Insuring Consumption Using Income-Linked Assets

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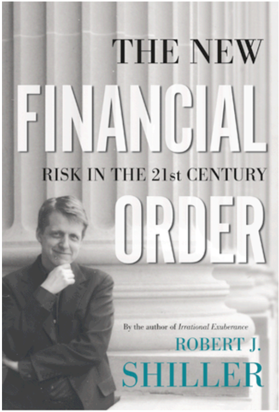
Introduction

- Human capital is the largest component of total household wealth for much of life
- It is also risky: income volatility is high (and supposedly has increased over past decades)
- Much evidence that this leads to consumption volatility, due to imperfect risk-sharing
- Not too surprising: risk-sharing is generally difficult because of
  - informational asymmetries (moral hazard)
  - limited commitment

Introduction

- Yet, part of human capital risk is *group-specific* and *cross-sectional*
- Such risk could be hedged through financial assets with payoffs linked to group-level income indices
  - and without requiring a risk premium for aggregate risk
- Shiller (2003) and others have advocated the introduction of new financial assets to allow households to better insure against human capital risk (among others)
- Our goal is to evaluate the potential use and usefulness of such assets for households' income risk management over the life cycle

Motivation



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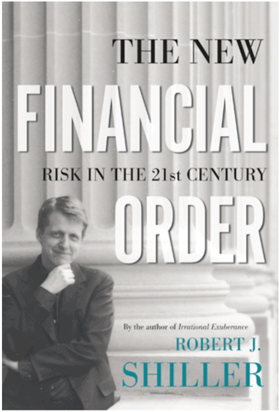

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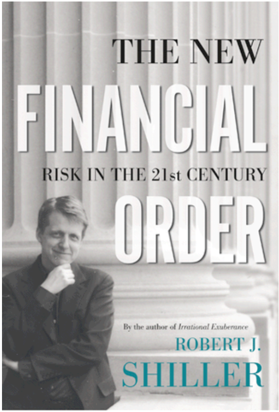

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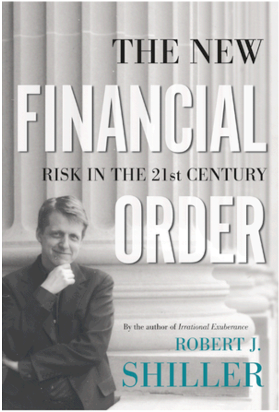

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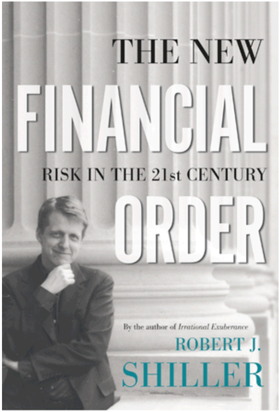

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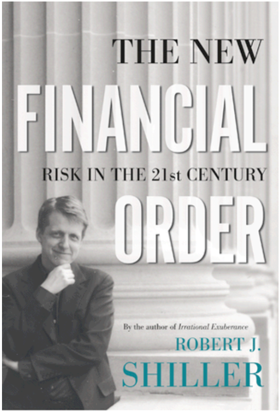

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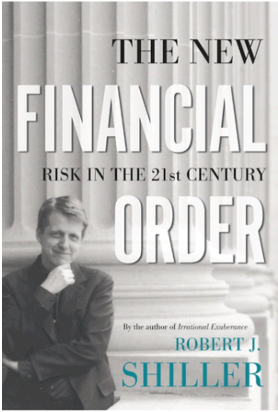

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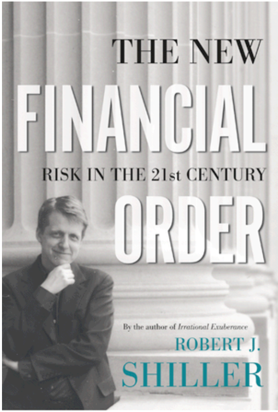

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In this paper, we consider (an example of) 1/3, and 4

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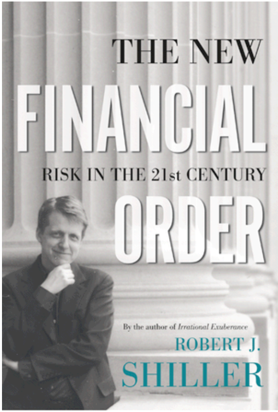

