AN EMPIRICAL APPROXIMATION
OF THE NATURAL RATE
OF INTEREST AND POTENTIAL
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“There is a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor to lower them”. Wicksell (1898, p102).

“So long as prices remain unaltered the [central] banks’ rate of interest is to remain unaltered. If prices rise, the rate of interest is to be raised; and if prices fall, the rate of interest is to be lowered; and the rate of interest is henceforth to be maintained at its new level until a further movement of prices calls for a change in one direction or the other”. Wicksell (1898, p189).
Abstract

The aim of this paper is to isolate the long run movements on equilibrium interest rate (or natural rate of interest) and potential growth. This estimations has been compute for US and Germany using a methodology developed by Laubach and Williams that is based on a Kalman Filter estimation of this two unobserved variables in a reduce structural model. The results match properly with the generally accepted periods of recessions and points to a reduced variation of the natural rate of interest, the potential growth and the business cycle during the last decade. This lower variation question the effects on the policy design from recent events like the “new economy”. Moreover, we find that recently the natural rate of interest had rose in US and exhibits a moderate deceleration on Germany, that seems compatible with the different effect of new technologies for both economies. Moreover, the paper reached some aspects of the current monetary policy stance in both countries.
1 Introduction

The introductory quote to this paper defines the equilibrium interest rate, also known as the natural rate of interest, as the rate consistent with an economy being in intertemporal equilibrium or, in monetary terms, as that compatible with a constant inflation rate and with output at its potential level. However, the natural rate of interest is not directly observable and depends on factors, such as the potential growth of the economy and the individual intertemporal discount factor, whose measurement is not obvious. Nevertheless, estimating the natural rate of interest and its possible variations over time is important in the conduct of monetary policy, since this concept can, if properly approached, provide a yardstick for assessing the expansionary or restrictive tendency of real interest rates at any given time. In fact, transformations in financial markets have reduced the importance of monetary aggregates as useful intermediate targets for interpreting how the decisions of monetary authorities are related to price stability and economic growth targets. Against this background, direct comparison of the interest rate set by the central bank with the natural rate of interest has taken on greater importance in assessing the monetary policy stance. A case in point here is the success of monetary policy rules as an analytical instrument for a posteriori interpretation of central bank decisions.

Obviously, although these measurements seem to reflect fairly well the interest rate behaviour in various developed countries, a central bank’s decision-making process involves more factors than those included in the measurements. However, various authors have pointed out that the misalignment in certain periods of time of the observed interest rates with those that would result from monetary policy rules may be related to the uncertainty faced by the authorities in approximating certain variables included in those rules. This debate, which has traditionally centred on the measurement of potential output, has spread recently to the assessment of errors that may result from not taking into account the fluctuations in the natural rate of interest. Notable in this respect are the studies by Laubach and Williams (2003) (hereafter “LW”) and by Orphanides and Williams (2002), which illustrate the costs of mismeasuring equilibrium real interest rates.

Against this background, this study uses methodology developed by LW based on the Kalman filter to estimate the natural rate and potential growth for the United States and Germany. The behaviour of these variables in the two areas is compared and the degree of variability shown by them is analysed. These two countries were chosen because of their importance in the global economy. Also, the stability of their economic policies and the stable behaviour of their main macroeconomic variables make the process of estimation less complex than for other countries.

This paper is structured as follows: Section 2 defines, from a theoretical standpoint, the concept of natural rate of interest that it is intended to identify; Section 3 discusses the estimation procedure used and presents the results; and Section 4 sets forth the main conclusions drawn from this work.

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1. Notable in this respect are the papers by Orphanides (2003), Judd and Rudesbuch (1999) and Clarida et al. (2000) discussing the possible monetary policy implications of the uncertainty faced when approximating unobservable variables such as potential output.
The equilibrium interest rate or natural rate of interest is a classical concept in economic literature introduced by Wicksell more than a century ago. However, the pre-eminent role of short-term interest rates in the implementation of monetary policy by various central banks and the increasing dissemination of Taylor rules among analysts – whose implementation required to compute an equilibrium interest rate among analysts have contributed to the resurgence of interest in this variable in both the theoretical and the empirical fields.

To understand the role played by the natural rate of interest in recent academic debate, it is useful to start by establishing a general definition of what is understood by it. The natural rate of interest is interpreted as "the interest rate consistent with output converging to potential, where potential is the level of output consistent with stable inflation" (Bomfim 1997). This definition is similar to that of other authors such as Woodford (2003) and Blinder (1998), and is a concept that is somewhat analogous to the natural rate of unemployment (Friedman 1968). The natural rate of interest is equal to the return on capital that would be expected if there was no friction in the economy, which should also be the interest rate set by monetary policy in the absence of rigidities.

