Housing, Adjustment Costs, and Endogenous Risk Aversion

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The utility function known as “constant relative risk aversion” (CRRA) is generally considered more plausible than specifications that imply certainty equivalence or constant absolute risk aversion, and for this reason has become the dominant specification of one-period utility in both macro and finance models. The identification of the power utility specification, for example, the utility function

$$u(c_t) = \frac{c_t^{1-\rho}}{1-\rho}$$

with constant relative risk aversion is based on Robert Merton’s result in finance models that the functional form of the value function (as a function of wealth) is the same as the function form of the utility function (as a function of consumption). Therefore, risk aversion, appropriately defined as the curvature of the value function, coincides with the curvature of the utility function, which for the power utility specification is given by the parameter rho . In these early finance models, utility was assumed to depend on a single, costlessly adjustable good. Since the power utility function, in conjunction with the assumption of a costlessly adjustable good, implies that the household’s degree of relative risk aversion is constant and equal to the curvature of the utility function, the power utility specification acquired the CRRA name, and the parameter governing the curvature of the utility function is commonly referred to as the coefficient of relative risk aversion.

This terminology misleadingly suggests that preferences toward risk are completely determined by the specification of the utility function. Following a line of research initiated by Grossman and Laroque [1990], this paper studies risk preference in a model that, like the standard model, assumes a constant curvature ("CRRA") utility function, but, unlike the standard model, assumes that one of the goods that appears as an argument of the utility function is subject to a nonconvex (lumpy) adjustment cost. When the adjustment cost is introduced to the model, Merton's result that the value function immediately inherits the curvature of the utility function no longer goes through; that is, we can not show analytically that relative risk aversion will be constant.

The model assumes that current period utility depends on two goods, a nondurable consumption good and housing. Adjusting the consumption of housing services requires the payment of a cost proportional to the value of the house sold and therefore not convex in the size of the adjustment. The household maximizes expected lifetime utility, optimally choosing the timing of each house sale, the size of any new house, the level of nondurable consumption, and the allocation of wealth among housing, equities and a riskless asset. After numerical solution of the household's problem to obtain the value function, we can quantitatively characterize the household's degree of relative risk aversion. The preliminary numerical results indicate that the introduction of the adjustment cost results in variation in the household's degree of relative risk aversion as a function of the ratio of wealth to housing, even though a constant curvature utility function is assumed. Further, the magnitude of the variation in risk aversion is large.

The household's problem

Households maximize the expected lifetime utility:

$$\widetilde{U} = E_0 \int_0^{\infty} e^{\delta_t} U(H_t, C_t) dt$$

The instantaneous utility function is depends on two goods: nondurable consumption, denoted C_t, and housing, denoted H_t. δ is the rate of time preference. The flow of housing services is assumed proportional to the stock of housing, H_t. The rate of physical depreciation of the house is assumed to be zero.

$$U(C, H) = \frac{(C^{\alpha} + \gamma H^{\alpha})^{\frac{\mathbb{a}}{\alpha}}}{\mathbb{a}}$$

The parameter α determines the degree of substitutability of the two goods; the parameter $\mathbb{a}$ determines the curvature of the utility function with respect to the composite good, i.e. for $\mathbb{a} = 1 - \rho$ the curvature of the utility function with respect to the composite good is ρ .

The instantaneous utility function can be rewritten as $U(C, H) = \frac{\left[H^{\alpha}\left(\left(\frac{C}{H}\right)^{\alpha} + \gamma\right)\right]^{\frac{\mathbb{a}}{\alpha}}}{\mathbb{a}} = \frac{H^{\mathbb{a}}\left[\left(\frac{C}{H}\right)^{\alpha} + \gamma\right]^{\frac{\mathbb{a}}{\alpha}}}{\mathbb{a}}$

Using lower case "c" to denote the ratio of C to H, define $u(c)$ as $u(c) = \frac{[c^{\alpha} + \gamma]^{\frac{\mathbb{a}}{\alpha}}}{\mathbb{a}}$. Thus the original utility function $U(C, H)$ can be expressed in intensive form as a function of the stock of housing and the ratio of nondurable goods to housing:

$U(C. H) = H^{\mathbb{a}} u(c)$.

