When Growth-at-Risk Hits the Fan: Comparing Quantile-Regression Predictive Densities with Committee Fan Charts

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The views expressed here do not necessarily reflect the position of the Bank of England.

This Paper: Motivation

- **Quantile regression growth-at-risk models** have become an important part of macroprudential policymakers' toolkit for monitoring financial stability risks
  - Focus is typically on estimating the tails of the GDP distribution (in line with a financial-stability objective), but methodology can be used to estimate entire conditional GDP growth density

[Adrian et al., 2019; Prasad et al., 2019]

- Separately, monetary policymakers have traditionally published "**fan charts**" to convey density estimates around point forecasts, constructed using a range of **judgement** and **linearised macroeconomic models**

[Britton et al., 1998]

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How do these density estimates compare? Can central bank fan charts be improved by using insights from quantile regression techniques?

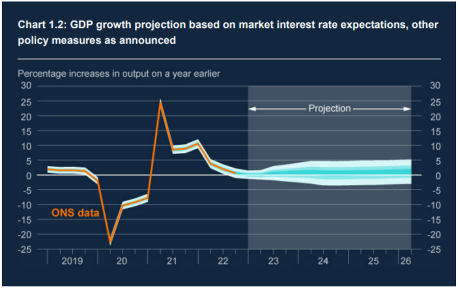
This Paper: Growth-at-Risk vs. Fans

1. Run forecasting horse race between growth-at-risk models and the Bank of England's MPR GDP fan charts
   - Compare quantile-specific "goodness of fit" statistics as well as tests of overall calibration

2. Growth-at-risk models provide worse forecasts of overall GDP-growth densities, but have superior forecasting power in the left tail specifically

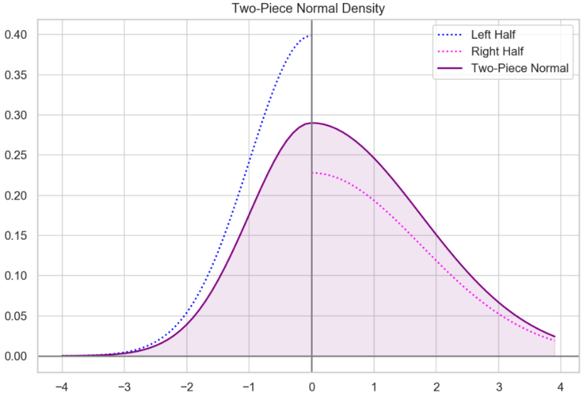
3. Simple combination methods provide the best density forecasts overall
   - ⋆ Combining higher growth moment estimates from growth-at-risk model with estimates of the mean from the MPR delivers improved forecasts over the MPR fan charts

Bank of England Monetary Policy Report (MPR) Fan Charts



Constructing the Fan Chart

- GDP growth assumed to follow a two-piece normal distribution
  - Governed by three parameters: the mode, a measure of uncertainty, and a measure of the balance of risks: $s(x|\mu, \sigma, \gamma)$



Constructing the Fan Chart

- GDP growth assumed to follow a two-piece normal distribution
  - Governed by three parameters: the mode, a measure of uncertainty, and a measure of the balance of risks: $s(x|\mu, \sigma, \gamma)$
- Calibration of the fan chart parameters μ, σ, γ is informed by a combination of statistical tools and judgements by the MPC [Britton et al., 1998]
  - **Mode** μ: central forecast is constructed using a small-open economy New Keynesian DSGE model (combined with other models and MPC judgement) [Burgess et al., 2013]
  - **Uncertainty** σ **and Skew** γ: informed by historical forecast errors, as well as forward-looking judgements and scenario analysis

Fan Chart Parameters Response to Exogenous Shocks

Growth-at-Risk Models

Growth-at-Risk Framework

Quantile regression for h-quarter-ahead GDP growth $\Delta^h y_{i,t+h}$ in country i at time t: [Koenker and Bassett, 1978]

$$Q_{\Delta^h y_{i,t+h}}(\tau|\mathbf{X}_{i,t}) = \alpha_i^h(\tau) + \beta^h(\tau)\mathbf{X}_{i,t}$$

where:

