Insuring Consumption Using Income-Linked Assets

Andreas Fuster and Paul Willen

Harvard University and Federal Reserve Bank of Boston

Conference on Household Finance and Macroeconomics
Banco de España, Madrid
October 16, 2009
Introduction

- Human capital is the largest component of total household wealth for much of life
- It is also risky: income volatility is high (and supposedly has increased over past decades)
- Much evidence that this leads to consumption volatility, due to imperfect risk-sharing
- Not too surprising: risk-sharing is generally difficult because of
  - informational asymmetries (moral hazard)
  - limited commitment
Yet, part of human capital risk is *group-specific* and *cross-sectional*. Such risk could be hedged through financial assets with payoffs linked to group-level income indices and without requiring a risk premium for aggregate risk.

Shiller (2003) and others have advocated the introduction of new financial assets to allow households to better insure against human capital risk (among others).

Our goal is to evaluate the potential use and usefulness of such assets for households’ income risk management over the life cycle.
Motivation

Shiller proposes six types of insurance:
Motivation

Shiller proposes six types of insurance:

1. Livelihood insurance
Motivation

Shiller proposes six types of insurance:

1. Livelihood insurance
2. Home equity insurance
Motivation

Shiller proposes six types of insurance:

1. Livelihood insurance
2. Home equity insurance
3. Macro markets
Motivation

Shiller proposes six types of insurance:

1. Livelihood insurance
2. Home equity insurance
3. Macro markets
4. Income-linked loans
Motivation

Shiller proposes six types of insurance:

1. Livelihood insurance
2. Home equity insurance
3. Macro markets
4. Income-linked loans
5. Inequality insurance
Shiller proposes six types of insurance:

1. Livelihood insurance
2. Home equity insurance
3. Macro markets
4. Income-linked loans
5. Inequality insurance
6. Intergenerational social security
Motivation

1. Livelihood insurance
2. Income-linked loans
3. Macro markets

In this paper, we consider (an example of) 1/3, and 4
Motivation

“Imagining the social and economic achievement that could come from a new financial order is difficult because we have not seen such an alternate world.”

Andreas Fuster (Harvard)
“Imagining the social and economic achievement that could come from a new financial order is difficult because we have not seen such an alternate world.”

⇒ Need to use a model
Motivation

“Imagining the social and economic achievement that could come from a new financial order is difficult because we have not seen such an alternate world.”

⇒ Need to use a model

“Making such [assets] more widely available would entail work from both the private sector and the government.”
Motivation

“Imagining the social and economic achievement that could come from a new financial order is difficult because we have not seen such an alternate world.”

⇒ Need to use a model

“Making such [assets] more widely available would entail work from both the private sector and the government.”

⇒ How large are the benefits? Is it worth it?
What we do

- Consider a life-cycle portfolio choice model with realistic borrowing and investment opportunities
  - Key feature: borrowing rate > lending rate
What we do

- Consider a life-cycle portfolio choice model with realistic borrowing and investment opportunities
  - Key feature: borrowing rate > lending rate
- Introduce new assets: *income-hedging instrument* or *income-linked loans*
What we do

- Consider a life-cycle portfolio choice model with realistic borrowing and investment opportunities
  - Key feature: borrowing rate > lending rate
- Introduce new assets: *income-hedging instrument* or *income-linked loans*
  - IHI: limited liability asset with returns negatively correlated with income shock
What we do

- Consider a life-cycle portfolio choice model with realistic borrowing and investment opportunities
  - Key feature: borrowing rate > lending rate
- Introduce new assets: *income-hedging instrument* or *income-linked loans*
  - IHI: limited liability asset with returns negatively correlated with income shock
  - ILL: loan with required repayment positively correlated with income shock
What we do

- Consider a life-cycle portfolio choice model with realistic borrowing and investment opportunities
  - Key feature: borrowing rate > lending rate
- Introduce new assets: *income-hedging instrument* or *income-linked loans*
  - IHI: limited liability asset with returns negatively correlated with income shock
  - ILL: loan with required repayment positively correlated with income shock
- Look at demand for these assets over the life cycle, and predicted welfare gains that their availability would generate for households
What we find
What we find

1. Usefulness of income-linked assets depends strongly on how they are implemented:
What we find

Usefulness of income-linked assets depends strongly on how they are implemented:

- ILL generally more beneficial than IHI
What we find

1. Usefulness of income-linked assets depends strongly on how they are implemented:
   - ILL generally more beneficial than IHI
   - Correlation with income shocks
What we find

Usefulness of income-linked assets depends strongly on how they are implemented:

- ILL generally more beneficial than IHI
- Correlation with income shocks
- Volatility
What we find

1. Usefulness of income-linked assets depends strongly on how they are implemented:
   - ILL generally more beneficial than IHI
   - Correlation with income shocks
   - Volatility

2. The income-linked assets (in particular ILL) can produce non-negligible welfare gains (>1%)
What we find

1. Usefulness of income-linked assets depends strongly on how they are implemented:
   - ILL generally more beneficial than IHI
   - Correlation with income shocks
   - Volatility

2. The income-linked assets (in particular ILL) can produce non-negligible welfare gains (>1%)

3. But difficult to reduce a large fraction of the welfare costs from labor income risk with the assets we consider
“Asset Pricing”
“Asset Pricing”

- What would be...
  - $E(r)$?
  - $\sigma(r)$?
  - $corr(r, income)$?
"Asset Pricing"

- What would be...
  - $E(r)$?
  - $\sigma(r)$?
  - $corr(r, income)$?

- We remain relatively agnostic & try various assumptions
“Asset Pricing”

- What would be...
  - $E(r)$?
  - $\sigma(r)$?
  - $\text{corr}(r, \text{income})$?

- We remain relatively agnostic & try various assumptions

- Baseline assumption for $|\text{corr}(r, \text{income})|$: 0.5, based on CPS occupation-level income series (Davis et al. 2009)
“Asset Pricing”

What would be...
- \( E(r) \)?
- \( \sigma(r) \)?
- \( corr(r, income) \)?

We remain relatively agnostic & try various assumptions

Baseline assumption for \( |corr(r, income)| \): 0.5, based on CPS occupation-level income series (Davis et al. 2009)

Baseline assumption for \( E(r) \): “actuarial fairness”
- \( E(\tilde{r}_{IHI}) = r_l \) (risk-free saving rate)
- \( E(\tilde{r}_{ILL}) = r_b \) (risk-free borrowing rate)
“Asset Pricing”

- What would be...
  - \( E(r) \)?
  - \( \sigma(r) \)?
  - \( corr(r, income) \)?

