## CHAPTER 8

# SEASONAL ADJUSTMENT AND SIGNAL EXTRACTION IN ECONOMIC TIME SERIES

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CONTENTS

# 1. INTRODUCTION

Seasonal adjustment has a long and well-documented tradition; see, for example, Nerlove, Grether and Carvalho (1979), Zellner (1978), Moore et al (1981), Den Butter and Fase (1991), and Hylleberg (1992). In essence, it consists in the removal of the seasonal variation from a time series. Since neither the seasonally adjusted (SA) series nor the seasonal component are directly observed, both can be seen as "unobserved components" (UC) of the series, and seasonal adjustment becomes a problem of UC estimation. Because the SA series is supposed to provide a cleaner signal of the underlying evolution of the variable, seasonal adjustment can also be viewed as a signal extraction problem in a "signal plus noise" decomposition of the series, where the noise is the seasonal component.

The widespread use of seasonal adjustment reflects powerful reasons. The most basic one is simply the need to understand better our present situation and to adjust our forecasts. As an example, in Cervantes (1605), Sancho Panza, overwhelmed by the disasters that befall upon them, asks (the senior) Don Quijote whether their misfortunes occur randomly or at periodic, forecastable, intervals. Of course, seasonal adjustment is also performed because of more sophisticated purposes. For example, in the preamble of the Federal Reserve Act of 1913, the US Congress sets as one of the main objectives of the Federal Reserve to accommodate seasonal variations in credit so as to maintain interest rates stable (Federal Reserve Board, 1915). The fact is that seasonal adjustment of economic series has become a nearly universal practice and millions of series are routinely adjusted. Moreover, economic analysis and research make heavy use of SA series, in the belief that they help interpretation and simplify modelling.

This chapter is not an attempt to summarize some of the last research developments, still at an early testing stage, but to present what we see as the "state of the art" concerning seasonal adjustment methods that satisfy two general constraints: a) that the method be of general availability, and b), that they can be, at present, reliably and efficiently used in large-scale applications by data-producing agencies. An implication of these two general requirements is that they restrict us to a world of univariate analysis. Multivariate extensions are few, still of limited capacity, and at an experimental stage (an interesting example is contained in the program STAMP; see Koopman et al, 1996).

It is a fact that the methods used to estimate UC in applied research often have little to do with the methods used by official data producing agencies, and this is a source of problems. The method presented in this chapter provides a relatively powerful tool that can be of interest in both cases. But, first, a word of caution may be appropriate.

The idea of living in a SA world is somewhat dangerous. It would, of course, cure Seasonal Auto Depression afflictions. But for a family of colibris whose brain size varies seasonally (enlarging for the winter, so as to be able to remember the places where food was stored,) seasonal adjustment of the brain size would prove disastrous. Within the economic field, the economics of seasonality (and some implications for seasonal adjustment) has attracted some attention; see, for example, Ghysels (1993a), Maravall (1983), Plosser (1978), Canova (1992), and Miron (1986). We shall not pursue this issue further, except to stress an important conclusion that will also emerge from our discussion, namely, that, as was the case with the brain of colibris, data used in econometric models should not be, as a rule, seasonally adjusted. (Further arguments that favour this conclusion can be found, for example, in Wallis, 1974; Osborn, 1988; Ghysels and Perron, 1993; Maravall, 1995; Findley et al, 1998).

There are several seasonal adjustment methods that satisfy the two general requirements mentioned above (see, for example, Fisher, 1995, and Balchin, 1995). We shall not survey them, but center on a particular class whose origins can be found in Nerlove, Grether and Carvalho (1979), Cleveland and Tiao (1976), Engle (1978), Harrison and Stevens (1976), Box, Hillmer and Tiao (1978), Piccolo and Vitale (1981), Burman (1980), Hillmer and Tiao (1982), Harvey and Todd (1983), and Gersh and Kitagawa (1983), to quote some important contributions. This class of methods is based on parametric models for the series and components, and computes the latter as the minimum mean squared error (MMSE) estimators given the observations (this is the "signal extraction" procedure). The models used are linear stochastic processes, often parametrized in the ARIMA-type format (Box and Jenkins, 1970). The methods that fall into this class will be called model-based signal-extraction (MBSE) methods.

A linear stochastic process is understood to mean a linear filter of gaussian innovations. Therefore, we shall not deal with nonlinear extensions, such as the ones in Harvey, Ruiz and Sentana (1992), Kitagawa (1987), Nelson (1996), Sheppard (1994), among others. Since what we have in mind is monthly (or lower frequency) data, nonlinearity is seldom a serious problem and, as seen in Fiorentini and Maravall (1996), proper outlier correction seems powerful enough to linearize most of those series. Moreover, one of the convenient features of the MBSE approach is that it permits to solve, in an internally coherent way, additional problems that might be relevant for the correct extraction of the signal. Examples are outlier correction, interpolation of missing values, trading day and easter effect correction, incorporation of regression or intervention variable effects, and, of course, forecasting; see, for example, Hillmer, Bell, and Tiao (1983) and Harvey (1989).

## 2. SOME REMARKS ON THE EVOLUTION OF SEASONAL ADJUSTMENT METHODS

### 2.1 The Evolution of the Methodological Approach

The crucial problem underlying the evolution of seasonal adjustment methods is the lack of a precise answer to the question of "what is seasonality?". The absence of a well-defined and generally accepted definition has fostered proliferation of procedures, and made it difficult to find common grounds for comparison. We shall briefly review some basic features of some approaches that provide the evolutionary line of the MBSE approach. In so doing, we leave aside important methods such as, for example, the bayesian BAYSEA procedure developed by Akaike and Ishiguro (1980), or the nonparametric SABL and STL procedures of the Bell Laboratories (see Cleveland, Dunn and Terpenning, 1978, and Cleveland, Mc Rae and Terpenning, 1990). Description and/or discussion of various of these methods can be found in Zellner (1978, 1983), Den Butter and Fase (1991), Hylleberg (1992), Ghysels (1993b), and Eurostat (1998a).

It will prove helpful to establish first some simple definitions. One is that of a deterministic model, which is meant to denote a model that can be forecast without error if the parameters are known. The second is the concept of white noise, which will denote a zero mean, finite variance, normally identically independently distributed (niid) variable. Finally, a moving-average (MA) filter applied to the observations will mean a linear combination of the latter.

The simplest way to model the seasonal component is as a deterministic function with seasonal dummy variables, as in (for monthly data) $s_t = \sum_{i=1}^{12} \beta_i d_{it}$, where $d_{it} = 1$ for month i and 0 otherwise, and the β-coefficients satisfy $\beta_1 + \ldots + \beta_{12} = 0$. An equivalent formulation uses cosine functions with the seasonal harmonics as frequencies. What characterizes these deterministic components is that

$$s_t + s_{t-1} + \ldots + s_{t-11} = 0, \tag{2.1}$$

that is, their sum over 12 consecutive months is zero. The SA series may be further decomposed into a deterministic function of time (the trend) and a noise or irregular component. The trend $(p_t)$ may be some polynomial in time, in its simplest form $p_t = a + bt$, which would imply

$$p_t - p_{t-1} = b, \quad \text{or} \tag{2.2}$$

$$(p_t - p_{t-1}) - (p_{t-1} - p_{t-2}) = 0. \tag{2.3}$$

The performance of these deterministic model proved unsatisfactory. The estimators of the β-parameters were typically unstable and did not seem to converge as observations increased. Residual seasonality could often be detected, and the out-of-sample forecasting performance of the overall model was poor. Although some extensions of the deterministic regression model have been developed (see, for example, Stephenson and Farr, 1972; Nourney, 1986; and Statistisches Bundesampt, 1997) attention moved in a different direction. Fixed deterministic components seemed to be inadequate because components "move" in time (an obvious example of a moving seasonal component is the weather, precisely one of the major causes of seasonality.) Attention shifted to MA filters, which seemed capable of capturing some of the moving features of the components. MA filters could be rationalized in several ways. First, as "local" approximations to deterministic functions of time (see, for example, Kendall, 1976). Second, since the moving features can be seen as the result of randomness, a natural way to think about the components is in the frequency domain. Obviously, the spectrum (by this term we also refer to the pseudo-spectrum when unit autoregressive roots are present; see Harvey, 1989) of a seasonal component would basically consist of peaks for the seasonal frequencies. The trend component, in turn, would be a peak around the zero frequency and, in general, a peak in the spectrum of the series for a cyclical frequency would indicate the presence of a periodic cyclical component. It follows that one could design "band-pass" filters in the frequency domain that would only capture the variation of the series within a specific frequency band. MA filters are also obtained as the time domain representation of band-pass filters (see, for example, Oppenheim and Schaffer, 1989). Since proper timing of events, and in particular of turning points requires that the complete filter induces a zero-phase effect in the adjusted series, and this, in turn, implies symmetric and centered filters, for now, we shall restrict our attention to this type of filters. Additionally, symmetric MA filters are also derived from optimizing some criterion that attempts to balance a trade-off between fitting and smoothness (see, for example, Gourieroux and Monfort, 1990).

As we shall see later, the three rationalizations of MA filters are closely linked, and the design of the filter requires, in all cases, "a priori" decisions. For example: what function should be used as local approximation? Which width should be selected for the frequency band? Which should be the penalty function? Once these "a priori" decisions have been taken, a so-called "ad-hoc" MA filter can be derived. The filter will have a fixed structure, independent of the structure of the series to which it is being applied.

In the field of seasonal adjustment, the most important filter designed has been unquestionably the one in the program X11 (Shiskin, Young and Musgrave, 1967). X11 basically consists of a linear filter, to which some additional features (for example, possible trimming of presumed outliers) and options (mostly, the selection of a few alternatives concerning the length of the filters) have been added. X11 has generated a family of programs (X11 ARIMA and X12 ARIMA; see Dagum, 1980, Bureau of the Census, 1997, and Findley et al, 1998) where the basic seasonal adjustment filter still is the linear filter in X11; we shall refer to the default value of this filter as the X11 filter. Figure 1a and 1b display the gain of the X11 filter, i.e., the way X11 filters the frequencies of the series spectrum, for a quarterly and monthly series, respectively. When the gain is 1, the frequency is fully transmitted; when the gain is zero, the frequency is ignored. If applied to a series with the spectrum of Figure 1c, the filter removes the variation around the seasonal frequencies, and provides a SA series with the spectrum of Figure 1d.

The empirical fact that many economic series have a similar dynamic structure and that this structure is broadly adequate for the X11 filter, evidences the ingenuity of the X11 designers and explains the success of the X11 program. But as the number of series treated increased and experience accumulated, the limitations of the filter became more apparent. The main limitation, in essence, is the rigidity implied by its fixed character. For some series, spurious results will be obtained, in particular those associated with under and overadjustment.

For a series containing a highly stochastic seasonal component, as evidenced by the width of the seasonal peaks in the series spectrum of Figure 2a, the width of the dips in the squared gain of the X11 filter seem too narrow. Application of the filter to the series yields a SA series with the spectrum of Figure 2b. The underadjustment causes the ackwards peaks for frequencies that are in the neighbourhood of the seasonal ones. On the other hand, for a series containing a close to deterministic seasonal component, as evidenced by the narrow peaks in the spectrum of Figure 2c, the width of the dips in the filter gain are too wide and, as seen in Figure 2d, X11 removes variance that is not a associated with the seasonal peaks of the series. In this case the result is overadjustment.

Clearly, the filter to seasonally adjust white-noise should simply be 1, since there is no seasonality. Alternatively, the filter to seasonally adjust a purely seasonal series (perhaps a seasonal component produced by X11) should simply be zero. The conclusion that the filter should depend on the structure of the series seems obvious. The MBSE approach solves this problem by tailoring the filter according to the model fit to the series.

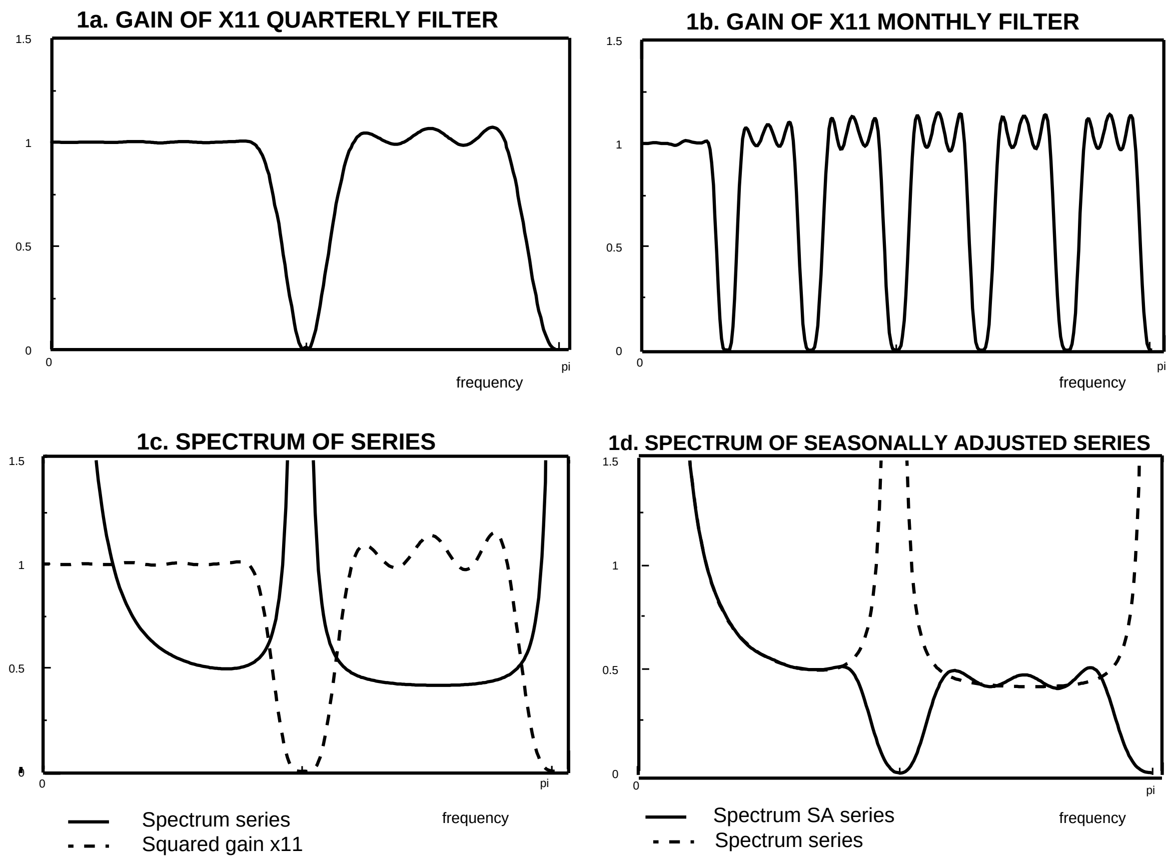
## 2.2 The situation at present

Although, as mentioned before, we do not review seasonal adjustment methods, it is of interest to make a brief reference to the main ones presently being used by data producing agencies. First, there are some isolated uses of several methods at specific institutions, which are in the process of being replaced sometime in the near future. Examples are the program GLAS, based on a spectral polishing of the series and used at the Bank of England (see Balchin, 1995), program SABL and program DAINTIES, the latter based on one -sided moving regressions (see Eurostat, 1998a), both still used at some sections of Eurostat, and program BAYSEA used at the Bank of Japan. Statistics Germany uses the moving-regression type method Berlin BV4 (see Statistiches Bundesamt, 1997), although this is not the case for the Bank of Germany which, as the vast majority of agencies and institutions, uses a member of the X11 family of programs. In many cases the standard X11 is used; in many other cases, the Statistics Canada modification X11 ARIMA is used; in some cases (Organization of National Statistics and Bank of Germany, for example) X11 with some added modifications is used. The US Bureau of the Census has just made available a new member of the family, X12 ARIMA, which presumably will replace in many cases the older members of the family. Besides incorporating additional tools for diagnosis in both cases, X11 ARIMA improved upon X11 by incorporating ARIMA forecasts and backcasts, so as to obtain better estimates at both ends of the series. X12 ARIMA has added a preadjustment program (REGARIMA), which deals with outliers and special effects (such as trading day) by means of a regression-ARIMA type model. Further, the number of ad-hoc filters available is larger, and the selection of the appropriate filter should depend on the particular series being adjusted. We mentioned before that, over the 40 years of the X11-empire, awareness of its limitations had inevitably increased. The extensions of X11 are attempts at solving some of the main limitations. It is worth noticing that the basic tools employed are ARIMA-model based tools, often reflecting the need to adjust the filter to the structure of the particular series.

