

MEASURING EURO-AREA INFLATION
RISK WITH OPTIONS: MARKET-IMPLIED
DISTRIBUTION AND BALANCE OF RISKS

2026

BANCO DE **ESPAÑA**
Eurosistema

Documentos Ocasionales
N.º 2625

Diego Bonelli

MEASURING EURO-AREA INFLATION RISK WITH OPTIONS: MARKET-IMPLIED DISTRIBUTION AND BALANCE OF RISKS

Diego Bonelli (*)

BANCO DE ESPAÑA

(*) diego.bonelli@bde.es

Documentos Ocasionales. N.º 2625

October 2026

<https://doi.org/10.53479/44845>

The Occasional Paper Series seeks to disseminate work conducted at the Banco de España in the performance of its functions that may be of general interest.

The opinions and analyses in the Occasional Paper Series are the responsibility of the authors and, therefore, do not necessarily coincide with those of the Banco de España or the Eurosystem.

The Banco de España disseminates its main reports and most of its publications online at: <https://www.bde.es>.

Reproduction for educational and non-commercial purposes is permitted provided that the source is acknowledged.

© BANCO DE ESPAÑA, Madrid, 2026

ISSN: 1696-2230 (online edition)

Abstract

We recover risk-neutral spot distributions of euro-area inflation from caps and floors using two non-normal models – the parametric SABR smile and a nonparametric estimator – benchmarked against a normal model. We construct forward distributions from the spot marginals using a copula. Both non-normal models imply skewed and fat-tailed distributions, while the normal benchmark assigns less probability to extreme outcomes. We summarise the distributions with a balance-of-risks index, which varies sharply over short horizons but shows more limited movement over longer horizons. On matched dates, the option-implied balance of risks is generally lower than the corresponding measure reported by professional forecasters.

Keywords: inflation options, risk-neutral density, SABR, balance of risks, tail risk, euro area.

JEL classification: E31, E44, G13, C58.

Resumen

Este documento obtiene distribuciones de probabilidad neutrales al riesgo de la inflación del área del euro a partir de los precios de opciones sobre inflación. Para ello, se comparan dos enfoques que permiten capturar las asimetrías y los riesgos extremos en la distribución de las expectativas de mercado, un modelo paramétrico basado en SABR y un método no paramétrico, frente a un modelo normal utilizado como referencia. A partir de estas distribuciones se construyen las distribuciones a plazo que permiten analizar cómo evolucionan las percepciones de riesgo a distintos horizontes temporales. Los resultados muestran que los enfoques no normales asignan una probabilidad significativamente mayor a los escenarios extremos de inflación. Asimismo, se propone un indicador sintético de balance de riesgos que resume la información contenida en las distribuciones y cuya evolución revela cambios importantes en la valoración de los riesgos inflacionistas, especialmente a corto plazo. En comparación con las previsiones de los analistas profesionales, las distribuciones implícitas en los mercados tienden a reflejar, de promedio, una valoración menos elevada de dichos riesgos.

Palabras clave: opciones de inflación, densidad neutral al riesgo, modelo SABR, balance de riesgos, riesgo de cola, área del euro.

Códigos JEL: E31, E44, G13, C58.

Contents

Abstract 5

Resumen 6

1 Introduction 8

2 Data and Empirical Design 11

3 From Option Prices to Inflation Distributions 13

3.1 Spot models: the SABR smile and the normal benchmark 14

3.2 Risk-neutral densities 16

3.3 A nonparametric alternative 17

3.4 Forward distributions via copulas 18

4 The Market-Implied Distribution of Euro-Area Inflation 20

4.1 Densities and moments 20

4.2 Location, shape, and the balance of risks 23

4.3 Comparing the two non-normal models 24

4.4 The term structure of the balance of risks 24

4.5 Market versus survey balance of risks 26

5 Robustness 27

6 Conclusion 30

References 31

Appendices 32

A SABR Calibration and Density Extraction: Details 32

B The Nonparametric Estimator: Details 33

1 Introduction

Assessments of the inflation outlook typically combine model forecasts, survey expectations, and market-based indicators. These measures provide information about the expected path of inflation, but describe the distribution around that path only imperfectly. Inflation options add a different margin of information because prices across strikes reflect how markets value upside and downside outcomes at different horizons. They can therefore be used to recover risk-neutral spot distributions and to track changes in asymmetry, tail risk, and the balance of risks over time.

This paper documents a framework for doing so with euro-area inflation options and for constructing forward distributions from the estimated spot marginals. We recover risk-neutral inflation distributions using two alternative approaches: a parametric SABR specification and a nonparametric estimator. The paper focuses on the measurement of inflation risk, providing a systematic comparison of alternative extraction methods and assessing the robustness of the resulting distributions and risk measures. We show that SABR-based estimates provide a more stable representation of the inflation-option smile through time while preserving the key features of the underlying distributions, making them especially suitable for the construction and monitoring of continuous inflation-risk indicators.

The framework is intended to complement, rather than replace, existing measures of the inflation outlook. Point forecasts and survey medians summarise central expectations, while inflation-swap rates provide a market-based measure of inflation compensation. Neither describes how probability mass is distributed around the centre. Option prices do, although the resulting distributions combine beliefs about future inflation with compensation for inflation risk. They should therefore be interpreted as measures of priced inflation risk rather than as direct estimates of the physical distribution of inflation.

The recovery of risk-neutral distributions from option prices follows a well-established no-arbitrage result. As shown by Breeden and Litzenberger (1978), the density can be obtained from the second derivative of the option-price schedule with respect to the strike. Subsequent work develops non- and semi-parametric implementations and shows that the quality of the recovered density depends on the treatment of no-arbitrage restrictions, interpolation, and the tails of the price schedule (Aït-Sahalia and Lo, 2000; Bliss and Panigirtzoglou, 2004; Jackwerth, 2000). In inflation markets, Kitsul and Wright (2013) show that caps and floors contain information about the distribution of future inflation that is not captured by surveys or swaps and that has predictive content for realised outcomes. Related studies examine the pricing of deflation and other tail events in inflation derivatives (Fleckenstein, Longstaff, and Lustig, 2014; Mercurio, 2005; Gimeno and Ibáñez, 2018; Ghaderi, Seo, and

Shaliastovich, 2025), as well as the use of inflation-option-implied risk-neutral densities to assess risks to price stability and their propagation across the term structure of inflation expectations (Garcia, Gimeno, Hinds, Su, and Tretyakov, 2026a,b). While this paper is closest to the latter literature in its use of inflation-option-implied distributions, it differs in its primary objective. Rather than focusing on the information content of inflation risks and their propagation across horizons, we focus on the measurement of those risks by comparing alternative procedures for recovering spot and forward inflation distributions and assessing how the choice of methodology affects the resulting risk measures.

This distributional perspective is closely related to the “inflation-at-risk” approach, which considers the likelihood and severity of outcomes above and below the central projection (López-Salido and Loria, 2024; Adrian, Boyarchenko, and Giannone, 2019). Tail probabilities, skewness, and kurtosis describe different features of the distribution, while the balance of risks provides a compact summary of the relative weight placed on upside and downside outcomes. The horizon also matters. Near-term distributions respond to current inflation developments and short-run uncertainty, whereas forward distributions such as two-years-in-two-years (2y2y) and five-years-in-five-years (5y5y) provide information about risks priced beyond the immediate outlook. These horizons are commonly used when assessing medium-term inflation expectations and the extent to which they remain anchored (Grishchenko, Mouabbi, and Renne, 2019).

Recovering these measures requires a choice of how to represent the option-price schedule. A normal benchmark with one volatility parameter per maturity is straightforward to estimate and maintain, but it imposes symmetry and thin tails. Because a single volatility must account for prices at all strikes, it cannot reproduce the skew and curvature observed in an implied-volatility smile. The resulting density may therefore not fully capture the balance between upside and downside outcomes, particularly in the tails of the distribution.

We consider two alternatives that allow for non-normality. The first is the SABR model of Hagan, Kumar, Lesniewski, and Woodward (2014), which introduces stochastic volatility and parameters governing the skew and curvature of the smile. SABR is widely used in fixed-income and inflation markets because it can fit non-flat smiles while remaining tractable for regular recalibration. The second is a nonparametric estimator that imposes monotonicity and convexity restrictions directly on the option-price schedule and recovers the density from its curvature, following Aït-Sahalia and Duarte (2003) and, for inflation options, Ghaderi et al. (2025). The two approaches differ in how much structure they impose: SABR restricts the smile to a parametric form, whereas the nonparametric estimator allows the data to determine its shape subject to these restrictions. Comparing them helps assess whether the main features of the recovered distribution are common across methods or depend on a

particular specification.

We apply these methods to daily euro-area inflation cap and floor quotes. At spot horizons, we estimate risk-neutral densities using SABR, the nonparametric estimator, and a normal benchmark fitted to the same quotes. At the forward horizons, we construct distributions from the corresponding spot marginals using a common copula approach in the spirit of Nelsen (2006). The forward distributions combine option-implied marginals with an estimated dependence structure and should therefore be interpreted as option-based constructions rather than as joint risk-neutral distributions identified directly from option prices. Using the same underlying quotes and forward construction across models keeps the comparison focused on the treatment of the option smile rather than on differences in data or implementation.

For each distribution, we calculate standard shape measures and a balance-of-risks indicator. The latter is defined as the probability that inflation exceeds the 2% target minus the probability that it falls below it. The measure is positive when the priced distribution places more weight above the target and negative when it places more weight below. It does not distinguish whether an imbalance reflects the location, dispersion, or higher-order shape of the distribution, but it provides a consistent summary that can be compared across dates, horizons, and extraction methods.

The results point to three broad findings. First, the option-implied distributions depart from normality under both non-normal methods. Excess kurtosis is positive at all horizons, while several medium-term horizons are right-skewed. Relative to the two non-normal estimates, the normal benchmark assigns less probability to upper-tail outcomes in the sample, with larger differences at forward horizons and during periods of elevated volatility. The parametric and nonparametric estimates are close in their main distributional features: the mean absolute difference in excess kurtosis across horizons is 0.54. Their similarity suggests that the departures from normality are present in the option-price data rather than being generated by one particular method. The methods differ more markedly in their degree of fit and smoothness. In sparse and illiquid cross-sections, the additional flexibility of nonparametric estimators may amplify idiosyncratic quote noise into apparently meaningful distributional features.

Second, the balance of risks is more responsive at short horizons than at longer forward horizons. At the one-year horizon, it moves from -60.8 percentage points during 2009–2020 to 40.1 during the 2021–22 inflation surge, a change of roughly one hundred percentage points. At the 5y5y horizon, it moves from -15.3 to 2.4. The smaller shift at the longer horizon indicates that the priced distribution of inflation beyond the near term changed less over this period. This pattern is consistent with more limited repricing of longer-horizon inflation

risk, although the option-based nature of the measure prevents a separate identification of expectations and risk premia.