- We remain relatively agnostic & try various assumptions

- Baseline assumption for \( |corr(r, income)| \): 0.5, based on CPS occupation-level income series (Davis et al. 2009)

- Baseline assumption for \( E(r) \): “actuarial fairness”
  - \( E(\tilde{r}_{IHI}) = r_l \) (risk-free saving rate)
  - \( E(\tilde{r}_{ILL}) = r_b \) (risk-free borrowing rate)

- This assumes that risks are cross-sectional (not aggregate), and in that sense stacks the deck in favor of these assets
Related Literature

- Quantitative dynamic macro models that consider welfare costs of income shocks
  - Storesletten, Telmer, Yaron (2004), Heathcote, Storesletten, Violante (2008)
- Risk-sharing and partial insurance
  - Attanasio and Davis (1996), Krueger and Perri (2006), Blundell et al. (2008)
- Optimal portfolio choice over the life cycle
  - Cocco, Gomes, Maenhout (2005), Gomes and Michaelides (2005)
  - Our model builds on Davis, Kübler, Willen (2006)
  - Close in spirit: De Jong, Driessen, Van Hemert (2008) on housing futures; Cocco and Gomes (2009) on longevity bonds
Outline

1. Two-period example
   - Goal: provide intuition for what determines demand for and welfare gains from income-linked assets
2. Life-cycle model
   - Goal: show that intuition carries over; quantitatively assess use and usefulness of assets over life cycle
3. Discussion / Conclusion
Two-Period Example: Setup

- CRRA=2 investor lives for 2 periods
- Objective: \( \max u(c_1) + E u(c_2) \)
- Has some cash-on-hand in period 1
- Receives stochastic income in period 2 with mean 8
  - \( Y_2 \in \{5.4, 8, 10.6\} \) with \( p = \{1/6, 2/3, 1/6\} \)
Two-Period Example: Setup

- CRRA=2 investor lives for 2 periods
- Objective: \( \max u(c_1) + E u(c_2) \)
- Has some cash-on-hand in period 1
- Receives stochastic income in period 2 with mean 8
  - \( Y_2 \in \{5.4, 8, 10.6\} \) with \( p = \{1/6, 2/3, 1/6\} \)
- Benchmark: Investor can...
  - save at \( r_l = 2\% \)
  - invest in equity with \( E(\tilde{r}_e) = 6\% \text{ and } \sigma(\tilde{r}_e) = 16\% \)
  - borrow at \( r_b = 8\% \)
Two-Period Example: Setup

- CRRA=2 investor lives for 2 periods
- Objective: \( \max u(c_1) + E u(c_2) \)
- Has some cash-on-hand in period 1
- Receives stochastic income in period 2 with mean 8
  - \( Y_2 \in \{5.4, 8, 10.6\} \) with \( p = \{1/6, 2/3, 1/6\} \)
- Benchmark: Investor can...
  - save at \( r_l = 2\% \)
  - invest in equity with \( E(\tilde{r}_e) = 6\% \) and \( \sigma(\tilde{r}_e) = 16\% \)
  - borrow at \( r_b = 8\% \)
- Constraints: \( b, l, e \geq 0 \)
- No default in model, so positive lower bound on \( Y_2 \) important (otherwise no borrowing possible)
Benchmark

\[ E(Y_2) = 8, \ r_l = 0.02, \ r_b = 0.08, \ E(\tilde{r}_e) = 0.06, \ \sigma_e = 0.16 \]
Income-Hedging Instrument

Now, investor can additionally invest in income-hedging instrument

- $\mathbb{E}(\tilde{r}_{IHI}) = r_l = 2\%$
- $\sigma(\tilde{r}_{IHI}) = 25\%$
- $corr(\tilde{r}_{IHI}, Y_2) = -0.5$

⇒ IHI provides some insurance benefits, but not perfect insurance
IHI: Optimal Asset Holdings

- Optimal IHI holdings nonlinear in cash-on-hand
  - Over some range of cash-on-hand, no IHI holdings
  - Relatively poor and relatively rich investors find IHI most attractive
IHI: Optimal Asset Holdings

- Optimal IHI holdings nonlinear in cash-on-hand
  - Over some range of cash-on-hand, no IHI holdings
  - Relatively poor and relatively rich investors find IHI most attractive

- Compared with benchmark:
  - Borrowing by poor investor increases
  - Equity holdings by rich decrease
What determines optimal asset holdings?

- How much a household holds of each asset depends on the \textit{risk-adjusted returns} $E_Q(\tilde{R}_i)$ of all assets
  - $E_Q(\tilde{R}_i)$ higher if $i$ pays off a lot in states of the world with high $u'(c)$
What determines optimal asset holdings?

- How much a household holds of each asset depends on the risk-adjusted returns $E_Q(\tilde{R}_i)$ of all assets
  - $E_Q(\tilde{R}_i)$ higher if $i$ pays off a lot in states of the world with high $u'(c)$
- For IHI, have that

\[
E_Q(\tilde{R}_{IHI}) > E(\tilde{R}_{IHI}) = R_l
\]
What determines optimal asset holdings?

- How much a household holds of each asset depends on the *risk-adjusted returns* $E_Q(\tilde{R}_i)$ of all assets
  - $E_Q(\tilde{R}_i)$ higher if $i$ pays off a lot in states of the world with high $u'(c)$
- For IHI, have that

$$E_Q(\tilde{R}_{IHI}) > E(\tilde{R}_{IHI}) = R_l$$

- So if household could borrow at $R_l$, would always hold IHI
What determines optimal asset holdings?

- How much a household holds of each asset depends on the risk-adjusted returns $E_Q(\tilde{R}_i)$ of all assets
  - $E_Q(\tilde{R}_i)$ higher if $i$ pays off a lot in states of the world with high $u'(c)$
- For IHI, have that
  
  $$E_Q(\tilde{R}_{IHI}) > E(\tilde{R}_{IHI}) = R_l$$

- So if household could borrow at $R_l$, would always hold IHI
- However, for households who must borrow at higher rate, only buy IHI if $E_Q(\tilde{R}_{IHI}) \geq R_b$
  - What determines whether a household borrows? Expected future consumption growth $\Rightarrow$ borrow if relatively poor today
What determines optimal asset holdings?

- How much a households holds of each asset depends on the risk-adjusted returns $E_Q(\tilde{R}_i)$ of all assets
  - $E_Q(\tilde{R}_i)$ higher if $i$ pays off a lot in states of the world with high $u'(c)$
- For IHI, have that
  \[ E_Q(\tilde{R}_{IHI}) > E(\tilde{R}_{IHI}) = R_l \]
- So if household could borrow at $R_l$, would always hold IHI
- However, for households who must borrow at higher rate, only buy IHI if $E_Q(\tilde{R}_{IHI}) \geq R_b$
  - What determines whether a household borrows? Expected future consumption growth $\Rightarrow$ borrow if relatively poor today
- And for households who save, IHI “competes” against equity $\Rightarrow$ only buy IHI if $E_Q(\tilde{R}_{IHI}) \geq E_Q(\tilde{R}_e)$
IHI: Optimal Asset Holdings

- Optimal IHI holdings nonlinear in cash-on-hand
  - Over some range of cash-on-hand, no IHI holdings
  - Relatively poor and relatively rich investors find IHI most attractive

