Idiosyncratic Shocks and Investment Irreversibility: Capital Misallocation over the Business Cycle

Tatsuro Senga
Queen Mary University and Keio University

Iacopo Varotto
Bank of Spain

16th Research Workshop BdE-CEMFI
Banco de España - April 8, 2025

The opinions in this presentation are those of the authors and do not necessarily reflect the views of the Banco de España or the Eurosystem

Motivation

- **Does capital irreversibility matter for aggregate dynamics in a granular world?**
  - High asset specificity for nonfinancial firms (Kermani and Ma, 2023)
  - At Micro-level, a small amount of irreversibility can generate a large range of inaction

Motivation

- **Does capital irreversibility matter for aggregate dynamics in a granular world?**
    - High asset specificity for nonfinancial firms (Kermani and Ma, 2023)
    - At Micro-level, a small amount of irreversibility can generate a large range of inaction
- Benchmark RBC economy: no, same aggregate volatility as its frictionless counterpart
    - At macro level, irrelevant given the small volatility of the aggregate shock (Veracierto, 2002)

Motivation

- **Does capital irreversibility matter for aggregate dynamics in a granular world?**
  - High asset specificity for nonfinancial firms (Kermani and Ma, 2023)
  - At Micro-level, a small amount of irreversibility can generate a large range of inaction

- Benchmark RBC economy: no, same aggregate volatility as its frictionless counterpart
  - At macro level, irrelevant given the small volatility of the aggregate shock (Veracierto, 2002)

- This paper: yes, different aggregate volatility compared to its frictionless counterpart

  1. Fluctuations depend on ergodic $\mathrm{Var}\text{-}\mathrm{Cov}$ of $\mu(\text{capital}, \text{productivity})$
  2. Build a quantitative model with granular shocks for the US economy
  3. Show implications for the aggregate fluctuations + quantification of granular components
  4. Validation of model predictions
  5. Welfare implications of subsidies to downsize firms

Our contributions

1. Fluctuations depend on the second-order ergodic moments of $\mu(\text{capital}, \text{productivity})$

    - Volatility increasing in the ergodic dispersion of capital-productivity, and their covariance
    - granular components:
        - *Time-varying capacity* – fluctuations of $\mu(\textit{productivity})$ and *K*
        - *Time-varying misallocation* – fluctuations of $\mathrm{Cov}(\text{capital}, \text{productivity})$

Our contributions

1. Fluctuations depend on the second-order ergodic moments of $\mu(\text{capital}, \text{productivity})$

2. Build a quantitative model with granular shocks for the US economy
   - Accounting for the micro/macro regularities of the ergodic economy
   - Global non-linear solution method with firm heterogeneity and aggregate uncertainty
     - Shocks replicating the fluctuations of the productivity distribution

Our contributions

1. Fluctuations depend on the second-order ergodic moments of $\mu(\text{capital}, \text{productivity})$

2. Build a quantitative model with granular shocks for the US economy

3. Show implications for the aggregate fluctuations + quantification of granular components
    - 35 % irreversibility shrinks granular volatility by 22%
        - *Time-varying misallocation* drives 36% of the granular volatility (50% without irreversibility)
    - Granular shocks explain around 30 % of the aggregate volatility for the period 1980-2019

Our contributions

1. Fluctuations depend on the second-order ergodic moments of $\mu(\text{capital}, \text{productivity})$

2. Build a quantitative model with granular shocks for the US economy

3. Show implications for the aggregate fluctuations $+$ quantification of granular components

4. Granular shocks cause fluctuations in capital misallocation

Our contributions

1. Fluctuations depend on the second-order ergodic moments of $\mu(\text{capital}, \text{productivity})$
2. Build a quantitative model with granular shocks for the US economy
3. Show implications for the aggregate fluctuations + quantification of granular components
4. Granular shocks cause fluctuations in capital misallocation
5. Subsidies to downsize unproductive firms improve welfare

Related Literature

Aggregate implications of lumpy investment

- Veracierto (2002); Khan and Thomas (2003,2008); Gourio and Kashyap (2007); Bachmann, Caballero, and Engel (2013); Bachmann and Ma (2016); Lantieri (2018); Wimberry (2021); Baley and Blanco (2024);

Micro-origins of aggregate fluctuations

- Carvalho (2010); Gabaix (2011); Acemoglu et al. (2012); di Giovanni and Levchenko (2012); Carvalho and Gabaix (2013); Stella (2015); di Giovanni, Levchenko and Mejean (2018); Carvalho and Grassi (2019); Yeh (2023); Ifergane (2024);

High moment shocks

- Bloom et al. (2018); Salgado et al. (2023);

Our contributions

1. Fluctuations depend on the second-order ergodic moments of $\mu(\text{capital}, \text{productivity})$
2. Build a quantitative model with granular shocks for the US economy
3. Show implications for the aggregate fluctuations + quantification of granular components
4. Granular shocks cause fluctuation in capital misallocation
5. Welfare implications of subsidies to downsize firms

Our contributions

1. **Fluctuations depend on the $\mu(\text{capital}, \text{productivity})$ second-order ergodic moments**

2. Build a quantitative model with granular shocks for the US economy

3. Show implications for the aggregate fluctuations + quantification of granular components

4. Granular shocks cause fluctuation in capital misallocation

5. Welfare implications of subsidies to downsize firms

Granularity in a Simple Model

- **Small open economy**
  - r interest rate

- **Heterogeneous firms** indexed by $i \in (0, N]$ with $N < \infty$
  - Perfectly competitive
  - Produce $y_{i,t}(\varepsilon, k_{i,t}) = \varepsilon_{i,t} k_{i,t}^{\alpha}$ with $\alpha < 1$
  - $\varepsilon_{i,t}$ stochastic firm-level productivity
  - Invest capital one period ahead $k_{i,t+1} = i_{i,t} + (1 - \delta) k_t$