The nondurable consumption good is numeraire. The price of housing services relative to nondurable consumption is assumed constant, further, the units of measurement for H are chosen so that the relative price of housing services is unity.

Let $W_t =$ total household wealth. That is,

$$W_t \equiv H_t + B_t + \underline{X_t}\underline{\ell}$$

Were B_t denotes the holding of riskless bonds, $\underline{X_t}$ denotes the vector of holdings of n risky financial assets, and $\underline{\ell}_t$ is a vector of ones. The household can either borrow or lend at the riskless interest rate; that is, B_t can be negative. No borrowing constraint is imposed.

The return to the riskless asset (and the interest rate for borrowing) is denoted r_f . Using b_{it} to denote the value per share of the ith risky financial asset, asset prices are assumed to follow a Brownian motion process:

$$db_{i,t} = b_{i,t}\big((\mu_i + r_f)dt + d\omega_{i,t}\big)$$

where μ_i is the expected excess return on risky asset i.

The change in wealth (assuming no sale of the house) is given by

$$dW_t = r_f B_t dt - C_t dt + \underline{X}_t((\underline{\mu} + r_f)dt + d\omega_t)$$

Rewriting to eliminate the term in bonds,

$$dW_t = r_f(W_t - H_t)dt - C_t dt + \underline{X}_t(\underline{\mu} dt + d\underline{\omega}_t)$$

where $\underline{\mu}$ represents the expected excess return to the vector of risky assets.

It is assumed that there are no adjustment costs incurred in buying or selling financial assets, or in adjusting the level of consumption of the nondurable good. However, in order to adjust the quantity of housing, the household sell the existing house and purchase a different one, incurring an adjustment cost which is proportional to the value of the house sold. Thus the adjustment cost is nonconvex in the size of the adjustment.

At the instant that the house is sold, wealth drops discontinuously by an amount proportional to the value of the house sold,

$$W_\tau = W_{\tau-} - \lambda H_{\tau-}$$

The magnitude of the adjustment cost is denoted by λ.

In this version of the model, there are only two state variables: total wealth, and the current size of the house. Thus the value function is:

$$V(W_0, H_0) = \underset{\widetilde{H},\tau,\{\underline{X}_t\},\{c_t\}}{Sup} E\left[\int_0^\tau e^{-\delta t}\,\frac{H_0^{\text{ª}}(c_t^\alpha + \gamma)^{\frac{\text{ª}}{\alpha}}}{\text{ª}}\,dt + e^{-\delta\tau} V\big(W_\tau - \lambda H_0, \widetilde{H}\big)\right]$$

One element of the household's problem is to choose the next time that the house is sold, denoted τ. Until the house is sold at time τ, the stock of housing is constant at its current level, H_0. The size of the next house, purchased at τ, is denoted $\widetilde{H}$. The household also chooses the vector of holdings of risky assets, and the level of nondurable consumption at every instant.

By a change of variables, the problem may be stated in terms of one state variable, y:

$$y_t = \frac{W_t}{H_t} - \lambda$$

Similarly, state the vector of risky asset holdings and the level of nondurable consumption as a ratio relative to the quantity of housing:

$$\underline{x}_t = \frac{\underline{X}_t}{H_t}$$

$$c_t = \frac{C_t}{H_t}$$

With the change of variables, the Bellman equation is:

$$H_0^{\text{ª}} h(y_0) = \underset{\widetilde{H},\tau,\{\underline{x}_t\},\{c_t\}}{Sup} E\left[\int_0^\tau e^{-\delta t}\,\frac{H_0^{\text{ª}}(c_t^\alpha + \gamma)^{\frac{\text{ª}}{\alpha}}}{\text{ª}}\,dt + e^{-\delta\tau}\widetilde{H}^{\text{ª}} h\left(\frac{W_{\tau-} - \lambda H_0}{\widetilde{H}} - \lambda\right)\right]$$

Where $H^{\text{ª}} h(y) = V(W, H)$.