This general definition of the natural rate of interest thus coincides with the ideas introduced by Wicksell and can be found in a wide range of recent economic models. Specifically, the only features required for models to coincide with this neo-Wicksellian interpretation of monetary policy are as follows:

— The behaviour of prices or of inflation can be expressed as a function of the difference between the interest rates set by monetary policy and the natural rate of interest.

— The natural rate of interest depends on real fundamentals (marginal productivity of capital, intertemporal elasticity of substitution in consumption and population growth, among others).

— The behaviour of monetary policy can be summarised through an operating rule.

While the last two conditions are explicitly included in most economic models, the first condition, which deals with the relationship of price behaviour to the difference between the interest rate and the natural rate, is not so obvious. However, as shown by Woodford (2003), it is easy to verify that this relation is implicitly included in most models.

Nevertheless, when estimating the equilibrium interest rate, certain details of how this concept is interpreted may vary depending on the formulation chosen in the model. Thus, if the framework in which the natural rate is estimated focuses on fluctuations around a constant mean level (known as high frequency movements) we obtained the response of natural interest rate to temporary shocks. This movement, by construction shows an important cyclical component. If, on the contrary, the natural rate is identified under a scenario that considers both the movements around a mean and the possible changes in this mean level, this generates a natural rate that includes mainly low-frequency movements and will not have an important cyclical nature.

Estimates can thus be found in the literature that remove low frequency movements by using detrended series, such as those made by Rotemberg and Woodford (1997), Neiss and Nelson (2001), Giammarioli and Valla (2003) and Andrés et al. (2003), which usually avoid low-frequency movements and use trendless series. In these studies the natural rate of interest has a markedly cyclical profile and provides some indication of a monetary policy.

2 These two terms are used indistinctly in this paper.
3 See J. B. Taylor (1993)
stance statistically related with the business cycle\textsuperscript{4}. This kind of approach is usually based on structural models and therefore, as shown by Levin, Wieland and Williams (2001), yields results that depend strongly on the assumptions used in the models.

The estimates aimed at identifying a natural rate more closely related to trend changes, such as those proposed by LW, provide a metric for monetary policy that is less dependent on cyclical shocks. However, the structural nature of these variables is not directly linked to any observable phenomenon, which means that these estimates usually suffer from accuracy and robustness problems.

The aim of this work is precisely to identify not only temporary fluctuations on natural interest rate but also more permanent changes. To do this, the estimation procedure used has to have certain properties. First, it has to allow the estimates of the natural rate of interest to vary with time. This characteristic, which is obvious from the theoretical angle, can be appreciated in any description of real interest rates for sufficiently long periods of time. For example, Manrique and Marqués (2002) discuss how the historical averages of real rates for different time periods have changed significantly.

Next, the results have to be robust to different specifications of the model implicit in the estimate and take into account not only temporary fluctuations but also more permanent changes. In fact, given the structural nature of this concept it seems natural that these low frequency movements prevail. For this reason the approach based on general dynamic equilibrium models such as in Rotemberg and Woodford (1997) and Neiss and Nelson (2003) is not used, since in this framework the estimates can be highly sensitive to the parameters calibrated in them and in which, in general in this models is imposed a constant mean for natural interest rate.

In addition, as noted in the preceding section, the natural rate of interest has to be directly related to real factors and should reflect shifts in the economic production structure. Therefore, it is unsuitable here to follow the approach of Siklos (2001), who used an ARIMA model in which the variance of the real interest rate follows a GARCH process, or that of Crespo-Cuaresma et al. (2003), which describes a multivariate technique for unobservable components in which only the statistical properties of the series are used. Similarly, estimates based on forward rates obtained in the interest rate term structure, such as those of Bomfin (2001) and of Kozicki and Tinsley (2001), generate low-frequency estimates which, however, are not necessarily related to structural characteristics of the economy and in which, furthermore, it is complicated to incorporate time variation into the risk premium.

In this respect the methodology developed by LW provides a more suitable framework to provide an estimate of the natural rate of interest useful for the monetary policy analysis. In fact, this methodology overcome the model dependency of the dynamic general equilibrium models and offers a natural rate of interest evolution compatible with production structural changes. These authors developed a reduced-form structural model in which the natural rate of interest is explained by the potential growth of the economy. Both factors are considered to be unobserved and they are estimated, in the framework of the model described in the following section, using Kalman filter methodology, which allows changes in state variables. This structure allows estimates able to identify a long-term movement in equilibrium real interest rates linked to real characteristics of the economy. Moreover, the reduced-form specification of the model avoids the high sensitivity of results to slight changes that is seen in more highly structured models.

