

Equity allocation and risk-taking in the intermediation chain

Anatoli Segura
Bank of Italy

Alonso Villacorta
University of California, Santa Cruz

April 2019

Abstract

We build an equilibrium model of the securitization process in presence of absolute demand for safety by some investors. The equity allocation along the intermediation chain is endogenous and trades off benefits from skin-in-the-game for loan origination and aggregate risk loss absorption to create safe securitized assets. The model predictions are consistent with the saving glut narrative of the expansion of securitization in the run-up to the crisis. The emergence of securitization increases risk-taking at origination but Pareto improves the economy when demand for safety is very high. Fiscally neutral guarantees to support the issuance of safe securitized assets allow to Pareto improve the economy but may increase risk-taking at origination.

JEL Classification: G01, G20, G28

Keywords: capital structure, risk-taking, originate-to-distribute, diversification, spreads, saving glut

* The views expressed in this paper are our own and do not necessarily coincide with those of the Bank of Italy. We would like to thank Saki Bigio, Elena Carletti, Arvind Krishnamurthy, David Martinez-Miera, Robert Marquez, Claudio Michelacci, Facundo Piguillem, Rafael Repullo, Tano Santos, Martin Schneider, Javier Suarez, Victoria Vanasco, Pierre-Olivier Weill, Guillermo Ordoñez, as well as seminar audiences at Stanford Institute for Theoretical Economics Workshop 2018, Cemfi, UCSC, Universidad de los Andes, and University of Vienna for helpful comments. Contact e-mails: anatoli.seguravelez@bancaditalia.it; avillaco@ucsc.edu.

1 Introduction

Securitization has changed financial intermediation from an originate-to-hold to an originate-to-distribute model in the last decades. The increase in the demand for safe assets observed since the 2000s is considered an important driver for this transformation (Bernanke (2005), Bernanke et al. (2011)). Yet, following the Global Financial Crisis concerns have mounted on the welfare implications of the emergence of securitization and the role of Government Sponsored Enterprises in fostering it. How does the demand for safe assets lead to a securitization boom, and what are the implications for the capital structure and risk-taking in the intermediation chain? Who are the winners and losers from the emergence of securitization? Should the government support the securitization process, and how?

To answer these questions, this paper develops an equilibrium model of the securitization process. The model features absolute demand for safety by some investors and limited endowment by equity investors that is endogenously allocated along the intermediation chain. The capital structure and risk-taking in the intermediation chain, the expected returns of the funding sources, aggregate lending and the amount of securitized assets are all determined in equilibrium. In particular, the equity allocation along the chain in equilibrium trades-off the gains from improving risk-taking incentives at origination (*skin-in-the-game*) with those from providing the loss absorption against aggregate risk necessary for the creation of safe securitized assets (*credit enhancement*).

The paper makes three main contributions to the existing literature. First, the paper provides a simple yet rich competitive equilibrium framework of the financial architecture of the securitization process whose empirical predictions are consistent with the saving glut narrative of the run-up to the Global Financial Crisis. Second, the paper shows that while the emergence of securitization increases risk-taking at origination, it may lead to Pareto improvements in welfare when demand for safe assets is high. Third, the paper finds that when a government has safe resources, fiscally neutral public guarantees to the issuance of securitized assets allow to Pareto improve welfare in the economy but sometimes achieve so through an increase in risk-taking at origination.

We model a two date competitive economy with two types of investors: savers and experts. Investors have one unit of endowment at the initial date, derive linear utility from consumption at either date and have a zero discount rate. Aggregate endowment is normal-

ized to one. Savers have absolute preference for safety so that their measure determines the demand for safety in the economy.¹ Experts are skilled agents that can set-up and invest their wealth in the equity of one out of two financial firms: originators and intermediaries. Originators can issue loans under a constant returns to scale technology. The expert managing an originator can monitor the loans in order to increase the probability that they yield a high return. Monitoring is not contractible and involves a convex disutility cost for the expert, which leads to a moral hazard problem as in [Holmstrom and Tirole \(1997\)](#). Originators can expand lending and increase their equity return by issuing both non-contingent securities (*safe securities*) and contingent securities (*risky securities*) that are placed to savers and intermediaries, respectively. Intermediaries, which are the other type of financial firm that experts can set-up and invest in, purchase the risky securities issued by many originators. Intermediaries have access to a “pooling” technology that allows them to diversify away the originators’ idiosyncratic risks and create additional securitized safe assets that can, in turn, be placed to savers. The “manufacturing” of new safe securities allows intermediaries to expand their balance sheets by purchasing more risky securities from originators, and to increase the return on their equity. The presence of aggregate risk that cannot be diversified imposes a maximum leverage constraint on intermediaries, as the equity provided by the expert needs to provide sufficient credit enhancement to ensure the safety of the securitized assets sold to savers.

Experts’ equity investment in the intermediation chain serves two different purposes. At origination, it provides experts skin-in-the game that increases their incentives to monitor the loans. The amount of risky securities distributed to intermediaries thus trades-off the gains from expanding lending (and increasing effective leverage) and the costs from more risk-taking as monitoring incentives get reduced. The lower the (endogenous) cost of funding from intermediaries, the larger the part of its loan payoffs that the originator sells to intermediaries, increasing both originators’ leverage and their risk-taking. At intermediation, the expert’s equity is a cushion for aggregate risk losses. In equilibrium, free entry of experts induces the return of originators and intermediaries’ equity to be equal, so that experts’ choice ends-up trading off the risk-taking gains at origination and the diversification benefits at intermediation. Experts’ frictionless capability to allocate their skills and funds between

¹This assumption has been used, e.g., in [Gennaioli et al. \(2013\)](#).

originators and intermediaries and the existence of competitive markets for safe and risky securities ensure the validity of constrained versions of the welfare theorems.

The demand for safety in the economy determines when securitization emerges. If demand for safety is low, originators' capability to create safe securities directly from their loans is sufficient for the market for safe securities to clear at a high return that is equal to the return on originator's equity. With a zero equity spread, incentives to expand the supply of safe assets through securitization are absent and intermediaries do not enter the economy. The risky part of originated loans is entirely held at originators and risk-taking is minimum. If the demand for safety is higher, the originators' capability to issue safe securities is not sufficient to keep the safe rate at its maximum level. Safe securities become scarce, the safe rate falls and a positive equity spread arises, which gives experts incentives to set-up intermediaries to exploit it. In fact, intermediaries can purchase risky securities from originators and resell their securitized safe tranche to savers who require a low rate, delivering a high equity return. Equity is thus reallocated from originators to intermediaries, and the distribution of risk out of originators leads to more risk-taking.

As the demand for safety keeps on increasing, the safe rate falls further. The widening equity spread, allows intermediaries to increase leverage and also leads experts to reallocate their funds and skills from originators to intermediaries. With higher leverage and overall equity, the intermediary sector expands. At the same time, the cheap financing offered by intermediaries leads originators to increase the risky part of their loans that is distributed, which increases their leverage but also leads to more risk-taking. Summing up: following an increase in the demand for safety, the model predicts a securitization boom, an increase in leverage along the intermediation chain and more risk-taking, which is consistent with the saving glut narrative of the run-up to the crisis.

We analyze the welfare implications of the emergence of securitization by comparing the utility of investors in the originate-to-distribute economy relative to that in a traditional originate-to-hold benchmark. Securitization involves the distribution by originators of risky securities that are pooled by intermediaries to expand safe securities supply, which leads to the following general welfare trade-off. On the one hand, the distribution of risk out of originators leads them to take more risk, which reduces aggregate surplus. On the other hand, when not all savers' endowment can be channeled to finance loans in the traditional

economy, the expansion of safe securities through securitization increases aggregate lending and surplus. We find that the aggregate lending effect on total surplus dominates if and only the demand for safety is sufficiently large. Besides, the modern intermediation chain always (weakly) increases the welfare of savers because securitization expands the supply of safe securities, which constitute their only investment opportunity. In contrast, experts' welfare gets reduced because of securitization when demand for safety is not too large. In these cases, the possibility for experts to engage in securitization leads to more competition in the supply of safe securities and ends up depriving experts of some of the scarcity rents they enjoyed in the traditional financial sector.

We extend the model to consider a risk-neutral government with some safe assets and analyze whether and how fiscally neutral public guarantees to the issuance of safe securities by financial firms can improve welfare. Given that originators' loans are exposed to both aggregate and idiosyncratic risk while intermediaries' assets are only exposed to aggregate risk, the *net* injection of funds by the government following negative aggregate shocks is larger when only guarantees to intermediaries are issued. Guarantees to intermediaries thus maximize the capability of the economy to issue safe securities, and are thus preferable. We then show that the issuance of guarantees to intermediaries always leads to an increase in the safe rate and welfare gains to the savers. When demand for safety is so large that the economy is not able to achieve full investment, these guarantees increase the size of securitization, aggregate investment and experts' welfare. Yet, they also lead to an increase in risk-taking at origination. When instead demand for safety is low and the economy achieves full investment, guarantees on intermediaries may lead to welfare losses for experts. In this case, combining guarantees with a lump-sum transfer from savers to experts at the initial date allows to achieve Pareto gains and also decreases risk-taking at origination. These results shed new light on the equilibrium effects of public guarantees to the financial sector, the need to combine them with other redistributive policies and their interplay with risk-taking at origination.

The paper is organized as follows. Section 2 describes the related literature. Section 3 presents the model. Section 4 characterizes the equilibrium of a benchmark economy with no intermediaries. Section 5 analyzes the partial equilibrium of the economy with an exogenous safe rate. Section 6 characterizes the general equilibrium of the model, discusses the welfare

effect from the emergence of securitization and shows that constrained versions of the Welfare Theorems hold. Section 7 analyzes optimal public guarantees to financial firms. Section 8 concludes. All the proofs of the formal results in the paper are in the Appendix.

2 Related literature

This paper belongs to vast literature on securitization. In practice, the securitization process involves several rounds of pooling, tranching and distribution of cash-flows generated by loans along an intermediation chain that exhibits different entities ([Ashcraft et al. \(2008\)](#), and [Pozsar et al. \(2013\)](#)). The objective of the pooling activity is to diversify the idiosyncratic risks of the underlying assets and that of the tranching activity is to split cash-flows into securities with different risk profiles. Early research focuses on the tranching and distribution of loan pay-offs and shows that it emerges as the optimal security design to overcome adverse selection problems ([Gorton and Pennacchi \(1990\)](#), [DeMarzo and Duffie \(1999\)](#)). [DeMarzo \(2004\)](#) extends this literature and develops an optimal security design model that features pooling and tranching along a longer intermediation chain. The paper exhibits endogenous risk retention along the chain but the risk of the originated loans is exogenous. The latter issue is addressed in a number of papers that analyze how moral hazard problems at origination are affected by the tranching and distribution of loans ([Gorton and Pennacchi \(1995\)](#), [Parlour and Plantin \(2008\)](#), [Chemla and Hennessy \(2014\)](#), and [Daley et al. \(2017\)](#)). These papers, though, only focus on the origination part of the originate-to-distribute intermediation chain and abstract from the diversification benefits associated with securitization. Another set of papers focus on the manufacturing of safe collateral by pooling risky securities and analyze how demand for safety drives the emergence of securitization ([Gennaioli et al. \(2013\)](#), [Moreira and Savov \(2017\)](#)). The issue of origination incentives is not addressed in these papers.²

Our paper contributes to the securitization literature by providing an equilibrium model of the intermediation chain that exhibits endogenous risk-taking at origination, safe collateral manufacturing through diversification, and endogenous risk retentions along the chain. To

²A final strand of the literature, stresses the role of regulatory arbitrage for the the emergence of securitization ([Calomiris and Mason \(2004\)](#), [Acharya et al. \(2009\)](#), [Acharya et al. \(2013\)](#)). These aspects are absent in our model.

the best of our knowledge this is the first paper to provide an equilibrium theory of the entire intermediation chain. Our focus, which is novel in the literature, is on how the endogenous equity allocation along the chain affects both origination incentives and the creation of safe securitized assets.

Our paper is also related to a literature that analyzes how moral hazard problems shape risk-taking by financial intermediaries. The equilibrium relationship between bank capital requirements and risk-taking is analyzed in [Repullo \(2013\)](#) and in [Martinez-Miera and Repullo \(2018\)](#). The implications of saving gluts or low interest rate environments for monitoring and origination incentives are analyzed in [Dell Ariccia et al. \(2014\)](#), [Martinez-Miera and Repullo \(2017\)](#) and [Bolton et al. \(2018\)](#).³ We embed similar agency frictions in a model of the financial architecture of the securitization business that captures the role played by the different players in the intermediation chain.

Some recent papers analyze the endogenous capital structure of non-financial firms and banks ([Allen et al. \(2015\)](#), [Gornall and Strebulaev \(2018\)](#), [Diamond \(2016\)](#)). Although the focus of these papers is different from ours, we share the interest on how market forces shape the equity allocation in the economy. A contribution of our paper to this literature is to endogenize the risk of the real assets in the economy, which in those papers is taken as exogenous and in our model affects and is affected by the equity allocation.

Finally, our paper also contributes to a large literature that studies the need and implications of public support to the financial sector following negative shocks (for recent contributions see, e.g., [Diamond and Rajan \(2012\)](#); [Farhi and Tirole \(2012\)](#); [Keister \(2015\)](#)). Most of the literature has highlighted a time consistency problem that makes public support optimal ex post but inefficient ex ante due to moral hazard. Our findings challenge this view because we show that public guarantees to securitized assets may lead to higher leverage and more risk-taking but still Pareto improve welfare from an ex ante perspective. Due to the market segmentation, public guarantees may have redistributive effects in our economy, which resembles the results in [Carletti et al. \(2017\)](#) on the welfare effects of changes in capital requirements. A final novel result we obtain is the optimality of concentrating the government's resources on granting guarantees on the issuance of the safe securities most exposed to aggregate risk.

³Other theories of the relationship between demand for safety and financial fragility are pursued by [Caballero and Krishnamurthy \(2009\)](#) or [Ahnert and Perotti \(2017\)](#).

3 The model

Consider an economy with two dates $t = 0, 1$ and two types of investors endowed at $t = 0$ with one unit of funds: experts and savers. Aggregate endowment is normalized to one. The overall wealth of savers is denoted with $\mu \in [0, 1]$, and that of experts is $1 - \mu$. Investors derive linear utility from consumption at either date and have a zero discount rate. At $t = 0$, each expert can set-up and manage one out of two types of financial firms, called *originators* and *intermediaries*. Both types of financial firm have access to some constant return to scale investment possibilities that are funded as described below. Finally, at $t = 0$, the expert decides how to allocate his endowment as (inside) equity in his own firm, investment in securities issued by other financial firms, or consumption. At $t = 0$, savers can either invest in *safe securities* issued by financial firms or consume their endowment.

Since investors have linear utility and all their investment possibilities are scalable, for the ease of exposition, we focus on investment strategies in which the entire endowment of each investor is either allocated to exactly one of the investment possibilities or consumed.

We describe each of the financial firms that experts can create next.

Originators An originator is a financial firm that has access to a constant returns to scale project whose stochastic payoff is realized at $t = 1$. The per unit return of the project, that we denote A_z , can be either high ($z = H$) or low ($z = L$), where $A_H > A_L \geq 0$. We also refer to A_L as the safe return of the project and to $\Delta \equiv A_H - A_L$ as its risky return. The probability that the high return is realized coincides with the monitoring intensity $p \in [0, p_{\max}]$ exerted by the expert that sets up and manages the originator, where $p_{\max} < 1$.⁴ We henceforth refer to p both as the monitoring and risk choice of the expert, and assume that it is not observable and entails the expert a disutility cost per unit of the project given by a function $c(p) \geq 0$ satisfying:

Assumption 1 $c(p) = 0$, $c'(0) = 0$, $c'(p_{\max}) \geq \Delta$, $c''(p) > 0$, and $c'''(p) \geq 0$.

We denote with \bar{p} the efficient risk choice, which is given by:

$$\bar{p} = \arg \max_p \{E[A_z|p] - c(p)\}. \quad (1)$$

⁴Notice that since $p_{\max} < 1$, the risky payoff Δ is in fact never realized with probability one.

Assumption 1 implies that $\bar{p} \in (0, p_{\max}]$ and is determined by the first order condition:

$$c'(\bar{p}) = \Delta. \quad (2)$$

We assume that:

Assumption 2 $E[A_z|\bar{p}] - c(\bar{p}) > 1$.

Assumption 3 $A_L < 1$.

The first assumption states that undertaking the project creates a surplus if efficient risk is chosen. The second assumption implies that the safe return of the project is not sufficient to allow for its funding to rely exclusively on safe securities.

At $t = 0$, an expert that starts an originator invests his unit of wealth in the firm (*inside equity*), and can increase the originator's investment in the project by issuing non-contingent claims (*safe securities*) and state-contingent claims (*risky securities*) whose repayment depends on the realization $z \in \{H, L\}$ of the originator's project. We use from now on the subindexes S and I to refer to safe and risky securities, respectively, where the subindex I refers to risky securities because they are purchased by intermediaries (the other type of financial firm, described next). The originator takes as given the prices of safe and risky securities, which we express as their market expected returns R_S, R_I . The overall notional promise at $t = 1$ on the safe securities issued by the originator and their market price at $t = 0$ are denoted with D_S and x_S , respectively. The variables $D_I = (D_{I,z})_{z \in \{H, L\}}$ and x_I denote analogous objects for the risky securities issued by the originator. Finally, we denote with $x \geq 1$ the total investment by the originator.

After setting up an originator, the expert's problem consists of maximizing the return from its inside equity net of monitoring costs. As we will see, this amounts to a trade-off between maximizing leverage and limiting excessive risk-taking due to moral hazard on the project risk choice. Formally, for given returns R_S, R_I , on the two securities, the problem of the originator at $t = 0$ consists of the choice of a balance sheet tuple $(x, D_S, x_S, D_I, x_I, p)$ solving the maximization problem

$$\max_{(x, D_S, x_S, D_I, x_I, p)} R_{E,O} \equiv E[A_z x - D_S - D_{I,z}|p] - c(p)x, \quad (3)$$

subject to the *budget constraint*

$$1 + x_S + x_I = x, \quad (4)$$

the *state contingent overall repayment constraints*

$$D_S + D_{I,L} \leq A_L x, \quad (5)$$

$$D_S + D_{I,H} \leq A_H x, \quad (6)$$

the *securities' pricing constraints*

$$x_S = \frac{D_S}{R_S}, \quad (7)$$

$$x_I = \frac{E[D_{I,z}|p]}{R_I}, \quad (8)$$

and the *optimal risk choice constraint*

$$p = \arg \max_{p'} \{E[A_z x - D_S - D_{I,z}|p'] - c(p')x\}. \quad (9)$$

The objective function $R_{E,O}$ in (3) is the expected utility the expert obtains from investing its wealth in the originator, which amounts to the value of the residual equity claim net of the monitoring costs. We will henceforth refer to $R_{E,O}$ as the originator's equity return. The maximization of the equity return is subject to the following constraints. The budget constraint (4) states how the originator finances the x units of the project with its own funds and those obtained by issuing safe and risky securities. Constraints (5) and (6) ensure that the securities issued by the originator are repaid in each state $z \in \{H, L\}$. Constraints (7) and (8) provide the pricing equation of the securities given their market returns. Finally, constraint (9) characterizes the risk choice that maximizes the residual payoff of the expert managing the originator taking into account the repayments to the holders of the securities issued to obtain external funding.