- **Firm-level productivities** evolve according to

$$\ln(\varepsilon_{i,t+1}) = \rho_{\varepsilon} \xi_{i,t} + (1 - \rho_{\varepsilon}) \sigma_{\epsilon} \epsilon_{i,t+1}$$

  - $\xi_{i,t} \sim \mathcal{N}(\bar{\xi}, \sigma_{\xi})$ known after the production at t and $\epsilon_t \sim \mathcal{N}(0, 1)$
  - $\sigma_{\epsilon} > 0$ and $\rho_{\varepsilon} \in [0, 1]$

Granularity in a Simple Model

- **Lemma 1** Idiosyncratic output y_i is distributed as $\log(y_i) \sim \mathcal{N}(\mu_y, \sigma_y)$

$$\begin{aligned} \mu_y &= \mathrm{E}(\ln(\varepsilon_{i,t+1})) + \alpha \mathrm{E}(\ln(k_{i,t+1})) \\ \sigma_y^2 &= \mathrm{V}(\ln(\varepsilon_{i,t+1})) + \alpha^2 \mathrm{V}(\ln(k_{i,t+1})) + 2\alpha \mathrm{Cov}((\ln(\varepsilon_{i,t+1})), \ln(k_{i,t+1})), \end{aligned} \tag{1}$$

Granularity in a Simple Model

- **Lemma 1** Idiosyncratic output y_i is distributed as $\log(y_i) \sim \mathcal{N}(\mu_y, \sigma_y)$

$$\begin{aligned}\mu_y &= \mathrm{E}(\ln(\varepsilon_{i,t+1})) + \alpha \mathrm{E}(\ln(k_{i,t+1})) \\ \sigma_y^2 &= \mathrm{V}(\ln(\varepsilon_{i,t+1})) + \alpha^2 \mathrm{V}(\ln(k_{i,t+1})) + 2\alpha \mathrm{Cov}((\ln(\varepsilon_{i,t+1})), \ln(k_{i,t+1})),\end{aligned} \tag{1}$$

- **Proposition 1** Aggregate output Y_t be can approximated by a $\log(Y) \sim \mathcal{N}(\mu_Y, \sigma_Y)$

$$\mu_Y = \ln N + \mu_y + \frac{\sigma_y^2}{2} - \frac{\sigma_Y^2}{2} \quad \text{and} \quad \sigma_Y^2 = \ln\left[\frac{e^{\sigma_y^2} - 1}{N} + 1\right], \tag{2}$$

and the coefficient of variation of aggregate output is:

$$\mathrm{CV}(Y) = \frac{\mathrm{V}(Y)^{\frac{1}{2}}}{\mathrm{E}(Y)} = \sqrt{\frac{e^{\sigma_y^2} - 1}{N}}. \tag{3}$$

Granularity in a Simple Model

- **Lemma 1** Idiosyncratic output y_i is distributed as $\log(y_i) \sim \mathcal{N}(\mu_y, \sigma_y)$

$$\sigma_y^2 = \mathrm{V}(\ln(\varepsilon_{i,t+1})) + \alpha^2 \mathrm{V}(\ln(k_{i,t+1})) + 2\alpha \mathrm{Cov}((\ln(\varepsilon_{i,t+1})), \ln(k_{i,t+1})), \tag{1}$$

- **Proposition 1** Aggregate output Y_t be can approximated by a $\log(Y) \sim \mathcal{N}(\mu_Y, \sigma_Y)$

and the coefficient of variation of aggregate output is:

$$\mathrm{CV}(Y) = \frac{\mathrm{V}(Y)^{\frac{1}{2}}}{\mathrm{E}(Y)} = \sqrt{\frac{e^{\sigma_y^2} - 1}{N}}. \tag{2}$$

Granularity in a Simple Model

- **Proposition 2** The ergodic aggregate output Y can be expressed as:

$$\mathrm{E}(Y) = N \exp\left(\underbrace{\mathrm{E}(\ln(\varepsilon)) + \mathrm{E}(\ln(k^{\alpha})) + \frac{\mathrm{V}(\ln(\varepsilon))}{2} + \frac{\mathrm{V}(\ln(k^{\alpha}))}{2}}_{\text{static capacity}} + \underbrace{\mathrm{C}ov(\ln(\varepsilon), \ln(k^{\alpha}))}_{\text{misallocation}} \right) \tag{3}$$

- **Granular components** determine the realization of Y:

$$Y_t = \underbrace{\mathrm{Cov}_t(\varepsilon, k^{\alpha})}_{\text{time-varying misallocation}} + \underbrace{\mathrm{E}_t(\varepsilon)\mathrm{E}_t(k^{\alpha})}_{\text{time-varying capacity}} \tag{4}$$

Our contributions

1. Fluctuations depend on the second-order ergodic moments of $\mu(\text{capital}, \text{productivity})$
2. **Build a quantitative model with granular shocks for the US economy**
3. Show implications for the aggregate fluctuations + quantification of granular components
4. Granular shocks cause fluctuation in capital misallocation
5. Welfare implications of subsidies to downsize firms