Dividing by $H_0^{\text{ª}}$ gives:

$$h(y_0) = \underset{\tau,\{\underline{x}_t\},\{c_t\}}{Sup} E\left[\int_0^\tau e^{-\delta t}\,\frac{(c_t^\alpha + \gamma)^{\frac{\text{ª}}{\alpha}}}{\text{ª}}\,dt + e^{-\delta\tau} \underset{\widetilde{H}}{Sup} \left(\frac{W_{\tau-} - \lambda H_0}{\widetilde{H}}\right)^{-\text{ª}} h\left(\frac{W_{\tau-} - \lambda H_0}{\widetilde{H}} - \lambda\right)\right]$$

Define M as:

$$M \equiv \sup_{y} (y+\lambda)^{-\mathbb{a}} h(y)$$

M represents the optimal value of the program at a stopping time, τ, when the house is sold, and incorporates the household's optimal choice of a new house. Denote the value of y that achieves the supremum as y^* .

$$M = (y^* + \lambda)^{-\mathbb{a}} h(y^*)$$

At a stopping time, the optimal choice of new house is the quantity $\widetilde{H}$ such that, for the current stock of wealth,

$$\frac{W}{\widetilde{H}} = y^* + \lambda$$

Thus the optimization problem becomes:

$$h(y_0) = \sup_{\tau, \{\underline{x}_t\}, \{c_t\}} E\left[\int_0^{\tau} e^{-\delta t} \frac{(c_t^{\alpha} + \gamma)^{\frac{\mathbb{a}}{\alpha}}}{\mathbb{a}} dt + e^{-\delta\tau} y_{\tau-}^{\mathbb{a}} M\right]$$

In general, the change in the state variable, y, is given by:

$$dy_t = \frac{dW}{H} - \frac{WdH}{H^2}$$

Using the equation for the evolution of wealth (and simplifying the notation by denoting the riskless rate as r), gives

$$dW_t = r[W_t - H_t]dt - C_t dt + \underline{X}_t(\underline{\mu} + d\underline{\omega}_t)$$

Dividing dW_t by H and noting that $dH = 0$ within the inaction region gives:

$$dy_t = r\left[\frac{W_t}{H_t} - 1\right] dt - c_t dt + \underline{x}_t(\underline{\mu} + d\underline{\omega}_t)$$

Restated in terms of the state variable y_t results in the differential equation

$$dy = r[y_t + \lambda - 1]dt - c_t dt + \underline{x}_t(\underline{\mu} + d\underline{\omega}_t)$$

To keep the household out of bankruptcy, we require that

$\frac{W_t}{H_t} \geq \lambda$ or equivalently, $y_t \geq 0$.

The interpretation of the bankruptcy condition is that if the household ever lets its net wealth fall to λH_t , the cost of selling the current house, the household is forced to use its remaining wealth to pay the adjustment cost, sell the house, and ends up with zero future utility (i.e., the household dies).

At any instant, it is always feasible to sell the house immediately and achieve the value

$y_t^{a} M$.

Since the household can, but is not required to, sell the house at any moment, it is always the case that

$h(y_t) \geq y_t^{a} M$.

As long as the inequality is strict, i.e.,

$h(y_t) > y_t^{a} M$

it is not optimal to sell the house. When, however,

$h(y_t) = y_t^{a} M$

t is a stopping time and the house is sold.