\textsuperscript{4} In certain cases, such as in Neiss and Nelson (2001), the difference between this natural rate and the real interest rate extracted from the market prices of indexed bonds is used as an indicator of the degree of monetary restrictiveness and for prediction purposes.
3 Estimation of a time-varying natural rate of interest

3.1 Specification of the Model

The model is based on the system of equations of LW (2003), although in certain robustness exercises slight changes have been made to it when measuring real interest rates. These authors use the Kalman filter to jointly estimate the natural rate of interest, potential output and potential output growth rate. To do this, they propose a system of equations that jointly characterises the behaviour of inflation and of the output gap. Although this multi-equation specification entails a larger number of parameters to be estimated, it provides a more appropriate framework for estimating the medium-term movements of the natural rate of interest, since it avoids the bias introduced into single-equation methods when there are prolonged inflationary or disinflationary processes. The reduced form of the model thus consists of the following equations:

\[
\begin{align*}
\tilde{y}_t &= A_y(L) \tilde{y}_{t-1} + A_r(L) (r_{t-1} - r^*_t) + \varepsilon_{1t} \quad (1) \\
\pi_t &= B\pi_t(L) \pi_{t-1} + B\tilde{y}_t(L) \tilde{y}_{t-1} + Bx(L) x_t + \varepsilon_{2t} \quad (2) \\
r^*_t &= cgt + z_t \quad (3) \\
z_t &= Dz_t(L) z_{t-1} + \varepsilon_{3t} \quad (4) \\
y^*_t &= y^*_{t-1} + g_{t-1} + \varepsilon_{4t} \quad (5) \\
g_t &= g_{t-1} + \varepsilon_{5t} \quad (6)
\end{align*}
\]

Equation (1) explains the output gap, defined as the percentage deviation of real GDP from potential GDP \(\tilde{y}_t = y_t - y^*_t\), as a function of its own lags, the lags in the differential between the short-term real interest rate \(r_{t-1}\) and the natural rate of interest \(r^*_t\) and an uncorrelated error term \(\varepsilon_{1t}\). In this equation, the output gap lag structure and the error term are intended to reflect short-term dynamics and temporary shocks, while structural shifts in this relationship between the output gap and the real interest rate are reflected in variations in the natural rate \(r^*\). Finally, the inclusion in this equation of the difference between the real and natural rates of interest means that the estimate of the latter has to be compatible with the real interest rate that would be obtained in a stationary state if the output gap were closed.

Equation (2) represents the inflation dynamic \(\pi_t\). Against a background of price rigidity, it is a function of its own lags, lagged output gap and an exogenous term \(x_t\) that reflects relative energy prices. In short, equations (1) and (2) are the measurement equations of the model in state space.

Equation (3) indicates that the natural rate of interest \(r^*_t\) depends only on real factors, specifically the trend growth in the economy \(g_t\) and a series of random factors (associated, for example, with changes in financial deregulation or with changes in the rate of intertemporal substitution of households), which are denoted by the variable \(z_t\), which in turn follows a stochastic process determined by equation (4).

Finally, equation (5) refers to potential output. It is assumed to follow a random walk and time variation is permitted in its growth rate, which in turn behaves according to a random walk model [equation (6)]. The model allows temporary shocks to both the level of

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5. The simplest example of univariate approximation consists of applying the Hodrick Prescott filter to the observed real interest rate series with a suitable lambda which generates a medium-term variation. In this case it is implicitly assumed that the inflation rate is stationary, so the resulting natural rate increases in periods in which inflation shows a downward trend.

6. This specification is similar to that used by Rudesbuch and Svensson (1999), except that they assumed constant behaviour for \(r^*\).

7. A summary of the additional factors that can affect the natural rate of interest can be found in N. Bjorkstend and O. Karagedikli (2003).
potential output and to its growth rate through inclusion of the error terms $\epsilon_4$ and $\epsilon_5$. Hence equations (3)-(6) are the transition equations of the state-space model.

In short, the system of equations (1)-(6) proposed by LW is fairly similar to that used by other authors to decompose GDP into its trend-cycle components [see Watson (1986), Clark (1987) and Kuttner (1994)], with the novelty that it includes the interest rate, thus enabling the natural rate of interest to be estimated. Recently Larsen and McKeown (2003) have used a model similar to that of LW for the UK without imposing the relationship between the natural rate of interest and potential growth. This, given the large number of parameters to be estimated, causes considerable difficulties in arriving at satisfactory results.

To specify the lags built into the model, in the case of the United States the restrictions included in LW were taken as the initial reference. Thus the output gap and inflation equations have a fairly broad lag structure that includes two lags for the output gap polynomial $-A_y(L)-$, consistent with the characterisation of the output gap as an AR(2) in the foregoing trend-cycle analyses. A structure of two lags is defined for the real interest rate gap $-A_r(L)-$ with the restriction that the coefficients have to be equal (this assumption is not rejected by the data in any of the exercises carried out). In the inflation equation (2), eight inflation lags are included and the restriction that the coefficients sum to unity is imposed. Further, for reasons of parsimony, an additional restriction requires the coefficients of lags two and four to be equal, and likewise for those of lags five to eight. A single lag is added for the output gap regressor in this equation, and the same for the energy relative price regressor. Finally, different autoregressive structures were tested with the interest rate term $z_t$, since the specification thereof can significantly influence the estimate of the natural interest rate.