Third, on matched dates, the option-implied balance of risks is generally lower than the corresponding measure reported by professional forecasters. This difference may arise because market prices include risk compensation, because market participants and forecasters use different information sets, because the survey and option horizons are not matched exactly, or because the two measures assign different probabilities to tail outcomes. The comparison does not distinguish among these channels, but it shows that survey- and option-based measures describe different aspects of the inflation outlook.

The paper focuses on measurement, documenting the construction, properties, and interpretation of the indicators rather than offering a structural or causal explanation of their movements. In the spirit of Coibion and Gorodnichenko (2015), it treats expectations data as objects that require careful measurement and interpretation. The resulting indicators provide an additional market-based view of the distribution of inflation risks and can be used alongside survey- and model-based measures. Section 2 describes the data; Section 3 presents the models and estimation methods; Section 4 reports the results; Section 5 presents the robustness checks; and Section 6 concludes.

2 Data and Empirical Design

Instruments. The raw data consist of daily mid-market premium quotes for euro-area zero-coupon inflation caps and floors from Bloomberg. The contracts are written on the Eurostat HICP excluding tobacco and are quoted at annual maturities from one to twenty years, with strikes ranging approximately from 0% to 6%. We report distributions at the 1-, 2-, 5-, and 10-year spot horizons, while retaining the intermediate maturities required for the forward construction in Section 3.4. The 2y2y forward, for example, is constructed from the two- and four-year marginals.

A zero-coupon inflation cap of maturity T is a European option on cumulative inflation, measured by the price-index ratio $G_T = I(T)/I(0)$, rather than on inflation realised in any single year. Its payoff depends on the difference between realised cumulative inflation and the annualised strike compounded to maturity. At each maturity, the cross-section of cap and floor prices therefore provides call and put schedules on cumulative inflation from which the risk-neutral density can be recovered.

The forward inflation rate at each maturity is proxied by the corresponding zero-coupon inflation-linked swap (ILS) rate from Bloomberg, and cash flows are discounted using the overnight-index-swap (OIS) curve. The sample runs from 5 October 2009 to 30 June 2026

and contains 4367 trading days.

Estimation design. We estimate each specification separately for every date–maturity pair. The resulting spot distributions are therefore daily objects and do not use information from adjacent dates. For each cross-section, we calibrate the SABR model, estimate the nonparametric specification, and fit the normal benchmark using the same underlying quotes. The resulting spot marginals are then passed through a common copula construction at the forward horizons and evaluated using the same set of distributional summaries. This design keeps the comparison focused on the treatment of the option-price cross-section rather than on differences in the data or forward-horizon construction.

Before estimation, we remove quotes recorded as missing or as placeholders and impose monotonicity and convexity restrictions on the call-price schedule. We retain a date–maturity cross-section when it contains enough quoted strikes to identify the smile and produces a well-behaved density. After these filters, the sample contains a broad daily panel of fitted spot distributions across the reported maturities, together with the three option-based forward distributions constructed from them.

Surveys. For the descriptive comparison between market- and survey-based distributions in Section 4, we use the euro-area forecasts collected in the ECB Survey of Professional Forecasters (SPF). The SPF reports aggregate probabilities over a set of inflation intervals. For the 1- and 2-year comparisons, we use the fixed-calendar-year distribution whose effective horizon is closest to the relevant option horizon. For the 5-year comparison, we use the SPF long-term distribution.

We calculate the survey balance of risks as the probability of inflation above 2% minus the probability below 2%. When an SPF interval contains the 2% threshold, which occurs because the survey grid changes over the sample, we allocate probability uniformly within that interval and split its mass at the threshold. This adjustment uses only the reported bucket probability and does not smooth across intervals. Each survey release is matched to the nearest trading day and compared with the option-implied distribution at the closest available horizon.

The horizon match is approximate rather than exact. The one- and two-year survey distributions refer to fixed calendar years, so their effective horizons can differ from the corresponding option horizons by up to about one year. The five-year comparison also pairs different objects: the SPF long-term distribution refers to inflation around five years ahead, whereas the option-implied distribution refers to average inflation over the coming five years. The comparison is therefore descriptive, and differences in reference periods may contribute

to the gap between the two measures.

3 From Option Prices to Inflation Distributions

This section describes how observed cap and floor quotes are converted into estimated inflation distributions. We first define the inflation variables and the displacement used to keep the inputs to the SABR and Black formulas positive. We then present the three spot specifications: the SABR smile and a nonparametric estimator, which allow for non-normality, and a normal benchmark. Finally, we recover risk-neutral spot densities from the fitted price schedules and construct forward distributions using a copula. Comparing the parametric and nonparametric models allows us to assess which features of the priced distribution are common across extraction methods.

Inflation variables and strikes. Work on a filtered probability space and let $\mathbb{Q}^T$ denote the forward pricing measure associated with maturity T . Let $I(t)$ be the price index and define the gross cumulative inflation factor as

$$G_T = \frac{I(T)}{I(0)}.$$

The corresponding T -year annualised inflation rate $\pi_{0,T}$, expressed in percent, satisfies

$$G_T = \left(1 + \frac{\pi_{0,T}}{100}\right)^T.$$

We express the option payoff in terms of cumulative inflation,

$$X_T \equiv 100 (G_T - 1), \tag{1}$$

measured in percentage points. An annualised market strike k therefore corresponds to the unshifted cumulative strike

$$K_T^0(k) = 100 \left[\left(1 + \frac{k}{100}\right)^T - 1 \right]. \tag{2}$$

Let $F_{0,T} = \mathbb{E}^{\mathbb{Q}^T}[X_T]$ denote the time-zero forward value of cumulative inflation. To keep the quantities entering the SABR and Black formulas positive, we apply the same constant

displacement s to the cumulative payoff, forward, and strike:

$$\tilde{X}_T = X_T + s, \quad \tilde{F}_{0,T} = F_{0,T} + s, \quad \tilde{K}_T(k) = K_T^0(k) + s. \quad (3)$$

Because the displacement is applied equally to the payoff and the strike,

$$(X_T - K_T^0(k))^+ = (\tilde{X}_T - \tilde{K}_T(k))^+,$$

so it does not change the economic payoff. The map $k \mapsto \tilde{K}_T(k)$ is strictly increasing. Conservation of probability mass,

$$f_{\pi_{0,T}}(k) dk = f_{\tilde{X}_T}(\tilde{K}_T(k)) d\tilde{K}_T,$$

therefore gives

$$f_{\pi_{0,T}}(k) = f_{\tilde{X}_T}(\tilde{K}_T(k)) \frac{d\tilde{K}_T}{dk}, \quad \frac{d\tilde{K}_T}{dk} = T \left(1 + \frac{k}{100}\right)^{T-1}. \quad (4)$$

We use this transformation to express all reported densities in annualised-inflation space.

3.1 Spot models: the SABR smile and the normal benchmark

SABR. To retain the information contained in prices across strikes, we use the SABR model of Hagan et al. (2014). Under the model, the displaced forward $\tilde{F}_{t,T}$ and its volatility α_t evolve as

$$d\tilde{F}_{t,T} = \alpha_t \tilde{F}_{t,T}^\beta dW_t, \quad d\alpha_t = \nu \alpha_t dZ_t, \quad dW_t dZ_t = \rho dt, \quad (5)$$

where W and Z are Brownian motions under $\mathbb{Q}^T$. The exponent $\beta \in [0, 1]$ governs how volatility varies with the level of the displaced forward, $\nu > 0$ controls volatility of volatility and hence the curvature of the smile, and $\rho \in [-1, 1]$ governs its skew.

Rather than pricing directly under (5), we use the SABR approximation to Black implied volatility. For compactness, let $\tilde{F} \equiv \tilde{F}_{0,T}$ and $\tilde{K} \equiv \tilde{K}_T(k)$. The approximation is

$$\begin{aligned} \sigma(\tilde{F}, \tilde{K}) &= \frac{\alpha}{(\tilde{F}\tilde{K})^{(1-\beta)/2} \left[1 + \frac{(1-\beta)^2}{24} \log^2\left(\frac{\tilde{F}}{\tilde{K}}\right) + \frac{(1-\beta)^4}{1920} \log^4\left(\frac{\tilde{F}}{\tilde{K}}\right) \right]} \frac{z}{\chi(z)} \\ &\times \left[1 + \left(\frac{(1-\beta)^2 \alpha^2}{24(\tilde{F}\tilde{K})^{1-\beta}} + \frac{\rho\beta\nu\alpha}{4(\tilde{F}\tilde{K})^{(1-\beta)/2}} + \frac{(2-3\rho^2)\nu^2}{24} \right) T \right], \end{aligned} \quad (6)$$

where

$$z = \frac{\nu}{\alpha} (\tilde{F}\tilde{K})^{(1-\beta)/2} \log\left(\frac{\tilde{F}}{\tilde{K}}\right), \quad \chi(z) = \log\left(\frac{\sqrt{1-2\rho z + z^2} + z - \rho}{1-\rho}\right). \quad (7)$$

The ratio $z/\chi(z)$ introduces the dependence of implied volatility on moneyness and converges to one at the money, where $\tilde{F} = \tilde{K}$.

We fix the backbone at $\beta = 0.5$ and estimate the remaining three parameters. In daily cross-sections containing six to ten strikes, β and ρ are weakly identified separately because both affect the fitted skew. Allowing the backbone to vary therefore creates a trade-off with ρ without materially improving the fit. The distributional features considered in the paper — dispersion, skewness, and the tails — are governed mainly by α , ρ , and ν , while alternative fixed values of β change the parameterisation but have limited effects on the fitted smile.

We estimate (α, ρ, ν) by minimising a weighted sum of squared implied-volatility residuals, with a light penalty on volatility of volatility:

$$(\hat{\alpha}, \hat{\rho}, \hat{\nu}) = \arg \min_{\alpha, \rho, \nu} \left\{ \sum_j w_j \left[\sigma(\tilde{F}, \tilde{K}_j; \alpha, \rho, \nu) - \sigma_j^{\text{obs}} \right]^2 + \lambda_\nu \nu^2 \right\}, \quad \lambda_\nu = 0.01. \quad (8)$$

The penalty regularises ν when the smile is sparsely quoted while retaining curvature supported by the cross-section. The weights w_j combine a vega component, a near-the-money proximity profile, and a base component that assigns less influence to synthetic strikes added to stabilise sparsely quoted smiles, as described in Appendix A.

We convert the fitted implied volatilities into a smooth call-price schedule using the Black formula:

$$C(\tilde{K}, T) = D(0, T) \left[\tilde{F} \Phi(d_1) - \tilde{K} \Phi(d_2) \right], \quad d_1 = \frac{\log(\tilde{F}/\tilde{K}) + \frac{1}{2}\hat{\sigma}^2 T}{\hat{\sigma}\sqrt{T}}, \quad d_2 = d_1 - \hat{\sigma}\sqrt{T}, \quad (9)$$

where $D(0, T)$ is the OIS discount factor and

$$\hat{\sigma} = \sigma(\tilde{F}, \tilde{K}; \hat{\alpha}, \hat{\rho}, \hat{\nu}).$$

The fitted price schedule is evaluated on a dense strike grid before the density is recovered.

