Design-Robust Two-Way-Fixed-Effects Regression For Panel Data

Dmitry Arkhangelsky

CEMFI

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Joint work with Lihua Lei, Guido Imbens, and Xiaoman Luo

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- Fundamentally based on a regression model rather than design assumptions
- Nevertheless, design-based thinking is invoked for "identification" arguments
- We show how to combine regression methods with design knowledge for experimental and observational data

Motivating example

- In Brazil the federal government **randomly** audits the local ones to prevent corruption
- Randomization is **staggered**: some municipalities are audited earlier, some – later
- Staggered rollout $\Rightarrow$ DiD-type analysis (Colonnelli and Prem'21)
- Alternatively, can do a fully experimental analysis (?)
- We show how to conduct both simultaneously

Intuition

- Literature on experiments:
  1. Regressions applied to experimental data can recover causal parameters even if misspecified
  2. But need to be careful if experiments are complicated
- Literature on regressions:
  1. If OLS works, then weighted OLS should also work
  2. But there can be a price in terms of standard errors
- This is the logic behind the paper, but there are a lot of details

Two-way fixed effect (TWFE) regression model and estimator

$$\text{TWFE model}: \underbrace{Y_{it}}_{\text{outcome}} = \underbrace{\alpha_i}_{\text{unit FE}} + \underbrace{\lambda_t}_{\text{time FE}} + \underbrace{\tau}_{\text{effect}} \cdot \underbrace{W_{it}}_{\text{treatment}} + \beta \cdot \underbrace{X_{it}}_{\text{covariates}} + \epsilon_{it}$$

$$\text{TWFE estimator}: \hat{\tau}_{\text{TWFE}} \leftarrow \texttt{OLS}(Y_{it} \sim \text{unit dummy} + \text{time dummy} + W_{it} + X_{it})$$

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- DiD estimator $\Longleftrightarrow$ TWFE (with $T = 2$ and no covariates)
- $\hat{\tau}_{\text{TWFE}}$ is unbiased for τ under the TWFE model
- Biased with heterogeneous treatment effect or violation of parallel trends Borusyak et al '17, Goodman-Bacon '17, de Chaisemartin and d'Haultfoeuille '18, Athey and Imbens '18, Sun and Abraham '18
- Many alternative methods recently Imai and Kim '16, Athey et al. '17, Borusyak et al. '17, Callaway and Sant'Anna '18, de Chaisemartin and d'Haultfoeuille '18, Sun and Abraham '18, Arkhangelsky and Imbens '19, Arkhangelsky et al. '19, Ben-Michael et al. '19, Roth and Sant'Anna '20, ...

Part I: DATE, RIPW, and design-based inference

Potential outcomes and doubly average treatment effect (DATE)

- Balanced panel: n units and T time periods; fixed T
- Binary treatment: $\mathbf{W}_i = (W_{i1}, \ldots, W_{iT})$; $\mathbf{W}_i \sim \boldsymbol{\pi}_i$ generalized propensity score Imbens '00
- Potential outcomes: $(Y_{it}(1), Y_{it}(0))_{t=1}^{T}$; observed outcome $Y_{it} = Y_{it}(W_{it})$ (SUTVA)
- For simplicity no covariates for this part
- Causal estimand: average effect with **user-specified** weights $\xi = (\xi_1, \ldots, \xi_t)$

$$\tau_{\text{DATE}}(\xi) = \sum_{t=1}^{T} \xi_t \left(\frac{1}{n} \sum_{i=1}^{n} (Y_{it}(1) - Y_{it}(0)) \right) \triangleq \sum_{t=1}^{T} \xi_t \tau_t, \quad \text{e.g., } \tau_{\text{eq}} = \frac{1}{T} \sum_{t=1}^{T} \tau_t$$

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How to leverage the treatment assignment mechanism to estimate DATE?