Owing to the lack of references in the literature, the structure chosen for Germany is similar to that imposed for the US, although a more reduced lag structure was specified in order to reduce the number of parameters to estimate. In particular, a single inflation lag seems sufficient in the inflation equation, whereas two lags are included in both the output gap and the interest rate gap. This specification seems to be sufficient to characterise the cyclical episodes in Germany on the basis of the proposed model. Finally, various assumptions were made about the movement of the component $z_t$.

3.2 Data
Quarterly data from various sources for a sample period running from 1962 Q2 to 2001 Q4 were used. For the variable $y_t$, log GDP at constant prices and seasonally adjusted was obtained from Eurostat. The nominal interest rate was proxied by the three-month interbank rate (unlike LW, who used the intervention rate). The reason is that, although this interest rate is closely linked to the monetary policy rate, its greater variability makes for readier estimation.

The inflation measure used was the year-on-year growth rate of GDP excluding unprocessed food and energy, on information from OECD databases. The ex-post real interest rate is the difference between the nominal interest rate and the rate of inflation as measured by the general CPI. The price of energy relative to that of other goods was measured by means of the producer price index for oil and oil products.

3.3 Estimation methodology
As in LW, the estimate was made by the maximum likelihood method using Kalman filter methodology. However, the variance of the potential growth and of the natural rate is presumably a good deal less than that of the data used to make the estimate. This means
that the maximum likelihood estimates of these variances are biased towards zero\(^9\). Therefore the estimate is made in two stages. In the first, the median-unbiased estimator proposed by Stock and Watson (1998) is used to estimate the relation between the standard deviations of the trend growth \((\sigma_5)\) and of the potential output \((\sigma_4)\) and that between the standard deviations of the term \(z_t\) \((\sigma_3)\) and of the output gap \((\sigma_1)\) by means of the terms denoted \(\lambda_g\) and \(\lambda_z\):

\[\lambda_g = \frac{\sigma_5}{\sigma_4}\]  

(7)

\[\lambda_z = \frac{\sigma_3}{\sigma_1} \sqrt{2}\]  

(8)

In the second stage these ratios are imposed in the estimation of the remaining model parameters by maximum likelihood.

To estimate \(\lambda_g\), potential output must previously be estimated. This is done by the method of Kuttner (1994), which consists of applying the Kalman procedure to a model in which the trend growth rate of the economy \(-g_t\) is constant and the output gap does not depend on the interest rate gap. Based on this potential output estimation exercise, the exponential Wald statistic of Andrews and Ploberger (1994) is calculated for a structural break at an unknown time in the growth of potential output. Then the Stock and Watson estimator is used to convert this statistic into an estimate of \(\lambda_g\).

Next the term \(\lambda_z\) is estimated under the initial model, but assuming that \(z_t\) is constant. Also, to estimate \(\lambda_z\), the value of \(\sigma_3\) is set to that obtained in the previous step.\(^{10}\) Once the value of \(\lambda_g\) and \(\lambda_z\) have been estimated, restrictions (7) and (8) are imposed in the estimation of (1)-(6). These restrictions on variances play a fundamental role in the estimation process, which is why in the annex 1 we include a robustness exercises that include some sensitivity analysis for this variances exercises in this respect based on the values obtained.

The model was estimate by the Kalman filter methodology. On this respect, in annex 2 it could be found some details of the estimation process.

### 3.4 Results

In the US, the result of the exponential Wald test, which tests for a structural change in the first difference of potential output, shows little variability over time. In fact, it reaches a value of 2.26, which is larger than the critical value of 1.55 at the 10\% significance level. Moreover, using the unbiased Stock and Watson estimator, the weak evidence of change in the potential growth rate of the economy manifests itself in an estimate of the parameter \(\lambda_g\) of 0.049. In the case of Germany, the exponential Wald test yields a value of 4.70 significant at the 1\% level, which results in an estimate of \(\lambda_g\) of 0.19.

The value given by the exponential Wald test for a structural change in the constant of the natural rate is 1.27 for the US and the unbiased estimator of \(\lambda_z\) is 0.03. In the case of Germany, the exponential Wald test yields a value of 0.71, with \(\lambda_z\) estimated at 0.02.