- $\Delta^h y_{i,t+h}$: h-quarter ahead 4-quarter real-time real GDP growth
- $\alpha_i^h(\tau)$: (potentially) quantile- and country-specific country fixed effect
- $\mathbf{X}_{i,t}$: set of covariates, including (lagged/real-time) domestic and foreign variables
- $\beta^h(\tau)$: association between covariates and τ-th quantile of h-quarter-ahead 4-quarter real GDP growth

Search over range of models to choose 'best' model using 'quantile score' criterion

Specific Growth-at-Risk Model

$$Q_{\Delta^h y_{i,t+h}}(\tau|\mathbf{X}_{i,t}) = \alpha_i^h + \beta^h(\tau)\mathbf{X}_{i,t}$$

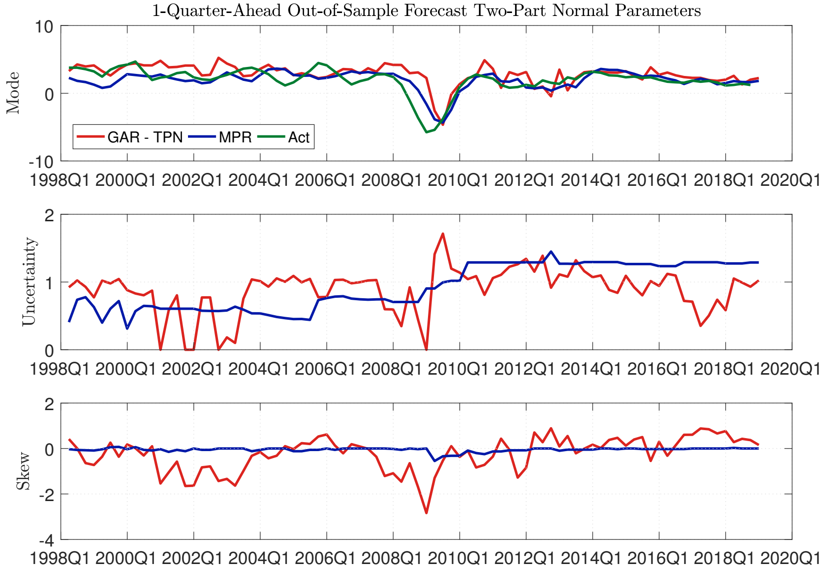
- Time span: 1981Q1-2018Q4
- Panel: 10 **advanced economies** [Aus, Can, Fra, Ger, Ita, Spa, Swe, Swi, UK, US]
- Country fixed effects as locational shifts for the entire distribution [Canay, 2011]
- Explanatory variables
  - **Domestic Macro**: 1q-lagged real-time quarterly real GDP growth, 1q-lagged annual CPI inflation
  - **Domestic Near-Term**: realised quarterly equity-price vol. [Adrian et al., 2019]
  - **Domestic Medium-Term**: 1q-lagged 3y change in debt-to-GDP, 1q-lagged 3y house-price growth [Aikman et al., 2019]
  - **Global**: 1q-lagged foreign-weighted real-time quarterly real GDP growth, foreign-weighted realised quarterly equity-price volatility [Lloyd et al., 2023]
- Back-test the model to construct **real-time out-of-sample** estimates of conditional UK GDP-growth quantiles from 1998Q1

How do I get Fan Charts from GaR?

To recap:

- From the previous model I get a vector of quantile-forecasts for GDP $Q_{\Delta^h y_{i,t+h}}$
- This is not a density forecast yet! Just Q points of the distribution
- Which parametric distribution shall I pick? Since we want a Fan Chart from this model, we fit a Two-piece-Normal distribution on the quantiles,
- How? By estimating the TPN moments μ, σ, γ that better fit the quantiles $Q_{\Delta^h y_{i,t+h}}$.

Comparison III: Moment Estimates



Comparing MPR Fan Charts to Growth-at-Risk

Density Forecast Evaluation

- Relative Evaluation:
  - Quantile Score (QS):

$$QS_t^h(\tau) = \frac{1}{N_\nu} \sum_\nu \rho_\tau \left(y_{t+h} - F_{\nu,h}^{-1}(\tau) \right) \tag{1}$$

    where:

    - N_ν denotes the number of forecast vintages
    - $\rho_\tau \equiv \rho_\tau(u) + u[\tau - 1(u < 0)]$ is the check function
    - $F_{\nu,h}^{-1}$ is the cdf.

