Tracking Trend Output Using Expectations Data

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Banco de Espana, Oct. 2023

Measuring trend output suffers from two pervasive problems

- separating permanent output movements from short-lived fluctuations in output, and
- accommodating the inherent instabilities in the drivers of the trend and changes in the nature of shocks to output over time.

This paper

- uses surveys on output expectations to distinguish innovations with transitory effects on output from innovations with permanent effects
- uses a 'meta modelling' time series technique, based on model averaging involving the survey data, to address the time-variation in the determination of the trend
- derives a real-time trend output series for UK manufacturing based on survey expectations from the Confederation of British Industry (CBI)

This paper uses the BN trend as a measure of permanent output change

- defined as the long-horizon expectation of the series (minus any deterministic drift) given current information
- natural interpretation as the 'steady-state' outcome that will occur in the absence of any further shocks
- every possible permanent measure must converge on the BN trend in expectation as the forecast horizon increases and the corresponding transitory element goes to zero.

But, when based on univariate AR models supported by the data, the BN trend is excessively volatile (more volatile than the series itself!).

- Kamber et al. (2018) suggest a solution where the AR model parameters are restricted to limit the size of the signal-to-noise ratio - i.e. the variance of the trend shocks relative to the variance of the overall forecast errors - to a 'realistic' level.

This paper measures the BN trend using survey measures of expected future outputs alongside actual output data in VAR models.

- the two series deliver a more sophisticated time series representation of actual output than can be obtained in a univariate context.
- the expected output growth series is relatively stable compared to the actual output growth series and together they provide an objective measure of the signal-to-noise ratio.
- the expectations series show what survey respondents consider to be the transitory and permanent components of a shock (since short-lived shocks will affect today's output but not expected future output)

Direct measures of expectations can also be used to investigate the role of uncertainty in business cycles and its contribution to the trend over time.

- Uncertainty is the extent to which something is not known and this is reflected directly by measures based on individuals' stated understanding of output growth.

The second problem in measuring trend output arises if there is substantial time variation in the processes underlying the determination of output.

- The transitory and permanent shocks to the macroeconomy associated with the GFC and the covid pandemic appear very different in nature to those experienced during the low levels of output volatility of the 'Great Moderation'
- There is a broad consensus that there has been a slowdown in productivity growth across advanced economies.
    - The timing is unclear: many associate the slowdown with GFC but others note that the data show a slowdown prior to GFC
    - Many explanations: demographic changes; changes in inequality and education; globalisation and international trade; credit constraints; role of intangibles; deterioration in technological advance; etc.
- Any or all of these explanations could be true, with some influences affecting trend growth slowly and incrementally over time and some more abruptly following a specific structural change.

We adopt a model averaging approach to deal with structural instability

- we focus on the time frame for which the model is relevant, following Pesaran and Timmermann (2007) to apply model averaging techniques to models estimated over different estimation windows.
- at each point in the sample, we consider models of actual output growth and survey responses on growth estimated over different sample lengths and averaged using weights that change over time.
- this provides a very flexible form for capturing structural change; we describe it as ‘meta modelling’ to highlight our emphasis on regime uncertainty compared to standard model averaging
- weights used in VAR models of actual and expected future outputs to obtain BN trends that can accommodate structural change
- the methods are illustrated using UK manufacturing data including the CBI's survey of UK manufacturing firms.

A simple model can illustrate some useful features of direct measures of expectations:

- Survey measures of expected future output growth - which tend to be relatively stable over time compared to actual growth - can be used with actual output to distinguish permanent from transitory shocks.
- Measures of the size of shocks based on actual output data alone will overstate the true extent of the uncertainty surrounding output (being unable to take account of the known-to-be-transitory element)
- Univariate models estimated using only actual data will overstate the consequences of the shocks for future output movements. This translates into measures of trend output that are excessively volatile.

For example, denote output growth $y_t - y_{t-1}$, and assume growth is determined according to the following

$$y_t - y_{t-1} = \rho(y_{t-1} - y_{t-2}) + v_t + \omega_t - \omega_{t-1} \tag{1}$$

Assuming (for illustrative purposes) that expectations are formed with FIRE, then the expected value of $(y_{t+1} - y_t)$ formed in time t is

$${}_t y^e_{t+1} - y_t = \rho(y_t - y_{t-1}) - \omega_t \tag{2}$$

where ${}_t y^e_{t+1}$ is the observed time-t expectation of y_{t+1}, and

$$(1-\rho)(y_t - y_{t-1}) + \left({}_t y^e_{t+1} - y_t\right) - \left({}_{t-1} y^e_t - y_{t-1}\right) = v_t. \tag{3}$$

The 'news' arriving on $(y_t - y_{t-1})$ at time t is the expectational error $y_t - {}_{t-1}y^e_t = v_t + \omega_t$ and the uncertainty surrounding growth - describing the extent to which output in t is not known in $t-1$, i.e. $\sigma^2_v + \sigma^2_\omega$ - can be readily calculated and split into its component parts.

If the modelling is based on the actual output series alone we would have the moving average (MA) process

$$y_t - y_{t-1} = \rho(y_{t-1} - y_{t-2}) + u_t + \theta u_{t-1} \quad (4)$$

where $\theta \in [-1, 0]$ ($\longrightarrow 0$ when permanent shocks dominate and $\frac{\sigma_v^2}{\sigma_\omega^2} \longrightarrow \infty$, and $\longrightarrow -1$ when short-lived shocks are relatively important and $\frac{\sigma_v^2}{\sigma_\omega^2} \longrightarrow 0$).