Normal benchmark. As a reference, we also estimate a normal model in which the displaced cumulative payoff is Gaussian under $\mathbb{Q}^T$:

$$\tilde{X}_T = \tilde{F}_{0,T} + \sigma_B \sqrt{T} Z, \quad Z \sim \mathcal{N}(0, 1), \quad (10)$$

where $\tilde{F}_{0,T} = \mathbb{E}^{\mathbb{Q}^T}[\tilde{X}_T]$ and σ_B is a single volatility parameter for each maturity. We estimate σ_B separately for each date and maturity by minimising squared pricing errors across the quoted strikes. Because one parameter must account for the entire cross-section, the normal model can match the overall level of dispersion but cannot reproduce skewness or excess kurtosis. It therefore imposes a symmetric, thin-tailed density. Comparing this benchmark with the two non-normal specifications shows how the treatment of the volatility smile affects the estimated asymmetry and tail probabilities.

3.2 Risk-neutral densities

The fitted spot models produce call-price schedules from which the risk-neutral density can be recovered. A European call on $\tilde{X}_T$ with strike $\tilde{K}$ has price

$$C(\tilde{K}, T) = D(0, T) \mathbb{E}^{\mathbb{Q}^T} \left[(\tilde{X}_T - \tilde{K})^+ \right].$$

Differentiating twice with respect to the strike gives the Breeden and Litzenberger (1978) identity

$$f_{\tilde{X}_T}^{\mathbb{Q}^T}(\tilde{K}) = \frac{1}{D(0, T)} \frac{\partial^2 C(\tilde{K}, T)}{\partial \tilde{K}^2}, \quad (11)$$

which follows from no-arbitrage and does not require a particular process for inflation.

In implementation, we impose the shape restrictions

$$\frac{\partial C}{\partial \tilde{K}} \leq 0, \quad \frac{\partial^2 C}{\partial \tilde{K}^2} \geq 0,$$

so that the call-price schedule is non-increasing and convex in the strike. For the normal benchmark, we apply finite differences directly to the fitted price schedule. For SABR, where second derivatives can be numerically sensitive, we first approximate the CDF:

$$F_{\tilde{X}_T}^{\mathbb{Q}^T}(\tilde{K}_{i-1/2}) \approx 1 + \frac{1}{D(0, T)} \frac{C(\tilde{K}_i, T) - C(\tilde{K}_{i-1}, T)}{\tilde{K}_i - \tilde{K}_{i-1}}, \quad (12)$$

where $\tilde{K}_{i-1/2}$ denotes a point between the two adjacent strikes. We then difference the resulting CDF to recover the density. This two-step procedure reduces numerical noise relative to differentiating the price schedule twice directly.

The density in displaced cumulative space is mapped into annualised-inflation space using (4):

$$f_{\pi_{0,T}}^{\mathbb{Q}^T}(k) = f_{\tilde{X}_T}^{\mathbb{Q}^T}(\tilde{K}_T(k)) T \left(1 + \frac{k}{100} \right)^{T-1}. \quad (13)$$

The resulting density is normalised to integrate to one.

For reporting, we then align its mean with the corresponding inflation swap. The annualised density is translated rigidly so that its implied mean coincides with the swap-implied mean and is renormalised over the numerical grid. This adjustment corrects a small numerical discrepancy between the mean of the finite-difference density and the swap-implied forward. It changes the location of the distribution but, away from the grid boundaries, leaves centred moments such as skewness and excess kurtosis unchanged. It does, however, affect tail probabilities and the balance of risks defined relative to fixed thresholds. Since the translation is applied after extraction, the mean-aligned density is not constrained to reprice the original option quotes exactly.

3.3 A nonparametric alternative

SABR imposes a three-parameter functional form on the implied-volatility smile. As an alternative representation of the same option prices, we estimate the density nonparametrically, following Aït-Sahalia and Duarte (2003) and its application to inflation options by Ghaderi et al. (2025). The estimator does not impose a parametric functional form on the smile, although it retains shape restrictions that improve numerical stability and keep the recovered density non-negative. Agreement between the two methods therefore provides evidence that the estimated departures from normality are present in the option-price cross-section rather than arising solely from the SABR parameterisation.

The estimator proceeds in two steps. First, we project the observed call prices $\{C(\tilde{K}_j)\}$ onto the set of price schedules that are non-increasing and convex in the strike. Convexity ensures that discrete butterfly spreads are non-negative. The projection solves

$$\min_{\{c_j\}} \sum_j \left[c_j - C(\tilde{K}_j) \right]^2 \quad \text{s.t.} \quad c_{j+1} \leq c_j, \quad \frac{c_{j+1} - c_j}{\tilde{K}_{j+1} - \tilde{K}_j} \geq \frac{c_j - c_{j-1}}{\tilde{K}_j - \tilde{K}_{j-1}}. \quad (14)$$

We solve this constrained least-squares problem as a quadratic program using a Dykstra-type projection onto the intersection of the monotonicity and convexity cones.

Second, we convert the projected prices into implied volatilities, smooth the resulting volatility smile using a local-linear kernel estimator (Bliss and Panigirtzoglou, 2004), and reprice the options on a dense strike grid. Smoothing the implied-volatility schedule before repricing is more stable than differentiating the raw option prices twice. Since repricing may reintroduce small local violations of convexity in the wings, we impose monotonicity and convexity again on the dense price schedule using isotonic regression on its slopes.

We recover the density from the final price curve using (11) and map it into annualised-

inflation space using (13). Since the final call-price schedule is convex, the resulting density is non-negative before the final smoothing step. We then impose a single mode and apply light smoothing to remove small numerical steps left by the convexification, as described in Appendix B. The nonparametric results are reported only over the observed strike support; any small numerical extension used during smoothing is not retained in the reported density. SABR, by contrast, is evaluated on a wider grid and therefore includes extrapolated tail mass. Section 4.3 compares the two non-normal specifications directly.

3.4 Forward distributions via copulas

Forward inflation rates such as 2y2y and 5y5y cannot be obtained directly from a single spot option-price cross-section because they depend on inflation realised between two future dates. We construct their distributions by coupling overlapping spot marginals.

For $0 < T_1 < T_2$, let

$$R_1 = \frac{I(T_1)}{I(0)}, \quad R_2 = \frac{I(T_2)}{I(0)}$$

be the gross cumulative inflation factors. Their marginal distribution functions F_1 and F_2 , and densities f_1 and f_2 , are obtained from the option-implied annualised-inflation distributions using

$$R_i = \left(1 + \frac{\pi_{0,T_i}}{100}\right)^{T_i}.$$

Each marginal is recovered under the forward pricing measure associated with its own maturity. We use the two extracted marginals as inputs to an option-based forward construction rather than treating them as marginals of a joint risk-neutral distribution identified directly from option prices.

The gross forward inflation factor over $[T_1, T_2]$ is

$$Y = \frac{R_2}{R_1} = \frac{I(T_2)}{I(T_1)}. \quad (15)$$

The distribution of Y depends on both spot marginals and their dependence. A chosen copula C couples the two marginal distributions and defines the constructed joint CDF

$$F_{R_1, R_2}(r_1, r_2) = C(F_1(r_1), F_2(r_2)), \quad (16)$$

with joint density

$$f_{R_1, R_2}(r_1, r_2) = c(F_1(r_1), F_2(r_2))f_1(r_1)f_2(r_2), \quad (17)$$

where c is the copula density.

Under the transformation

$$(r_1, r_2) \mapsto (r_1, y), \quad r_2 = yr_1,$$

the Jacobian is $|r_1|$. Since gross inflation factors are positive, $|r_1| = r_1$, and the density of the gross forward factor is

$$q_Y(y) = \int_0^\infty r_1 c(F_1(r_1), F_2(yr_1)) f_1(r_1) f_2(yr_1) dr_1. \quad (18)$$

We evaluate this integral numerically on the R_1 grid. The marginals f_1 and f_2 determine the probability mass at each spot horizon, while the copula determines how outcomes at the two horizons are associated.

We use a bivariate Student- t copula:

$$c(u_1, u_2; \rho_c, \eta) = \frac{f_t^{(2)}(q_1, q_2; \rho_c, \eta)}{f_t^{(1)}(q_1; \eta) f_t^{(1)}(q_2; \eta)}, \quad q_i = t_\eta^{-1}(u_i), \quad (19)$$

with $\eta = 4$ degrees of freedom. Relative to a Gaussian copula, the Student- t specification allows for dependence in the joint tails (Nelsen, 2006).¹

We use the rolling correlation of daily inflation-swap rate changes at the two relevant maturities as a calibration proxy for the dependence parameter ρ_c . It should not be interpreted as an estimate of a latent risk-neutral copula parameter. The resulting forward distribution is therefore a hybrid object: its marginals are recovered from option prices under maturity-specific pricing measures, whereas dependence across horizons is calibrated from historical swap-rate movements. We interpret it as an option-based forward distribution rather than as a joint risk-neutral distribution identified entirely from option prices.

Let

$$\Delta = T_2 - T_1.$$

The annualised forward inflation rate over $[T_1, T_2]$ is

$$\pi_{T_1, T_2} = 100 (Y^{1/\Delta} - 1). \quad (20)$$

Equivalently,

$$Y = \left(1 + \frac{\pi_{T_1, T_2}}{100}\right)^\Delta.$$

¹For numerical stability, we bound the copula arguments away from zero and one.

The density of annualised forward inflation is therefore

$$f_{\pi_{T_1, T_2}}(p) = q_Y \left[\left(1 + \frac{p}{100} \right)^\Delta \right] \frac{\Delta}{100} \left(1 + \frac{p}{100} \right)^{\Delta-1}. \quad (21)$$

We construct the 1y1y forward from the 1Y and 2Y marginals, the 2y2y forward from the 2Y and 4Y marginals, and the 5y5y forward from the 5Y and 10Y marginals. We then align the mean of each annualised forward density with the corresponding forward inflation swap.²

Distributional summaries. From each daily density $f_{t,T}(\pi)$, we compute the mean, standard deviation, skewness, and excess kurtosis by numerical integration. We also calculate the tail probabilities

$$P_{t,T}(\pi < 1\%), \quad P_{t,T}(\pi > 3\%),$$

and the *balance of risks* relative to the target:

$$\text{BoR}_{t,T} = P_{t,T}(\pi > 2\%) - P_{t,T}(\pi < 2\%). \quad (22)$$

The measure is positive when the distribution places more probability above 2% than below it and negative when the reverse is true. We apply the same definitions to the SABR and nonparametric densities, the normal benchmark, the option-based forward distributions, and the survey distributions so that the comparisons use a common set of statistics.