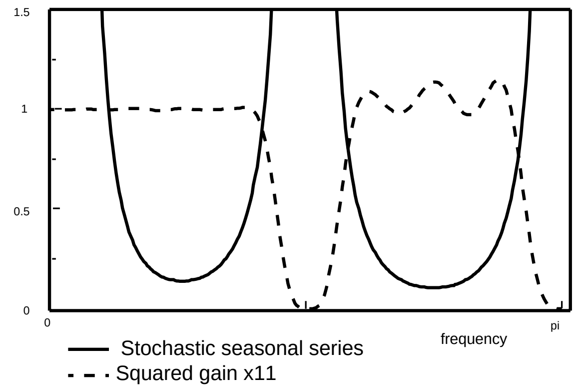
During this last decade work has been done on developing methods based on the (so-called) model-based approach. This work followed the basic methodology of Burman (1980), Hillmer and Tiao (1982), and Harvey and Todd (1983). Two directions emerged: one that begins by directly specifying the model for the components, which has been termed the structural time series (STS) approach (see Engle, 1978, and Harvey, 1989); the other approach, termed "ARIMA-model-based" (AMB) method, starts by identifying a model for the observed series, and derives from that the appropriate models for the components (see Box, Hillmer, and Tiao, 1978 and Bell and Hillmer, 1984). The models are linear stochastic processes, often parametrized in the ARIMA format. Fruit of that work are, within the STS approach, program STAMP, and, within the AMB approach, the pair of programs TRAMO-SEATS (TRAMO is a preadjustment program; part of SEATS emerged from an original program of Burman; see Gómez and Maravall, 1996). Their use has spread beyond academic and research applications to the production of official statistics. At a small scale, STAMP is used at some agencies (examples are the ONS and the Statistical Institute of Cantabria, Spain). TRAMO and SEATS have been used routinely on large data sets at Eurostat since 1994, and their use extends at present to various european countries. The simultaneity of the appearance of X12 ARIMA and of the first large-scale experiences with an AMB method has fostered a renewed interest in the topic of seasonal adjustment (interest now extends to trend-cycle estimation and to preadjustment of the series). This interest has been further reinforced by the effort of european countries to harmonize the production of data. The outcome of this interest has concentrated mostly on comparisons of X12 ARIMA with TRAMO-SEATS, and considerable amount of information on this work can be found at the internet site *http://europa.eu.int/en/comm/eurostat/research/noris4/*. Two recent task forces created for the purpose of reaching a recommendation concerning seasonal adjustment methods presented their reports at two conferences, in June 1998, at Rome, organized by the Italian Statistical Institute, and in October 1998 at Bucharest, organized by Eurostat and the International Statistical Institute. The first task force ("Seasonal Adjustment Research Appraisal" committee) was formed by representatives from different fields of professional activity and institutions; the second one was a Eurostat task force on seasonal adjustment policy. Both committees recommended the use of the AMB (TRAMO-SEATS) method; see Eurostat (1998b) and SARA (1998). Although the issue of selecting a seasonal adjustment method is far from being universally settled, a trend seems discernable. The model-based method has come of age and this may eventually lead to the replacement of the X11 paradigm.

## 3. THE NEED FOR PREADJUSTMENT

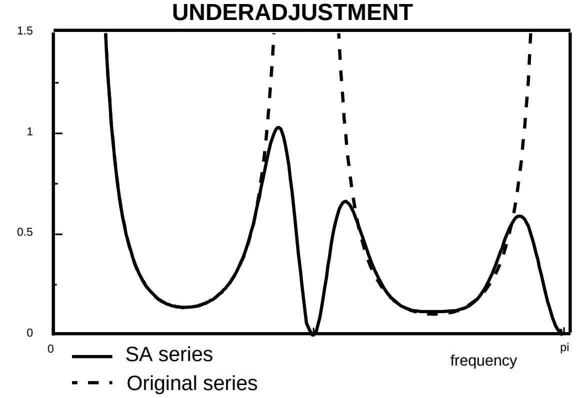
The model used, as already mentioned, is that of a linear stochastic process. Before


1a. GAIN OF X11 QUARTERLY FILTER
1b. GAIN OF X11 MONTHLY FILTER
1.5
1
0.5
0
pi
frequency
1c. SPECTRUM OF SERIES
1d. SPECTRUM OF SEASONALLY ADJUSTED SERIES
Spectrum series
Squared gain x11
Spectrum SA series
Spectrum series

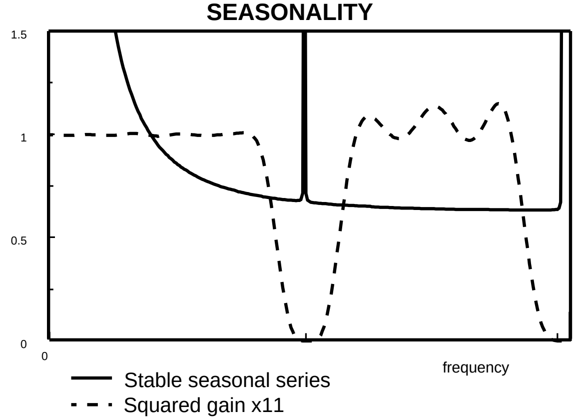
**2a. SPECTRUM OF SERIES WITH A HIGHLY STOCHASTIC SEASONALITY**



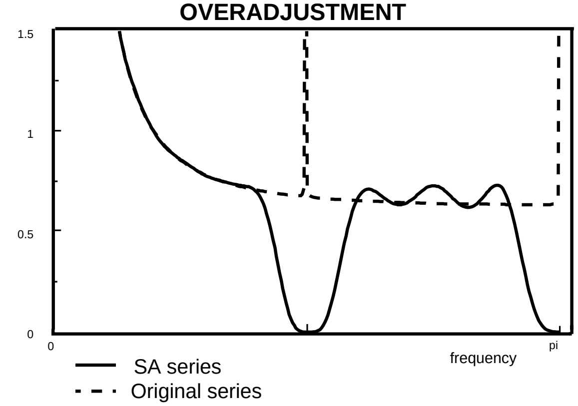

**2b. SPECTRUM OF ADJUSTED SERIES: UNDERADJUSTMENT**




**2c. SPECTRUM OF SERIES WITH STABLE SEASONALITY**




**2d. SPECTRUM OF ADJUSTED SERIES: OVERADJUSTMENT**



this assumption can be made, some modifications to the series often are needed, i.e., the series needs preadjustment. Some of these modifications are:

* Interpolation of missing values.
* Outlier correction.
* Removal of special effects, such as trading day and easter effects. The first refers to the difference in the number of weekdays per month; the second to the location of the easter period in different years.
* Correction for special events known a-priori. These effects will be referred to as "intervention variable" effects (Box and Tiao, 1975).
* Correction for the effect of other variables (examples can be national and regional festivities, or some indicator whose effect one wishes to remove).

Those types of effects (including missing values), traditionally neglected or dealt with by some empirical procedure, can all be expressed as regression variables. In the MBSE approach, a convenient tool is the regression-ARIMA model

$$y_t = W_t \beta + x_t , \tag{3.1}$$

where $y_t$ is the observed series, $W_t$ is the matrix with rows the regression variables, $\beta$ is a vector of coefficients, and $x_t$ follows a possibly nonstationary (NS) ARIMA model. (For the case of missing observations, an equivalent procedure is to leave them out of the likelihood, and estimate them with a fixed-point smoother; see Gómez, Maravall and Peña, 1999). The series $x_t = y_t - W_t \beta$ is the "linearized" series, in the sense that it can be assumed to be generated by a linear process.

For the general case of possible missing observations and possibly NS $x_t$ series, estimation of model (3.1) has been discussed in previous chapters (see also Gómez and Maravall, 1994). For this type of preadjustment to be operational in large-scale use, it requires an automatic model identification and outlier correction procedure. At present, these requirements can be met in a straightforward manner (see Chapter 8). Two programs that perform preadjustment based on models of the type (3.1) are the programs REGARIMA and TRAMO.

In presenting the MBSE method we shall assume that the ARIMA model is known and that the observed series is a linear series.

## 4. MODEL SPECIFICATION

We consider the additive decomposition (perhaps for the log of the series)

$$x_t = s_t + n_t , \tag{4.1}$$

where $s_t$ denotes the SA series (the "signal",) and $n_t$ the seasonal component (the "noise"). Often, the SA series is expressed as

$$s_t = p_t + u_t ,$$

where $p_t$ is denoted the trend (or trend-cycle), and $u_t$ is the irregular component. This last component is supposed to absorb highly erratic variation, often, simply white noise. In so far as the main purpose of removing seasonality is to obtain a better signal of the underlying evolution of the series, and since the addition of white noise will hardly improve the signal, for the rest of the paper, we assume that the irregular component $u_t$ is white noise. Proceeding in this way, the trend is defined as the residual after removal of the seasonal and the white-noise components. It follows that an AR(2) factor associated, for example, with a two-year cycle would be part of the trend, as would be an AR factor with a relatively small modulus. These factors, that cause short-term and transitory movements in $p_t$ can be separated from the trend, as in

$$p_t = m_t + c_t ,$$

where $c_t$ represents a stationary transitory component and $m_t$ the smoother trend. What should enter $c_t$ and how smooth the trend should be depends on the analyst horizon. Until Section 11, our perspective will be a short-term use and hence we consider short-term trends, also called trend-cycle components. We shall refer to them simply as trends; their aim is to provide a smoothed SA series; the smoothing removes the noise and perhaps some, relatively small, autocorrelation.

In the MBSE approach, the components are modelled as parametric linear stochastic processes, chosen so as to capture the spectral peaks associated with each component. Denote by B the backward operator (such that $B^j x_t = x_{t-j}$,) and let $\nabla = 1 - B$ and $S = 1 + B + ... + B^{\tau-1}$ denote the differencing and the annual-aggregation operators, respectively ($\tau$ = number of periods per year). The parametric model expressions can be rationalized as follows.

A stochastic trend can be seen as the equilibrium relationships (2.2) or (2.3), that characterize a deterministic trend, perturbated every period by some random disturbance with zero mean and moderate variance. Thus (2.2) may become the random-walk-plus-drift trend model

$$\nabla p_t = \mu + a_{pt} , \qquad a_{pt} \sim \text{niid}\ (0, \sigma_p^2)$$

while (2.3) could become the model

$$\nabla^2 p_t = a_{pt} ,$$

or, more generally, the IMA (2,1) model

$$\nabla^2 p_t = (1 + \theta_p B) a_{pt} ,$$

all of them well-known models for the trend (see, for example, Stock and Watson, 1988; Gersch and Kitagawa, 1983; and Harvey and Todd, 1983). More generally, one can think of models for the trend of the type

$$\phi_p(B) \nabla^d p_t = \theta_p(B) a_{pt} , \tag{4.2}$$

where $\phi_p(B)$ and $\theta_p(B)$ are low-order polynomials, with all roots of $\phi_p(B)$ real, positive, and stable, and d = 1, 2, or, very occasionally, 3 (see Maravall, 1993).

Concerning the seasonal component, $n_t$, condition (2.1), satisfied by a deterministic seasonal component, can be restated as S $n_t$ = 0. Perturbating every period this equilibrium with zero-mean random shocks of moderate variance, a stochastic component is obtained, with model

$$S n_t = w_t , \tag{4.3}$$

where $w_t$ is a stationary process, often a finite MA. Examples can be found, in Harvey and Todd, 1983, Burridge and Wallis, 1984, Gersch and Kitagawa, 1983, Aoki, 1990, and Kohn and Ansley, 1987. More generally one can think of models of the type

$$\phi_n(B) S n_t = \theta_n(B) a_{nt} , \qquad a_{nt} \sim niid\ (0, \sigma_n^2) \tag{4.4}$$

where the roots of $\phi_n(B)$ are associated with seasonal frequencies (see Maravall, 1989).

The irregular component is assumed white-noise. When a separate stationary transitory component is included, we shall simply assume an ARMA expression

$$\phi_c(B) c_t = \theta_c(B) a_{ct} , \qquad a_{ct} \sim niid\ (0, \sigma_c^2).$$

On some relatively rare occasions, the polynomial $\phi_c(B)$ has roots associated with a fixed-period cyclical component (examples are found in Crafts, Leybourne and Mills, 1989, and in Jenkins, 1979). In economics, however, the term cycle is often used to denote the seasonally adjusted and detrended series (see, for example, Stock and Watson, 1988). What is relevant to our purpose is that, while the very concepts of trend and seasonality imply a persistence or a regularity associated with nonstationarity, the transitory and irregular components are associated with stationary behaviour.

In general, if k components are present, the model will consist of the set of equations

$$x_t = x_{1t} + \dots + x_{kt} , \tag{4.5}$$

$$\phi_i(B)x_{it} = \theta_i(B)a_{it}, \quad i=1,\dots,k \tag{4.6}$$

where $\phi_i(B)$ and $\theta_i(B)$ are finite polynomials in B of orders $p_i$ and $q_i$, respectively, with no root in common and with all roots on or outside the unit circle, and the variable $a_{it}$ is a $(0,\sigma_i^2)$ white noise. The following assumptions are made:

Assumption A. The variables $a_{it}$ and $a_{jt'}$, $i \neq j$, are uncorrelated for all values of (t,t´).