- here $\sigma_u^2 > \sigma_v^2 + \sigma_\omega^2$ so that the uncertainty surrounding growth as obtained from the univariate MA specification overstates the ‘true’ uncertainty surrounding growth.

- Applying the definitions of the BN trend, we have

$$\bar{y}_t = \bar{y}_{t-1} + \frac{v_t}{1-\rho} \qquad \text{if } v_t \text{ and } \omega_t \text{ can be distinguished; and}$$

$$\bar{y}_t = \bar{y}_{t-1} + \frac{1+\theta}{1-\rho} u_t \qquad \text{according to the univariate representation}$$

- The overstatement of the uncertainty surrounding growth by the univariate MA specification translates into an overstatement of the persistent effect of shocks to output - and hence the BN trend.
- A 1% change in output growth, based on the composite shock $v_t + \omega_t$, will typically involve a v_t of size $\frac{\sigma_v^2}{\sigma_v^2+\sigma_\omega^2}$% and so the size of the increase in BN trend is $\frac{1}{1-\rho} \times \frac{\sigma_v^2}{\sigma_v^2+\sigma_\omega^2}$. The corresponding 1% shock in the univariate MA representation results in output $\frac{1+\theta}{1-\rho}$% higher at the infinite horizon. Since $\frac{1+\theta}{1-\rho} > \frac{1}{1-\rho} \times \frac{\sigma_v^2}{\sigma_v^2+\sigma_\omega^2}$, MA overstates the persistent effect of shocks.

- To gain a sense of the orders of magnitudes: in the .U.S., the volatility of actual quarterly output growth is 0.60 compared to 0.23 for that of the expected one-quarter-ahead growth provided by the Survey of Professional Forecasters. If growth was characterised by the model of (1) with $\rho = 0.8$, this conservatism in the expectations series - suggesting $\frac{\sigma_v}{\sqrt{\sigma_v^2+\sigma_\omega^2}} \approx 0.4$ - implies that the transitory shocks are five times more volatile than the permanent shocks and that we have $\theta = -0.64$ in the ARMA specification of (4).
- Here a 1% increase in output on impact would raise output by 0.84% eventually and a gap measure based on the true BN trend (where the two types of shock are identified separately) are sensibly pro-cyclical. But the estimate of the long-run effect based on the univariate MA representation would be 1.80% and the BN trend would show more volatility than the actual output series itself.

A VAR in actual and expected output growth is appropriate if output is stationary in differences and expectational errors are stationary (which will be true for any reasonable expectation formation process). For example, consider the first-order model

$$\begin{bmatrix} y_t - y_{t-1} \\ {}_t y^e_{t+1} - y_t \end{bmatrix} = \begin{bmatrix} b_{01} \\ b_{02} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} - y_{t-2} \\ {}_{t-1} y^e_t - y_{t-1} \end{bmatrix} + \begin{bmatrix} \xi_{1t} \\ \xi_{2t} \end{bmatrix}. \tag{5}$$

- This model can clearly accommodate the illustrative example of (1), with $b_{12} = 1$, $b_{22} = \rho$ and $b_{11} = b_{21} = 0$, and with $\xi_{1t} = v_t + \omega_t$ and $\xi_{2t} = \rho v_t - (1-\rho)\omega_t$.
- The FIRE assumption of the illustrative model is embedded within (5) by imposing the restriction that $b_{12} = 1$ and $b_{11} = 0$ but more sophisticated assumptions on the expectation formation process could be captured through an appropriate set of (less-rigid) restrictions in the first row of the VAR.

The VAR-E of (5) is readily written in levels form

$$\mathbf{z}_t = \Phi_0 + \Phi_1 \mathbf{z}_{t-1} + \Phi_2 \mathbf{z}_{t-2} + \boldsymbol{\varepsilon}_t$$

where $\mathbf{z}_t = (y_t, \ \ _t y^e_{t+1})'$, $\boldsymbol{\varepsilon}_t = (\xi_{1t}, \ \xi_{2t} + \zeta_{2t})'$

- The Φ_i contain functions of the parameters in (5) reflecting that, with expectational errors being stationary, actual and expected output series are cointegrated with cointegrating vector $(1, \ -1)'$.
- The associated MA representation

$$\Delta \mathbf{z}_t = \mathbf{g} + \mathbf{C}(L)\boldsymbol{\varepsilon}_t \qquad (6)$$

  has the property that $\mathbf{C}(1)$ is reduced rank, so actual and expected outputs are both driven by the same single stochastic trend and, as we'll see, share the same BN trend.
- The element of shocks that have a permanent effect on actual and expected output can be identified following the standard methods of Blanchard and Quah (1989)

The vector of BN trends, $\bar{\mathbf{z}}_t$, is defined by the long-horizon expectations of the series (minus any deterministic drift) given current information

$$\bar{\mathbf{z}}_t = \lim_{h \to \infty} E\left[{}_t\mathbf{z}_{t+h}\right] - \mathbf{g}h \tag{7}$$

where $\mathbf{g}$, the element of deterministic growth, is typically a vector of constants.

- If $\Delta\mathbf{z}_t$ can be given a stationary moving average representation of the form

  $$\Delta\mathbf{z}_t = \mathbf{g} + \mathbf{C}(L)\varepsilon_t$$

  with $\mathbf{C}(L)$ a lag polynomial and ε_t a vector of iid shocks, then the BN trends can be expressed as

  $$\Delta\bar{\mathbf{z}}_t = \mathbf{g} + \mathbf{C}(1)\varepsilon_t \tag{8}$$

  where $\mathbf{C}(1)\varepsilon_t$ represents the persistent or 'infinite horizon' effect of the shock experienced at t.