- Compared with benchmark:
  - Borrowing by poor investor increases
  - Equity holdings by rich decrease
Bottom line:

- Whether and how extensively investor will use income-linked asset will depend on
Bottom line:

- Whether and how extensively investor will use income-linked asset will depend on
  - his financial wealth
Bottom line:

- Whether and how extensively investor will use income-linked asset will depend on
  - his financial wealth
  - his life-cycle income profile
Bottom line:

- Whether and how extensively investor will use income-linked asset will depend on
  - his financial wealth
  - his life-cycle income profile
  - the risk-adjusted returns of other investment opportunities
Bottom line:

- Whether and how extensively investor will use income-linked asset will depend on
  - his financial wealth
  - his life-cycle income profile
  - the risk-adjusted returns of other investment opportunities
- The welfare gain from an income-linked asset will depend on its (opportunity) cost
  - High for IHI
Income-Linked Loan

Now, instead, investor can additionally borrow through income-linked loan

- \( E(\tilde{r}_{ILL}) = r_b = 8\% \)
- \( \sigma(\tilde{r}_{ILL}) = 25\% \)
- \( corr(\tilde{r}_{ILL}, Y_2) = +0.5 \)
ILL: Optimal Asset Holdings

- Optimal ILL borrowing nonlinear in cash-on-hand
  - Goes to zero as cash-on-hand increases
ILL: Optimal Asset Holdings

- Optimal ILL borrowing nonlinear in cash-on-hand
  - Goes to zero as cash-on-hand increases

- Compared with benchmark:
  - ILL substitutes for unsecured borrowing
  - Over some range, additional borrowing & investment in equity
    \[
    \mathbb{E}_Q(\tilde{R}_{ILL}) = \mathbb{E}_Q(\tilde{R}_e), \text{ even though } \mathbb{E}(\tilde{R}_{ILL}) > \mathbb{E}(\tilde{R}_e)
    \]
Welfare Gains over Benchmark

- Welfare measure:
  - certainty-equivalent consumption \( \bar{c} \) s.th.
  - \( u(c_1) + E u(c_2) = 2u(\bar{c}) \)
Welfare Gains over Benchmark

- Welfare measure: certainty-equivalent consumption $\bar{c}$ s.th.
  $$u(c_1) + Eu(c_2) = 2u(\bar{c})$$
- ILL provides larger gains over wide range of cash-on-hand
Welfare Gains over Benchmark

- **Welfare measure**: certainty-equivalent consumption $\bar{c}$ s.th.
  
  $u(c_1) + EU(c_2) = 2u(\bar{c})$

- **ILL provides larger gains over wide range of cash-on-hand**

- **Intuitively**: lower (opportunity) cost

![Graph showing welfare gains from two assets](image-url)
Welfare Gains over Benchmark

- Welfare measure:
  certainty-equivalent consumption $\bar{c}$ s.th.
  $u(c_1) + Eu(c_2) = 2u(\bar{c})$

- ILL provides larger gains over wide range of cash-on-hand

- Intuitively: lower (opportunity) cost

- Welfare gain small as compared to having $Y_2=8$ for sure
  - 9.21% for c-o-h=0
  - 2.81% for c-o-h=5
  - 1.40% for c-o-h=10
Life-Cycle Model
Life-Cycle Model

- Households in partial equilibrium, live from 20 to 80, retire at 65
- Receive stochastic income $Y_t$ during working life
Life-Cycle Model

- Households in partial equilibrium, live from 20 to 80, retire at 65
- Receive stochastic income $Y_t$ during working life
- Can trade three or four financial assets.
Life-Cycle Model

- Households in partial equilibrium, live from 20 to 80, retire at 65
- Receive stochastic income $Y_t$ during working life
- Can trade three or four financial assets.
  - equity with stochastic net return $\tilde{r}_e$, 
Life-Cycle Model

- Households in partial equilibrium, live from 20 to 80, retire at 65
- Receive stochastic income $Y_t$ during working life
- Can trade three or four financial assets.
  - equity with stochastic net return $\tilde{r}_e$,
  - save at a net risk-free rate $r_l$, 

Andreas Fuster (Harvard)
Life-Cycle Model

- Households in partial equilibrium, live from 20 to 80, retire at 65
- Receive stochastic income $Y_t$ during working life
- Can trade three or four financial assets.
  - equity with stochastic net return $\tilde{r}_e$,
  - save at a net risk-free rate $r_l$,
  - engage in uncollateralized borrowing at the rate $r_b > r_l$
Life-Cycle Model

- Households in partial equilibrium, live from 20 to 80, retire at 65
- Receive stochastic income $Y_t$ during working life
- Can trade three or four financial assets.
  - equity with stochastic net return $\tilde{r}_e$,
  - save at a net risk-free rate $r_l$,
  - engage in uncollateralized borrowing at the rate $r_b > r_l$

and either
Life-Cycle Model

- Households in partial equilibrium, live from 20 to 80, retire at 65
- Receive stochastic income $Y_t$ during working life
- Can trade three or four financial assets.
  - equity with stochastic net return $\tilde{r}_e$,
  - save at a net risk-free rate $r_l$,
  - engage in uncollateralized borrowing at the rate $r_b > r_l$

and either

- invest in income-hedging instrument with stochastic net return $\tilde{r}_{IHI}$,
Life-Cycle Model

- Households in partial equilibrium, live from 20 to 80, retire at 65
- Receive stochastic income $Y_t$ during working life
- Can trade three or four financial assets.
  - equity with stochastic net return $\tilde{r}_e$,
  - save at a net risk-free rate $r_l$,
  - engage in uncollateralized borrowing at the rate $r_b > r_l$
  and either
    - invest in income-hedging instrument with stochastic net return $\tilde{r}_{IHI}$,
    or
    - borrow through income-linked loans at the stochastic rate $\tilde{r}_{ILL}$.
Life-Cycle Model

- Households in partial equilibrium, live from 20 to 80, retire at 65
- Receive stochastic income $Y_t$ during working life
- Can trade three or four financial assets.
  - equity with stochastic net return $\tilde{r}_e$,
  - save at a net risk-free rate $r_l$,
  - engage in uncollateralized borrowing at the rate $r_b > r_l$

and either
  - invest in income-hedging instrument with stochastic net return $\tilde{r}_{IHI}$,
  - or
  - borrow through income-linked loans at the stochastic rate $\tilde{r}_{ILL}$.
- No explicit limit on borrowing; have to be able to repay with prob. 1
Life-Cycle Model

- Households in partial equilibrium, live from 20 to 80, retire at 65
- Receive stochastic income $Y_t$ during working life
- Can trade three or four financial assets.
  - equity with stochastic net return $\tilde{r}_e$,
  - save at a net risk-free rate $r_l$,
  - engage in uncollateralized borrowing at the rate $r_b > r_l$
- and either
  - invest in income-hedging instrument with stochastic net return $\tilde{r}_{IHI}$,
  - or
  - borrow through income-linked loans at the stochastic rate $\tilde{r}_{ILL}$.
- No explicit limit on borrowing; have to be able to repay with prob. 1
- Short-sale constraints:
  $$e_t \geq 0, l_t \geq 0, b_t \geq 0, IHI_t \geq 0, ILL_t \geq 0$$
Life-Cycle Model