  - Continuous ranked probability score (CRPS):

$$CRPS_t^h(F_{\nu,h}^{-1}, y) = \int_0^1 QS_t^h(F_{\nu,h}^{-1}(\tau), t_{t+h}) d\tau \tag{2}$$

- Calibration with PITs plots for the best model.

Comparison: Forecast Accuracy with Quantile Scores

Table: Scores for GDP-at-risk Model relative to MPR forecasts

	$\tau = 0.05$	$\tau = 0.25$	$\tau = 0.5$	$\tau = 0.75$	$\tau = 0.95$
h=1	0.779	1.075	1.072	0.945	0.562
h=4	0.931	1.196	1.358	1.248	0.982
h=8	0.859	1.384	1.470	1.325	1.140
h=12	0.932	1.330	1.435	1.195	1.020

- A relative quantile score $< 1 \Rightarrow$ improved forecast accuracy for GaR vs MPR

⋆ Growth-at-risk model performs better at the 5th and 95th percentile, but performs worse at other quantiles.

Summary of Comparisons

- ⋆ On average, growth-at-risk model appears to perform worse than the MPR fan chart in the mass of the distribution
- ⋆ But growth-at-risk model performs better at the left tail, and e.g. picks-up run-up to GFC as time of heightened uncertainty and downside skew

Could density forecast from the MPR be improved by *combining* them with forecasts from growth-at-risk model?

Combining MPR Fan Charts and Growth-at-Risk

Optimal Combination: Quantile combination

$$\underset{1\times Q}{y^{qc}_{t+h}} = diag\left(\underset{Q\times K}{w_{q,k,t}}\underset{K\times Q}{f(q)_{t+h,k}}\right) \tag{3}$$

where:

- y^{qc}_{t+h}: h-quarter ahead combined forecast for real GDP growth
- K: number of forecasts combined: here $K = 2$: MPR and GaR
- $f(q)_{t+h,k}$: set of forecasts to be combined here : MPR and GaR

and

- $w_{q,k,t}$: quantile-specific combination weights

$$\underset{Q\times K}{w_{q,k,t}} = \frac{1/QS^h_{t,q,k}}{\sum_{k=1}^{K} 1/QS^h_{t,q,k}} \tag{4}$$

- A two-piece normal distribution is fitted on the combined quantiles $s(y^{qc}_{t+h})$ following [Aastveit, Mantoan (WP)].

Alternative Combinations

- **Simple Combination Method**: Combine the modal estimate from the MPR with the estimates of the skew and uncertainty from growth-at-risk model; i.e:

$$y_{t+h}^{SIMP} \sim s(\mu_{t+h}^{MPR}, \sigma_{t+h}^{GaR}, \gamma_{t+h}^{GaR})$$

- **Moment Average Combination Method**: Combine the modal estimate from the MPR with the average estimates of the skew and uncertainty from both MPR and growth-at-risk model

$$y_{t+h}^{AVG} \sim s\left(\mu_{t+h}^{MPR}, \frac{\sigma_{t+h}^{GaR} + \sigma_{t+h}^{MPR}}{2}, \frac{\gamma_{t+h}^{GaR} + \gamma_{t+h}^{MPR}}{2}\right)$$

- **Equal Weight**:

$$y_{t+h}^{EQ} \sim 0.5 * s(y_{t+h}^{MPR}) + 0.5 * s(y_{t+h}^{GaR})$$

Combination I: Forecast Accuracy at each Quantile

Table: Relative Quantile Scores for Combined Model Relative to MPR Forecasts.

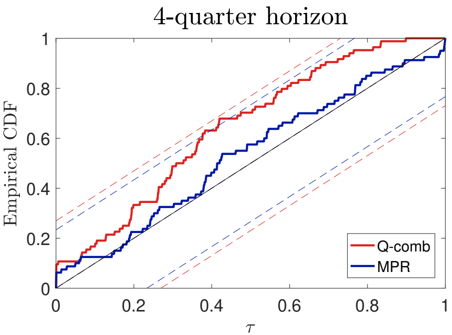
Q	GaR	Simp	Aver	EQ	Q-comb
Q=0.05	0.779	0.626	0.698	0.812	0.747
Q=0.25	1.075	1.007	0.956	0.903	0.805
Q=0.50	1.072	1.105	1.018	0.947	0.794
Q=0.75	0.945	1.008	0.974	0.837	0.703
Q=0.95	0.563	0.814	0.845	0.519	0.455