Assumption B. The $\phi_i -$ polynomials are prime.

Assumption C. The $\theta_i -$ polynomials do not share unit roots in common.

Assumption A is based on the "a priori" belief that what causes, for example, seasonal fluctuations (weather, holidays) has little to do with what causes the evolution of the trend (productivity, technology). Of course, the assumption may be questioned on some applications (as an example, Ghysels, 1994, finds possible correlation between seasonality and cycle for US GNP). Assumption B seems sensible given that different components are associated with different spectral peaks (violation of the assumption, besides, would produce estimators with unbounded MSE; see Pierce, 1979). Finally, assumption C guarantees invertibility of the model for $x_t$. This last assumption could be relaxed, but it is rather innocuous and simplifies considerably notation.

Since aggregation of ARIMA models yields ARIMA models, the series $x_t$ will also follow an ARIMA model, say

$$\phi(B)x_t = \theta(B)a_t, \tag{4.7}$$

where $a_t$ is white noise with variance $\sigma_a^2$ and $\phi(B)$ – but not $\theta(B)$ – may contain unit roots. From (4.5), (4.6) and (4.7), it is straightforward to show that the AR polynomial in the model for $x_t$ satisfies

$$\phi(B) = \phi_1(B)\phi_2(B)\dots\phi_k(B), \tag{4.8}$$

and the MA one can be obtained from the relationship

$$\theta(B)a_t = \sum_{i=1}^{k} \phi_{ni}(B)\theta_i(B)a_{it}, \tag{4.9}$$

where $\phi_{ni}(B)$ is the product of all $\phi_j(B)$, $j=1,\dots,k$, not including $\phi_i(B)$. [Thus, for example, $\phi_{n1}(B) = \phi_2(B)\dots\phi_k(B)$.]

The model consisting of equations (4.5) and (4.6), together with assumptions A, B and C will be referred to as an Unobserved Component ARIMA (UCARIMA) model. It will prove convenient to express the UCARIMA model also in a more compact way, as the signal-plus-noise model (4.1), where $s_t$ is the signal of interest and $n_t$ groups all other components.

The specification of the UCARIMA model has followed two main directions. As mentioned earlier, the STS approach starts by directly specifying the models for the components, and avoids thus identification problems; as a counterpart, it assumes a particular structure for the time series at hand. (Identification of a component is typically ensured by restricting the order of its MA polynomial, $q_i$, to be smaller than that of its AR polynomial, $p_i$.)

To avoid possible misspecification problems, the AMB approach starts by identifying the ARIMA model for the observed $x_t$, and derives the components from the structure of that model. For the "trend + seasonal + irregular" components case, the AMB approach, in essence, does the following. Given the ARIMA model for the observed data (4.7), factorization of the AR polynomial yields the AR polynomials for the component models, which are of the type (4.2) and (4.4). Most often, the model for the seasonal component is given by (4.3) with $w_t$ an MA process of order $(\tau - 1)$, which is exactly the structure a seasonal component should have according to Roberts and Harrison (1984). If the spectra of all components are nonnegative the decomposition is called admissible. For a given observed ARIMA model (4.7), in general there is not a unique UCARIMA representation that can generate it. The AR polynomials can be obtained from the factorization of $\phi$ (B), but the $\theta_i$ - polynomials and the innovation variances $(\sigma_i^2)$ are not identified. The AMB approach solves this underidentification problem by, first, assuming $q_i \leq p_i$. Then it can be seen that the different (admissible) decompositions differ in the way white noise is allocated among the components (see Hillmer and Tiao, 1982, and Bell and Hillmer, 1984). By adding all additive white noise to the irregular component, a unique decomposition is achieved. This decomposition is termed canonical and, in it, all components except the irregular have a spectral minimum of zero, and are thus noninvertible. Hillmer and Tiao (1982) show that the canonical decomposition maximizes the variance of the irregular and minimizes the variance of the other component innovations, providing thus components as stable as possible given the model for the series.

Although the specifications vary, the models in the STS and the AMB approaches are both UCARIMA-type models and are closely related (see Maravall, 1985). Table 1 contains some examples of model specification for monthly series. While in the STS approach the models for the components are parsimonious and the ARIMA model for the observed model is not, the inverse is true for the AMB approach. For the rest of the paper only the UCARIMA structure is of relevance; the additional assumptions made to identify the particular models used for the components play no role.

Table 1

LES OF MODEL SPECIFICATION (Monthly series)

Structural Model (Harvey-Todd, 1984); ARIMA specification.

A-Model-Based decomposition of Airline model (Hillmer-Tiao, 1982)

A-Model-Based interpretation of X11 (Cleveland, 1972)

| | A | B | C |
|---|---|---|---|
| ent | $\nabla^2 p_t = (1+\alpha B) a_{pt}$ | $\nabla^2 p_t = (1+\alpha B)(1+B) a_{pt}$ | $\nabla^2 p_t = (1+.26B+.30B^2-.32B^3) a_{pt}$ |
| | $S(B) s_t = a_{st}$ | $S(B) s_t = \theta_s(B)\, a_{st}$<br>$\theta_s(B)$ of order 11 | $S(B) s_t = (1+.26B^{12})\, a_{st}$ |
| | w.n. | w.n. | w.n. |
| | $\nabla\nabla_{12}\, x_t = \theta(B)\, a_t$<br>$\theta(B)$ of order 13<br>3 parameters | $\nabla\nabla_{12} x_t = (1+\theta_1 B)(1+\theta_{12} B^{12}) a_t$<br>$\theta(B)$ of order 13<br>2 parameters | $\nabla\nabla_{12}\, x_t = \theta(B)\, a_t$<br>$\theta(B)$ of order 14<br>0 parameters |

## 5. ESTIMATION OF THE COMPONENTS

Using the two-component representation of the UCARIMA model, let $s_t$ be the signal of interest and $n_t$ the rest of the series ("the noise"). The model is given by equation (4.1), the models

$$\phi_s(B)\,s_t = \theta_s(B)\,a_{st} \tag{5.1}$$

$$\phi_n(B)\,n_t = \theta_n(B)\,a_{nt} \tag{5.2}$$

where $a_{st}$ and $a_{nt}$ are white noises with variances $\sigma_s^2$ and $\sigma_n^2$, plus assumptions A, B, and C of the previous section.

The model for the observed series is given by (4.7) and the aggregation relationships (4.8) and (4.9) become

$$\phi(B) = \phi_s(B)\,\phi_n(B)$$

$$\theta(B)\,a_t = \phi_n(B)\,\theta_s(B)\,a_{st} + \phi_s(B)\,\theta_n(B)\,a_{nt} \ .$$

Our purpose is, given $X_T$, a particular realization of the time series $x_t$, to obtain the estimator $\hat{s}_t$ such that $E[(s_t - \hat{s}_t)^2 \mid X_T]$ is minimized, i.e., the MMSE estimator of $s_t$. Under the joint normality assumption, $\hat{s}_t$ is also equal to the conditional expectation $E(s_t \mid X_T)$, and hence, a linear function of the elements in $X_T$.

The Model Based Signal Extraction (MBSE) procedure consists of estimating the signal by its MMSE estimator within the UCARIMA framework described above. Since nonnormality should have been dealt with at the preadjustment level, in the paper we shall stick to the normality assumption. (When the series is not normal, the estimators remain the best linear projections.)

### 5.1. Stationary Case

Rewrite the models in their MA expression as $s_t = \psi_s(B)\,a_{st}$, $n_t = \psi_n(B)\,a_{nt}$, and $x_t = \psi(B)\,a_t$, where $\psi_s(B) = \theta_s(B) / \phi_s(B)$, and similarly for $\psi_n(B)$ and $\psi(B)$.

a) Projection on a complete realization $X = [x_{-\infty} \dots x_t \dots x_\infty]$.

Let $j_t$ denote one of the components (hence j = s, n, p, u). For the rest of the paper the ratio of variances $\sigma_i^2/\sigma_a^2$ will be denoted $k_i$ (i.e., $k_s = \sigma_s^2/\sigma_a^2$). Denote by F the "forward" operator, $F = B^{-1}$, such that $F^j x_t = x_{t+j}$ . As shown in Whitle (1963), $\hat{s}_t$

is obtained with the symmetric filter

$$\hat{s}_t = \left[ k_s \frac{\psi_s(B)\psi_s(F)}{\psi(B)\psi(F)} \right] x_t$$

$$= \upsilon\,(B,F)\,x_t \;=\; \left[ \upsilon_0 + \sum_{j=1}^{\infty} \upsilon_j \,(B^j + F^j) \right] x_t \qquad (5.3)$$

The filter $\upsilon(B,F)$ is the so-called Wiener-Kolmogorov (WK) filter.

Let ACGF(z) denote the autocovariance generating function of the variable z, and $g_z(\omega)$ its associated spectrum $(\omega$ is measured in radians and defined in the interval $-\pi < \omega \leq \pi)$. The filter can be expressed as

$$\upsilon(B,F) = ACGF(s_t) \,/\, ACGF(x_t)$$

or, in the frequency domain, as

$$\tilde{\upsilon}(\omega) = g_s(\omega) \,/\, g_x(\omega)\ .$$

The function $\tilde{\upsilon}(\omega)$ is also referred to as the gain of the filter. Thus, for the spectrum of the estimator of the signal

$$g_{\hat{s}}(\omega) = \left[ \frac{g_s(\omega)}{g_x(\omega)} \right]^2 g_x(\omega)\,, \qquad (5.4)$$

so that the squared gain of the filter determines how the variance of the series contributes to the variance of the signal for the different frequencies. Notice that since $g_x(\omega) = g_s(\omega) + g_n(\omega)$, the gain can also be expressed as

$$\tilde{\upsilon}(\omega) = \left( 1 + \frac{1}{r(\omega)} \right)^{-1},$$

where $r(\omega) = g_s(\omega)/g_n(\omega)$ is the signal-to-noise ratio. When for some frequency the signal dominates the noise, $\tilde{\upsilon}$ approaches 1; when the noise dominates the signal, $\tilde{\upsilon}$ approaches zero.

For the two-component model we consider, the WK filter can be expressed after simplification, as

$$\upsilon(B,F) = k_s \frac{\theta_s(B)\phi_n(B)\theta_s(F)\phi_n(F)}{\theta(B)\theta(F)}\ . \qquad (5.5)$$

Notice that invertibility of the model for $x_t$ guarantees convergence of the filter (in B and in F,) irrespectively of the $\phi$ – polynomials.

b) Projection on a finite realization $X_T = [x_1, x_2, \dots, x_T]$.

Having already $\hat{s}_t$, the projection of $s_t$ onto X, we can now project $\hat{s}_t$ onto the subset $[x_1, \dots, x_T]$. One way to do it (Cleveland and Tiao, 1976) is by extending $X_T$ with backcasts and forecasts, and then applying the WK filter to the "extended series". The Burman-Wilson algorithm (Burman, 1980) allows for the full projection to be efficiently computed with just a few forecasts and backcasts. Proceeding in that way yields $\hat{s}_{t|T} = E(s_t \mid X_T)$.

An alternative way of computing $\hat{s}_{t|T}$ is by means of the Kalman Filter (KF); see, Harvey (1993) or Anderson and Moore (1979). First, the model is put into a State Space representation (many are available,) consisting of an observation equation, say, $x_t = H' z_t$, and a transition equation of the type $z_{t+1} = F z_t + G v_t$, where the vectors $z_t$ and $v_t$, and the matrices H, F, G have been appropriately defined. Then the KF is run with starting conditions derived from the marginal distribution of the variables in the model. Finally a smoother is applied (fixed point or fixed interval smoother) to obtain $E(s_t \mid X_T)$. For stationary series, proofs of the equivalence between the WK filter and the KF can be found in Kailath (1976) and in Burridge and Wallis, (1988).

## 5.2. Nonstationary series

By their very nature, concepts such as a trend or seasonality imply a time-varying mean associated with NS series. For example, the sum of the seasonal component over 12 consecutive months should not be far from zero. The model based expression of this condition is given by an expression of the type (4.3), which implies the presence of the S operator in the AR part of the model for the seasonal. The type of nonstationary we consider is the one associated with Unit Roots (UR) in AR polynomials, such as the ones implied for example by a $\nabla\nabla_{12} (= \nabla^2 S)$ differencing. These roots will capture the NS behaviour of trends and of seasonal components.

Bell (1984) shows that under standard assumptions for computing ARIMA forecasts for NS series (see Brockwell and Davis, 1987), the WK filter given by (5.5) still provides the optimal (MMSE) estimator of the signal $s_t$ for the $\infty$ realization X in the NS case. For a finite realization $X_T$, since $X_T$ is a subset of X, it follows that $E(s_t \mid X_T) = E[E(s_t \mid X) \mid X_T] = E(\hat{s}_t \mid X_T)$, and the MMSE estimator of $s_t$ for the finite realization can be obtained by projecting $\hat{s}_t$ onto $X_T$. This is equivalent to replacing the unknowns $x_t$ in X by their forecasts or backcasts (given the observations in $X_T$). Further, the projection onto the finite series $X_T$ can still be obtained following the Burman-Wilson algorithm.

The frequency domain representation of the filter remains also valid, with $g(\omega)$ denoting the pseudo-spectrum. Despite the $\infty$ peaks of $g_s(\omega)$

corresponding to UR in $\phi_s(B)$, $\tilde{\upsilon}(\omega)$ is everywhere well-defined. In fact, (5.5) shows that $\upsilon(B,F)$ is the ACGF of the stationary (finite variance) ARMA model.

$$\theta(B)y_t = [\theta_s(B)\phi_n(B)]\, b_t \,,$$

where $b_t \sim w.n.(0, k_s)$. The gain $\tilde{\upsilon}(\omega)$ is thus the spectrum of this model.

Extension of the Kalman filter approach to signal extraction in NS series poses a problem with starting conditions, since nonstationarity prevents the use of the marginal distributions. Several solutions have been developed; see for example, Harvey (1993), Ansley-Kohn (1985), de Jong (1991), and de Jong and Chu-Chun Lin (1994). Very broadly, starting conditions are modelled as a random vector $(\alpha)$ with an unknown distribution. A modified KF and a modified smoother is applied to the first observations to get rid of the starting conditions, after which the filter collapses to the ordinary KF and smoother. In brief, if $\hat{\alpha}$ denotes an assumed value for the starting conditions, the KF provides the estimator $E(s_t \mid X_T, \hat{\alpha})$. By letting $\hat{\alpha}$ be a GLS projection of $\alpha$ onto $X_T$, the above conditional expectation becomes simply $E(s_t \mid X_T)$, that is, the estimator provided by the WK filter. Thus extension to NS components (and hence NS series) is, under both approaches, straightforward. As in the stationary case, if properly applied, the WK filter and the KF yield the same result (for a general proof, see Gómez, 1998). The WK approach is enforced, for example, in the programs PROPHET (see Burman, 1995) and SEATS. The KF approach is enforced, for example, in the program STAMP.

