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Abstract

Exchange rate targets in a stabilization game are considered. The targeting strategy consists on the choice of a desired level for the exchange and the weight assigned to such target in the loss function. The exchange rate target appears then as an intermediate objective and acts as a surrogate to policy coordination. The targeting solution reveals that the targeting strategy can be embedded on a straight line in the policy-instruments space (the respective money supplies), which greatly facilitates the analysis. It turns out that the targeting strategy is optimal when the reaction of the countries exert a positive externality on the other country. In this case, policymakers have some flexibility in the choice of the target as long as the optimal commitment to such target is selected accordingly.
Introduction

This article deals with the design of exchange rate targets and their use as stabilizing devices in the face of economic shocks. The exchange rate plays in our framework a role of intermediate target, acting as an instrument for implicit cooperation. Countries agree on the exchange rate target and conditionally upon it, maximise (non-cooperatively) their individual loss function.

Incentive compatibility turns out to be the necessary condition to render credible the arrangement, otherwise private agents would realise the incentive to renege that a return to a non-cooperative equilibrium provides. Pareto optimality is a complementary condition which maximises the quality of the arrangement and, in our reference model, delivers the explicit coordination solution.

After presenting the model and the targeting framework in the first two sections, the search for an optimal targeting strategy is developed in two stages. In the first stage (section III), we explore the possibility of targeting the exchange rate for different types of shocks. This is formally done by delimiting the type of shocks for which the exchange rate arrangement is incentive compatible and Pareto optimal. It turns out that the feasibility of targeting the exchange rate depends on the type of shock and that it may be counterproductive in certain circumstances.

The second stage (section IV) directly tackles the question of designing optimal exchange rate targets within the relevant shock subset. We will observe that there is some scope for the discretion for the policymakers who can choose between a wide range of exchange rate targets, provided that they also choose the optimal commitment to the selected target. Section V interprets the results and the conclusions sum up our results and compares them with other targeting schemes.

I-Model and definitions

We consider two identical and interdependent economies (home and foreign countries,
labelled with subscripts 1,2). The policy makers aim at minimising their respective loss functions \(L_1, L_2\) which penalise the deviations of employment \((n)\) and consumer prices \((q)\) from their desired levels, which we assume, without loss of generality, that are set to zero:

\[
L_1 = \frac{1}{2} [\sigma n_1^2 + \eta q_1^2]
\]
\[
L_2 = \frac{1}{2} [\sigma n_2^2 + \eta q_2^2]
\]

\(\sigma > 0, \ \eta > 0\)

The instrument for the policy makers is the money supply \(\{m_1, m_2\}\). The characteristics of the economy are defined by the model developed in Canzoneri & Henderson (1988,91), which is summed up in appendix A.

At the beginning of the game, workers and firms enter into wage contracts which specify nominal wages and employment. Firms employ labour up to the point that real wages equal the marginal product of labour. Workers agree to supply whatever quantity of labour firms want at the nominal wages specified in the contracts. The result is that nominal wages are set so that the expected employments are at their full-employment levels of zero. Since, as it is shown in appendix B, the expected money supplies equal zero, the employment level in each country is determined by the respective money supply surprises \(\{m_1, m_2\}\).

After contracts are set, these economies may suffer shocks on the demand \((u_1, u_2)\) or the supply side \((x_1, x_2)\). Consumer prices and the real exchange rate \(z\) are affected by these shocks, as it can be observed in the reduced forms of the model:

\[
n_1 = m_1; \quad n_2 = m_2
\]
\[
q_1 = (\sqrt{\eta})^{-1} \left[ m_1 - 2\theta m_2 + (1 - \tau)x_1 + \tau x_2 - (u_1 - u_2) \right]
\]
\[
q_2 = (\sqrt{\eta})^{-1} \left[ m_2 - 2\theta m_1 + (1 - \tau)x_2 + \tau x_1 - (u_1 - u_2) \right]
\]
\[
z = (\sqrt{\eta})^{-1} \left[ 2\theta (m_1 - m_2) - \tau x_1 - x_2 - (u_1 - u_2) \right]
\]

\(0 < \theta < \frac{1}{2}; \quad 0 < \tau < \frac{1}{2}; \quad 0 < (\sqrt{\eta})^{-1} < 1; \quad \xi < \frac{1}{2}\)

For instance, a negative symmetric supply shock \((x_1 = x_2 > 0)\), due for instance to a commodity shock, will increase domestic and foreign consumer prices, because it reduces labour
productivity in both countries and wages are already set; the real exchange rate remains at the same level though. The case of a demand shock is less intuitive in this model. A shift in demand from country two to country one \((u_2 > 0)\) does not modify nominal incomes, but it creates an excess demand on country one and an excess supply in country two. The currency of the excess demand country must appreciate in real terms \((\pi > 0)\) to eliminate the imbalance. This reduces consumer prices at home and increases them abroad.

Consequently, shocks lead to a welfare loss which can be observed in figures one and two. The welfare loss just after the shock, that is, before any reaction of the authorities, corresponds to the origin. The quadratic loss function defines elliptical indifference curves in the instrument space \([m_1, m_2]\). Policy makers will make then use of monetary policy to minimise welfare losses, offsetting the effects of the shocks on the domestic consumer prices. In particular, for the supply shock we are considering both countries would contract their money supplies to reduce the consumer price inflation; in the case of a shift in demand, the foreign country would contract its money supply, while the home country would expand. This policy action has a cost, however in terms of employment. Furthermore, the effects of the monetary action will spillover to the other country and this is taken into account by both policymakers.

At this point, the outcome of the game depends on the strategic position of the countries. If countries do cooperate, they would use their money supplies to minimise the joint loss function given by:

\[ L^c = \delta L_1 + (1-\delta) L_2 \]

where \(\delta, 1-\delta\) are the weights assigned to each country. An Pareto optimal outcome is obtained for each value \(\delta\), which are embedded in the contract curve \(C(\delta)\). Setting \(\delta = 1, (\delta = 0)\), the instruments of both countries are chosen to maximise the welfare of the first (second) country. These solutions are known as bliss points \((B_1, B_2)\). Since the bliss points correspond to zero welfare losses, \(B_1, B_2\) are placed at the center of the ellipses.

When countries act non-cooperatively, each country takes the actions of the other player as given and try to minimise their own welfare loss, for each possible choice of the second country.

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1-The following solutions are derived in appendix C.
This set of outcomes is contained in the reaction function for each country \( (R^N_1, R^N_2) \). The Nash or non-cooperative equilibrium \( (N) \) is given by the intersection of these reaction functions.