Model Ingredients

- **Representative Household** $u(C, L) = \ln C + \eta L$

- **Fixed mass** of firms $i \in [0, 1]$

- **Production Technology** $y_{i,t} = \varepsilon_{i,t} k_{i,t}^{\alpha} n_{i,t}^{\nu}$ with $\alpha + \nu < 1$
  - Idiosyncratic shock $\log(\varepsilon) = \xi + \epsilon$
    - $\theta_{i,t+1} = \begin{cases} \theta_{i,t} & \text{with probability} \quad 1 - \pi \\ \tilde{\theta}_{i,t} \sim \mathcal{P}(e, e_m) & \text{with probability} \quad \pi \end{cases}$
    - $\epsilon_{i,t} \sim \mathcal{N}(0, \sigma_{\epsilon})$

- **Invest** $\hat{i}_{i,t} = k_{i,t+1} - (1 - \delta) k_{i,t}$ subject to partial irreversibility

$$i_{i,t} = \hat{i}_{i,t}(1 - \psi \mathbb{1}_{\hat{i}_{i,t} < 0})$$

Heterogeneous Firms: Bellman Equation

- **Equation**

$$V(\varepsilon, k; \mu) = \max_{n,k'} \varepsilon F(k, n) - \omega(\mu) n - i + \sum_{\mu'} \Pi^{\mu}(\mu'|\mu) d(\mu, \mu') \sum_{\varepsilon'} \Pi^{\varepsilon}_{\mu'|\mu}(\varepsilon'|\varepsilon) V(\varepsilon', k'; \mu')$$

subject to

$$\hat{i} = k' - (1-\delta)k$$
$$i = \hat{i}(1 - \psi \mathbb{1}_{\hat{i}<0})$$
$$k', n \in \mathbb{R}_+$$

Heterogeneous Firms: Bellman Equation

- **Equation**

$$V(\varepsilon, k; \mu) = \max_{n,k'} \varepsilon F(k,n) - \omega(\mu)n - i + \sum_{\mu'} \Pi^{\mu}(\mu'|\mu) d(\mu,\mu') \sum_{\varepsilon'} \Pi^{\varepsilon}_{\mu'|\mu}(\varepsilon'|\varepsilon) V(\varepsilon', k'; \mu')$$

subject to

$$\hat{i} = k' - (1-\delta)k$$
$$i = \hat{i}(1 - \psi \mathbb{1}_{\hat{i}<0})$$
$$k', n \in \mathbb{R}_+$$

- **Aggregate state** $\mu(\varepsilon, k)$
  - $\mu' \sim G(\mu)$ follow a Markov process
  - $\Pi^{\varepsilon}_{\mu'}(\varepsilon'|\varepsilon)$ conditional transition probability for the idiosyncratic productivity
  - $\Pi^{\varepsilon}_{\mu'|\mu}(\varepsilon'|\varepsilon)$ and $\Pi^{\mu}(\mu'|\mu)$ replicate the fluctuations of $\mu_{\varepsilon}(\varepsilon)$ consistent with $\mathbf{\Pi}^{\varepsilon}$ and **N firms**

Aggregate State

Stochasticity

- Failure of L.L.N. causes stochastic fluctuations in $\mu_\varepsilon(\varepsilon)$
- Shock to transition probability Π^ε replicating the $\mu_\varepsilon(\varepsilon)$ fluctuations

Aggregate State

Stochasticity

- Failure of L.L.N. causes stochastic fluctuations in $\mu_\varepsilon(\varepsilon)$
- Shock to transition probability Π^ε replicating the $\mu_\varepsilon(\varepsilon)$ fluctuations

Characterization of μ

- Moments of K aggregate capital stock
- $\widetilde{h} \in \widetilde{H}$ distribution of the current and previous period productivities

Aggregate State

Stochasticity

- Failure of L.L.N. causes stochastic fluctuations in $\mu_\varepsilon(\varepsilon)$
- Shock to transition probability Π^ε replicating the $\mu_\varepsilon(\varepsilon)$ fluctuations

Characterization of μ

- Moments of K aggregate capital stock
- $\widetilde{h} \in \widetilde{H}$ distribution of the current and previous period productivities

$\widetilde{\boldsymbol{H}}$ and $\boldsymbol{\Pi}^{\widetilde{\boldsymbol{h}}}(\widetilde{\boldsymbol{h}}'|\widetilde{\boldsymbol{h}})$

- Replicate the fluctuation of $\boldsymbol{\mu}_{\boldsymbol{\varepsilon}}$
- Consistently with $\boldsymbol{\Pi}^{\boldsymbol{\varepsilon}}$ and the total number of firms $\boldsymbol{N}$

Calibration

Targeted Ergodic Moments			
Description	Parameter	Data	Model
Capital-Output Ratio ($\frac{K}{Y}$)	α, e_m, Π	2.300	2.23
Total Hours Worked (N^h)	η	0.333	0.336
TOP 100 Std. Dev. $\Delta\varepsilon$ (perc.)	e, e_m	12.00	11.90
Sales share top 500 firms (perc.)	e_m	40.00	39.50
Walmart Employment Share (perc.)	e, e_m	1.000	1.087
$Pr(\lvert\frac{i}{k}\rvert \leqslant 0.01)$	ψ, α, e	$0.08 - 0.11$	0.095

Untargeted Ergodic Moments		
Description	Data	Model
Tail index of Firm size dist.	1.097	1.078
Sales share top 50 Firms (perc.)	24.00	22.40
Sales share top 100 Firms (perc.)	29.00	27.55

Calibration

Fixed parameters		
Description	Parameter	Value
Discount factor	β	0.96
Labor share	ν	0.60
Depreciation rate	δ	0.07
Std. dev. transitory component	σ_e	0.022
Number of firms	N	4.5 mil.