Consider a small interval of time (0,s) such that

$h(y_t) > y_t^{a} M$

for 0<t<s, that is, a small interval of time within which it is optimal not to sell the house. At time t=0,

$$h(y_0) = \sup_{\{\underline{x}_t\},\{c_t\}} E\left[\int_0^s e^{-\delta t}\,\frac{(c_t^{\alpha} + \gamma)^{\frac{a}{\alpha}}}{a}\,dt + e^{-\delta s} h(y_s)\right]$$

Subtract $h(y_0)$ from both sides, divide by t and take the limit as t approaches zero:

$$0 = \lim_{t \to 0} \sup_{\{\underline{x}_t\},\{c_t\}} E\left[\frac{1}{t}\int_0^s e^{-\delta t}\,\frac{(c_t^\alpha + \gamma)^{\frac{\mathrm{a}}{\alpha}}}{\mathrm{a}}\,dt + e^{-\delta s} h(y_s)\right]$$

Using Ito's lemma,

$$0 = \sup_{\underline{x}_0, c_0} E\left[\frac{(c_0^\alpha + \gamma)^{\frac{\mathrm{a}}{\alpha}}}{\mathrm{a}} - \delta h(y_0) + h'(y_0)\left\{r(y_0 + \lambda - 1) + \underline{x}_0^T \underline{\mu} - c_0\right\} + \frac{1}{2} h''(y_0)\underline{x}_0^T \Sigma \underline{x}_0\right]$$

After taking the limit as the size of the time interval approaches zero, the decision variables become simply the vector of risky asset holdings, and the level of nondurable consumption at the current time, rather than the paths of these variables over time. By differentiating the expression in square brackets with respect to the vector of risky asset holdings, one finds that the optimal choice of risky asset holdings, $\underline{x}_0$, is:

$$\underline{x}_0 = \frac{-h'(y_0)}{h''(y_0)}\Sigma^{-1}\mu$$

where Σ is the (nxn) covariance matrix of the innovations to the vector of risky assets.

Note that, in the equation for the optimal holding of risky assets, the product

$\Sigma^{-1}\mu$ is an (nx1) vector with values that depend purely on the stochastic process generating asset returns and is therefore common to all households. The (nx1) vector $\Sigma^{-1}\mu$ is multiplied by a scalar that represents the (inverse) of the curvature of the value function h(y):

$$\frac{-h'(y_0)}{h''(y_0)}$$

Note that since all households hold the same proportions of the risky assets in their portfolio, the mutual fund theorem of the CAPM holds, and all households hold risky assets in the same proportion as the market portfolio. Since holdings of risky assets are always proportional to the market portfolio, only the expected excess return to the market portfolio, denoted μ, and the variance of the return to the market, denoted σ^2 are required to characterize the risk characteristics of financial assets. While the composition of the portfolio or risky assets is the same for all households, the quantity (or scale factor) of the holdings of risky assets depends on the curvature of the individual household's value function at the current value of its state variable y_0. This implies that even if all households are identical in terms of their fundamental preference parameters and their beliefs about the stochastic process generating asset returns, any heterogeneity across households in terms of the current value of their state variable will in general lead to heterogeneity across households in the quantity of risky assets held in the portfolio.

To consider the household's optimal choice of current nondurable consumption, substitute in the solution for $\underline{x}_0$ to obtain

$$0 = \sup_{c_0} \left[\frac{(c_0^\alpha + \gamma)^{\frac{\mathrm{a}}{\alpha}}}{\mathrm{a}} - \delta h(y_0) + h'(y_0)\{r(y_0 + \lambda - 1) - c_0\} - \frac{1}{2} h''(y_0) \frac{\left(h'(y_0)\right)^2}{h''(y_0)} \frac{\mu^2}{\sigma^2}\right]$$

Denote the value of c_0 that achieves the supremum as $c(y_0)$.

Thus for any value of y within the inaction region (i.e., for any value of y such that $h(y_t) \geq y_t^{\mathrm{a}} M$, the following equation must hold:

$$0 = \frac{([c(y_t)]^\alpha + \gamma)^{\frac{\mathrm{a}}{\alpha}}}{\mathrm{a}} - \delta h(y_t) + h'(y_t)[r(y_t + \lambda - 1) - c(y_t)] - \frac{1}{2} \frac{\left(h'(y_t)\right)^2}{h''(y_t)} \frac{\mu^2}{\sigma^2}$$

