Discussion of "Estimating Nonlinear Heterogeneous Agent Models with Neural Networks " by Hanno Kase, Leonardo Melosi and Matthias Rottner

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Overview

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- Linearized small-scale DSGE model - can compare to the analytical solution.
- A RANK model with a ZLB - compare with other nonlinear estimation methods.
- A nonlinear HANK model (ZLB + aggregate risk) - **main contribution**.

Overview: Model Solution

- Treat the estimable parameters $\tilde{\Omega}$ as pseudo-state variables.
- Build two networks

$$\psi_t^i = \psi^I(\mathbb{S}_t^i, \mathbb{S}_t, \tilde{\Omega}|\bar{\Omega}) \qquad \psi_t^A = \psi^A(\mathbb{S}_t, \tilde{\Omega}|\bar{\Omega})$$

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- Curse of dimensionality
  - Make draws from the parameter space *instead* of solving on the grid .
  - The relevant share of the state space shrinks as the number of state variables $\uparrow$.
  - Make draws from the simulated model.

Overview: Model Estimation

$$\mathcal{L}(\mathbb{Y}_{1:T}|\tilde{\Omega}) = \prod_{t=1}^{T} p_{\tilde{\Omega}}\left(y_t|y_{t-1},...,y_1,\tilde{\Omega}\right)$$

When the model is nonlinear:

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When the model is nonlinear:

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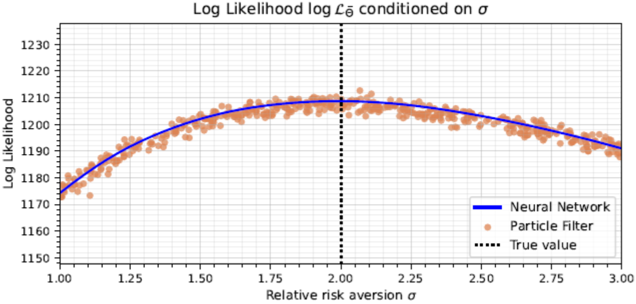
Instead: Use the ANN to approximate the mapping between $\mathcal{L}(\mathbb{Y}_{1:T}|\tilde{\Omega})$ and $\tilde{\Omega}$, dubbed as the ANN *particle filter*. This allows for fast and precise likelihood evaluation.



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We want a network that approximates well BUT allows for a *fast* convergence: 1. trains fast, 2. stable across iterations.

Stability especially relevant because:

- In the context of model solution, data is *endogenous* to the network weights.
- Every iteration there is a *new* sample draw.
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Wide or Narrow?

- Deep networks generalize better, making them more stable when the new data arrives. Wide networks memorize and may be less stable to new data.
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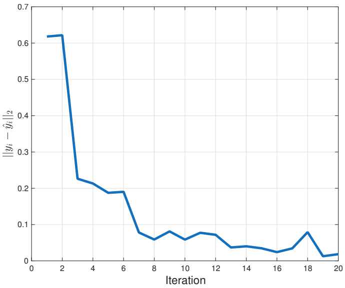
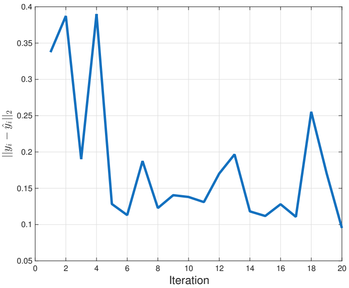
Experiment:

- Consider a eide shallow network (27 neurons, 1 layer) and a deep narrow network (9 layers 3 neurons each).
- Each iteration: learn the mapping $y_i = f(X_i)$ with the new data draws.



(a) Narrow; Av. training time 23s.



(b) Wide; Av. training time 44s.

Discretization or Moments?

Discretization

Currently, the model is solved by assuming that the economy consists of a discrete number L agents:

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- Requires a large L for a precise solution when the wealth distribution matters,
- Large L complicates the ANN stability and the algorithm convergence .

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Distribution Moments

Alternatively, one can use selected moments of the wealth distribution in $\mathbb{S}$.

- Simulate a histogram (Young, 2010) and compute selected moments as elements of $\mathbb{S}$,
- Potentially easier to train the network and converge the algorithm and NOT more costly to simulate,
- Which moments to use?
- OR one could use the histogram as part of $\mathbb{S}$.

Discretized distribution when individual nonlinearity is relevant

This paper makes significant progress in handling aggregate nonlinearity.
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Additionally:

- Ramsey problems with heterogeneous agents - one needs to keep track of the whole distribution, discrete L does the trick!

Thank You!

References I

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