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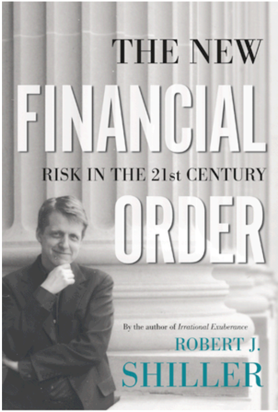

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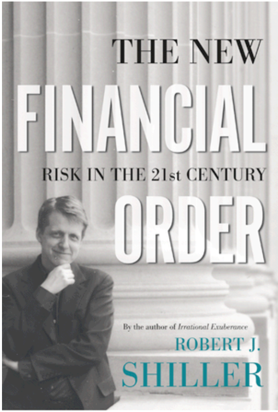

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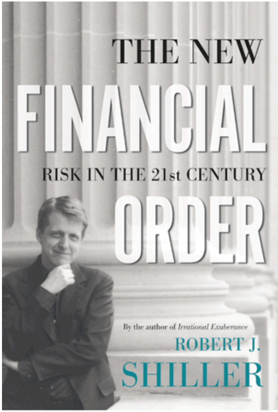

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"Making such [assets] more widely available would entail work from both the private sector and the government."

$\Rightarrow$ *How large are the benefits? Is it worth it?*

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  - IHI: limited liability asset with returns negatively correlated with income shock
  - ILL: loan with required repayment positively correlated with income shock
- Look at demand for these assets over the life cycle, and predicted welfare gains that their availability would generate for households

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1. Usefulness of income-linked assets depends strongly on how they are implemented:
   - ILL generally more beneficial than IHI
   - Correlation with income shocks
   - Volatility
2. The income-linked assets (in particular ILL) can produce non-negligible welfare gains (>1%)
3. But difficult to reduce a large fraction of the welfare costs from labor income risk with the assets we consider

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- Baseline assumption for $\mathrm{E}(r)$: "actuarial fairness"
  - $\mathrm{E}(\tilde{r}_{IHI}) = r_l$ (risk-free saving rate)
  - $\mathrm{E}(\tilde{r}_{ILL}) = r_b$ (risk-free borrowing rate)

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  - $\mathrm{E}(\tilde{r}_{IHI}) = r_l$ (risk-free saving rate)
  - $\mathrm{E}(\tilde{r}_{ILL}) = r_b$ (risk-free borrowing rate)
- This assumes that risks are cross-sectional (not aggregate), and in that sense stacks the deck in favor of these assets

Related Literature

- Quantitative dynamic macro models that consider welfare costs of income shocks
  - Storesletten, Telmer, Yaron (2004), Heathcote, Storesletten, Violante (2008)
- Risk-sharing and partial insurance
  - Attanasio and Davis (1996), Krueger and Perri (2006), Blundell et al. (2008)
- Optimal portfolio choice over the life cycle
  - Cocco, Gomes, Maenhout (2005), Gomes and Michaelides (2005)
  - Our model builds on Davis, Kübler, Willen (2006)
  - Close in spirit: De Jong, Driessen, Van Hemert (2008) on housing futures; Cocco and Gomes (2009) on longevity bonds

Outline

1. Two-period example
   - Goal: provide intuition for what determines demand for and welfare gains from income-linked assets
2. Life-cycle model
   - Goal: show that intuition carries over; quantitatively assess use and usefulness of assets over life cycle
3. Discussion / Conclusion

Two-Period Example: Setup

- CRRA=2 investor lives for 2 periods
- Objective: max $u(c_1) + Eu(c_2)$
- Has some cash-on-hand in period 1
- Receives stochastic income in period 2 with mean 8
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- Benchmark: Investor can...
  - save at $r_l = 2\%$
  - invest in equity with $\mathrm{E}(\tilde{r}_e) = 6\%$ and $\sigma(\tilde{r}_e) = 16\%$
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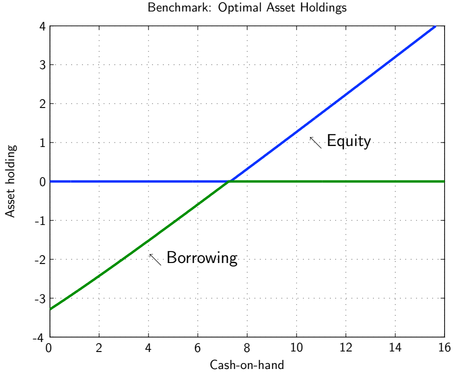
- Constraints: b, l, $e \geq 0$
- No default in model, so positive lower bound on Y_2 important (otherwise no borrowing possible)