4 The Market-Implied Distribution of Euro-Area Inflation

4.1 Densities and moments

The two extraction methods fit the same option quotes while imposing different amounts of structure on the volatility smile. Figure 1 illustrates the fitted smiles at the two- and five-year horizons on a date with a relatively rich cross-section of quotes. SABR restricts the smile to a smooth three-parameter form, whereas the nonparametric estimator follows the quoted volatilities more closely subject to monotonicity and convexity restrictions. Both provide a close representation of the observed cross-section but differ in the degree of smoothing

²This adjustment changes the location of the distribution but does not mechanically determine centred shape statistics such as skewness or excess kurtosis. It does, however, affect tail probabilities and the balance of risks defined relative to fixed thresholds.

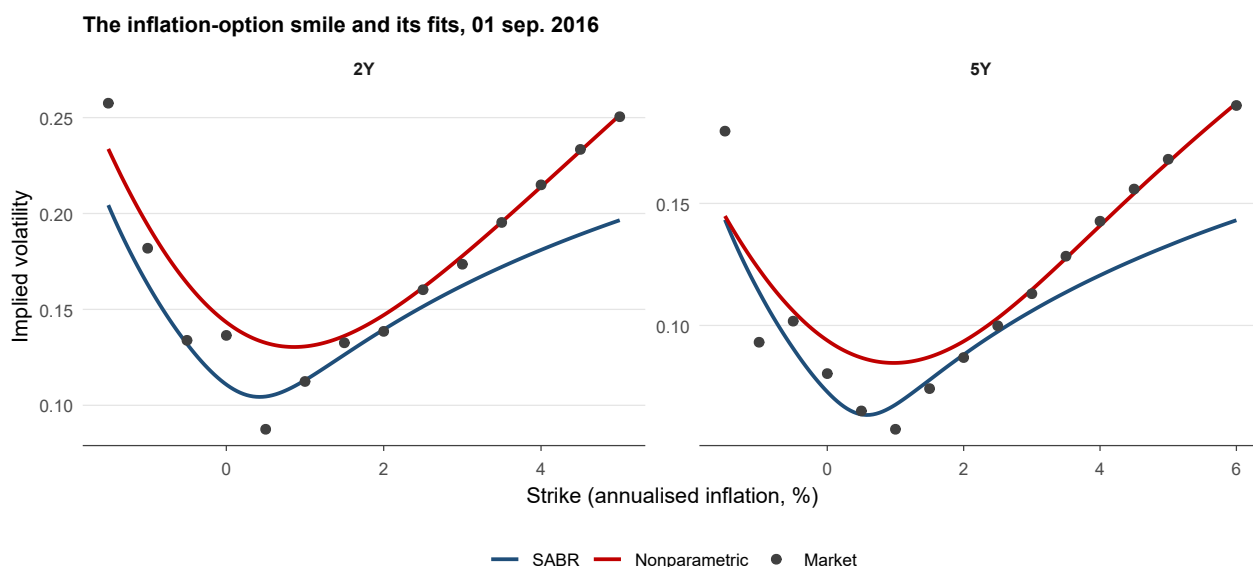


Figure 1: The euro-area inflation-option volatility smile and its fits on a richly quoted date, at the 2Y and 5Y horizons: market quotes (points), the SABR parametric fit (blue), and the nonparametric fit (red). The nonparametric model smooths the quoted volatilities subject to monotonicity and convexity restrictions, whereas SABR imposes a three-parameter functional form.

Figure 2 shows the corresponding SABR, nonparametric, and normal distributions at four horizons. The two non-normal distributions are close at each horizon, indicating that their main features are not specific to one extraction method. Both place more mass in the tails than the normal benchmark, with the difference most visible at the five-year and 5y5y horizons, where the normal distribution is more concentrated around its centre. The comparison therefore suggests that the normal specification provides a limited representation of the shape implied by prices across strikes.

The moments obtained from the two non-normal methods are also similar. Both distributions are right-skewed at the medium horizons and closer to symmetric at the shortest and longest horizons, while excess kurtosis is positive throughout. Skewness has the same sign under both methods at every horizon, and the mean absolute difference in excess kurtosis across horizons is 0.54. The departures from normality therefore appear to reflect the option-price cross-section rather than being specific to the SABR functional form. The probability of inflation below 1% is larger at short horizons and declines with maturity, whereas the probability above 3% remains between approximately one tenth and one fifth. The option-based distributions thus assign non-negligible mass to high-inflation outcomes even when their central tendency lies closer to the target.

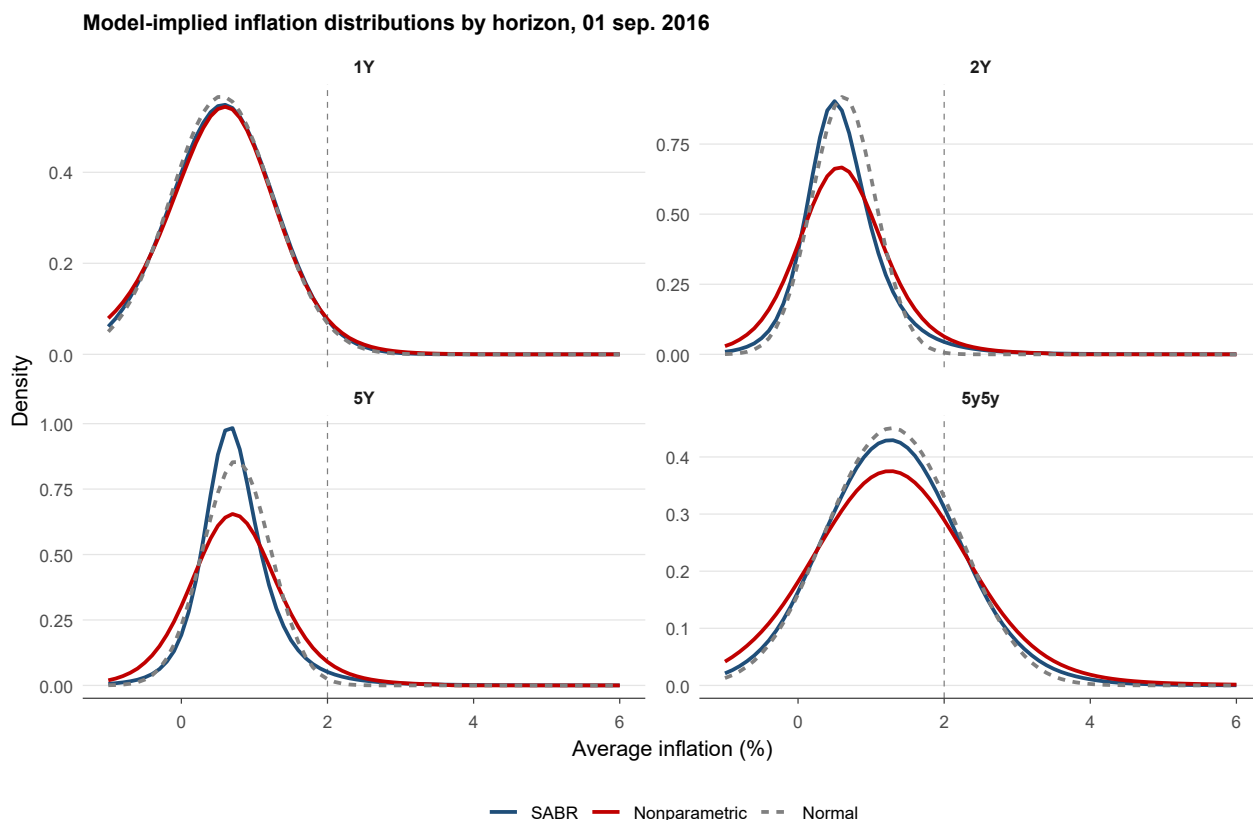


Figure 2: The three model-implied distributions of euro-area inflation, one panel per horizon, on a representative richly quoted date: SABR (blue), the nonparametric estimator (red), and the normal benchmark (grey, dashed). The 1Y, 2Y, and 5Y panels show risk-neutral spot distributions, while the 5y5y panel shows the option-based forward distribution. The dashed vertical line marks the 2% target. The date is the same as in Figure 1; the distributions are obtained from the fitted smiles shown there and include the mean-alignment step described in Section 3.2.

Figure 3 displays the evolution of the estimated distributions over the sample at the 5Y spot and 5y5y forward horizons. The SABR and nonparametric surfaces remain close over time and both exhibit a right-hand shoulder that is absent from the symmetric normal benchmark.

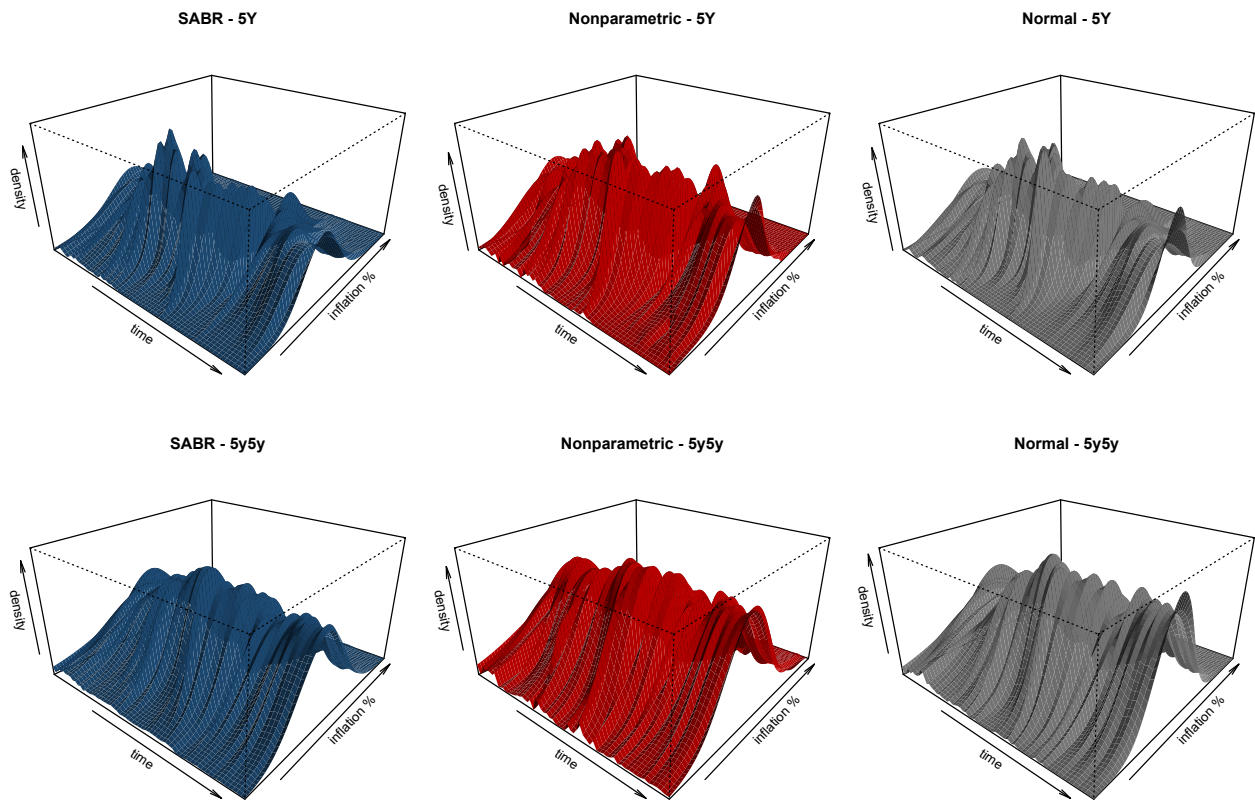


Figure 3: Inflation distributions over time under the three models — SABR (blue), the nonparametric estimator (red), and the normal benchmark (grey) — at the 5Y spot horizon (top row) and the 5y5y forward horizon (bottom row). Each surface shows the daily density over the inflation grid. The 5Y panels are risk-neutral spot distributions, while the 5y5y panels are option-based forward distributions.