- Households in partial equilibrium, live from 20 to 80, retire at 65
- Receive stochastic income $Y_t$ during working life
- Can trade three or four financial assets.
  - equity with stochastic net return $\tilde{r}_e$,
  - save at a net risk-free rate $r_l$,
  - engage in uncollateralized borrowing at the rate $r_b > r_l$
  and either
    - invest in income-hedging instrument with stochastic net return $\tilde{r}_{IHI}$,
    - or
    - borrow through income-linked loans at the stochastic rate $\tilde{r}_{ILL}$.
- No explicit limit on borrowing; have to be able to repay with prob. 1
- Short-sale constraints:
  $$e_t \geq 0, l_t \geq 0, b_t \geq 0, IHI_t \geq 0, ILL_t \geq 0$$
- Finite-horizon dynamic program, solved computationally
Our Strategy

- Start with model that only features $e, l, b$
  - Calibrate to match wealth/income before retirement
  - Demonstrate that this model makes reasonable predictions regarding equity holdings and borrowing over the LC
  - Use this as benchmark model
Our Strategy

- Start with model that only features $e, l, b$
  - Calibrate to match wealth/income before retirement
  - Demonstrate that this model makes reasonable predictions regarding equity holdings and borrowing over the LC
  - Use this as benchmark model

- Then, add either income-hedging instrument or income-linked loan, with various assumptions about return process
  - Look at effect on asset holdings over the LC
  - Evaluate welfare gain from having access to new asset
Income Process

- Income process as standard in consumption/pf choice literature (following Gourinchas-Parker 2002, Cocco et al. 2005):

\[
\log(Y_{it}) = \tilde{y}_t = d_t + \tilde{\eta}_t + \tilde{\epsilon}_t
\]
Income Process

- Income process as standard in consumption/pf choice literature (following Gourinchas-Parker 2002, Cocco et al. 2005):
  \[ \log(Y_{it}) = \tilde{y}_t = d_t + \tilde{\eta}_t + \tilde{\epsilon}_t \]

- Deterministic component \(d_t\)
Income Process

- Income process as standard in consumption/pf choice literature (following Gourinchas-Parker 2002, Cocco et al. 2005):

  \[ \log(Y_{it}) = \tilde{y}_t = d_t + \tilde{\eta}_t + \tilde{\epsilon}_t \]

- Deterministic component \( d_t \)
- Permanent (random walk) component

  \[ \tilde{\eta}_t = \eta_{t-1} + \tilde{\eta}_t, \text{ with } \tilde{\eta}_t \sim N(-\sigma_u^2/2, \sigma_u^2) \]
Income Process

- Income process as standard in consumption/pf choice literature (following Gourinchas-Parker 2002, Cocco et al. 2005):

\[ \log(Y_{it}) = \tilde{y}_t = d_t + \tilde{\eta}_t + \tilde{\epsilon}_t \]

- Deterministic component \( d_t \)
- Permanent (random walk) component

\[ \tilde{\eta}_t = \eta_{t-1} + \tilde{u}_t, \text{ with } \tilde{u}_t \sim N(-\sigma_u^2/2, \sigma_u^2) \]

- Temporary component

\[ \tilde{\epsilon}_t \sim N(-\sigma_\varepsilon^2/2, \sigma_\varepsilon^2) \]
Income Process

- Income process as standard in consumption/pf choice literature (following Gourinchas-Parker 2002, Cocco et al. 2005):

\[
\log(Y_{it}) = \tilde{y}_t = d_t + \tilde{\eta}_t + \tilde{\varepsilon}_t
\]

- Deterministic component \(d_t\)

- Permanent (random walk) component

\[
\tilde{\eta}_t = \eta_{t-1} + \tilde{u}_t, \text{ with } \tilde{u}_t \sim N(-\sigma_u^2/2, \sigma_u^2)
\]

- Temporary component

\[
\tilde{\varepsilon}_t \sim N(-\sigma_\varepsilon^2/2, \sigma_\varepsilon^2)
\]

- Shocks are effectively bounded; no zero-income temporary shock
Income Process

- Income process as standard in consumption/pf choice literature (following Gourinchas-Parker 2002, Cocco et al. 2005):
  \[
  \log(Y_{it}) = \tilde{y}_t = d_t + \tilde{\eta}_t + \tilde{\varepsilon}_t
  \]

- Deterministic component \( d_t \)
- Permanent (random walk) component
  \[
  \tilde{\eta}_t = \eta_{t-1} + \tilde{u}_t, \text{ with } \tilde{u}_t \sim N(-\sigma_u^2/2, \sigma_u^2)
  \]
- Temporary component
  \[
  \tilde{\varepsilon}_t \sim N(-\sigma_\varepsilon^2/2, \sigma_\varepsilon^2)
  \]
- Shocks are effectively bounded; no zero-income temporary shock
- Retirement income: \( \tilde{y}_t = \log(\lambda) + d_{tR} + \eta_{tR}, t > t_R \)
Income Process

Use parameters from Cocco et al. for HS grads: $\sigma_u = 0.103$, $\sigma_\varepsilon = 0.272$, $\lambda = 0.682$. Enter at 20, retire at 65, die at 80.
Income Process

Use parameters from Cocco et al. for HS grads: $\sigma_u = 0.103$, $\sigma_\varepsilon = 0.272$, $\lambda = 0.682$. Enter at 20, retire at 65, die at 80.

Income over the Lifecycle (Thousands of 1992 USD)
Other Parameters for Benchmark

- CRRA utility with curvature $\gamma = 2$
- Taste-shifter s.th. consumption drops 10% at retirement
- Risk-free saving rate: $r_l = 0.02$
- Risk-free borrowing rate: $r_b = 0.08$ (Davis et al. 2006)
- Equity returns: $E(\tilde{r}_e) = 0.06$, $\sigma_e = 0.16$
- Discount factor: $\beta = 0.936$. Chosen to match $\frac{W}{Y} = 2.6$ of households with head aged 50 to 59 (Laibson et al. 2007)
Benchmark Results

Investment and Borrowing over the LC (means)

Borrowing & Stock Market Participation over the LC

Thousands of 1992 USD

Equity

Borrowing

% of households

Equity

Borrowing

age

age
Benchmark Results

- Successes: general pattern of borrowing and risky asset holdings (and participation) over the LC
- Failures: no bond holdings, and almost no borrowing late in life
Income-Hedging Instrument

Add IHI to benchmark setting. Parameters:

- $r_l = 0.02$, $r_b = 0.08$, $E(\tilde{r}_e) = 0.06$, $\sigma_e = 0.16$
Income-Hedging Instrument

