{N_h-1}, h_{N_h}\},$$

by iterating steps 1 and 2 for F times, in order to ultimately implement step 3:

1. Construct

$$H^f = \{\mu^f_{\varepsilon,1}, \mu^f_{\varepsilon,2}, \ldots, \mu^f_{\varepsilon,N_h}\}$$

to reflect the discretized distribution m^{ε}.

2. Given Π^{ε}, implement a constrained draw of h such that each element of the set

$$H^f = \{h_1^f, h_2^f, \ldots, h^f_{l+(m^{\varepsilon}-1)N_{\mu_\varepsilon}}, h^f_{l+(m^{\varepsilon}-1)N_{\mu_\varepsilon}+1}, \ldots, h^f_{N^2_{\mu_\varepsilon}}\}$$

satisfies the following conditions for all $p, q = 1, 2, \ldots, N_{\mu_\varepsilon}$ and $i, j = 1, 2, \ldots, N_\varepsilon$:

$$\mu^f_{\varepsilon,q}(\varepsilon_i) = \sum_j h^f_{p+(q-1)N_{\mu_\varepsilon}}(\varepsilon_i, \varepsilon_j),$$
$$\mu^f_{\varepsilon,p}(\varepsilon_j) = \sum_i h^f_{p+(q-1)N_{\mu_\varepsilon}}(\varepsilon_i, \varepsilon_j),$$

where $N_{\mu_\varepsilon} = (\text{number of moments})^{d_{m^\varepsilon}}$.

3. Compute the average distribution:

$$H = \frac{1}{F}\sum_{f=1}^{F} H^f.$$

C.2 Model Simulation

C.2.1 Firm's Problem

We can rewrite the firm's problem (Equation 9) using marginal utility, $p \equiv U'_c(c, n^h)$, as follows:

$$\widehat{v}(\varepsilon, k; \mu) = \max_{n,i} p(\mu)\left[\varepsilon F(k,n) - \omega(\mu)n - i\right] + \beta \sum_{\mu'} \Pi^{\mu}(\mu'|\mu) \sum_{\varepsilon'} \Pi^{\varepsilon}_{\mu'|\mu}(\varepsilon'|\varepsilon)\widehat{v}(\varepsilon', k'; \mu') \quad (49)$$

The evolution of the aggregate equilibrium is characterized by the following mappings:

$$\begin{aligned} p &= \Gamma_p(\mu), \\ \mu' &\sim G(\mu). \end{aligned} \quad (50)$$

Given the transition probabilities defined above, we express the Bellman equation in terms of h and K, explicitly substituting these for μ:

$$v(\varepsilon, k; h, K) = \max_{n,i} p(h, K) [\varepsilon F(k, n) - \omega(h, K)n - i] + \beta \sum_{h'} \Pi^h(h'|h) \sum_{\varepsilon'} \Pi^{\varepsilon}_{h'}(\varepsilon'|\varepsilon) v(\varepsilon', k'; h', K') \tag{51}$$

We approximate the equilibrium mappings using log-linear rules:

$$\ln(\widehat{p}) = \alpha_p(h) + \beta_p(h) \ln(K), \tag{52}$$

$$\ln(\widehat{K}') = \alpha_K(h) + \beta_K(h) \ln(K) \tag{53}$$

These log-linear rules are used to forecast the future values of the price level p and aggregate capital stock K'. The coefficients α and β depend on the current aggregate state h.

C.2.2 Solution Algorithm

The solution algorithm consists of an inner and outer loop:

- **Inner Loop:** Solve the firm's value function, $v(\varepsilon, k; h, K)$, via value function iteration, holding the forecasting rules for aggregate variables (Equations 52) fixed.

- **Outer Loop:** Simulate the economy forward. In each period, draw a new aggregate state h' based on Π^h. Use the firm's policy functions (from the inner loop) and the transition probabilities $\Pi^{\varepsilon}_{h'}$ to determine firms' optimal decisions and update the aggregate capital stock and the distribution of firms. Then, update the forecasting rules.

The algorithm iterates between the inner and outer loops until the coefficients of the forecasting rules converge. This provides an approximate solution to the recursive equilibrium.

C.3 Performance

Our procedure to build firm distributions across transitions delivers an error of 0.0000830440, computed as follows:

$$\frac{1}{N_h}\sum_{j=1}^{N_h}\frac{1}{3}\left(\frac{\left|m_{1,j}^{\text{target}}-\text{st.dev.}(h_j)\right|}{m_{1,j}^{\text{target}}}+\frac{\left|m_{2,j}^{\text{target}}-\text{skew}(h_j)\right|}{m_{2,j}^{\text{target}}}+\frac{\left|m_{3,j}^{\text{target}}-\text{Tail Mass}(h_j)\right|}{m_{3,j}^{\text{target}}}\right) \quad (54)$$

Figure 4 presents the kernel density estimates of the R^2 values for the 138 forecasting rules applied to price and capital. For the capital forecasting rules, the mean and minimum R^2 values are 0.9992 and 0.9891, respectively. Similarly, for the price forecasting rules, the mean and minimum R^2 values are 0.9992 and 0.9962.