We can observe in the figures that the outcomes are in this case inefficient: there are other solutions which would be welfare-improving for both countries. This set of incentive compatible points is called the bargaining area, labelled \( A \) in the figures. It is delimited by the ellipses intersecting at the Nash equilibria and it obviously includes a segment of the contract curve.

II. The exchange rate as intermediate target

While both countries would gain from cooperation, this solution faces a "cheating problem" (Canzoneri & Gray (1985)), because monetary cooperative arrangements are difficult to verify and they are easily altered in subtle ways.

Our strategy attempts to overcome this problem, emphasising the value of exchange rate targets as adequate surrogates for explicit cooperation because of its direct observability. In this sense, Kenen (1989,p.54) notes that "governments are prone to cheat and will not engage in optimal coordination because they cannot trust each other. A government cannot cheat on a firm commitment to exchange rate pegging without being caught. Therefore, exchange rate pegging is viewed as a viable alternative to full-fledged coordination". Notwithstanding this, in the policy coordination literature, the choice of the exchange rate usually appears as a by-product of the coordination solution and it is not explicitly considered. We follow here Hughes Hallet et. al (1989) and, in particular, Hughes Hallet (1993) where exchange rate targets are included in the policymakers' optimization problem. We can justify this inclusion more formally using an analogy with the optimal contract literature, adapted to the context of policy coordination

Two countries on an equal strategic footing decide to commit to a contract in order to minimise the welfare losses derived from unanticipated shocks. Of course, they will only stick to the contract if they expect to gain from it, hence the need for the arrangement to be incentive compatible. Reneging on the arrangement opens up the possibility of retaliation by the second

\[\text{---See Rogoff (1985) and, for more recent developments, Baron (1989), Walsh (1995) and Persson & Tabellini (1993).}\]
country and the suspension of an agreement which is in general beneficial for both countries. This punishment strategy is assumed to eliminate the incentive to renege, so that the time inconsistency problem does not arise and the game can be considered as static. Finally, we have to specify the content of the contract, denoted by V. The contract is defined in terms of deviations from the desired real exchange rate (z'), which enters in the loss function of both countries with a weight equal to β:

\[ V = \frac{1}{2} \beta (z - z')^2 \]

Thus, the function which each country considers is then modified to become:

\[ W_1 = L_1 + V; \quad W_2 = L_2 + V \]

Note that the exchange rate is just an intermediate target in the modified loss functions \((W_1, W_2)\). The values of \( \beta \) and \( z' \) should be chosen so as to induce the optimal response of policymakers to attain their final goals: consumer prices and employment stability.

Following Rogoff (1985) we can define the parameter or weight \( \beta \) as the optimal degree of commitment to the intermediate target, in this case the desired real exchange rate. The parameter \( \beta \) is constrained to be positive, otherwise what is being targeted is the exchange rate to avoid!.

When \( \beta \) equals zero, no constraint is imposed on the exchange rate and the result corresponds to the non-cooperative free-float solution\(^1\). The second element to determine is the choice of the exchange rate target \((z')\). How do players agree on the desired level for the real exchange rate?,

We take as benchmarks the Exchange Rate Mechanism (ERM) of the EMS—a nominal exchange rate target—and the target zone proposed by Williamson (1985)—a real exchange rate target, and allow for a continuum of exchange rate targets, spanning between both alternatives. This setup allows for flexibility in the design of the contract, adding new insights to the question of exchange rate targeting.

Let us take the real exchange rate identity, in terms of purchasing power parity:

\[ z = e^{-\left(\rho_1 \rho_2\right)} \]

where \( p_1 \) is the price of goods in the respective country and the nominal exchange rate \( e \), is

---

\(^1\)Since positive values of \( \beta \) penalize deviations from the desired values, it represents a soft band of fluctuation for the desired exchange rate target; the larger the value of \( \beta \), the narrower will be the implied band. This specification allows us to think of the targeting strategy as a target zone with soft bands, where \( z' \) represents the central parity.
defined as the price of country 2 currency in terms of country 1 currency. The ERM regime aims at maintaining a fixed nominal exchange rate parity, i.e. $e'=0$, which is equivalent to a real exchange rate target equal to the negative of price differentials: $z'=-(p_1-p_2)$. The Williamson target zone on the contrary implies a desired value for the real exchange rate equal to zero $z'=0$ or, equivalently, a depreciation of the nominal exchange rate to completely offset price differentials, that is, $e'=(p_1-p_2)$.

Let us now define the parameter $\rho$, such that $e'=(1-\rho)(p_1-p_2)$, where $(1-\rho)$ is then the offsetting degree of price differentials. Thus, we can write the exchange rate target in general form as a function of the price differentials

$$ z'=e'-(p_1-p_2)=-\rho(p_1-p_2) $$

It immediately follows that $\rho=1$ corresponds to a nominal exchange rate target, and $\rho=0$ corresponds to a real exchange rate target. The intermediate values present special interest because they will provide flexibility in the choice of exchange rate target, according to the preferences of policymakers.

We can observe that the design of the targeting strategy is then determined by the choice of just two parameters: $\rho$ and $\beta$. The range of parameters is constrained to positive values of $\beta$ and to values of $\rho$ between zero and one, i.e. between real and nominal exchange rate targets. The combination of exchange rate targets and values for $\beta$ represents the set of targeting strategies ($\lambda$):

$$ \lambda=\{[\beta \times \rho], \beta \geq 0, 0 \leq \rho \leq 1\} $$

which, for latter convenience, can be seen as a a subset of $\Lambda=\{[\beta \times \rho], \forall \beta, \rho\}$.

Since, as mentioned above, the exchange rate arrangement must be incentive compatible, the choice of $\rho$ and $\beta$ must deliver an equilibrium laying inside the bargaining area $\Lambda$ in the figures. Moreover, it would be desirable that the targeting strategy places the economies on the contract curve of Pareto optimal outcomes. Now we are in the position to explore whether targeting the exchange rate pays when countries are placed on an equal strategic footing.

### III-Economic shocks and optimal targets

The introduction of an exchange rate target in the optimization problem implies that the
exchange rate target acts as an indirect cooperation device. It is indirect because the optimization problem facing each country is equivalent to the non-cooperative case; each country minimises its own welfare loss, making as given the actions of the foreign country, but also taking into account the common exchange rate target, that is, the respective money supplies are chosen so as to minimise $W_1, W_2$.