Benchmark

$$E(Y_2) = 8,\ r_l = 0.02,\ r_b = 0.08,\ \mathrm{E}(\tilde{r}_e) = 0.06,\ \sigma_e = 0.16$$



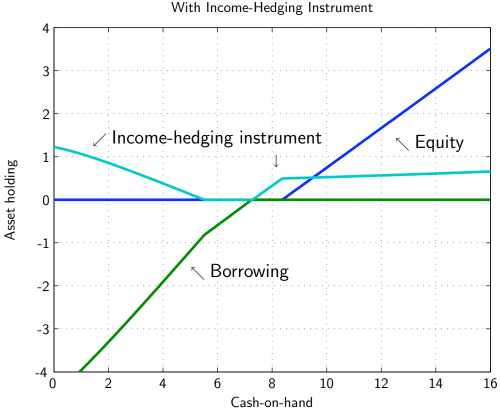
Income-Hedging Instrument

Now, investor can additionally invest in income-hedging instrument

- $\mathrm{E}(\tilde{r}_{IHI}) = r_l =$ 2%
- $\sigma(\tilde{r}_{IHI}) =$ 25%
- $corr(\tilde{r}_{IHI}, Y_2) =$ –0.5

$\Rightarrow$ IHI provides some insurance benefits, but not perfect insurance

IHI: Optimal Asset Holdings



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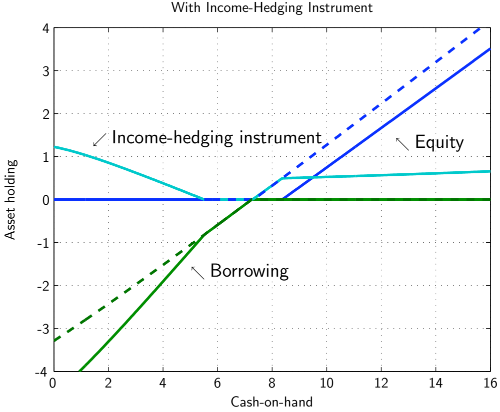

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  - What determines whether a household borrows? Expected future consumption growth $\Rightarrow$ borrow if relatively poor today
- And for households who save, IHI "competes" against equity $\Rightarrow$ only buy IHI if $\mathrm{E}_Q(\tilde{R}_{IHI}) \geq \mathrm{E}_Q(\tilde{R}_e)$

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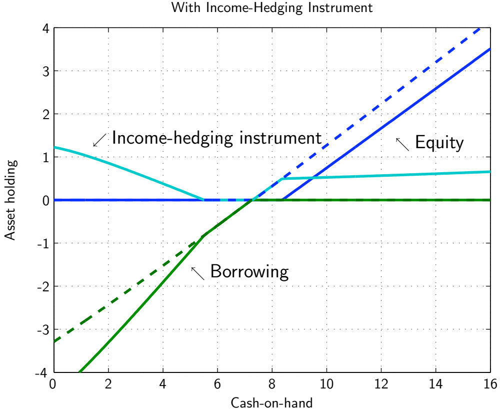

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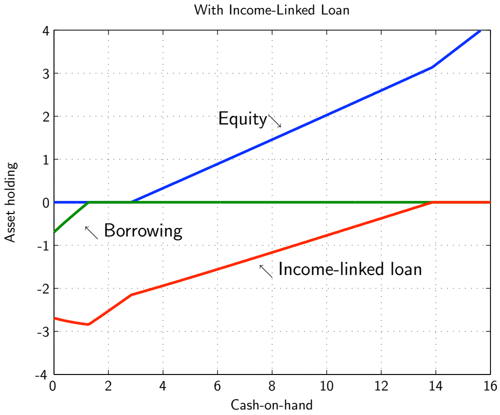
  - his life-cycle income profile
  - the risk-adjusted returns of other investment opportunities
- The welfare gain from an income-linked asset will depend on its (opportunity) cost
  - High for IHI

Income-Linked Loan

Now, instead, investor can additionally borrow through income-linked loan

- $\mathrm{E}(\tilde{r}_{ILL}) = r_b = 8\%$
- $\sigma(\tilde{r}_{ILL}) = 25\%$
- $corr(\tilde{r}_{ILL}, Y_2) = +0.5$