Add IHI to benchmark setting. Parameters:

- $r_l = 0.02$, $r_b = 0.08$, $E(\tilde{r}_e) = 0.06$, $\sigma_e = 0.16$
- $E(\tilde{r}_{IHI}) = r_l = 0.02$
Income-Hedging Instrument

Add IHI to benchmark setting. Parameters:

- $r_l = 0.02$, $r_b = 0.08$, $\mathbb{E}(\tilde{r}_e) = 0.06$, $\sigma_e = 0.16$
- $\mathbb{E}(\tilde{r}_{IHI}) = r_l = 0.02$
- $corr(\tilde{r}_{IHI}, \tilde{u}) = \{-0.25, -0.5, -0.75, -1\}$
  - Return negatively correlated with permanent shock to income
Income-Hedging Instrument

Add IHI to benchmark setting. Parameters:

- $r_l = 0.02$, $r_b = 0.08$, $\mathbb{E}(\tilde{r}_e) = 0.06$, $\sigma_e = 0.16$
- $\mathbb{E}(\tilde{r}_{IHI}) = r_l = 0.02$
- $\text{corr}(\tilde{r}_{IHI}, \tilde{u}) = \{-0.25, -0.5, -0.75, -1\}$
  - Return negatively correlated with permanent shock to income
- $\sigma(\tilde{r}_{IHI}) = \{0.3, 0.5\}$
Add IHI to benchmark setting. Parameters:

- $r_l = 0.02$, $r_b = 0.08$, $E(\tilde{r}_e) = 0.06$, $\sigma_e = 0.16$
- $E(\tilde{r}_{IHI}) = r_l = 0.02$
- $corr(\tilde{r}_{IHI}, \tilde{u}) = \{-0.25, -0.5, -0.75, -1\}$
  - Return negatively correlated with permanent shock to income
- $\sigma(\tilde{r}_{IHI}) = \{0.3, 0.5\}$

Focus on welfare gains from introducing IHI (in paper, look at LC profiles in detail)
Welfare Gains from IHI

\[
\begin{array}{c}
\text{Gain in CE consumption (% over Benchmark)} \\
\text{Correlation}
\end{array}
\]

- \( \sigma = 0.5 \)
- \( \sigma = 0.3 \)
Welfare Gains from IHI

Compare to
- gain from reducing permanent income shock variance by 25%: 3.5%
- gain from eliminating all income risk: 16.4%
IHI: Conclusions

- Unless returns very highly correlated with income shock and very volatile, IHI not very useful
IHI: Conclusions

- Unless returns very highly correlated with income shock and very volatile, IHI not very useful
- Too “expensive” for young households, who would benefit most from hedge
IHI: Conclusions

- Unless returns very highly correlated with income shock and very volatile, IHI not very useful
- Too “expensive” for young households, who would benefit most from hedge
- Richer (older) households hold more of IHI, but at expense of equity
IHI: Conclusions

- Unless returns very highly correlated with income shock and very volatile, IHI not very useful
- Too “expensive” for young households, who would benefit most from hedge
- Richer (older) households hold more of IHI, but at expense of equity
- Welfare gains convex in $\text{corr}(\tilde{r}_{IHI}, \tilde{u})$ (and even in $\text{corr}^2$)
Income-Linked Loans

Now instead add ILL to benchmark setting. Parameters:

- $r_l = 0.02$, $r_b = 0.08$, $E(\tilde{r}_e) = 0.06$, $\sigma_e = 0.16$
Income-Linked Loans

Now instead add ILL to benchmark setting. Parameters:

- \( r_l = 0.02, \ r_b = 0.08, \ E(\tilde{r}_e) = 0.06, \ \sigma_e = 0.16 \)
- \( E(\tilde{r}_{ILL}) = r_b = 0.08 \)
Income-Linked Loans

Now instead add ILL to benchmark setting. Parameters:

- \( r_l = 0.02, \ r_b = 0.08, \ E(\tilde{r}_e) = 0.06, \ \sigma_e = 0.16 \)
- \( E(\tilde{r}_{ILL}) = r_b = 0.08 \)
- \( corr(\tilde{r}_{ILL}, \tilde{u}) = \{0.25, 0.5, 0.75, 1\} \)
  - Rate positively correlated with permanent shock to income
Income-Linked Loans

Now instead add ILL to benchmark setting. Parameters:

- \( r_l = 0.02, r_b = 0.08, \ E(\tilde{r}_e) = 0.06, \sigma_e = 0.16 \)
- \( E(\tilde{r}_{ILL}) = r_b = 0.08 \)
- \( corr(\tilde{r}_{ILL}, \tilde{u}) = \{0.25, 0.5, 0.75, 1\} \)
  - Rate positively correlated with permanent shock to income
- \( \sigma(\tilde{r}_{ILL}) = \{0.3, 0.5\} \)
Welfare Gains from ILL

\[ \sigma = 0.3 \]

\[ \sigma = 0.5 \]
ILL: Conclusions

- ILL offer more potential for welfare gains than IHI
ILL: Conclusions

- ILL offer more potential for welfare gains than IHI
  - Even for relatively moderate $\rho$, $\sigma$
ILL: Conclusions

- ILL offer more potential for welfare gains than IHI
  - Even for relatively moderate $\rho, \sigma$
- Young households (who would borrow anyway) would use it extensively and benefit from improved insurance
ILL: Conclusions

- ILL offer more potential for welfare gains than IHI
  - Even for relatively moderate $\rho, \sigma$
- Young households (who would borrow anyway) would use it extensively and benefit from improved insurance
- They invest part of their ILL borrowing in high-return equity
ILL: Conclusions

- ILL offer more potential for welfare gains than IHI
  - Even for relatively moderate $\rho$, $\sigma$
- Young households (who would borrow anyway) would use it extensively and benefit from improved insurance
- They invest part of their ILL borrowing in high-return equity
- Yet, still far from hypothetical welfare gain achieved by reducing income risk to zero
Robustness – Alternative Investment Option

- Our model does not generate enough (any) riskfree asset holdings
Robustness – Alternative Investment Option

- Our model does not generate enough (any) riskfree asset holdings
- Consequence: may understate benefits from IHI; overstate benefits from ILL
Robustness – Alternative Investment Option

- Our model does not generate enough (any) riskfree asset holdings
- Consequence: may understate benefits from IHI; overstate benefits from ILL
- Check: version of the model where agent can only invest in 50/50 stocks-bonds fund (expected return $(E(\tilde{r}_e) + r_l)/2$, st. dev. $0.5\sigma_e$)
Robustness – Alternative Investment Option

- Our model does not generate enough (any) riskfree asset holdings
- Consequence: may understate benefits from IHI; overstate benefits from ILL
- Check: version of the model where agent can only invest in 50/50 stocks-bonds fund (expected return \( \frac{E(\tilde{r}_e) + r_l}{2} \), st. dev. \( 0.5\sigma_e \))
- Set \( \beta = 0.947 \) to match \( \frac{W}{Y} \)
Robustness – Alternative Investment Option