The CBI Survey asks"Excluding seasonal variations, what has been the trend over the past three months and what are the expected trends for the next three months, with regard to volume of output (i.e. production)?".

Survey participants respond that the trend has been one of ‘Up’, ‘Same’, ‘Down’ or ‘n/a’ over the two time frames. Then

$$
\begin{aligned}
{}_tR_t = \frac{n_{Rt}}{n_t} &= \text{ prop. firms, at } t\text{, responding ‘Up’ for previous quarter} \\
{}_tS_t = \frac{n_{St}}{n_t} &= \text{ prop. firms, at } t\text{, responding ‘Same’ for previous quarter} \\
{}_tF_t = \frac{n_{Ft}}{n_t} &= \text{ prop. firms, at } t\text{, responding ‘Down’ for previous quarter}
\end{aligned}
$$

while the corresponding variables referring to the expected trend over the next three months are denoted ${}_tR_{t+1}$, ${}_tS_{t+1}$ and ${}_tF_{t+1}$.

3. Survey Expectations and Structural Change

3.1 Deriving quantitative expectations series (cont.)

If the average increase in output for those reporting a rise is α and the decrease for those reporting a fall is $-\beta$, then the average across all individuals is

$$
\begin{aligned}
{}_t\bar{y}_t &= \frac{1}{n_t}\left[\sum^{n_{Rt}}\alpha + \sum^{n_{St}} 0 - \sum^{n_{Ft}}\beta\right] \\
&= \frac{1}{n_t}\left[(n_{Rt}\times\alpha) + (n_{St}\times 0) - (n_{St}\times\beta)\right] \\
&= (\,{}_tR_t\times\alpha) - (\,{}_tF_t\times\beta). \qquad (9)
\end{aligned}
$$

Using the same α and β, expected future growth is given by

$$
{}_t\bar{y}_{t+1} = (\,{}_tR_{t+1}\times\alpha) - (\,{}_tF_{t+1}\times\beta). \qquad (10)
$$

This also motivates the use of the balance statistic ${}_tB_t = {}_tR_t - {}_tF_t$ as an indicator of growth in the special case where $\alpha = \beta$, so that ${}_t\bar{y}_t = \alpha\times\,{}_tB_t$ and ${}_t\bar{x}_{t+1} = \alpha\times\,{}_tB_{t+1}$; i.e. growths move in proportion with the backward- and forward-looking balance statistics.

In practice there is no reason to suppose $\alpha = \beta$ and so the more general forms of (9) and (10) are likely to be more useful than the expressions involving the simple balance statistic.

Estimated values for α and β can be obtained by treating (9) as a relationship that holds over time with error and regressing ${}_t\bar{x}_t$ on ${}_tR_t$ and ${}_tF_t$.The estimated values can then be applied to (10) to obtain a time series for expected growth.

Realistically, α will be higher and β will be lower in good times, and vice versa in recessions. They are also likely to change with shift in the balance of transitory to permanent shocks. But the formula at (9)-(10) remain relevant even if these averages change over time if we replace α and β with time-varying α_t and β_t. Two possibilities are:

- use a rolling sample window of, say, six years and obtaining values α_t and β_t for $t = 2000q1, ..., 20120q1$ by regressing ${}_s\overline{x}_s$ on ${}_sR_s$ and ${}_sF_s$.for $s = t - 24, ..., t$; i.e. obtain time-varying values for α and β at time T by estimating the models $M_{s,T}$ defined by.

$$M_{s,T}: \quad x_t = \alpha_{s,T} \; R_t - \beta_{s,T} \; F_t + \varepsilon_{s,T} \qquad \text{for} \qquad t = T - s, ..., T. \tag{11}$$

  so that the coefficient attached to each period are based on the regression estimated over the most recent s periods.

..... or:

- use a 'meta model', rolling through the sample, but allowing more flexibility by estimating models of duration 8 quarters to 24 quarters and using a weighted average of the models with weights determined by their abilities to fit the recent data; i.e. the set of models

$$\overline{M}_{\cdot,\cdot} = \{M_{s,T}, \quad w_{s,T} \quad \text{for} \quad s = 8, .., 24, \quad T = \underline{T}, ..., \overline{T}\} \qquad (12)$$

with weights $w_{s,T}$ capturing the relevance of the candidate model at each point in the sample. One approach to deriving weights is

$$w_{s,T-1} \rightarrow \begin{cases} w_{s+1,T} & \text{if } M_{s+1,T} \text{ is not rejected for } M_{r,T} \ , r < s, \\ w_{r,T} & \text{if } M_{s+1,T} \text{ is rejected for } M_{r,T} \ , r < s \end{cases} \qquad (13)$$

Here, the weight assigned at time $T-1$ to the model based on data $T-1-s$ to $T-1$ is either transferred to the model with one additional observation - i.e. using data $T-1-s$ to T - or to a new model based on the shorter sample of data $T-r$ to T.