Intermediaries An intermediary is a financial firm that issues safe securities by “pooling and tranching” the risky securities purchased from multiple originators. This securitization process allows the expert that manages the intermediary to lever up its investment in the firm. Yet, the presence of *aggregate risk* in the economy, which is described next, limits the intermediaries' leverage because the issuance of safe securities requires of the loss-absorption capacity against non-diversifiable risk provided by the intermediary's equity.

At $t = 1$ an aggregate shock θ that affects the return of the originators' projects is realized. The support of the shock is $[1 - \lambda, 1/p_{\max}]$, with $\lambda \in (0, 1)$. The distribution $F(\theta)$ of the aggregate shock has positive density (at least) in a neighborhood of $\theta = 1 - \lambda$ and satisfies $E[\theta] = 1$. We assume that conditional on the realization of the aggregate shock θ , the high payoff of the project of an originator with risk choice p is θp . Hence, when $\theta > 1$ ($\theta < 1$) the conditional probability of a high payoff is larger (lower) than its unconditional value.⁵ In addition, we assume that conditional on the realization of θ , the project payoffs are independent across originators. The *aggregate risk* parameter λ thus determines the diversification possibilities in the economy: when $\lambda \rightarrow 0$, the risk in the originators' projects is totally diversifiable, while when $\lambda \rightarrow 1$, it is not diversifiable at all.

Besides, we have assumed that the risky securities issued by originators are contingent *only* on the realization of project payoffs. This exposes the pools of risky securities purchased by intermediaries to aggregate risk and forces them to have sufficient equity to be able to issue safe securities.

We next describe the intermediaries formally. At $t = 0$, an expert that starts an intermediary has access to a technology that allows to purchase well diversified pools of risky securities issued by originators and to issue safe securities backed by the payoffs of the portfolio of risky securities.⁶ The intermediary takes as given the market returns and the design of the securities in the market. For the sake of expositional simplicity, we assume that all originators design the same risky security, which, for a given market return R_I , is described by a tuple (x_I, D_I, p) satisfying the pricing equation (8).⁷ The state-contingent return of the risky securities in the market, that we denote with $R_{I,z}$ for $z \in \{H, L\}$, is thus given by:

$$R_{I,z} = \frac{D_{I,z}}{x_I}. \quad (10)$$

We can thus more compactly describe the risky securities in the market by a tuple $(R_{I,H}, R_{I,L}, p)$

⁵Notice that the assumption $\theta \leq 1/p_{\max}$ ensures that the conditional probability of the high return is upper bounded by 1. Besides, using that $E[\theta] = 1$, for an originator with risk choice p we have:

$$\Pr[A_z = A_H] = \int_{1-\lambda}^{1/p_{\max}} \Pr[A_z = A_H | \theta] dF(\theta) = \int_{1-\lambda}^{1/p_{\max}} \theta p dF(\theta) = p E[\theta] = p,$$

as expected.

⁶Allowing intermediaries to issue (outside) equity would not affect our results since these securities should be bought by experts, who can set-up and invest in their own intermediary.

⁷This is the case in equilibrium because as we will see, given market returns R_S, R_I , the maximization problem of the originator described in (3) - (9) has a unique solution.

satisfying:

$$E[R_{I,z}|p] = R_I. \quad (11)$$

The expert managing and intermediary decides at $t = 0$ the amount y of funds to invest in a well-diversified pool of risky securities. This purchase is funded with the unit of wealth of the expert (equity) and with the funds x_S obtained from the issuance of safe securities with an overall notional promise B_S at $t = 1$. For given market returns R_S, R_I and risky securities described by the tuple $(R_{I,H}, R_{I,L}, p)$ satisfying (11), the problem of the intermediary at $t = 0$ consists of choosing a balance sheet tuple (y, B_S, y_S) solving the maximization problem

$$\max_{(y, B_S, y_S)} R_{E,I} \equiv \int_{1-\lambda}^{1/p_{\max}} (E[R_{I,z}|p, \theta] y - B_S) dF(\theta) = R_I y - B_S, \quad (12)$$

subject to the *budget constraint*

$$1 + y_S = y, \quad (13)$$

the *repayment constraint*

$$B_S \leq \min_{\theta} E[R_{I,z}|p, \theta] y, \quad (14)$$

and the *pricing constraint*

$$y_S = \frac{B_S}{R_S}. \quad (15)$$

The objective function $R_{E,I}$ in (12) is the utility of the expert that sets-up an intermediary, which equals the expected residual payoff of the firm. We refer to $R_{E,I}$ as the intermediary's equity return. Notice that the latter expression for $R_{E,I}$ in (12) immediately results from (11) and (14). The maximization of the equity return is subject to the following constraints. The budget constraint (13) states how the intermediary finances its purchase of risky securities from originators with its own funds and those obtained by issuing safe securities. The constraint (14) ensures that the safe securities issued by the intermediary are repaid always in full and takes into account that, by the law of large numbers, the payoff of the intermediary's pool of risky securities at $t = 1$ is a function of the risk choice of the originators p and the realization of the aggregate shock θ . Constraint (15) provides the pricing equation of the safe securities given their market return.

We denote E_O, E_I the measures of experts that set-up at $t = 0$ an originator and an intermediary, respectively. E_O, E_I also represent the aggregate amounts of (inside) equity in each sector.

Equilibrium definition A competitive equilibrium consists of choices for active originators and intermediaries described by balance sheet tuples $(x^*, D_S^*, x_S^*, D_I^*, x_I^*, p^*), (y^*, B_S^*, y_S^*)$, respectively, overall amounts E_O^*, E_I^* of equity in originators and intermediaries, respectively, and expected returns R_S^*, R_I^*, R_E^* on safe debt, risky funding to originators, and financial firms' equity, respectively, such that:

1. The choices of originators and intermediaries satisfy the maximization problems in (3) - (9) and (12) - (15), respectively.
2. The return on equity obtained by an expert that sets-up any financial firm is R_E^* and the experts' decision to set-up a financial firm instead of investing on safe securities or consuming is optimal.
3. Savers' investment and consumption decisions are optimal.
4. The markets for safe and risky securities clear.

Figure 1 graphically illustrates the equilibrium funding structures, the financing and securities flows in the economy and the market clearing conditions.

4 Benchmark: equilibrium without securitization

We consider in this section a benchmark economy in which experts cannot set-up intermediaries. Consider an expert that has set-up an originator and has to decide its project size (or leverage) x at $t = 0$. In absence of intermediaries, the originator can raise external funding only by issuing safe securities. For a given safe rate R_S , the originator's problem is as described in (3) - (9) with the additional constraints $x_I = D_{I,L} = D_{I,H} = 0$.

Using (1), the optimal risk choice condition in (9) satisfies:

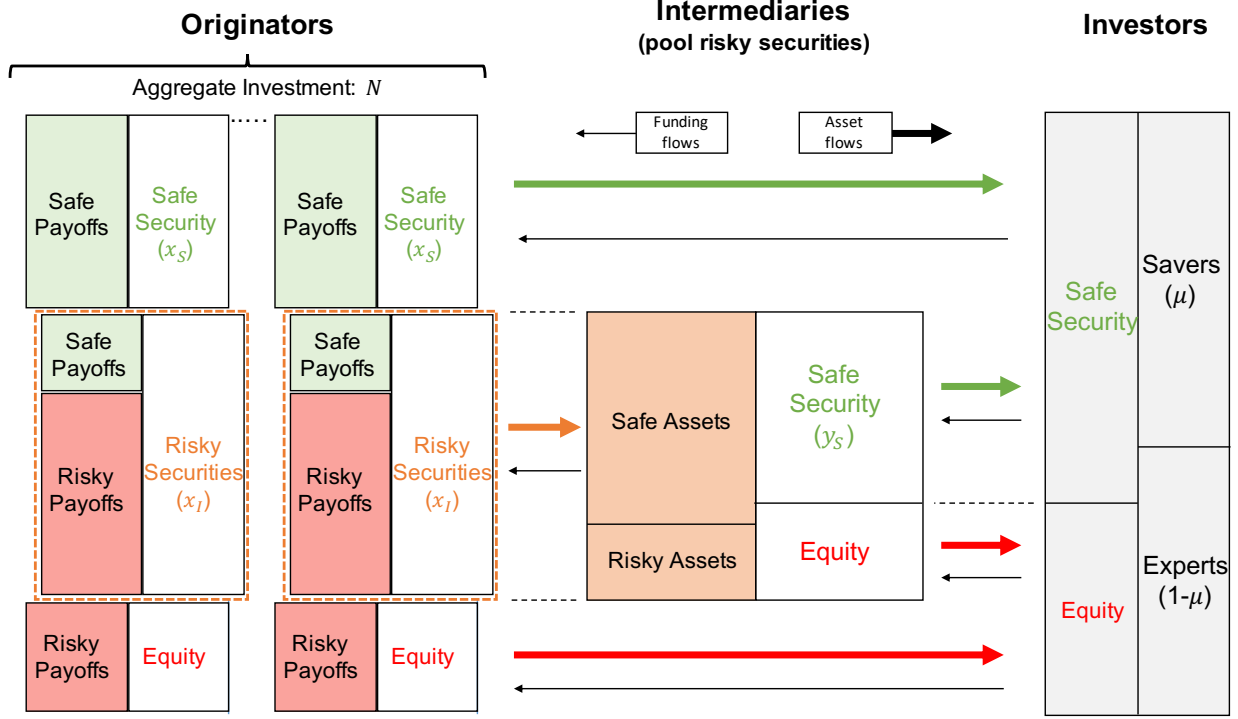
$$p = \arg \max_{p'} \{E[A_z x - D_S | p'] - c(p')x\} = \arg \max_{p'} \{[E[A_z | p'] - c(p')]x - D_S\} = \bar{p}. \quad (16)$$

Since safe debt is always repaid in full, the expert fully appropriates the marginal benefits from monitoring and thus his risk choice is efficient.

We denote with

$$R_A(p) \equiv E[A_z | p] - c(p), \quad (17)$$

Figure 1: Asset distribution and flow of funds



Note. Illustration of the asset distribution and funding flows along the intermediation chain.

the expected return of the project of an originator with risk choice p net of monitoring costs, and refer to $R_A(p)$ as the return of the originators' assets. Using this definition, constraints (4) and (7) in the originator's problem and equation (16), the expression for the return on the originator's equity in (3) can be written as:

$$R_{E,O} = R_S + (R_A(\bar{p}) - R_S)x. \quad (18)$$

The expression states that the originator's return on equity exceeds the safe rate by an amount that is proportional to leverage (x) and the spread between the return on the originator's assets and the safe rate ($R_A(\bar{p}) - R_S$). The next lemma, which results from the market clearing for safe securities, states that such spread is positive in equilibrium (Notice that from here on we denote equilibrium variables in this benchmark economy with a b supraindex):

Lemma 1 *The equilibrium safe rate R_S^b satisfies*

$$1 \leq R_S^b \leq R_A(\bar{p}). \quad (19)$$

We informally illustrate next how the equilibrium is determined when the inequalities in Lemma 1 are strict. Suppose that:

$$1 < R_S^b < R_A(\bar{p}). \quad (20)$$

From the expression for $R_{E,O}$ in (18) we have that the expert finds optimal to issue as much safe securities as possible to maximize project size. In particular, we have that $R_{E,O}^b > R_S^b > 1$ and the entire wealth of experts is invested in originators' equity and that of savers in safe securities. This in particular implies that there is full investment: the entire endowment of the economy is invested in originator's projects and $N^b = 1$. The clearing of the market for safe securities can be written as:

$$\mu = \frac{A_L N^b}{R_S^b}. \quad (21)$$

The LHS in the expression above is the demand for safe securities and its RHS is its supply by originators. The latter takes into account that each unit of investment has a safe pay-off A_L , that overall investment is N^b , and that investors discount safe pay-offs at the rate R_S^b . From (21), and using that $N^b = 1$, we have that:

$$R_S^b = \frac{A_L}{\mu}, \quad (22)$$

which states that the equilibrium safe rate equals the ratio of the overall safe return of originators' projects and savers' wealth. This expression implies that the safe rate is decreasing on savers' wealth μ , and the equilibrium returns satisfy the inequalities in (20) if and only if the savers' wealth lays in an intermediate region. Otherwise, one of the inequalities in Lemma 1 is binding.

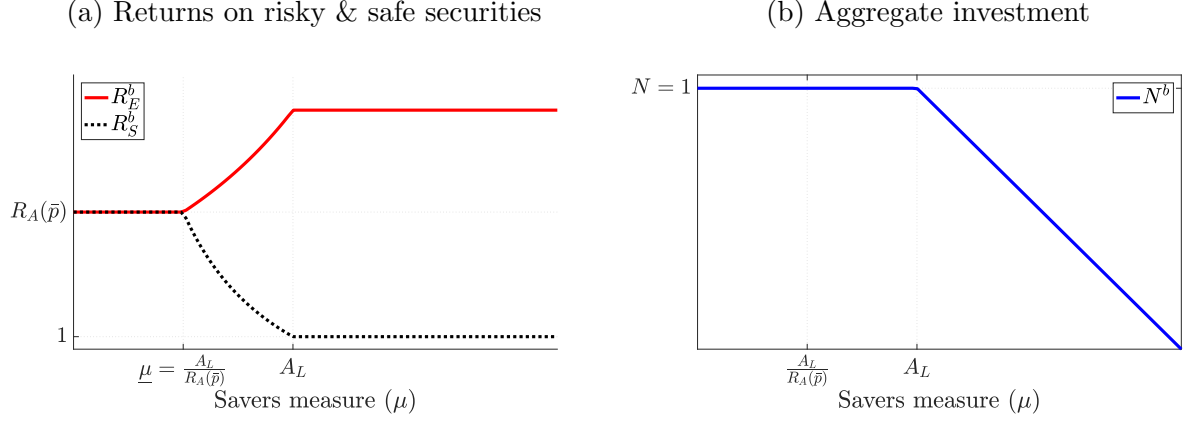
Proposition 2 provides a complete formal characterization of the equilibrium in the economy without securitization.

Proposition 2 *The equilibrium of the benchmark economy without securitization is unique up to Modigliani-Miller type of indifference when there is no equity spread. Let μ be savers' overall wealth and $\underline{\mu} \equiv \frac{A_L}{R_A(\bar{p})}$. Let R_S^b and $R_{E,O}^b$ be the equilibrium return on safe debt and originator's equity, respectively, and N^b the aggregate investment in the economy. We have:*

(i) *If $\mu \leq \underline{\mu}$ then:*

$$R_S^b = R_{E,O}^b = R_A(\bar{p}), \text{ and } N^b = 1.$$

Figure 2: Equilibrium with intermediaries



(ii) If $\mu \in (\underline{\mu}, A_L]$ then:

$$R_S^b < R_A(\bar{p}) < R_{E,O}^b, \text{ and } N^b = 1.$$

(iii) If $\mu > A_L$ then:

$$1 = R_S^b < R_A(\bar{p}) < R_{E,O}^b, \text{ and } N^b = \frac{1 - \mu}{1 - A_L} < 1.$$

The proposition describes how the equilibrium of the economy depends on the savers' wealth, which can be interpreted as a measure of the demand for safety in the economy. Figure 2 illustrates the results in the proposition. When the demand for safety is low ($\mu \leq \underline{\mu}$), the safe payoff of the originators' project is sufficiently large to deliver in equilibrium a high safe rate that equals the expected net return of the originators' project and there is no equity spread. As a result, all the endowment in the economy is invested in originators' projects, there is Modigliani-Miller indifference in the capital structure of originators and in some equilibria a fraction of the experts invest their endowment in safe securities. For an intermediate demand for safety ($\mu \in (\underline{\mu}, A_L]$), the available safe-payoffs in the economy become scarce relative to the savers wealth and the equilibrium safe rate falls. This in turn leads to an increase in the equity return that induces experts to invest their entire endowment in originators' equity. In this region, the scarcity of safe payoffs leads to a positive equity spread but full investment is still achieved. When the demand for safety is large ($\mu > A_L$), the safe rate is one, some savers opt to consume their endowment and full investment is not achieved.

5 Partial equilibrium: exogenous safe rate

In this Section, we consider the baseline economy with intermediaries in a *partial equilibrium* context with an exogenously fixed safe rate R_S . We start the analysis with the following lemma that provides the relevant range of values for R_S and its equilibrium relationship with the returns on risky securities and equity:

Lemma 3 *The general equilibrium value of the safe rate, R_S^* , must satisfy*

$$1 \leq R_S^* \leq R_A(\bar{p}).$$

Besides, for a given exogenous safe rate $R_S \leq R_A(\bar{p})$, if a partial equilibrium exists, then the equilibrium returns on risky securities, $R_I^(R_S)$, and equity, $R_E^*(R_S)$, satisfy*

$$R_S \leq R_I^*(R_S) \leq R_E^*(R_S).$$

Finally, there is equality in either of the inequalities above if and only if $R_S = R_A(\bar{p})$.

The lemma makes three statements. First, it provides bounds on the general equilibrium safe rate that result from savers' possibility to consume at $t = 0$ and the maximum expected return at $t = 1$ of the productive assets in the economy. Second, for an exogenously fixed safe rate below its maximum equilibrium value, the lemma states that, in partial equilibrium, the expected return on risky securities lays between between the return on safe securities and that on equity. This results from the fact that the assets of the intermediaries consist of risky securities, while their funding sources consist of safe securities and equity. Third, there is no spread between the return on equity, risky securities and safe securities if and only if the exogenous safe rate satisfies $R_S = R_A(\bar{p})$. When that is the case, a Modigliani-Miller type of capital structure indifference arises and the payoff the two investor types coincides with that in no intermediaries benchmark described in the previous section. To avoid the notational complexities of having to deal with equilibrium indeterminacy, we conduct the partial equilibrium analysis in the rest of Section 5 under the assumption that $R_S < R_A(\bar{p})$.

The partial equilibrium analysis is split in three steps. First, we consider the originator's problem and describe its dependence on the ratio between the return on equity and that of risky securities, R_E/R_I , which we call the *intermediary funding discount*. Second, we derive from the intermediary's problem a *funding discount pass-through equation* that provides a

relationship between the returns of the three funding sources in the economy (equity, risky securities and safe securities). Third, we analyze the determination of the equilibrium returns on the risky funding forms (equity and risky securities) and the other partial equilibrium variables.

5.1 Originators' problem and the intermediary funding discount

In this section we consider the originators' optimal balance sheet tuple $(x, D_S, x_S, D_I, x_I, p)$ problem (3) - (9) for given returns $R_S < R_A(\bar{p})$ and $R_I > R_S$. Using the pricing equations (7) and (8), the return on the originator's equity is given by:

$$R_{E,O} = R_A(p) + (R_A(p) - R_S)x_S + (R_A(p) - R_I)x_I. \quad (23)$$

This expression extends that in (18) by including a third term that captures the (positive or negative) spread experts obtain by issuing risky securities to expand project size. Finally, $R_{E,O}$ also depends on the risk choice of the originator, which we analyze next.