## 6. HISTORICAL OR FINAL ESTIMATOR

The WK filter given by (5.5) is symmetric and centered, convergent in B and in F and, unless the observed model is a pure AR, the filter will extend from -∞ to +∞. Convergence, however, guarantees that it can always be approximated by a finite 2-sided filter. Although estimation uses the full filter, its finite approximation is useful for discussion. We assume that the WK filter (5.5) can be approximated by the $(2L+1)-$ term centered and symmetric filter:

$$\upsilon(B,F) = \upsilon_0 + \sum_{j=1}^{L} \upsilon_j (B^j + F^j)\,. \qquad (6.1)$$

In practice, for seasonal adjustment, L typically expands between 3 and 5 years; trends usually converge faster. Therefore, when $T > 2L + 1$, final estimators can be assumed for the central observations of the series.

### 6.1 Properties of Final Estimator

From (5.3), (5.5), and (4.7), it is obtained that

$$\phi_s(B)\,\hat{s}_t = k_s\,\theta_s(B)\,\frac{\theta_s(F)\,\phi_n(F)}{\theta(F)}\,a_t\,, \tag{6.2}$$

or, in short,

$$\phi_s(B)\,\hat{s}_t = \theta_s(B)\,\alpha_s(F)\,a_t\,. \tag{6.3}$$

Thus the model generating $\hat{s}_t$ is known. It will prove helpful to write (6.3) in the (symbolic) representation

$$\hat{s}_t = \xi_s(B,F)\,a_t \tag{6.4}$$

where the weights of $\xi_s(B,F)$ can be obtained from the identity

$$\phi_s(B)\,\theta(F)\,\xi_s(B,F) = k_s\,\theta_s(B)\,\theta_s(F)\,\phi_n(F)\,; \tag{6.5}$$

see Maravall (1994). The filter $\xi_s(B,F)$ is divergent in B, and convergent in F; only the part in F will be of relevance.

### 6.2 Component versus estimator

As pointed out in Nerlove, Grether, and Carvalho (1979), the shape of the spectrum of the MMSE estimator of a component in UCARIMA models is different from that of the component. This is a consequence of the fact that, while the component $s_t$ follows model (5.1), its MMSE estimator $\hat{s}_t$ follows model (6.3). Comparison of the two models shows that, although they share the same polynomials in B and the same stationarity inducing transformation, their ACGFs and spectra will be different. The most noticeable differences are the following. First, expression (5.4) can be rewritten as

$$g_{\hat{s}}(\omega) = \left(\frac{g_s(\omega)}{g_x(\omega)}\right) g_s(\omega)\,. \tag{6.6}$$

Since $g_s(\omega)/g_x(\omega) \le 1,$ the estimator will always underestimate the variance of the component. Relatively more stochastic components will imply smaller underestimation, and hence the estimator displays a bias towards stability.

The second noticeable difference between the component and estimation spectra is the presence of "dips" in the spectrum of the estimator. In the usual case of a seasonal component satisfying (4.3), from (5.2) and (6.3),

$$\alpha_s(F) = k_s \frac{\theta_s(F) S(F)}{\theta(F)} .$$

Thus the unit roots in S will show up as unit MA roots in the model generating $\hat{s}_t$ and will produce spectral zeroes for the associated seasonal frequencies. The frequency domain derivation also explains the appearance of the spectral zeroes in the estimator model. Consider the case where the signal $s_t$ is the SA series and the noise is a NS seasonal component. Let $\omega_0$ denote a seasonal frequency; then $g_s(\omega_0)$ is finite, while $g_n(\omega_0) \rightarrow \infty$, and from (5.4),

$$g_{\hat{s}}(\omega_0) = \frac{g_s(\omega_0)^2}{g_s(\omega_0) + g_n(\omega_0)}$$

will be zero. These spectral zeroes are the frequency counterpart of the unit MA roots. More generally, the spectral zeroes in the spectrum of the estimator of the SA series will be a feature of any method that removes a nonstationary seasonal component.

The difference between the models for the signal and for its estimator has some relevant implications. The first one has to do with the standard practice of building models on seasonally adjusted data. This practice is based on the belief that, by removing seasonality, model dimensions can be reduced. Yet this belief is unjustified. While the model for the SA series $s_t$ is of the type (5.2), the estimator of the SA series, $\hat{s}_t$, has the structure (6.3), more complicated than the one for $s_t$ or $x_t$. Table 2 compares the MA expansions of (the stationary transformation of) the three variables $x_t$, $s_t$, and $\hat{s}_t$ for the model $\nabla\nabla_{12} x_t = (1 - .4B)(1 - .6B^{12}) a_t$, a relatively common model, and one for which AMB seasonal adjustment yields results similar to those of X11 (Cleveland and Tiao, 1976). As seen in the table, which only lists the lags for which there are some nonzero coefficients, relatively high coefficients may appear for large lags (as was actually detected in Ghysels and Perron, 1993).

Table 2

| LAG | ORIGINAL SERIES | SA | SA SERIES |
|---|---|---|---|
| 1 | -.4 | -1.37 | -1.33 |
| 2 | 0 | .39 | .38 |
| 12 | -.6 | 0 | -.40 |
| 13 | .24 | 0 | .53 |
| 14 | 0 | 0 | -.15 |
| 24 | 0 | 0 | -.24 |
| 25 | 0 | 0 | .32 |
| 26 | 0 | 0 | -.09 |

Hence no reduction in dimension can be expected from using the SA series.

As for the second implication, consider the difference between (5.1) and (6.3), that is the factor $\alpha_s(F)$. Direct inspection shows that when $n_t$ is NS or $s_t$ noninvertible, $\alpha_s(F)$ will induce unit MA roots, and hence the estimator will be a non-invertible series. In particular, the estimator $\hat{s}_t$ is noninvertible if $n_t$ is NS, and $\hat{n}_t$ is noninvertible if $s_t$ is NS. Hence in a standard trend + seasonal + irregular decomposition, with NS trend and NS seasonality, the estimators of the 3 components, as well as that of the SA series, will be noninvertible. An important consequence of the previous result is that the estimators of the SA series, trend, seasonal and irregular components will not accept, in general, an AR (or VAR) approximation to its Wold representation.

The third implication is that, in the MBSE approach, knowledge of the theoretical model for the optimal estimator offers a natural tool for additional diagnostics. To illustrate the point we use as example the white-noise $(0, \sigma_u^2)$ irregular component. Proceeding as before, the model for its MMSE estimator $\hat{u}_t$ is found to be the "inverse" model of the ARIMA model for the series (see Bell and Hillmer, 1984), that is

$$\theta(F)\,\hat{u}_t = \phi(F)\,a'_t, \qquad (a'_t = k_u a_t). \tag{6.7}$$

In practice, $\hat{u}_t$ is obtained as the residual, once the other components have been estimated. But, if, in an application, the irregular is to be used for residual diagnostics, its ACF and variance should not be compared to those of the component $u_t$, but to those of the theoretical estimator, given by model (6.7). Large departures from white noise in the ACF of $\hat{u}_t$ may be acceptable. Significant differences, however, between the theoretical and empirical ACF of $\hat{u}_t$ would indicate misspecification. The structure of the differences, besides, may provide a clue as to the type of misspecification. For example, if the theoretical ACF of the stationary transformation of the trend has $\rho_1 = -.4$, positive autocorrelation for low lags in the empirical ACF would clearly point towards underestimation of the trend (see Maravall, 1987).

## 6.3 Covariance between estimators

The models for $\hat{s}_t$ and $\hat{n}_t$ can also be used to derive the joint distribution of the estimators. In particular, the Crosscovariance Generating Function (CCGF) for a stationary series is straightforward to obtain from (6.2) and the equivalent expression for $\phi_n(F)\,\hat{n}_t$. It is seen that $\mathrm{CCGF}(\hat{s}_t, \hat{n}_t)$ is the ACF of the ARIMA model

$$\theta(B)\,y_t = \theta_s(B)\,\theta_n(B)\,b_t, \tag{6.8}$$

with $b_t$ white noise, and $\sigma_b^2 = k_s k_n$. Thus the CCGF is symmetric and the lag-0 covariance between the estimators will always be positive, despite the fact that the theoretical components are orthogonal. This positive covariance between the estimators is the time-domain explanation of the underestimation of the components covariance mentioned in Section 6.2.

Expression (6.8) does not contain the AR polynomials $\phi_s$ (B) and $\phi_n$ (B). If, say, the first polynomial contains one (or more) unit root, we proceed as follows: First, replace this root by one with the same frequency and modulus m<1, and denote by $\hat{s}_t(m)$ the estimator obtained after having replaced the root. By defining

$$CCGF(\hat{s}_t, \hat{n}_t) = \lim_{m \to 1} CCGF\left(\hat{s}_t(m), \hat{n}_t\right),$$

expression (6.8) is once more obtained. In this sense, in the standard case of NS trend and seasonal components, since the two estimators cannot be cointegrated (the unit AR roots are different), they will diverge in time, each one with a NS variance, but their covariance will remain stationary. In practice, thus, the crosscorrelation between the estimates of NS components will typically be small. Finally, as was the case with the autocovariances, comparison between the crosscovariances of the theoretical estimators and of the estimates actually obtained may provide an additional tool for diagnostics.

## 7. ESTIMATORS FOR RECENT PERIODS

The properties of the estimators have been derived for the final (or historical) estimators. For a finite (long enough) realization, they can be assumed to characterize the estimators for the central observations of the series, but for periods close to the beginning or the end, the filter cannot be completed and some preliminary estimator has to be used. Let the observed series be $X_T = [x_1 \ldots x_t \ldots x_T]$. As shown by Cleveland and Tiao (1976), the MMSE signal estimator (given $X_T$) can be expressed as

$$\hat{s}_{t|T} = \nu(B,F)\, \hat{x}^e_{t|T} ,$$

where $\nu(B,F)$ is the WK filter (5.5), and $\hat{x}^e_{t|T}$ denotes the series extended with forecasts and backcasts. Seasonal adjustment of the time series $X_T = [X_1, \ldots, X_T]$ yields the SA series $[\hat{s}_{1|T}, \ldots, \hat{s}_{T|T}]$, where $\hat{s}_{j|T}$ denotes the estimator of $s_j$ obtained with $X_T$. Using the finite filter approximation (6.1), assume that $T > 2L + 1$, so that the estimator for the central observations of the series can be considered final, and that the part in B of the filter can be completed when applied to the last observation. (Thus, for the second half of the series we can ignore starting conditions and the estimator of the signal can be seen as the projection onto the semi-infinite realization $[x_{-\infty}, \dots, x_{T-1}, x_T]$ .) This projection will be represented by the operator $E_T$.

We center on preliminary estimators for recent periods. Let $k = T - t$, $0 \le k < L$; applying $E_T$ to expression (6.1),

$$\hat{s}_{T-k|T} = \upsilon_L x_{T-L-k} + \dots + \upsilon_k x_T + \upsilon_{k+1} \hat{x}_{T+1|T} + \dots + \upsilon_L \hat{x}_{T+L-k|T} ,$$

or, since $\hat{x}_{T+j|T} = \pi_1^{(j)} x_T + \pi_2^{(j)} x_{T-1} + \dots ,$ in terms of the observations,

$$\hat{s}_{T-k|T} = \left[\upsilon_k + \sum_{j=1}^{L-k} \upsilon_{k+j}\, \pi_1^{(j)}\right] x_T + \left[\upsilon_{k-1} + \sum_{j=1}^{L-k} \upsilon_{k+j-1}\, \pi_2^{(j)}\right] x_{T-1} + \dots ,$$

and hence the preliminary estimator can be expressed as

$$\hat{s}_{T-k|T} = \upsilon_k(B,F,k)\, x_{T-k} = \sum_{j=-L}^{k} \upsilon_{jk}\, x_{T-k+j,} \ ,$$

where $\upsilon_k(B,F,k)$ is finite and asymmetric, of degree L in B and k in F. The coefficients $\upsilon_{jk}$ depend on k, as does the length of the filter. It follows that the models that generate the different preliminary estimators $(\hat{s}_{t|t}, \hat{s}_{t|t+1}, \dots, \hat{s}_{t|t+L-1})$ will all be different, different also from the model for the final estimator, given by (6.3), and from the model for the component, given by (5.1). Bell (1995) shows, for example, that the model for the concurrent estimator is always of the form

$$\phi_s(B)\, \hat{s}_{t|t} = \lambda(B)\, a_t ,$$

where the order of $\lambda(B) = \max(p_s - 1, q_s)$. It is worth noticing that estimators and component share the AR polynomial $\phi_s(B)$, and hence the same stationary transformation. They differ in the MA part.

As a consequence, the SA series available at a certain time, $\left[\hat{s}_{1|T}, \dots, \hat{s}_{t|T}, \dots, \hat{s}_{T|T}\right]$, are non-homogenous. The elements at the beginning, at the end, and in the middle of the series are generated by different models; the SA series has a nonlinear structure, with time-varying parameters (for other nonlinearities in SA series, see Ghysels, Granger, and Siklos, 1996).

As a simple example, consider the UCARIMA model with $\nabla s_t = a_{st}$ and $n_t$ white noise (a "random-walk plus noise" model). Trivially the model for $x_t$ is $\nabla x_t = (1 + \theta B) a_t$, $-1 < \theta < 0$, and the parameters $\theta$ and $\sigma_a^2$ are determined from $(1 + \theta B) a_t = a_{st} + \nabla n_t$. The model for the component $s_t$ is an IMA (1,0); the model for the final estimator is, from (6.2), the ARIMA (1,1,0) model

$$(1+\theta F)\nabla \hat{s}_t = k_s a_t , \tag{7.1}$$

while for the concurrent estimator the model is also an IMA(1,0), with the innovation a constant fraction of $a_t$.

## 8. REVISIONS IN THE ESTIMATOR

### 8.1 The Structure of the Revision

Starting with the concurrent estimator, $\hat{s}_{t|t}$, as new observations become available the estimator of $s_t$ is revised, yielding the sequence $(\hat{s}_{t|t}, \hat{s}_{t|t+1}$, ..., $\hat{s}_{t|t+k}$, ...). As $k \to \infty$ (in practice, $k > L$) $\hat{s}_{t|t+k}$ converges to $\hat{s}_t$, the final or historical estimator. To look at the revision the concurrent estimator will undergo, write expression (6.4) as

$$\hat{s}_t = \xi_s(B)^- a_t + \xi_s(F)^+ a_{t+1} . \tag{8.1}$$

When $x_t$ is the last observation, the first term in (8.1) contains the effect of the starting conditions and of the present and past innovations in the series. The second term reflects the effect of future innovations. Taking conditional expectations at time t, $\hat{s}_{t|t} = \xi_s(B)^- a_t$ and the revision in the concurrent estimator $(\hat{s}_t - \hat{s}_{t|t})$ is given by

$$r_t = \xi_s(F)^+ a_{t+1} , \tag{8.2}$$

a zero-mean stationary process. Hence the distribution of $r_t$ can be derived. Similar derivation applies to other preliminary estimators, $\hat{s}_{t+k|t}$, including forecasts; see Pierce (1980).