ILL: Optimal Asset Holdings



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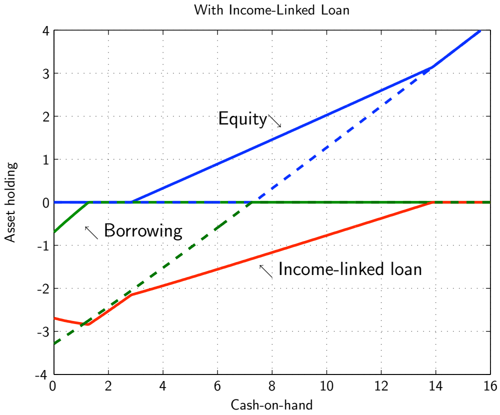

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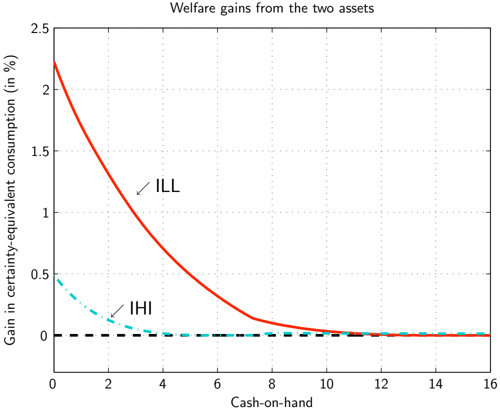
  - Goes to zero as cash-on-hand increases
- Compared with benchmark:
  - ILL substitutes for unsecured borrowing
  - Over some range, additional borrowing & investment in equity ($\mathrm{E}_Q(\tilde{R}_{ILL}) = \mathrm{E}_Q(\tilde{R}_e)$, even though $\mathrm{E}(\tilde{R}_{ILL}) > \mathrm{E}(\tilde{R}_e)$)

Welfare Gains over Benchmark



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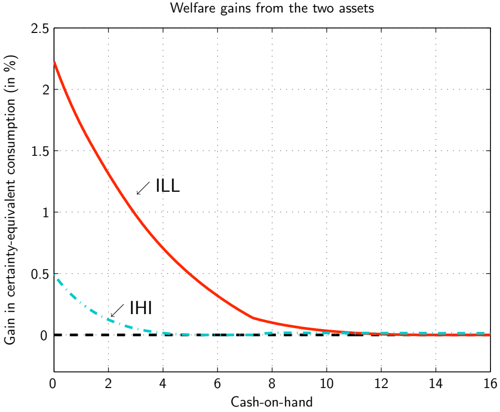

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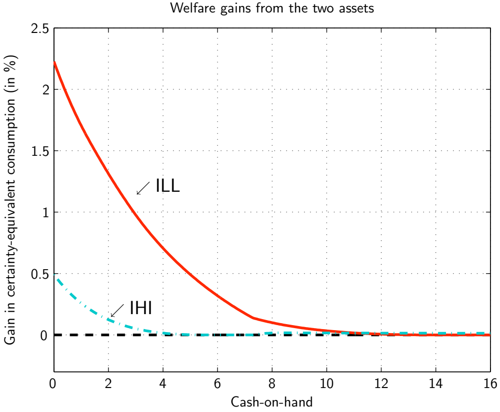

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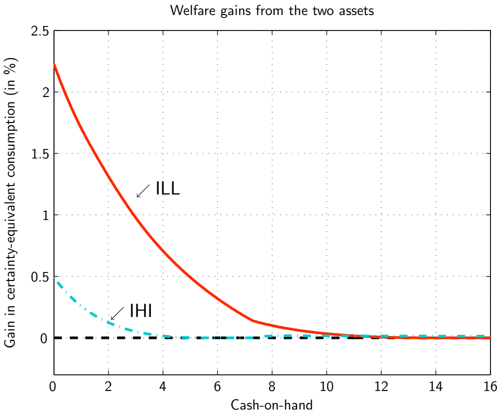

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- Welfare gain small as compared to having Y_2=8 for sure
  - 9.21% for c-o-h=0
  - 2.81% for c-o-h=5
  - 1.40% for c-o-h=10