A useful measure of (*ex post*) uncertainty at time t over outcomes at time $t+1$ is provided by the size of the expectation error given by

$$ {}_{t}x_{t+1}^{uc} = \left(x_{t+1} - {}_{t}\overline{x}_{t+1}\right)^{2} $$

- The measure will depend on how the expected output series is derived
- This measure is dominated by the actual outcome though and does not properly measure the uncertainty that existed at t.
- A way to deal with this is to estimate an ARCH model explaining the squared error with an intercept and error that has time-varying standard deviation (driven by squared errors dated at t), using the fitted value of the estimated standard deviation as measure of *ex ante* uncertainty.

The interplay between a variable's actual outcomes, individuals' expectations of the variable over different horizons and the associated uncertainties can be captured in a 'meta VAR-E' model of order q relating to the sample $T - s_{\max}, ..., T$:

$$M_T : \quad \begin{bmatrix} \widetilde{y}_t - \widetilde{y}_{t-1} \\ {}_t\widetilde{y}^e_{t+1} - \widetilde{y}_t \\ {}_t\widetilde{y}^u_{t+1} \end{bmatrix} = \mathbf{B}_{0,T} + \sum_{s=1}^{q} \mathbf{B}_{i,T} \begin{bmatrix} \widetilde{y}_{t-i} - \widetilde{y}_{t-i-1} \\ {}_{t-i}\widetilde{y}^e_{t-i+1} - \widetilde{y}_{t-i} \\ {}_{t-i}\widetilde{y}^u_{t-i+1} \end{bmatrix} + \begin{bmatrix} \xi_{1t} \\ \xi_{2t} \\ \xi_{3t} \end{bmatrix}, \tag{14}$$

for $t = T - s_{\max}, ..., T$, and where $\widetilde{y}_{t-s} = \sqrt{w_{s,T}} \times y_{t-s}$ and ${}_t\widetilde{y}^u_{t+1} = \sqrt{w_{s,T}} \times {}_ty^u_{t+1}$, again for $t = T - s_{\max}, ..., T$.

- Here each of the equations in the VAR is estimated using weighted least squares with weights defined by the survey-based exercise above.

- The model can be estimated working recursively through the data, delivering estimates of $\mathbf{B}_{i,T}$, $i = 0, .., q$ and for $T = \underline{T}, ..., \overline{T}$. Then we obtain the date-specific MA representation

$$\Delta \mathbf{z}_t = \mathbf{g}_T + \mathbf{C}_T(L)\varepsilon_t, \qquad t = T - s_{\max}, ..., T$$

  with the $\mathbf{C}_T(L)$ obtained from the parameters of the $\mathbf{B}_{i,T}$.
- The BN trend is then defined by

$$\Delta \bar{\mathbf{z}}_T = \mathbf{g}_T + \mathbf{C}_T(1)\varepsilon_T \ . \tag{15}$$

  The BN trend in T is equal to its value in the previous period plus the accumulated future effects of the time$-T$ shock based on the parameters of the estimated meta VAR-E model
- This assumes that the infinite horizon effect of past shocks - embedded within last period's BN trend - remains unchanged.

- Figure 1: shows the quarterly output growth series and associated qualitative survey responses
- Figure 2 shows the quantitative series derived using various quantification methods:
  - the 'balance statistic' approach, assuming α and β in (9) and (10) are constant over time;
  - the 'rolling modelling' approach in which α and β are allowed to vary over time with their estimates based on a rolling regression analysis of 25 quarters; and
  - the 'meta modelling' approach in which α and β vary over time with their estimates based on estimated model averages.
- Figure 3 plots 'duration' statistics
- Figure 4 plots the derived the uncertainty measure