Discussion I

- All randomness comes from $\{\mathbf{W}_i\}_{i=1}^n$, which can be dependent (experimental analysis)
- Static model: potential outcomes depend only on current treatments
- Treatment effects can vary over units and periods:

$$\tau_{it} \triangleq Y_{it}(1) - Y_{it}(0)$$

Discussion II

- In simple experiments $\boldsymbol{\pi}_i = \boldsymbol{\pi}$ – the same protocol for all units
- Often not the case, e.g., different municipalities in Brazil have different chances of being audited
- Two questions:
  1. If $\boldsymbol{\pi}_i = \boldsymbol{\pi}$ does the TWFE estimator work?
  2. How should we adjust it if $\boldsymbol{\pi}_i$ varies over units?
- Answer both questions simultaneously

First thought: IPW estimator

For cross-sectional data, the Hájek-IPW estimator is given by

$$\hat{\tau} = \frac{\sum_{W_i=1} Y_i/\mathbb{P}(W_i = 1)}{\sum_{W_i=1} 1/\mathbb{P}(W_i = 1)} - \frac{\sum_{W_i=0} Y_i/\mathbb{P}(W_i = 0)}{\sum_{W_i=0} 1/\mathbb{P}(W_i = 0)} \xrightarrow{p} \mathrm{ATE}$$

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Numerically equivalent to an IP-weighted LS estimator:

$$\hat{\tau} \triangleq \arg\min_{\tau} \sum_{i=1}^{n} \underbrace{(Y_i - \mu - W_i\tau)^2}_{\text{least squares objective}} \underbrace{\frac{1}{\pi_i(W_i)}}_{\text{propensity score}}$$

Key idea: reweighting the objective function via the treatment assignment mechanism

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Analog in the panel data:

$$\hat{\tau}_{\text{IPW}} \triangleq \arg\min_{\tau} \sum_{i=1}^{n} \sum_{t=1}^{T} \underbrace{(Y_{it} - \alpha_i - \lambda_t - W_{it}\tau)^2}_{\text{TWFE objective}} \qquad \frac{1}{\underbrace{\boldsymbol{\pi}_i(\mathbf{W}_i)}_{\text{generalized propensity score}}}$$

Reduces to the TWFE if $\boldsymbol{\pi}_i(\cdot) = \boldsymbol{\pi}(\cdot) = \text{const}$

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Two examples with 3 time periods

Transient treatments

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$$\mathbf{W}_i \in \{(0,0,0),(0,0,1),(0,1,1),(1,1,1)\}$$

$$\hat{\tau}_{\text{IPW}} \xrightarrow{p} 0.3\tau_1 + 0.4\tau_2 + 0.3\tau_3$$

Effective estimand of IPW-TWFE estimator

Theorem (Arkh., Imbens, Lei, and Luo '21)

Under regularity conditions (**overlap**, **limited dependence**, **bounded moments***), as* $n \to \infty$,

$$\hat{\tau}_{IPW} \xrightarrow{p} \sum_{t=1}^{T} \xi_t \tau_t$$

and for transient/staggered designs

$$\xi_t \propto \eta_t(1 - \eta_t), \quad \textit{where } \eta_t = \frac{|w \in \mathbb{S} : w_t = 1|}{|\mathbb{S}|},$$

where $\mathbb{S} = \bigcup_i \mathrm{Supp}(\mathbf{W}_i)$

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What if we want DATE with pre-specified weights (e.g., τ_{eq})?

Reshaped IPW estimator

Given a data-independent distribution $\mathbf{\Pi}$ on $\mathbb{S}$:

$$\text{RIPW estimator: } \hat{\tau}_{\text{RIPW}}(\mathbf{\Pi}) \triangleq \arg\min_{\tau} \sum_{i=1}^{n} \sum_{t=1}^{T} (Y_{it} - \alpha_i - \lambda_t - W_{it}\tau)^2 \frac{\mathbf{\Pi}(\mathbf{W}_i)}{\boldsymbol{\pi}_i(\mathbf{W}_i)}$$

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- The IPW-TWFE estimator is the RIPW-TWFE estimator with $\mathbf{\Pi} \sim \mathrm{Unif}(\mathbb{S})$
- When $\boldsymbol{\pi}_i = \mathbf{\Pi}$, the RIPW-TWFE estimator reduces to the TWFE estimator

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For what $\mathbf{\Pi}$ does $\hat{\tau}_{\text{RIPW}}(\mathbf{\Pi}) \xrightarrow{p} \tau_{\text{DATE}}(\xi)$?