4.2 Location, shape, and the balance of risks

A symmetric distribution has a balance of risks equal to zero only when it is centred exactly on the target. Because the normal benchmark is centred on the swap-implied mean, which is generally away from the 2% target, its balance of risks is not zero and moves with the gap between the priced mean and the target in much the same way as the non-normal measures. The balance of risks at a fixed 2% threshold is therefore strongly influenced by the location of the priced distribution, although dispersion and higher-order shape also matter. The

measure,

$$P(\pi > 2\%) - P(\pi < 2\%),$$

varies widely over the sample.

Figure 4 shows the balance of risks by horizon. At short horizons, the measure moves from close to -100 percentage points during the 2015–2020 disinflation period, when the priced distribution was concentrated below the target, to close to $+100$ during the 2021–22 inflation surge. At the 5y5y horizon, it remains much closer to zero. The close correspondence between the SABR and nonparametric series indicates that the measure is not driven by either method alone.

Because the three models are aligned to the same swap-implied mean, differences between the non-normal measures and the normal benchmark do not reflect differences in location. They may, however, reflect both dispersion and higher-order shape. Empirically, the difference is small, at no more than about one and a half percentage points across horizons. The balance of risks therefore mainly summarises where the priced distribution lies relative to the target. Differences between the normal and non-normal models are more visible in excess kurtosis and tail probabilities, for which the normal benchmark assigns less mass to extreme outcomes in the sample.

4.3 Comparing the two non-normal models

The agreement between SABR and the nonparametric estimator on the broad shape of the distribution does not imply that the two methods are interchangeable. Both methods produce skewness with the same sign and positive excess kurtosis, although the nonparametric densities are generally less peaked. SABR, in turn, produces smoother densities with less day-to-day variation. The comparison reflects the different roles of the two specifications: the nonparametric estimator follows each daily cross-section more closely, whereas SABR provides a more parsimonious and stable representation of the smile.

4.4 The term structure of the balance of risks

The balance of risks in (22) becomes less responsive as the horizon increases. At the one-year horizon, it moves from approximately -60.8 percentage points during 2009–2020 to 40.1 points during the 2021–22 inflation surge, a change of roughly one hundred percentage points. At the 5y5y forward horizon, it moves only from -15.3 to 2.4 points. The smaller movement at the longer horizon indicates that the priced distribution of inflation beyond the near term changed less over this episode. This pattern is consistent with more limited

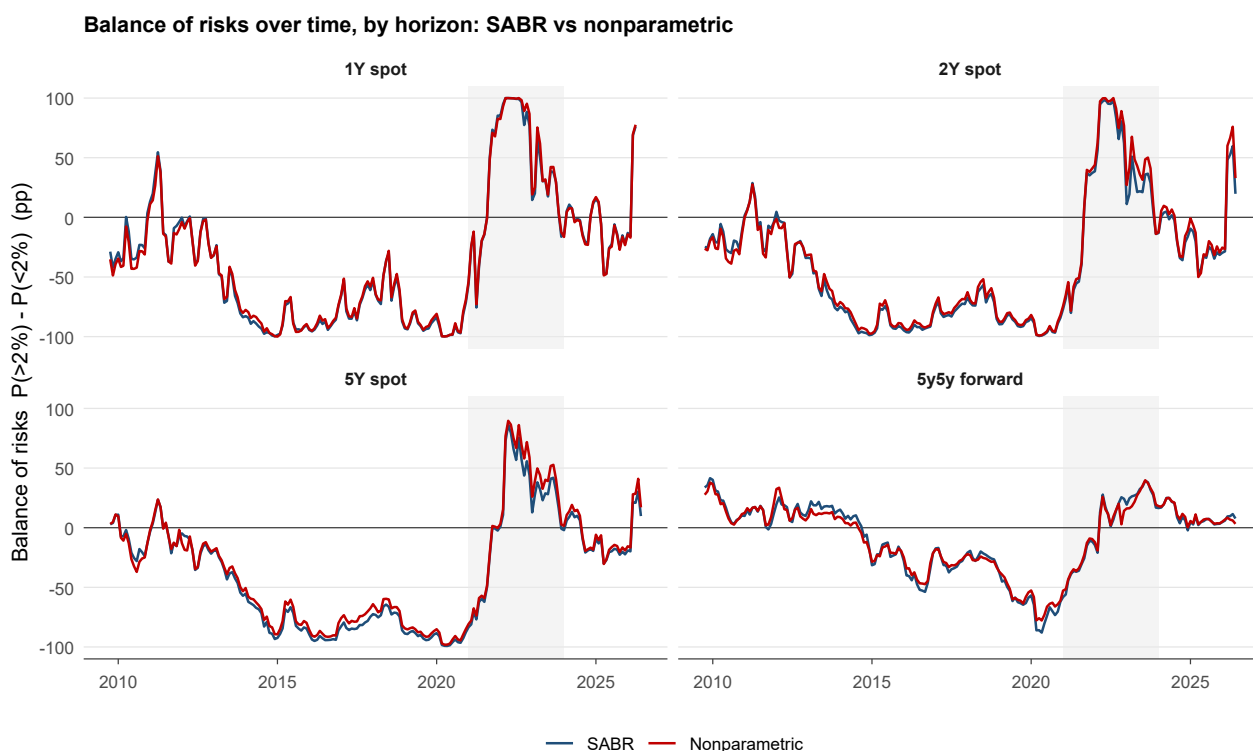


Figure 4: Option-based balance of risks, $P(\pi > 2\%) - P(\pi < 2\%)$, in percentage points over time at four horizons (1Y, 2Y, 5Y spot, and 5y5y forward) under the two non-normal models — SABR (blue) and the nonparametric estimator (red). Positive values indicate more probability above the target than below it, while negative values indicate the reverse; the horizontal line at zero marks a neutral balance. The shaded band marks the 2021–2023 surge.

repricing of longer-horizon inflation risk, but it is not a direct test of the anchoring of physical expectations.

Figure 5 shows the probability mass underlying the balance-of-risks measure. For each horizon and model, it divides the distribution into four regions: below 1%, between 1% and 2%, between 2% and 3%, and above 3%. At the one-year horizon, probability shifts from the lower-inflation regions during the 2015–2020 period to the higher-inflation regions during the 2021–22 surge. At the 5y5y horizon, the allocation changes less and remains concentrated in the 1–3% range. The SABR and nonparametric estimates give a similar account of these movements.

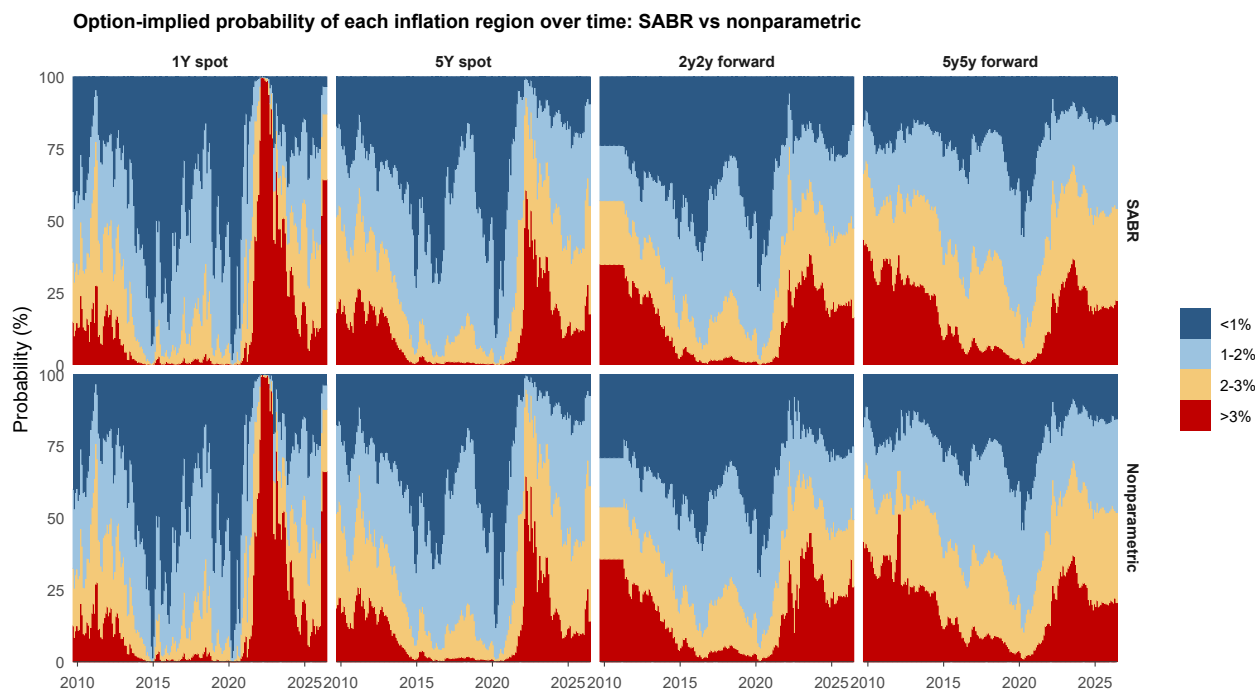


Figure 5: Probability assigned to each inflation region over time (monthly), at four horizons (1Y and 5Y spot, 2y2y and 5y5y forward, columns) under both non-normal models (SABR top row, nonparametric bottom row). Each band is the probability that average inflation falls below 1%, between 1% and 2%, between 2% and 3%, or above 3%.

4.5 Market versus survey balance of risks

Table 1 and Figure 6 compare the option-implied balance of risks with the corresponding measure from the ECB Survey of Professional Forecasters on matched dates. The survey measure is calculated directly from the reported histogram probabilities, with the mass in any bucket containing the 2% target divided uniformly at the threshold. The comparison therefore does not require fitting a continuous density to the survey responses.