Computed parameters		
Description	Parameter	Value
Capital share	α	0.26
Preference parameter	η	2.15
Probability of reset	Π	0.02
Pareto scale parameter	e_m	6.89
Curvature of Pareto distribution	e	6.90
Investment irreversibility	ψ	0.35

Our contributions

1. Fluctuations depend on the second-order ergodic moments of $\mu(\text{capital}, \text{productivity})$
2. Build a quantitative model with granular shocks for the US economy
3. **Show implications for the aggregate fluctuations + quantification of granular components**
4. Granular shocks cause fluctuation in capital misallocation
5. Welfare implications of subsidies to downsize firms

Effects of Irreversibility in a Granular world

	$\psi = 0.35$ and $\Sigma_{\sigma^{MPK}} = 2.44$			$\psi = 0.00$ and $\Sigma_{\sigma^{MPK}} = 3.32$			DATA		
	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$
$\ln Y$	0.295	1.000	1.000	0.377	1.000	1.000	1.103	1.000	1.000
$\ln C$	0.170	0.575	0.787	0.119	0.316	0.716	0.791	0.716	0.902
$\ln I$	1.210	4.098	0.905	1.962	5.209	0.975	5.408	4.902	0.913
$\ln H$	0.193	0.653	0.839	0.303	0.805	0.962	1.637	1.486	0.881

Effects of Irreversibility in a Granular world

	$\psi = 0.35$ and $\Sigma_{\sigma^{MPK}} = 2.44$			$\psi = 0.00$ and $\Sigma_{\sigma^{MPK}} = 3.32$			DATA		
	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$
$\ln Y$	0.295	1.000	1.000	0.377	1.000	1.000	1.103	1.000	1.000
$\ln C$	0.170	0.575	0.787	0.119	0.316	0.716	0.791	0.716	0.902
$\ln I$	1.210	4.098	0.905	1.962	5.209	0.975	5.408	4.902	0.913
$\ln H$	0.193	0.653	0.839	0.303	0.805	0.962	1.637	1.486	0.881

Effects of Irreversibility in a Granular world

	$\psi = 0.35$ and $\Sigma_{\sigma^{MPK}} = 2.44$			$\psi = 0.00$ and $\Sigma_{\sigma^{MPK}} = 3.32$			DATA		
	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$
$\ln Y$	0.295	1.000	1.000	0.377	1.000	1.000	1.103	1.000	1.000
$\ln C$	0.170	0.575	0.787	0.119	0.316	0.716	0.791	0.716	0.902
$\ln I$	1.210	4.098	0.905	1.962	5.209	0.975	5.408	4.902	0.913
$\ln H$	0.193	0.653	0.839	0.303	0.805	0.962	1.637	1.486	0.881

Effects of Irreversibility in a Granular world

	$\psi = 0.35$ and $\Sigma_{\sigma MPK} = 2.44$			$\psi = 0.00$ and $\Sigma_{\sigma MPK} = 3.32$			DATA		
	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$
$\ln Y$	0.295	1.000	1.000	0.377	1.000	1.000	1.103	1.000	1.000
$\ln C$	0.170	0.575	0.787	0.119	0.316	0.716	0.791	0.716	0.902
$\ln I$	1.210	4.098	0.905	1.962	5.209	0.975	5.408	4.902	0.913
$\ln H$	0.193	0.653	0.839	0.303	0.805	0.962	1.637	1.486	0.881

Effects of Irreversibility with Aggregate TFP shocks

	$\psi = 0.35$ with $y = Z\varepsilon k^{\alpha} l^{\nu}$			$\psi = 0.00$ with $y = Z\varepsilon k^{\alpha} l^{\nu}$		
	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$
$\ln Y$	1.888	1.000	1.000	1.898	1.000	1.000
$\ln C$	0.981	0.520	0.924	0.957	0.504	0.934
$\ln I$	6.775	3.589	0.963	7.412	3.905	0.965
$\ln H$	1.051	0.557	0.934	1.062	0.559	0.946

Effects of Irreversibility with Aggregate TFP shocks

	$\psi = 0.35$ with $y = Z\varepsilon k^{\alpha} l^{\nu}$			$\psi = 0.00$ with $y = Z\varepsilon k^{\alpha} l^{\nu}$		
	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$
$\ln Y$	1.888	1.000	1.000	1.898	1.000	1.000
$\ln C$	0.981	0.520	0.924	0.957	0.504	0.934
$\ln I$	6.775	3.589	0.963	7.412	3.905	0.965
$\ln H$	1.051	0.557	0.934	1.062	0.559	0.946

Effects of Irreversibility with Aggregate TFP shocks

	$\psi = 0.35$ with $y = Z\varepsilon k^{\alpha} l^{\nu}$			$\psi = 0.00$ with $y = Z\varepsilon k^{\alpha} l^{\nu}$		
	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$
$\ln Y$	1.888	1.000	1.000	1.898	1.000	1.000
$\ln C$	0.981	0.520	0.924	0.957	0.504	0.934
$\ln I$	6.775	3.589	0.963	7.412	3.905	0.965
$\ln H$	1.051	0.557	0.934	1.062	0.559	0.946

Effects of Irreversibility with Aggregate TFP shocks

	$\psi = 0.35$ with $y = Z\varepsilon k^{\alpha} l^{\nu}$			$\psi = 0.00$ with $y = Z\varepsilon k^{\alpha} l^{\nu}$		
	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$
$\ln Y$	1.888	1.000	1.000	1.898	1.000	1.000
$\ln C$	0.981	0.520	0.924	0.957	0.504	0.934
$\ln I$	6.775	3.589	0.963	7.412	3.905	0.965
$\ln H$	1.051	0.557	0.934	1.062	0.559	0.946