For the random walk plus noise example, from the identity

$$\frac{1}{\nabla(1+\theta F)} = \frac{1}{1+\theta}\left(\frac{1}{1-B} - \frac{\theta F}{1+\theta F}\right),$$

we can write, considering (7.1),

$$\hat{s}_t = c\left[\frac{1}{1-B} - \frac{\theta F}{1+\theta F}\right] a_t ,$$

where $c = k_s/(1+\theta)$. Therefore,

$$\xi_s(F)^+ = -c\,\theta/(1+\theta F) , \tag{8.3}$$

and, from (8.2),

$$(1 + \theta F)\, r_t = a'_t\,, \qquad a'_t = -c\,\theta\, a_{t+1}\,. \tag{8.4}$$

Hence the revision $r_t$ has the ACF of a stationary AR(1) process.

## 8.2 Optimality of the Revisions

Revisions in preliminary estimators are implied by the use of a two-sided filter, as in

$$\hat{s}_t = \ldots + \upsilon_1 x_{t-1} + \upsilon_0 x_t + \upsilon_1 x_{t+1} + \upsilon_2 x_{t+2} + \ldots\,. \tag{8.5}$$

Starting with the concurrent estimator, if the observations are $[x_1, \ldots, x_t]$,

$$\hat{s}_{t|t} = \ldots + \upsilon_0 x_t + \upsilon_1 \hat{x}_{t+1|t} + \upsilon_2 \hat{x}_{t+2|t} + \ldots\,, \tag{8.6}$$

and when the new observation $(x_{t+1})$ arrives, the revised estimator is

$$\hat{s}_{t|t+1} = \ldots + \upsilon_0\, x_t + \upsilon_1\, x_{t+1} + \upsilon_2\, \hat{x}_{t+2|t+1} + \ldots\,,$$

and so on. Two-sided filters are necessary to avoid phase effects; they are also implied by MMSE ("optimal") estimation of the components. Of course, to revise series is always disturbing and an inconvenience, and revisions can indeed be large (for a case study, see Maravall and Pierce, 1983). But revisions simply reflect the fact that knowledge of the future will help in understanding the present, a very basic fact of life. (Concurrent estimators are, like "first impressions", usually insufficient to form an accurate judgement). Thus revisions are necessary, and to suppress them is to ignore relevant information, to refuse to improve our knowledge, and to distort our timing of events.

From (8.5) and (8.6), the revision $r_t = \hat{s}_t - \hat{s}_{t|t}$ can be expressed as

$$r_t = \upsilon_1 (x_{t+1} - \hat{x}_{t+1|t}) + \upsilon_2 (x_{t+2} - \hat{x}_{t+2|t}) + \ldots = \sum_{j=1}^{\infty} \upsilon_j\, e_t(j) \tag{8.7}$$

where $e_t(j)$ is the j-th-period-ahead forecast error of the series. Expression (8.7) shows that the revision depends on the forecast errors and the weights of the WK filter. This justifies the interest in "small" forecast errors (in essence, the rationale behind the X11ARIMA modification of X11), but revisions still depends on the $\upsilon_j$´s, which depend, in turn, on the stochastic structure of the series (i.e., on the ARIMA model). For some series, the revisions should be large; for other series, they should be small. Also, for some series the revisions will last long; for others, they will disappear fast. Thus, for a given series, there is an appropriate amount of revision. The revision should not be larger than that, nor should it be smaller.

In the MBSE approach, the revisions are "optimal" (both, in terms of size and duration) in the following way. They are implied by optimal ( MMSE ) forecasting, and optimal ( MMSE ) estimation of the components. Since the former implies minimum forecast errors, revisions will tend to be small. But the vague (and often made) recommendation of "small revisions" should be replaced by that of "optimal revisions", associated with optimal estimation of the components.

## 9. INFERENCE

### 9.1 Optimal forecasts of the components

Similarly to the case of preliminary estimation, the k-periods-ahead forecast is given by

$$\hat{s}_{T+k|T} = \ldots \ \nu_k x_T + \nu_{k-1} \hat{x}_{T+1|T} + \ldots \ + \nu_0 \hat{x}_{T+k|T} + \ldots + \nu_L \hat{x}_{T+k+L|T} \,,$$

hence, in practice, one simply needs to further extend the series with some additional ARIMA forecasts. The properties of the forecast error $s_{T+k} - \hat{s}_{T+k|T}$ can be obtained in exactly the same way as the error in the preliminary estimator which we discuss in the next section. Since, on occasion, one may wish to forecast the trend rather than the original series, a convenient feature of the MBSE method is that it provides optimal forecasts of the components, as well as their associated MSE.

### 9.2 Estimation error

An issue of considerable applied concern has been to obtain a measure of the precision of the component estimator, in particular of the SA series (see Bach et al, 1976; Moore et al, 1981; Bank of England, 1992). This need is specially felt for key variables that are (explicitly or implicitly) being subject to some type of targeting (for example, a monetary aggregate or a consumer price index). In these cases, intrayear monitoring and policy reaction is based on the SA series (for an example, see Maravall, 1988). We consider now the precision of the concurrent, successively revised, and final estimators, and of the forecasts. Bell and Hillmer (1984), Burridge and Wallis (1985), and Hillmer (1985) have shown how to obtain standard errors for the component MMSE estimators in UCARIMA models of the type we consider. Here we sketch how, under the semi-∞ realization assumption, the models for the errors can be obtained and used in inference.

Because of the stochastic nature of $s_t$, its final estimator $\hat{s}_t$ contains an error, $e_t = s_t - \hat{s}_t \quad (= \hat{n}_t - n_t)$, to be denoted "final estimation error". Although $e_t$ is unobservable, it can be seen (Pierce, 1979) as the output of the stationary ARMA model (6.8). Therefore the distribution of $e_t$ is easily obtained. Since the ACGF of $e_t$ is identical to the CCGF of the estimators $\hat{s}_t$ and $\hat{n}_t$, the final estimation error variance is equal to the lag-0 covariance between the estimators.

For the concurrent estimator, the one of most applied relevance, let $r_T = \hat{s}_T - \hat{s}_{T|T}$ denote the "revision error"; we already saw how its distribution can be obtained. The total estimation error, $\varepsilon_T$, is

$$\varepsilon_T = s_T - \hat{s}_{T|T} = (s_T - \hat{s}_T) + (\hat{s}_T - \hat{s}_{T|T}) = e_T + r_T .$$

Since $e_T$ and $r_T$ are orthogonal (Pierce, 1980), the model for $\varepsilon_T$ is immediately obtained.The derivation of the model for the error in any preliminary estimator or forecast, $\varepsilon_{t|T} = s_t - \hat{s}_{t|T}$ can be done in an identical manner. From this model, the variance and ACF of the error can be obtained.

For a given ARIMA model for the observed series, analytical expressions for the component estimation error as a function of the particular decomposition chosen is found in Maravall and Planas (1996).

## 9.3 The precision of the rates of growth

Short-term analysis of the evolution of economic variables, as well as the setting of targets, is often based on rates of growth, rather than levels. Assume we wish to obtain the MSE of the error in the concurrent estimator of the rate of growth over the last m months of a SA series. Since more often than not, ARIMA models are appropriate for the log of macroeconomic time series, let $S_t$ = SA series and $s_t = \log(S_t)$. The rate of growth of the SA series over the last m months is given by $R_t = (S_t - S_{t-m}) / S_{t-m}$. Using the linear approximation $R_t = s_t - s_{t-m}$, the concurrent estimator of $R_t$ is $\hat{R}_{t|t} = \hat{s}_{t|t} - \hat{s}_{t-m|t}$. To compute the estimation error variance, consider the identity

$$\hat{s}_{t|t} - \hat{s}_{t-m|t-m} = (\hat{s}_{t|t} - \hat{s}_{t-m|t}) + (\hat{s}_{t-m|t} - \hat{s}_{t-m|t-m}) \qquad (9.1)$$

The left hand side is the difference in two concurrent estimators. Let $\varepsilon_t$ and $\varepsilon_{t-m}$ be the associated estimation errors. We saw how to derive their variance and ACF. As for the right hand side, the first term is $\hat{R}_{t|t}$, and the second term is the m-period revision in the concurrent estimator.

Replacing t by t-m in (8.1), letting $\xi(F)^+ = \sum_{i=0}^{\infty} \xi_j F^j$, and applying the operators $E_{t-m}$ and $E_t$ yields, after simplification,

$$\hat{s}_{t-m|t} - \hat{s}_{t-m|t-m} = \sum_{i=1}^{m} \xi_{i-1} a_{t-m+i} ,$$

and the identity (9.1) can be rewritten as

$$\hat{s}_{t|t} - \hat{s}_{t-m|t-m} = \hat{R}_{t|t} + \sum_{i=1}^{m} \xi_{i-1} a_{t-m+i} . \tag{9.2}$$

Denote the error of interest by $D_t = \hat{R}_{t|t} - R_t$. Subtracting $s_t - s_{t-m}$ from both sides of (9.2) yields

$$\varepsilon_t - \varepsilon_{t-m} = D_t + \sum_{i=1}^{m} \xi_{i-1} a_{t-m+i} . \tag{9.3}$$

Because $D_t$ is a function of $a_{t+j}$, $j > 0$, the two terms in the right hand side of (9.3) are orthogonal, and hence

$$\sigma^2(D_t) = 2\sigma^2_\varepsilon(1 - \rho^\varepsilon_m) - \sum_{i=1}^{m} \xi^2_{i-1}\sigma^2_a , \tag{9.4}$$

where $\rho^\varepsilon_m$ denotes the m - lag autocorrelation of $\varepsilon_t$. Expression (9.4) can be derived from the UCARIMA model; in the AMB approach, simply from the ARIMA model for the series.

As an example, consider the random-walk plus noise model, and assume we are interested in $\sigma^2(D)$ for m = 1, i.e., in the variance of the error in the measurement of the signal rate of growth for the last period.

To compute (9.4) we need $\sigma^2_\varepsilon$, $\rho^\varepsilon_m$ and $\xi_0$. The models for the uncorrelated $r_t$ and $e_t$ processes are (8.4) and, from (6.8), $(1 + \theta F) e_t = b_t$, with $\sigma^2_b = k_s k_n$. From these two AR(1) models one trivially obtains $\sigma^2_e$, $\gamma^e_1$, $\sigma^2_r$, $\gamma^r_1$ where $\gamma^e_1$ and $\gamma^r_1$ are the lag-1 autocovariances of $e_t$ and $r_t$, respectively. Thus $\sigma^2_\varepsilon = \sigma^2_e + \sigma^2_r$, $\rho^\varepsilon_1 = (\gamma^e_1 + \gamma^r_1)/\sigma^2_\varepsilon$, and $\xi_0$ is the coefficient in the expansion of (8.3), that is, $\xi_0 = -\theta k_s/(1 + \theta)$.

## 9.4 The gain from concurrent adjustment

A point of concern for data-producing agencies is the frequency at which seasonal adjustment should be performed. Since concurrent adjustment is costly and implies changing the data frequently, seasonal adjustment is often performed once a year (or twice a year), and forecasted seasonal factors are used until the next seasonal adjustment is done.

Naturally, the use of forecasted factors increases the MSE of the SA series, and the question of what would be gained in practice moving from a once a year adjustment to a concurrent one is important. The MBSE approach provides a simple answer to the question. From (8.1), it is seen that

$$MSE(\hat{s}_{t+k|t} - \hat{s}_{t|t}) = \sigma_a^2 \sum_{j=0}^{k-1} \xi_{-j}^2 ,$$

where $\xi_0, \dots, \xi_{-k+1}$ are the first k coefficients in the polynomial $\xi_s(B)^+$. Thus the loss in precision due to the use of forecasted factors can be easily measured.

### 9.5 Innovations in the components (pseudo-innovations)

If the UCARIMA model parameters are known and the semi-∞ realization is considered, then $a_t$, the 1-period-ahead forecast error of $x_t\ (= x_t - \hat{x}_{t|t-1})$ is eventually observed. But since $s_t$ and $n_t$ are never observed, neither will be $a_{st}$ and $a_{nt}$, the innovations in the components of the models (5.1) and (5.2). We refer to them as "pseudo-innovations" (Harvey and Koopman, 1992, use the term "cuasi-residuals"). Although unobservable, their MMSE estimators can be obtained. Taking conditional expectations in (5.1) yields

$$\phi_s(B)\hat{s}_t = \theta_s(B)\hat{a}_{st} , \tag{9.5}$$

where $\hat{a}_{st} = E(a_{st} \mid X)$. Using (5.2), (9.5) can be expressed as

$$\hat{a}_{st} = k_s \frac{\theta_s(F)\phi_n(F)}{\theta(F)} a_t , \tag{9.6}$$

Compared to (5.5), expression (9.6) shows that the filter that provides the MMSE estimator of the standarized pseudo-innovation $\hat{a}_{st}/\sigma_s$ is the one-sided WK filter for obtaining $\hat{s}_t$. In other words, $ACF\left(\hat{a}_{st}/\sigma_s\right) = \upsilon_s(B,F)$, therefore, although $a_{st}$ is white noise, the estimator $\hat{a}_{st}$ can be highly correlated. Care should be taken thus when interpreting the series $\hat{a}_{st}$; for example, when testing for randomness of $a_{st}$ or for detecting outliers. For the semi-∞ realization, applying $E_T$ to (9.6) yields

$$\theta(F)\hat{a}_{st|T} = k_s \theta_s(F)\phi_n(F)\hat{a}_{t|T} ,$$

where $\hat{a}_{t|T} = a_t$ when $T \geq t$, and 0 otherwise.Therefore, the concurrent estimators are given by $\hat{a}_{st|t} = k_s a_t$, and $\hat{a}_{nt|t} = k_n a_t$, so that both are a fraction of the series innovation. It is worth noticing that the models for the final estimation error, the revision error, the irregular estimator, and the p-innovation estimator, all have $\theta(F)$ as the AR polynomial. As a consequence, as a general rule, large MA roots in the model for the observed series are associated with slowly converging revisions, and highly autocorrelated irregular and p-innovations.