Conditional on the current value of the state variable, y_t, the optimal value of c_t satisfies $\frac{\partial u}{\partial c} = h'(y_t)$. Since nondurable consumption, unlike housing, is costlessly adjustable, the household continuously sets the marginal utility of nondurable consumption equal to the marginal value of wealth. In the general case that the instantaneous utility function is nonseparable in the two goods, the marginal utility of nondurable consumption will depend on the current stock of housing. For the particular utility function we have assumed, the optimal level of nondurable consumption, C_t , is the level of consumption such that given the current stock of housing H_0 , the ratio c_t satisfies the following first order condition:

$$[c_t^{\alpha} + \gamma]^{\frac{\mathbb{a}-\alpha}{\alpha}} c_t^{\alpha-1} = h'(y_t)$$

The solution to the household's problem consists of the mapping $h(y)$ and the three critical values of the state variable $y_1 < y^* < y_2$ such that

$h(y_1) = y_1^{\mathbb{a}} M$

$h(y_2) = y_2^{\mathbb{a}} M$

$h^{'}(y_1) = \mathbb{a} y_1^{\mathbb{a}-1} M$

$h^{'}(y_2) = \mathbb{a} y_2^{\mathbb{a}-1} M$

$h(y) = y^{\mathbb{a}} M$ for $y < y_1$ or $y > y_2$ and $h(y)$ satisfies the differential equation

$$0 = \frac{([c(y_t)]^{\alpha} + \gamma)^{\frac{\mathbb{a}}{\alpha}}}{\mathbb{a}} - \delta h(y_t) + h^{'}(y_t)[r(y_t + \lambda - 1) - c(y_t)] - \frac{1}{2}\frac{\left(h'(y_t)\right)^2}{h''(y_t)}\frac{\mu^2}{\sigma^2}$$

for $y_1 < y < y_2$.

Recall that M is defined as $M \equiv \sup_{y} (y + \lambda)^{-\mathbb{a}} h(y)$ and that y^* denotes the value of y that achieves the supremum, that is, $M = (y^* + \lambda)^{-\mathbb{a}} h(y^*)$.

To solve the problem numerically, first pick a value for M. Make an initial guess for the lower critical value, y_1. This determines $h(y_1)$ and $h'(y_1)$, and, using the first order condition for nondurable consumption determines $c(y_1)$. Using these values and the assumed parameter values, the differential equation then determines $h''(y_1)$.

For a small increment, s, in y , consider the point $y_n = y_1 + s$. Using Taylor's theorem, we can calculate the level and slope of the value function at y_n:

$$h(y_n) = h(y_1) + h'(y_1)s + \frac{1}{2}h''(y_1)s^2$$

$h'(y_n) = h'(y_1) + h''(y_1)s$

The level of nondurable consumption $c_n = c(y_n)$ is the value which satisfies

$[c_n^\alpha + \gamma]^{\frac{\mathbb{a}-\alpha}{\alpha}} c_n^{\alpha-1} = h'(y_n)$

Having determined the values of $h(y_n)$, $h'(y_n)$, and $c(y_n)$, the differential equation can be solved for $h''(y_n)$. With the level and first and second derivatives of the h function at y_n , we can then determine the level and derivatives of h , as well as nondurable consumption, at the next point, $y_{\tilde{n}} = y_n + s$. In this manner, we can continue to increase the value of *y* by adding the increment *s* and calculating the corresponding values of h, h', and h'' . As long as $h(y) > y^{\mathbb{a}}M$ the household remains within the inaction region, that is, $y_1 < y < y_2$. However, when a value of y is reached such that $h(y) = y^{\mathbb{a}}M$, this is our provisional value of the upper boundary, y_2 . We then check whether the provisional value of y_2 satisfies the smooth pasting condition, that is, whether this value of y_2 satisfies

$h'(y_2) = \mathbb{a} y_2^{\mathbb{a}-1} M$.

If the smooth pasting condition is not satisfied, the initial guess for y_1 was not correct. We pick a new guess for y_1 use the same recursive process to find a new provisional value of y_2, and continue searching over y_1 until we find the pair (y_1, y_2) such that y_2 satisfies the smooth pasting condition.