We focus in this section on the set $\Lambda$, with no constraint on the target parameters. Each combination of $\beta$ and $\rho$ defines a targeting equilibrium where the modified reaction functions $R_1(\Lambda), R_2(\Lambda)$ intersect. We define this set of solutions in the instrument space: $T(\Lambda) = \{m_1^T, m_2^T\}$; when $\beta=0, \forall \rho$, the exchange rate is not targeted, corresponding to the Nash non-cooperative solution $N=T(0)=\{m_1^N, m_2^N\}$. It turns out that $T(\Lambda)$ is a straight line (target line, hereafter), passing through the Nash equilibrium. More formally, in appendix D we firstly show that:

**Proposition 1**: The set of targeting equilibria, $T(\Lambda)$, is contained in a straight line with slope equal to -1 in the $[m_1, m_2]$ space, where:

$$T(\Lambda): \{ (m_1^T, m_2^T) \mid m_2^T = \frac{-x_1 + x_2}{1 + a - 2\theta} m_1^T \}$$

This proposition is central to the derivation of our results. On the one hand, recall that incentive compatible strategies must be Pareto superior to the Nash solution, that is, the target line must pass through the bargaining area. On the other hand, it would be desirable that the targeting strategy is Pareto optimal. This requirements lead to the following conditions for an optimal exchange rate arrangement, where incentive compatibility is the necessary condition:

**INCENTIVE COMPATIBILITY**: $\exists \Lambda \mid T(\Lambda) \cap A \neq \{ \emptyset \}$

**PARETO OPTIMALITY**: $\exists \Lambda, \delta \mid T(\Lambda) = C(\delta)$

Now we will explore the feasibility of optimal targets, applying these two optimality conditions to the shocks arising on the demand and on the supply side; note however that the figures advance the conclusions of our formal analysis below for certain types of shocks.

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4-Therefore, strictly speaking the propositions below are necessary but not sufficient conditions for targeting the exchange rate appropriately. Only when the value of of the targeting parameters are specified, the propositions will be completed. In the next section we will focus in more detail on the conditions for which $\beta$ is positive when $\rho$ is between zero and one.
Effect of demand shocks. Asymmetric and idiosyncratic shocks

Figure 1 - Strategic behaviour and demand shocks
Effect of supply shocks. Opposite shocks

Figure 2.a-Strategic behaviour and opposite supply shocks
Effect of supply shocks. Idiosyncratic shock

Figure 2.b-Strategic behaviour and idiosyncratic supply shocks
Figure 2.c Strategic behaviour and symmetric supply shocks
Supply and demand shocks have different effects on welfare as an inspection of [1,2] reveals. Consequently, the scope for targeting may crucially depend on the type of the shock hitting the economy. We consider every type of shock, although in the figures we just display the cases of symmetric \((x_1=x_2)\), idiosyncratic \((x_1>x_2=0, u_1>u_2=0)\) and opposite \((x_1=-x_2<0, u_1=-u_2<0)\) shocks. From the observation of the reduced forms in [2], we can see that symmetric demand shocks \((u_1=u_2)\) have no effect whatsoever on the economy.

Figure 1 suggests the feasibility of a targeting strategy in the case of demand shocks, since the target line crosses the bargaining area \((A)\) and the contract curve \((C(\theta))\) for both asymmetric and idiosyncratic shocks. This suggests that an optimal targeting strategy can be specified. This intuition is confirmed by the formal analysis of the model, which is carried out in the appendix D. The results can be summed up in the following three propositions:

**Proposition 2:** For demand shocks of any type and magnitude, there exists at least one exchange rate arrangement which is incentive compatible.

**Proposition 3:** For demand shocks of any type and magnitude, there exists a Pareto optimal exchange rate arrangement.

**Proposition 4:** For demand shocks of any type and magnitude, the Pareto optimal exchange rate arrangement is incentive compatible and corresponds to the point \(C(\bar{\theta})\), where

\[
C(\bar{\theta}) = \{m_1^*, m_2^*\} \in T(A)
\]

\[
m_i^* = \frac{(1+2\theta)(u_i-u_2)}{\sigma(1+2\theta)^2} = -m_i^*
\]

Therefore, it is formally shown that the targeting equilibrium which intersects the contract curve belongs to the bargaining area, so that the Pareto optimal equilibrium is also incentive compatible, hence making optimal targets feasible for any type of demand shock.

The question is not so straightforward in the case of supply shocks. The plots in figure 2.a-c display a quite different picture: only in the case of opposite supply shocks, the target line crosses \(A\) and \(C(\theta)\). As above, the results are formalised in three propositions:
**Proposition 5:** For supply shocks at least one incentive compatible exchange rate target will exist if

\[
\frac{(1+\sigma)-(1+\sigma-2\theta)\tau}{2\theta+(1+\sigma-2\theta)\tau} \leq x_i \leq \frac{2\theta+(1+\sigma-2\theta)\tau}{(1+\sigma)-(1+\sigma-2\theta)\tau}, \quad \forall x_i > 0 \quad \{i,j\} = \{1,2\}, i \neq j
\]

**Proposition 6:** For supply shocks there exists an optimal exchange rate target only if

\[
\frac{-(1+\sigma)-(1+\sigma-2\theta)\tau}{2\theta+(1+\sigma-2\theta)\tau} \leq x_j \leq \frac{-2\theta+(1+\sigma-2\theta)\tau}{(1+\sigma)-(1+\sigma-2\theta)\tau}, \quad \forall x_j > 0 \quad \{i,j\} = \{1,2\}, i \neq j
\]

Noting in [2] that the parameters \(\theta\) and \(\tau\) are positive and less than \(\frac{1}{2}\) and \(\sigma\) is positive, it follows that

\[
\frac{-2\theta+(1+\sigma-2\theta)\tau}{(1+\sigma)-(1+\sigma-2\theta)\tau} < 0\quad < \frac{2\theta+(1+\sigma-2\theta)\tau}{(1+\sigma)-(1+\sigma-2\theta)\tau} < 0
\]

This result supports the existence of optimal targets for opposite supply shocks but rules them out for symmetric and idiosyncratic supply shocks. More precisely,

**Proposition 7:** For opposite supply shocks \((x_i = -x_j)\), the Pareto optimal exchange rate arrangement is incentive compatible and corresponds to the point \(C(\frac{1}{2})\):

\[
\forall x_i = -x_j \Rightarrow C(\frac{1}{2}) = \{m_1^*, m_2^*\} \in \mathcal{T}(\Lambda)
\]

\[
m_1^* = \frac{(1+\sigma)(1-2\gamma)x_1}{\sigma(1+2\theta)} = -m_2^*
\]