Figure 1
Quarterly Output Growth and Survey Responses

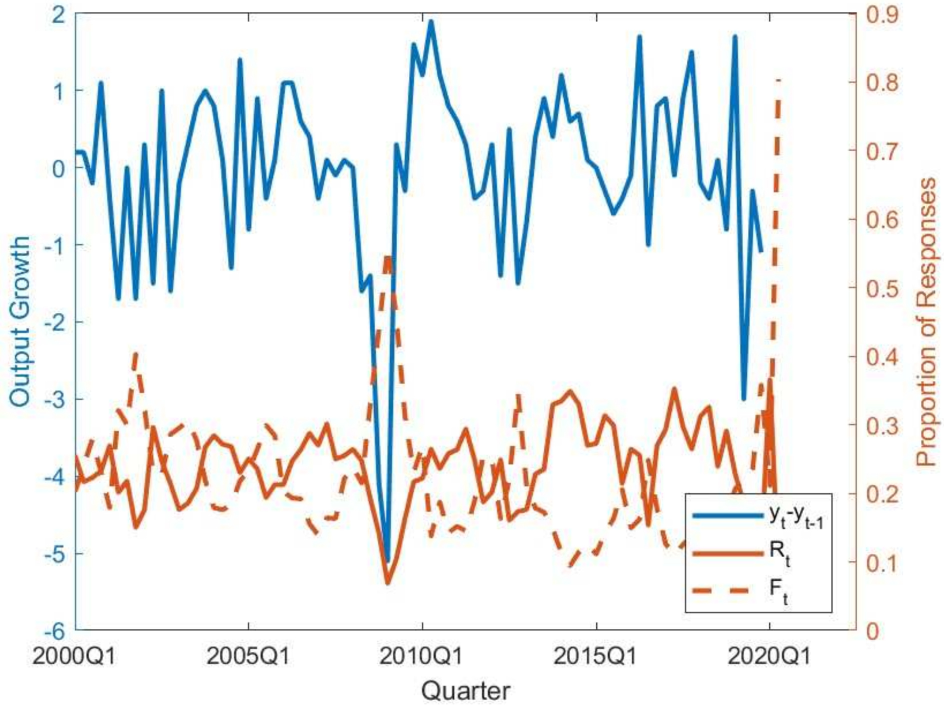


Figure 2
Actual and Expected Output Growth

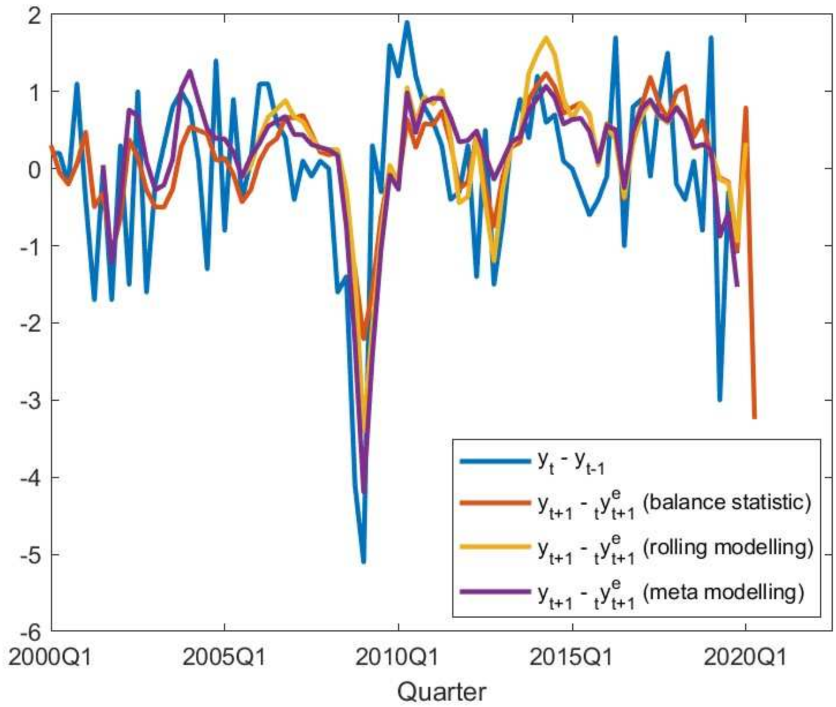


Figure 3
Sample Duration and Differences in Time-Varying Parameters

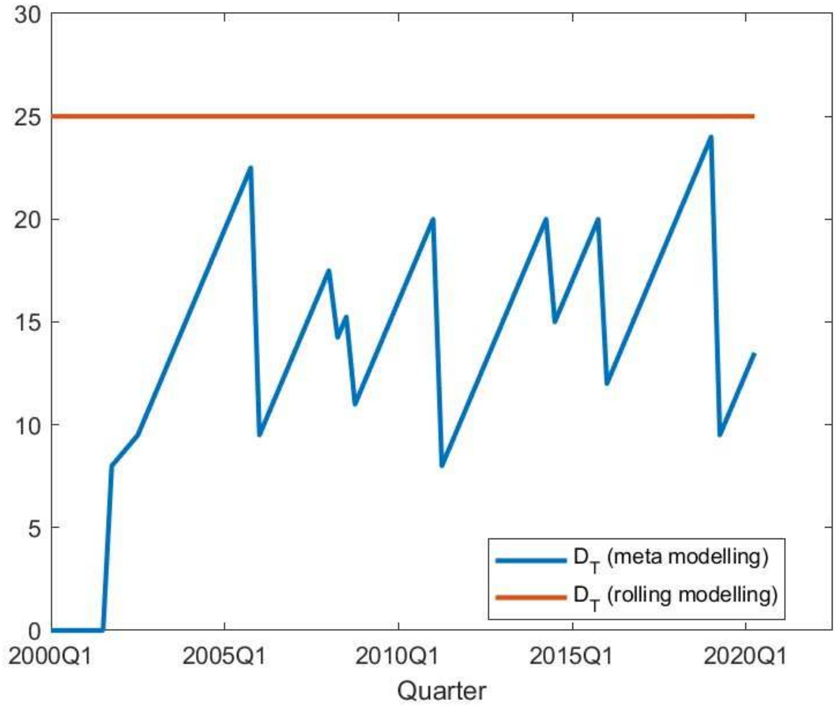
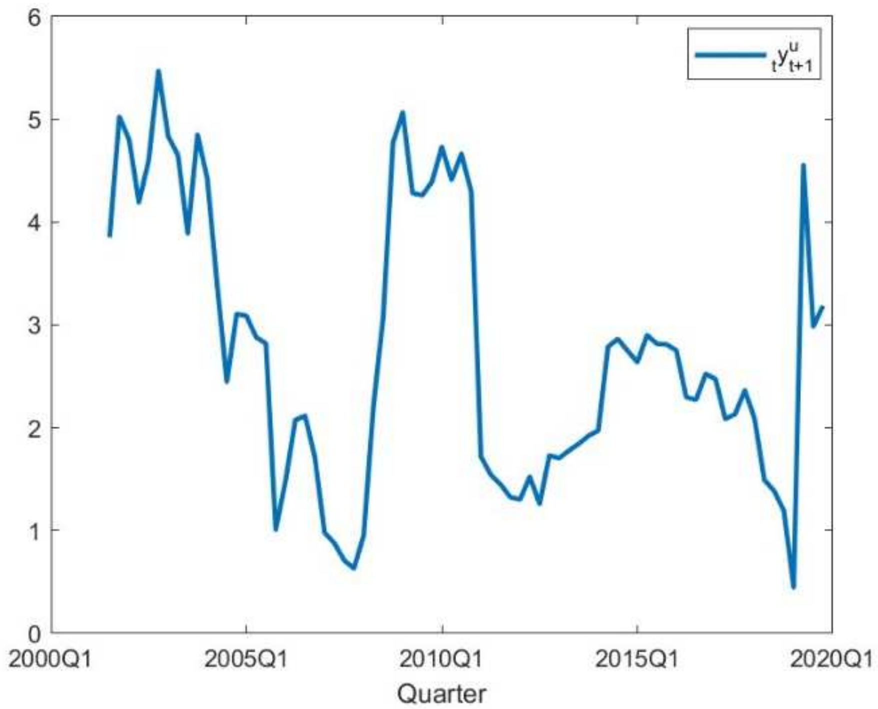