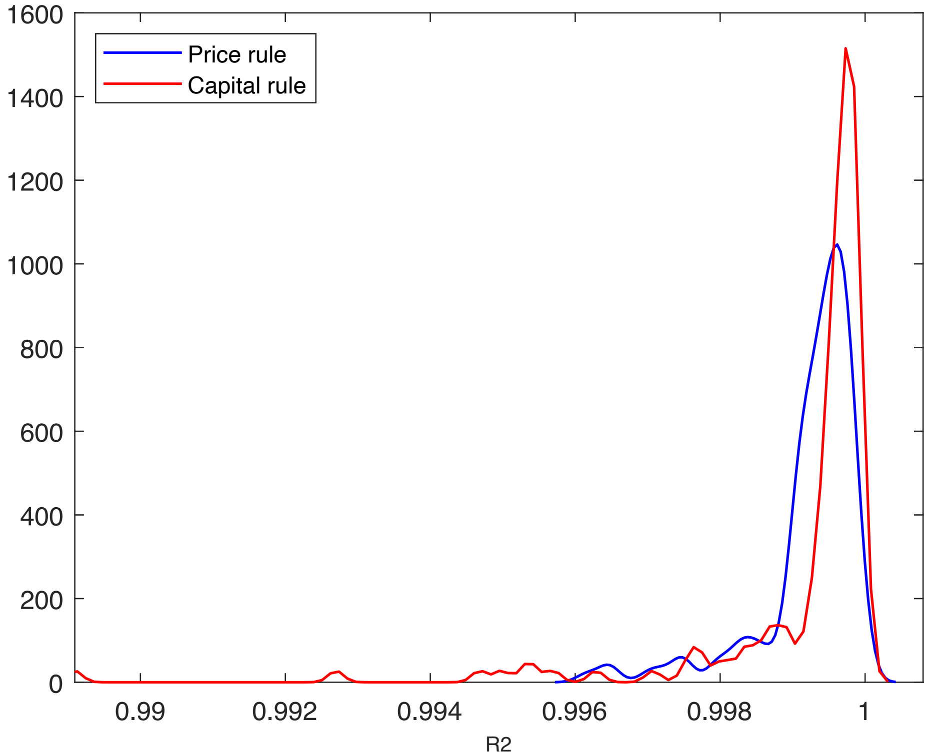


Figure 4: Distribution of the R^2 of the Forecasting Rules

Notes: The figure reports the kernel densities of the R^2 values for the 138 forecasting rules. The blue line corresponds to price forecasting rules, and the red line to capital forecasting rules.

D Data

Sample Selection.— We construct an annual panel dataset by merging firm-level TFP estimates from İmrohoroğlu and Tüzel (2014) with Compustat data from 1964 to 2019. Using Standard Industry Classification (SIC) codes, we exclude firms in the oil, energy, and financial sectors from our sample. Specifically, we exclude:

- **Oil and oil-related firms:** SIC codes 2911, 5172, 1311, 4922, 4923, 4924, and 1389;
- **Energy firms:** SIC codes ranging from 4900 to 4940;
- **Financial firms:** SIC codes ranging from 6000 to 6999.

We also eliminate firms with missing data to ensure valid sales observations across the sample. The TFP estimates are adjusted using industry-specific time dummies to remove industry-year effects.

Variable Construction.—We define the variables used in our empirical analysis as follows:

1. *Gross investment rate:* Ratio of capital expenditures (CAPX) to plant, property, and equipment (PPEGT). We control for 2-digit sector-by-year fixed effects.
2. *Inaction rate:* Mass of firms such that $|i/k| \leqslant 0.01$.
3. *Net investment rate:* Growth rate of PPEGT. We control for 2-digit sector-by-year fixed effects.
4. *Negative investment spike:* Among the 500 largest firms (by sales) in the previous period, defined as the mass of firms such that
$$\Pr\left(\frac{\Delta k'}{k} \leqslant -0.2\right).$$
5. *Marginal product of capital* (MPK): Logarithm of the ratio of sales (SALE) to physical capital (PPEGT). We control for 2-digit sector-by-year fixed effects.
6. *Idiosyncratic shocks:* Approximated by the productivity growth rate:
$$\Delta\varepsilon_{i,t} = \frac{\varepsilon_{i,t} - \varepsilon_{i,t-1}}{\varepsilon_{i,t-1}}. \tag{55}$$

7. *Granular residual:* Difference between the sales-weighted and equally weighted average of idiosyncratic shocks among the 200 largest firms in the previous period:

$$\Theta_t^{\text{Gabaix}} = \sum_{i=1}^{100} \frac{\text{Sale}_{i,t-1}}{\text{Sale}_{t-1}^{100}} \Delta\varepsilon_{i,t} - \frac{1}{100}\sum_{i=1}^{100} \Delta\varepsilon_{i,t}. \tag{56}$$

8. *Granular capital misallocation:* Dispersion of MPK among the 200 largest firms:

$$\sigma_t^{\text{MPK,200}} = \sqrt{\frac{1}{200}\sum_{i=1}^{200} (MPK_{i,t} - MPK_t^{\text{mean}})^2}. \tag{57}$$