**IV. The design of optimal targeting strategies**

The feasibility of optimal targeting strategies has just been shown, but we have not yet constrained the target parameters to belong to the relevant targeting set \((\Lambda \subset \Lambda)\), where \(\beta\) is positive and \(0 \leq \rho \leq 1\). This constraint is now introduced, so that the issue of designing exchange

---

5. For the rest of supply shocks, we had to proceed by numerical simulation. The outcome depends on the values of \(\sigma\) and \(\theta\). In particular, for values close to the extremes of the range the Pareto optimal point does not fall within the bargaining area. In any case, for these latter situations, an exchange rate agreement could be reached because incentive compatibility is the sufficient condition.
rate targets can be tackled. We claim that

**Proposition 8:** When an optimal targeting strategy exists, the optimal degree of commitment \( \beta^* \) to an exchange rate target is a function of \( \rho \) and the parameters of the model, \( \beta^* = f(\rho, \ldots) \) and it follows that

\[
\forall u_1 - u_2, \forall \rho \in [0, 1] = \beta^* > 0, \quad \beta^*_r > 0
\]

\[
\forall x_1 = -x_2, \forall \rho > \frac{2\theta}{\sigma - \frac{1 - 2\theta}{1 - 2\theta \sqrt{\eta}}} > 0 \Rightarrow \beta^* > 0, \quad \beta^*_r < 0
\]

This result shows that the exchange rate target (and the optimal commitment to it) depends on the type of shock affecting the economy, but not on the magnitude or the sign of the shocks. This is a result which is also found in the existing policy coordination literature, and it greatly facilitates the design of the optimal targeting strategy.

Secondly, for demand shocks nominal and real exchange rates can be targeted, while for opposite supply shocks real exchange rate targets are ruled out \( (\rho > 0) \) and, for certain parameter values, nominal exchange rate targets could be inadequate, too.

The third conclusion of this section is that there is not an unique optimal design for the exchange rate target. It ultimately depends on the preferences of policy makers, who can choose between different combinations of commitment and exchange rate targets given the function \( f(\rho, \ldots) \). This trade-off is conveyed in \( \beta_\rho \), the derivative of \( \beta^* \) with respect to \( \rho \). If the policymaker’s goal is to design a target zone which minimises exchange rate volatility relative to the desired target, the higher \( \beta^*_\rho \), the better. On the contrary, if the aim is to provide exchange rate flexibility reaping the full benefits of coordination, the value of \( \rho \) which allows for the highest exchange rate flexibility will be chosen. According to this second criterion, a real exchange rate target would be the optimal choice in the case of demand shocks while for the case opposite supply shocks, a

---

\( ^6 \) We can observe in [5] that the optimal response to an opposite supply shock \( (x_1 = -x_2 < 0) \) requires a change of the opposite sign in the money supplies \( (m_1 = -m_2 > 0) \). From the expression of the real exchange rate in [2], this in turn implies further deviations from the real exchange rate target.
nominal peg would be the solution.

V-Interpretation of the results

The cases for which a targeting strategy is feasible have some features in common. Comparison of figure 1 (demand shocks) with figure 2.c (opposite supply shocks) actually reveals an equivalent outcome in graphical terms. Note that in both cases, the solution requires manipulation of the money supplies in different directions. This causes a positive externality because, from [2], both countries are moving the exchange rate in the same direction. It is in these cases when targeting the exchange rates pays. More formally:

**Proposition 2:** The necessary condition for the existence of an optimal exchange rate arrangement is that, for any type of shocks

$$\text{sign}(m_1') = \text{sign}(m_2') \neq \text{sign}(m_1^2) = \text{sign}(m_2^2)$$

Let us explain why an exchange rate target is optimal in the case of a shift in demand towards the home good ($u_1 - u_2 > 0$). As we have mentioned above, the shift in demand provokes an exchange rate appreciation which reduces (increases) consumer prices at home (abroad). Consequently, the home (foreign) country will expand (contract) its money supply, but if countries do not cooperate, this individual effort to reverse the exchange rate appreciation derived from the shock is too cautious with respect to the optimal solution. In other words, a smaller deviation from the pre-shock zero exchange rate levels is required to attain an efficient equilibrium. Since including an additional target in the loss function has the effect of reducing the deviation of the new target from its desired level, targeting the real or nominal exchange rate in the loss function will induce the right response.

Note that this implies a more activist role for monetary policy derived from the targeting strategy. Comparing the expressions for the Nash solution (expression [VI] in the appendix) and the optimal targeting strategy for demand and opposite supply shocks (expressions [4] and [5] respectively), we can express the latter as a function of the Nash solution. It turns out that:
on a real target or vice-versa has not been shown. Indeed, we have proved that in certain cases 
(demand shocks) both are valid and in others neither of them is (a wide range of supply shocks).

The assumption of an identical strategic role for each country in the targeting strategy 
is central to our results. This is revealed when comparing them with the conclusions of Canzoneri 
and Henderson (1991), who use an identical model. In their case however an asymmetric targeting 
strategy is considered, where one country (the follower) pegs the exchange rate to the leader. The 
leader sets the value of its money supply to minimise its own welfare function and the follower 
only cares about maintaining the parity. This asymmetric strategy would pay in the case of 
symmetric supply shocks. In this case, as it is apparent in figure 2.a both countries non- 
cooperatively respond by changing their money supplies in the same direction, provoking an 
overshooting of the exchange rate with respect to the efficient solution. Therefore, the result of 
a leader-follower strategy is to offset this negative externality and place the economy on the point 
\( H \), which is optimal.

In this case, the existence of a leader exerts a disciplinary effect on the actions of the 
follower because changes in the money supplies are smaller. Thus, when the optimal response to 
a shock requires a restraint or discipline in the management of the money supply a leader-follower 
strategy is desirable because the leader provides an anchor to the monetary policy. However, when 
countries act on an equal strategic basis no disciplinary effect can be attained. Therefore, the 
optimal contract strategy may only be beneficial when a more activist response is required. This 
implies that our alternative dominates for demand and opposite supply shocks.

All in all then the optimal arrangement depends on the type of shock hitting the economy, 
highlighting the case for flexibility in the design of exchange rate arrangements. In any case, it is 
the economic and not the political environment which should dictate the strategic environment.
\[
|m_{i}^{*}| = \frac{(1-2\gamma)x_{i} + u_{i} + u_{j}}{1 + \sigma + 2\theta} < |m_{i}^{*}| = \frac{(1-2\theta)(1 + \sigma + 2\theta)}{\sigma + (1 + 2\theta)^{2}} |m_{i}^{*}| < (1-2\gamma)x_{i} + u_{i} - u_{j}
\]

\{i,j\} = \{1,2\}, i \neq j

Therefore, the optimal strategy will always imply a larger change in the money supply, both for the expansionary and the deflationary country. This result is confirmed by the graphical analysis where we can observe that, in the relevant figures, the optimal solution is more distant from the origin than the Nash solution.