At the matched horizons, both non-normal option models generally imply distributions that are more tilted towards outcomes below the target than the survey distributions, and the option-minus-survey difference increases with the horizon. This gap should be interpreted descriptively. The option-implied spot distributions are risk-neutral and therefore combine beliefs about future inflation with compensation for inflation risk, whereas the survey distributions record reported probabilities under the forecasters' own assessments. The comparison does not identify the contribution of risk premia, differences in information sets, differences in beliefs about the tails, or the residual mismatch between the survey and option reference periods.

Table 1: Market versus survey balance of risks by horizon, on matched survey dates, in percentage points. *SABR* and *Nonparam.* are the two non-normal models (SABR parametric and the nonparametric estimator); *Survey* is the ECB SPF, with the balance of risks calculated from the histogram buckets (1Y and 2Y from the calendar-year distribution nearest each horizon, and 5Y from the long-term distribution). $Gap = SABR - Survey$; N is the number of matched releases. A negative entry means that the distribution places more probability below the 2% target than above it.

Horizon	N	Survey	SABR	Nonparam.	Gap
1Y	106	-28.3	-32.2	-36.8	-3.9
2Y	106	-26.1	-34.4	-32.9	-8.3
5Y	103	-15.3	-24.3	-22.0	-9.0

5 Robustness

The results for the tails and the balance of risks are generally insensitive to the main numerical choices used in the extraction. Since the conclusions rely in part on the tails of the estimated distributions, we first examine whether they are driven by the outer edges of the density grid. Recomputing the balance of risks and the tail probabilities on progressively trimmed supports, with the density renormalised after each trim, produces only small changes. At the five-year and 5y5y horizons, the balance of risks changes by only hundredths of a percentage point. The results therefore do not depend materially on probability mass assigned near the grid boundaries.

We next vary the threshold used to define the balance of risks. The baseline measure is centred on the 2% target, but its level necessarily depends on the chosen cutoff. Recomputing the measure at 1.5% and 2.5% changes its level while preserving the ordering across regimes at each threshold. In particular, the balance of risks during the inflation surge remains above

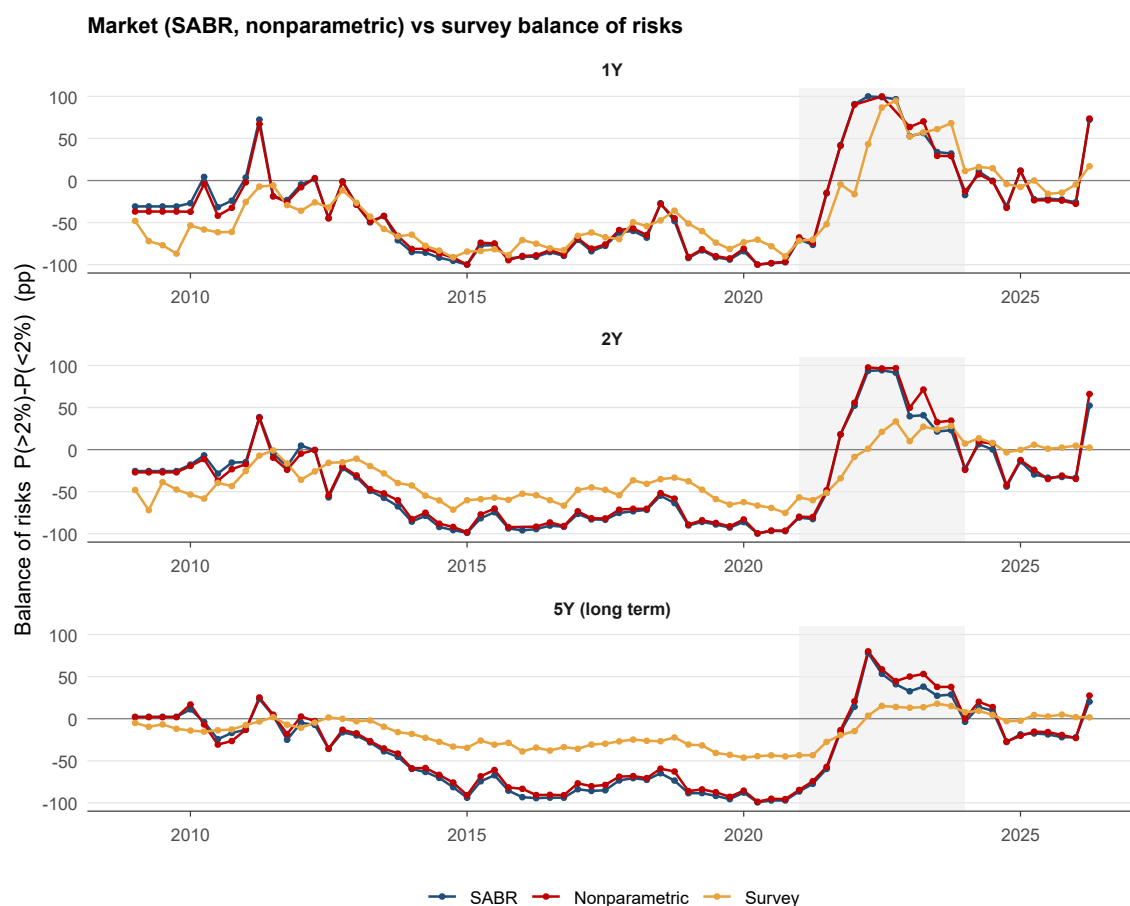


Figure 6: Balance of risks over time by horizon on matched survey dates: SABR (blue) and nonparametric (red) option-implied measures compared with the survey measure (gold). The shaded band marks the 2021–2023 surge. Both non-normal models imply a lower balance of risks than the survey, and the gap widens with the horizon.

its value in the calm period regardless of the cutoff. The difference across regimes is therefore not specific to a threshold of exactly 2%.

The extraction also requires choices for the SABR backbone and the copula specification. We fix the SABR parameter at $\beta = 0.5$. As discussed in Section 3.1, β and ρ are weakly identified separately within a single-date cross-section, while the reported features of the density are governed mainly by α , ρ , and ν . Alternative fixed values of β change the parameterisation but have little effect on the fitted smile or the resulting distributional measures.

The forward distributions are constructed using a Student- t copula with $\eta = 4$ degrees of freedom. The mean-alignment step fixes the location of each forward density and therefore affects statistics defined relative to fixed thresholds, including the balance of risks. The following comparisons should consequently be interpreted conditional on the common anchoring rule. Replacing the Student- t copula with a Gaussian copula, or varying its degrees of freedom between four and eight, changes the mean 5y5y balance of risks by less than one-tenth of a percentage point. Varying the rolling window used to calibrate the dependence parameter between roughly six months and two years changes it by less than half a percentage point. Conditional on the mean-alignment procedure, the 5y5y balance of risks is therefore not sensitive to the main choices governing the copula construction.

Because the nonparametric estimator is reported only over the observed strike support while SABR is evaluated on a wider grid, we also recompute the shape statistics for both models on their common support. The mean absolute difference in excess kurtosis then falls to about one-tenth, while their balance-of-risks estimates differ by only a few percentage points. The agreement between the methods is therefore not an artefact of SABR's wider evaluation grid. If anything, the extrapolated SABR tails modestly raise its excess kurtosis relative to the common-support estimate.

The comparison with the nonparametric estimator provides a broader robustness check. The two methods impose different structures and follow different numerical procedures, yet they produce similar estimates of skewness, excess kurtosis, and the balance of risks, as shown in Section 4.3. The nonparametric procedure is also unaffected by one source of short-run instability in the SABR implementation. The agreement between the two methods despite these differences indicates that the estimated departures from normality are not driven solely by a particular modelling or numerical choice.

6 Conclusion

This paper documents a framework for extracting risk-neutral spot distributions of euro-area inflation from inflation option prices. The framework recovers spot distributions using two non-normal specifications — the parametric SABR smile and a nonparametric estimator — through the Breeden–Litzenberger identity, constructs option-based forward distributions using a Student- t copula, and compares both with a normal benchmark estimated from the same quotes. It thereby extends the information available from central forecasts and market measures of inflation compensation to the asymmetry and tails of the priced distribution.

The results show that the normal benchmark provides an incomplete description of the inflation risks contained in option prices. Under both non-normal specifications, the estimated distributions are skewed at several horizons and exhibit positive excess kurtosis throughout. Relative to these estimates, the normal model assigns less probability to extreme outcomes in the sample, with larger differences at forward horizons and during periods of elevated volatility. The close agreement between the SABR and nonparametric estimates indicates that these features are present in the option-price cross-section rather than being specific to a particular extraction method. The two approaches differ mainly in their fit and smoothness: the nonparametric estimator follows the daily smile more closely subject to its shape restrictions, whereas SABR provides a more parsimonious and stable representation.

The balance of risks summarises these distributions as the difference between the probabilities of inflation above and below the 2% target. At the one-year horizon, the measure moves from -60.8 to 40.1 percentage points over the 2009–2026 sample, reflecting the shift from the low-inflation period to the 2021–22 surge. At the 5y5y horizon, it moves only from -15.3 to 2.4, indicating more limited changes in the priced distribution beyond the near term. On matched dates, the option-implied balance of risks is also generally lower than the corresponding measure reported by professional forecasters.

These results require three qualifications. First, the spot distributions are risk-neutral, while the forward distributions combine option-implied marginals with a dependence structure calibrated from historical swap-rate movements. Differences from survey distributions may therefore reflect risk compensation, differences in information sets, and differences in the construction of the measures. Second, the comparison across regimes is based largely on one inflation cycle and remains in-sample. Third, the survey comparison is descriptive, involves imperfectly matched horizons, and does not assess relative forecasting performance. Subject to these qualifications, the balance-of-risks measure provides a compact market-based summary of the distribution around the inflation outlook and can complement existing survey- and model-based indicators.