Table 6: Summary Statistics

x	Observations	mean	sd	min	max
Θ_t^{Gabaix}	38	0.000	0.017	-0.031	0.056
$\ln(\sigma^{MPK,200})$	39	0.000	0.026	-0.055	0.0771
$\ln(\sigma^{MPK,-200})$	39	0.000	0.017	-0.048	0.047
$Pr(\lvert\frac{i}{k}\rvert \leqslant 0.01)$ TOP 500	39	0.111	0.021	0.075	0.186
$Pr(\frac{\Delta k'}{k} \leqslant -0.2)$ TOP 500	39	0.024	0.0282	0.000	0.197

Notes: This table reports descriptive statistics for the variables used in Section 3. All series are HP-filtered with a smoothing parameter of 6.25, except for the inaction rate and the negative investment spike.

E Equilibrium Wage Dynamics and Business Cycles

Table 7: The Impact of Frictions and General Equilibrium on Business Cycles

Variable	**Statistic**	**Benchmark (GE)** ($\psi = 0.35$)	**Frictionless (GE)** ($\psi = 0.00$)	**Partial Equilibrium** **(Frictionless)**
Output (Y)	$\sigma(x)$	0.294	0.377	0.858
	$\sigma(x)/\sigma(Y)$	1.000	1.000	1.000
	$\rho(x, Y)$	1.000	1.000	1.000
Consumption (C)	$\sigma(x)$	0.171	0.119	2.702
	$\sigma(x)/\sigma(Y)$	0.582	0.316	3.150
	$\rho(x, Y)$	0.786	0.716	0.442
Investment (I)	$\sigma(x)$	1.200	1.962	14.706
	$\sigma(x)/\sigma(Y)$	4.088	5.209	17.146
	$\rho(x, Y)$	0.901	0.975	0.050
Hours (H)	$\sigma(x)$	0.191	0.303	0.858
	$\sigma(x)/\sigma(Y)$	0.651	0.805	1.000
	$\rho(x, Y)$	0.834	0.962	1.000

Notes: This table compares business cycle moments across three model specifications to isolate the effects of capital frictions and general equilibrium wage adjustments. (a) Column 1 reports the benchmark model with capital irreversibility ($\psi = 0.35$); (b) Column 2 reports the general equilibrium model without capital irreversibility ($\psi = 0.00$); (c) Column 3 reports a frictionless model in partial equilibrium, where wages do not adjust to clear the aggregate labor market.

F Cyclicality of the Real Interest Rate over Time

Table 8: Cyclical Dynamics of Risk-Free Rate

	$\sigma(\ln(1+r))$	$\rho(\ln(1+r), \ln Y)$
Whole sample	1.310	-0.286
Pre-1980	1.131	-0.828
Post-1980	1.178	0.189

Notes: Real interest rate is measured as the nominal return on 1-year Treasury bills (DGS1 taken from FRED) adjusted for realized CPI inflation (CPIAUCSL_PC1 taken from FRED). Output measured as real gross domestic product (GDPCA taken from FRED). All series are HP-filtered in logs with a smoothing parameter of 6.25, following Ravn and Uhlig (2002). Whole sample refers to the 1964–2019 series. Pre-1980 refers to the 1964:1979 sample. Post-1980 refers to the 1980–2019 sample.

G Decomposition of Irreversibility's Impact

Table 9: Decomposition on the Effect of Capital Irreversibility and Misallocation

	Benchmark		*No T.V. Depreciation*		*Household Renting Capital*	
	$\frac{\sigma(x)}{\sigma(x^{\text{No-irrev.}})}$	$\rho(x, \ln Y)$	$\frac{\sigma(x)}{\sigma(x^{\text{No-irrev.}})}$	$\rho(x, \ln Y)$	$\frac{\sigma(x)}{\sigma(x^{\text{No-irrev.}})}$	$\rho(x, \ln Y)$
$\ln Y$	0.780	1.000	0.947	1.000	0.499	1.000
$\ln C$	1.437	0.786	1.050	0.702	0.882	0.702
$\ln I$	0.612	0.901	0.978	0.967	0.386	0.967
$\ln H$	0.629	0.834	0.936	0.950	0.383	0.950
$\ln(1+r)$	1.443	-0.496	1.110	-0.148	0.889	0.823

Notes: This table compares the business cycle moments of the benchmark model, the model with only the *time-varying misallocation component*, and the model without both the *time-varying misallocation* and *time-varying depreciation* components. $\sigma(x)$ denotes the percentage standard deviation of variable x; $\sigma(x)/\sigma(x^{\text{No-irrev.}})$ is the relative standard deviation with respect to its counterpart in the model with $\psi = 0$; and $\rho(x, \ln Y)$ is the contemporaneous correlation of x with output Y. All series are HP-filtered with a smoothing parameter of 6.25, following Ravn and Uhlig (2002).