Figure 4
Uncertainty over Output



- VAR-E models, explaining actual output, expected output and uncertainty about output, estimated three ways:
  - M^{WHOL}, estimated using the whole sample;
  - M^{ROLL}, estimated recursively using a rolling window of 25 observations (with all observations given equal weight); and
  - M^{META}, estimated recursively using a rolling window of 25 observations with the three series adjusted according to the weights obtained in the meta modelling exercise
- Figure 5(a)-(c) provide an indication of the system dynamics obtained from the models, focusing on M^{WHOL}
- Figures 6 and 6*ℓ* illustrate the system dynamics obtained from the M^{META} analysis
  - Figure 6 plots impulse responses for four specific dates
  - Figure 6*ℓ* shows impulse responses for every point in the sample

Figure 5(a)

Response of Actual Output, Expected Output and Uncertainty over Output to an Uncertainty Shock Using M^{WHOL}

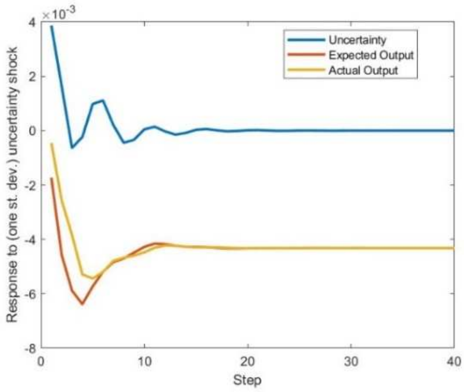


Figure 5(b)

Response of Actual Output, Expected Output and Uncertainty over Output to an Actual Output Shock Using M^{WHOL}

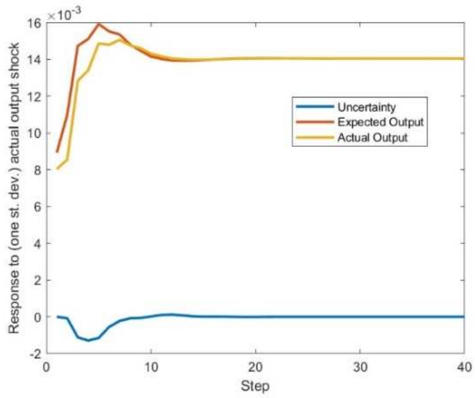


Figure 5(c)

Response of Actual Output, Expected Output and Uncertainty over Output to an Expected Output Shock Using M^{WHOL}

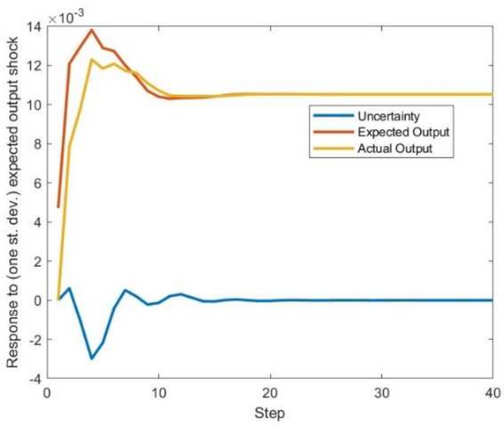


Figure 6
Response of Actual Output to an Actual Output Shock at Different Times Using M^{META}