Granular Components with Irreversibility

	$\psi = 0.35$ and $\Sigma_{\sigma^{MPK}} = 2.44$			$\psi = 0.35$ and $\Sigma_{\sigma^{MPK}} = 0.00$			$K = 0.0$ with $y = \varepsilon l^{\alpha+\nu}$		
	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$
$\ln Y$	0.295	1.000	1.000	0.188	1.000	1.000	0.129	1.000	1.000
$\ln C$	0.170	0.575	0.787	0.105	0.559	0.836	0.129	1.000	1.000
$\ln I$	1.210	4.098	0.905	0.760	4.054	0.921			
$\ln H$	0.193	0.653	0.839	0.116	0.618	0.868	0.000	0.000	0.000

Granular Components with Irreversibility

	$\psi = 0.35$ and $\Sigma_{\sigma MPK} = 2.44$			$\psi = 0.35$ and $\Sigma_{\sigma MPK} = 0.00$			$K = 0.0$ with $y = \varepsilon l^{\alpha+\nu}$		
	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$
$\ln Y$	0.295	1.000	1.000	0.188	1.000	1.000	0.129	1.000	1.000
$\ln C$	0.170	0.575	0.787	0.105	0.559	0.836	0.129	1.000	1.000
$\ln I$	1.210	4.098	0.905	0.760	4.054	0.921			
$\ln H$	0.193	0.653	0.839	0.116	0.618	0.868	0.000	0.000	0.000

Granular Components with Irreversibility

	$\psi = 0.35$ and $\Sigma_{\sigma MPK} = 2.44$			$\psi = 0.35$ and $\Sigma_{\sigma MPK} = 0.00$			$K = 0.0$ with $y = \varepsilon l^{\alpha+\nu}$		
	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$
$\ln Y$	0.295	1.000	1.000	0.188	1.000	1.000	0.129	1.000	1.000
$\ln C$	0.170	0.575	0.787	0.105	0.559	0.836	0.129	1.000	1.000
$\ln I$	1.210	4.098	0.905	0.760	4.054	0.921			
$\ln H$	0.193	0.653	0.839	0.116	0.618	0.868	0.000	0.000	0.000

Granular Components with Irreversibility

	$\psi = 0.35$ and $\Sigma_{\sigma^{MPK}} = 2.44$			$\psi = 0.35$ and $\Sigma_{\sigma^{MPK}} = 0.00$			$K = 0.0$ with $y = \varepsilon l^{\alpha+\nu}$		
	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$
$\ln Y$	0.295	1.000	1.000	0.188	1.000	1.000	0.129	1.000	1.000
$\ln C$	0.170	0.575	0.787	0.105	0.559	0.836	0.129	1.000	1.000
$\ln I$	1.210	4.098	0.905	0.760	4.054	0.921			
$\ln H$	0.193	0.653	0.839	0.116	0.618	0.868	0.000	0.000	0.000

Granular Components without Irreversibility

	$\psi = 0.00$ and $\Sigma_{\sigma MPK} = 3.32$			$\psi = 0.00$ and $\Sigma_{\sigma MPK} = 0.00$			$K = 0.0$ with $y = \varepsilon l^{\alpha+\nu}$		
	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$
$\ln Y$	0.377	1.000	1.000	0.188	1.000	1.000	0.129	1.000	1.000
$\ln C$	0.119	0.316	0.716	0.105	0.559	0.836	0.129	1.000	1.000
$\ln I$	1.962	5.209	0.975	0.760	4.054	0.921			
$\ln H$	0.303	0.805	0.962	0.116	0.618	0.868	0.000	0.000	0.000

Granular Components without Irreversibility

	$\psi = 0.00$ and $\Sigma_{\sigma MPK} = 3.32$			$\psi = 0.00$ and $\Sigma_{\sigma MPK} = 0.00$			$K = 0.0$ with $y = \varepsilon I^{\alpha+\nu}$		
	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$
$\ln Y$	0.377	1.000	1.000	0.188	1.000	1.000	0.129	1.000	1.000
$\ln C$	0.119	0.316	0.716	0.105	0.559	0.836	0.129	1.000	1.000
$\ln I$	1.962	5.209	0.975	0.760	4.054	0.921			
$\ln H$	0.303	0.805	0.962	0.116	0.618	0.868	0.000	0.000	0.000

Granular Components without Irreversibility

	$\psi = 0.00$ and $\Sigma_{\sigma MPK} = 3.32$			$\psi = 0.00$ and $\Sigma_{\sigma MPK} = 0.00$			$K = 0.0$ with $y = \varepsilon l^{\alpha+\nu}$		
	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$
$\ln Y$	0.377	1.000	1.000	0.188	1.000	1.000	0.129	1.000	1.000
$\ln C$	0.119	0.316	0.716	0.105	0.559	0.836	0.129	1.000	1.000
$\ln I$	1.962	5.209	0.975	0.760	4.054	0.921			
$\ln H$	0.303	0.805	0.962	0.116	0.618	0.868	0.000	0.000	0.000

Granular Components without Irreversibility

	$\psi = 0.00$ and $\Sigma_{\sigma_{MPK}} = 3.32$			$\psi = 0.00$ and $\Sigma_{\sigma_{MPK}} = 0.00$			$K = 0.0$ with $y = \varepsilon l^{\alpha+\nu}$		
	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$
$\ln Y$	0.377	1.000	1.000	0.188	1.000	1.000	0.129	1.000	1.000
$\ln C$	0.119	0.316	0.716	0.105	0.559	0.836	0.129	1.000	1.000
$\ln I$	1.962	5.209	0.975	0.760	4.054	0.921			
$\ln H$	0.303	0.805	0.962	0.116	0.618	0.868	0.000	0.000	0.000