In Section 4.2, we discussed how investment irreversibility mitigates output volatility by distinguishing between its direct effects on firm-level decision-making and its broader general equilibrium implications. To assess the relative contribution of these two channels to aggregate volatility, we consider a counterfactual scenario: an economy with *no time-varying capital depreciation*, where capital adjustment costs are purely virtual. In this setting, while adjustment costs continue to influence firms' optimal capital choices, they do not entail any actual reallocation of real resources. Consequently, this setup partially removes the indirect general equilibrium effect, as the absence of real adjustment costs enhances the representative household's capacity to smooth consumption over time.[27]

[27] The indirect general equilibrium effect may still influence volatility in the *no time-varying capital depreciation* economy. Specifically, firms that remain within the inaction region and retain excessively large

Table 9 presents the relative volatilities of the *Benchmark* and *No Time-Varying Depreciation* economies, expressed relative to the *Frictionless* economy. The findings suggest that the indirect general equilibrium effect accounts for at least three-quarters of the total reduction in aggregate volatility.

We conclude by quantifying the contribution of capital misallocation fluctuations to granular volatility by analyzing a third counterfactual scenario: the *Household Renting Capital* economy. In this setting, capital is rented by the representative household to firms each period, ensuring that the marginal product of capital is equalized across firms and eliminating factor misallocation. As shown in Table 9, while irreversibility partially dampens the aggregate volatility arising from factor misallocation, it still accounts for more than one-third of granular volatility.

capital stocks reduce the representative household's capacity to smooth consumption over time.

H Aggregate TFP Shock Economy

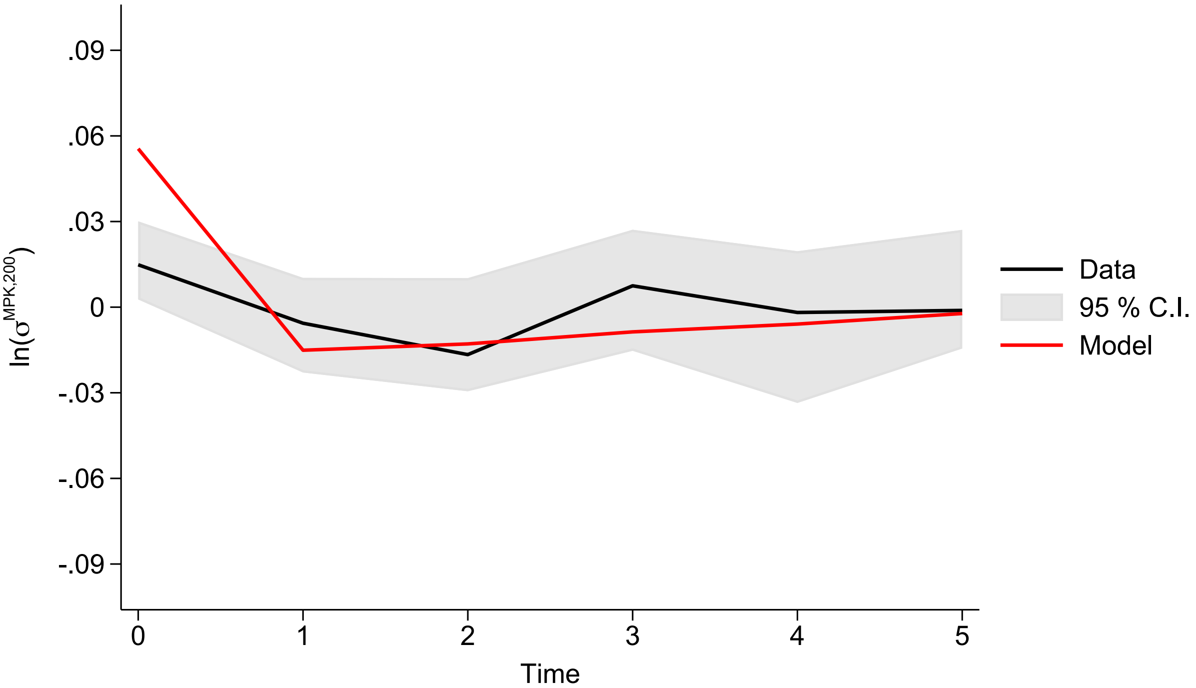
Table 10: Business Cycle Moments: Aggregate TFP Shock

	$\psi = 0.35$ with $y = Z\varepsilon k^{\alpha} l^{\nu}$			$\psi = 0.00$ with $y = Z\varepsilon k^{\alpha} l^{\nu}$		
	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, \ln Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, \ln Y)$
$\ln Y$	1.888	1.000	1.000	1.898	1.000	1.000
$\ln C$	0.981	0.520	0.924	0.957	0.504	0.934
$\ln I$	6.775	3.589	0.963	7.412	3.905	0.965
$\ln H$	1.051	0.557	0.934	1.062	0.559	0.946
$\ln(1+r)$	0.169	0.089	0.848	0.151	0.079	0.871

Notes: The table compares the business cycle moments of model economies whose volatility is exclusively driven by aggregate TFP shock. In particular, $\psi > 0$ refers to the economy with partial capital irreversibility, and $\psi = 0$ refers to the economy without partial irreversibility. $\sigma(x)$ is the percentage standard deviation of x, and $\sigma(x)/\sigma(\ln Y)$ is the relative standard deviation to that of Y, and $\rho(x, \ln Y)$ is the contemporaneous correlation of x with Y. All series are HP-filtered with a smoothing parameter of 6.25, following Ravn and Uhlig (2002).