- Our model does not generate enough (any) riskfree asset holdings
- Consequence: may understate benefits from IHI; overstate benefits from ILL
- Check: version of the model where agent can only invest in 50/50 stocks-bonds fund (expected return \( \frac{\mathbb{E}(\tilde{r}_e) + r_l}{2} \), st. dev. 0.5\( \sigma_e \))
- Set \( \beta = 0.947 \) to match \( \frac{W}{Y} \)
- Gain from IHI with \( \rho = -0.5 \) and \( \sigma = 0.5 \) is now 0.33% instead of 0.04%
Robustness – Alternative Investment Option

- Our model does not generate enough (any) riskfree asset holdings
- Consequence: may understate benefits from IHI; overstate benefits from ILL
- Check: version of the model where agent can only invest in 50/50 stocks-bonds fund (expected return \( (E(\tilde{r}_e) + r_l)/2 \), st. dev. \( 0.5\sigma_e \))
- Set \( \beta = 0.947 \) to match \( W/Y \)
- Gain from IHI with \( \rho = -0.5 \) and \( \sigma = 0.5 \) is now 0.33% instead of 0.04%
- Gain from ILL with \( \rho = +0.5 \) and \( \sigma = 0.5 \) is now 0.85% instead of 1.36%
Robustness – Alternative Investment Option

- Our model does not generate enough (any) riskfree asset holdings
- Consequence: may understate benefits from IHI; overstate benefits from ILL
- Check: version of the model where agent can only invest in 50/50 stocks-bonds fund (expected return $(E(\tilde{r}_e) + r_l)/2$, st. dev. $0.5\sigma_e$)
- Set $\beta = 0.947$ to match $\frac{W}{Y}$
- Gain from IHI with $\rho = -0.5$ and $\sigma = 0.5$ is now 0.33% instead of 0.04%
- Gain from ILL with $\rho = +0.5$ and $\sigma = 0.5$ is now 0.85% instead of 1.36%

$\Rightarrow$ ILL still generates larger welfare gain than IHI, but difference smaller
Robustness – Borrowing Cost

- We assume an interest rate wedge between borrowing and lending of 6%, based on Davis et al. (2006)
  - Adjust for tax considerations and charge-offs ⇒ remaining wedge due to transaction costs etc.
Robustness – Borrowing Cost

- We assume an interest rate wedge between borrowing and lending of 6%, based on Davis et al. (2006)
  - Adjust for tax considerations and charge-offs ⇒ remaining wedge due to transaction costs etc.
- What happens if households had access to cheaper borrowing, at 5%?
Robustness – Borrowing Cost

- We assume an interest rate wedge between borrowing and lending of 6%, based on Davis et al. (2006)
  - Adjust for tax considerations and charge-offs ⇒ remaining wedge due to transaction costs etc.
- What happens if households had access to cheaper borrowing, at 5%?
- Welfare gain from baseline IHI: 0.8%
Robustness – Borrowing Cost

- We assume an interest rate wedge between borrowing and lending of 6%, based on Davis et al. (2006)
  - Adjust for tax considerations and charge-offs ⇒ remaining wedge due to transaction costs etc.
- What happens if households had access to cheaper borrowing, at 5%?
- Welfare gain from baseline IHI: 0.8%
- Welfare gain from baseline ILL with \( \mathbb{E}(\tilde{r}_{ILL}) = \tilde{r}_b = 0.05 \): 3%
Robustness – Borrowing Cost

- We assume an interest rate wedge between borrowing and lending of 6%, based on Davis et al. (2006)
  - Adjust for tax considerations and charge-offs ⇒ remaining wedge due to transaction costs etc.
- What happens if households had access to cheaper borrowing, at 5%?
- Welfare gain from baseline IHI: 0.8%
- Welfare gain from baseline ILL with $E(\tilde{r}_{ILL}) = r_b = 0.05$: 3%
- Welfare gain from baseline ILL but with $E(\tilde{r}_{ILL}) = 0.08 > r_b$: 0.5%
Robustness – Borrowing Cost

- We assume an interest rate wedge between borrowing and lending of 6%, based on Davis et al. (2006)
  - Adjust for tax considerations and charge-offs ⇒ remaining wedge due to transaction costs etc.
- What happens if households had access to cheaper borrowing, at 5%?
  - Welfare gain from baseline IHI: 0.8%
  - Welfare gain from baseline ILL with $E(\tilde{r}_{ILL}) = r_b = 0.05$: 3%
  - Welfare gain from baseline ILL but with $E(\tilde{r}_{ILL}) = 0.08 > r_b$: 0.5%
- Thus, if households had access to borrowing at a cheaper rate than what they would pay on the ILL, result that ILL generates larger gains than IHI may be reversed
Robustness – Preferences

- With higher risk aversion, welfare gains increase
Robustness – Preferences

- With higher risk aversion, welfare gains increase
- Try $\gamma = 3$, $\beta = 0.92$ (and equity as earlier, not 50/50)
Robustness – Preferences

- With higher risk aversion, welfare gains increase
- Try $\gamma = 3$, $\beta = 0.92$ (and equity as earlier, not 50/50)
- Gain from IHI with $\rho = -0.5$ and $\sigma = 0.5$ is now 0.42% ($\gamma = 2$: 0.04%)
Robustness – Preferences

- With higher risk aversion, welfare gains increase
- Try $\gamma = 3$, $\beta = 0.92$ (and equity as earlier, not 50/50)
- Gain from IHI with $\rho = -0.5$ and $\sigma = 0.5$ is now 0.42% ($\gamma = 2$: 0.04%)
- Gain from ILL with $\rho = +0.5$ and $\sigma = 0.5$ is now 2.42% ($\gamma = 2$: 1.36%)
Robustness – Preferences

- With higher risk aversion, welfare gains increase
- Try $\gamma = 3$, $\beta = 0.92$ (and equity as earlier, not 50/50)
- Gain from IHI with $\rho = -0.5$ and $\sigma = 0.5$ is now 0.42% ($\gamma = 2$: 0.04%)
- Gain from ILL with $\rho = +0.5$ and $\sigma = 0.5$ is now 2.42% ($\gamma = 2$: 1.36%)

$\Rightarrow$ Gains significantly larger with higher risk aversion (as is the welfare cost from life cycle income shocks in the benchmark: 24.5%)
Usefulness of income-linked assets depends strongly on how they are implemented:
Usefulness of income-linked assets depends strongly on how they are implemented:

- ILL vs. IHI
Usefulness of income-linked assets depends strongly on how they are implemented:

- ILL vs. IHI
- Correlation with income shocks
Usefulness of income-linked assets depends strongly on how they are implemented:

- ILL vs. IHI
- Correlation with income shocks
- Volatility
Discussion & Conclusion