Finally, the slope of the target line being equal to -1 implies that the global money supply does not change when the solution shifts from the Nash to the targeting equilibrium. More formally, the expression for the target line in [3] reveals that at the targeting equilibrium the global money supply remains constant and equal to the Nash solution. For supply shocks \(m_{i} + m_{j} = -(1-2\gamma)(x_{i} + x_{j})/(1 + \sigma + 2\theta)\) and for demand shocks the global money supply is simply zero, that is, the effect of the targeting strategy is to allocate more efficiently a given global money supply than in a non-cooperative situation, which reminds McKinnon’s proposal for monetary stabilization (MacKinnon 1984, 88).

VI-Conclusions. The need for flexibility

Welfare considerations should be the basis for any exchange rate arrangement among countries. Consequently, exchange rate targets which do not benefit to each participant cannot be sustained. Upon this idea we have set up a framework to analyse exchange rate targeting in the form of an optimal contract between countries which are on an equal strategic basis.

The exchange rate target is viewed as an intermediate objective on which policymakers agree (optimal contract). As intuition suggests, targeting the exchange rate in such a way may only be appropriate when both countries are interested in moving the exchange rate in the same direction. While for demand shocks of any type or magnitude an optimal target can be devised, for supply shocks the answer depends on their differential impact on each country.

The optimal contract, when feasible, allocates a fixed global money supply more efficiently than in a non-cooperative Nash situation. We have also identified a certain room for discretion in the choice of the exchange rate target but a general dominance of a nominal exchange rate target
Appendix


Let us consider two economies (home and foreign, subscripts one and two, respectively) with identical structures, but for the good they produce ($y_1$, $y_2$). These economies are subject to shocks on the demand ($u_1, u_2$) and the supply side ($x_1, x_2$). Rational expectations are assumed, so that only unanticipated shocks can affect equilibrium. All the variables except the interest rates are expressed in logs and represent deviations of actual values from equilibrium. The disaggregation of shocks and the treatment of the exchange rates introduce some minor modifications into the original model.

The output of each country ($y_j$) is obtained through a Cobb-Douglas production function. It is an increasing function of domestic employment $n_j$ and it decreases when some adverse supply shock $x'_j$, hits the economy:

$$ y_j = (1-\alpha)n_jx'_j ; \quad y_2 = (1-\alpha)n_2x'_2 $$

Firms hire labour up to the point in which real wages equal the marginal product of labour:

$$ w_1 = -\alpha n_1x'_1 ; \quad w_2 = -\alpha n_2x'_2 $$

where $w_1$ and $p_1$ are nominal wages and prices, respectively. Contracts are signed at the beginning of each period, so that shocks are unanticipated. These contracts specify nominal wages and employment rules and workers agree to supply whatever quantity of labour firms want at the nominal wage specified in the contracts.

Consumer price indexes $q_j$ are weighted averages of domestic and foreign goods prices:

$$ q_1 = (1-\xi)p_1 + \xi(e+p_2) = p_1 + \xi z ; \quad q_2 = (1-\xi)p_2 + \xi(e-p_2) = p_2 + \xi z $$

The market equilibrium conditions for the demands of goods are:

$$ y_1 = \delta z + (1-\xi)e y_1 + \xi e y_2 - (1-\xi)\nu r_1 - \xi \nu r_2 + u'_1 $$

$$ y_2 = -\delta z + \xi e y_1 + (1-\xi)e y_2 - \xi \nu r_1 - (1-\xi)\nu r_2 + u'_2 $$

where $r_j$ are the real interest rates and $u_j$ are positive demand shocks. Uncovered interest parity holds, so that $r_1 - r_2 = \xi z - z$. The superscript stands for expected value. Finally, the equilibrium in the

---

1. The parameters which appear in the model are all positive and take the following values: $\alpha < 1$, capital coefficient in the production function; $\delta$, real exchange rate elasticity, $e < 1$, marginal propensity to spend; $\delta < 1$, demand elasticity to the real exchange rate; $\nu$, interest rate elasticity; $\xi < \nu$, share of import goods on domestic basket. The rest of parameters are combinations of these: $\gamma = (2\delta + (1-2\xi)^2\nu)^{-1} < 1$; $\pi = 1-(1-2\xi)e < 1$; $\tau = \xi \gamma \pi < \nu$; $\phi = \nu(1-\alpha) < \nu$; $\sqrt{\eta} = (\alpha + \phi)^{-1} > 1$; $\theta = \nu \phi < \nu$. 

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money market is given by the Cambridge equations: 

\[ m_1 = p_1 + y_1; \quad m_2 = p_2 + y_2. \]

Nominal wages are set as follows. From the output and the wages equations above, and using the money market equilibrium equations, employment can be expressed as a function of the money supplies and nominal wages: 

\[ n_1 = m_1 - w_1; \quad n_2 = m_2 - w_2. \]

Firms and workers choose the nominal wage that minimizes the expected square deviations of employments from the full-employment value, set equal to zero. Optimizing the square of the expression above, we observe that the respective nominal wages are set equal to the expected money supplies:

\[
\frac{\partial (n_1^2)}{\partial w_1} = w_1 - m_1^* = 0; \quad \frac{\partial (n_2^2)}{\partial w_2} = w_2 - m_2^* = 0
\]

The reduced forms are obtained by expressing all the variables of interest in terms of the instruments and the shocks. It is shown below that the expected money supplies are zero. Taking this into account and redefining the shocks: 

\[ x_i = \sqrt{\eta} x_i' \quad \text{and} \quad u_i = \sqrt{\eta} \gamma u_i', \quad i = 1, 2, \]

the reduced forms for the policy objectives are:

\[ n_1 = m_1; \quad n_2 = m_2; \]

\[ q_1 = (\sqrt{\eta})^{-1}[m_1 - 2\theta m_2 + (1 - \tau)x_1 + \tau x_2 - (u_1 - u_2)] \]

\[ q_2 = (\sqrt{\eta})^{-1}[m_2 - 2\theta m_1 + (1 - \tau)x_2 + \tau x_1 + (u_1 - u_2)] \]

and for the relevant exchange rates:

\[ z = \omega \gamma (1 - \alpha)(m_1 - m_2) - (\sqrt{\eta})^{-1}[\tau (x_1 - x_2) + (u_1 - u_2)] \]

\[ z^d = \rho [\alpha (m_1 - m_2) + (\sqrt{\eta})^{-1}(x_1 - x_2)] \]

\[ z - z^d = \frac{\sqrt{\rho'}}{(m_1 - m_2)(\sqrt{\eta})^{-1}[\rho - \gamma](x_1 - x_2) - \frac{u_1 - u_2}{\gamma}} \]

where \( \sqrt{\rho'} = [\rho \alpha + \omega \gamma (1 - \alpha)] \) and \( \{i,j\} = \{1,2\}, \quad i \neq j. \)