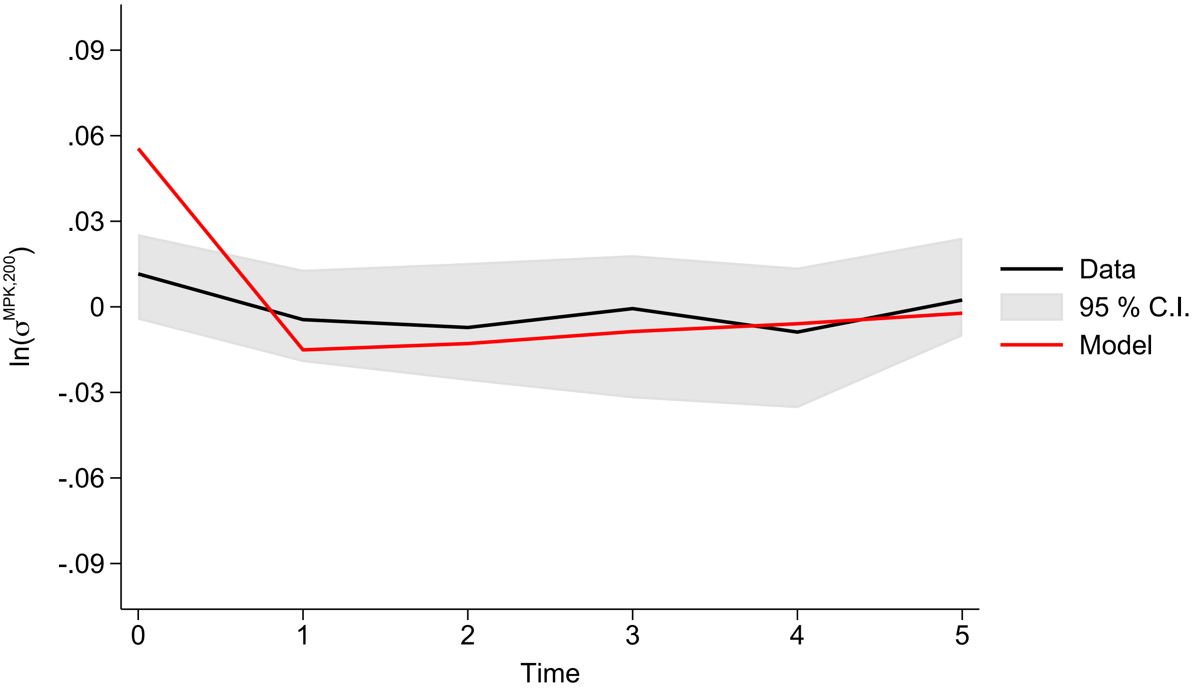
I Granular shocks and Misallocation: Robustness Checks

Figure 5: Effect of Granular Residual Shock on Granular Misallocation



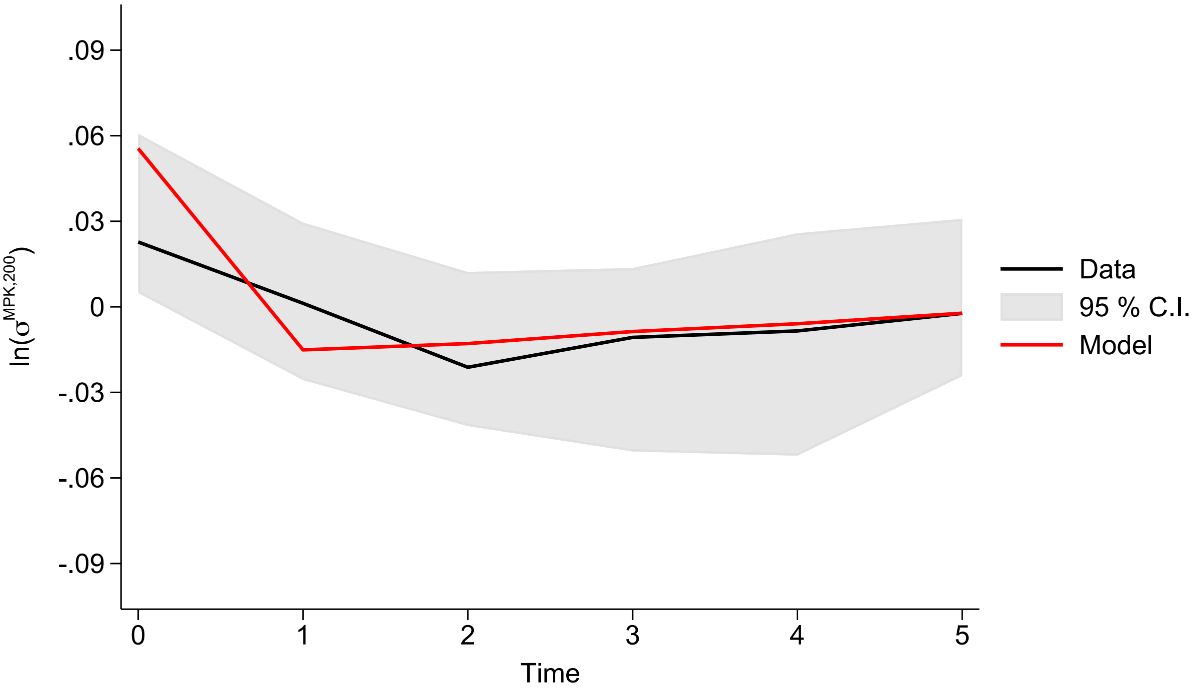
Notes: Responses of the logarithm of the granular dispersion of the marginal product of capital to a negative one standard deviation granular residual shock. The black line represents the response estimated from the data. The red line represents the response obtained from the simulated data. The 95 percent confidence intervals of the empirical estimations are computed with a wild bootstrap of 1000 repetitions. All series are HP-filtered using a smoothing parameter of 6.25, following Ravn and Uhlig (2002). The sample period spans from 1980 to 2019. Granular misallocation is estimated conditional on 2-digit sector-by-year fixed effects. For further details on data construction, see Appendix D.