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- Short-sale constraints:

$$e_t \geq 0, l_t \geq 0, b_t \geq 0, IHI_t \geq 0, ILL_t \geq 0$$

Life-Cycle Model

- Households in partial equilibrium, live from 20 to 80, retire at 65
- Receive stochastic income Y_t during working life
- Can trade three or four financial assets.
  - equity with stochastic net return $\tilde{r}_e$,
  - save at a net risk-free rate r_l,
  - engage in uncollateralized borrowing at the rate $r_b > r_l$

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- Finite-horizon dynamic program, solved computationally

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- Start with model that only features e, l, b
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- Then, add either income-hedging instrument or income-linked loan, with various assumptions about return process
  - Look at effect on asset holdings over the LC
  - Evaluate welfare gain from having access to new asset

Income Process

- Income process as standard in consumption/pf choice literature (following Gourinchas-Parker 2002, Cocco et al. 2005):

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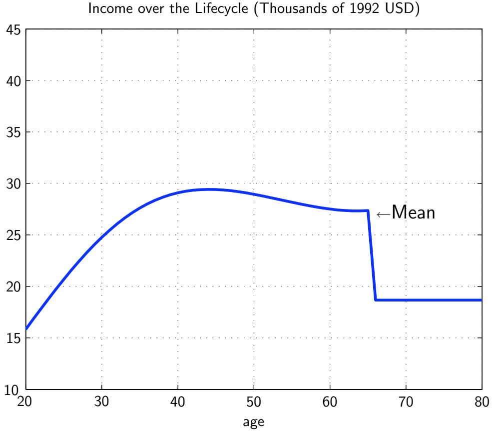
- Retirement income: $\tilde{y}_t = \log(\lambda) + d_{t_R} + \eta_{t_R}, t > t_R$

Income Process

Use parameters from Cocco et al. for HS grads: σ_u =0.103, σ_ε =0.272, λ =0.682. Enter at 20, retire at 65, die at 80.



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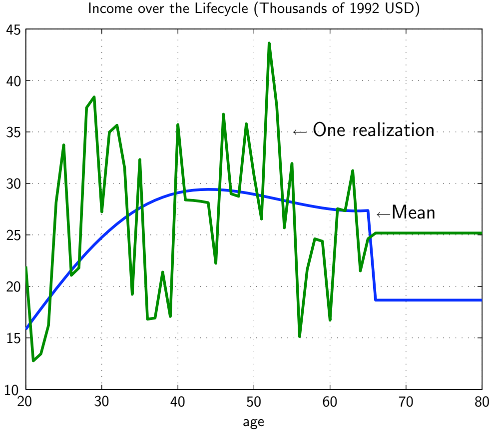
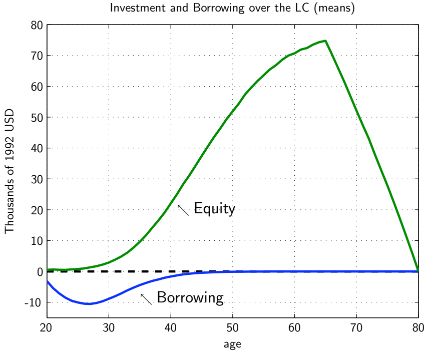
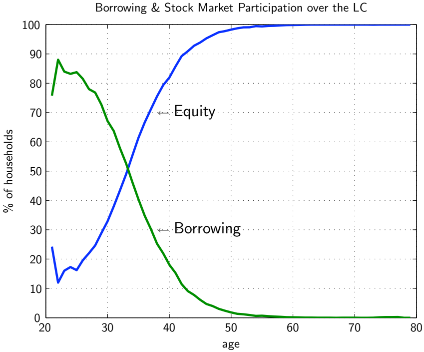


Other Parameters for Benchmark

- CRRA utility with curvature γ =2
- Taste-shifter s.th. consumption drops 10% at retirement
- Risk-free saving rate: r_l =0.02
- Risk-free borrowing rate: r_b =0.08 (Davis et al. 2006)
- Equity returns: $\mathrm{E}(\tilde{r}_e)$ =0.06, σ_e =0.16
- Discount factor: β =0.936. Chosen to match $\overline{W/Y}$ =2.6 of households with head aged 50 to 59 (Laibson et al. 2007)