DATE equation

Theorem (Arkh., Imbens, Lei, and Luo '21)

Given $\mathbb{S}$ *and* $\mathbf{\Pi}$ *with* $\mathrm{Supp}(\mathbf{\Pi}) = \mathbb{S}$, $\hat{\tau}_{TWFE} \xrightarrow{p} \tau_{DATE}(\xi)$ *iff*

$$\mathbb{E}_{\mathbf{W}\sim\mathbf{\Pi}}\left[(\mathrm{diag}(\mathbf{W}) - \xi\mathbf{W}^{\top})J(\mathbf{W} - \mathbb{E}_{\mathbf{W}\sim\mathbf{\Pi}}[\mathbf{W}])\right] = 0 \quad (\textit{DATE equation}),$$

where $J = I - \mathbf{1}_T\mathbf{1}_T^{\top}/T$.

- Only depends on the support
- Quadratic equations on $(\mathbf{\Pi}(w) : w \in \mathbb{S})$ with linear constraints (simplex, positivity)
- Closed-form solutions exist in many examples (DiD, cross-over, staggered rollouts, transient, ...)

An interpretation of DATE equation

- When $\boldsymbol{\pi}_i = \boldsymbol{\Pi}$, $\hat{\tau}_{\text{TWFE}} = \hat{\tau}_{\text{RIPW}}(\boldsymbol{\Pi}) \xrightarrow{p} \tau_{\text{DATE}}(\xi)$

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- DATE equation gives a completely randomized experiment for which TWFE "works"!
- Conflict with the literature that TWFE has negative weights?
- Not really! $\mathbf{W}_i$'s are treated as fixed in the literature but as random in our work
- When talking about "weights", important to specify the sources of randomness

Negative weighting

- The weights discussed in the literature:

$$\underbrace{\mathbb{E}[\hat{\tau}_{\text{TWFE}} \mid \mathbf{W}]}_{\text{conditional estimand}} = \sum_{i=1}^{n} \sum_{t=1}^{T} \underbrace{\zeta_{it}(\mathbf{W})}_{\text{conditional weight}} \tau_{it}$$

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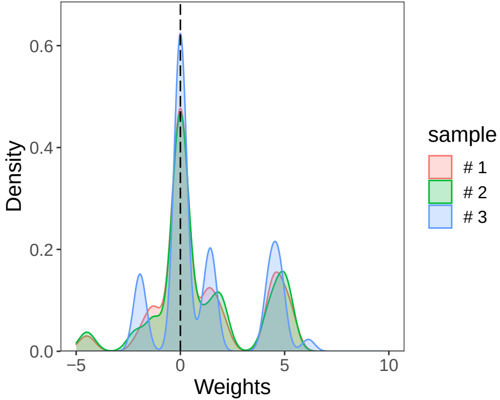
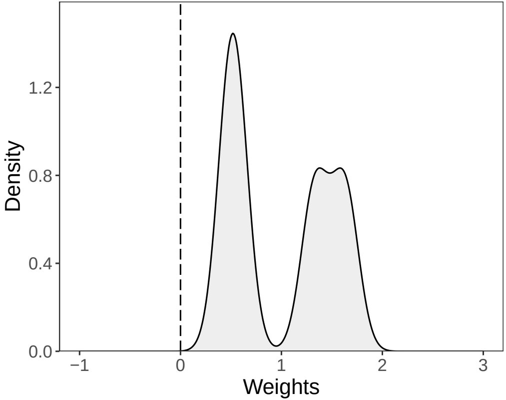
The result proved in our work: for any design for which the DATE equation has a solution:

$$\forall (i, t) : \mathbb{E}[\zeta_{it}(\mathbf{W})] > 0$$

Negative weighting for TWFE: simulation example



Conditional weights $\mathbb{E}[\zeta_{it} \mid \mathbf{W}]$ (histogram)



Unconditional weights $\mathbb{E}[\zeta_{it}]$ (histogram)

$n = 1000$, $T = 4$, staggered rollout

Design-based inference for RIPW estimator

$$\text{RIPW estimator: } \hat{\tau}(\mathbf{\Pi}) \triangleq \arg\min_{\tau} \sum_{i=1}^{n}\sum_{t=1}^{T}(Y_{it} - \alpha_i - \lambda_t - W_{it}\tau)^2 \frac{\mathbf{\Pi}(\mathbf{W}_i)}{\boldsymbol{\pi}_i(\mathbf{W}_i)}$$

