Housing, Adjustment Costs, and Endogenous Risk Aversion

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Most macro models use the "CRRA" utility function:

$$u(c) = \frac{c^{1-\rho}}{1-\rho}$$

ρ is the parameter governing the curvature of the utility function

The household's problem:

The household's expected lifetime utility is given by:

$$(1) \quad \tilde{U} = E\int_0^{\infty} e^{-\delta t} U(H_t, C_t)\,dt$$

H_t = stock of housing

C_t = nondurable consumption (numeraire)

$$U(C,H) = \frac{\left(C^{\alpha} + \gamma H^{\alpha}\right)^{\frac{a}{\alpha}}}{a} \qquad a = 1 - \rho$$

$$W_t = H_t + \underline{X}_t \underline{\ell} + B_t$$

$\underline{X}_t =$ (1xn) vector of amounts held of the risky assets financial assets

$\underline{\ell} =$ (nx1) vector of ones.

$B_t =$ amount held of riskless asset

In other papers, I have imposed a borrowing, or collateral constraint that says that the household cannot borrow more than the value of the house:

$$-H_t \leq B_t$$

In this paper, however, there is no constraint on the holding of the riskless asset; B can be either positive or negative.

Wealth evolves according to:

$$dW_t = \left(H_0\mu_H + \underline{X}_t\underline{\mu} - C_t\right)dt + \underline{X}_t d\underline{\omega}_{Ft} + H_0 d\omega_{Ht}$$

when the house is not sold.

At the instant the house is sold, the Household pays an adjustment cost equal to λH . The adjustment costs is “lumpy” in the sense that it is nonconvex in the size of the adjustment.

The Bellman equation is:

$$V(H_0, W_0) = \sup_{\underline{X}_s, C_s, \tau} E\left[\int_0^{\tau} e^{-\delta s} u(H_0, C_s)\, ds + e^{-\delta\tau} V(H_\tau, W_\tau)\right]$$

where τ is the next stopping time (that is, the next time that the house is sold.

Problem can be stated in terms of one state variable, instead of two, with a change of variables. The state variable becomes

$$y = \frac{W}{H} - \lambda$$

Asset holdings and nondurable consumption are also stated as a ratio to H

$$\underline{x} = \frac{\underline{X}}{H}$$

$$c = \frac{C}{H}$$

$$U(C,H) = \frac{\left(C^{\alpha} + \gamma H^{\alpha}\right)^{\frac{a}{\alpha}}}{a}$$

$$= H^{a} u(c)$$

$$u(c) = \frac{\left[c^{\alpha} + \gamma\right]^{\frac{a}{\alpha}}}{a}$$

The Bellman equation is:

$$V(H_0, W_0) = \sup_{\underline{X}_s, C_s, \tau} E\left[\int_0^{\tau} e^{-\delta s} U(H_0, C_s)\, ds + e^{-\delta\tau} V(H_\tau, W_\tau)\right]$$

With the change of variables, the Bellman equation can be written as:

$$h(y_0) = \sup_{\underline{x}_s, c_s, \tau} E\left[\int_0^{\tau} e^{-\delta s} u(c_s)\, ds + e^{-\delta\tau} h(y_t)\right]$$

where $h(y) = H^{-a} V(H, W)$

The first order conditions imply:

the marginal utility of consumption is equal to the marginal value of wealth:

$$\frac{\partial u}{\partial c} = h'(y)$$

nondurable consumption

the vector of risky asset holdings is:

$$\underline{x} = \left[\frac{-h'(y)}{h''(y)}\right]\Sigma^{-1}\underline{\mu}$$

portfolio allocation

Relative risk aversion $= \dfrac{-h'(y)}{h''(y)y}$

risk aversion

The solution to the problem consists of

$$y_1 < y^* < y_2$$

$$h(y_1) = y_1^a M \qquad h(y_2) = y_2^a M$$

$$h'(y_1) = a y_1^{a-1} M \qquad h'(y_2) = a y_2^{a-1} M$$

$$h(y) = y^a M \qquad y < y_1 \quad \text{and} \quad y > y_2$$

$$M \equiv \sup_y (y+\lambda)^{-a} h(y)$$

$h(y)$ satisfies the differential equation

$$0 = \frac{([c(y_t)]^{\alpha}+\gamma)^{\frac{a}{\alpha}}}{a} - \delta h(y_t) + h'(y_t)[r(y_t+\lambda-1) - c(y_t)] - \frac{1}{2}\frac{(h'(y_t))^2}{h''(y_t)}\frac{\mu^2}{\sigma^2}$$