References

- Adrian, Tobias, Nina Boyarchenko and Domenico Giannone. (2019). "Vulnerable Growth". *American Economic Review*, 109(4), pp. 1263-1289. <https://doi.org/10.1257/aer.20161923>
- Ait-Sahalia, Yacine, and Jefferson Duarte. (2003). "Nonparametric option pricing under shape restrictions". *Journal of Econometrics*, 116(1-2), pp. 9-47. [https://doi.org/10.1016/S0304-4076\(03\)00102-7](https://doi.org/10.1016/S0304-4076(03)00102-7)
- Ait-Sahalia, Yacine, and Andrew W. Lo. (2000). "Nonparametric risk management and implied risk aversion". *Journal of Econometrics*, 94(1-2), pp. 9-51. [https://doi.org/10.1016/S0304-4076\(99\)00016-0](https://doi.org/10.1016/S0304-4076(99)00016-0)
- Bliss, Robert R., and Nikolaos Panigirtzoglou. (2005). "Option-Implied Risk Aversion Estimates". *The Journal of Finance*, 59(1), pp. 407-446. <https://doi.org/10.1111/j.1540-6261.2004.00637.x>
- Breeden, Douglas T., and Robert H. Litzenberger. (1978). "Prices of State-Contingent Claims Implicit in Option Prices". *The Journal of Business*, 51(4), pp. 621-651. <https://doi.org/10.1086/296025>
- Coibion, Olivier, and Yuriy Gorodnichenko. (2015). "Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts". *American Economic Review*, 105(8), pp. 2644-2678. <https://doi.org/10.1257/aer.20110306>
- Fleckenstein, Matthias, Francis A. Longstaff and Hanno Lustig. (2013). "The TIPS-Treasury Bond Puzzle". *The Journal of Finance*, 69(5), pp. 2151-2197. <https://doi.org/10.1111/jofi.12032>
- Garcia, Juan Angel, Ricardo Gimeno, Piers Hinds, Haozhe Su and Michael Tretyakov. (2026a). "The propagation of inflation risks". Working paper. Forthcoming.
- Garcia, Juan Angel, Ricardo Gimeno, Piers Hinds, Haozhe Su and Michael Tretyakov. (2026b). "Understanding inflation: Insights from the term structure of inflation risks". ECB Working Paper Series, 3294, European Central Bank. <https://www.ecb.europa.eu/pub/pdf/scpwps/ecb.wp3294~343f6d6c31.en.pdf?5d895632fcad43806bd50e5383bd4cbe>
- Ghaderi, Mohammad, Sang Byung Seo and Ivan Shaliastovich. (2025). "Learning and Subjective Beliefs about Good and Bad Inflation Ranges". SAFE Working Paper, 478, Sustainable Architecture for Finance in Europe. <https://doi.org/10.2139/ssrn.4980898>
- Gimeno, Ricardo, and Alfredo Ibáñez. (2018). "The eurozone (expected) inflation: An option's eyes view". *Journal of International Money and Finance*, 86, pp. 70-92. <https://doi.org/10.1016/j.jimonfin.2018.03.018>
- Grishchenko, Olesya V., Sarah Mouabbi and Jean-Paul Renne. (2019). "Measuring Inflation Anchoring and Uncertainty: A U.S. and Euro Area Comparison". *Journal of Money, Credit and Banking*, 51(5), pp. 1053-1096. <https://doi.org/10.1111/jmcb.12622>
- Hagan, Patrick S., Deepak Kumar, Andrew S. Lesniewski and Diana E. Woodward. (2014). "Arbitrage-Free SABR". *Wilmott*, 2014(69), pp. 60-75. <https://doi.org/10.1002/wilm.10290>
- Jackwerth, Jens Carsten. (2000). "Recovering Risk Aversion from Option Prices and Realized Returns". *The Review of Financial Studies*, 13(2), pp. 433-451. <https://doi.org/10.1093/rfs/13.2.433>
- Kitsul, Yuriy, and Jonathan H. Wright. (2013). "The economics of options-implied inflation probability density functions". *Journal of Financial Economics*, 110(3), pp. 696-711. <https://doi.org/10.1016/j.jfineco.2013.08.013>
- López-Salido, David, and Francesca Loria. (2024). "Inflation at risk". *Journal of Monetary Economics*, 145S(103570). <https://doi.org/10.1016/j.jmoneco.2024.103570>
- Mercurio, Fabio. (2005). "Pricing inflation-indexed derivatives". *Quantitative Finance*, 5(3), pp. 289-302. <https://doi.org/10.1080/14697680500148851>
- Nelsen, Roger B. (2006). *An Introduction to Copulas* (2nd ed.). Springer. <https://doi.org/10.1007/0-387-28678-0>

A SABR Calibration and Density Extraction: Details

Objective weights. The calibration objective (8) assigns each strike the weight

$$w_j = w_j^{\text{base}} \cdot w_j^{\text{atm}} \cdot w_j^{\text{vega}}.$$

The vega component w_j^{vega} is proportional to Black vega, normalised by its maximum within each cross-section, with a floor of 0.25 so that far-out-of-the-money strikes with small but non-zero vega retain some influence on the fit. The at-the-money component

$$w_j^{\text{atm}} = 0.15 + \exp \left\{ - \left(\frac{|\log(\tilde{K}_j/\tilde{F})|}{0.18} \right)^2 \right\}$$

places greater weight near the money, where the analytical approximation is most accurate. The base component w_j^{base} equals one for quoted strikes and takes smaller values for synthetic strikes added to stabilise sparsely quoted smiles. The volatility-of-volatility penalty $\lambda_\nu \nu^2$, with $\lambda_\nu = 0.01$, regularises the estimate of ν on days with sparse strike coverage. The penalty is kept small because a stronger restriction would suppress the smile curvature that the model is intended to capture.

Optimisation and retention. We initialise the parameters from the most recent previous-day estimate at the same maturity, or from a default when none is available, and optimise with a derivative-free simplex method (Subplex) subject to the bounds $\alpha > 0$, $|\rho| \leq 0.90$, and $0 < \nu \leq \nu_{\text{max}}$, with $\nu_{\text{max}} = 2.5$. The optimisation uses at most 1200 function evaluations and a relative tolerance of 10^{-5} . The objective includes the penalty $\lambda_\nu \nu^2$, with $\lambda_\nu = 0.01$.

We retain a date–maturity cross-section when the calibration succeeds on the quoted strikes and the extracted density is non-degenerate. Specifically, we require a standard deviation of at least 0.10 percentage points, which removes rare near-spike solutions. We do not impose an additional screen based on the calibration fit error. Before extracting the density, we apply a shape filter that enforces non-negativity, monotonicity, and convexity of the call-price schedule.

Numerical grid and forwards. We compute the densities on the compounded-strike grid and map them into annualised-inflation space using (4). The resulting densities are evaluated on a uniform grid over $[-3\%, 10\%]$ with a 0.1-percentage-point step. Halving the step to 0.05 percentage points leaves the reported tail probabilities and balance-of-risks values unchanged to within 10^{-4} of probability mass, indicating that the grid is sufficiently

fine for the reported quantities.

The option-based forward densities in (18) combine the spot marginals using the Student- t copula in (19), with $\eta = 4$. The dependence input is the correlation of daily inflation-swap rate changes at the two relevant maturities over a rolling window of 252 trading days. For numerical stability, the copula arguments are bounded away from zero and one. We apply mild Gaussian kernel binning to the mapped forward mass to reduce numerical ridge artefacts.

The mean of each forward density is then aligned with the corresponding forward inflation swap, subject to a small numerical tolerance. This adjustment changes the location of the density and therefore affects tail probabilities and the balance of risks defined relative to fixed thresholds. Away from the grid boundaries, it leaves centred moments such as skewness and excess kurtosis unchanged.

B The Nonparametric Estimator: Details

We solve the shape-constrained projection in (14) as a quadratic program in the observed call prices. The projection finds the closest price vector in least-squares distance subject to consecutive prices being non-increasing and discrete second differences, or butterfly spreads, being non-negative. We then convert the projected prices into Black implied volatilities and smooth the resulting volatility smile with a local-linear degree-one kernel estimator. The bandwidth is set equal to the larger of twice the median strike spacing and one-sixth of the quoted strike range.

The smoothed smile is repriced on a dense strike grid spanning the observed support, with a small volatility-flat extension used for numerical smoothing. Because repricing can reintroduce local non-convexity in the wings, we re-impose monotonicity and convexity on the dense price curve using isotonic regression on its slopes before differencing. The density obtained from the second difference of the convexified price curve is therefore non-negative at the extraction stage.

We then impose a single mode and apply light triangular smoothing to remove small steps left by the convexification. The density is mapped into annualised-inflation space using (4), evaluated only over the observed strike support, and renormalised there. The numerical extension used during smoothing is therefore not retained in the reported density. The SABR density, by contrast, is evaluated over the full grid and carries some extrapolated tail mass that the nonparametric estimator does not. This difference should be borne in mind when comparing tail probabilities and kurtosis across the two methods.

Forward densities are constructed from the nonparametric spot marginals using the same Student- t copula and mean-alignment procedure as for the SABR distributions.