For the (arbitrarily chosen) value of M, we now have the associated mapping between *y* and *h*, and the values of the lower and upper bounds, y_1 and y_2. Since the mapping is conditional on a specific value of M, denote the (candidate) solution $h(y; M)$. Before a candidate solution can be accepted as the actual solution to the problem, we must check whether the candidate solution satisfies the definition of M:

$M \equiv \sup_y \, (y + \lambda)^{-\mathbb{a}} h(y)$

For the candidate solution $h(y; M)$ calculate

$$G = \sup_{y} \, (y + \lambda)^{-\mathbb{a}} h(y; M)$$

If $G = M$, we have found the fixed point

$$M = \sup_{y} \, (y + \lambda)^{-\mathbb{a}} h(y; M)$$

and the candidate solution is a complete solution to the problem. If $G \neq M$, the initial choice of M was incorrect. For a new guess for M, for example $\widetilde{M}$, the same computational procedure is used to obtain the candidate solution $h\left(y; \widetilde{M}\right)$. The search over values for M continues until we find the value of M that satisfies $M = \sup_{y} (y + \lambda)^{-\mathbb{a}} h(y; M)$. Having found the value of M that satisfies the fixed point, we can suppress the dependence of the solution on M, and denote the h mapping as simply $h(y)$.

<u>Parameterization</u>

The parameters representing the expected excess return and the variance of the return to the market portfolio were chosen to coincide with the benchmark values used in Grossman and Laroque [1990], with the expected excess real return set equal to .059 and the standard deviation equal to .22. Also following the benchmark parameter values in Grossman and Laroque, the riskfree rate was assumed to be 0.01, and the rate of time preference to be 0.01. The size of the adjustment cost, λ, was set at 5%. Given that selling a house and buying a different one entails very substantial nonpecuniary costs (that is, time and hassle costs) in addition to the explicit costs of realtor fees, legal fees, and moving expenses, 5% is a conservative value for the adjustment cost.

The parameters of the instantaneous utility function are based on the estimation of the Euler equation for nondurable goods using household level data in Flavin and Nakagawa [2008]. The curvature of the utility function with respect to the composite good is set at $\rho = 2$. Thus $a \equiv 1 - \rho = -1$. The utility weight on housing, γ, is set equal to unity, and the parameter governing intratemporal substitution of the two goods is $\alpha = -8$.

Numerical solution of the value function

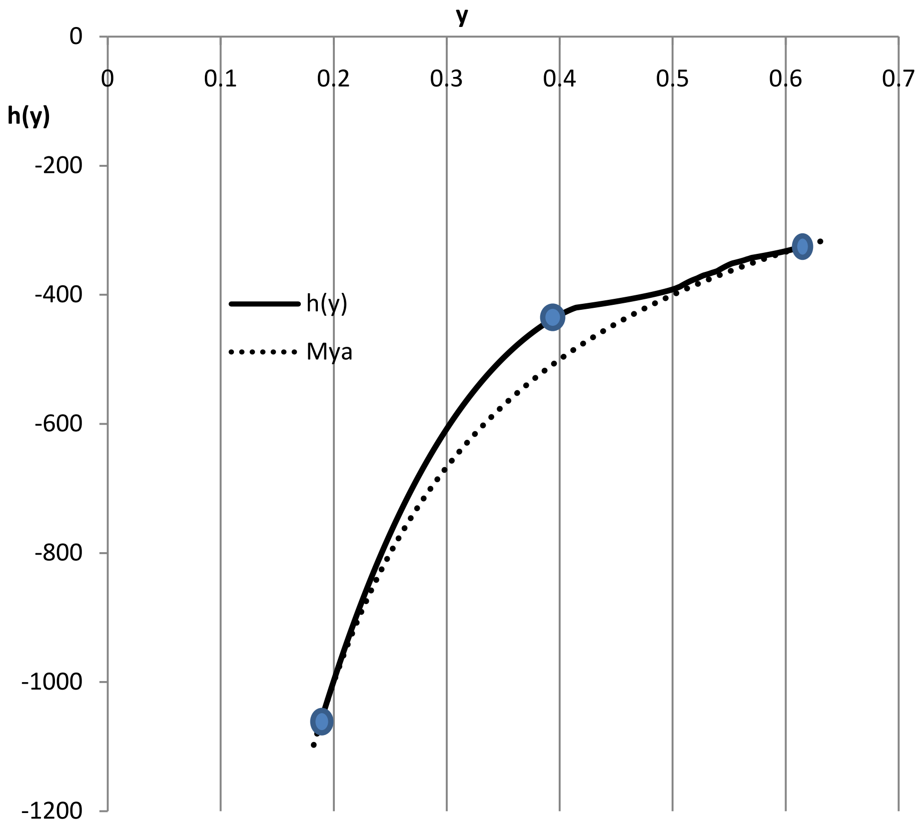
The complete solution to the household's problem is given by the three critical values of y, y_1, y^*,y_2, and the mapping h(y). A basic attribute of the value function, h(y) is that for the interval $y_1 < y < y_2$, it must be the case that