Using (5), the optimal risk choice condition in (9) takes the form:

$$p = \arg \max_{p'} \left\{ p' \left(\Delta - \left(\frac{D_{I,H} - D_{I,L}}{x} \right) \right) - c(p') \right\}. \quad (24)$$

Notice that the risk choice does not depend on the promise on safe securities D_S . Since the issuance of safe securities is cheaper than that of risky securities, we have that:

Lemma 4 *For given $R_S < R_A(\bar{p})$ and $R_I > R_S$, any solution to the originator's problem satisfies:*

$$D_S = A_L x \text{ and } D_{I,L} = 0.$$

The lemma states that the originator exhausts its capability to issue safe securities. As a result, the risky securities are described by their promise under the H payoff of the project or, equivalently, by their promise per unit of the project:

$$d_I \equiv D_{I,H}/x, \quad (25)$$

which we refer to as the *risky security promise*. We have from Lemma 4 and (8) that $d_I \in [0, \Delta]$.

Using Lemma 4, we obtain from the optimality condition (24) the following first order condition for the optimal risk choice p given d_I :

$$\Delta - d_I = c'(p). \quad (26)$$

The optimality condition above implies that:

Lemma 5 *For given $R_S < R_A(\bar{p})$ and $R_I > R_S$, the originators' optimal risk choice is a function $\hat{p}(d_I)$ of the risky security promise $d_I \in [0, \Delta]$ satisfying*

$$\frac{d\hat{p}(d_I)}{dd_I} < 0, \hat{p}(0) = \bar{p} \text{ and } \hat{p}(\Delta) = 0. \quad (27)$$

The lemma states that as the risky security promise d_I increases the originator's project becomes riskier (p decreases). The reason is that when d_I is larger, the expert's incentives to undertake the costly monitoring get reduced, since the value created by this action is to a larger extent appropriated by the holders of the risky securities. The non-observability of the monitoring intensity thus creates a moral hazard problem that increases the project risk when risky securities are issued.

Using constraints (4) - (8), Lemma 4 and Lemma 5, it is possible to rewrite the originator's return on equity in (23) as:

$$R_{E,O}(d_I) = \frac{R_A(\hat{p}(d_I)) - A_L - \hat{p}(d_I)d_I}{1 - A_L/R_S - \hat{p}(d_I)d_I/R_I}. \quad (28)$$

The numerator of this expression captures the expected residual cash-flow generated by each unit of the project after repayment of safe and risky securities issued and net of the monitoring costs. The denominator represents the funding provided by the expert to each unit of the project.

From our discussion so far, the originator's problem (3) - (9) can be written as the following optimal choice of d_I :

$$\max_{d_I \in [0, \Delta]} R_{E,O}(d_I). \quad (29)$$

The following lemma characterizes the solution to the originator's problem.

Lemma 6 *For given $R_S < R_A(\bar{p})$, let $\bar{R}_I > \underline{R}_I$ be the positive constants given by*

$$\bar{R}_I = \frac{R_A(\bar{p}) - A_L}{1 - A_L/R_S} \text{ and } \underline{R}_I = \frac{\max_{d_I} (\hat{p}(d_I)d_I)}{1 - A_L/R_S}.$$

Suppose that $R_I > R_S$. Then if $R_I \in (\underline{R}_I, \bar{R}_I)$, the solution d_I^* to (29) is unique, satisfies

$$(R_{E,O}(d_I) - R_I) \left(\frac{1}{R_I} \frac{d(\hat{p}(d_I)d_I)}{dd_I} \right) + \frac{dR_A(\hat{p}(d_I))}{dd_I} = 0, \quad (30)$$

and leads to $R_{E,O}(d_I^*) > R_I$. Besides, if $R_I \geq \bar{R}_I$ then $d_I^* = 0$ is the unique solution to (29), while if $R_I \leq \underline{R}_I$, then $R_{E,O}(d_I)$ can grow unboundedly.

The lemma states that for an intermediate range of values of R_I , the optimal risky security promise is characterized by the first order condition in (30). The first term in this expression captures the *leverage gains*: a marginal increase dd_I in d_I allows the originator to raise additional funds dx_I from risky securities amounting to $dx_I = (1/R_I)d(\hat{p}(d_I)d_I)$ per unit of the project. The additional funds have a cost R_I for the originator, but free up an equal amount of equity that (in combination with external financing) allows to increase project size and to obtain a return $R_{E,O}$, so that the originator obtains a spread $R_{E,O} - R_I$ on the additional funds. The second term in (30), which from Lemma 5 is negative, accounts for the *incentives costs*: a marginal increase dd_I in d_I weakens the originator's incentives to monitor, which entails a reduction in the net return of each unit of the project of $dR_A < 0$.

Lemma 6 also characterizes when the originators' risky external funding problem exhibits corner solutions. If the return on risky securities is sufficiently high, originators do not rely on this funding form, while if it is sufficiently low, the financial constraints are so weak that the originator can unboundedly increase leverage and its return on equity. The latter, though, cannot happen in general equilibrium since securities' markets would not clear.

We next consider the optimality condition (30) *in equilibrium*. Denoting $\chi^*(R_S) \equiv R_E^*(R_S)/R_I^*(R_S)$, which from Lemma 3 satisfies $\chi^*(R_S) > 1$, such condition in equilibrium can be written as

$$\underbrace{(\chi^*(R_S) - 1) \frac{d(\hat{p}(d_I)d_I)}{dd_I}}_{\text{Leverage gains (+)}} + \underbrace{\frac{dR_A(\hat{p}(d_I))}{dd_I}}_{\text{Incentives costs (-)}} = 0, \quad (31)$$

The decomposition highlights the key role played by the ratio $\chi^*(R_S)$ for the relative importance of the leverage and incentive effects in the determination of the optimal d_I : When $\chi^*(R_S)$ is large, risky securities constitute a much cheaper funding source than equity, and thus the incentives for the originator to expand leverage by switching, for each unit of the project, some equity funding with risky securities funding are strong. We can interpret the

ratio $\chi^*(R_S)$ as the *funding discount* offered by the intermediaries for the financing of the risky payoffs of the originators

We obtain from (30) the following result:

Proposition 7 *There exists a function $\widehat{d}_I(\chi)$ defined for any $\chi \geq 1$ and independent from the safe rate R_S , such that if an equilibrium exists for a given $R_S < R_A(\bar{p})$ then the equilibrium variables $d_I^*(R_S), p^*(R_S)$ and $\chi^*(R_S)$ satisfy*

$$d_I^*(R_S) = \widehat{d}_I(\chi^*(R_S)) \text{ and } p^*(R_S) = \widehat{p}\left(\widehat{d}_I(\chi^*(R_S))\right), \quad (32)$$

where $\widehat{p}(d_I)$ is defined in Lemma 5. Besides, we have that

$$\frac{d\widehat{d}_I(\chi)}{d\chi} > 0, \frac{d\widehat{p}\left(\widehat{d}_I(\chi)\right)}{d\chi} < 0, \frac{d\left(\widehat{p}\left(\widehat{d}_I(\chi)\right)\widehat{d}_I(\chi)\right)}{d\chi} > 0. \quad (33)$$

Finally, $\widehat{d}_I(1) = 0$.

The proposition has three results. First, it states that the the originators' choices (d_I^*, p^*) are given by functions of the intermediary funding discount χ^* , so that the effect of changes on R_S on the originators' decisions is totally determined by the effect of this change on χ^* . This important property allows us to derive the partial equilibrium of the economy sequentially by determining first the intermediary funding discount χ^* given R_S , and after that the remaining variables. Second, the proposition states that changes in R_S that lead to an increase in χ^* also necessarily induce an increase in the risky debt promise d_I^* , its expected payoff $p^*d_I^*$, and risk-taking (p^* decreases). The intuition stems from the equilibrium optimality condition for d_I in (30), that shows that when χ^* is larger the leverage effect becomes more important and the originator has stronger incentives to substitute the financing of the risky part of its project from equity to risky securities, even though this increases risk-taking. Third, as the intermediary funding discount χ^* approaches 1, the issuance of risky securities becomes negligible.

5.2 Intermediary's problem and the funding discount pass-through equation

In this section we analyze the intermediary's problem and derive an equation that links the intermediary funding discount, $\chi^*(R_S) = R_E^*(R_S)/R_I^*(R_S)$, and the ratio of return on equity and safe securities, $R_E^*(R_S)/R_S$.

Suppose that $R_I > R_S$ and recall the definition of the return $R_{I,z}$ of risky securities in (10). From Lemma 4, we have that

$$R_{I,L} = 0, R_{I,H} = R_I/p. \quad (34)$$

An expert setting up and investing its wealth in an intermediary chooses at $t = 0$ a balance sheet tuple (y, B_S, y_S) solving the maximization problem (12) - (15). Using (13) and (15), the intermediary's return on equity $R_{E,I}$ can be written as the following function of its asset size (or leverage) y :

$$R_{E,I} = R_S + (R_I - R_S)y. \quad (35)$$

The expression implies that the intermediary makes a spread $R_I - R_S > 0$ on each unit of investment in risky securities. The intermediary's return on equity is thus maximized with maximum leverage and from (14) and (34), the optimal promise B_S on safe securities satisfies:

$$\begin{aligned} B_S &= \min_{\theta} E[R_{I,z}|p, \theta] y = \min_{\theta} [\theta p R_{I,H} + (1 - \theta p) R_{I,L}] y = \\ &= (1 - \lambda) R_I y. \end{aligned} \quad (36)$$

Notice that the lowest return of each of the securities purchased by the intermediary is zero, but due to diversification it is able to pledge a fraction $1 - \lambda$ of the return of its portfolio of securities to safe security investors. Securitization effectively expands the supply of safe securities in the economy.

Using (36), the pricing constraint for the intermediary's safe securities in (15) can be rewritten as:

$$R_S y_S = (1 - \lambda) R_I y, \quad (37)$$

and from this expression, $R_{E,I}$ in (35) can be rewritten as:

$$R_{E,I} = \lambda R_I y. \quad (38)$$

Equations (37) and (38) capture how the safe and risky parts of the payoffs of the intermediary's pool of securities are pledged to safe security investors and the expert who holds the intermediary's equity, respectively.

Using (13), (37) and (38), we get the following accounting identity that captures how the intermediary “passes” its funding cost to originators:

$$\frac{1}{R_I} = (1 - \lambda) \frac{1}{R_S} + \lambda \frac{1}{R_{E,I}}. \quad (39)$$

In fact, taking into account that the cost of a funding form equals the inverse of its expected return, the LHS in the expression above coincides with the cost of intermediary funding to originators and the RHS captures the average cost of the funding raised by the intermediary. Notice that the latter captures that the return of the intermediary’s assets is pledged to investors in safe securities and equity in proportions $1 - \lambda$ and λ , respectively.

Using that in equilibrium the return on equity on both originators and intermediaries equals $R_E^*(R_S)$, we immediately obtain from (39) the following result:

Proposition 8 *For a given safe rate $R_S < R_A(\bar{p})$, suppose an equilibrium exists and let $R_E^*(R_S)$ and $\chi^*(R_S)$ denote the associated equilibrium variables. They satisfy:*

$$\chi^*(R_S) = (1 - \lambda) \frac{R_E^*(R_S)}{R_S} + \lambda. \quad (40)$$

The proposition provides an equilibrium *funding discount pass-through equation* that states that the funding discount (relative to the cost of equity funding) χ^* offered by the intermediary to originators amounts to the weighted average of the “discounts” with which the intermediary finances its portfolio of risky securities. In fact, in equilibrium a fraction $1 - \lambda$ of the return of the intermediary’s assets are used to issue safe securities which have a cost advantage relative to equity of R_E^*/R_S , while the residual fraction λ is used to compensate the expert holding the intermediary’s equity at no discount relative to the originators’ equity.

From the funding discount pass-through equation in (40), we have that if $\lambda \rightarrow 1$ and diversification possibilities in the economy disappear, then $\chi^* \rightarrow 1$ and intermediary funding is not cheaper than equity. If in contrast $\lambda \rightarrow 0$ and all the risk in the economy is diversifiable, then $\chi^* \rightarrow R_E^*/R_S$ and intermediary funding becomes as cheap as the issuance of safe securities. Similarly, if changes in the exogenous safe rate R_S lead to an increase in the relative equity spread, R_E^*/R_S , the funding discount pass-through equation implies that the intermediary can offer in equilibrium a larger funding discount χ^* .

5.3 Equilibrium with exogenous safe rate

In this Section we finish the characterization of the equilibrium of the economy for an exogenous safe rate. The analysis is split as follows: We first focus on the determination of the equilibrium returns R_E^* and R_I^* . We then describe the main properties of the financial firms' balance sheets and their dependence on R_S . After that, we describe experts' wealth allocation across the equity of originators and intermediaries and aggregate investment. For the sake of notational simplicity we will drop for the remaining of this section the dependence of the equilibrium variables on R_S except in the statement of formal results.

The returns of the risky funding sources For given R_S , let $\chi^* = R_E^*/R_I^*$ be the equilibrium intermediary funding discount. Using the functions $\widehat{p}(d_I)$ defined by the optimality condition for the risk choice in (26), and $\widehat{d}_I(\chi)$ defined in Proposition 7 and capturing the optimal risky promise made by the originator as a function of the intermediary funding discount, we can use (28) to rewrite the equilibrium return on equity as the following function of R_S and χ^* :

$$R_E^*(R_S, \chi^*) = \frac{\widehat{p}(\widehat{d}_I(\chi^*))\Delta - c(\widehat{p}(\widehat{d}_I(\chi^*))) + (\chi^* - 1)\widehat{p}(\widehat{d}_I(\chi^*))\widehat{d}_I(\chi^*)}{1 - A_L/R_S}. \quad (41)$$

Using this function, we can rewrite the funding discount pass-through equation (40) as:

$$\chi^* = (1 - \lambda) \frac{R_E^*(R_S, \chi^*)}{R_S} + \lambda. \quad (42)$$

This equation provides an equilibrium relationship between R_S and χ^* , from which we can derive the following result:

Proposition 9 *There exists $\underline{R}_S \in (A_L, R_A(\bar{p}))$, such that for a given safe rate $R_S < R_A(\bar{p})$ an equilibrium exists if and only if $R_S > \underline{R}_S$, in which case the equilibrium is unique. For $R_S > \underline{R}_S$, the functions $R_E^*(R_S), \chi^*(R_S)$ describing the equilibrium return on equity and intermediary funding discount, respectively, satisfy*

$$\begin{aligned} \frac{dR_E^*(R_S)}{dR_S} &< 0, \lim_{R_S \rightarrow \underline{R}_S} R_E^*(R_S) = \infty, \text{ and } \lim_{R_S \rightarrow R_A(\bar{p})} R_E^*(R_S) = R_A(\bar{p}), \\ \frac{d\chi^*(R_S)}{dR_S} &< 0, \lim_{R_S \rightarrow \underline{R}_S} \chi^*(R_S) = \infty, \text{ and } \lim_{R_S \rightarrow R_A(\bar{p})} \chi^* = 1. \end{aligned}$$

The proposition states that the partial equilibrium of the economy with exogenous safe rate exists and is unique unless the safe rate is too low, in which case financial constraints would not be binding and equity returns would be infinity. The proposition also states that in equilibrium both the intermediary funding discount and the return on equity are strictly decreasing in the safe rate. The intuition is as follows. An increase in R_S , increases the funding cost of the originator and decreases its leverage which, for given χ^* , reduces R_E^* . As a result, the relative equity spread R_E^*/R_S falls and the funding discount pass-through equation (40) implies that χ^* decreases. This amplifies the initial reduction in R_E^* as the reduction in χ^* makes the funding of the intermediary less advantageous, which further reduces R_E^* and, through (40), leads to an additional reduction of χ^* .

Financial firms balance sheets Combining the analysis in Section 5.1 and 5.2 with Proposition 9, we can prove the following result:

Proposition 10 *For a given safe rate $R_S \in (\underline{R}_S, R_A(\bar{p}))$, the functions $d_I^*(R_S), p^*(R_S), x^*(R_S)$ describing the originator's equilibrium risky security promise, risk choice, and leverage, respectively, satisfy*

$$\begin{aligned} \frac{dd_I^*(R_S)}{dR_S} &< 0, \frac{dp^*(R_S)}{dR_S} > 0, \frac{d(p^*(R_S)d_I^*(R_S))}{dR_S} < 0, \frac{dx^*(R_S)}{dR_S} < 0 \text{ and} \quad (43) \\ \lim_{R_S \rightarrow R_A(\bar{p})} d_I^*(R_S) &= 0, \lim_{R_S \rightarrow R_A(\bar{p})} p^*(R_S) = \bar{p} \text{ and } \lim_{R_S \rightarrow R_A(\bar{p})} x^*(R_S) = 1 \end{aligned}$$

Besides, the function $y^(R_S)$ describing the intermediary's equilibrium leverage satisfies*

$$\frac{dy^*(R_S)}{dR_S} < 0 \text{ and } \lim_{R_S \rightarrow R_A(\bar{p})} y^*(R_S) = 1.$$

The proposition describes how the financial firms' equilibrium balance sheets respond to an increase in the safe rate. When R_S increases, the equity spread falls and the intermediary is less capable of offering cheap funding to originators (χ^* decreases, from Proposition 9). This in turn leads originators to pledge a lower part of their risky payoffs for the issuance of risky securities, improving incentives at origination (d_I^* and $p^*d_I^*$ decrease, and p^* increases, from Proposition 7). Finally, the increase in R_S reduces the external funding raised by the originator both with safe and risky securities, which reduces its leverage (x^* decreases). In addition, the reduction in the intermediary's funding discount associated with an increase in R_S also leads, in equilibrium, to a reduction in the intermediary's leverage (y^* decreases, from (38)).

The equity allocation Recall that E_O^* and E_I^* denote the aggregate amount of experts' funds invested in the equity of originators and intermediaries, respectively. The determination of the equity allocation across the two sectors results from two equilibrium conditions. First, the clearing of the market for risky securities, which can be written as:

$$E_O^* x_I^* = E_I^* y^*, \quad (44)$$

where the LHS captures the overall supply of risky securities by originators and the RHS accounts for its overall demand by intermediaries.

Second, in equilibrium the return experts obtain from investing in the equity of each of the financial firms must be the same, which implies that:

$$[R_A(p^*) - A_L - p^* d_I^*] x^* = \lambda R_I^* y^*. \quad (45)$$

The LHS corresponds to the return of the originators' equity and is expressed as the product of the expected residual payoff of each unit of the project and project size. The RHS captures the return of the intermediary's equity, which corresponds to the fraction λ of the expected payoff of the intermediary's assets that is risky and thus pledged to equity investors.