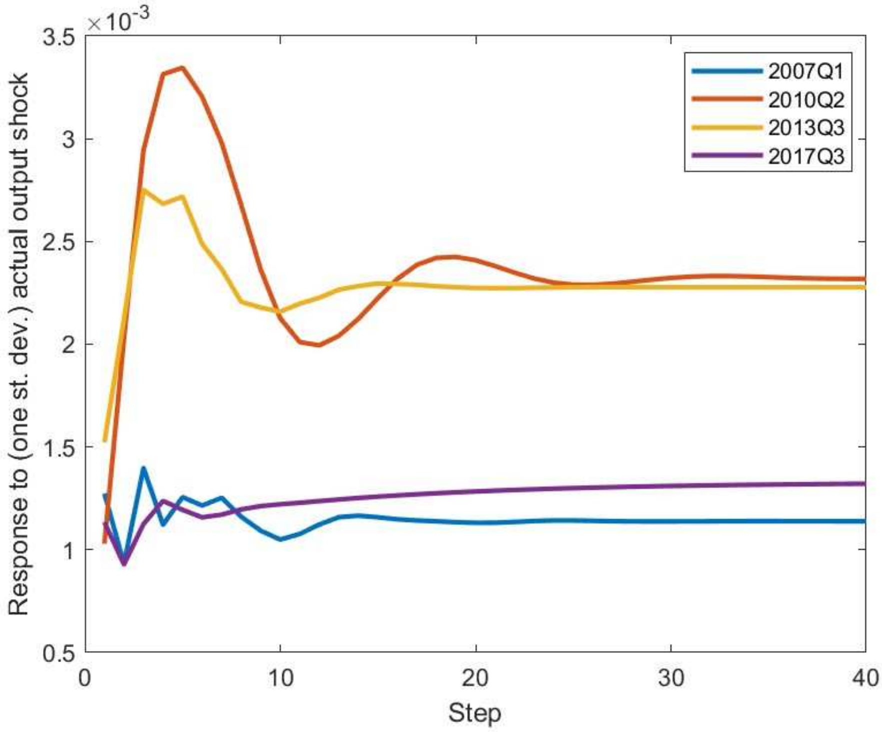
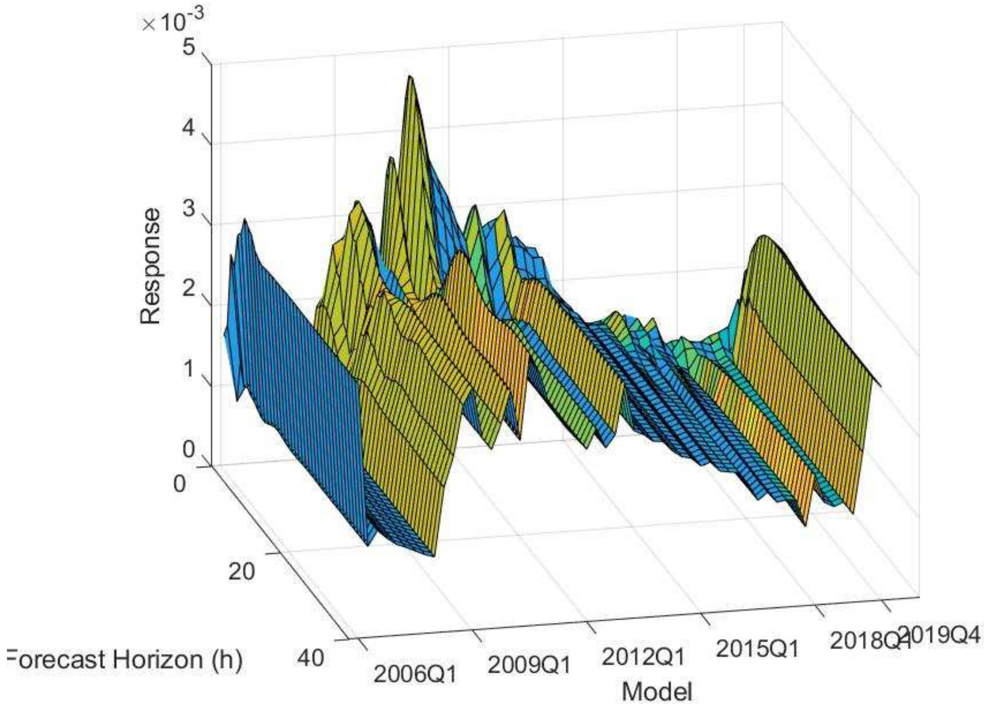


Figure 6'
Response of Actual Output to an Actual Output Shock across Times Using M^{META}



- Figures 7 plots the three variants of the BN output trend obtained from M^{WHOL}, M^{ROLL} and M^{META} alongside actual output.
    - the correlations between the three series all exceeding 0.7 but they are qualitatively and quantitatively distinct
- Figure 8 plots the output gap measures obtained from the three methods, smoothed by averaging the gap over the previous year.
- As a thought experiment, Figure 9 plots again the actual output level and the trend based on M^{META} but now also plots the BN trend that would be obtained from that model assuming no uncertainty shocks occurred after 2008q2 (assuming a Choleski ordering in which uncertainty shocks come last).