Our contributions

1. Fluctuations depend on the second-order ergodic moments of $\mu(\text{capital}, \text{productivity})$

2. Build a quantitative model with granular shocks for the US economy

3. Show implications for the aggregate fluctuations $+$ quantification of granular components

4. **Granular shocks cause fluctuation in capital misallocation**

5. Welfare implications of subsidies to downsize firms

Recap

- Failure of the L.L.N. causes firm churning
- *Time-varying misallocation* still drives more than one third of the granular volatility
- Empirical validation

$\Longrightarrow$ **Does Gabaix Residual cause capital misallocation among the largest firms?**

Data

- **Idiosyncratic shocks**
  - $\Delta\varepsilon_{i,t} = \frac{\varepsilon_{i,t}-\varepsilon_{i,t-1}}{\varepsilon_{i,t-1}}$ from 1980-2019, İmrohoroğlu and Tüzel(2014)

$$\Theta_t^{Gabaix} = \sum_{i=1}^{100} \frac{\text{Sale}_{i,t-1}}{\text{Sale}_{t-1}^{100}} \Delta\varepsilon_{i,t} - \frac{\sum_{i=1}^{100} \Delta\varepsilon_{i,t}}{100};$$

- **Accounting information**
  - Compustat sales and gross physical capital excluded financial and utility
  - $MPK_{i,t} = \ln(\frac{\text{SALES}_{i,t}}{\text{PPEGT}_{i,t}})$ with $SEC \times YEAR$ f.e.

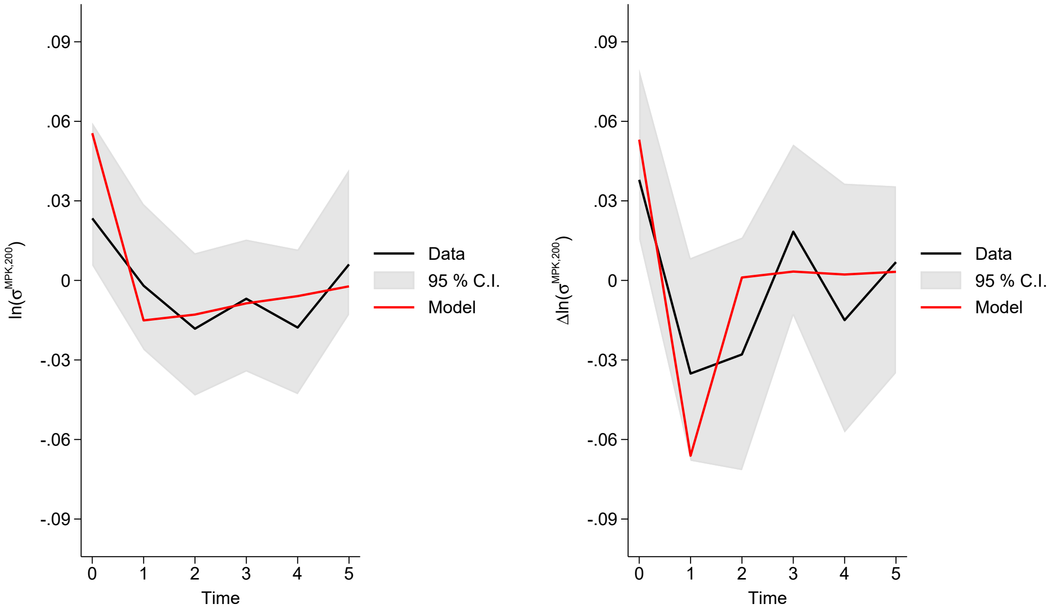
$$\sigma^{MPK,200} = \sqrt{\frac{\sum_{i=1}^{200}(MPK_{i,t} - MPK_t^{mean})^2}{200}}$$

- **Local Projection**

$$\ln(\sigma_{t+\tau}^{MPK,200}) = \alpha_\tau + \phi_\tau(L)z_{t-1} + \beta_\tau\Theta_t^{Gabaix} + v_{t+\tau} \quad \text{for} \quad \tau = 0, 1, 2, \ldots, \qquad (5)$$

  - Real GDP, 10-year T-bill, term spread , CPI, and $\ln(\sigma_{t+\tau}^{MPK,-200})$ as controls
  - HP-filtered variables

Gabaix Residual and Dispersion of MPK



Our contributions

1. Fluctuations depend on the second-order ergodic moments of $\mu(\text{capital}, \text{productivity})$
2. Build a quantitative model with granular shocks for the US economy
3. Show implications for the aggregate fluctuations + quantification of granular components
4. Granular shocks cause fluctuation in capital misallocation
5. **Welfare implications of subsidies to downsize firms**

Exercise

- Irreversibility lowers capital reallocation: consumption drops and its volatility increases

- Assume that government fully absorbs the irreversibility costs
  - Firms act as if there is no capital irreversibility

- Government finances the expenditure through a lump-sum tax on RH

$\Longrightarrow$ **What are the welfare implications of such policy?**

Subsides to Downsize Unproductive Firms

	$\psi = 0.35$			Investment subsidies		
	$\exp x$	$\sigma(x)$	$\rho(x, Y)$	$\exp x$	$\sigma(x)$	$\rho(x, Y)$
$\ln Y$	0.778	0.295	1.000	0.802	0.320	1.000
$\ln C$	0.646	0.170	0.787	0.656	0.161	0.796
$\ln I$	0.131	1.210	0.905	0.131	3.375	0.862
$\ln H$	0.336	0.193	0.839	0.341	0.215	0.892
$\ln \hat{\delta}$	0.076	0.155	-0.781	0.078	0.161	-0.746
$\ln \sigma^{MPK}$	0.022	2.439	-0.522	0.019	3.399	-0.392
E(U)	25.052	0.674	0.459	25.250	0.300	-0.203