Table 1 shows estimates of the other parameters under different specifications of the \(z_t\) process. It can be seen that, for the United States, the parameters are fairly robust to the two specifications of \(z_t\). The parameters for Germany also seem to be fairly stable to the various specifications, except for the autoregressive components of the output gap. There is a strong relation between the natural rate and potential growth (parameter \(c\)) in both the United States and in Germany. This result has already been reported by LW for the United

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\(^9\) This problem, known as “pile-up”, is discussed by Stock (1994). It arises in non-stationary processes with little relative variability.

\(^{10}\) This restriction should be imposed only when \(z_t\) is non-stationary. However, in cases when it is not, the restriction appreciably improves the estimation results.
States and potential growth and the natural rate of interest, given the low level of component \( z_t \), points to a clear relation between average.

<table>
<thead>
<tr>
<th>ESTIMATION RESULTS</th>
<th>UNITED STATES</th>
<th>GERMANY</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Z_t \sim AR(1) )</td>
<td>( \lambda_g = 0.049 )</td>
<td>( \lambda_g = 0.049 )</td>
</tr>
<tr>
<td>( Z_t \sim AR(2) )</td>
<td>( \lambda_g = 0.19 )</td>
<td>( \lambda_g = 0.19 )</td>
</tr>
<tr>
<td>( \lambda_z = 0.03 )</td>
<td>( \lambda_z = 0.02 )</td>
<td>( \lambda_z = 0.02 )</td>
</tr>
<tr>
<td>( \Sigma )</td>
<td>0.999</td>
<td>0.999</td>
</tr>
<tr>
<td>( \Sigma )</td>
<td>0.510</td>
<td>0.270</td>
</tr>
<tr>
<td>( \lambda_r )</td>
<td>-2.299</td>
<td>-2.299</td>
</tr>
<tr>
<td>( \lambda_r )</td>
<td>-2.320</td>
<td>-2.320</td>
</tr>
<tr>
<td>( c )</td>
<td>0.846</td>
<td>0.846</td>
</tr>
<tr>
<td>( c )</td>
<td>0.610</td>
<td>0.610</td>
</tr>
<tr>
<td>Log F.V.</td>
<td>368.397</td>
<td>368.360</td>
</tr>
<tr>
<td>Log F.V.</td>
<td>258.130</td>
<td>258.600</td>
</tr>
</tbody>
</table>

Chart 1 shows output gap for both countries. It can be observed how the recessions in both economies are set out fairly accurately and how the estimated fluctuation in the output gap seems to be reasonable enough.
Chart 2 shows the natural rate of interest for both countries. This variable exhibits a reasonable stability but also has a somewhat cyclical nature, especially in recession periods. This characteristic has been discussed in many empirical papers and in particular, Kim and Murray (2002) point out how, in the US recessions, temporary or cyclical factors explain between 77% and 96% of the sample variance of output, the rest being due to more structural movements, which may therefore affect the natural rate of interest.

As can be seen in Chart 2, the levels of the natural rate were very different in the two series during the sample considered, although, as noted, there are certain common patterns. In particular, the natural rate of interest seems to be lower than the historical average in both countries, and in the last decade it seems to be more stable. However, the natural rate of interest is more volatile in the United States, an observation that is consistent with the greater variability of potential growth in this area.
To make these patterns more clearly visible, Table 2 depicts the mean and standard deviation of the natural rate of interest and of potential growth, along with the standard deviation of the output gap in different decades. As regards the business cycle, a reduction in volatility is seen in the last part of the sample, a phenomenon that various papers have reported in the US economy [see MacConnell and Pérez-Quirós (2000)] and that Stock and Watson (2003) have found recently in the cycles of various countries, including Germany. This reduction in volatility has reached almost the 60% on Germany (the standard deviation of the output gap fell from 1.02 between 1964 and 1980 to 0.41 between 1980 and 2003) and the 46% on US (from 2.62 to 1.41 for the same periods), in line with the results of Stock and Watson (2003).

As regards the level of the standard deviation of the output gap, the results for the United States are very similar to those found in the literature [see, for example, LW or Stock and Watson (2003)]. The estimated volatility of the German output gap is clearly lower than that of the US output gap, reflecting the more volatile nature of the US economy. In this respect, Larsen and Mckeown (2003), using a similar methodology, also found the UK cycle to have an appreciably lower volatility than that of the US.

The natural rate of interest in both countries has declined in both level and volatility. The differential between the natural rates of interest of the two areas has, on average,
increased from 90 basis points between 1964 and 1980 to 120 basis points in the 1990s. This result seems consistent with recent estimates for the euro area [see, among others, Cuaresma et al. (2003)] and with the results of LW for the United States. Moreover, this performance of the differential between the two areas seems compatible with the divergent behaviour of productivity—probably related to the different uses of new technologies—and population growth in the two countries. Moreover, the natural rate of interest during the last part of the sample indicate an important degree of easy from monetary policy stance on United States and a moderate ease on Germany.