B. Money supply expectations

Substituting the reduced forms in which expectations explicitly appear in the functions to optimize and taking the derivative with respect to the instruments, we obtain:

\[
\frac{\partial W}{\partial m_i} = \omega (m_i - m_i^*) + \eta (\phi + \alpha) (m_1 - m_i^*) (m_i - m_i^*) + \phi (m_i - m_i^*) + (1 - \tau)x_i' + \tau x_i' - \gamma (u_i' - u_i') \\
+ \beta \{(m_i - m_i) - (m_i^* - m_i^*)\} + (\rho - \gamma)(x_i' - x_i') - \gamma (u_i' - u_i') = 0
\]
Taking expectations, and noting that \( x_i' = u_i' = 0, \ i = 1, 2 \), because they represent unanticipated shocks, it is straightforward to conclude that the expected money supplies equal zero. Setting \( m_i' = m_i = 0 \) the reduced forms for the modified loss function are, for \( \{i,j\} = \{1, 2\}, \ i \neq j \):

\[
W_i = L_i + V = \frac{1}{2} \left[ \sigma (m_i) + (m_i - 2\theta m_i + (1 - \tau)x_i + \tau x_i' - (u_i - u_j))^2 + \left[ \beta \right] \right] \\
+ \frac{\rho(\xi)}{\eta}(x_i - x_j)^2 - \frac{1}{\xi(\eta)} (u_i - u_j)^2
\]

\[ I \]

C. Solutions to the model

The standard solutions are obtained setting \( \beta = 0 \), such that \( W^C = L^C \). The explicit cooperative solution is obtained by the minimization of the weighted joint loss function, \( L^C = \delta L_1 + (1 - \delta) L_2 \) where \( \delta, 1 - \delta \) are the weights assigned to each country. The solutions are contained in the contract curve \( (C(\delta)) \) whose rate of marginal substitution is equal to minus one:

\[
dm_i/\eta dm_j \mid_{\delta = 0} = -1
\]

\[ D \]

Setting \( \delta = 1, (\delta = 0) \), the instruments of both countries are chosen to maximize the welfare of the first (second) country. These solutions are known as bliss points \( (B_1, B_2) \)

\[
C(1) = B_1 \mid \delta = 1 = \{ m_1, m_2 \} = \{ 0, (2\theta)^{-1}[(1 - \tau)x_1 + \tau x_2' - (u_1 - u_2)] \};
\]

\[
C(0), B_2 \mid \delta = 0 = \{ m_1', m_2' \} = \{ (2\theta)^{1/2}[(1 - \tau)x_1 + \tau x_2 + (u_1 - u_2)] , 0 \}
\]

Since the bliss points correspond to zero welfare losses, \( B_1, B_2 \) are placed at the center of the ellipses. It is convenient to define the segment which joins the bliss points as the bliss line \( (B) \):

\[ -25 - \]
Minimising each country loss function with respect to the respective instrument, and taken
the action of the other country as given, we obtain the (modified) reaction functions:

\[
R_l(A), R_r(A)
\]

The points where these modified reaction functions intersect represent the targeting solutions,
\[T(A) = \{m_l, m_r\}:
\]

The non-cooperative Nash solution is a particular case of the general targeting solution,
when \[\beta = 0\] and \[R_l(A) = R_l^N, R_r(A) = R_r^N : N = T(0) = \{m_l^N, m_r^N\}.\] In this case, where \[\psi_1 = 1 + \sigma, \psi_2 = 2\theta, \psi_3 = 0, \psi_4 = 1\], it is straightforward to see by direct substitution into \[V\] that:

\[
m_l^N = \frac{[\psi_4(1-\tau) + \psi_3]x_1(1-\tau) + \psi_2(\sigma - \tau) + (1 + \sigma)(1-\tau)(x_1 - x_2)}{(1-\tau)^2 - (2\theta)^2}
\]

\[\text{D. Proof to the propositions}\]

\[\text{Proof to proposition 1 (Target line).}\] The slope of \[T(\Lambda)\] in the \([m_l, m_r]\) space is obtained by
the cocient of the derivatives of the target solutions appearing in \[V\] with respect to \(\Lambda\):

\[
dm_l^2/dm_r^2 = \frac{\partial m_l^2/\partial \Lambda / \partial m_r^2/\partial \Lambda}
\]

\[\text{Canzonieri & Henderson (1991, pgs. 21 and 37) only consider symmetric supply shocks}\]
\((x_1 = x_2)\) and opposite demand shocks \((u_1/2 = -u_2/2)\). Substituting in [3] we obtain the same expressions as theirs.
Now we will show that the partial derivatives are of different sign but equal value. Let us consider the different terms in equation [V]. When \( i = 1, j = 2 \) the last two terms in the expression, and consequently their partial derivatives, are equal and of opposite sign than when \( i = 2, j = 1 \), but the first term is different. Let us denote the respective numerators of this first term by \( Z_1, Z_2 \) and let us express \( x_2 \) in terms of \( x_1, x_2 = Kx_1, K \in \mathbb{R} \). Simplifying this expression we obtain

\[
Z_1 = (a\Psi_1 + b\Psi_2) x_1; \quad Z_2 = (a\Psi_1 + b\Psi_2) x_1
\]

where \( a = K + \tau(1-K), b = 1 - \tau(1-K) \). Adding and subtracting \( a_1 x_1 \) from \( Z_1 \) and \( b_1 x_2 \) from \( Z_2 \), we obtain that

\[
Z_1 = [a(\Psi_1 + \Psi_2)] x_1; \quad Z_2 = [b(\Psi_1 + \Psi_2) - c\Psi_2] x_1
\]

where \( c = (1-K)(1-2\tau) \). Again, the last term of this expression is equal and of opposite sign, so that we can concentrate on the first part of the expression. Taking now also into account the denominator, the relevant expression simplifies as follows:

\[
\frac{a(\Psi_1 + \Psi_2)}{\Psi_1^2 - \Psi_2^2} = \frac{a}{1 + 2\theta} x_1; \quad \frac{b(\Psi_1 + \Psi_2)}{\Psi_1^2 - \Psi_2^2} = \frac{b}{1 + 2\theta} x_1
\]