## 10. AN EXAMPLE

We consider, as an example, the quarterly series of the Spanish Industrial Production Index (IPI) for the period 1981/1 - 1997/1; the series is displayed in Figure 3a. To specify the UCARIMA model following the AMB approach we start with the ARIMA model for the observed series. A good fit is provided by the model

$$\nabla\nabla_4 x_t = (1 - .11B)(1 - .96B^4) a_t, \quad (\sigma_a = 2.03).$$

Direct inspection of the MA parameters indicates the presence of a fairly stochastic trend and a very small or very stable seasonality. In the factorization of the AR polynomial,

$$\nabla\nabla_4 = \nabla^2 S, \qquad (S = 1 + B + B^2 + B^3),$$

the factors $\nabla^2$ and S imply the presence of a trend and a seasonal component, respectively. Therefore we can decompose the series into

$$x_t = p_t + n_t + u_t,$$

where $\phi_p(B) = \nabla^2$, $\phi_n(B) = S$, $u_t$ is white noise, and $\theta_p(B)$ and $\theta_n(B)$ are polynomials in B of degree 2 and 11, respectively, which satisfy the identity

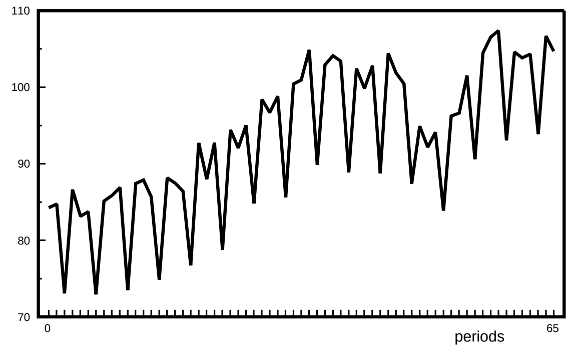
$$(1 - .11B)(1 - .96B^4) a_t = \theta_p(B) S a_{pt} + \theta_n(B) \nabla^2 a_{nt} + \nabla\nabla_4 u_t.$$

A simple and efficient procedure to obtain the canonical decomposition (with noninvertible trend and seasonal components) is given in Burman (1980), using a partial fraction expansion of the model in the frequency domain. Easy procedures to compute the ACGF of an ARMA model and to factorize the spectrum of an MA model are given in Box, Hillmer and Tiao (1978) and in Maravall and Mathis (1994), respectively. The UCARIMA model obtained is given by

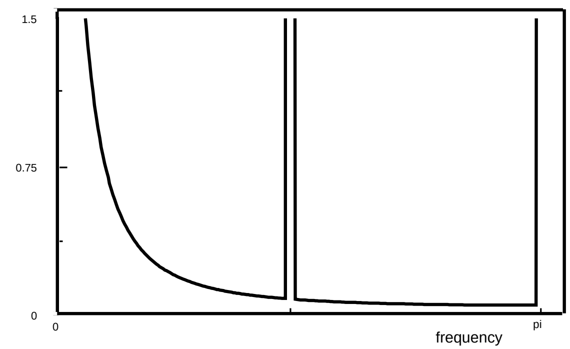
$$\nabla^2 p_t = (1 + .01B - .99B^2) a_{pt}, \quad (k_p = .19),$$

$$S n_t = (1 + .50B - .35B^2 - .94B^3) a_{nt}, \quad (k_n = .0001)$$

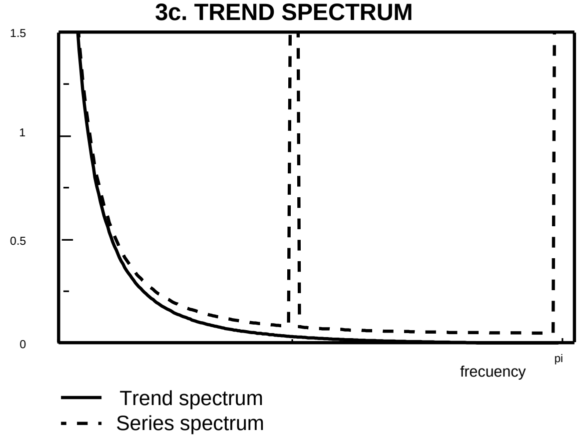
**3a. ORIGINAL SERIES**



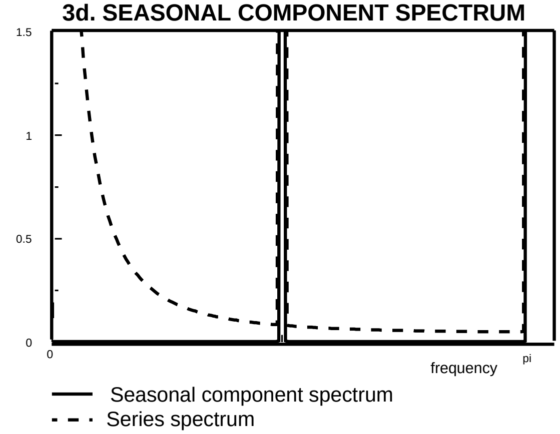

**3b .SERIES SPECTRUM**




**3c. TREND SPECTRUM**




**3d. SEASONAL COMPONENT SPECTRUM**



and, for, the irregular component $k_u = .30$. Notice that $\theta_p(B=-1)=0$, which implies a spectral zero for the trend at the $\pi$ (twice-a-year) frequency, while the seasonal component displays a spectral zero for a frequency between the two seasonal ones. Looking at the variance of the component innovations, it is clear that seasonality will be very stable, the trend fairly stochastic, and the irregular relatively important. The SA series, equal to $(p_t + u_t)$, follows the model

$$\nabla^2 s_t = (1 - 1.10B + .11B^2)\, a_{st}, \quad (k_s = .97), \tag{10.1}$$

which can be expressed as

$$\nabla^2 s_t = (1 - .11B)(1 - .99B)\, a_{st},$$

and hence the model is seen to be very close to the random-walk-plus-drift process. Further, since $k_s$ is close to 1 seasonal adjustment will not reduce much the stochastic nature of the series. The spectra of the series, the trend, and the seasonal component are displayed in Figures 3b, c, and d.

From (5.5), the WK filters to obtain the final estimators of the SA series and seasonal component are given by

$$\nu_s(B,F) = .97 \, \frac{||(1 - 1.10B + .11B^2)S||^2}{||(1 - .11B)(1 - .96B^4)||^2},$$

$$\nu_n(B,F) = .0001 \, \frac{||(1 + .50B - .35B^2 - .94B^3)\nabla^2||^2}{||(1 - .11B)(1 - .96B^4)||^2},$$

where, if $\varrho$ (B) denotes a polynomial in B, $||\varrho(B)||^2 = \varrho(B)\varrho(F)$. The two filters and the associated squared gains are displayed in Figure 4. The narrowness of the dip for the gain function of the SA series and of the peak for the gain of the seasonal component reflects the fact that the seasonality in the series is of a highly stable nature.

Using expression (6.2), the process generating the estimator of the SA series is given by:

$$(1 - .11F)(1 - .96F^4)\nabla^2 \hat{s}_t = .97(1 - 1.10B + \\ + .11B^2)(1 - 1.10F + .11F^2)S(F)\, a_t. \tag{10.2}$$

The spectrum of the SA series ($s_t$) and of its estimator $(\hat{s}_t)$ are shown in Figure 5a. It is seen how estimation induces spectral zeros for the seasonal frequencies, and hence noninvertibility of the estimator. The associated spectral dips imply a very small underestimation of the variance of the seasonal component. In particular, from (10.1) and (10.2), $Var(\nabla^2 s_t) = 2.17\sigma_a^2$, while $Var(\nabla^2 \hat{s}_t) = 2.12\sigma_a^2$.

The spectrum of the irregular component estimator is shown in figure 5b. Given that the theoretical irregular component is white noise, MMSE is seen to distort considerably its spectrum. The variance underestimation is now more pronounced and Table 3 also exhibits the value of the lag-1 and lag-4 autocorrelations ($\rho_1$ and $\rho_4$) for the irregular component, its theoretical MMSE estimator, and the estimate actually obtained. (One year has been removed at both end of the series to decrease the distortion due to preliminary estimators).

Table 3

IRREGULAR COMPONENT

Comparison of the second moments

| | COMPONENT | MMSE ESTIMATOR | ESTIMATE |
|---|---|---|---|
| $\rho_1$ | 0 | -.44 | -.52 |
| $\rho_4$ | 0 | .02 | .07 |
| Variance (in units of $\sigma_a^2$) | .30 | .16 | .15 |

MMSE estimation induces a negative and large lag-1 autocorrelation. Comparison of the theoretical estimator of the irregular with the estimate actually obtained (computed as the residual) can be used as a diagnostic tool; the close agreement between estimator and estimate points towards validation of the results.

Expression (6.8) can be used to derive the covariance between the component estimators. Table 4 displays the correlations between the (stationary transformations) of the estimators and of the estimates actually obtained.

Table 4

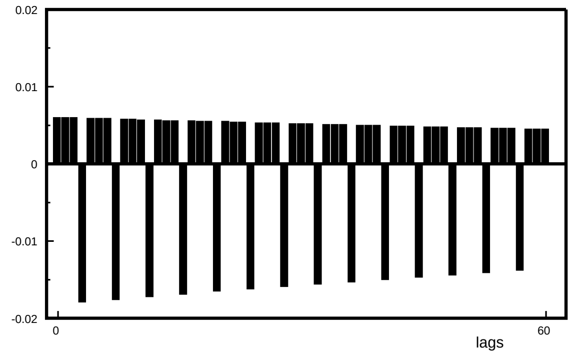
CORRELATION BETWEEN ESTIMATORS

| | Trend and Seasonal Component | Seasonal and Irregular Components | Trend and Irregular Component |
|---|---|---|---|
| Estimator | -.06 | .03 | -.04 |
| Estimate | -.09 | .08 | .01 |

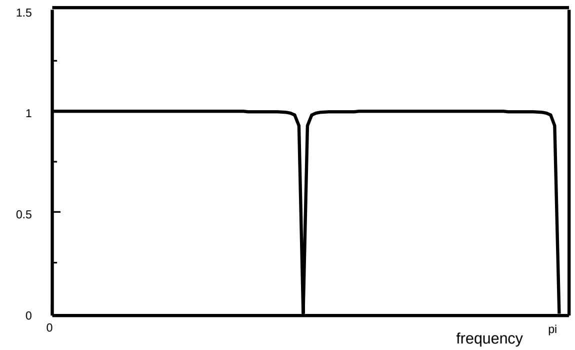
The correlations are, in all cases, negligible, the estimator and estimate provide, again, similar results.

As for the estimation errors, the variance of model (6.8), particularized for the three components, yields the variance of the final estimation error. To look at the

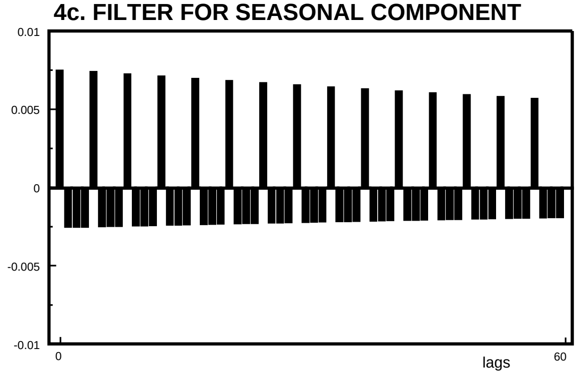
**4a. SEASONAL ADJUSTMENT FILTER**



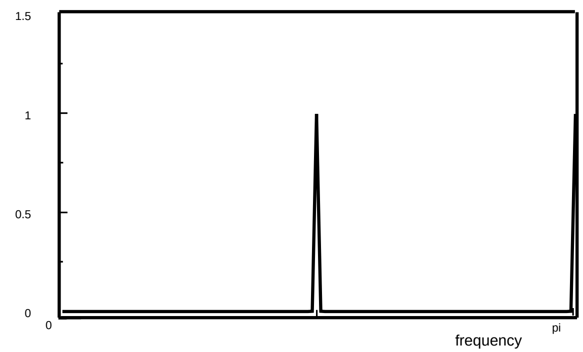

**4b. SQUARED GAIN OF SEASONAL ADJUSTEMENT FILTER**



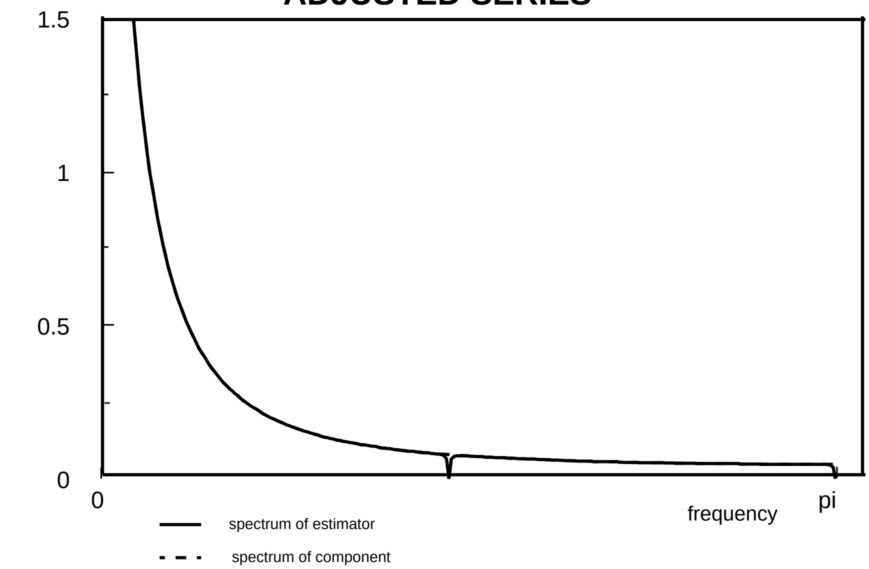

**4c. FILTER FOR SEASONAL COMPONENT**



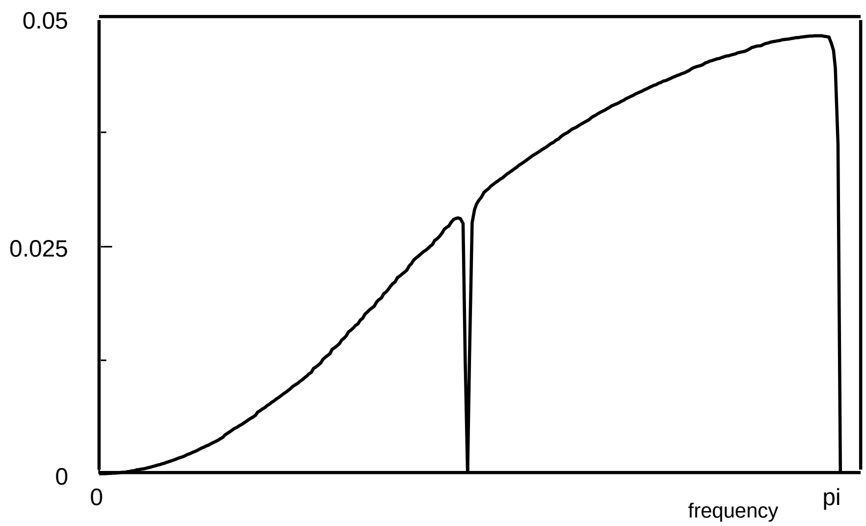

**4d. SQUARED GAIN OF FILTER FOR SEASONAL COMPONENT**



**5a. SPECTRUM OF MMSE ESTIMATOR OF ADJUSTED SERIES**




**5b. SPECTRUM OF MMSE ESTIMATOR OF IRREGULAR COMPONENT**



revision errors, the weights of the filter $\xi(B,F)$ can be obtained through (6.5). Table 5 presents the estimation error variances of the trend and SA series, for the final and concurrent estimators.