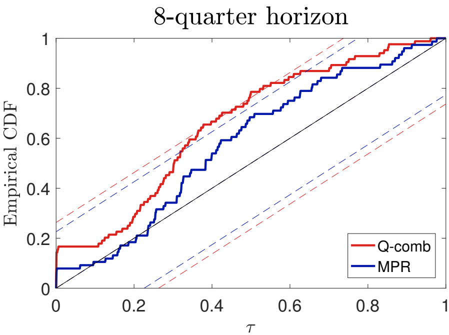
Combination II: Forecast Accuracy overall

Table: CRPS with emphasis on different parts of the support

				Combinations			
		GaR	MPR	Simp	Aver	EQ	Q-comb
$h=1$	Uniform	0.792	0.794	0.801	0.769	0.694	0.591
	Left Tail	0.250	0.240	0.233	0.226	0.218	0.191
$h=4$	Uniform	1.197	0.963	0.992	0.958	1.021	0.863
	Left Tail	0.397	0.324	0.328	0.318	0.338	0.294
$h=8$	Uniform	1.355	1.012	1.084	0.993	1.079	0.993
	Left Tail	0.452	0.354	0.398	0.362	0.37	0.333
$h=12$	Uniform	1.692	1.317	1.210	1.218	1.312	1.306
	Left Tail	0.591	0.471	0.457	0.442	0.470	0.445

Combination III: Forecast Calibration





Blue line: PIT from Bank of England MPR fan chart.

Red line: PIT from **combined** growth-at-risk and MPR fan chart density, with 'Quantile combination' method.

Summary of Combinations' Comparison

- ⋆ Building from the evidence that GaR and MPR fancharts accuracy change across part of the distribution, we decide to combine them.
- ⋆ We combine the two with an "optimal combination" (Q-comb), providing the highest forecast accuracy.
- ⋆ Moreover, we combine the two with a set of more intuitive combinations. Despite not being more accurate than the optimal, they are often a good alternative to one model only.

Conclusions

- **What We Do**: Compare GDP-growth density forecasts from Bank of England's MPR to growth-at-risk model estimates

- **What We Find**:
  - ⋆ Forecasting of growth-at-risk worse than MPR, apart from at left-tail!
  - ⋆ Simple combination of growth-at-risk and fan chart performs best

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⇒ Central banks can improve fan-chart calibration using quantile regression techniques to calibrate width and skew of fans

⋆ Simple methods provide opportunity to unify framework within which monetary policymakers and financial-stability policymakers analyse macroeconomic developments within same institution

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What Makes the Combination 'Better'?

Benchmark Growth-at-Risk Model for Combination

Estimate 'restricted' growth-at-risk model:

$$Q_{\Delta^h y_{i,t+h}}(\tau|\mathbf{X}_{i,t}) = \alpha_i^h + \beta^h(\tau)\mathbf{X}_{i,t}$$

with only domestic macro explanatory variables $\mathbf{X}_{i,t}$: 1q-lagged real-time quarterly real GDP growth, 1q-lagged annual CPI inflation

Table: Quantile Scores for GaR Model Relative to MPR Forecasts.

	$\tau = 0.05$	$\tau = 0.25$	$\tau = 0.5$	$\tau = 0.75$	$\tau = 0.95$
h=1	1.066	1.142	1.090	0.947	0.588
h=4	1.057	1.083	1.179	1.217	0.947
h=8	0.965	1.009	1.129	1.150	1.151
h=12	1.020	1.032	0.951	0.992	0.964

How much accuracy do I gain?

Table: CRPS with emphasis on different parts of the support

		MPR	$GaR_{CPI+GDP}$	$GaR_{For-Aug}$	Q-comb
$h=1$	Uniform	0.794	0.817	0.792	0.591
	Left Tail	0.240	0.267	0.250	0.191
$h=4$	Uniform	0.963	1.097	1.197	0.863
	Left Tail	0.324	0.366	0.397	0.294
$h=8$	Uniform	1.012	1.101	1.355	0.993
	Left Tail	0.354	0.366	0.452	0.333
$h=12$	Uniform	1.317	1.307	1.692	1.306
	Left Tail	0.471	0.471	0.591	0.445

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$\Rightarrow$ Central banks can improve fan-chart calibration using quantile regression techniques to calibrate width and skew of fans

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