But note that the derivative of these expressions with respect to \( \lambda \) is just zero. Hence we infer that \( \partial m^T/\partial \lambda = \partial m^T/\partial \lambda \) and \( \partial m^T/\partial \lambda \), \( -1 \). Finally, the Nash solution is known to represent one point in this line, so that we can derive the equation of the straight line:

\[
T(\Lambda) := \{(m_1^T, m_2^T) \mid m_2^T = \frac{x_1 + x_2}{1 + 2\theta} - m_1^T \}
\]  

(VII)

Proofs to propositions 2 and 5. (Incentive compatibility) Taking as reference the \( m_1 \)-axis, the slope of the target line is equal to \(-1\), so that \( t g(\omega_T) = -1 \), where \( \omega_T = 135^\circ, 315^\circ \). The bargaining area, \( A \), is formed by the area within the ellipses crossing at the Nash solution \( N \). Thus, \( A \) is placed between the tangent lines to the two ellipses at \( N \). Secondly, \( N \) is known to be a point on the target line. Therefore, as the figures suggest, if the target line lies between the angle formed by those two tangents: \( \omega_m < \omega_T < \omega_{np} \{i, j\} = \{1, 2\}, i \neq j \), the target line will cross the bargaining area. The general expressions for the ellipses slope at Nash equilibrium are:

\[
\frac{dm_2}{dm_1} \bigg|_{\omega = \omega_T} = -\frac{\partial W/\partial m_1}{\partial W/\partial m_2} = \frac{(1+\sigma)m_2^N - 2\theta m_1^N + (1-\tau)x_1 + r_1 - (u_i - u_j)}{2\theta[m_1^N - 2\theta m_j^N + (1-\tau)x_1 + r_1 - (u_i - u_j)]}
\]

Let us consider first demand shocks \((x_1 = x_2 = 0)\). Substituting the Nash solution [VI] into

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the previous expression, we obtain

\[ \frac{dm_2}{dm_1} \bigg|_{\omega_n=0} = -\frac{\sigma}{2}(u_1-u_2)=0 ; \quad \frac{dm_1}{dm_2} \bigg|_{\omega_n=0} = \frac{\sigma}{2}(u_1-u_2)=\infty \]

The angles formed by these tangent lines depend on the sign of \((u_1-u_2)\). In particular, for the ellipse corresponding to \(W_i\):

\[ \forall u_1, u_2, \lim_{m_1 \to m_n} \frac{dm_2}{dm_1} \bigg|_{\omega_n=0} = 0 \Rightarrow \tan(\omega_1)=0 \]

and \(\exists \epsilon > 0 \mid \forall u_1-u_2 < 0, m_1 \in [m_1^N-e, m_1^N], \Rightarrow \frac{dm_2}{dm_1} \bigg|_{\omega_n=0} > 0 ; \quad \forall u_1-u_2 > 0, m_1 \in [m_1^N-e, m_1^N], \Rightarrow \frac{dm_2}{dm_1} \bigg|_{\omega_n=0} < 0.\]

Thus \(\forall u_1-u_2 < 0, \omega_1=0^\circ; \forall u_1-u_2 > 0, \omega_1=180^\circ\)

and for the second ellipse \(W_2\):

\[ \forall u_1, u_2, \lim_{m_1 \to m_n} \frac{dm_2}{dm_1} \bigg|_{\omega_n=0} = \infty \Rightarrow \tan(\omega_1)=\infty \]

and \(\exists \epsilon > 0 \mid \forall u_1-u_2 < 0, m_2 \in [m_2^N-e, m_2^N], \Rightarrow \frac{dm_1}{dm_2} \bigg|_{\omega_n=0} < 0 ; \quad \forall u_1-u_2 > 0, m_2 \in [m_2^N-e, m_2^N], \Rightarrow \frac{dm_1}{dm_2} \bigg|_{\omega_n=0} > 0.\]

Thus \(\forall u_1-u_2 < 0, \omega_2=90^\circ; \forall u_1-u_2 > 0, \omega_2=270^\circ\)

It follows then that

\(\forall u_1-u < 0 \Rightarrow \omega_1 < \omega_2 < \omega_2; \quad \forall u_1-u > 0 \Rightarrow \omega_1 < \omega_2 < \omega_2;\)

and the target line is precisely the bisectrix of the angle formed by the tangents to both ellipses.

Thus, the target zone will always cross the bargaining area.

Proceeding as before we obtain that, for supply shocks \((u_1=u_2=0)\):

\[ \frac{dm_2}{dm_1} \bigg|_{\omega_n=0} = \frac{0}{\sigma \theta((1+\sigma)x_1+2\theta x_1+(1+\sigma-2\theta)\tau(x_1-x_1))} = 0 \]

\[ \frac{dm_1}{dm_2} \bigg|_{\omega_n=0} = \frac{0}{\sigma \theta((1+\sigma)x_1+2\theta x_1+(1+\sigma-2\theta)\tau(x_1-x_1))} = \infty \]

For \(W_j\) and \(W_2\), we have now:
Thus, the following cases arise:

$\forall x_i < 0$:

1. $x_i < \frac{2(1 + \sigma - 2\theta)}{1 + \sigma - (1 + \sigma - 2\theta)^T} x_i \rightarrow \omega_{W_i} = 90^\circ$;
2. $x_i \leq -\frac{2(1 + \sigma - 2\theta)}{1 + \sigma - (1 + \sigma - 2\theta)^T} x_i \rightarrow \omega_{W_i} = 270^\circ$

$2\theta + (1 + \sigma - 2\theta)^T \tau < (1 + \sigma) - (1 + \sigma - 2\theta)^T \tau$. Thus, the following cases arise:

$\forall x_i > 0$:

1. $x_i \geq \frac{2(1 + \sigma - 2\theta)}{1 + \sigma - (1 + \sigma - 2\theta)^T} x_i \rightarrow \omega_{W_i} = 90^\circ$;
2. $x_i \leq \frac{2(1 + \sigma - 2\theta)}{1 + \sigma - (1 + \sigma - 2\theta)^T} x_i \rightarrow \omega_{W_i} = 270^\circ$

Thus, the target line constitutes the bisectrix of the angle formed by the tangents to both ellipses in cases labelled (a); for the rest of cases, the target line is perpendicular to the bisectrix and consequently are ruled out. The range of shocks for which the target line crosses the bargaining area can then be established:

$\forall x_i > 0, \forall j = 1, 2, i \neq j, \text{if } \frac{(1 + \sigma - (1 + \sigma - 2\theta)^T \tau}{2\theta + (1 + \sigma - 2\theta)^T \tau} x_i \leq x_j \leq \frac{2(1 + \sigma - 2\theta)}{1 + \sigma - (1 + \sigma - 2\theta)^T} x_j \Rightarrow \exists A \mid T(A) \cap A$

**Proof of propositions 3 and 6 (Pareto optimal points).** Let us consider the bliss line $B$, instead of the contract curve, since the latter is too complex to work with. As a previous step, it is claimed that if the target line intersects the bliss line it also implies intersects the contract curve. We prove this claim as follows.