Subsidies to Downsize Unproductive Firms

	$\psi = 0.35$			Investment subsidies		
	$\exp x$	$\sigma(x)$	$\rho(x, Y)$	$\exp x$	$\sigma(x)$	$\rho(x, Y)$
$\ln Y$	0.778	0.295	1.000	0.802	0.320	1.000
$\ln C$	0.646	0.170	0.787	0.656	0.161	0.796
$\ln I$	0.131	1.210	0.905	0.131	3.375	0.862
$\ln H$	0.336	0.193	0.839	0.341	0.215	0.892
$\ln \hat{\delta}$	0.076	0.155	-0.781	0.078	0.161	-0.746
$\ln \sigma^{MPK}$	0.022	2.439	-0.522	0.019	3.399	-0.392
E(U)	25.052	0.674	0.459	25.250	0.300	-0.203

Subsidies to Downsize Unproductive Firms

	$\psi = 0.35$			Investment subsidies		
	$\exp x$	$\sigma(x)$	$\rho(x, Y)$	$\exp x$	$\sigma(x)$	$\rho(x, Y)$
$\ln Y$	0.778	0.295	1.000	0.802	0.320	1.000
$\ln C$	0.646	0.170	0.787	0.656	0.161	0.796
$\ln I$	0.131	1.210	0.905	0.131	3.375	0.862
$\ln H$	0.336	0.193	0.839	0.341	0.215	0.892
$\ln \hat{\delta}$	0.076	0.155	-0.781	0.078	0.161	-0.746
$\ln \sigma^{MPK}$	0.022	2.439	-0.522	0.019	3.399	-0.392
E(U)	25.052	0.674	0.459	25.250	0.300	-0.203

Subsidies to Downsize Unproductive Firms

	$\psi = 0.35$			Investment subsidies		
	$\exp x$	$\sigma(x)$	$\rho(x, Y)$	$\exp x$	$\sigma(x)$	$\rho(x, Y)$
$\ln Y$	0.778	0.295	1.000	0.802	0.320	1.000
$\ln C$	0.646	0.170	0.787	0.656	0.161	0.796
$\ln I$	0.131	1.210	0.905	0.131	3.375	0.862
$\ln H$	0.336	0.193	0.839	0.341	0.215	0.892
$\ln \hat{\delta}$	0.076	0.155	-0.781	0.078	0.161	-0.746
$\ln \sigma^{MPK}$	0.022	2.439	-0.522	0.019	3.399	-0.392
E(U)	25.052	0.674	0.459	25.250	0.300	-0.203

Subsidies to Downsize Unproductive Firms

	$\psi = 0.35$			Investment subsidies		
	$\exp x$	$\sigma(x)$	$\rho(x, Y)$	$\exp x$	$\sigma(x)$	$\rho(x, Y)$
$\ln Y$	0.778	0.295	1.000	0.802	0.320	1.000
$\ln C$	0.646	0.170	0.787	0.656	0.161	0.796
$\ln I$	0.131	1.210	0.905	0.131	3.375	0.862
$\ln H$	0.336	0.193	0.839	0.341	0.215	0.892
$\ln \hat{\delta}$	0.076	0.155	-0.781	0.078	0.161	-0.746
$\ln \sigma^{MPK}$	0.022	2.439	-0.522	0.019	3.399	-0.392
E(U)	25.052	0.674	0.459	25.250	0.300	-0.203

$$\Longrightarrow \text{CEV} = \mathbf{0.8\%}$$

Conclusion

Investment irreversibility dampens granular volatility

1. Lumpy changes in the mass of firms binding the irreversibility constraint
   - Raise in the dispersion of MPK and fall in its volatility
2. *Time-varying misallocation* is an important component of the granular volatility
   - Failure of L.L.N. generates fluctuations in misallocation among the largest firms
3. Subsidies that accelerate capital reallocation enhance welfare

APPENDIX

Calibration

Ergodic Moments

Description	$\psi = 0.00$	$\psi = 0.35$
Capital-Output Ratio ($\frac{K}{Y}$)	2.33	2.23
Total Hours Worked (N^h)	0.332	0.336
Std. Dev. TOP 100 Firm-Level TFP (perc.)	11.90	11.90
Sales share top 500 firms (perc.)	41.30	39.50
Walmart Employment Share (perc.)	1.170	1.087
$Pr(\lvert\frac{i}{k}\rvert \leqslant 0.01)$	0.000	0.095
Tail index of Firm size dist.	1.139	1.078
Sales share top 50 Firms (perc.)	23.88	22.40
Sales share top 100 Firms (perc.)	29.17	27.55