Figure 6: Effect of Granular Residual Shock on Granular Misallocation



Notes: Responses of the logarithm of the granular dispersion of the marginal product of capital to a negative one standard deviation granular residual shock. The black line represents the response estimated from the data. The red line represents the response obtained from the simulated data. The 95 percent confidence intervals of the empirical estimations are computed with a wild bootstrap of 1000 repetitions. All series are HP-filtered using a smoothing parameter of 6.25, following Ravn and Uhlig (2002). The sample period spans from 1964 to 2019. Granular misallocation is estimated conditional on 3-digit sector-by-year fixed effects. For further details on data construction, see Appendix D.

Figure 7: Effect of Granular Residual Shock on Granular Misallocation



Notes: Responses of the logarithm of the granular dispersion of the marginal product of capital to a negative one standard deviation granular residual shock. The black line represents the response estimated from the data. The red line represents the response obtained from the simulated data. The 95 percent confidence intervals of the empirical estimations are computed with a wild bootstrap of 1000 repetitions. All series are HP-filtered using a smoothing parameter of 6.25, following Ravn and Uhlig (2002). The sample refers to the period spanning from 1980 to 2019. Granular misallocation is estimated conditional on 3-digit sector-by-year fixed effects. For further details on data construction, see Appendix D.

J Policy Implications

Table 11: Business Cycle Moments

	Benchmark			*Subsidy*			*No-Price Gap*		
	$\exp(x)$	$\sigma(x)$	$\rho(x, \ln Y)$	$\exp(x)$	$\sigma(x)$	$\rho(x, \ln Y)$	$\exp(x)$	$\sigma(x)$	$\rho(x, \ln Y)$
$\ln Y$	0.778	0.295	1.000	0.802	0.320	1.000	0.787	0.377	1.000
$\ln C$	0.646	0.170	0.787	0.656	0.161	0.796	0.659	0.119	0.716
$\ln I$	0.132	1.210	0.905	0.131	1.263	0.862	0.128	1.962	0.975
$\ln H$	0.336	0.193	0.839	0.341	0.215	0.892	0.333	0.303	0.962
$\ln(1+r)$	1.042	0.025	-0.496	1.042	0.023	-0.332	1.042	0.018	-0.022

Notes: The table reports the business cycle moments of the *Benchmark* economy and the economies under the two downsize policies. $\sigma(x)$ is the percentage standard deviation of x, and $\rho(x, \ln Y)$ is the contemporaneous correlation of x with $\ln Y$. The model moments are obtained from a 15,000-period unconditional simulation using the solution of the model. The reported standard deviations and correlations refer to HP-filtered series in logarithms, using a smoothing parameter of 6.25, following Ravn and Uhlig (2002).