Table 5

ESTIMATION ERROR VARIANCE

(in units of $\sigma_a^2$)

| | Final Estimation Error | Revision Error | Concurrent Estimation Error | % Reduction in revision S.E. after one year of data |
|---|---|---|---|---|
| Trend | .13 | .08 | .21 | 91 |
| SA series | .01 | .01 | .02 | 4 |

Because of the close-to-deterministic nature of the seasonal component, the estimation error of the SA series, be that the final estimation error or the revision, is very small. The error in estimating the trend is larger due to the fact that a relatively important irregular component has been removed. Still, the revision is of a moderate size, and the variance of the error in the concurrent estimator is approximately 1/5 of the innovation variance of the series. Concerning convergence of the revision, as is typically the case, the very small revision in the concurrent estimator of the SA series converges very slowly, while in just one year the trend has practically converged to the final estimator. The slow convergence of the SA series estimator to the final estimator suggests that very little would be gained from moving from a once-a-year adjustment to a concurrent one, and in fact the average decrease in Root-MMSE would be 1.5%. For this series, infrequent adjustment would imply little loss in precision for the SA series.

Figure 6a displays the last two years of observations for the series and the next two years of forecasts, with the associated 95% confidence intervals. Figures 6b and 6c exhibit for the trend and seasonal component, the estimates for the last two years and the forecasts for the next two years, together with the 95% confidence interval. Seasonality is seen to be highly significant and stable, and its forecast fairly precise. As for the trend, although the forecasts are more precise than those of the original series, they deteriorate fast and would only be useful for short-term horizons.

Finally, analysis of the short-term evolution of the series is mostly based on changes, not on levels. Expressions (8.2) and (6.8) permit us to obtain the ACGF of the revision and final estimation errors, from which, proceeding as in Section 9.3, it is straightforward to find that, for example, 90% confidence intervals for the quarterly change implied by the last observation are equal to (±.47) when the trend is used, and to (±.19) when the SA series is used. Further, if the present rate of annual growth is measured as the rate of change over a year period centered at the present month (a measure that uses 2 forecasts), the standard error of the annual rate of growth is 3.64 when measured with the original or the SA series, and 3.46 when measured with the trend. For longer spans, thus, the trend signal turns out to be more precise.

## 11. RELATIONSHIP WITH FIXED FILTERS

The MBSE approach we have outlined provides a rich procedure for the derivation of linear filters to estimate signals of interest and, often, fixed-type filters can be seen as the result of a particular MBSE application, at least to a reasonable approximation. A well known case is the approximation to the X11 filter developed by Cleveland and Tiao (1976) and Burridge and Wallis (1984). The model found in these approximations for the aggregate observed series is broadly similar to a class of ARIMA models often found in practice, namely those of the type $\nabla\nabla_{12} x_t = \theta(B) a_t$, where $\theta(B)$ displays moderately large negative values of $\rho_1$ and $\rho_{12}$. The spectral shape of this type of model presents the stylized features of the typical spectra of economic time series, as noticed by Granger (1966). Other model-based interpretations of some ad-hoc filters can be found in Tiao (1983), Tiao and Hillmer (1978), King and Rebelo (1993), and Watson (1986).

To illustrate the relationship we consider a family of fixed filter of the low-pass type (aimed at capturing low frequency signals, i.e., long-term trends) namely the Butterworth family of filters, popular in electrical engineering (often in the one-sided expression). For the 2-sided filter, the gain is defined by

$$G(\omega) = \frac{1}{1 + \left(\frac{\sin(\omega/2)}{\sin(\omega_c/2)}\right)^{2d}}, \qquad (0 \le \omega \le \pi) \tag{11.1}$$

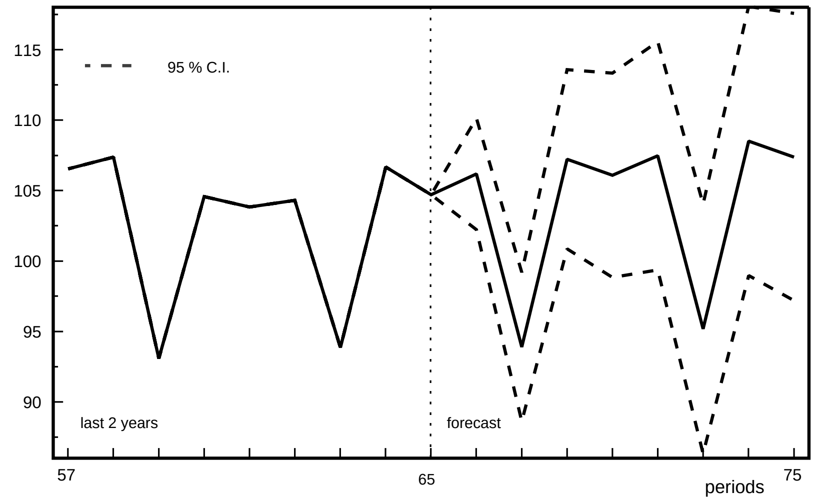
when based on the sine function (BFS), and by the same expression (11.1), with "sin" replaced by "tan", when based on the tangent function (BFT). The filter depends on two-parameters: $\omega_c$, the frequency for which $G(\omega_c) = 1/2$, and d = 1, 2, 3..., where larger values of d produce sharper filters.

To obtain the time domain expression of (11.1), we use the identity $2\sin^2(\omega/2) = (1 - e^{-i\omega})(1 - e^{i\omega})$, and replace $e^{-i\omega}$ by B. This yields

$$g(B,F) = \frac{1}{1 + k[(1-B)(1-F)]^d},$$

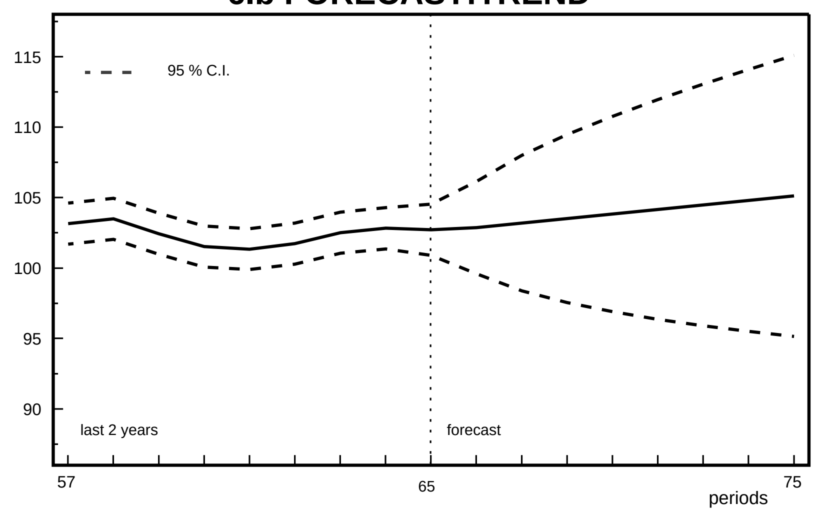
where $k = [2 \sin^2 (\omega_c/2]^{-d}$. It is easily seen that g(B,F) is the WK filter for estimating $s_t$ in the decomposition (4.1), with $\nabla^d s_t = a_{st}$ and $n_t$ white noise $(k = \sigma_n^2 / k_s^2)$. For the BFT version of the filter, using $\tan^2 (\omega/2) = (1-e^{-i\omega}) (1-e^{i\omega})/(1+e^{-i\omega}) (1+e^{i\omega})$, the

## 6.a FORECAST:ORIGINAL SERIES

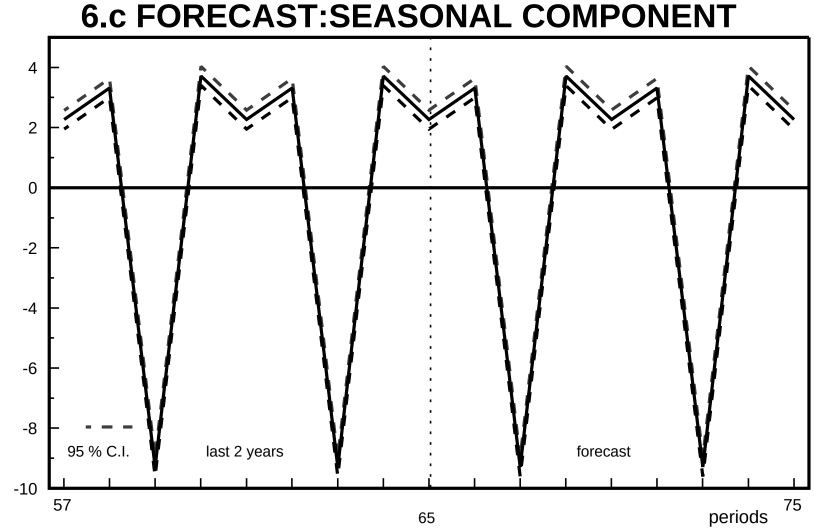



65 = period for the last observed value of series

## 6.b FORECAST:TREND




## 6.c FORECAST:SEASONAL COMPONENT



time domain expression becomes

$$g(B,F) = \left\{ 1 + k \left[ \frac{(1-B)(1-F)}{(1+B)(1+F)} \right]^d \right\}^{-1},$$

which is the WK filter to estimate $s_t$ in the decomposition (4.1), with $\nabla^d s_t = (1+B)^d a_{st}$ and $n_t$ white noise. Therefore, both versions of the Butterworth filter accept simple MBSE interpretations. Notice that the signal provided by the BFT will be "canonical" in the sense of displaying a spectral zero for $\omega = \pi$.

When $d = 1$, the BFS yields the "random walk plus noise" decomposition. When d = 2, from results in King and Rebelo (1993), the BFS yields the popular Hodrick-Prescott (HP) filter (Hodrick and Prescott, 1980). Since the HP was derived originally from the minimization of a function that attempts to balance the trade-off between fitting and smoothness criteria, the example also illustrates the relationship between MA filters derived in this way and the MBSE method. It is worth noticing that, although the same filter is obtained with the different approaches, only the MBSE one provides MSE of the estimators as well as forecasts, (for a more complete discussion, see Gómez, 1998).

## 12. SHORT- VERSUS LONG-TERM TRENDS; MEASURING ECONOMIC CYCLES

In the MMBE approach we have followed, the trend can be seen as a smoothed SA series, since it is obtained by removing additive white noise and perhaps some highly transitory effect as described in Section 4. As a consequence, the trend will, in general, have power over the range of cyclical frequencies (i.e., the range between the zero and the fundamental seasonal frequency). Trends of this type are also called trend-cycle components or short-term trends.

From a long-term perspective, short-term trend are of little use since they will not separate the long-term growth from cyclical oscillations. In economics, the study of cycles is an important field, and a simple and standard way to estimate cycles has been by using some low-pass fixed filter, often the HP filter, to detrend an X11-SA series. As mentioned in the previous section, the HP filter can be seen as the minimization of an ad-hoc function that attempts to balance fitting versus smoothing. It can also be seen as an optimal signal extraction filter in the UCARIMA model

$$x_t = m_t + c_t, \tag{12.1}$$

$$\nabla^2 m_t = b_t, \tag{12.2}$$

where $c_t$ and $b_t$ are mutually orthogonal white-noise variables, with variances $\sigma_c^2$ and $\sigma_b^2$; the standard application of the filter to quarterly series sets $\sigma_c^2 = 1600\,\sigma_b^2$. Algorithms to obtain the filter based on the minimization approach and on the Kalman filter estimation of the signal can be found in Danthine and Girardin (1989) and in Harvey and Jaeger (1993), respectively. An alternative algorithm estimates the signal through the WK filter. First, it is straightforward to find that (12.1) and (12.2) imply that the observed series follows the model $\nabla^2 x_t = \theta_H(B)\,a_t$, where $\theta_H(B) = 1 - 1.7771B + .7994B^2$, and $\sigma_a^2 = 2000\sigma_b^2$. Denoting the HP filter to estimate the trend by $H_p(B,F)$ and applying expression (5.5), it is found that

$$H_p(B,F) = \frac{1}{2000}\,\frac{1}{\theta_H(B)}\,\frac{1}{\theta_H(F)}\,. \tag{12.3}$$

Trivially, the detrended series are obtained through the filter $H_c(B,F) = 1 - H_p(B,F)$. The squared gain of this last filter is displayed in Figure 7a. The filter is seen to remove the variation associated roughly with the interval $\omega\ \varepsilon\ [0, \pi/20]$, and hence will remove cycles with periods of 10 or more years. The numerical results obtained with the three algorithms are indistinguishable; the KF and WK procedure are about equally fast, and considerably faster than the Danthine and Girardin procedure. The WK representation (12.3) is convenient for analytical discussion.

The HP filter to compute the cycle cannot be applied to the observed series, since the seasonality would be included in the cycle. It needs to be applied either to the SA series or to the (short-term) trend. Therefore, in general, the 2-step estimator of the cycle can be written as

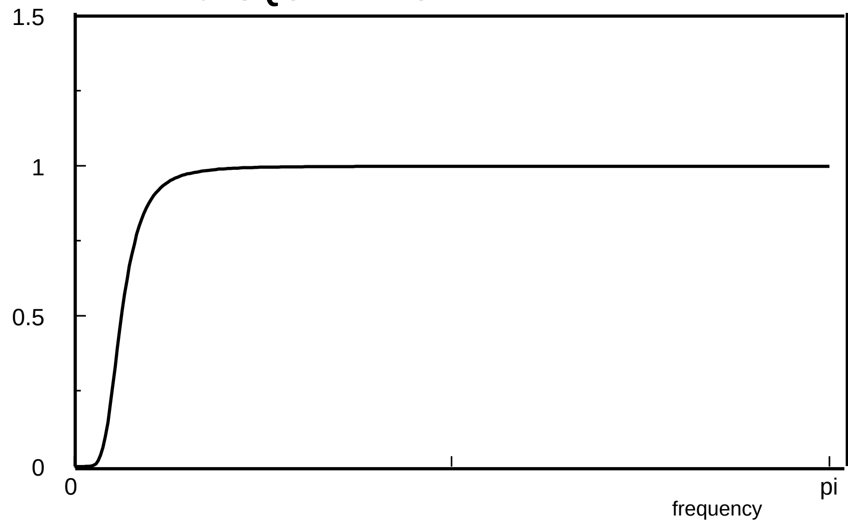
$$\hat{c}_t = H_c(B,F)\,\nu(B,F)\,x_t = \eta(B,F)\,x_t\,, \tag{12.4}$$

where $H_c(B,F)$ denotes the HP filter, $\nu(B,F)$ the WK filter that provides the SA series or the trend, and $\eta(B,F)$ the convolution of the two. This last filter will be symmetric and centered and using the model for $x_t$, given by (4.7), one can proceed with model-based analysis in a straightforward manner.