OCCASIONAL PAPERS

- 2430 JOSE GONZÁLEZ MÍNGUEZ: The Letta report: a set of proposals for revitalising the European economy. (There is a Spanish version of this edition with the same number).
- 2431 MARIYA MELNYCHUK and JAVIER MENCÍA: A taxonomy of macro-financial risks and policies to address them.
- 2432 DMITRY KHAMETSHIN, DAVID LÓPEZ RODRÍGUEZ and LUIS PÉREZ GARCÍA: El mercado del alquiler de vivienda residencial en España: evolución reciente, determinantes e indicadores de esfuerzo.
- 2433 ANDRÉS LAJER BARON, DAVID LÓPEZ RODRÍGUEZ and LUCIO SAN JUAN: El mercado de la vivienda residencial en España: evolución reciente y comparación internacional.
- 2434 CARLOS GONZÁLEZ PEDRAZ, ADRIAN VAN RIXTEL and ROBERTO PASCUAL GONZÁLEZ: Navigating the boom and bust of global SPACs.
- 2435 PATROCINIO TELLO-CASAS: El papel de China como acreedor financiero internacional.
- 2436 JOSÉ RAMÓN MARTÍNEZ RESANO: CBDCs, banknotes and bank deposits: the financial stability nexus.
- 2501 PEDRO DEL RÍO, PAULA SÁNCHEZ, MARÍA MÉNDEZ, ANTONIO MILLARUELO, SUSANA MORENO, MANUEL ROJO, JACOPO TIMINI and FRANCESCA VIANI: La ampliación de la Unión Europea hacia el este: situación e implicaciones para la economía española y la Unión Europea.
- 2502 BANCO DE ESPAÑA: In-person access to banking services in Spain. 2024 monitoring report. (There is a Spanish version of this edition with the same number).
- 2503 ANDRÁS BORSOS, ADRIAN CARRO, ALDO GLIELMO, MARC HINTERSCHWEIGER, JAGODA KASZOWSKA-MOJSA and ARZU ULUC: Agent-based modeling at central banks: recent developments and new challenges.
- 2504 ANDRES ALONSO-ROBISCO, ANDRES AZQUETA-GAVALDON, JOSE MANUEL CARBO, JOSE LUIS GONZALEZ, ANA ISABEL HERNAEZ, JOSE LUIS HERRERA, JORGE QUINTANA and JAVIER TARANCON: Empowering financial supervision: a SupTech experiment using machine learning in an early warning system.
- 2505 JÉSSICA GUEDES, DIEGO TORRES, PAULINO SÁNCHEZ-ESCRIBANO and JOSÉ BOYANO: Incertidumbre en el mercado de bonos: una propuesta para identificar sus narrativas con GDELT.
- 2506 LAURA JIMENA GONZÁLEZ GÓMEZ, FERNANDO LEÓN, JAIME GUIXERES PROVINCIALE, JOSÉ M. SÁNCHEZ and MARIANO ALCANIZ: Evolución de la investigación neurocientífica del efectivo: revisión y perspectivas actuales.
- 2507 LUIS FERNÁNDEZ LAFUERZA, IRENE ROIBÁS and RAQUEL VEGAS SÁNCHEZ: Indicadores de desequilibrios de precios del mercado inmobiliario comercial.
- 2508 PANA ALVES and OLIVIER HUBERT: ¿Influye la eficiencia energética en el precio de la vivienda en España?.
- 2509 ALEJANDRO FERRER and ANA MOLINA: The interaction of liquidity risk and bank solvency via asset monetisation mechanisms. (There is a Spanish version of this edition with the same number).
- 2510 ISABEL ALCALDE and PATRICIA STUPARIU: Financial education at an early age. (There is a Spanish version of this edition with the same number).
- 2511 ALEJANDRO GONZÁLEZ FRAGA, AITOR LACUESTA GABARAIN, JOSÉ MARÍA LABEAGA AZCONA, MARÍA DE LOS LLANOS MATEA ROSA, SOLEDAD ROBLES ROMERO, MARÍA VALKOV LORENZO and SERGIO VELA ORTIZ: Estructura del mercado de electrolineras.
- 2512 FERNANDO ARRANZ GOZALO, CLARA I. GONZÁLEZ MARTÍNEZ and MERCEDES DE LUIS LÓPEZ: Sovereign assets and sustainable and responsible investment: the importance of climate metrics. (There is a Spanish version of this edition with the same number).
- 2513 IRMA ALONSO-ÁLVAREZ and DANIEL SANTABÁRBARA: Decoding Structural Shocks in the Global Oil Market.
- 2514 JOSÉ MANUEL CARBÓ, CLAUDIA TOLEDO and ÁNGEL IVÁN MORENO: Hacia un diccionario panhispánico de sentimiento de la estabilidad financiera.
- 2515 IGNACIO FÉLEZ DE TORRES, CLARA I. GONZÁLEZ MARTÍNEZ and ELENA TRIEBSKORN: The puzzle of forward-looking climate transition risk metrics.
- 2516 MIGUEL GARCÍA-POSADA: Un análisis de los efectos de la introducción del procedimiento especial de insolvencias para microempresas sobre la propensión a concursar.
- 2517 ADRIAN VAN RIXTEL: Whatever it takes? Economic policymaking in China in the context of a possible deflationary spiral.
- 2518 PATRICIA STUPARIU and JUAN RAFAEL RUIZ: Adding up the benefits: education, numeracy and financial literacy. (There is a Spanish version of this edition with the same number).
- 2519 DAVID CUBERES, AITOR LACUESTA, MARÍA DE LOS LLANOS MATEA and DANIEL OTO-PERALÍAS: El efecto de la regulación sobre el tamaño de las plantas fotovoltaicas.
- 2520 MARINA GARCÍA GIL and DMITRY KHAMETSHIN: Ayudas directas de la línea COVID-19: los determinantes de la asignación y el efecto sobre el crédito bancario.
- 2521 LUCÍA CUADRO-SÁEZ, CORINNA GHIRELLI, MAXIMILIANO MORENO-LÓPEZ and JAVIER J. PÉREZ: Monitoring and forecasting food prices in the euro area.

- 2522 LAURA HOSPIDO, JÚLIA MARTÍ LLOBET and CARLOS SANZ: ¿Qué políticas son efectivas para reducir la exclusión social? Evaluación de cinco proyectos piloto de inclusión social a través de ensayos aleatorizados.
- 2523 MARIO ALLOZA, MARÍA ELENA CRISTÓBAL RODRÍGUEZ, JULIA GARCÍA-ROYO DÍAZ, BEATRIZ GONZÁLEZ, ALBERTO MARTÍN DEL CAMPO SOLA, ANE MARTÍN UGARTE, ENRIQUE MORAL-BENITO and IRENE PINILLA MELGAREJO: Diagnóstico y consecuencias económicas del grado de competencia en las licitaciones públicas.
- 2524 ADRIÁN CARRO, JORGE E. GALÁN, ENRIC MARTORELL and RAQUEL VEGAS: A literature review on ex-ante and ex-post analysis of the implications of borrower-based macroprudential measures.
- 2525 PAULA SEMPERE and RAQUEL PANTOJA: Efectivo y *neuromarketing*: Concepto, funciones y aplicación.
- 2601 RODOLFO G. CAMPOS, JACOPO TIMINI, FRANCESCA VIANI and ELENA VIDAL: The EU-Mercosur agreement: analysis of its characteristics from a sectoral perspective. (There is a Spanish version of this edition with the same number).
- 2602 PABLO AGUILAR, CORINNA GHIRELLI and SAMUEL HURTADO: MTBE v2025: new version of the Quarterly Model of the Banco de España.
- 2603 IRMA ALONSO-ALVAREZ, EKATERINA BUKINA, MARINA DIAKONOVA, NINO KHITARISHVILI, JAVIER J. PÉREZ and PEDRO PIQUERAS: Geopolitical risk: a database of general and bilateral indices.
- 2604 RUBÉN DOMÍNGUEZ-DÍAZ, MARTA GARCÍA-RODRÍGUEZ, JAVIER QUINTANA and RUBÉN VEIGA-DUARTE: Estimación del crecimiento potencial de la economía española: una revisión metodológica.
- 2605 CARLES MANERA, FERRAN NAVINÉS, JAVIER FRANCONETTI, MIQUEL QUETGLAS and JOSÉ A. PÉREZ-MONTIEL: Regional outlook on labor productivity in Spain, 2000–2022.
- 2606 RICARDO BARAHONA: Current trends in performance and flows in the Spanish pension funds industry.
- 2607 FRANCISCO GONZÁLEZ RODRÍGUEZ, ALBERTO ORTOS TORRES, JOSÉ MARÍA SERENA GARRALDA and MIQUEL TARÍ SÁNCHEZ: Issuance yield of MREL-eligible bank debt: a benchmarking model.
- 2608 MÓNICA CORREA-LÓPEZ, MAR DELGADO-TÉLLEZ and MARTA SUÁREZ-VALERA: Europa en transición energética: respuestas y desafíos tras dos crisis consecutivas.
- 2609 AITOR LACUESTA, MARÍA VALKOV and MICOLE DE VERA: Un análisis de los mecanismos de protección de los operadores económicos establecidos por la Ley de garantía de la unidad de mercado.
- 2610 Encuesta Financiera de las Familias (EFF) 2024: métodos, resultados y cambios desde 2022.
- 2611 ALFONSO CAMBA, MATÍAS COVARRUBIAS, JOSÉ LUÍS RODRÍGUEZ and RAQUEL TÁRREGA: El coste en bienestar de los márgenes en mercados de productos en España: un análisis con empresas heterogéneas.
- 2612 MICAELA ARIAS, LUCÍA CRIADO, JAVIER GARCÍA-VERDUGO, EDUARDO GUTIÉRREZ, ALMUDENA KESSLER, JOSÉ MANUEL MONTERO, SARA PEREIRA and PAU ROLDÁN-BLANCO: Evolución de la estructura de mercado y de los indicadores de competencia en España en las dos últimas décadas.
- 2613 ANDRÉS ALONSO-ROBISCO, JOSÉ MANUEL CARBÓ, CARLOS JOSÉ GARCÍA, JORGE QUINTANA and JAVIER TARANCÓN: Retriever or reasoner? Decomposing retrieval-augmented generation performance in external audit supervision.
- 2614 RUBÉN VEIGA-DUARTE, EDUARDO GUTIÉRREZ, ALBERTO MARCOS-FERNÁNDEZ, ENRIQUE MORAL-BENITO and JAVIER QUINTANA: Las fuentes del crecimiento en España y las principales economías europeas: productividad total de los factores y heterogeneidad sectorial.
- 2615 MATEO CACHO UZAL, M. DANIELA FILIP and DIANA POSADA RESTREPO: Regional convergence dynamics in the main european economies. (There is a Spanish version of this edition with the same number).
- 2616 IRMA ALONSO-ALVAREZ, IVÁN KATARYNIUK, DAVID LÓPEZ-SALIDO and JAVIER J. PÉREZ: Geopolitical Oil-Supply Scenarios and Macroeconomic Risks: A Bayesian Toolkit for Policy Analysis.
- 2617 MIHAI-VASILE CÎRJĂ and JORDI ROMEU GRANADOS: Transparency and accountability as drivers of cultural change. A comparative analysis of central banks and national competent authorities.
- 2618 LUCÍA CANALEJO GUIJOSA, FERNANDO LEÓN MARTÍNEZ and JOSÉ MARÍA SÁNCHEZ ECHAVE: The voice as a neurophysiological marker: applications in implicit emotion perception for Neurocash® and cash design.
- 2619 DANIEL DEJUAN-BITRIA: El proceso de desapalancamiento de las sociedades no financieras españolas en el período 2007-2024.
- 2620 MIGUEL GARCÍA-POSADA GÓMEZ y MARÍA RODRÍGUEZ MORENO: Determinantes de la modalidad del tipo de interés de las hipotecas y evolución de sus márgenes durante el período de endurecimiento de la política monetaria 2022-2023.
- 2621 MARÍA ALEJANDRA AMADO GARFIAS, JOÃO AMADOR, SÓNIA FÉLIX, JOANA M. GARCIA, VANDA GERALDES DA CUNHA, CORINNA GHIRELLI, ÁNGEL LUIS GÓMEZ JIMÉNEZ, EDUARDO GUTIÉRREZ CHACÓN, CRISTINA MANTEU, LUIS MOLINA SÁNCHEZ, ENRIQUE MORAL-BENITO, SERGIO PUENTE DÍAZ, JAVIER QUINTANA GONZÁLEZ, MARTÍN SALDÍAS, SHARMIN SAZEDJ, SOFIA TERLICA and JACOPO TIMINI: Perspectives on Iberian Economic Integration.
- 2622 CLARA I. GONZÁLEZ, PEDRO LINARES, SOLEDAD NÚÑEZ, ÍÑIGO LOSADA, ANIL MARKANDYA, AVINASH PERSAUD, FRANK ELDERSON, KEVIN STIROH, ROB PATALANO, ENRIQUE SERRANO, CASPAR SIEGERT, JAMES TALBOT, JUAN AYUSO, ELENA TRIEBSKORN, LUIS ÁNGEL MAZA, AGNIESZKA TRZCINSKA, CARMEN NUZZO and YANN MARIN: Climate Risk and Sustainable Finance: Bridging Policy, Research and Practice.
- 2623 IRENE PABLOS: The AI investment boom: capital formation, financing structures, interconnections and vulnerabilities.
- 2624 JACOB HESS and DIANA POSADA RESTREPO: El uso de la inteligencia artificial por parte de las empresas españolas: avances recientes y factores asociados a su uso.
- 2625 DIEGO BONELLI: Measuring Euro-Area Inflation Risk with Options: Market-Implied Distribution and Balance of Risks.