BANCO DE ESPAÑA PUBLICATIONS

WORKING PAPERS

2430 MIGUEL GARCÍA-POSADA and PETER PAZ: The transmission of monetary policy to credit supply in the euro area.
2431 KLODIANA ISTREFI, FLORENS ODENDAHL and GIULIA SESTIERI: ECB communication and its impact on financial markets.
2432 FRUCTUOSO BORRALLO, LUCÍA CUADRO-SÁEZ, CORINNA GHIRELLI and JAVIER J. PÉREZ: "El Niño" and "La Niña": Revisiting the impact on food commodity prices and euro area consumer prices.
2433 VÍCTOR CABALLERO, CORINNA GHIRELLI, ÁNGEL LUIS GÓMEZ and JAVIER J. PÉREZ: The public-private wage GAP in the euro area a decade after the sovereign debt crisis.
2434 LIDIA CRUCES, ISABEL MICÓ-MILLÁN and SUSANA PÁRRAGA: Female financial portfolio choices and marital property regimes.
2435 RODOLFO G. CAMPOS, ANA-SIMONA MANU, LUIS MOLINA and MARTA SUÁREZ-VARELA: China's financial spillovers to emerging markets.
2436 LUDOVIC PANON, LAURA LEBASTARD, MICHELE MANCINI, ALESSANDRO BORIN, PEONARE CAKA, GIANMARCO CARIOLA, DENNIS ESSERS, ELENA GENTILI, ANDREA LINARELLO, TULLIA PADELLINI, FRANCISCO REQUENA and JACOPO TIMINI: Inputs in Distress: Geoeconomic Fragmentation and Firms' Sourcing.
2437 DANIEL DEJUAN-BITRIA, WAYNE R. LANDSMAN, SERGIO MAYORDOMO and IRENE ROIBÁS: How do changes in financial reporting standards affect relationship lending?
2438 ALICIA AGUILAR and RICARDO GIMENO: Discrete Probability Forecasts: What to expect when you are expecting a monetary policy decision.
2439 RODOLFO G. CAMPOS, JESÚS FERNÁNDEZ-VILLAVERDE, GALO NUÑO and PETER PAZ: Navigating by Falling Stars: Monetary Policy with Fiscally Driven Natural Rates.
2440 ALEJANDRO CASADO and DAVID MARTÍNEZ-MIERA: Local lending specialization and monetary policy.
2441 JORGE ABAD, DAVID MARTÍNEZ-MIERA and JAVIER SUÁREZ: A macroeconomic model of banks' systemic risk taking.
2442 JOSEP PIJOAN-MAS and PAU ROLDAN-BLANCO: Dual labor markets and the equilibrium distribution of firms.
2443 OLYMPIA BOVER, LAURA HOSPIDO and ANA LAMO: Gender and Career Progression: Evidence from the Banco de España.
2444 JESÚS FERNÁNDEZ-VILLAVERDE, GALO NUÑO and JESSE PERLA: Taming the curse of dimensionality: quantitative economics with deep learning.
2445 CLODOMIRO FERREIRA and STEFANO PICA: Households' subjective expectations: disagreement, common drivers and reaction to monetary policy.
2446 ISABEL MICÓ-MILLÁN: Inheritance Tax Avoidance Through the Family Firm.
2447 MIKEL BEDAYO, EVA VALDEOLIVAS and CARLOS PÉREZ: The stabilizing role of local claims in local currency on the variation of foreign claims.
2501 HENRIQUE S. BASSO, MYROSLAV PIDKUYKO and OMAR RACHEDI: Opening the black box: aggregate implications of public investment heterogeneity.
2502 MARCO BARDOSCIA, ADRIAN CARRO, MARC HINTERSCHWEIGER, MAURO NAPOLETANO, LILIT POPOYAN, ANDREA ROVENTINI and ARZU ULUC: The impact of prudential regulations on the UK housing market and economy: insights from an agent-based model.
2503 IRINA BALTEANU, KATJA SCHMIDT and FRANCESCA VIANI: Sourcing all the eggs from one basket: trade dependencies and import prices.
2504 RUBÉN VEIGA DUARTE, SAMUEL HURTADO, PABLO A. AGUILAR GARCÍA, JAVIER QUINTANA GONZÁLEZ and CAROLINA MENÉNDEZ ÁLVAREZ: CATALIST: A new, bigger, better model for evaluating climate change transition risks at Banco de España.
2505 PILAR GARCÍA and DIEGO TORRES: Perceiving central bank communications through press coverage.
2506 MAR DELGADO-TÉLLEZ, JAVIER QUINTANA and DANIEL SANTABÁRBARA: Carbon pricing, border adjustment and renewable energy investment: a network approach.
2507 MARTA GARCÍA RODRÍGUEZ: The role of wage expectations in the labor market.
2508 REBECA ANGUREN, GABRIEL JIMÉNEZ and JOSÉ-LUIS PEYDRÓ: Bank capital requirements and risk-taking: evidence from Basel III.
2509 JORGE E. GALÁN: Macroprudential policy and the tail risk of credit growth.

2510 PETER KARADI, ANTON NAKOV, GALO NUÑO, ERNESTO PASTÉN and DOMINIK THALER: Strike while the Iron is Hot: Optimal Monetary Policy with a Nonlinear Phillips Curve.

2511 MATTEO MOGLIANI and FLORENS ODENDAHL: Density forecast transformations.

2512 LUCÍA LÓPEZ, FLORENS ODENDAHL, SUSANA PÁRRAGA and EDGAR SILGADO-GÓMEZ: The pass-through to inflation of gas price shocks.

2513 CARMEN BROTO and OLIVIER HUBERT: Desertification in Spain: Is there any impact on credit to firms?

2514 ANDRÉS ALONSO-ROBISCO, JOSÉ MANUEL CARBÓ, PEDRO JESÚS CUADROS-SOLAS and JARA QUINTANERO: The effects of open banking on fintech providers: evidence using microdata from Spain.