We can obtain from (44), (45) and Proposition 10 the following result:

Proposition 11 *For a given safe rate $R_S \in (\underline{R}_S, R_A(\bar{p}))$, the functions $E_O^*(R_S)$, $E_I^*(R_S)$ describing the equilibrium amounts of equity invested in the originators and intermediaries, respectively, satisfy the following relationship*

$$\frac{E_I^*(R_S)}{E_O^*(R_S)} = \frac{p^*(R_S) d_I^*(R_S)}{R_A(p^*(R_S)) - A_L - p^*(R_S) d_I^*(R_S)} \lambda. \quad (46)$$

and the properties

$$\frac{dE_O^*(R_S)}{dR_S} < 0, \quad \lim_{R_S \rightarrow R_A(\bar{p})} E_O^*(R_S) = 1 - \mu \text{ and } \frac{dE_I^*(R_S)}{dR_S} > 0, \quad \lim_{R_S \rightarrow R_A(\bar{p})} E_I^*(R_S) = 0.$$

Equation (46) states that the ratio of equity invested in intermediaries relative to that in originators is the product of two factors. The first one captures how the expected risky payoff of the originators' projects net of monitoring costs, $R_A(p^*) - A_L$, is tranchsed into risky securities sold to intermediaries, $p^* d_I^*$, and inside equity placed to experts, $R_A(p^*) - A_L - p^* d_I^*$. The second factor is the aggregate risk parameter λ , that accounts for the fraction of the tranche placed to intermediaries that is funded with equity.

Proposition 11 also describes how the equity allocation along the chain is affected by the safe rate. An increase in R_S leads originators to reduce the part of the risky payoffs of their projects that backs the issuance of risky securities to intermediaries ($p^*d_I^*$ decreases, from Proposition 10), and hence to increase the part which contributes to the retribution of inside equity. The “retranching” of the risky payoffs of the originators’ projects must be accompanied in equilibrium with a reallocation of experts’ funds from the equity of intermediaries to that of originators (first factor in the RHS of (46) decreases).

Aggregate investment Overall investment in the economy, N^* , equals the aggregate amount of funding raised by originators from their three sources of financing, that is:

$$N^* = E_O^* + E_O^*x_S^* + E_O^*x_I^*. \quad (47)$$

Using the market clearing for risky securities (44) and the intermediary’s budget constraint (13), we can rewrite N^* as:

$$N^* = (E_O^* + E_I^*) + (E_O^*x_S^* + E_I^*y_S^*). \quad (48)$$

This equation captures the *end financing flow* from investors to projects in the economy. The first term in parentheses accounts for overall investment in financial firms’ equity, and the second term includes the overall issuance of safe securities by financial firms.

Since $R_E^* > R_I^* > R_S$, each expert finds optimal to set up a financial firm and invest its entire endowment in its equity, so that

$$E_O^* + E_I^* = 1 - \mu. \quad (49)$$

Besides, each financial firm exhausts its capacity to issue safe securities, so that the overall funding raised with safe securities by originators and intermediaries is given by the following aggregate pricing constraints:

$$E_O^*x_S^* = \frac{A_L N^*}{R_S}, \quad (50)$$

$$E_I^*y_S^* = \frac{(1 - \lambda)p^*d_I^*N^*}{R_S}. \quad (51)$$

The expressions discount at the rate R_S the overall safe part of the assets’ payoff of the two types of financial firms.

Combining (48) - (51) we obtain the following result:

Proposition 12 *For a given safe rate $R_S \in (\underline{R}_S, R_A(\bar{p}))$, the equilibrium aggregate investment $N^*(R_S)$ is given by*

$$N^*(R_S) = \frac{1 - \mu}{1 - (A_L + (1 - \lambda)p^*(R_S)d_I^*(R_S)) / R_S}, \quad (52)$$

and satisfies

$$\frac{dN^*(R_S)}{dR_S} < 0, \quad \lim_{R_S \rightarrow \underline{R}_S} N^*(R_S) = \infty.$$

The intuition for the expression for aggregate investment N^* in (52) is as follows. The numerator accounts for experts' aggregate wealth, which is invested as equity in financial firms. The denominator is an *overall leverage* multiplier that captures how much equity funding is needed as “downpayment” per unit of the project. It accounts for the sum of the safe securities raised per unit of final investment directly by originators, A_L/R_S , and indirectly by intermediaries, $(1 - \lambda)p^*d_I^*/R_S$. As R_S increases, the part of the risky pay-off that is pledged to the intermediary to back safe securities issuance decreases ($p^*d_I^*$ decreases, from Proposition 10), and the rate at which investors discount safe payoffs increases. The two effects lead to a reduction in the overall value of the safe securities issued by the financial sector, which decreases the overall leverage of the experts' endowment and aggregate investment.

6 Equilibrium and welfare analysis

In this Section we determine the equilibrium of the economy with an endogenous safe rate, analyze the welfare implications of the emergence of securitization, and show that the equilibrium of the economy is Pareto constrained efficient.

6.1 Equilibrium with endogenous safe rate

We first characterize the equilibrium of the economy with an endogenous safe rate and its dependence on savers' aggregate wealth, μ , which can be interpreted as the demand for safety in the economy. Using the partial equilibrium results in the previous section, we only need to determine the value of the safe rate that ensures that aggregate investment in the economy equals investors' overall investment in financial firms.⁸

⁸Equivalently, the equilibrium safe rate can also be found by imposing the clearing of the market for safe securities.

Consider an equilibrium with a safe rate R_S^* satisfying

$$1 < R_S^* < R_A(\bar{p}), \quad (53)$$

which, from Proposition 9, implies that in equilibrium $R_E^* > R_S^*$. In such an equilibrium savers and experts find optimal to invest all their endowment in safe securities and equity, respectively, so that aggregate investment satisfies $N^* = 1$. We obtain thus from (52) the following expression for R_S^* that extends that in (22) for the benchmark economy:

$$R_S^* = \frac{A_L + (1 - \lambda)p^*(R_S^*)d_I^*(R_S^*)}{\mu}. \quad (54)$$

The equation states that the equilibrium safe rate equals the ratio of overall safe payoffs in the economy and savers' wealth. Using Proposition 10, the equation implies that R_S^* is decreasing in μ and the safe rate satisfies (53) for an intermediate region of values for μ . Otherwise, one of the two inequalities in (53) is binding.

Building on these intuitions, the next proposition formally characterizes the equilibrium of the economy.

Proposition 13 *The equilibrium of the economy is unique up to Modigliani-Miller type of indifference when there is no equity spread. Let μ be savers' overall wealth and $\underline{\mu}, \bar{\mu} \in (0, 1]$ with $\underline{\mu} < \bar{\mu}$ the constants defined as*

$$\underline{\mu} = \frac{A_L}{R_A(\bar{p})}, \bar{\mu} = \begin{cases} \frac{A_L + (1 - \lambda)p^*(1)d_I^*(1)}{1} & \text{if } \underline{R}_S < 1 \\ 1 & \text{otherwise} \end{cases},$$

Let R_S^, R_E^*, p^*, N^* be the equilibrium safe rate, return on equity, originator's risk choice, and aggregate investment, respectively. We have:*

(i) *If $\mu \leq \underline{\mu}$, then intermediaries do not enter and:*

$$R_S^* = R_E^* = R_A(\bar{p}), p^* = \bar{p} \text{ and } N^* = 1.$$

(ii) *If $\mu \in (\underline{\mu}, \bar{\mu}]$, then intermediaries enter and:*

$$R_S^* < R_A(\bar{p}) < R_E^*, p^* < \bar{p} \text{ and } N^* = 1.$$

(iii) If $\mu > \bar{\mu}$, which requires that $\bar{\mu} < 1$, then intermediaries enter and:

$$1 = R_S^* < R_A(\bar{p}) < R_E^*, p^* < \bar{p} \text{ and } N^* = \frac{1 - \mu}{1 - \bar{\mu}} \in (N^b, 1),$$

where N^b is the equilibrium aggregate investment in the benchmark economy with no intermediaries.

The proposition describes how the main equilibrium variables depend on the demand for safety in the economy. Figure 3 illustrates the results in the proposition and also exhibits some other equilibrium variables not discussed in the proposition. When the demand for safety is low ($\mu \leq \underline{\mu}$), the originators' safe payoffs are enough to deliver a high return on safe securities. There is no equity spread and thus no securitization. As the demand for safety increases ($\mu \in (\underline{\mu}, \bar{\mu}]$), the safe securities supplied by originators become scarce, which gives rise to a positive equity spread. When that happens, intermediaries can use their ability to create safe securities through securitization to exploit the equity spread, so intermediaries endogenously emerges. As originators pledge to intermediaries a fraction of their risky payoffs, the supply of safe securities increases but monitoring incentives at origination deteriorate, which leads to more risk-taking ($p^* < \bar{p}$). As the demand for safety keeps on increasing in this region, the safe rate drops and the equity spread widens. This leads to a reallocation of experts' wealth (equity) from originators to intermediaries, which contributes to the increase in risk-taking. In this intermediate region, the financial sector is able to create sufficient safe securities to achieve full investment of the economy's endowment. This is not anymore the case when demand for safety becomes very large ($\mu > \bar{\mu}$) because the safe rate falls to one and some savers opt to consume their endowment.

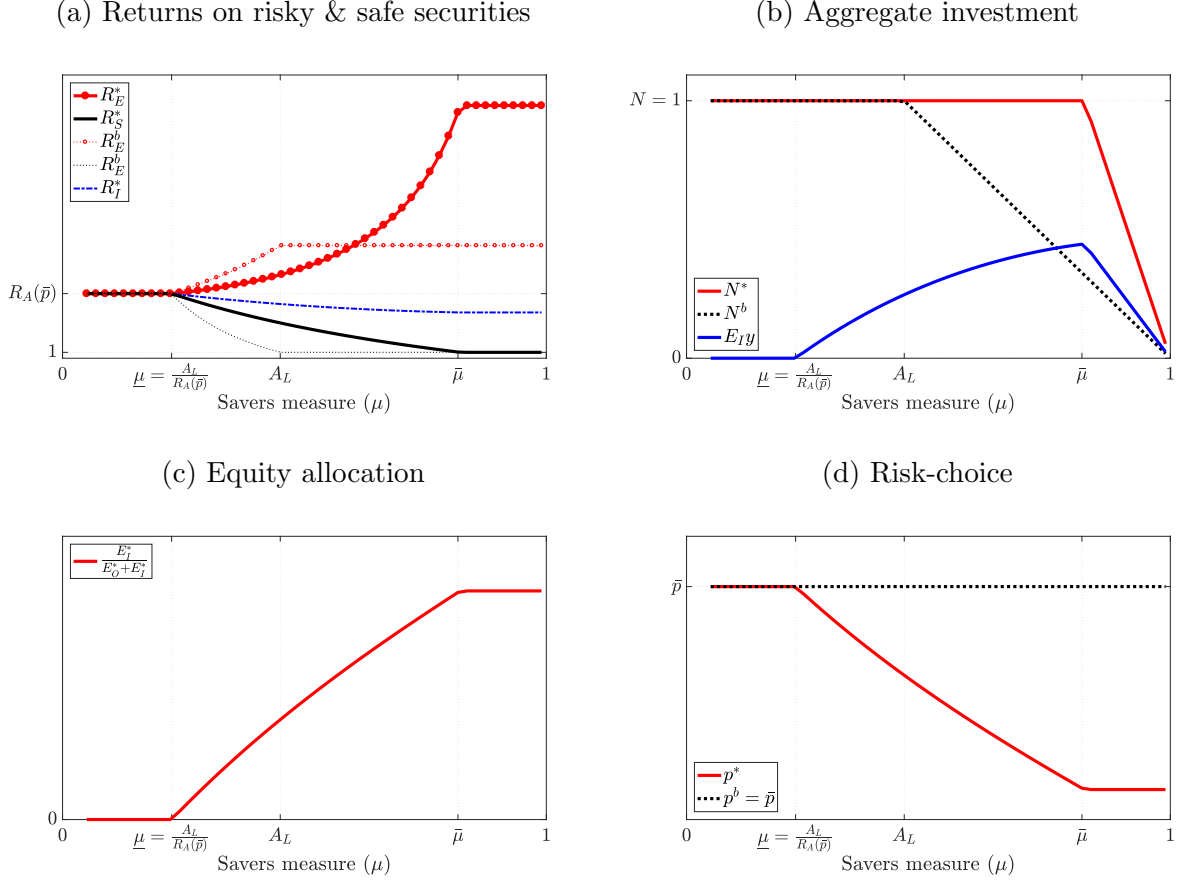
6.2 Welfare effects from emergence of securitization

We next address the welfare effects associated with the emergence of securitization. We do so by comparing the equilibrium utility of savers and experts with that in the benchmark economy without intermediaries.

Since investors have linear utilities with zero discount, the equilibrium expected utility of savers and experts in the economy with intermediaries coincides with the expected return on safe securities, R_S^* , and equity, R_E^* , respectively.⁹ Aggregate welfare in the economy, which

⁹Notice that the statement is true also if $R_S^* = 1$ or if $R_S^* = R_E^*$.

Figure 3: Equilibrium with intermediaries



Note. In each panel, dotted lines correspond to the benchmark economy without securitization.

is defined as $W^* \equiv \mu R_S^* + (1 - \mu) R_E^*$, can be written as:

$$W^* = (1 - N^*) + N^* E[A_z | p^*] - N^* c(p^*). \quad (55)$$

The expression results from two observations. First, aggregate welfare coincides with expected aggregate consumption by investors net of the monitoring costs incurred by some experts. Second, aggregate consumption at $t = 0$ coincides with the amount of funds that are not invested in originators' projects and at $t = 1$ coincides with the payoff of those projects.

The welfare variables in the benchmark economy with no intermediaries can be described in an analogous manner and are denoted, as in Section 4, with a superscript b . The welfare effect due to the emergence of securitization for savers, experts and the aggregate economy,

are defined as:

$$\Delta R_S = R_S^* - R_S^b, \Delta R_E = R_E^* - R_E^b \text{ and } \Delta W = W^* - W^b.$$

Using (17), we can write ΔW as:

$$\Delta W = \underbrace{(N^* - N^b)(R_A(p^*) - 1)}_{\text{Gain from increase in investment (+)}} - \underbrace{N^b(R_A(\bar{p}) - R_A(p^*))}_{\text{Cost from increase in risk (-)}}. \quad (56)$$

The expression for the aggregate welfare effect from securitization is the difference of two terms. The first one captures the value created by the expansion in investment allowed by the additional safe securities created by intermediaries ($N^* \geq N^b$, from Propositions 2 and 13). The second term accounts for the costs implied by the increase in originators' risk induced by the emergence of intermediaries ($p^* \leq p^b = \bar{p}$, from Proposition 13).

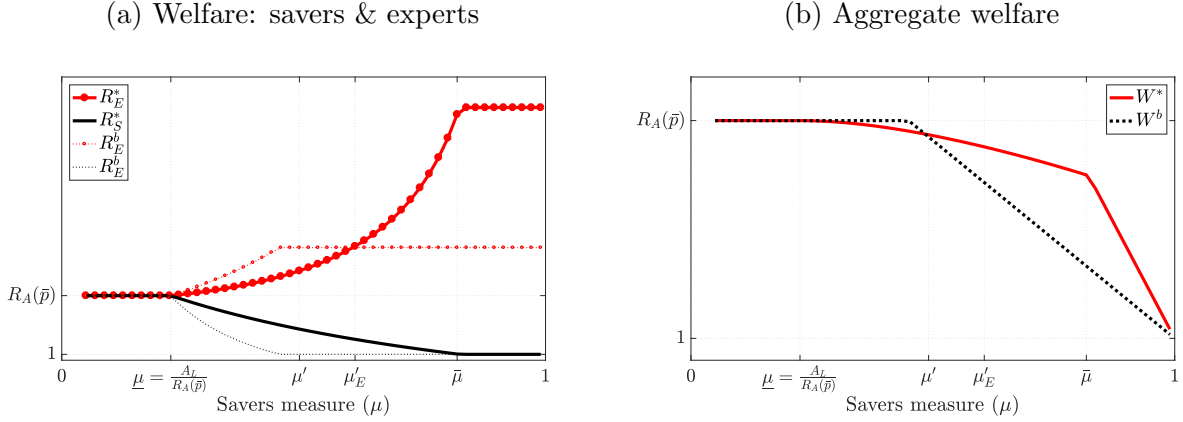
The following formal result describes the welfare effects from securitization.

Proposition 14 *Let μ be savers' overall wealth and $\underline{\mu}, \bar{\mu}$ the constants defined in Proposition 13, which satisfy $\underline{\mu} < A_L < \bar{\mu} \leq 1$. Let ΔR_S , ΔR_E , and ΔW be the welfare gains for savers, experts, and the aggregate economy due to the emergence of securitization. They satisfy:*

- (i) *Savers: $\Delta R_S \geq 0$ for any $\mu \geq 0$, and $\Delta R_S > 0$, if and only if $\mu \in (\underline{\mu}, \bar{\mu})$.*
- (ii) *Experts: there exists $\mu'_E \in (A_L, \bar{\mu}]$ such that $\Delta R_E < 0$ if $\mu < \mu'_E$ and $\Delta R_E > 0$ if $\mu > \mu'_E$. Besides, if $\bar{\mu} < 1$ then $\mu'_E < \bar{\mu}$.*
- (iii) *Aggregate: there exists $\mu' \in (A_L, \bar{\mu}]$ with $\mu' \leq \mu'_E$ such that $\Delta W < 0$ if $\mu < \mu'$ and $\Delta W > 0$ if $\mu > \mu'$. Besides, if $\bar{\mu} < 1$ then $\mu' < \mu'_E < \bar{\mu}$.*

The proposition describes the welfare effects associated with the emergence of intermediation. Figure 4 illustrates these results. Savers' always weakly benefit from the entry of intermediaries and experts only benefit if the demand for safety is sufficiently high. More precisely, for a low safety demand ($\mu \leq \underline{\mu}$), the equity spread is zero and the equilibria of the two economies coincide. If demand for safety increases sufficiently ($\mu > \underline{\mu}$), a positive spread arises, intermediaries enter in the economy and this leads to an increase in originators' risk-taking ($p^* < \bar{p}$) and the welfare effects on experts and the aggregate economy depend on how much more investment is undertaken thanks to securitization. For a medium

Figure 4: Equilibrium with intermediaries



Note. In each panel, dotted lines correspond to the benchmark economy without securitization.

safety demand ($\underline{\mu} < \mu < \mu'$), the increase in aggregate investment is not very large (if at all) and aggregate welfare falls. The increase in saver's utility implied by the expansion of the supply of safe securities is thus more than offset by a reduction in the utility of experts. When the demand for safety is sufficiently large ($\mu > \mu'$), the investment expansion effect dominates and the entry of intermediaries increases aggregate welfare. At higher values of the demand for safety ($\mu > \mu'_E > \mu'$), the investment expansion effect is so important that experts' welfare increases with the entry of intermediaries. In this region, securitization leads to a Pareto improvement in the economy despite the increase in originators' risk-taking it induces.