Sketch of the Implementation

- **Construct** $\Pi(\widetilde{h}'|\widetilde{h})$
  1. Given Π^{ε}, simulate $\mu_{\varepsilon}(\varepsilon)$ for T to recover the time series of its moments $\bar{M}_T$.
  2. Approximate $\bar{m}_t$ with a multivariate first order markov chain to recover $\Pi^{\bar{M}}$ and $\bar{M}$
  3. Obtain $\Pi^{\widetilde{h}} = \Pi^{\bar{m}}$ since ε follows a first order markov process
- **Build** $\widetilde{H} = \{\widetilde{h}_1, \widetilde{h}_2, \ldots, \widetilde{h}_{N_h-1}, \widetilde{h}_{N_h}\}$ iterating 1-2 for I times, to finally calculate 3.
  1. Construct $\bar{H}^i = \{\mu^i_{\varepsilon,1}, \mu^i_{\varepsilon,2}, \ldots, \mu^i_{\varepsilon,N_{\bar{h}}}\}$ that reflects $\bar{M}$.
  2. Given Π^{ε}, implement a constrained draw of $\widetilde{h}$ and such that each element of the set

$$\widetilde{H}^i = \{\widetilde{h}^i_1, \widetilde{h}^i_2, \ldots, \widetilde{h}^i_{l+(m-1)N_{\hat{h}}-1}, \widetilde{h}^i_{l+(m-1)N_{\hat{h}}}, \widetilde{h}^i_{l+(m-1)N_{\hat{h}}+1}, \ldots, \widetilde{h}^i_{N^2_{\bar{h}}-1}, \widetilde{h}^i_{N^2_{\bar{h}}}\}$$

     is such that $\forall l, m = 1, 2, \ldots, N_{\bar{h}}$ and $\forall i, j = 1, 2, \ldots, N_{\varepsilon}$

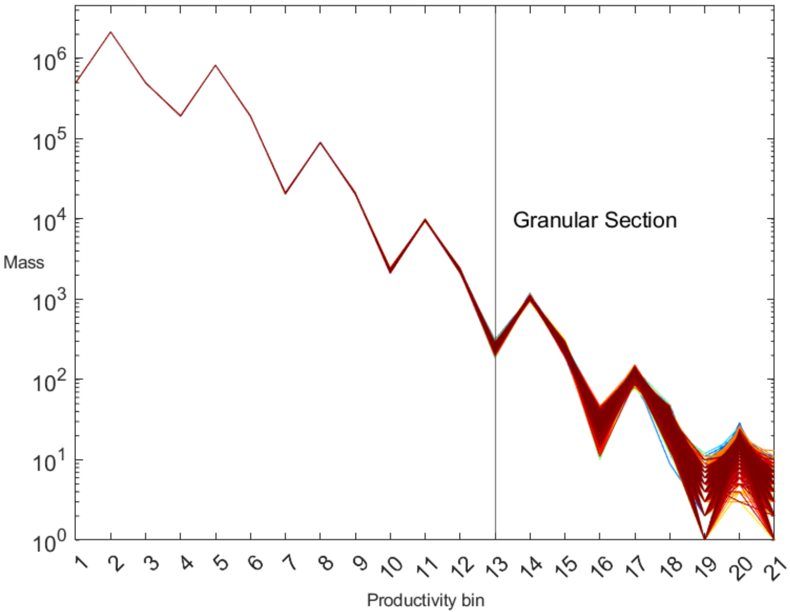
$$\mu^i_{\varepsilon,m}(\varepsilon_i) = \sum_j \widetilde{h}^i_{l+(m-1)N_{\bar{h}}}(\varepsilon_i, \varepsilon_j)$$

$$\mu^i_{\varepsilon,l}(\varepsilon_j) = \sum_i \widetilde{h}^i_{l+(m-1)N_{\bar{h}}}(\varepsilon_i, \varepsilon_j)$$

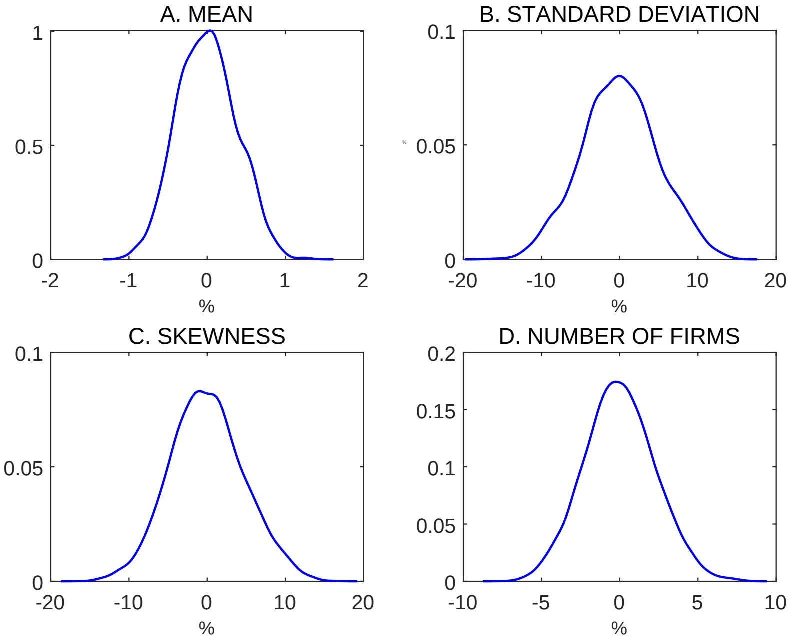
  3. Calculate the average

$$\widetilde{H} = \frac{\widetilde{H}^i}{I}$$

Simulated Productivities



Tail Moments



General Equilibrium Frictionless Model

	GE with $\psi = 0.00$			PE with $\psi = 0.00$		
	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, Y)$
$\ln Y$	0.377	1.000	1.000	0.858	1.000	1.000
$\ln C$	0.119	0.316	0.716	2.702	3.150	0.442
$\ln I$	1.962	5.209	0.975	14.706	17.146	0.050
$\ln H$	0.303	0.805	0.962	0.858	1.000	1.000
$\ln \sigma^{MPK}$	3.32	8.849	-0.441	3.399	3.969	-0.392