2515 RODOLFO G. CAMPOS and JACOPO TIMINI: Trade bloc enlargement when many countries join at once.

2516 CORINNA GHIRELLI, JAVIER J. PÉREZ and DANIEL SANTABÁRBARA: Inflation and growth forecast errors and the sacrifice ratio of monetary policy in the euro area.

2517 KOSUKE AOKI, ENRIC MARTORELL and KALIN NIKOLOV: Monetary policy, bank leverage and systemic risk-taking.

2518 RICARDO BARAHONA: Index fund flows and fund distribution channels.

2519 ALVARO FERNÁNDEZ-GALLARDO, SIMON LLOYD and ED MANUEL: The Transmission of Macroprudential Policy in the Tails: Evidence from a Narrative Approach.

2520 ALICIA AGUILAR: Beyond fragmentation: unraveling the drivers of yield divergence in the euro area.

2521 RUBÉN DOMÍNGUEZ-DÍAZ and DONGHAI ZHANG: The macroeconomic effects of unemployment insurance extensions: A policy rule-based identification approach.

2522 IRMA ALONSO-ALVAREZ, MARINA DIAKONOVA and JAVIER J. PÉREZ: Rethinking GPR: The sources of geopolitical risk.

2523 ALBERTO MARTÍN, SERGIO MAYORDOMO and VICTORIA VANASCO: Banks vs. Firms: Who Benefits from Credit Guarantees?

2524 SUMIT AGARWAL, SERGIO MAYORDOMO, MARÍA RODRÍGUEZ-MORENO and EMANUELE TARANTINO: Household Heterogeneity and the Lending Channel of Monetary Policy.

2525 DIEGO BONELLI, BERARDINO PALAZZO, and RAM YAMARTHY: Good inflation, bad inflation: implications for risky asset prices.

2526 STÉPHANE BONHOMME and ANGELA DENIS: Fixed Effects and Beyond. Bias Reduction, Groups, Shrinkage and Factors in Panel Data.

2527 ÁLVARO FERNÁNDEZ-GALLARDO and IVÁN PAYÁ: Public debt burden and crisis severity.

2528 GALO NUÑO: Three Theories of Natural Rate Dynamics.

2529 GALO NUÑO, PHILIPP RENNER and SIMON SCHEIDEGGER: Monetary policy with persistent supply shocks.

2530 MIGUEL ACOSTA-HENAO, MARÍA ALEJANDRA AMADO, MONTSERRAT MARTÍ and DAVID PÉREZ-REYNA: Heterogeneous UIPDs across Firms: Spillovers from U.S. Monetary Policy Shocks.

2531 LUIS HERRERA and JESÚS VÁZQUEZ: Learning from news.

2532 MORTEZA GHOMI, JOCHEN MANKART, RIGAS OIKONOMOU and ROMANOS PRIFTIS: Debt maturity and government spending multipliers.

2533 MARINA DIAKONOVA, CORINNA GHIRELLI and JAVIER J. PÉREZ: Political polarization in Europe.

2534 NICOLÁS FORTEZA and SERGIO PUENTE: Measuring non-workers' labor market attachment with machine learning.

2535 GERGELY GANICS and LLUC PUIG CODINA: Simple Tests for the Correct Specification of Conditional Predictive Densities.

2536 HENRIQUE S. BASSO and OMAR RACHEDI: Robot adoption and inflation dynamics.

2537 PABLO GARCIA, PASCAL JACQUINOT, ČRT LENARČIČ, KOSTAS MAVROMATIS, NIKI PAPADOPOULOU and EDGAR SILGADO-GÓMEZ: Green transition in the Euro area: domestic and global factors.

2538 MARÍA ALEJANDRA AMADO, CARLOS BURGA and JOSÉ E. GUTIÉRREZ: Cross-border spillovers of bank regulations: Evidence of a trade channel.

2539 ALEJANDRO CASADO and DAVID MARTÍNEZ-MIERA: Banks' specialization and private information.

2540 CHRISTIAN E. CASTRO, ÁNGEL ESTRADA GARCÍA and GONZALO FERNÁNDEZ DIONIS: Diversifying sovereign risk in the Euro area: empirical analysis of different policy proposals.

2541 RAFAEL GUNTIN and FEDERICO KOCHEN: The Origins of Top Firms.

2542 ÁLVARO FERNÁNDEZ-GALLARDO: Natural disasters, economic activity, and property insurance: evidence from weekly U.S. state-level data.

2543 JOSÉ ELÍAS GALLEGOS, ESTEBAN GARCÍA-MIRALLES, IVÁN KATARYNIUK and SUSANA PÁRRAGA RODRÍGUEZ: Fiscal Announcements and Households' Beliefs: Evidence from the Euro Area.

2544 LUIS HERRERA, MARA PIROVANO and VALERIO SCALONE: From risk to buffer: Calibrating the positive neutral CCyB rate.

2545 ESTEBAN GARCÍA-MIRALLES et al.: Fiscal drag in theory and in practice: A European perspective.

2546 TATSURO SENGA and IACOPO VAROTTO: Investment Irreversibility in a Granular World.