Figure 7

Actual and Trend Output Using M^{WHOL}, M^{ROLL} and M^{META}

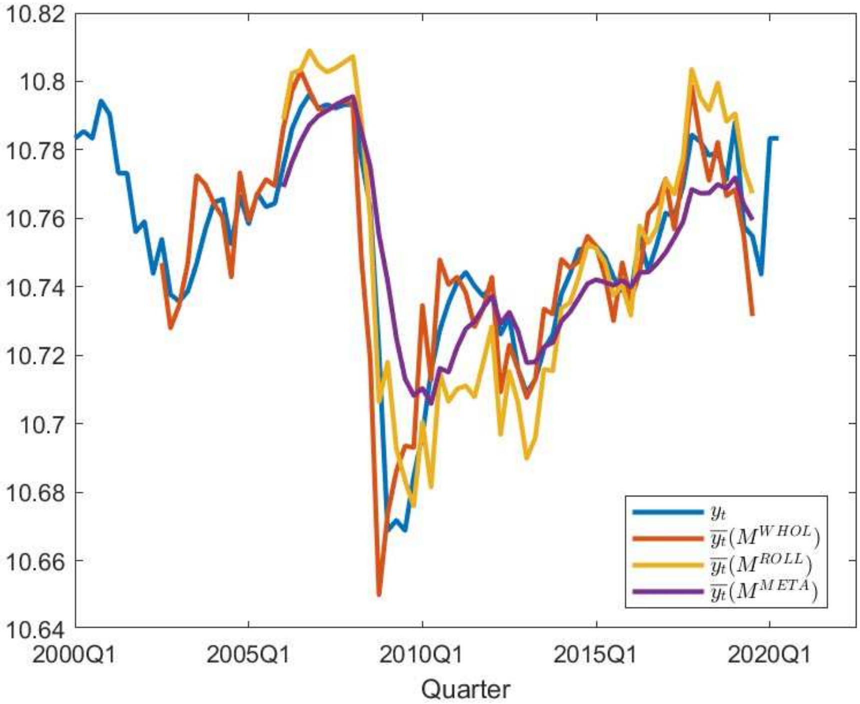


Figure 8
(Smoothed) Output Gaps Using M^{WHOL}, M^{ROLL} and M^{META}

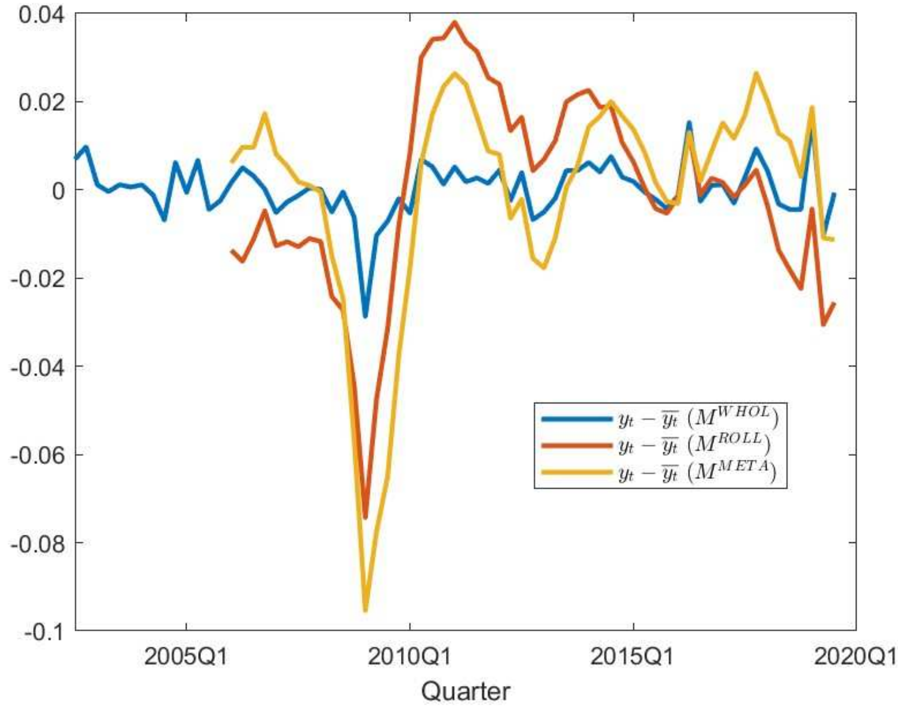
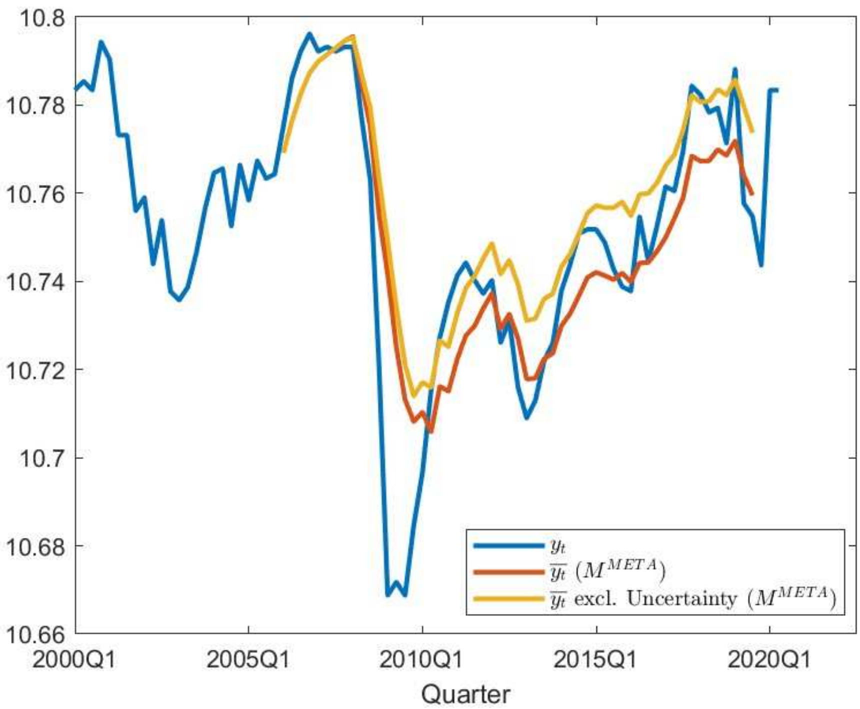


Figure 9
Actual and Trend Output Using M^{META} with and without Uncertainty Shocks



5. Concluding comments

The use of direct measures of expectations mitigates against the volatility of the actual expectations series, offsetting the effects of known-to-be-short-lived shocks, in defining the BN trend.

CBI data provide very credible direct measures of expectations when properly take into account structural change with meta modelling.

The VAR-E models estimated using time-varying weights deliver trend and output gap measures which are very sensible, are very different to models that fail to accommodate change and which provide useful characterisation of experiences over the last 20 years

Rise in uncertainty has a downward effect on output; not main source of output fluctuations but significant (esp at particular times).