for $y_1 < y < y_2$

parameter values

$$\rho = 2$$

$$a = 1 - \rho = -1$$

$$\alpha = -8$$

$$\mu = 0.059$$

$$\sigma = .22$$

$$r_f = .01$$

$$\delta = .01$$

$$\lambda = .05$$

Plot of value function: h(y)

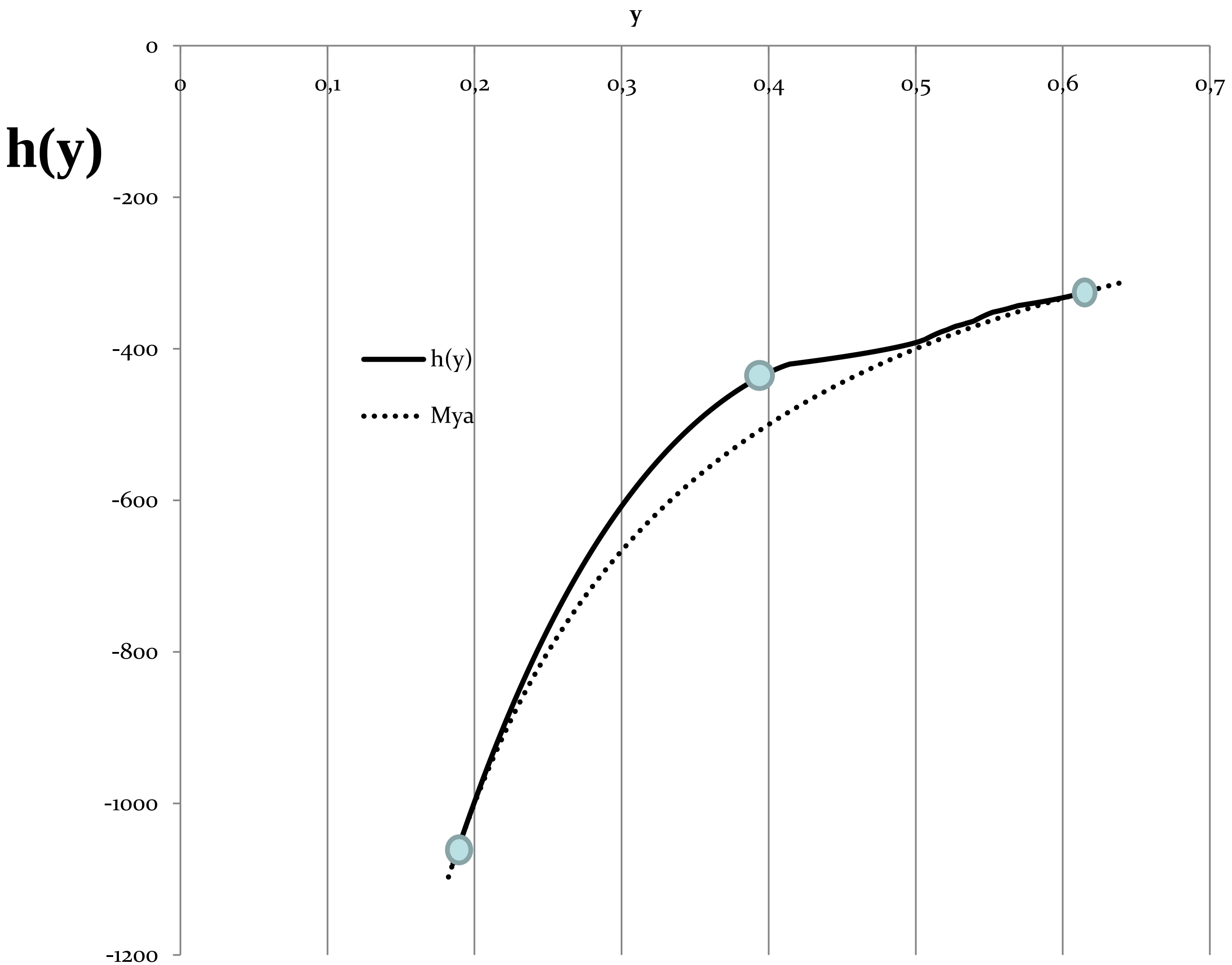
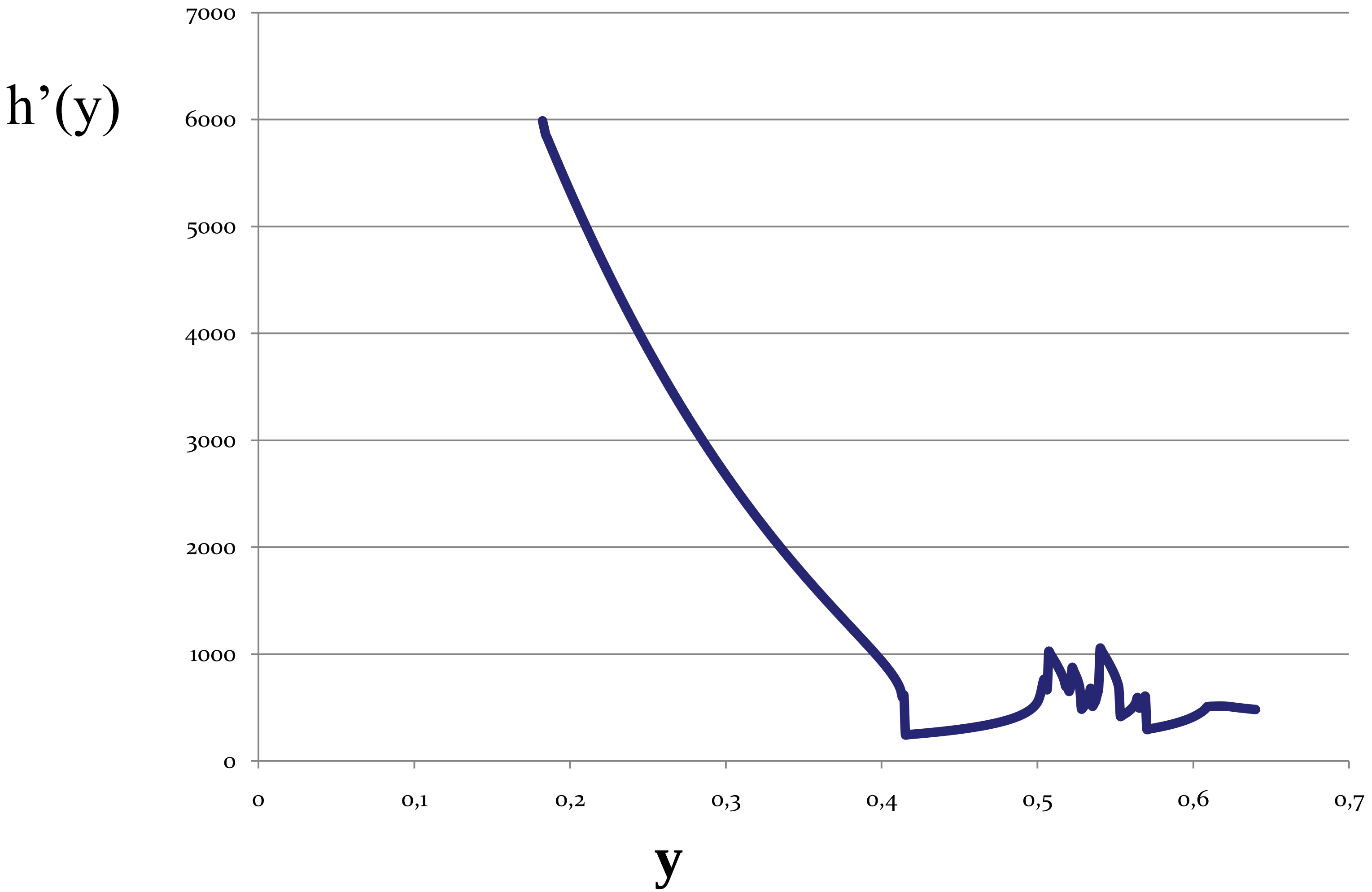