1. Usefulness of income-linked assets depends strongly on how they are implemented:
   - ILL vs. IHI
   - Correlation with income shocks
   - Volatility

2. The income-linked assets (in particular ILL) can produce non-negligible welfare gains
   - Baseline welfare gains with $|\rho| = 0.5$, $\sigma = 0.5$: $\text{IHI} \approx 0$, $\text{ILL} \approx 1.4\%$ (US$400/year)
Usefulness of income-linked assets depends strongly on how they are implemented:

- ILL vs. IHI
- Correlation with income shocks
- Volatility

The income-linked assets (in particular ILL) can produce non-negligible welfare gains

- Baseline welfare gains with $|\rho| = 0.5$, $\sigma = 0.5$: $\text{IHI} \approx 0$, $\text{ILL} \approx 1.4\%$ (US$400/year)
- Attractiveness of alternative investment options matters for relative gains from ILL vs. IHI
Discussion & Conclusion

1. Usefulness of income-linked assets depends strongly on how they are implemented:
   - ILL vs. IHI
   - Correlation with income shocks
   - Volatility

2. The income-linked assets (in particular ILL) can produce non-negligible welfare gains
   - Baseline welfare gains with $|\rho| = 0.5, \sigma = 0.5$: IHI ≈ 0, ILL ≈ 1.4% (US$400/year)
   - Attractiveness of alternative investment options matters for relative gains from ILL vs. IHI

3. But difficult to reduce a large fraction of the welfare costs from labor income risk with the assets we have considered
Discussion & Conclusion

1. Usefulness of income-linked assets depends strongly on how they are implemented:
   - ILL vs. IHI
   - Correlation with income shocks
   - Volatility

2. The income-linked assets (in particular ILL) can produce non-negligible welfare gains
   - Baseline welfare gains with $|\rho| = 0.5, \sigma = 0.5$: IHI $\approx 0$, ILL $\approx 1.4\%$ (US$400/year)
   - Attractiveness of alternative investment options matters for relative gains from ILL vs. IHI

3. But difficult to reduce a large fraction of the welfare costs from labor income risk with the assets we have considered
   - Unless they were highly correlated with shocks to permanent income...
   - or households had access to cheap borrowing
Usefulness of income-linked assets depends strongly on how they are implemented:

- ILL vs. IHI
- Correlation with income shocks
- Volatility

The income-linked assets (in particular ILL) can produce non-negligible welfare gains

- Baseline welfare gains with $|\rho| = 0.5$, $\sigma = 0.5$: IHI $\approx 0$, ILL $\approx 1.4\%$ (US$400/year)
- Attractiveness of alternative investment options matters for relative gains from ILL vs. IHI

But difficult to reduce a large fraction of the welfare costs from labor income risk with the assets we have considered

- Unless they were highly correlated with shocks to permanent income...
- or households had access to cheap borrowing
Using a model with realistic portfolio constraints & opportunity costs is key to evaluating new assets.
Using a model with realistic portfolio constraints & opportunity costs is key to evaluating new assets.

If instead assumed $r_b = r_l = E(\tilde{r}_{ILA}) = 0.02$, model would predict:
- IHI and ILL equivalent
- $\sigma$ does not matter
- even with $|\rho| = 0.5$, welfare gain > 4%
Discussion & Conclusion

THE END – THANK YOU!
Appendix: Welfare Measures

- Through large number of simulations of stochastic variables, find ex-ante lifetime expected utility $\bar{U}$
- Then, compute certainty equivalent consumption $\bar{c}$: constant level of consumption such that lifetime utility equals $\bar{U}$
Appendix: Welfare Measures

- Through large number of simulations of stochastic variables, find ex-ante lifetime expected utility $\bar{U}$
- Then, compute certainty equivalent consumption $\bar{c}$: constant level of consumption such that lifetime utility equals $\bar{U}$
- For benchmark parameters, eliminating income shocks would raise $\bar{c}$ by 16.4%
Appendix: Welfare Measures

Through large number of simulations of stochastic variables, find ex-ante lifetime expected utility $\bar{U}$

Then, compute certainty equivalent consumption $\bar{c}$: constant level of consumption such that lifetime utility equals $\bar{U}$

For benchmark parameters, eliminating income shocks would raise $\bar{c}$ by 16.4%

Alternative measure to assess effect of new assets: coefficient of partial insurance against shocks (Kaplan and Violante, 2008):

$$\phi^u_i = 1 - \frac{cov(\Delta c_{it}, u_{it})}{var(u_{it})}$$
Appendix: Welfare Measures

Through large number of simulations of stochastic variables, find ex-ante lifetime expected utility $\bar{U}$

Then, compute certainty equivalent consumption $\bar{c}$: constant level of consumption such that lifetime utility equals $\bar{U}$

For benchmark parameters, eliminating income shocks would raise $\bar{c}$ by 16.4%

Alternative measure to assess effect of new assets: coefficient of partial insurance against shocks (Kaplan and Violante, 2008):

$$\phi_t^u = 1 - \frac{cov(\Delta c_{it}, u_{it})}{var(u_{it})}$$

The lower this coefficient, the more an income shock translates into consumption changes. $\phi = 1$: perfect insurance.

In benchmark, $\bar{\phi}^u = 0.08$ and $\bar{\phi}^\varepsilon = 0.9$: easy to insure against transitory shocks, hard to insure against permanent ones.
Partial Insurance Coefficients with IHI

\[ \rho = -0.5 \]

\[ \rho = -0.75 \]

\[ \rho = -1 \]

\[ \text{Benchmark} \]

Insurance coefficients (against perm. shocks) during working life

\begin{figure}
\centering
\includegraphics[width=\textwidth]{chart}
\end{figure}
Partial Insurance Coefficients with ILL

![Graph showing insurance coefficients against permanent shocks during working life for different values of $\rho$. The graph includes lines for $\rho = 1$, $\rho = 0.75$, and $\rho = 0.5$, each representing insurance coefficients over age. The Benchmark line shows a steady increase.]
Computational Solution

- Closely follow Davis, Kübler, Willen (2006)
Computational Solution

- Closely follow Davis, Kübler, Willen (2006)
- Finite-horizon dynamic program, solved by backward induction
Computational Solution

- Closely follow Davis, Kübler, Willen (2006)
- Finite-horizon dynamic program, solved by backward induction
- Get rid of one state by exploiting scale-independence and dividing everything by permanent income
Computational Solution

- Closely follow Davis, Kübler, Willen (2006)
- Finite-horizon dynamic program, solved by backward induction
- Get rid of one state by exploiting scale-independence and dividing everything by permanent income
- Thus, state variables: normalized cash-on-hand \( (x_t) \) and age \( (t) \)
Computational Solution

- Closely follow Davis, Kübler, Willen (2006)
- Finite-horizon dynamic program, solved by backward induction
- Get rid of one state by exploiting scale-independence and dividing everything by permanent income
- Thus, state variables: normalized cash-on-hand ($x_t$) and age ($t$)
- 3 or 4 asset holding decisions, with short-sale constraints on all of them
Computational Solution