### Table 2: Natural Rate of Interest and Output Gap

<table>
<thead>
<tr>
<th></th>
<th>Natural Rate</th>
<th>Potential Growth</th>
<th>Output Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average level</td>
<td>Volatility (a)</td>
<td>Average level</td>
</tr>
<tr>
<td>UNITED STATES</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1964-2003</td>
<td>2.9</td>
<td>0.7</td>
<td>3.4</td>
</tr>
<tr>
<td>1964-1981</td>
<td>3.3</td>
<td>0.7</td>
<td>4.0</td>
</tr>
<tr>
<td>1982-1992</td>
<td>2.5</td>
<td>0.5</td>
<td>2.9</td>
</tr>
<tr>
<td>1993-2003</td>
<td>2.6</td>
<td>0.3</td>
<td>3.1</td>
</tr>
<tr>
<td>GERMANY</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1964-2003</td>
<td>1.9</td>
<td>0.5</td>
<td>3.2</td>
</tr>
<tr>
<td>1964-1981</td>
<td>2.4</td>
<td>0.4</td>
<td>4.0</td>
</tr>
<tr>
<td>1982-1992</td>
<td>1.7</td>
<td>0.1</td>
<td>2.7</td>
</tr>
<tr>
<td>1993-2003</td>
<td>1.4</td>
<td>0.1</td>
<td>2.4</td>
</tr>
</tbody>
</table>

**Source:** Author’s calculations.

a. Sample standard deviation.

In this country the estimates for the natural rate of interest on the recent past seems significantly lower than the results obtained for the euro area aggregate in other papers. In particular, Giammaroli and Valla (2003) estimate a dynamic general equilibrium for the euro area and find that the natural interest rate fluctuates between 3% and 3.7% during last decade. In the same vein but using an unobserved component approach, Crespo, Cuaresma and Ritzberger-Gruenwald (2003) obtained that natural interest rate for the euro area would result close to the 2% at the end of the 90s. This spread between the real equilibrium interest rate for Germany and for the euro area could create some restrictiveness of the euro area monetary policy for this country. However, this bias could be partially offset if the equilibrium inflation rate for Germany will be lower than the average since this creates less tolerance for the inflation than in other countries.12

Moreover, the lower variability of the "latent" variables (potential growth and natural rate of interest) in both countries seems to indicate that changes in these variables represent less uncertainty for the design of economic policy objectives than in previous periods. Specifically, in the case of monetary policy, Chart 2 plots the amount by which the interest rate that would result from applying a Taylor rule with the natural rate and output gap estimated in this study differs from that obtained by the traditional approach of assuming the equilibrium real interest rate to be constant and calculating the business cycle by the Hodrick-Prescott filter. As can be seen in the chart, in the last two decades the deviation between the two interest rates is barely one percentage point in absolute terms. This discrepancy, taking into account the error margin in estimating the two variables, does not

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12 In terms of the Taylor rule this would imply that the constant is inferior in the rule for Germany but, for the same level of inflation creates a higher response in Germany since the equilibrium inflation rate is reduced.
seem high. In the United States, the deviations between the two series exhibit certain inertia because of the greater amplitude of the business cycle in our estimate with respect to that obtained using a Hodrick-Prescott filter. By contrast, in Germany the differential is fairly erratic because the amplitude of the business cycle is very similar. In both countries the natural rate shows low variability in this period and this largely explains the results in Chart 3. In this respect, most of the differences reflected on the chart is due to the method used to estimate the amplitude of the business cycle.

3.5 Cyclical component of the real rate
As discussed in Section 2, the methodology used should identify low-frequency movements in the natural rate of interest. To ascertain whether this is certain or whether, rather, the possible mistaken identification of the cyclical component has affected the estimates of growth potential and the natural rate of interest in such a way that they incorporate more cyclical elements, the estimation exercise has been replicated, but using series for output in which a cyclical component has been previously extracted. The exercise seeks to identify the extent to which the estimation of the natural rate of interest and growth potential is sensitive to the way in which this dynamic is obtained.

The exercise conducted consisted of the following stages. Firstly, the series of seasonal components and outliers was adjusted using the automatic identification function of the SEATS-TRAMO program. Thereafter, with the trend-cycle series identified in the first
stage, the cyclical component was extracted, having been identified as the residual of an IMA (2,2) process where the parameters are obtained on the basis of the size of the cycle it is intended to estimate using the Kaiser-Maravall algorithm (2001). This process was followed for all series and has been calculated for cycles of 10 and 20 years.