Benchmark Results





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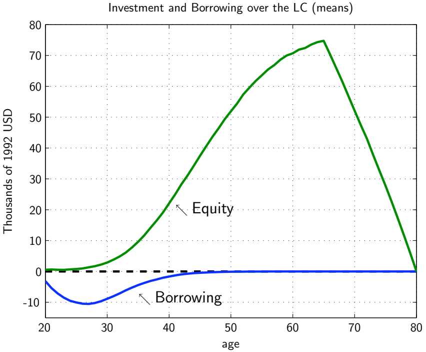

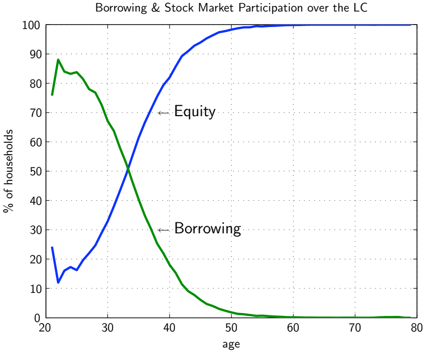

- Successes: general pattern of borrowing and risky asset holdings (and participation) over the LC
- Failures: no bond holdings, and almost no borrowing late in life

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Add IHI to benchmark setting. Parameters:

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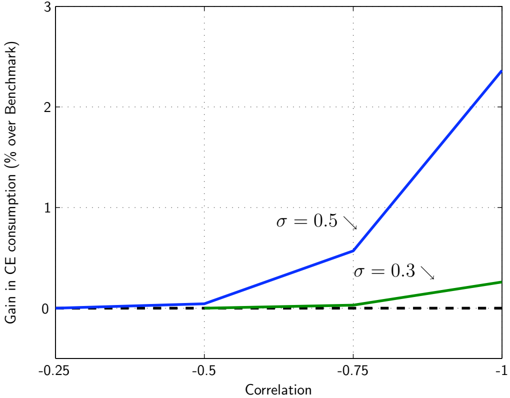
Focus on welfare gains from introducing IHI (in paper, look at LC profiles in detail)

Welfare Gains from IHI



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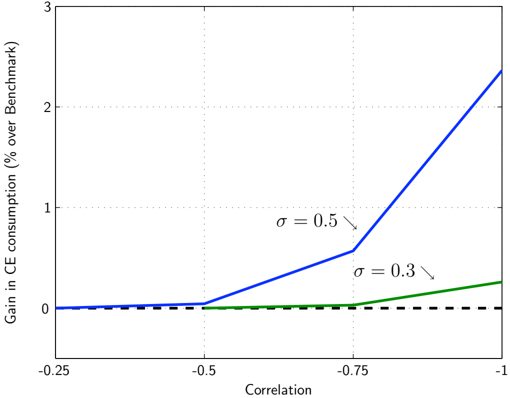

Compare to

- gain from reducing permanent income shock variance by 25%: 3.5%
- gain from eliminating all income risk: 16.4%

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- Welfare gains convex in $corr(\tilde{r}_{IHI}, \tilde{u})$ (and even in $corr^2$)

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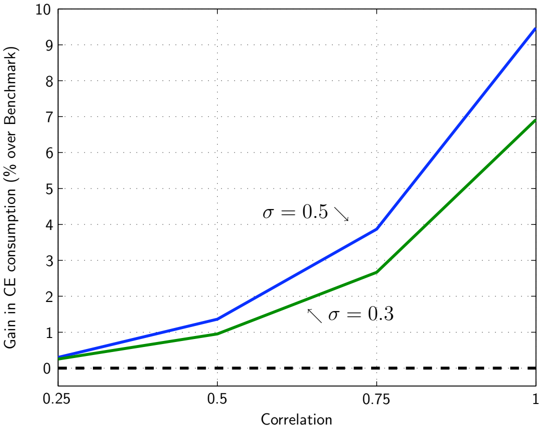
LC profiles

Welfare Gains from ILL



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- They invest part of their ILL borrowing in high-return equity
- Yet, still far from hypothetical welfare gain achieved by reducing income risk to zero

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$\Rightarrow$ ILL still generates larger welfare gain than IHI, but difference smaller

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- Welfare gain from baseline ILL but with $\mathrm{E}(\tilde{r}_{ILL}) = 0.08 > r_b$: 0.5%
- Thus, if households had access to borrowing at a cheaper rate than what they would pay on the ILL, result that ILL generates larger gains than IHI may be reversed

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$\Rightarrow$ Gains significantly larger with higher risk aversion (as is the welfare cost from life cycle income shocks in the benchmark: 24.5%)

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- If instead assumed $r_b = r_l = \mathrm{E}(\tilde{r}_{ILA}) =$0.02, model would predict
  - IHI and ILL equivalent
  - σ does not matter
  - even with $|\rho| =$0.5, welfare gain $>$ 4%

Discussion & Conclusion

THE END – THANK YOU!