- Closely follow Davis, Kübler, Willen (2006)
- Finite-horizon dynamic program, solved by backward induction
- Get rid of one state by exploiting scale-independence and dividing everything by permanent income
- Thus, state variables: normalized cash-on-hand \((x_t)\) and age \((t)\)
- 3 or 4 asset holding decisions, with short-sale constraints on all of them
- Solve by policy-function iteration, as FOCs necessary and sufficient
Computational Solution

- Use Garcia-Zangwill (1981) “trick” to transform Kuhn-Tucker conditions into system of nonlinear equations. E.g. for $e_t$: 
Computational Solution

- Use Garcia-Zangwill (1981) “trick” to transform Kuhn-Tucker conditions into system of nonlinear equations. E.g. for $e_t$:

\[
\begin{align*}
\frac{c_t}{u'(x_t + b_t - l_t - e_t) - \beta E[(1 + \tilde{r}_e)u'(c_{t+1})] - \mu_{e,t} = 0} \\
e_t &\geq 0, \quad \mu_{e,t} \geq 0, \quad e_t \mu_{e,t} = 0
\end{align*}
\]
Computational Solution

- Use Garcia-Zangwill (1981) “trick” to transform Kuhn-Tucker conditions into system of nonlinear equations. E.g. for $e_t$:

$$u'(c_t + b_t - l_t - e_t) - \beta E[(1 + \tilde{r}_e)u'(c_{t+1})] - \mu_{e,t} = 0$$

$$e_t \geq 0, \quad \mu_{e,t} \geq 0, \quad e_t \mu_{e,t} = 0$$

Define $e_t = (\max\{0, \lambda_e\})^\kappa$ and $\mu_{e,t} = \max\{0, -\lambda_e\})^\kappa$. 
Computational Solution

- Use Garcia-Zangwill (1981) “trick” to transform Kuhn-Tucker conditions into system of nonlinear equations. E.g. for $e_t$:

$$u'(x_t + b_t - l_t - e_t) - \beta \mathbb{E}[(1 + \tilde{r}_e)u'(c_{t+1})] - \mu_{e,t} = 0$$

$$e_t \geq 0, \mu_{e,t} \geq 0, e_t \mu_{e,t} = 0$$

Define $e_t = (\max\{0, \lambda_e\})^\kappa$ and $\mu_{e,t} = \max(\{0, -\lambda_e\})^\kappa$.

Then, $(\max\{0, \lambda_e\})^\kappa \geq 0$, $(\max\{0, -\lambda_e\})^\kappa \geq 0$, and

$(\max\{0, \lambda_e\})^\kappa \cdot (\max\{0, -\lambda_e\})^\kappa = 0$
Computational Solution

- Use Garcia-Zangwill (1981) “trick” to transform Kuhn-Tucker conditions into system of nonlinear equations. E.g. for $e_t$:

\[
u'(x_t + b_t - l_t - e_t) - \beta E[(1 + \tilde{r}_e)u'(c_{t+1})] - \mu_{e,t} = 0
\]
\[e_t \geq 0, \quad \mu_{e,t} \geq 0, \quad e_t \mu_{e,t} = 0\]

Define $e_t = (\max\{0, \lambda_e\})^\kappa$ and $\mu_{e,t} = \max\{0, -\lambda_e\}^\kappa$.

Then, $(\max\{0, \lambda_e\})^\kappa \geq 0$, $(\max\{0, -\lambda_e\})^\kappa \geq 0$, and $(\max\{0, \lambda_e\})^\kappa \cdot (\max\{0, -\lambda_e\})^\kappa = 0$

$\Rightarrow$ can solve equation that is differentiable of degree $\kappa - 1$ for $\lambda_e$ (and similarly for other assets and multipliers).
Computational Solution

- Discretization: 2 nodes for income shocks, 3 for equity, 4 for income-linked assets
  - Results not sensitive to adding more nodes, as long as lowest income shock “not too small”
Computational Solution

- Discretization: 2 nodes for income shocks, 3 for equity, 4 for income-linked assets
  - Results not sensitive to adding more nodes, as long as lowest income shock “not too small”
- Set bounds of grid for cash-on-hand s.th. never move out of bounds in simulations
Computational Solution

- Discretization: 2 nodes for income shocks, 3 for equity, 4 for income-linked assets
  - Results not sensitive to adding more nodes, as long as lowest income shock “not too small”

- Set bounds of grid for cash-on-hand s.th. never move out of bounds in simulations

- Denser grid at low values of cash-on-hand
Computational Solution

- Discretization: 2 nodes for income shocks, 3 for equity, 4 for income-linked assets
  - Results not sensitive to adding more nodes, as long as lowest income shock "not too small"
- Set bounds of grid for cash-on-hand s.th. never move out of bounds in simulations
- Denser grid at low values of cash-on-hand
- Solve in Matlab, using dogleg algorithm by H.B. Nielsen
Computational Solution

- Discretization: 2 nodes for income shocks, 3 for equity, 4 for income-linked assets
  - Results not sensitive to adding more nodes, as long as lowest income shock “not too small”
- Set bounds of grid for cash-on-hand s.th. never move out of bounds in simulations
- Denser grid at low values of cash-on-hand
- Solve in Matlab, using dogleg algorithm by H.B. Nielsen
- Average consumption-equivalent EE error of order $10^{-6}$
Optimal Asset Holdings with IHI

Equity

Borrowing

No IHI

Asset Holdings and Borrowing over the LC (means)
Optimal Asset Holdings with IHI

Asset Holdings and Borrowing over the LC (means)

Equity

Borrowing

\( \rho = -0.5 \)
Optimal Asset Holdings with IHI

Asset Holdings and Borrowing over the LC (means)

Equity

Borrowing

$\rho = -0.75$

Thousands of 1992 USD

20 30 40 50 60 70 80

-60 -40 -20 0 20 40 60

age
Optimal Asset Holdings with IHI

![Graph showing asset holdings and borrowing over the lifespan (LC means). The graph indicates the relationship between equity, IHI, and borrowing over time, with a negative correlation indicated by \( \rho = -1 \).]
Optimal Asset Holdings with ILL

Asset Holdings and Borrowing over the LC (means)

Thousands of 1992 USD

Equity

Borrowing

No ILL

age

Back
Optimal Asset Holdings with ILL

Asset Holdings and Borrowing over the LC (means)

Thousands of 1992 USD

Equity

Borrowing

ILL

\( \rho = 0.5 \)
Optimal Asset Holdings with ILL

Income-Linked Assets

Equity

Borrowing

\( \rho = 0.75 \)

Thousands of 1992 USD

age

Back

Andreas Fuster (Harvard)
Optimal Asset Holdings with ILL

Asset Holdings and Borrowing over the LC (means)

Thousands of 1992 USD age

Equity

Borrowing

\( \rho = 1 \)