Charts 4 and 5 offer the results for the United States and Germany, respectively, comparing them with those obtained previously. Moreover, the growth potential estimate is presented using the Hodrick-Prescott filter in the habitual fashion. The charts show how the results are very similar, although in some periods minor differences arise. In the case of the United States, these differences only appear slightly to affect the level of the natural rate in certain periods of time, while a somewhat less stable behaviour of the natural rate of interest is obtained in the case of Germany. In sum, this exercise suggests that the estimation of both components appears to be independent of the way in which the business cycle is characterised. In any event, the main regularities discussed for the natural rate and growth potential (reduction in the level and in volatility during the second half of the sample), are not affected in the case of extracting the cyclical component from output by means of a prior filter.

**UNITED STATES. ESTIMATES OF NATURAL RATE AND OF OUTPUT GAP. SENSITIVITY TO DIFFERENT TREND AMPLITUDES**

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**CHART 4**

**SOURCE:** Author’s calculations.

1. Recessions estimated by the National Bureau of Economic Research (NBER) are shaded.
a. The baseline estimate is AR(2), $\lambda_z = 0.19$ and $\lambda_g = 0.02$.

b. German recessions estimated by the Economic Cycle Research Institute (ECRI) are shaded.
4 Conclusions

This paper presents the estimates of the natural rate of interest and the output gap for the United States and Germany via the methodology developed by LW. Output gap values consistent with the periods of recession and expansion established for these countries are obtained and it may be concluded, with the necessary caveats inherent in these types of estimates, that the natural rate of interest and the output gap in both countries have shown volatility over the past decade below their historical mean. The greater stability of these variables allows the possible impact on the design of economic policies of recent phenomena such as the technology revolution or the “new economy” to be relativised, in the sense that recent changes in growth potential and the natural rate of interest do not appear to have come about in the immediate past more sharply than in previous periods.

Moreover, the positive differential of the US natural rate over that of Germany is found to have increased recently. That would appear to be consistent with the information available on the growth potential differential between both areas as a likely consequence of the different incorporation of new technologies in both economies.

Finally, these results could be use for the assessment of monetary policy stance in these areas. This exercise should be considered with some concern since the level for the natural rate of interest was obtained with some uncertainty and was specially affected by the estimation assumptions. Taking into account those caveats, the results points to some slackness for monetary conditions at the end of the sample. In fact, the natural rate of interest for US was situated on 2003 between 1.6% and 2.3% compared with the -0.6% of ex-post US real rate of interest. However in Germany natural rate of interest fluctuates around 0.5% and 1.7% closer to the 0.3% of observed real rate of interest on this country.
BIBLIOGRAPHY


ANNEX 1. ROBUSTNESS ANALYSIS

The main disadvantage of estimating the natural rate of interest and growth potential using a methodology in which specific behaviour equations are not detailed is that the results may show high volatility depending on the different changes to the assumptions included in the estimate. In this respect, the foreseeable low volatility of the series to be estimated (the natural rate and growth potential) compared with the actual series (inflation and output) means that several optimal points may be found in the estimate. In particular, given the high number of parameters estimated and the small number of variables observed, likelihood functions with several critical points can be obtained, in which it is complicated to select those that are most suitable. In the estimates presented, only those specifications whose result provided values for the unobserved parameters that were reasonable in economic terms were considered. In addition, the initial values selected for the parameters for which information was not available beforehand were established so as to satisfy the cyclical profile of each economy. In this connection, regard was had to the business cycle recessions dated by the NBER, in the case of the United States, and by the ECRI (Economic Cycle Research Institute, which uses the same methodology as the NBER) in the case of Germany.

Nonetheless, in such estimates it is advisable to perform robustness exercises in order to assess the sensitivity of the estimates to those factors of greater uncertainty, such as the restrictions imposed on the variance of growth potential and of the natural rate of interest, or a specification of the model in which real interest rates correspond to ex-ante interest rates consistent with the model.

Sensitivity to restrictions in the variance of growth potential and the natural rate of interest

As the estimate includes restrictions on the variance of growth potential and of the natural rate of interest, which have been obtained on the basis of a prior estimate, it would seem appropriate to estimate to what extent the uncertainty of this prior estimate may affect the final results. To do this, the extreme values of a confidence interval for the restriction of the variance of growth potential, $\lambda_g$, obtained by means of a Monte Carlo experiment, are used. In the case of the United States, the central estimate obtained by LW is also used. These extreme values are then used to estimate once more the restriction of the variance of the natural rate, $\lambda_z$, and the model is re-estimated with the new values of $\lambda_g$ and $\lambda_z$.

Charts 6 and 7 show the effect of different restrictions on the variance of the United States and Germany. The charts reveal how the introduction of very low variance has scant effects on the course of the natural interest rate in the United States and causes significant smoothing in the case of Germany. However, in the case of the output gap these low values of $\lambda_g$ and $\lambda_z$ give rise to a very high change in both countries\(^{13}\). Moreover, it is worth highlighting that although the course of the natural rate appears relatively unresponsive to how high the values of $\lambda_g$ and $\lambda_z$ are, its level, especially in the final segment depicted for the United States, and does change significantly on the basis of the restrictions imposed.