$$h(y) > My^{a}$$

Since the value of My^{a} (which represents the value of the program if the household were to sell the house immediately, pay the adjustment cost, buy a new house consistent with the optimal return point y^*, and optimize fully in the future) has constant curvature equal to ρ (the curvature of the utility function with respect to the composite good, it is useful to plot both
The vertical gap given by h(y) - My^{a} represents magnitude of the loss (in terms of the value of the program) if the household were to (non-optimally) sell the house immediately. Thus the magnitude of the vertical gap h(y) - My^{a} provides an indication of the quantitative importance of the effect of the adjustment cost on the household's behavior.

Both curves, $h(y)$ and My^{a}, are plotted in Figure 1.

Figure 1: Plot of $h(y)$ and My^{a}



The inaction region $(y_{1,}y_2)$ is between y=.185 and y=.609. The value of the state variable at the optimal return point, y^*, is .398. Since the state variable is defined as

$$y = \frac{W}{H} - \lambda$$

these critical values indicate that the household optimally sells the house to buy a smaller one when the wealth to house ratio falls to .235, sells the house to buy a larger one when the wealth to house value ratio rises to .659, and, when buying a new house, chooses the size house that results in a wealth to house value ratio of .448. Note that for the parameter values used in the numerical optimization, the ratio of net wealth (W) to house value (H) is always less than unity, which of course implies that ratio of house value to net wealth is always greater than unity.

While the result that the household always owns a house with value greater than its net wealth may seem troubling, there is nothing in the optimization problem that rules out that outcome. Net wealth, W, is defined as