Theorem

Under the same setting, as $n \to \infty$,

$$\sqrt{n}(\hat{\tau}(\mathbf{\Pi}) - \tau(\xi)) = \frac{1}{\sqrt{n}}\sum_{i=1}^{n}\mathcal{V}_i + o_{\mathbb{P}}(1) \quad (\textit{RIPW converges to DATE})$$

for some complicated but estimable "influence functions" $\mathcal{V}_i$

Lessons

- In simple experiments ($\boldsymbol{\pi}_i = \boldsymbol{\pi}$) the TWFE estimator delivers $\tau(\xi)$, where ξ depends on $\boldsymbol{\pi}$
- In complicated experiments, you need to adjust the TWFE (or rely on the model!)
- Adjustment is simple: the propensity weights + factor that fixes the estimand
- The RIPW estimator is asymptotically normal and can be used for inference

Part II: Double robustness of RIPW estimator

Discussion

- In academic research panel experiments are still rare and we use TFWE for observational data
- Nevertheless, we can construct $\hat{\boldsymbol{\pi}}_i$ and hope that it is close enough to $\boldsymbol{\pi}_i$
- Researches often do this informally, e.g., by relating adoption dates to observed covariates
- RIPW allows us to do this systematically

Estimating assignment models

- Different designs require for different models:
  1. Staggered rollout – duration models, e.g., Cox proportional hazard model
  2. Independent decisions – logit model
  3. Dynamic decisions – stationary Markov models
- In the first case, can use $\mathbf{X}_i = (X_{i1}, \ldots, X_{iT})$ to fit an assignment model $\hat{\boldsymbol{\pi}}_i(\cdot)$:
- In the last two cases, can additionally incorporate unobserved heterogeneity using sufficiency arguments

RIPW estimator is double robust for observational studies

$$\hat{\tau}(\mathbf{\Pi}) \triangleq \arg\min_{\tau} \sum_{i=1}^{n} \sum_{t=1}^{T} \Big(\underbrace{(Y_{it} - \hat{m}_{it})}_{\text{regression adjustment}} - \alpha_i - \lambda_t - W_{it}\tau \Big)^2 \frac{\mathbf{\Pi}(\mathbf{W}_i)}{\underbrace{\hat{\pi}_i(\mathbf{W}_i)}_{\text{assignment modeling}}}$$

- For the regression adjustment can use $\beta^\top X_{it}$ or something more flexible
- **Robustness**: RIPW$\xrightarrow{p}$DATE if
  - either the assignment model is well estimated
  - or the TWFE model is correct
- We also derive CI by **cross-fitting** $\hat{m}_{it}$ and $\hat{\pi}_i(\cdot)$

Intuition for robustness

- If the assignment model is well estimated ($\hat{\boldsymbol{\pi}}_i \approx \boldsymbol{\pi}_i$),

$$\text{Design-based results} \implies \text{Consistency}$$

- If the TWFE model is approximately correct after regression adjustment ($\hat{m}_{it} \approx m_{it}$),

$$\text{Weighted OLS} \implies \text{Consistency}$$

Part III: Empirical illustration

State of emergency in the early COVID-19 pandemic

Inslee issues COVID-19 emergency proclamation

February 29, 2020

Story

Gov. Jay Inslee today declared a state of emergency in response to new cases of COVID-19, directing state agencies to use all resources necessary to prepare for and respond to the outbreak.

A **state of emergency** is a situation in which a government is empowered to perform actions or impose policies that it would normally not be permitted to undertake

How the state of emergency affects economic activities in short term

- Interested in ATE of the state of emergency on dine-in rate during 02/29 – 03/13, 2020
  - State of emergency was less confounded; the first policy affecting the vast majority of the public
  - Restaurant industry is responding to the policy swiftly, thus immune to long-term confounders