6.3 Constrained efficiency of the equilibrium

In this section, we describe the problem of a “constrained” Social Planner (SP) and show that constrained versions of the Welfare Theorems hold in this economy. We consider a SP that at $t = 0$: *i*) allocates some experts to origination, others to intermediation, while the remaining ones remain passive; *ii*) allocates investors' funds into originators' projects and initial date consumption; and *iii*) designs the securities issued by the financial firms, which determine originators' risk choice and how the pay-off of the originators' projects at $t = 1$ is distributed across investors. Notice that the SP is constrained insofar as she cannot choose the originator's project risk. The SP decisions create three expert groups that we refer to as originators, intermediaries, and passive. Due to constant returns to scale in the financial

firms' technologies, the actual measures of experts in each group is irrelevant.¹⁰

Formally, a SP allocation is described by: the investment amount $N \in [0, 1]$ by originators, an incentive compatible originator project risk choice p , the per unit of the project promises $d_S, (d_{I,z})_z$ made by originators on safe and risky securities at $t = 1$, the per unit of the project promise b_S made by intermediaries on safe securities, aggregate consumption at $t = 0$ of savers, passive experts, originators and intermediaries, $(C_{S,0}, C_{P,0}, C_{O,0}, C_{I,0})$, aggregate consumption at $t = 1$ by savers, $C_{S,1}$, and passive experts, $C_{P,1}$, and for each aggregate shock θ aggregate consumptions at $t = 1$ net of monitoring costs by originators, $C_{O,1}(\theta)$, and intermediaries, $C_{I,1}(\theta)$.

An allocation $(N, p, d_S, (d_{I,z})_z, C_{S,0}, C_{P,0}, C_{O,0}, C_{I,0}, C_{S,1}, C_{P,1}, (C_{O,1}(\theta))_\theta, (C_{I,1}(\theta))_\theta)$ is *constrained feasible* if it satisfies the following conditions.

The state contingent promises of the originator are always repaid:

$$d_S + d_{I,L} \leq A_L, \quad (57)$$

$$d_S + d_{I,H} \leq A_H. \quad (58)$$

Originators optimally choose risk:

$$p = \arg \max_{p'} \{E[A_z - d_S - d_{I,z}|p'] - c(p')\}. \quad (59)$$

Intermediaries always repay safe securities:

$$b_S \leq \min_{\theta} (\theta p d_{I,H} + (1 - \theta p) d_{I,L}). \quad (60)$$

Consumption at $t = 0$ equals the amount of funds that are not invested in originators' projects:

$$C_{S,0} + C_{P,0} + C_{O,0} + C_{I,0} = 1 - N. \quad (61)$$

Aggregate consumption at $t = 1$ by savers' and passive experts' equals the overall payoff of safe securities

$$C_{S,1} + C_{P,1} = (d_S + b_S) N. \quad (62)$$

¹⁰The only restriction is that there is a continuum of originators and at least one intermediary to take advantage of the possibility to diversify idiosyncratic risks.

For each θ , originators' net consumption at $t = 1$ equals their residual claim net of monitoring costs:

$$C_{O,1}(\theta) = [\theta p (A_H - d_{I,H} - d_S) + (1 - \theta p) (A_L - d_{I,L} - d_S) - c(p)] N. \quad (63)$$

For each θ , intermediaries' consumption at $t = 1$ equals their residual claim:

$$C_{I,1}(\theta) = [\theta p d_{I,H} + (1 - \theta p) d_{I,L} - b_S] N. \quad (64)$$

We assume that the SP assigns weights ω_S to the utility of savers and ω_E to that of experts regardless of they being originators, intermediaries or passive. The weighted expected aggregate welfare of a constrained feasible allocation is given by:

$$W_{\omega_S, \omega_E} \equiv \omega_S (C_{S,0} + C_{S,1}) + \omega_E (C_{P,0} + C_{O,0} + C_{I,0} + C_{P,1} + E[C_{O,1}(\theta) + C_{I,1}(\theta)|p]).$$

Using the consumption constraints (61) - (64) we can rewrite the expression above in the following compact form:

$$W_{\omega_S, \omega_E} = \omega_E (1 - N + R_A(p)N) + (\omega_S - \omega_E) (C_{S,0} + C_{S,1}), \quad (65)$$

Notice that this expression only depends on investment N , the originators' project risk p , and the aggregate consumption allocated to savers at each date $C_{S,0}, C_{S,1}$. The interpretation of (65) is as follows. The first term gives weight ω_E to the aggregate utility in the economy, which coincides with aggregate expected consumption net of monitoring costs. The second term gives additional weight $\omega_S - \omega_E$ to the aggregate consumption of savers.

For given weights ω_S, ω_E , we say that a feasible SP allocation is *constrained efficient* if it maximizes the weighted welfare function W_{ω_S, ω_E} in (65) within the set of feasible SP allocations. Finally, the *Pareto constrained efficient set* of the economy is defined as the set of allocations that are constrained efficient for some weights ω_S, ω_E .

It is easy to prove following the arguments in Lemma 4 that restricting to allocations with $d_{I,L} = 0$ does not reduce weighted welfare. It follows from (59) that the risk-choice is given by the function $\hat{p}(\cdot)$ defined in Lemma 5. Taking this into account and using the expression for W_{ω_S, ω_E} in (65), a constrained efficient allocation can be described by a tuple $(N, d_{I,H}, C_{S,0}, C_{S,1})$ that solves the following problem:

$$\max_{(N, d_{I,H}, C_{S,0}, C_{S,1})} \omega_E (1 - N + R_A(\hat{p}(d_{I,H}))N) + (\omega_S - \omega_E) (C_{S,0} + C_{S,1}), \quad (66)$$

subject to

$$C_{S,0} \leq 1 - N, \quad (67)$$

$$C_{S,1} \leq [A_L + (1 - \lambda)\hat{p}(d_{I,H})d_{I,H}]N. \quad (68)$$

It is easy to prove that (67) is necessarily binding in any solution to the SP problem because the originators' project is valuable. This means that only savers might consume at $t = 0$. Besides, when the SP does not weight savers more than experts ($\omega_S \leq \omega_E$), then trivially $N = 1$, $d_{I,H} = 0$ and first best investment and risk-choice are achieved.

In contrast, when the SP weights more savers than experts ($\omega_S > \omega_E$), she faces a trade-off in its $d_{I,H}$ choice between between worsening origination incentives and creating safe payoffs that relax the safe pay-off constraint (68) and allow to increase savers' consumption at $t = 1$. Using that (68) is also necessarily binding in any solution, the FOC for an optimum $d_{I,H}$ is given by

$$\left(\frac{\omega_S}{\omega_E} - 1\right) \left[(1 - \lambda) \frac{d(\hat{p}(d_{I,H})d_{I,H})}{dd_{I,H}} \right] + \frac{dR_A(\hat{p}(d_{I,H}))}{dd_{I,H}} = 0. \quad (69)$$

Notice that using the equilibrium funding discount pass-through equation (40), the FOC above is equivalent to the equilibrium FOC for the optimal $d_{I,H}$ choice of the originator in (31) provided that the SP weights and the equilibrium returns satisfy $\omega_S/\omega_E = R_E^*/R_S^*$. This suggests that the competitive equilibrium outcome is Pareto constrained efficient. Conversely, if after some initial date transfers across investors any possible equity spread R_E^*/R_S^* can be induced, then all the Pareto constrained efficient allocations would be achieved as an equilibrium outcome of the economy.

Building on these intuitions we can formally prove that constrained versions of the Welfare Theorems hold in this economy:

Proposition 15 *The equilibrium of the economy leads a Pareto constrained efficient allocation. Any allocation in the Pareto constrained efficient set can be achieved as the equilibrium of the economy following some initial date transfers across investors at the initial date.*

The reason why Welfare Theorems hold in this economy is that experts can freely set-up and invest in the two financial firms, which leads the SP to face the same trade-off than experts in equilibrium. For the SP, the way to improve savers' utility is to create safe pay-offs, which implies deteriorating origination incentives and reducing the aggregate expected

pay-offs that can be allocated to experts. The possibility of experts to freely reallocate from originators to intermediaries, and viceversa, and trade risky and safe securities in competitive markets, implies that the relative gains from creating safe pay-offs are represented in prices and returns, which leads to constrained efficient allocations. In fact, it can be proved that when experts' investments are exogenously fixed the resulting equilibrium is not necessarily constrained efficient.

7 Public guarantees and risk-taking

In this Section, we consider a government with safe resources at $t = 1$ and analyze how it can optimally use them to provide fiscally neutral public guarantees to the issuance of safe securities. We find that Pareto optimal policies direct all the government resources to the provision of guarantees to the issuance of safe securities by intermediaries, instead of originators, due to their larger exposure to aggregate risk. Besides, the impact of these policies on originators' risk-taking is ambiguous.

7.1 Extended model set-up

We assume throughout the section that safety demand satisfies $\mu > \underline{\mu}$, which from Proposition 13 implies that safe securities are scarce in the baseline economy. We extend the baseline set-up to include a risk-neutral government with some assets whose payoff at $t = 1$ is $X > 0$. The government can use its assets to provide at the initial date guarantees to the issuance of safe securities by financial firms that must be repaid in expectation. We next describe with some detail the policies at the disposal of the government and its optimization problem.

Guarantee to intermediaries A guarantee to the intermediaries' issuance of safe securities is described by an aggregate shock threshold $\bar{\theta} \in [1 - \lambda, 1]$, and transfers $T_{\bar{\theta}}(\theta, y|R_I)$ at $t = 1$ from the government to each intermediary conditional on the aggregate shock θ , the intermediary's size y and the market return R_I given by

$$T_{\bar{\theta}}(\theta, y|R_I) = \min(\bar{\theta} - \theta, 0)R_I y. \quad (70)$$

By construction, the *after guarantees* safe payoff of an intermediary of size y satisfies

$$\min_{\theta \in [1 - \lambda, 1/p_{\max}]} (\theta R_I y + T_{\bar{\theta}}(\theta, y|R_I)) = \bar{\theta} R_I y. \quad (71)$$

A guarantee with threshold $\bar{\theta} \in [1 - \lambda, 1]$ thus allows the intermediary to pledge a fraction $\bar{\theta}$ of the return of its assets to issue safe. Notice that $\bar{\theta} = 1 - \lambda$ and $\bar{\theta} = 1$ correspond to the cases of no guarantees and full guarantees, respectively.

Intermediaries compensate the government for the guarantee out of their profits when the aggregate shock satisfies $\theta \geq \bar{\theta}$. It is easy to check that the residual claim of intermediaries conditional on $\theta \geq \bar{\theta}$ is sufficiently large to be able to compensate the government for any guarantee $\bar{\theta} \leq 1$. Since by construction the presence of a fiscally neutral guarantee does not affect the value of the intermediary's residual claim, we have that the only change to the intermediary's problem analyzed in Section 5.2 is the replacement of the maximum safe debt constraint in (36) with

$$B_S \leq \bar{\theta} R_I y. \quad (72)$$

The guarantee thus amounts to a “reduction” on the aggregate risk parameter to which the intermediaries are exposed from λ to $1 - \bar{\theta}$.

Guarantee to the originators' issuance of safe securities A guarantee to the originators' issuance of safe securities consists on a per unit of the project transfer $\sigma \in [0, \Delta]$ from the government to the originator when the L return of its project is realized. The guarantee is compensated with a per unit of the project tax $\sigma' \geq 0$ the originator pays to the government when the H return of the project is realized. For a given risk choice p , the fiscal neutrality of the guarantee can be written as:

$$p\sigma' = (1 - p)\sigma. \quad (73)$$

Following similar steps as in Section 5.1 and using (73), we can derive the following optimal risk-choice condition for given σ and d_I :

$$\Delta - d_I - \frac{\sigma}{p} = c'(p). \quad (74)$$

Notice that for a given d_I an increase in the guarantee σ increases risk-taking (reduces p). Since by construction the presence of a fiscally neutral guarantee does not affect the value of the originator's residual claim, we have that the only change to the originator's problem analyzed in Section 5.1 is the replacement of the optimal risk-choice condition in (26) with (74).

Initial date transfers The government can also conduct lump-sum transfers across agent types at $t = 0$ that can be described by the (positive or negative) amount of funds $\tau \in [-(1 - \mu), \mu]$ transferred from savers to experts.

Pareto optimal policies A feasible government policy consists of a tuple $(\bar{\theta}, \sigma, \tau)$ of guarantees and lump-sum transfer, such that the competitive equilibrium of the economy they induce satisfies the following government's resource constraint:

$$E_I^* (\bar{\theta} - 1 + \lambda) R_I^* y^* + E_O^* [1 - (1 - \lambda)p^*] \sigma x^* \leq X. \quad (75)$$

The LHS of this inequality account for the overall disbursements by the government from the guarantees to intermediaries and originators, respectively, conditional on the worst aggregate shock, $\theta = 1 - \lambda$. The RHS are simply the government safe payoffs. Since the government disbursements are decreasing in the realization of the aggregate shock θ , the government satisfies guarantees for any θ if and only if (75) holds.

Finally, a feasible policy is Pareto optimal if it induces an equilibrium outcome that: *i*) weakly Pareto improves the outcome of the no intervention policy; and *ii*) is not Pareto improved by the equilibrium induced by any other feasible policy.

7.2 Pareto optimal policies

In absence of a government intervention, the economy exhibits scarcity of safe securities or, equivalently, of experts' funds (recall that $\mu > \underline{\mu}$). This scarcity reduces aggregate surplus relative to first-best. A government that provides guarantees to financial firms provides additional loss absorption capacity in the economy, mitigating the scarcity of experts' funds and, because of this, the scarcity of safe securities.

The capability of the two types of guarantees to enhance welfare differs, as the next lemma formally states:

Lemma 16 *The restriction to policies that do not include guarantees to originators ($\sigma = 0$) does not affect the set of allocations achieved by Pareto optimal policies. Besides, if X is small using guarantees to originators is never part of a Pareto optimal policy.*

The intuition for the lemma stems from the different exposure of the two types of financial firms to aggregate risk. As a result of diversification, all the risk in the intermediaries' assets

is aggregate risk, which implies that in the worst aggregate shock the government has to satisfy the guarantee to all the intermediaries. In contrast, originators are exposed both to aggregate and idiosyncratic risk, so that in the worst aggregate shock the government satisfies the guarantee to some originators *and* gets repaid by others. As a result, the *net injection* of funds by the government into the financial sector following the worst aggregate shock is maximized when only guarantees to intermediaries are issued. This maximizes the capability to issue safe securities in the economy and makes this type of guarantees preferable.

Using Lemma 16, we restrict from now on to policies that only include guarantees to intermediaries. The next formal result provides some properties of the Pareto optimal policies.

Proposition 17 *Pareto optimal policies Pareto improve the baseline economy but never induce first-best allocations. Besides:*

- *If the baseline economy exhibits full investment ($\mu \leq \bar{\mu}$), Pareto optimal policies reduce risk-taking at origination.*
- *If the baseline economy does not exhibit full investment ($\mu > \bar{\mu}$), Pareto optimal policies increase investment and, for X small, increase risk-taking.*

The proposition states that Pareto optimal policies Pareto improve welfare in the economy. The reason is that guarantees to intermediaries are equivalent to a reduction in the maximum exposure to aggregate risk of intermediaries. This can be interpreted as a technological improvement that strictly expands the Pareto frontier of allocations in the economy. Using the constrained Second Welfare Theorem included in Proposition 15 we have that, after properly setting lump-sum transfers across investors at the initial date, guarantees to intermediaries Pareto improve the economy.

Proposition 17 also states that optimal policies never induce first-best allocations. This is because intermediaries' guarantees are a substitute for the need of expert's funds in the pooling and tranching activities that create safe assets through securitization. Yet, the expansion of safe securities supply beyond the safe return A_L of the originators' project necessarily involves a reduction of originators' exposure to the risk of their projects, which leads to a reduction on monitoring incentives. Since this cannot be avoided, the best that can do a government that has unlimited safe resources is to induce experts' endowment to be fully invested in originators.

Finally, Proposition 17 provides some results on the risk-taking effects of the optimal policies. When the baseline economy exhibits full investment ($\mu \leq \bar{\mu}$), Pareto optimal policies must necessarily reduce originators' risk-taking. This is because Pareto optimal policies increase aggregate surplus and investment being at its maximum level, the only way a policy can achieve so is by reducing risk-taking at origination. When the baseline economy does not exhibit full investment ($\mu > \bar{\mu}$) and the government resources are small, guarantees on intermediaries with no lump-sum transfer increase investment but are not able to increase the safe return above one. This implies that the equilibrium return on equity increases, and also does risk-taking at origination.¹¹ Pareto optimal policies in this case must thus include a weakly positive lump-sum transfer from experts to savers, which further increase risk-taking.

8 Conclusion

We present an equilibrium model of the capital structure and risk-taking in the originate-to-distribute intermediation chain in presence of absolute demand for safety by some investors and limited endowment by equity investors. Loan originators can finance the risky part of their assets through equity or by obtaining funding from intermediaries. The latter implies the off-balance sheet transfer of risk and worsens originators' risk-taking incentives. Intermediaries can pool the acquired idiosyncratic risks to issue safe securities and expand their balance sheets. Yet, the presence of aggregate risk implies that intermediaries rely on equity to do securitization. Equity investment in the intermediation chain serves two different purposes. At origination, it provides experts skin-in-the game that increases their incentives to monitor the loans. At intermediation, equity is a cushion for aggregate risk losses.

Following an increase in the demand for safety, the model predicts a securitization boom. The demand for safe assets leads to the reallocation equity from originators to intermediaries and implies an increase in leverage along the intermediation chain, the relative size of the intermediary sector and risk-taking at origination. We thus provide a single framework that captures the main features emphasized by the saving glut narrative of the run-up to the crisis.

¹¹Originators' risk-taking increases because the intermediary funding discount, which determines it, increases in equilibrium. The latter is the result of two effects that can be observed in the intermediary funding discount pass through equation in (40): first, as standard the increase in R_E^*/R_S^* increases χ^* ; second, the guarantee on the intermediary is equivalent to a reduction in λ , which also increases χ^* .

We show that the frictionless capability to allocate equity between originators and intermediaries and the existence of competitive markets for safe and risky securities ensure the validity of constrained versions of the welfare theorems. The competitive equilibrium of the economy is constrained Pareto efficient and any allocation in the constrained Pareto frontier can be achieved as the competitive equilibrium of the economy following some redistribution of wealth across investors' types at the initial date.

We analyze the welfare implications of the emergence of securitization by comparing the originate-to-distribute economy relative to a traditional originate-to-hold benchmark. Securitization leads to the following general welfare trade-off. On the one hand, the distribution of risks out of originators leads to more risk-taking and reduces aggregate surplus. On the other hand, the expansion of safe securities supply increases aggregate lending when in the traditional economy all endowment cannot be channeled to finance loans, which increases aggregate surplus. We find that the aggregate lending effect on total surplus dominates if and only if the demand for safety is sufficiently large. Instead, if the demand for safety is not large enough, excessive risk-taking leads to aggregate losses and implies redistributive effects. Safety investors always benefit from the increased supply of safe assets. In contrast, the possibility to engage in securitization increases competition in the supply of safe securities and ends up depriving equity investors of some of the scarcity rents they enjoyed in the traditional financial sector.