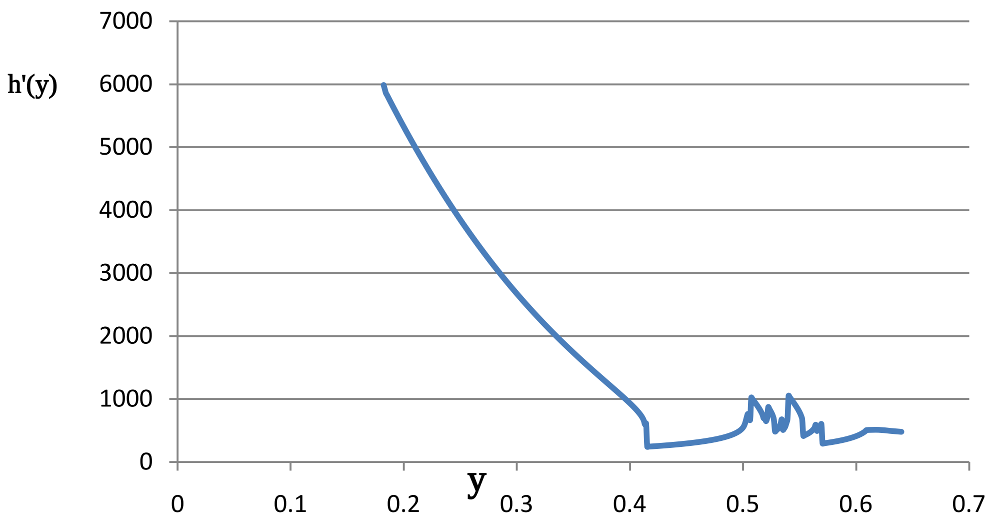
$$W = H + X + B$$

or the total value of the household's holdings of real estate, H, risky financial assets, X, and the risk-free asset, B. Since no borrowing constraint is imposed, and indeed the household can borrow or lend at the risk-free interest rate, the household can take a negative position in the riskless asset. Further, the value of the riskless rate was set at 0.01, which is a common parameterization of the real interest rate paid on government bonds in the US data. Given the opportunity to borrow, without any collateral constraint, at an interest rate of 1%, and the opportunity to invest in housing, which yields a utility flow, and risky financial assets with an average real return of 5.9%, the numerical results indicate that the household's optimal portfolio consists of positive holdings of housing and equities, and a (large) negative position in bonds.

In a previous paper, Flavin and Yamashita [2002], I have considered the household's optimal portfolio decision, conditional on its current holding of housing, when household borrowing cannot exceed the value of the house. That is, the household is permitted to borrow in the form of a mortgage up to 100% of the house value, but is otherwise unable to borrow. A robust result of that exercise was that for most households, the borrowing constraint was binding; i.e., the household optimally holds none of the riskless asset, and would choose to take a negative position in the riskless asset if borrowing at the riskless rate were allowed. The optimization problem considered in this paper is the partial equilibrium problem of the household. If one maintained the assumptions on the expected return and variance of return on the risky asset, and the utility return to housing, and imposed a general equilibrium condition of zero net borrowing in the economy as a whole, it seems clear that the equilibrium real return on the riskless asset would be greater than 1%. However, we are not attempting a general equilibrium analysis; the strategy is to parameterize the behavior of asset returns based on the data, and solve the household's partial equilibrium problem.

The numerical results presented in Figure 1 are preliminary. Over the range of y=.4 to y=.61, the plot of h(y) is notably nonsmooth, which suggests some instability in the numerical solution. In solving for h(y), our ultimate goal is to obtain the first and second derivatives of the function in order to determine how the adjustment cost affects the marginal value of wealth, and the degree of relative risk aversion over the inaction region. The marginal value of wealth, h'(y) is plotted in Figure 2.

Figure 2: plot of h'(y)



The presumed numerical problems which created a lack of smoothness in the level of h(y) result in jagged movements in the derivative of h(y) over the corresponding range of the state variable. While the numerical results in Figure 2 are clearly not reliable, there are some general conclusions about the behavior of the first derivative that can be drawn from the general features of the h(y) function. Given that h'(y) must equal the first derivative of the My^{a} function at the limits of the inaction region, and h(y) lies above the $My^{\mathbb{a}}$ function within the inaction region, the affect of the adjustment cost is to increase the marginal value of wealth near the lower value, y_1, and reduce the marginal value of wealth near the upper value, y_2, compared to the frictionless case.

The household's degree of relative risk aversion is given by

$$\frac{-h^{''}(y)y}{h^{'}(y)}$$

In the absence of an adjustment cost on housing, relative risk aversion would be constant and equal to the curvature of the utility function with respect to the composite good, which was set at a value of 2. However, in the presence of the adjustment cost, the degree of relative risk aversion is equal to 1.12 at y=.185 (the lower bound), 1.24 at y=.20, 2.34 at y=.30 and 7.44 at y=.40. Thus in comparison to the frictionless case, the household more risk averse for values of y near the return point, and less risk averse as the value of y approaches the lower bound. Note that even though the parameterization of the problem used a conservative value for the adjustment cost of 5%, the magnitude of the variation in relative risk aversion over the inaction region is large.

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