How the state of emergency affects economic activities in short term

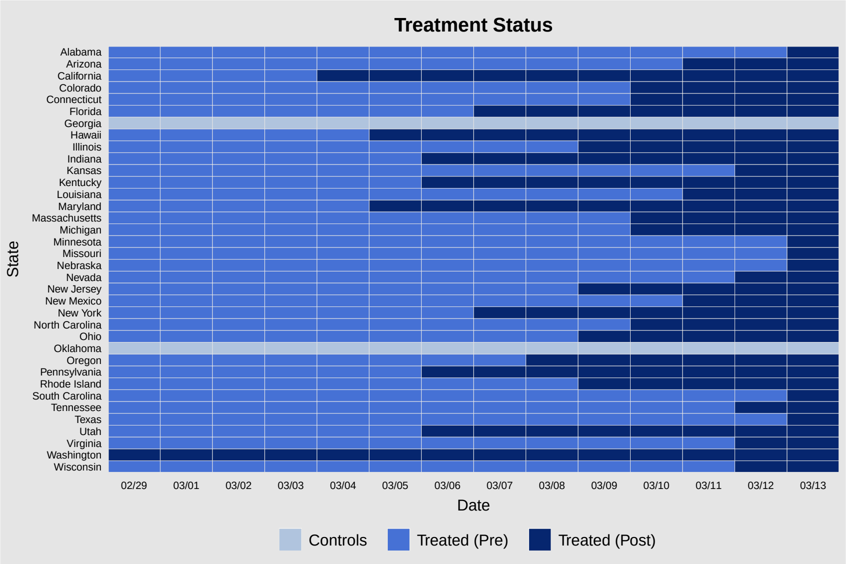
- Interested in ATE of the state of emergency on dine-in rate during 02/29 – 03/13, 2020

- Declaration time (assignment model) is easier to model than the dine-in rate (outcome model)
  - Dine-in rate is driven by many unmeasured behavioral variables
  - Declaration time is mainly driven by the progress of the pandemic and the authority's attitude

How the state of emergency affects economic activities in short term

- Interested in ATE of the state of emergency on dine-in rate during 02/29 – 03/13, 2020
- Declaration time (assignment model) is easier to model than the dine-in rate (outcome model)
- Covariates:
  - State-level accumulated confirmed cases
  - The vote share of Democrats based on the 2016 presidential election data
  - Number of hospital beds per-capita

Cox proportional hazard model for assignment model



Cox proportional hazard model for assignment model

- Treat the declaration of a state of emergency as an "event"
- Fit the distribution of declaration time T_i by the Cox model (related to Shaikh and Toulis '19)
- Let $\hat{F}_i$ be the estimated survival function of unit i,

$$\hat{\pi}_i(\mathbf{W}_i) = \begin{cases} \hat{F}_i(T_i) - \hat{F}_i(T_i + 1) & \text{(State } i \text{ declares before 03/13)} \\ 1 - \hat{F}_i(03/13) & \text{(otherwise)} \end{cases} .$$

RIPW estimate

- For outcome model, fit $\hat{m}_{it} = X_{it}^\top \hat{\beta}_{\text{TWFE}}$
- For both models, use 10-fold cross-fitting
- Estimand: equally-weighted DATE
- Estimate: -4.0% (95% CI $[-8.6\%, 0.6\%]$, 90% CI $[-7.9\%, -0.1\%]$)
- Unweighted TWFE: -1.1% (95% CI $[-4.3\%, 2.1\%]$, 90% CI $[-3.8\%, 1.6\%]$)

Part IV: Extensions

Dynamics

- The static model is restrictive and goes against common practice in applications
- Researchers usually estimate a more flexible event study specification:

$$Y_{it} = \alpha_i + \lambda_t + \sum_{j=0}^{k} \tau_j W_{it-j} + \epsilon_{it}$$

- It is important to understand if this presents a challenge for our strategy

Preliminary results on dynamics

- Potential outcomes now depend on histories:

$$(w_1, \ldots, w_t) \to Y_{it}(w_1, \ldots, w_t)$$

- Suppose we have a separable time-homogenous model:

$$Y_{it}(w_1, \ldots, w_t) = \alpha_{it} + \sum_{j=0}^{k} \tau_{ij} w_{t-j}$$

- Then the reweighted event-study specification is design-robust in the same sense as before

Conclusion

Summary

- **DATE equation** identifies all randomization schemes for which TWFE converges to DATE
- **RIPW estimators** permit valid design-based **inference** for most practical designs
- They are **double-robust** and work for **general designs** (not limited to staggered adoption)
- Practically, they empower the users to leverage the information from the assignment model