We also show that when a government has safe resources, fiscally neutral public guarantees to the issuance of securitized assets can reduce the scarcity of safe securities in the economy, and lead to Pareto improvements in welfare if properly combined with lump-sum transfers across investors' types. Besides, these policies are preferable to the introduction of guarantees to originators because of the higher exposure of intermediaries' assets to aggregate risk. Despite Pareto improving welfare in the economy, the effect of these policies on risk-taking at origination is ambiguous. These results shed new light on the equilibrium effects of public guarantees to the financial sector, the need to combine them with other redistributive policies and their interplay with risk-taking at origination.

References

- Acharya, V. V., Richardson, M. P., et al. (2009). *Restoring financial stability: how to repair a failed system*, volume 542. John Wiley & Sons.
- Acharya, V. V., Schnabl, P., and Suarez, G. (2013). Securitization without risk transfer. *Journal of Financial economics*, 107(3):515–536.
- Ahnert, T. and Perotti, E. (2017). Intermediaries as safety providers. Technical report, Technical report, University of Amsterdam Working Paper.
- Allen, F., Carletti, E., and Marquez, R. (2015). Deposits and bank capital structure. *Journal of Financial Economics*, 118(3):601–619.
- Ashcraft, A. B., Schuermann, T., et al. (2008). Understanding the securitization of subprime mortgage credit. *Foundations and Trends® in Finance*, 2(3):191–309.
- Bernanke, B. (2005). The global saving glut and the us current account deficit. Technical report, Board of Governors of the Federal Reserve System (US).
- Bernanke, B. S., Bertaut, C. C., Demarco, L., and Kamin, S. B. (2011). International capital flows and the return to safe assets in the united states, 2003-2007.
- Bolton, P., Santos, T., and Scheinkman, J. A. (2018). Savings gluts and financial fragility.
- Caballero, R. J. and Krishnamurthy, A. (2009). Global imbalances and financial fragility. *American Economic Review*, 99(2):584–88.
- Calomiris, C. W. and Mason, J. R. (2004). Credit card securitization and regulatory arbitrage. *Journal of Financial Services Research*, 26(1):5–27.
- Carletti, E., Petriconi, S., and Marquez, R. (2017). The redistributive effects of bank capital regulation. In *American Finance Association 2018 Meeting in Philadelphia, January*.
- Chemla, G. and Hennessey, C. A. (2014). Skin in the game and moral hazard. *The Journal of Finance*, 69(4):1597–1641.
- Daley, B., Green, B. S., and Vanasco, V. (2017). Securitization, ratings, and credit supply.

- Dell Ariccia, G., Laeven, L., and Marquez, R. (2014). Real interest rates, leverage, and bank risk-taking. *Journal of Economic Theory*, 149:65–99.
- DeMarzo, P. and Duffie, D. (1999). A liquidity-based model of security design. *Econometrica*, 67(1):65–99.
- DeMarzo, P. M. (2004). The pooling and tranching of securities: A model of informed intermediation. *The Review of Financial Studies*, 18(1):1–35.
- Diamond, D. W. and Rajan, R. G. (2012). Illiquid banks, financial stability, and interest rate policy. *Journal of Political Economy*, 120(3):552–591.
- Diamond, W. (2016). Safety transformation and the structure of the financial system.
- Farhi, E. and Tirole, J. (2012). Collective moral hazard, maturity mismatch, and systemic bailouts. *American Economic Review*, 102(1):60–93.
- Gennaioli, N., Shleifer, A., and Vishny, R. W. (2013). A model of shadow banking. *The Journal of Finance*, 68(4):1331–1363.
- Gornall, W. and Strebulaev, I. A. (2018). Financing as a supply chain: The capital structure of banks and borrowers. *Journal of Financial Economics*.
- Gorton, G. and Pennacchi, G. (1990). Financial intermediaries and liquidity creation. *The Journal of Finance*, 45(1):49–71.
- Gorton, G. B. and Pennacchi, G. G. (1995). Banks and loan sales marketing nonmarketable assets. *Journal of monetary Economics*, 35(3):389–411.
- Holmstrom, B. and Tirole, J. (1997). Financial intermediation, loanable funds, and the real sector. *the Quarterly Journal of economics*, pages 663–691.
- Keister, T. (2015). Bailouts and financial fragility. *The Review of Economic Studies*, 83(2):704–736.
- Martinez-Miera, D. and Repullo, R. (2017). Search for yield. *Econometrica*, 85(2):351–378.
- Martinez-Miera, D. and Repullo, R. (2018). Markets, banks, and shadow banks.

- Moreira, A. and Savov, A. (2017). The macroeconomics of shadow banking. *The Journal of Finance*, 72(6):2381–2432.
- Parlour, C. A. and Plantin, G. (2008). Loan sales and relationship banking. *The Journal of Finance*, 63(3):1291–1314.
- Pozsar, Z., Adrian, T., Ashcraft, A., and Boesky, H. (2013). Shadow banking. *Federal Reserve Bank of New York Economic Policy Review*, 19(2):1.
- Repullo, R. (2013). Cyclical adjustment of capital requirements: A simple framework. *Journal of Financial Intermediation*, 22(4):608–626.

A Appendix

This appendix contains the proofs of the formal results included in the body of the paper.

Proof of Lemma 1 Recall that $R_A(\bar{p}) > 1$ from Assumption 2. Suppose that $R_S^b < 1$. The demand for safe securities would be zero. Since $R_A(\bar{p}) > R_S^b$, we have from (18) that originators would borrow as much as possible and $R_{E,O}^b > R_A(\bar{p})$. Hence, all experts would find optimal to set-up originators and invest in them, so that the supply of safe securities would be strictly positive. The market for safe securities would not clear.

Suppose that $R_S^b > R_A(\bar{p})$. Since $R_A(\bar{p}) > 1$ savers would invest their entire endowment in safe securities and the demand for these assets would be strictly positive. From (18) originators would not find optimal to issue safe securities, so that their supply would be zero. The market for safe securities would not clear. ■

Proof of Proposition 2 We proceed in a sequence of steps. Recall that $\underline{\mu} = A_L/R_A(\bar{p})$.

a) $R_S^b \in (1, R_A(\bar{p}))$ is the safe rate of an equilibrium if and only if $R_S^b = \frac{A_L}{\underline{\mu}}$, and in that case $N^b = 1$.

Suppose $R_S^b \in (1, R_A(\bar{p}))$. If R_S^b is the safe rate in an equilibrium then the arguments in the main text preceding the proposition show that R_S^b satisfies (??), that is, $R_S^b = \frac{A_L}{\underline{\mu}}$, and that $N^b = 1$.

If $R_S^b = \frac{A_L}{\underline{\mu}}$, then those arguments can be reverted and R_S^b is the safe rate of an equilibrium in which $N^b = 1$.

b) If $\mu \in (\underline{\mu}, A_L]$ then the equilibrium is unique and satisfies the properties in statement ii) in the Proposition.

Suppose first that $\mu \in (\underline{\mu}, A_L)$. Define R_S^b as $R_S^b = \frac{A_L}{\underline{\mu}}$. By the definition of $\underline{\mu}$, we have that $R_S^b \in (1, R_A(\bar{p}))$ and a) shows the existence of an equilibrium.

Suppose there exists another equilibrium and denote $R_S^{b'}$ its safe rate. Using Lemma 1, it must be the case that $R_S^{b'} = 1$ or $R_S^{b'} = R_A(\bar{p})$.

If $R_S^{b'} = 1$ then reproducing the arguments in the main text preceding the proposition we have that the supply of safe assets amounts to $\frac{A_L(1-\mu)}{1-A_L}$, which satisfies

$$\frac{A_L(1-\mu)}{1-A_L} > \mu.$$

This implies that the market for safe assets does not clear because their demand is upper bounded by μ .

If $R_S^{b'} = R_A(\bar{p})$, then we have from (18) that $R_{E,O}^b = R_A(\bar{p})$. Experts would be indifferent between investing in originators and in safe securities. This implies that the supply of safe

assets is upper bounded by $\frac{\mu(1-\mu)}{1-\underline{\mu}}$, which satisfies

$$\frac{\mu(1-\mu)}{1-\underline{\mu}} < \mu.$$

This implies that the market for safe assets does not clear because their demand is lower bounded by μ .

Suppose that $\mu = A_L$. It suffices to reproduce arguments done above to show that the equilibrium is unique and satisfies $R_S^b = 1$ and $N^b = 1$.

c) If $\mu > A_L$ then the equilibrium is unique and satisfies the properties in statement iii) in the Proposition.

It suffices to reproduce arguments done in the proof of b).

d) If $\mu \leq \underline{\mu}$ then there exist Modigliani-Miller equilibria satisfying the properties in statement i) in the Proposition and all the equilibria are of this type.

It suffices to reproduce arguments done in the proof of b). ■

Proof of Lemma 3 In the proof we will make use of some expressions and results that are presented in the main text of the paper after Lemma 3. Each time we do so we include a footnote in which we explain why the arguments are not subject to circularity problems. The superscript $*$ will throughout the proof denote partial or general equilibrium variables.

We have to prove three results

i) If an equilibrium exists, then $1 \leq R_S^* \leq R_A(\bar{p})$

The inequality $R_S^* \geq 1$ is proven as in Lemma 1.

Suppose that $R_S^* > R_A(\bar{p})$ and an equilibrium exists. Since $R_A(\bar{p}) > 1$ savers would invest their entire endowment in safe securities. Since experts have the option to invest in safe securities, then $R_E^* \geq R_S^*$, because otherwise there would be no investment at all in the economy to back the repayment of safe securities. Suppose that aggregate investment by originators is N^* and their risk choice is p^* . At $t = 1$ all the payoffs in the economy are distributed to the measure N^* of savers and experts that have provided funding either directly or indirectly (through intermediaries) to originators. Since $R_E^* \geq R_S^*$, necessarily have that $R_A(p^*) \geq R_S^*$ which implies $R_A(p^*) > R_A(\bar{p})$, which contradicts (1).

ii) For given $R_S \leq R_A(\bar{p})$, if a partial equilibrium exists then $R_S \leq R_I^* \leq R_E^*$

Suppose $R_S \leq R_A(\bar{p})$ and a partial equilibrium exists. Suppose that $R_I^* > R_E^*$. From the expression for $R_{E,I}$ in (35),¹² we have that an expert that sets up an intermediary obtains a return on equity $R_{E,I}$ satisfying $R_{E,I} \geq R_I^*$, so that in equilibrium $R_E^* \geq R_{E,I} \geq R_I^*$.

¹²Equation (35) is presented in Section 5.2 but is derived directly from the objects and conditions in the maximization problem (12) - (15) presented in Section 3.

Suppose that $R_S > R_I^*$. From (35), we have that an expert that sets up an intermediary can obtain a return on equity $R_{E,I}$ satisfying $R_{E,I} = R_I^*$. Besides, an expert that sets-up an originator obtains a return $R_{E,O}$ satisfying $R_{E,O} \geq R_A(\bar{p}) \geq R_S > R_I^*$. All experts would thus find optimal to set-up originators, and the demand for risky securities (whose potential only buyers are intermediaries) would be zero. Market clearing in the market for risky securities then implies that originators do not issue risky securities. Yet, since $R_I^* < R_S \leq R_A(\bar{p})$, from (18) we have that originators would find optimal to issue safe securities in the market for risky securities.

iii) For given $R_S < R_A(\bar{p})$, if a partial equilibrium exists then $R_S < R_I^* < R_E^*$, and for $R_S = R_A(\bar{p})$, if a partial equilibrium exists and $R_S = R_I^* = R_E^*$

Suppose that $R_S < R_A(\bar{p})$ and a partial equilibrium exists. Since $R_S < R_A(\bar{p})$, the arguments in the main text preceding Proposition 2 imply that $R_S < R_E^*$ because an originator has the possibility not to issue risky securities. Suppose that $R_I^* = R_E^*$, which implies that $R_S < R_I^*$. Then using expression (39),¹³ we would have that $R_S = R_{E,I} = R_E^* = R_I^*$. Hence, we must have $R_I^* < R_E^*$. Suppose that $R_S = R_I^*$. From (35), we would have that $R_S = R_{E,I} = R_E^*$. Hence, we must have $R_S < R_I^*$.

Finally, suppose that $R_S = R_A(\bar{p})$. The same argument as at the end of *i)* and *ii)* implies that $R_S = R_I^* = R_E^*$. ■

Proof of Lemma 4 Suppose $R_S \in [1, R_A(\bar{p}))$ and $R_I > R_S$. Let $(x, D_S, x_S, D_I, x_I, p)$ be a balance sheet tuple solving the originator's maximization problem and let $R_{E,O}$ denote the return on equity under this tuple. We proceed in three steps.

i) If $D_{I,H} = 0$ then $D_{I,L} = 0$

Suppose that $D_{I,H} = 0$ and $D_{I,L} > 0$. From (24) and Assumption 1 we have that $p \geq \bar{p}$, so that $R_A(p) \leq R_A(\bar{p})$. Suppose the originator issues securities with notional promises $D'_S = D_S + D_{I,L}$, $D'_{I,L} = 0$, $D'_{I,H} = 0$. Let x', x'_S, x'_I, p' be the rest of the elements of the originator's balance sheet tuple, which are determined by the conditions (4), (7), (8) and (9) given $(D'_S, D'_{I,L}, D'_{I,H})$. Let $R'_{E,O}$ denote the return on equity under this alternative balance sheet tuple. It is immediate to check using that $R_I > R_S$ that

$$x'_S > x_S, x'_I < x_I, x' > x \text{ and } p' = \bar{p}.$$

Notice that since $x' > x$ and $A_H > A_L$, the fact that (x, D_S, D_I) satisfies (5) and (6) implies that (x', D_S, D'_I) also satisfies those constraints. Finally, using that $x'_S > x_S + x_I$, $R_A(p) \leq$

¹³Equation (39) is presented in Section 5.2. It can be checked that it only relies on the definitions in Section 3 and Lemma 4, which is presented in Section 5.1 and is stated in terms of exogenous returns $R_S < R_I$, and thus makes no use of the equilibrium results in Lemma 3.

$R_A(p')$ and $R_I > R_S$ we have from (23) that $R'_{E,O} > R_{E,O}$, which contradicts the optimality of $(x, D_S, x_S, D_I, x_I, p)$.

ii) $D_{I,L} = 0$

Suppose $D_{I,L} > 0$. Then i) implies that $D_{I,H} > 0$. Choose $\varepsilon > 0$ such that $\varepsilon \leq \min(D_{I,L}, D_{I,H})$. Suppose the originator issues securities with notional promises $D'_S = D_S + \varepsilon$, $D'_{I,L} = D_{I,L} - \varepsilon$, $D'_{I,H} = D_{I,H} - \varepsilon$. Let x', x'_S, x'_I, p' denote the rest of the elements of the originator's balance sheet tuple, which are determined as above. It is immediate to check using that $R_I > R_S$ that

$$x'_S > x_S, x'_I < x_I, x' > x \text{ and } p' = p.$$

And this leads to a contradiction as in the previous steps.

iii) $D_S = A_L x$

Suppose $D_S < A_L x$. Suppose the originator issues securities with notional promises $D'_S = A_L x$, $D'_{I,L} = 0$, $D'_{I,H} = \max(D_{I,H} - (D'_S - D_S), 0)$. Let x', x'_S, x'_I, p' denote the rest of the elements of the originator's balance sheet tuple, which are determined as above. It is immediate to check using that $R_I > R_S$ that

$$x'_S > x_S, x'_I \leq x_I, x' > x \text{ and } \bar{p} \geq p' \geq p.$$

And this leads to a contradiction as in the previous steps. ■

Proof of Lemma 5 The lemma is a direct implication of the optimality condition (26) and Assumption 1. ■

Proof of Lemma 6 We first present the following results which are an immediate consequence of (17), Lemma 5 and (26), and will be used without explicit reference throughout the proof of this lemma and the next proposition:

$$\frac{d(\hat{p}(d_I))}{dd_I} = -\frac{1}{c''(\hat{p}(d_I))}, \quad (76)$$

$$\frac{dR_A(\hat{p}(d_I))}{dd_I} \leq 0 \text{ with equality iff } d_I = 0. \quad (77)$$

Consider an exogenous $R_S < R_A(\bar{p})$ and $R_I > R_S$. Let $\bar{R}_I > \underline{R}_I$ be the constants defined in the Lemma. By definition we have that $\bar{R}_I = R_{E,O}(0)$. The originator's problem is described by (29). Denote with d_I^* any of its solutions in case they exist. After some algebra, we have

from (28) that:

$$\frac{dR_{E,O}(d_I)}{dd_I} = \frac{(R_{E,O}(d_I) - R_I) \left(\frac{1}{R_I} \frac{d(\widehat{p}(d_I)d_I)}{dd_I} \right) + \frac{dR_A(\widehat{p}(d_I))}{dd_I}}{1 - A_L/R_S - \widehat{p}(d_I)d_I/R_I}, \quad (78)$$

$$\left. \frac{dR_{E,O}(d_I)}{dd_I} \right|_{d_I=0} = \frac{(\overline{R}_I - R_I) \frac{\overline{p}}{R_I}}{1 - A_L/R_S}. \quad (79)$$

We proceed in a sequence of steps.

i) If $R_I \geq \overline{R}_I$ then $d_I^* = 0$ is the unique solution to (29)

If $R_I \geq \overline{R}_I$, then consider the function $G(a) = \frac{R_A(\overline{p}) - A_L - a}{1 - A_L/R_S - a/R_I}$. We have:

$$G'(a) = \frac{(R_A(\overline{p}) - A_L)/R_I - (1 - A_L/R_S)}{(1 - A_L/R_S - a/R_I)^2}.$$

Using the definition of \overline{R}_I and $R_I \geq \overline{R}_I$, we have from the expression above that $G'(a) \leq 0$. The following sequence of inequalities follows immediately for $d_I > 0$:

$$R_{E,O}(d_I) < \frac{R_A(\overline{p}) - A_L - \widehat{p}(d_I)d_I}{1 - A_L/R_S - \widehat{p}(d_I)d_I/R_I} = G(\widehat{p}(d_I)d_I) \leq G(0) = R_{E,O}(0),$$

which proves the claim.

ii) If $R_I \leq \underline{R}_I$ then a solution to (29) does not exist, because $R_{E,O}(d_I)$ can grow unboundedly

We have from Assumption 1 and (26) that for any $d_I \in [0, \Delta]$:

$$R_A(\widehat{p}(d_I)) - A_L - \widehat{p}(d_I)d_I = \widehat{p}(d_I)c'(\widehat{p}(d_I)) - c(\widehat{p}(d_I)) > 0. \quad (80)$$

By definition of \underline{R}_I we have that $1 = A_L/R_S + \max_{d_I} (\widehat{p}(d_I)d_I) / \underline{R}_I$. As a result, if $R_I \leq \underline{R}_I$ for d_I sufficiently close to $\arg \max_{d_I} (\widehat{p}(d_I)d_I)$ the originator could lever up unboundedly and from (80) its equity return would also do so.

iii) If $R_I \in (\underline{R}_I, \overline{R}_I)$ then any d_I^* satisfies (30)