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13. In the extreme case of not introducing the restriction, the results obtained prove rather unreasonable.
UNITED STATES: ESTIMATES OF NATURAL RATE AND OF OUTPUT GAP. SENSITIVITY TO λg AND λz.

CHART 6

SOURCE: Author’s calculations.

a. The baseline estimate is AR(2).
b. US recessions estimated by the National Bureau of Economic Research (NBER) are shaded.
Observing the real interest rates are proxied in the model by those based on observed inflation, i.e. ex-post real rates. However, the proposed model allows these to be replaced by ex-ante real interest rates consistent with the model. This resolves the inconsistency caused by using the general CPI to obtain the real rate and proxying inflation with a restricted CPI (see Footnote 10) and it overcomes a certain inconsistency of the model due to the use of ex-post real interest rates. In this reformulated model, observed real rates are replaced by the difference between the short-term rate in period $t-1$ and the inflation expectation for inflation period $t$, the latter determined on the basis of the expected value of Equation (6) at $t-1$. That is to say, the variable $r_{t-1}$, which in the estimates was taken as exogenous, is replaced by:

$$r_{t-1} = i_{t-1} - E_{t-1} [\pi_t]$$  \hspace{1cm} (9)

where

$$E_{t-1} [\pi_t] = B_n(L) \pi_{t-1} + B_y(L) \bar{y}_{t-1} + B_x(L) x_{t-1}$$  \hspace{1cm} (10)

Chart 8 shows that this formulation of the model yields very similar results, although in the first part of the sample the mid-1970s recession is not clearly identified. The behaviour of the natural rate of interest does not seem to be strongly affected by this modification, although, as occurred in the previous exercise, its level is slightly different. Chart 9 shows how
this specification gives rise to higher volatility of the trend variables, although the main regularities discussed in this paper seem to hold.

a. US recessions estimated by the National Bureau of Economic Research (NBER) are shaded.
To estimate the model by Kalman filter methodology, it must be rewritten in the form of state equations and observation equations:

\[ y_t = A' x_t + H' h_t + w_t \]  \hspace{1cm} (11)
\[ h_{t+1} = F' h_t + v_{t+1} \]  \hspace{1cm} (12)

Where \( h_{t}(rx1) \) denotes the unobservable variables, \( x_{t}(nx1) \) stands for the exogenous variables and the matrices \( A(nxn) \) and \( H(rxn) \) include the model parameters.

In the case at hand, after some algebra on the model presented in Section 3.2, the particular form taken by these matrices is as follows:

\[
H' = \begin{pmatrix}
1 & -a1 & -a2 & -a3 & 0 & 0 & 0 \\
0 & -b4 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

\[
A' = \begin{pmatrix}
A1 & a2 & a3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & …
\]
With the matrices of unobservable and observable variables given by:

\[
\begin{align*}
\begin{bmatrix}
Y^t & Y_{t-1} \\
Y^t_{t-1} & Y_{t-2} \\
Y^t_{t-2} & R_{t-1} \\
r^t_{t-1} & \pi_{t-1} \\
r^t_{t-2} & \pi_{t-2} \\
G_{t+1} & \pi_{t-3} \\
Z_t & \pi_{t-4} \\
G_t & \pi_{t-5} \\
Z_{t-1} & \pi_{t-6} \\
\end{bmatrix} & = \\
\begin{bmatrix}
Y_{t-1} \\
Y_{t-2} \\
R_{t-1} \\
R_{t-2} \\
\pi_{t-1} \\
\pi_{t-2} \\
\pi_{t-3} \\
\pi_{t-4} \\
\pi_{t-5} \\
\pi_{t-6} \\
\pi_{t-7} \\
\pi_{t-8} \\
\pi_{t-9} \\
x_{t-1} \\
\end{bmatrix}
\end{align*}
\]

The initial parameter values used for the United States were those of LW and those for Germany were the results of individual estimates of the equations by the least squares method. Different values of the autoregressive parameters of the unobservable term \( z_t \) were tested until reasonable estimates of the output gap and of the natural rate of interest were obtained. In this respect, it should be noted that, although the estimation procedure allows numerous critical points, when only the economically reasonable ones that reproduce the recessions in the two economies are considered, the results are fairly stable. However, the Annex discusses in greater detail the selection process used and analyses the robustness of the results reported in the next section.

The estimates of the initial parameter values are highly sensitive. However, if account is taken only of those combinations that provide a cyclical profile of the economy conforming with the cycles established by other methodologies and a reasonable natural rate in terms of level and variation, the sensitivity of the results to various initial parameters is low.
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