Appendix: Welfare Measures

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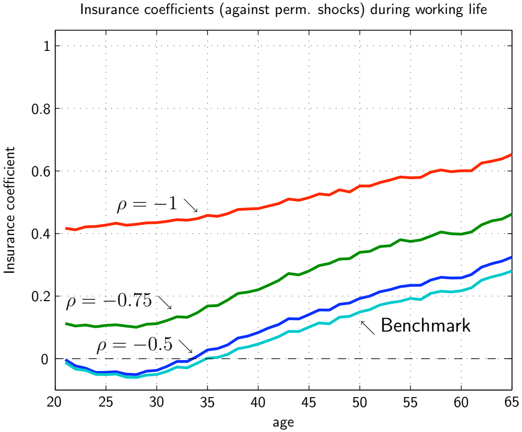
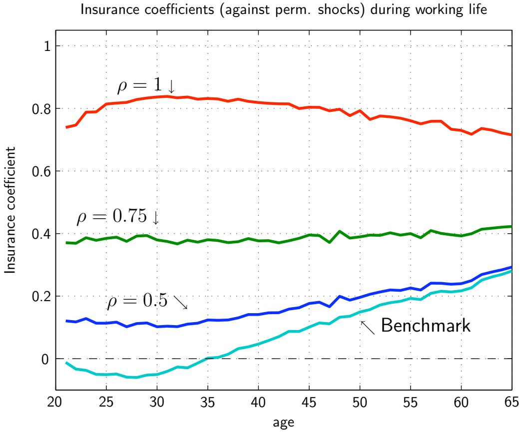
- The lower this coefficient, the more an income shock translates into consumption changes. $\phi = 1$: perfect insurance.
- In benchmark, $\bar{\phi^u} = 0.08$ and $\bar{\phi^\varepsilon} = 0.9$: easy to insure against transitory shocks, hard to insure against permanent ones.

Partial Insurance Coefficients with IHI



Partial Insurance Coefficients with ILL



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- Solve by policy-function iteration, as FOCs necessary and sufficient

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$\Rightarrow$ can solve *equation* that is differentiable of degree κ–1 for λ_e (and similarly for other assets and multipliers).

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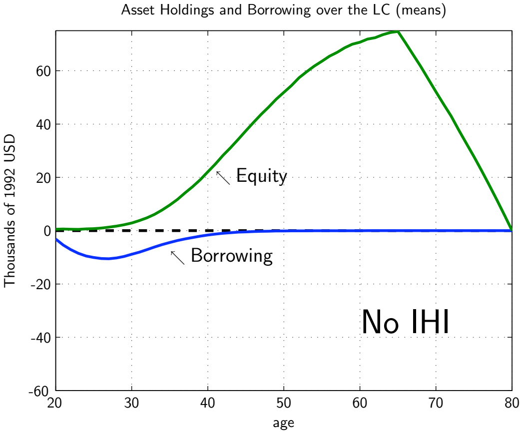
Optimal Asset Holdings with IHI



Back

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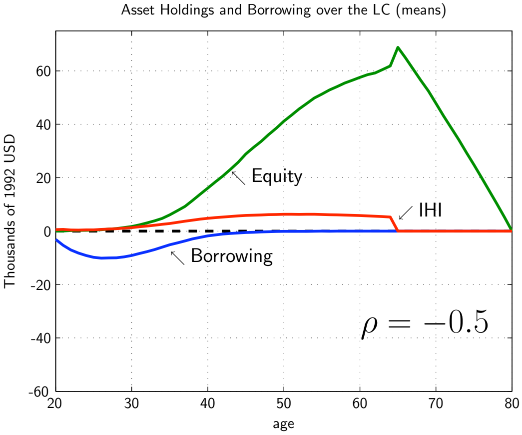