If $R_I \in (\underline{R}_I, \overline{R}_I)$ then $R_{E,O}(d_I)$ is bounded in the compact interval $[0, \Delta]$ and some d_I^* exists. From (79) we have that $\left. \frac{dR_{E,O}(d_I)}{dd_I} \right|_{d_I=0} > 0$. Besides, since $\widehat{p}(\Delta) = 0$, we have that $R_{E,O}(\Delta) = 0 < R_{E,O}(0)$. Hence any d_I^* must be interior and satisfy $\left. \frac{dR_{E,O}(d_I)}{dd_I} \right|_{d_I=d_I^*} = 0$, which from (78) is equivalent to (30).

iv) For given $\chi \geq 1$, the following equation in d_I has a unique solution in the interval $[0, \Delta]$:

$$(\chi - 1) \frac{d(\widehat{p}(d_I)d_I)}{dd_I} + \frac{dR_A(\widehat{p}(d_I))}{dd_I} = 0. \quad (81)$$

Using (17), (26), (81) can be rewritten after some straightforward algebra as

$$d_I = \frac{(\chi - 1)}{\chi} \widehat{p}(d_I) c''(\widehat{p}(d_I)), \quad (82)$$

so that it is sufficient to prove that this equation has a unique solution. From Assumption 1, we have that

$$\frac{d(\widehat{p}(d_I) c''(\widehat{p}(d_I)))}{dd_I} \leq -1. \quad (83)$$

If $\chi > 1$, from (83) we have that the RHS in (82) is decreasing in d_I . Besides from Assumption 1 and Lemma 5 it is strictly positive for $d_I = 0$ and is zero for $d_I = \Delta$. Hence it has a unique intersection with the line d_I in the interval $(0, \Delta)$, and (82) has a unique solution. If $\chi = 1$, we trivially have that $d_I = 0$ is the unique solution of (82).

v) If $R_I \in (\underline{R}_I, \overline{R}_I)$ then d_I^* is unique

Suppose $R_I \in (\underline{R}_I, \overline{R}_I)$ and there exist two solutions. From *iii*), they must satisfy (30). Let $R_{E,O}^*$ the originator's equity return they lead to. Define $\chi = R_{E,O}^*/R_I$. Since $\left. \frac{dR_{E,O}(d_I)}{dd_I} \right|_{d_I=0} > 0$ we have $R_{E,O}^* > \overline{R}_I$ and $\chi > 1$. By definition of equation (81) and χ , any solution to (30) is also a solution to (81), and conversely. From *iv*) the latter equation has a unique solution, which contradicts that the former has at least two. ■

Proof of Proposition 7 Recall partial result *iv*) in the proof of Lemma 6. For given $\chi \geq 1$, denote $\widehat{d}_I(\chi)$ the unique solution to (81), or equivalently to (82). We proceed in two steps:

i) The function $\widehat{d}_I(\chi)$ satisfies the properties in (33)

We have that $\frac{(\chi-1)}{\chi}$ is increasing in χ for $\chi \geq 1$. From (83) we immediately have that $\frac{d\widehat{d}_I(\chi)}{d\chi} > 0$, and hence from Lemma 5 that $\frac{d\widehat{p}(\widehat{d}_I(\chi))}{d\chi} < 0$. Besides, from (82) we have after some immediate algebra that

$$\frac{d\left(\widehat{p}(\widehat{d}_I(\chi)) \widehat{d}_I(\chi)\right)}{d\chi} = \frac{\widehat{p}(\widehat{d}_I(\chi))}{\chi} \frac{d\widehat{d}_I(\chi)}{d\chi}. \quad (84)$$

Moreover, from (82) and $\widehat{p}(d_I) = 0$ if and only if $d_I = \Delta$, we have that $\widehat{p}(\widehat{d}_I(\chi)) > 0$ for all $\chi \geq 1$. We hence have from (84) and $\frac{d\widehat{d}_I(\chi)}{d\chi} > 0$ that $\frac{d(\widehat{p}(\widehat{d}_I(\chi)) \widehat{d}_I(\chi))}{d\chi} > 0$.

Finally, (82) implies that $\widehat{d}_I(1) = 0$ and a continuity argument leads to $\lim_{\chi \rightarrow 1} \widehat{d}_I(\chi) = 0$.

ii) The function $\widehat{d}_I(\chi)$ corresponds to that defined in the proposition

For given $R_S < R_A(\overline{p})$, suppose an equilibrium exists and let d_I^*, p^*, χ^* denote the associated equilibrium variables. From (31) we have that

$$(\chi^* - 1) \frac{d(\widehat{p}(d_I^*) d_I^*)}{dd_I} + \frac{dR_A(\widehat{p}(d_I^*))}{dd_I} = 0.$$

Comparing with (81), we conclude that $d_I^* = \widehat{d}_I(\chi^*)$ and hence from Lemma 5 that $p^* = \widehat{p}(\widehat{d}_I(\chi^*))$. ■

Proof of Proposition 8 The proposition has been proven in the main text. ■

Proof of Proposition 9 We first present the following partial derivatives of the function $R_E^*(R_S, \chi^*)$ defined in (41):

$$\frac{\partial R_E^*(R_S, \chi^*)}{\partial R_S} < 0 \text{ and } \frac{\partial R_E^*(R_S, \chi^*)}{\partial \chi^*} = \frac{\widehat{p}(\widehat{d}_I(\chi^*))\widehat{d}_I(\chi^*)}{1 - A_L/R_S} > 0, \quad (85)$$

where for the partial derivative with respect to χ^* we have used the optimality condition in (31) and that $\frac{dR_A(p)}{dp} = \Delta - c'(p)$.

Let $R_S < R_A(\bar{p})$. Any equilibrium intermediary funding discount χ^* satisfies $\chi^* \geq 1$ and (42), and conversely. We denote with $G(\chi^*, R_S)$ the function of χ^* and R_S in the RHS of (42). Notice that we do not make explicit the dependence of $G(\chi^*)$ on R_S for the sake of notacional simplicity. Using (85) and (??), we have that

$$\frac{\partial G(\chi^*, R_S)}{\partial \chi^*} = \frac{(1 - \lambda)\widehat{p}(\widehat{d}_I(\chi^*))\widehat{d}_I(\chi^*)}{R_S - A_L}. \quad (86)$$

We proceed in a sequence of steps:

i) For any R_S , any solution $\chi^* \geq 1$ to (42) satisfies $\frac{\partial G(\chi^*, R_S)}{\partial \chi^*} < 1$.

Suppose that there exists a solution $\chi^* \geq 1$ to (42) with $\frac{\partial G(\chi^*, R_S)}{\partial \chi^*} \geq 1$. Let R_E^* and R_I^* denote the equilibrium returns in the economy with equilibrium intermediary funding discount χ^* . From (86) we have

$$\frac{\widehat{p}(\widehat{d}_I(\chi^*))\widehat{d}_I(\chi^*)}{R_S - A_L} \geq \frac{1}{1 - \lambda}. \quad (87)$$

Recall from Lemma 6 that if an equilibrium exists we must have $R_I^* > \underline{R}_I$, otherwise R_E^* would be infinity and χ^* as well. From the definition of \underline{R}_I and (87), we have that

$$\underline{R}_I = \frac{\max_{d_I} \widehat{p}(d_I)d_I}{1 - A_L/R_S} \geq \frac{\widehat{p}(d_I(\chi^*))d_I(\chi^*)}{1 - A_L/R_S} \geq \frac{R_S}{1 - \lambda}.$$

The equilibrium condition (40) and the inequality above imply that

$$\frac{1}{R_I^*} = (1 - \lambda)\frac{1}{R_S} + \frac{1}{R_E^*} > \frac{1 - \lambda}{R_S} \geq \frac{1}{\underline{R}_I},$$

which contradicts that $R_I^* < \underline{R}_I$.

ii) Equation (42) has at most one solution $\chi^* \geq 1$

Suppose that there exist two solutions $\chi_1^* < \chi_2^*$. Notice that the derivative with respect to χ^* of the LHS of (42) is equal to one. From Proposition 7 and (86) we have that $\frac{\partial^2 G(\chi^*, R_S)}{\partial^2 \chi^*} > 0$. And then the existence of two solutions $\chi_1^* < \chi_2^*$, implies that

$$\frac{\partial G(\chi_1^*, R_S)}{\partial \chi^*} < 1 < \frac{\partial G(\chi_2^*, R_S)}{\partial \chi^*}.$$

The second inequality contradicts *i*).

Before stating the next partial results, we denote $\Gamma = \{R_S < R_A(\bar{p}) \text{ st (42) has a solution } \chi^* \geq 1\}$. From *ii*) we can define for any $R_S \in \Gamma$ the unique solution to (42) as $\chi^*(R_S)$. We also introduce the function $F^*(R_S) = \frac{\partial G(\chi^*, R_S)}{\partial \chi^*} \Big|_{\chi^* = \chi^*(R_S)}$.

iii) Γ is non empty

We have from (86) and Proposition 7 that $\frac{\partial G(\chi^*=1, R_S)}{\partial \chi^*} = 0$. In addition, from (41) we have

$$\lim_{R_S \rightarrow R_A(\bar{p})} R_E^*(R_S, \chi^* = 1) = R_A(\bar{p}),$$

so that as $R_S \rightarrow R_A(\bar{p})$, we have that $G(1, R_S)$ tends to 1. Then equation (42) necessarily has a solution for R_S sufficiently close to $R_A(\bar{p})$.

iv) If $R_{S,1}, R_{S,2} < R_A(\bar{p})$ with $R_{S,1} < R_{S,2}$ and $R_{S,1} \in \Gamma$, then $R_{S,2} \in \Gamma$

This simply results from the fact that $G(\chi^*, R_S)$ is decreasing in R_S and that for all $R_S < R_A(\bar{p})$ we have $G(\chi^* = 1, R_S) > 1$.

v) There exists $\underline{R}_S < R_A(\bar{p})$ such that $\Gamma = (\underline{R}_S, R_A(\bar{p}))$

Let $\underline{R}_S = \inf(\Gamma)$. It suffices to prove that $\underline{R}_S \notin \Gamma$. Suppose that $\underline{R}_S \in \Gamma$. Then *i*) implies that $F^*(\underline{R}_S) < 1$. By definition this implies that for small $\varepsilon > 0$, we have that $\chi^* \in (\chi^*(\underline{R}_S), \chi^*(\underline{R}_S) + \varepsilon)$ implies $\chi^* > G(\chi^*, \underline{R}_S)$. And thus for small $\delta > 0$, we have that $R'_S \in (\underline{R}_S - \delta, \underline{R}_S)$ implies that $\chi^* > G(\chi^*, R'_S)$. Since we have that $1 < G(\chi^* = 1, R'_S)$, we conclude that $R'_S \in \Gamma$. But we have that $R'_S < \underline{R}_S = \inf(\Gamma) \leq R'_S$.

vi) $\chi^*(R_S)$ is strictly decreasing in R_S , with $\lim_{R_S \rightarrow R_A(\bar{p})} \chi^*(R_S) = 1$

The monotonicity of $\chi^*(R_S)$ can be obtained by derivating implicitly equation (42) and using *i*), and $\frac{\partial G(\chi_1^*, R_S)}{\partial R_S} < 0$. The other statement results from $\lim_{R_S \rightarrow R_A(\bar{p})} G(\chi^* = 1, R_S) = 1$ and *ii*).

vii) $R_E^*(R_S)$ is strictly decreasing in R_S , with $\lim_{R_S \rightarrow R_A(\bar{p})} R_E^*(R_S) = R_A(\bar{p})$

By definition we have $R_E^*(R_S) = R_E^*(R_S, \chi^*(R_S))$. The monotonicity of $R_E^*(R_S)$ is immediately obtained from (85) and *vi*). ■

Proof of Proposition 10 The results on $d_I^*(R_S)$, $p^*(R_S)$ and $p^*(R_S)d_I^*(R_S)$ are an immediate consequence of Proposition 7 and Proposition 9.

We have from (38) that

$$y^*(R_S) = \frac{\chi^*(R_S)}{\lambda},$$

and the results on $y^*(R_S)$ are an immediate consequence of Proposition 9.

From Lemma 4, we have that $x_S^*(R_S) = A_L x / R_S$, and plugging this expression into (4) we have that

$$x^*(R_S) = \frac{1}{1 - A_L/R_S} (1 + x_I^*(R_S)).$$

Using (44), the expression above can be rewritten as:

$$x^*(R_S) = \frac{1}{1 - A_L/R_S} \left(1 + \frac{E_I^*(R_S)}{E_O^*(R_S)} y^*(R_S) \right),$$

and the results on $x^*(R_S)$ follow from those for $y^*(R_S)$ and Proposition 11. (The arguments are not subject to circularity problems because neither (44) nor Proposition 11 rely on Proposition 10 despite being posterior to this result in the main text). ■

Proof of Proposition 11 From (8), (10) and (25) we have that:

$$R_I^*(R_S) x_I^*(R_S) = p^*(R_S) d_I^*(R_S) x^*(R_S). \quad (88)$$

Using the equation above, equation (46) is immediately obtained from from (44), (45). Proposition 9 implies that for $R_S < R_A(\bar{p})$ we have $R_E^*(R_S) > R_A(\bar{p})$, and thus $E_O^*(R_S), E_I^*(R_S)$ satisfy (49). Using that equation, the properties of $E_O^*(R_S)$ and $E_I^*(R_S)$ then result immediately from Proposition 9. ■

Proof of Proposition 12 The expression in (52) is obtained from (48) - (51) and the remaining results are an immediate consequence of Proposition 9. ■

Proof of Proposition 13 We first prove existence and then uniqueness of equilibrium. Each of the two claims is proven in a sequence of steps.

Recall that the results in Section 5.3 imply that for a given exogenous $R_S \in (\underline{R}_S, R_A(\bar{p}))$ the partial equilibrium of the economy exists, is unique and described by the formal results in that section.

Existence of general equilibrium

a) If $R_S \in (\underline{R}_S, R_A(\bar{p}))$ and $R_S > 1$ ($R_S = 1$), then $N^*(R_S) = 1$ ($N^*(R_S) \leq 1$) if and only if R_S is the safe rate in some general equilibrium

Suppose a given safe rate R_S satisfying $R_S \in (\underline{R}_S, R_A(\bar{p}))$ and $R_S > 1$. We know that a unique partial equilibrium of the economy exists for such R_S . From Proposition 9, it satisfies $R_E^* > R_S > 1$ which implies that savers find strictly optimal to invest in safe securities and experts in financial firms' equity. In order to prove the existence of a general equilibrium for the given R_S , it suffices to prove that the market for safe securities clears. Taking into

account that the existence of a partial equilibrium implies the clearing of the market for equity and risky securities, and that the two type of investors fully invest their endowment in financial firms, the clearing of the market for safe securities is equivalent to the entire endowment of the economy being invested (directly or indirectly) into originators' projects, that is, $N^*(R_S) = 1$. The result then follows.

The statement for $R_S = 1$ is proven analogously after noticing that savers are indifferent between investing in safe securities or consuming.

b) If $\mu \in (\underline{\mu}, \bar{\mu}]$ there exists an equilibrium satisfying $R_S^ < R_A(\bar{p}) < R_E^*, p^* < \bar{p}$ and $N^* = 1$. Moreover, the equilibrium is unique within the class of equilibria with $R_S^* < R_A(\bar{p})$.*

Suppose that $\mu \in (\underline{\mu}, \bar{\mu}]$. From (52), we have that

$$\lim_{R_S \rightarrow R_A(\bar{p})} N^*(R_S) < 1 \Leftrightarrow \mu > \underline{\mu}. \quad (89)$$

Using Proposition 12 and the definition of $\underline{\mu}$, we conclude that there exists a solution $R_S^* < R_A(\bar{p})$ such that $N^*(R_S^*) = 1$ iff $\mu > \underline{\mu}$, and in such a case the solution is unique. In addition, the solution R_S^* satisfies $R_S^* \geq 1$ iff $\underline{R}_S < 1$ and $N^*(1) \geq 1$, which from the definition of $\bar{\mu}$ is equivalent to $\mu \leq \bar{\mu}$. The result is then a consequence of *a)*, Proposition 9 and Proposition 10.

c) If $\mu > \bar{\mu}$ there exists an equilibrium satisfying $1 = R_S^ < R_A(\bar{p}) < R_E^*, p^* < \bar{p}$ and $N^* = \frac{1-\mu}{1-\bar{\mu}} \in (N^b, 1)$. Moreover, the equilibrium is unique within the class of equilibria with $R_S^* < R_A(\bar{p})$.*

Suppose that $\mu > \bar{\mu}$, which from the definition of $\bar{\mu}$ implies that $\underline{R}_S < 1$. From (52), we have also that $N^*(1) = \frac{1-\mu}{1-\bar{\mu}} < 1$. Then *a)* implies that $R_S^* = 1$ is the safe rate of a general equilibrium. The results for the associated equilibrium variables, except from $N^* > N^b$, are then a consequence of Proposition 9 and Proposition 10. The inequality $N^* > N^b$ results from Proposition 2. Finally, for any $R_S \in (1, R_A(\bar{p}))$ we have from Proposition 12 that $N^*(R_S) < 1$ and *a)* implies that R_S is not the safe rate in some general equilibrium.

d) If $\mu \leq \underline{\mu}$ there exists M-M equilibria with $R_S^ = R_E^* = R_A(\bar{p}), p^* = \bar{p}$ and $N^* = 1$*

Suppose that $\mu \leq \underline{\mu}$. The equilibria of the economy with no intermediaries are in the M-M indifference region and satisfy $R_S^* = R_E^* = R_A(\bar{p})$. Consider one such equilibrium and suppose the return of the risky securities is $R_I^* = R_A(\bar{p})$. It is easy to directly prove from the originator's problem (3) - (9) that for the pair of returns $R_S^* = R_I^* = R_A(\bar{p})$ it is weakly optimal for the originator to choose $D_{I,z} = 0$. If originators do not issue risky securities, then market clearing implies that the supply of risky securities is zero which means that intermediaries do not enter. This proves that the equilibrium of the no intermediary benchmark economy can be sustained when experts can set up intermediaries and they expect a return for risky securities $R_I^* = R_A(\bar{p})$ in that market.

e) An equilibrium exists

Immediate from *b)*, *c)* and *d)*.