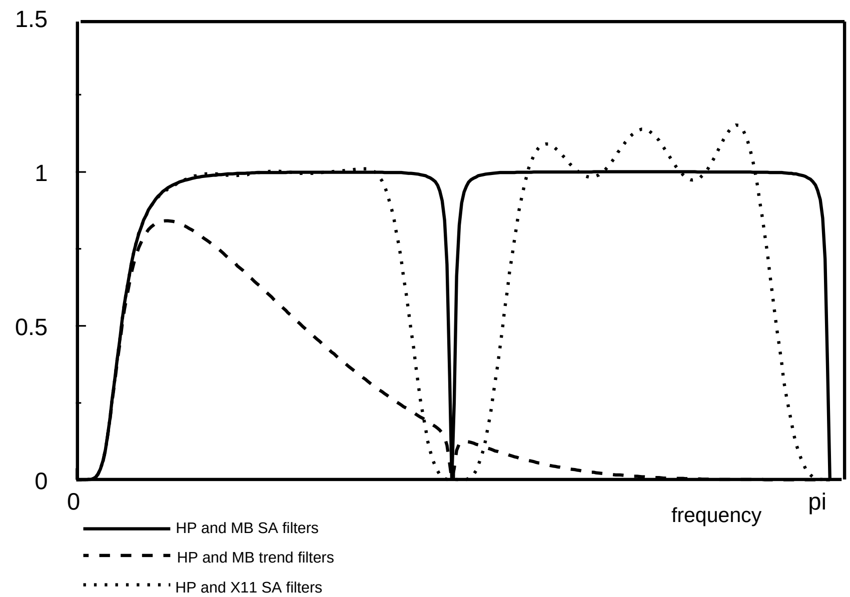
The squared gain of the $\eta(B,F)$ filter that estimates the cycle is given by the continuous line in Figure 7b when the SA series are used, and by the discontinuous line when the trend is considered. The dotted line in the figure displays the squared gain of the convolution of the X11 and HP filters. It is seen that the filter based on the trend is considerably more concentrated around the cyclical frequencies and ignores variation in the series of no cyclical interest. On the contrary, this variation would contaminate the cycle if the SA series is used as input.

As seen in expression (5.4), if the previous squared gain is applied to the spectrum

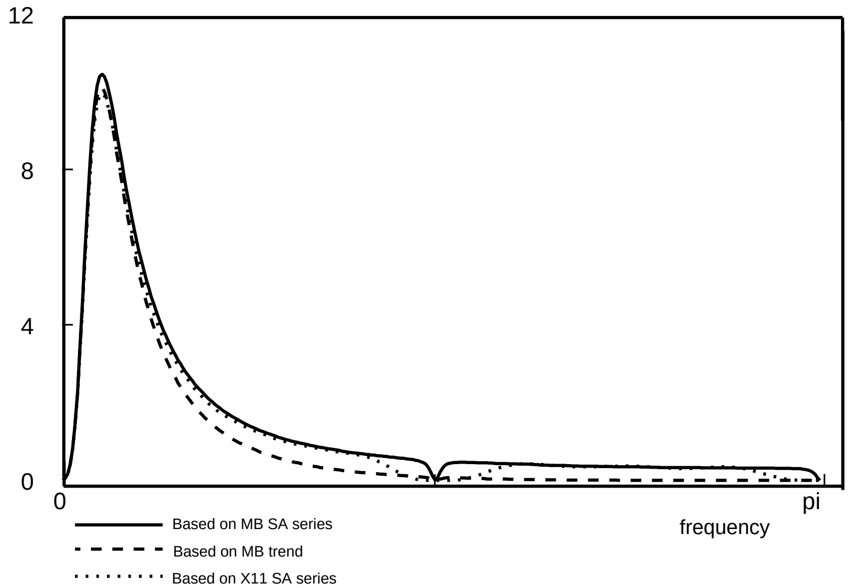
## 7a. SQUARED GAIN: HP FILTER



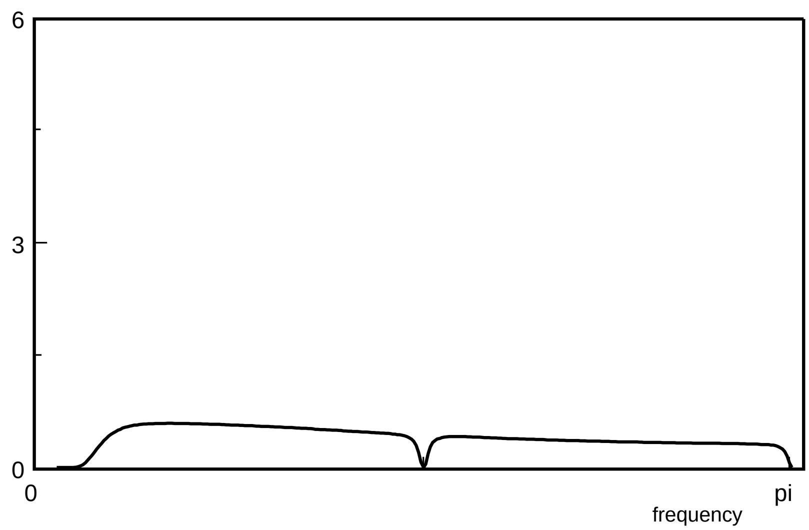

## 7b. SQUARED GAIN: CONVOLUTION OF HP AND SA FILTERS



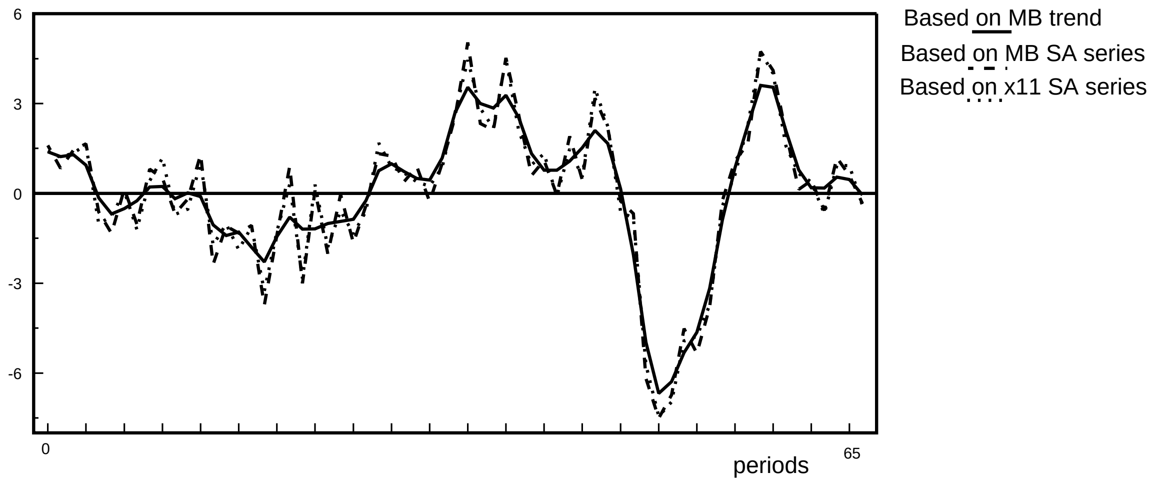
**8a. SPECTRUM OF CYCLICAL COMPONENT**



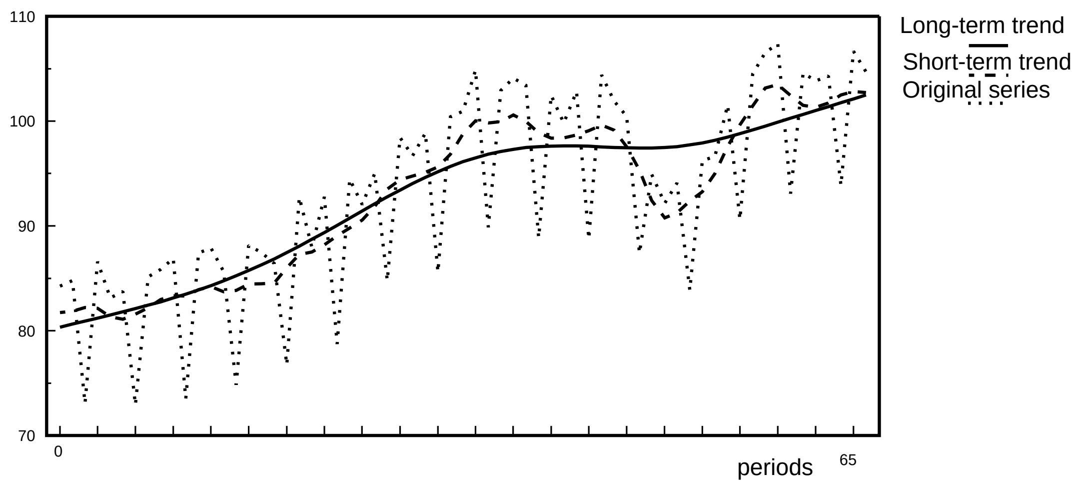

**8b. DIFFERENCE BETWEEN CYCLE OBTAINED WITH SA SERIES AND TREND**



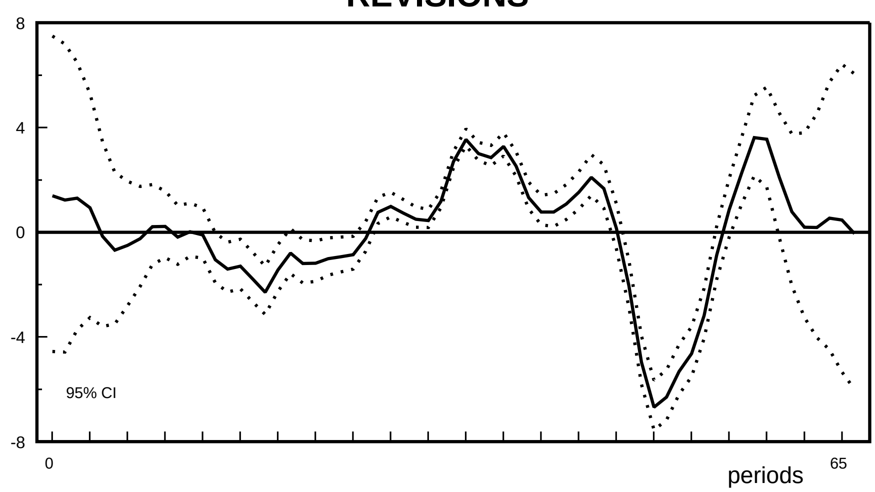
## 9a. ESTIMATE OF CYCLE




## 9b. SHORT-TERM AND LONG-TERM TREND




## 9c. CYCLE:CONFIDENCE INTERVALS BASED ON REVISIONS



of $x_t$, given in Figure 3b, the spectrum of the cycle estimator is obtained. Figure 8a displays the spectra obtained with the three inputs; the dotted line corresponds to the one based on the X11-SA series, while the continuous and discontinuous lines correspond to the ones based on the model-based SA series and trend, respectively. They are seen to be similar in shape and the peak is associated, in the 3 cases, with a (roughly) 8-year cycle. The spectrum of the difference between the two cyclical components computed with the model-based SA series and trend is displayed in Figure 8b; it is close to a white-noise spectrum, and hence the cycle computed using the SA series is approximately equal to the cycle computed using the trend plus some additional noise.

Figure 9a compares the cycle estimates obtained with the three inputs (X11 was applied, in the X11ARIMA spirit, to the series extended at both ends with 3 years of forecasts and backcasts). The difference between using the X11-SA series or the model-based SA one is seen to be minor. The difference between using the SA series or the trend-cycle is, on the contrary, remarkable; the cycle estimator obtained from the trend is considerably smoother. During the 65 quarters considered, the cycle based on the SA series crosses the zero ordinate line 21 times. The cycle estimator based on the short-term trend behaves in a more sensible manner: it crosses the zero line only 7 times and cyclical periods are neatly defined. Figure 9b plots the MB short-term trend of Section 10 (equal to the seasonally adjusted and noise clean series) and the long-term trend obtained by applying the HP filter to the previous short-term trend; the figure illustrates well the difference between the two and clearly indicates that the short-term trend is the signal of interest when looking at the quarter-to-quarter underlying growth of the series (i.e., the growth that results once the seasonal component and the noise have been removed). The long-term trend is of interest for a much larger horizon.

The model-based structure can be useful in more ways, as seen in Kaiser and Maravall (1998). But even if the analyst using and ad-hoc filter has no model for the component in mind, he/she will still worry about revisions in the estimator (implied by the two-sided structure of the filter.) Because it considers a larger information set, the final estimator will be more accurate than the concurrent one, and the difference between the two estimators (i.e., the revision) can be considered an estimation error. Proceeding as in Section 8, and assuming the HP filter is applied to the short-term trend, from (12.3), (5.5), and (4.7), expression (12.4) can be rewritten in terms of the observed series innovations, as

$$\hat{c}_t = k \, \frac{\theta_p(B)}{\theta_H(B)} \, \frac{\theta_p(F)(1-F^4)(1-F)}{\theta_H(F)\,\theta(F)} \, a_t = \delta(B,F)\, a_t \, , \qquad (12.5)$$

where k = .7994 $k_p$. It is straightforward to see that the revision in the preliminary estimator $\hat{c}_{t-k|t}$ can then be expressed as

$$d_{k,t} = \hat{c}_t - \hat{c}_{t-k|t} = \sum_{j=1}^{\infty} \delta_{k+j} a_{t+j} ,$$

where invertibility of the denominator of (12.5) implies that the variance of $d_{k,t}$ can be computed using a finite number of terms. For the IPI example, the standard deviation of the revision error was computed for the estimator of the cycle based on the model-based trend. The 95% confidence intervals are shown in Figure 9c from which two clear facts emerge. First, even for a series with only 65 quarterly observations, historical estimation of the cycle is fairly precise. Second, estimation for recent periods is unreliable. This poor performance is mostly due to the large revisions implied by the HP filter. One could exploit the model-based structure to obtain forecasts of the cycle (in a similar manner to that used in Section 9.1,) but considering the size of the associated standard errors, these forecasts are of little interest.

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