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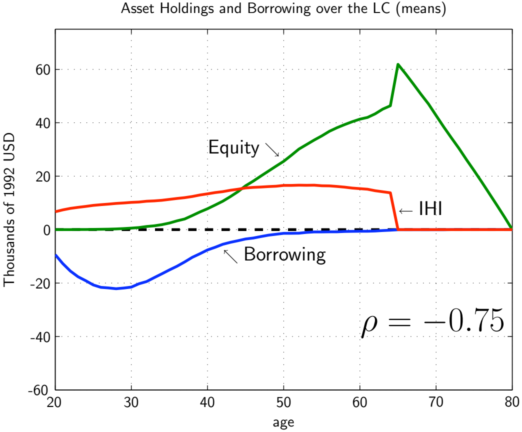

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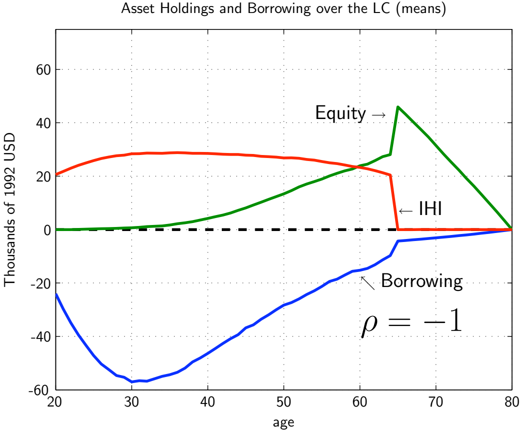

Back

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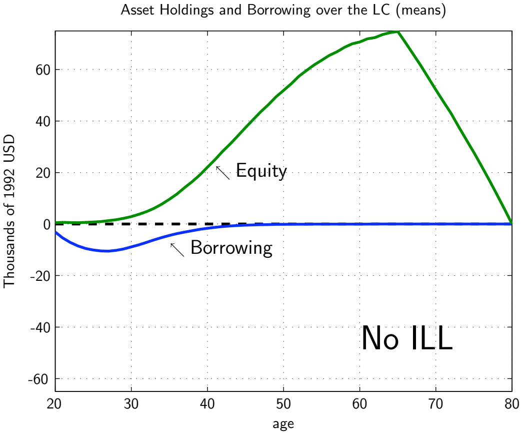

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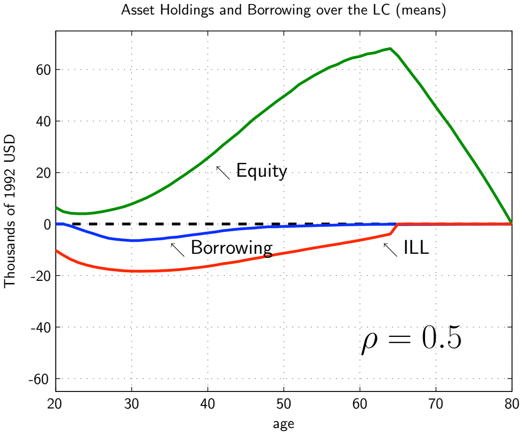

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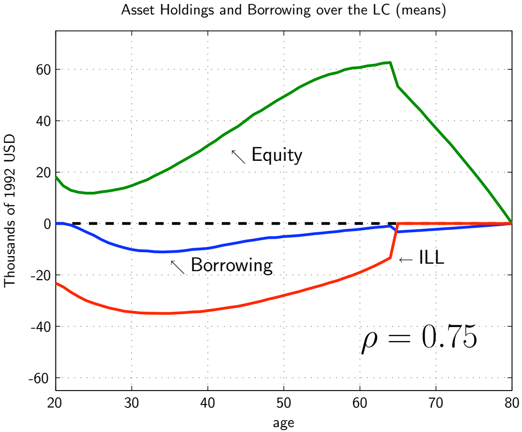

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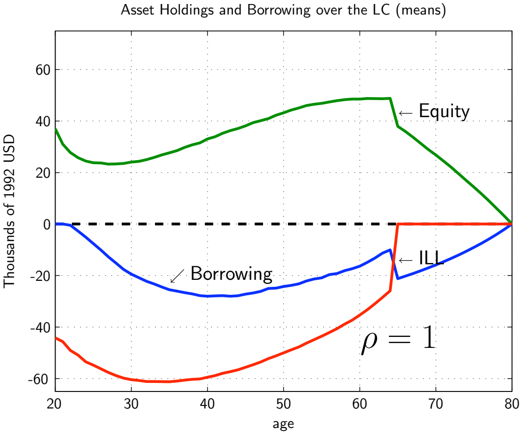

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