Uniqueness of equilibrium

f) If $\mu > \underline{\mu}$ the equilibrium is unique

Suppose that $\mu > \underline{\mu}$. We have from *b)* and *c)* that the economy has a unique equilibrium with a safe rate $R_S^* < R_A(\bar{p})$. Suppose that $R_S^* = R_A(\bar{p})$ is the safe rate in some general equilibrium. Then Lemma 3 implies that $R_S^* = R_I^* = R_E^* = R_A(\bar{p})$ and the arguments made in the proof of that lemma imply that originator's risk choice is $p^* = \bar{p}$. Besides, aggregate investment must be $N^* = 1$. Let $D_S^*, D_{I,z}^*$ be the equilibrium safe and risky promises made by originators. From (24) we have that $p^* = \bar{p}$ implies that $D_{I,H}^* = D_{I,L}^*$, and thus risky securities are in fact safe. This means that intermediaries, in case they enter in the economy, they do not expand the supply of safe securities by diversifying idiosyncratic risks. Formally, the supply of safe securities in this economy is necessarily upper bounded by

$$\frac{A_L N^*}{R_S^*} = \frac{A_L}{R_A(\bar{p})} = \underline{\mu}.$$

Besides, since $R_S^* > 1$, savers find strictly optimal to invest in safe securities and the demand for safe securities is at least $\underline{\mu}$. But then $\mu > \underline{\mu}$ implies that this market does not clear. We conclude that $R_S^* = R_A(\bar{p})$ cannot be the safe rate in some general equilibrium.

g) If $\mu \leq \underline{\mu}$ all the equilibria are M-M type with $R_S^ = R_E^* = R_A(\bar{p})$, $p^* = \bar{p}$ and $N^* = 1$*

Suppose there exists an equilibrium with $R_S^* < R_A(\bar{p})$. Then *a)* implies that $N^*(R_S^*) \leq 1$. From Proposition 12, we have that $\lim_{R_S \rightarrow R_A(\bar{p})} N^*(R_S) < 1$ and (89) states that $\mu > \underline{\mu}$. We conclude that any equilibrium must have $R_S^* = R_A(\bar{p})$. Reproducing arguments made in *f)* we get the result.

h) The equilibrium is unique up to M-M indifference if and only if $\mu \leq \underline{\mu}$

Immediate from *f)* and *g)*. ■

Proof of Proposition 14 We start with a preliminary observation. If the equilibrium returns in either of the economies are denoted with R'_E, R'_S , then aggregate welfare in that economy can be written as

$$W' = (1 - \mu)R'_E + \mu R'_S. \quad (90)$$

Notice the expression holds also if $R'_S = 1$ or if $R'_E = R'_S$.

We prove sequentially each of the statements in the proof.

For any given μ we refer in this proof to equilibrium variables in the no intermediaries economy with *b* superscript and to equilibrium variables in the baseline economy with a *** superscript. Moreover we will make explicit the dependence of these variables on μ

Statement i)

It follows immediately from Proposition 2, Proposition 13, and the expressions for $R_S^b(\mu)$ in (22) in the region $\mu \in (\underline{\mu}, A_L]$ and for $R_S^*(\mu)$ in (54) in the region $\mu \in (\underline{\mu}, \bar{\mu}]$.

Statement ii)

Let us consider three regions

First, $\mu > \bar{\mu}$. Notice that this requires that $\bar{\mu} < 1$. From Proposition 2 and Proposition 13, we have that $R_S^*(\mu) = R_S^b(\mu) = 1$. Besides, since $\chi^*(\mu) > 1$ we have from Proposition 7 that in the baseline economy the originator finds strictly optimal to issues a positive amount of risky securities, that is $d_I^*(\mu) > 0$. Notice that since $R_S^*(\mu) = R_S^b(\mu) = 1$ and the originator finds strictly suboptimal so set $d_I = 0$ in which case its return on equity would be equal to that in the no intermediaries economy, $R_E^b(\mu)$, we must have that $R_E^b(\mu) < R_E^*(\mu)$.

Second, $\mu \in (\underline{\mu}, A_L]$. From Proposition 2 and Proposition 13, we have that $N^*(\mu) = N^b(\mu) = 1$ and $p^*(\mu) < p^b(\mu) = \bar{p}$. Since all the consumption in the two economies is at the final date, we have from (90) in the two economies is invested (directly or indirectly) in originator's project, and their payoffs are consumed by savers and experts we have that

$$\begin{aligned} (1 - \mu)R_E^*(\mu) + \mu R_S^*(\mu) &= W^*(\mu) = R_A(p^*(\mu)) < \\ &< R_A(\bar{p}) = W^b(\mu) = (1 - \mu)R_E^b(\mu) + \mu R_S^b(\mu). \end{aligned}$$

Using from *i)* that $R_S^b(\mu) > R_S^*(\mu)$ we conclude from the inequality above that $R_E^*(\mu) < R_E^b(\mu)$.

Third, $\mu \in (A_L, \bar{\mu}]$. From Proposition 2 and Proposition 13 we have that $R_E^b(\mu) = 1$ is constant in all this region while $R_E^*(\mu)$ is strictly increasing.

The statement in *ii)* then results immediately from our results in the three regions.

Statement iii)

Using (90), it follows from the two previous statements. ■

Proof of Proposition 15 Recall the variables $\underline{R}_S \in (A_L, R_A(\bar{p}))$ defined in Proposition 9, and $\underline{\mu}, \bar{\mu}$, defined in Proposition 13. We rely extensively in this proof without explicit reference to the results in Proposition 13 and to the discussion in the main text preceding Proposition 15, in particular the equivalence between the FOC in (69) and that in (31) after plugging in the equilibrium equation (40).

We first describe the set of Pareto efficient allocations. For SP weights ω_S, ω_E with $\omega_E > 0$, we define $\omega \equiv \omega_S/\omega_E$ and adopt the convention that $\omega = \infty$ when $\omega_E = 0$. We have from (66) that if $\omega_E > 0$ then the associated optimal allocations depend only on ω . Besides, we have from the main text that optimal allocations are described by a tuple $(N, d_{I,H}, C_{S,0}, C_{S,1})$. For each value of ω , the optimal allocations are denoted with a superscript *SP*, can be obtained from (66), and are presented next (Details of the derivations are omitted):

I- For $\omega < 1$: $N^{SP} = 1, d_{I,H}^{SP} = 0, C_{S,0}^{SP} = 0, C_{S,1}^{SP} = 0$

II- For $\omega = 1$: $N^{SP} = 1, d_{I,H}^{SP} = 0, C_{S,0}^{SP} = 0, C_{S,1}^{SP}$ is any value satisfying (68)

For the rest of the allocation Pareto frontier, we distinguish two cases:

Case $\underline{R}_S \geq 1 (\Leftrightarrow \bar{\mu} = 1)$:

III- For $\omega > 1$: $N^{SP} = 1, d_{I,H}^{SP} \in (0, \Delta)$ is the unique solution to (69), $C_{S,0}^{SP} = 0, C_{S,1}^{SP}$ satisfies (68) with equality.

Case $\underline{R}_S < 1 (\Leftrightarrow \bar{\mu} < 1)$: Let $\chi^*(R_S = 1)$ denote the intermediary funding discount in the equilibrium of the economy with an exogenous $R_S = 1$. Define $\bar{\omega} = (1 - \lambda)\chi^*(R_S = 1)$.

III.a- For $\omega \in (1, \bar{\omega})$: $N^{SP} = 1, d_{I,H}^{SP} \in (0, \Delta)$ is the unique solution to (69), $C_{S,0}^{SP} = 0, C_{S,1}^{SP}$ satisfies (68) with equality

III.b- For $\omega = \bar{\omega}$: N^{SP} is any value in the interval $[0, 1]$, $d_{I,H}^{SP} \in (0, \Delta)$ is the unique solution to (69), $C_{S,0}^{SP} = 1 - N^{SP}, C_{S,1}^{SP}$ satisfies (68) with equality

III.c- For $\omega > \bar{\omega}$: $N^{SP} = 0, d_{I,H}^{SP}$ is irrelevant since there is no investment, $C_{S,0}^{SP} = 1, C_{S,1}^{SP} = 0$

We now proceed to the proof of the two constrained Welfare Theorems in the proposition. For the sake of brevity we restrict to the slightly more involved case of $\underline{R}_S < 1 \Leftrightarrow \bar{\mu} < 1$.

First Welfare Theorem:

For given μ , we need to prove that the (general) equilibrium of the economy is a Pareto efficient allocation. We distinguish three cases:

i) $\mu \leq \underline{\mu}$: The equilibrium is of the M-M type and thus belongs to the efficient allocation region **I** if $\mu = 0$ and **II** if $\mu > 0$.

ii) $\mu \in (\underline{\mu}, \bar{\mu}]$: Let $\omega = (1 - \lambda)\chi^*$ where χ^* denotes the general equilibrium value of this variable for the given μ . We have by construction that $\omega \leq \bar{\omega}$ and the equilibrium coincides with the efficient allocation in region **III.a** if $\omega < \bar{\omega}$ and the unique efficient allocation in the region **III.b** with $N^{SP} = 1$ if $\omega = \bar{\omega}$.

iii) $\mu > \bar{\mu}$: The equilibrium coincides with the unique efficient allocation in the region **III.b** with $N^{SP} = \frac{1-\mu}{1-\bar{\mu}}$

Second Welfare Theorem:

For given Pareto efficient allocation $(N^{SP}, d_{I,H}^{SP}, C_{S,0}^{SP}, C_{S,1}^{SP})$, we need to prove that there exists μ such that the allocation coincides with that induced by the equilibrium of the economy for such value of μ . We distinguish three cases:

i) $(N^{SP}, d_{I,H}^{SP}, C_{S,0}^{SP}, C_{S,1}^{SP})$ in regions **I** or **II**: Define $\mu = C_{S,1}^{SP}/R_A(\bar{p})$. Then we have by construction that $\mu \leq \underline{\mu}$ and the equilibrium of the economy for this value of μ induces the allocation.

ii) $(N^{SP}, d_{I,H}^{SP}, C_{S,0}^{SP}, C_{S,1}^{SP})$ in regions **III.a** or **III.b** with $N^{SP} = 1$: Let $\omega \leq \bar{\omega}$ be the SP weight ratio associated with the allocation. Taking into account the properties of the partial equilibrium function $\chi^*(R_S)$ described in Proposition 9, we have that there exists a unique

$R'_S \in [1, A_L, R_A(\bar{p})]$ such that $\chi^*(R'_S) = \omega/(1 - \lambda)$. Define $\mu = C_{S,1}^{SP}/R'_S$. Then we have by construction that $\mu \in (\underline{\mu}, \bar{\mu}]$ and the equilibrium of the economy for this value of μ induces the efficient allocation.

iii) $(N^{SP}, d_{I,H}^{SP}, C_{S,0}^{SP}, C_{S,1}^{SP})$ in regions **III.b** with $N^{SP} < 1$ or **III.c**: Define μ to be the unique solution to $N^{SP} = \frac{1-\mu}{1-\bar{\mu}}$. Then we have by construction that $\mu < \bar{\mu}$ and the equilibrium of the economy for this value of μ induces the efficient allocation. ■

Proof Proposition 17 We denote equilibrium variables of the economy with no intervention with a * superscript. We focus from Lemma 16 on feasible policies $(\bar{\theta}, \tau)$ and denote with $p(\bar{\theta}, \tau), N(\bar{\theta}, \tau), W(\bar{\theta}, \tau)$ the values of these variables induced by the policy $(\bar{\theta}, \tau)$. We denote with λ_0 the exogenous aggregate risk parameter in the baseline economy and refer to an economy with generic aggregate risk parameter λ as a λ -economy.

We prove the statements in the proposition in a sequence of steps.

i) The equilibrium of the economy with no intervention is not a first-best allocation

This results from $\mu > \underline{\mu}$ and Proposition 13.

ii) The first-best allocations in the Pareto frontier of a λ -economy are independent from λ

From the proof of Proposition 15, we have that first-best allocations in the Pareto frontier of the economy correspond to regions I and II, which are independent from λ .

iii) The non first-best part of the Pareto frontier of a λ -economy with positive investment is strictly shifted rightwards as λ decreases.

From the proof of Proposition 15, an allocation of the non first-best part of the Pareto frontier of a λ -economy can be described by a pair $(N, d_{I,H})$ satisfying the properties in III or in III-a-b-c, which in particular imply that $d_{I,H} > 0$. In either case, the overall welfare for savers and experts, W_S, W_E is given by:

$$W_S(N, d_{I,H}|\lambda) = (A_L + (1 - \lambda)\hat{p}(d_{I,H})d_{I,H})N + (1 - N), \quad (91)$$

$$W_E(N, d_{I,H}|\lambda) = (R_A(\hat{p}(d_I)) - A_L - (1 - \lambda)\hat{p}(d_{I,H})d_{I,H})N. \quad (92)$$

Notice in addition from (26) that the function $\hat{p}(d_I)$ does not depend λ .

Let $\lambda_1 < \lambda_2$ and consider an allocation in the non-first best Pareto frontier of the λ_2 -economy with positive investment. It is thus described by a pair $(N, d_{I,H})$ with $N > 0, d_{I,H}$, and induces welfare for savers and experts amounting to $W_S(N, d_{I,H}|\lambda_2), W_E(N, d_{I,H}|\lambda_2)$, respectively.

Consider the allocation of the λ_1 -economy described by $(N', d'_{I,H})$, where $N' = N$ and $d'_{I,H}$ is such that $(1 - \lambda_1)\hat{p}(d'_{I,H})d'_{I,H} = (1 - \lambda_2)\hat{p}(d_{I,H})d_{I,H}$. Since $d_{I,H}$ is part of the Pareto

Frontier of the λ_2 -economy, we have that $\hat{p}(d)d$ is increasing in d in the interval $[0, d_{I,H}]$.¹⁴ Using $\lambda_1 < \lambda_2$, we have that $d'_{I,H} < d_{I,H}$, $R_A(\hat{p}(d_{I,H})) < R_A(\hat{p}(d'_{I,H}))$ and:

$$W_S(N, d'_{I,H}|\lambda_1) = W_S(N, d_{I,H}|\lambda_2) \text{ and } W_E(N, d'_{I,H}|\lambda_1) > W_E(N, d_{I,H}|\lambda_2),$$

which shows that the allocation induced by $(N, d_{I,H})$ in the λ_2 -economy does not belong to the Pareto frontier of the λ_1 -economy.

iv) Let $(\bar{\theta}, \tau)$ be a Pareto optimal policy, then $W(\bar{\theta}, \tau) > W^*$.

Let $\bar{\theta}' = 1 - \lambda_0 + \varepsilon$ with $\varepsilon > 0$ sufficiently small to ensure that any policy $(\bar{\theta}', \tau')$ is feasible, and $\bar{\lambda}' = 1 - \bar{\theta}'$. From Proposition 15, the equilibrium of the economy with no intervention is in the Pareto frontier of the λ_0 -economy. Using that by construction $\bar{\lambda}' < \lambda_0$, claims *i)* and *iii)* imply that the equilibrium of the economy with no intervention does not belong to the Pareto frontier of the $\bar{\lambda}'$ -economy. Choose an allocation of such an economy that Pareto improves the equilibrium with no intervention. Using Proposition 15 for the $\bar{\lambda}'$ -economy, such allocation is the equilibrium of a $\bar{\lambda}'$ -economy after sum lump-sum transfers τ' . By construction, the policy $(\bar{\theta}', \tau')$ is feasible, induces the just described allocation and thus satisfies $W(\bar{\theta}', \tau') > W^*$. A fortiori, any Pareto optimal policy $(\bar{\theta}, \tau)$ satisfies $W(\bar{\theta}, \tau) > W^*$.

v) Let $(\bar{\theta}, \tau)$ be a Pareto optimal policy, then $\bar{\theta} > 1 - \lambda_0$

This can be proved by contradiction using the definition of a Pareto optimal policy, Proposition 15, and *iv)*.

vi) Pareto optimal policies cannot induce first-best allocations

Suppose a Pareto optimal policy $(\bar{\theta}, \tau)$ induces a first-best allocation. From *ii)* we have that $(\bar{\theta}, \tau)$ induces an allocation in the Pareto frontier of the λ_0 -economy. From here we can reproduce the arguments in *v)* to get a contradiction.

vii) For a feasible policy $(\bar{\theta}, \tau)$ we have that:

$$W(\bar{\theta}, \tau) - W^* = (N(\bar{\theta}, \tau) - N^*)(R_A(p(\bar{\theta}, \tau)) - 1) - N^*(R_A(p^*) - R_A(p(\bar{\theta}, \tau))) \quad (93)$$

Taking into account that the government breaks-even by construction under the allocation induced by a feasible policy, the equation above is analogous to (56) and can be derived in the same manner.

viii) If $\mu \leq \bar{\mu}$, then Pareto optimal policies induce less risk-taking

We have from Proposition 13 that $N^* = 1$. Let $(\bar{\theta}, \tau)$ be a Pareto optimal policy. From *iv)* and (93) we must necessarily have that $p(\bar{\theta}, \tau) > p^*$.

ix) For any λ , let $\chi^*(R_S, \lambda)$ denote the variable defined in Proposition 9. We have that $\frac{\partial \chi^*(R_S, \lambda)}{\partial \lambda} < 0$.

¹⁴Otherwise, the SP would reduce $d_{I,H}$ and improve both agents. [AV: Toli, crees que hay que probar es increasing? Easy to see that It must be the case that $\hat{p}(d)d < \hat{p}(d_{I,H})d_{I,H}$ for any $d < d_{I,H}$]

The partial equilibrium variable $\chi^*(R_S, \lambda)$ satisfies (42). Notice that the expression for $R_E^*(R_S, \chi^*)$ in (41) and its partial derivatives satisfy (85). The property $\frac{\partial \chi^*(R_S, \lambda)}{\partial \lambda} < 0$ then immediately results.

x) If $\mu > \bar{\mu}$, then Pareto optimal policies increase investment

We have from Proposition 13 that $N^* < 1$ and $R_S^* = 1$. Let $(\bar{\theta}, \tau)$ be a Pareto optimal policy and suppose that $N(\bar{\theta}, \tau) \leq N^*$. From *iv)* and (93) we must necessarily have that $p(\bar{\theta}, \tau) > p^*$.

Denote $\bar{\lambda} = 1 - \bar{\theta}$. From *v)* we have that $\bar{\lambda} < \lambda_0$. We have in addition that the equilibrium induced by $(\bar{\theta}, \tau)$ is an equilibrium of the $\bar{\lambda}$ -economy and since $N(\bar{\theta}, \tau) < 1$ we must have that its equilibrium safe rate is $R_S = 1$. We thus have from Proposition 7 that

$$p(\bar{\theta}, \tau) = \hat{p}\left(\hat{d}_I(\chi^*(R_S = 1, \bar{\lambda}))\right).$$

Notice in addition from the proof of Proposition 7 that the function $\hat{d}_I(\chi)$ does not depend on λ and that from (26) that the function $\hat{p}(d_I)$ neither depends on λ . Using *ix)* and the monotonicity properties of $\hat{d}_I(\chi)$ and $\hat{p}(d_I)$, described in Proposition 7 and Lemma 5, respectively, we have that $\bar{\lambda} < \lambda_0$ implies that

$$p(\bar{\theta}, \tau) = \hat{p}\left(\hat{d}_I(\chi^*(R_S = 1, \bar{\lambda}))\right) < \hat{p}\left(\hat{d}_I(\chi^*(R_S = 1, \lambda_0))\right) = p^*,$$

which contradicts that $p(\bar{\theta}, \tau) > p^*$.

xi) If $\mu > \bar{\mu}$ and X sufficiently small, then Pareto optimal policies increase risk-taking

We have from Proposition 13 that $N^* < 1$ and $R_S^* = 1$. For X sufficiently small, due to continuity arguments we have that a Pareto optimal policy $(\bar{\theta}, \tau)$ cannot induce full investment, that is $N(\bar{\theta}, \tau) < 1$ and $R_S(\bar{\theta}, \tau) = 1$. We can reproduce the arguments in the proof of claim *x)* to prove that $p(\bar{\theta}, \tau) < p^*$. ■