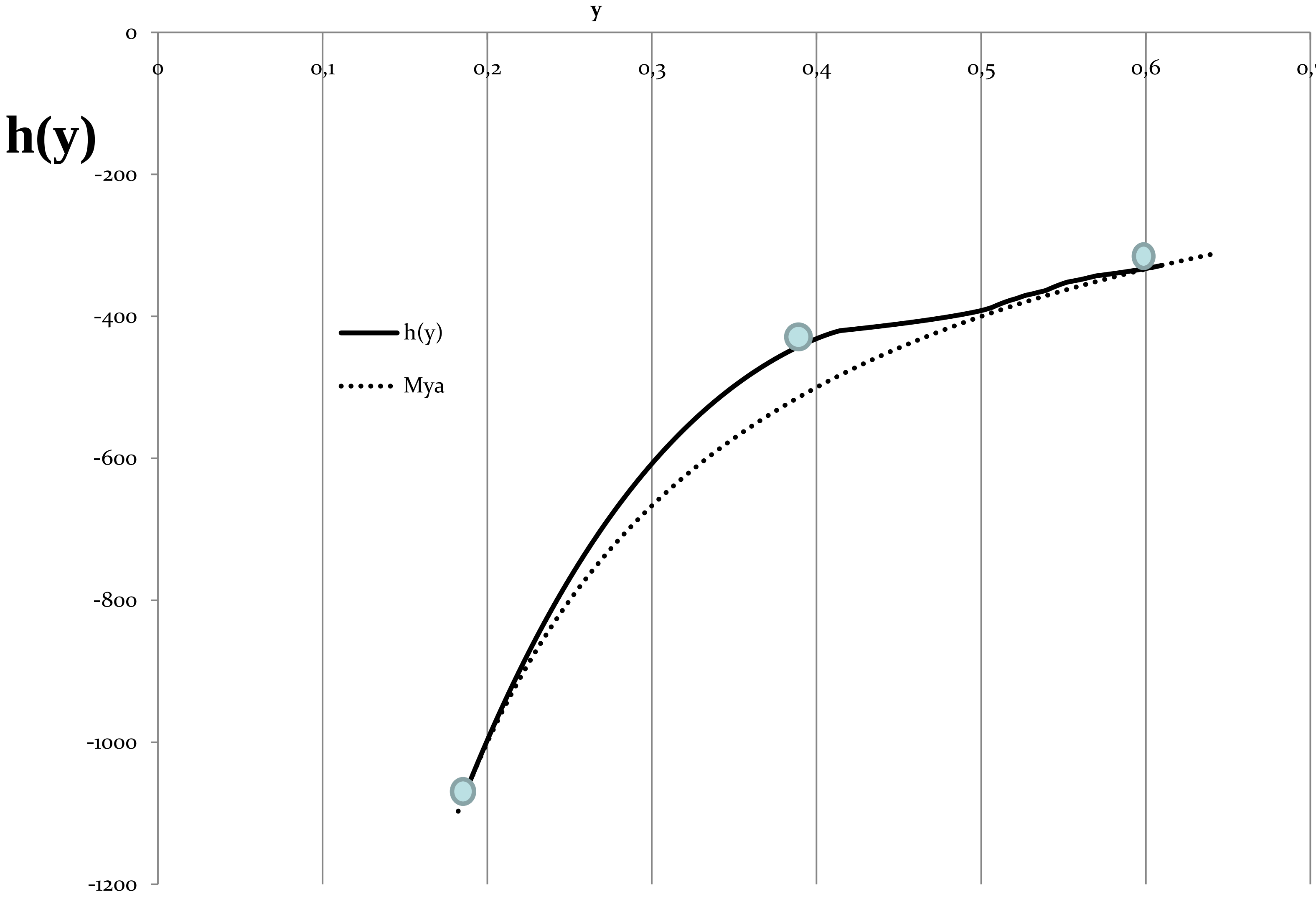


Figure 2: plot of h'(y)



Plot of value function: h(y)



RRA 1.24 2.34 7.44