Since the contract curve, the target line and the bliss line are continuous and differentiable, we can express:

- $m_i^c = f(\delta)$ as a function of $m_i^c = g(\delta)$: $m_i^c = f(g^c(m_i^c)) = C(m_i^c)$
- $m_i^T = f'(\Lambda)$ as a function of $m_i^T = g'(\Lambda)$: $m_i^T = f'(g^T(m_i^T)) = T(m_i^T)$
- $m_i^b$ as a function of $m_i^b$: $m_i^b = B(m_i^b)$

Then the lemma below, based on the Bolzano-Weierstrass theorem can be directly applied and our claim is proved.
The coordinates for which $T=B$ are now found for the two different types of shocks. The intersection between $B$ and $T$ is obtained by equating expressions $[IV]$ and $[V]$.

For demand shocks ($x_1=0, x_2=0$), we get

$$m_1^T=m_1^B=-m_2^B=-m_2^T=\frac{1}{2} \frac{u_1-u_2}{2\theta}$$

which corresponds precisely to the middle point of the bliss line. Thus, in the case of demand shocks the target line will always cross the contract curve.

The resolution is more complex when supply shocks hit the economies ($u_1=u_2=0$). Equating the target and bliss lines, the intersection is given by

$$m_1^T=m_1^B=-\frac{(1+\sigma)x_1+2\theta x_2}{2\theta(1+\sigma-2\theta)} + \frac{(1-\tau)x_1+\tau x_2}{2\theta}$$

$$m_2^T=m_2^B=-\frac{(1+\sigma)x_1+2\theta x_2}{2\theta(1+\sigma-2\theta)} + \frac{(1-\tau)x_1+\tau x_2}{2\theta}$$

where it is not straightforward to ascertain whether this point falls within the relevant segment of the bliss line. Thus, all the possible combinations of supply shocks are examined to obtain the range of shocks which permits the target line to intersect the bliss segment. The solution is given by the following range:

$$\forall \lambda_i > 0, i,j=1,2, i \neq j, \text{ if}$$

$$\frac{(1+\sigma)-(1+\sigma-2\theta)\tau}{2\theta+(1+\sigma-2\theta)\tau} x_i \leq x_j \leq \frac{(1+\sigma)-(1+\sigma-2\theta)\tau}{2\theta+(1+\sigma-2\theta)\tau} x_j \Rightarrow \exists \lambda, \delta \mid \gamma(\lambda) = C(\delta),$$

the same than in proposition 5 above.

**Lemma (Bolzano-Weierstrass)** The lines $C(\delta)$ and $B$ have two common points, at $\delta=0, \delta=1$. Then, if $\exists m_i \mid T(\lambda)=B \iff m_i \mid T(\lambda)=C(\delta)$.

**PROOF:** The claim is that $\exists m_i \mid (T'-C)m_i=0$. Adding and subtracting $B$, we get $(T'-B-B-C)m_i=0$. Let us assume that $\exists m_i \mid B(m_i)=T(m_i)=m_i$. Recall that $C(0)=B, C(1)=B'$ so that $(C-B)m_i^{B}=(C'-B)m_i^{B}$. Since $B, B'$ are the extremal points of the bliss line, this implies that if $B(m_i^{B})=C'(m_i^{B}) \mid T'(m_i^{B})$, $B(m_i^{B})=C'(m_i^{B}) < T'(m_i^{B})$, $i,j=1,2, i \neq j$

it follows that

$$\exists m_i \mid C'(m_i^{B'})=T'(m_i^{B'})$$

by Bolzano-Weierstrass, and the lemma is proved.

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Proof of proposition 4 (and 7). (Optimal and incentive compatible points). From [VI], we observe that \( m_t' = \text{optimal} \) on the target line in the absence of supply shocks. Substituting \( m_t \) for \(-m_2\) in the first order conditions of the cooperative solution (which corresponds to the left-side term and right-side term of the expression for \( C(\delta) \) in [II]) and substituting \( \Sigma_i \) for the respective values, the following equalities must hold:

\[
\begin{align*}
\delta (1 + \sigma + (1 - \delta) (2\delta)^2 + 2\theta) m_t' &= \delta ((1 - \delta) 2\theta) (u_1 - u_2); \\
(1 - \delta) (1 + \sigma + (1 - \delta) (2\delta)^2 + 2\theta) m_t' &= (\delta 2\theta + (1 - \delta))(u_1 - u_2)
\end{align*}
\]

Given the demand shocks, this is a non-linear system in \( \delta \) and \( m_t' \). However, we showed that \( T \) intersects the bliss line at the middle point; this suggest that a reasonable guess for solving the system is \( \delta = 1/2 \). Indeed, it is immediate to see that \( \delta = 1/2 \) satisfy both equations. The Pareto optimal point on the target line is then given by:

\[
C_2 = \{m_t^*, m_2^* \} \in T(\lambda) \mid m_t^* = \frac{(1 + \delta 2\theta)(u_1 - u_2)}{\sigma + (1 + 2\delta)^2} = -m_2^*.
\]

Substituting \( N \) and \( C(1/2) \) in the loss functions, the welfare loss is obtained:

\[
W_i^N = \frac{1 + \sigma}{2(1 + \sigma + 2\delta)} (u_i - u_2);
\]

\[
W_i^* = \frac{(1 + \delta 2\theta)}{(1 + \sigma + 2\theta)^2 + (2\theta)\sigma} W_i^N = W_i^N < W_i^N, \ i = 1, 2
\]

and consequently the Pareto optimal point is incentive compatible.

Proof of propositions 8 (optimal degree of commitment) The money supplies of the cooperative solution [II] in the cases of demand and opposite supply shocks must belong to the target solution [V]. Noting that \( x_1 = -x_2 \) and \((\psi_1, \psi_2)/(\psi_1^2 - \psi_2^2) = (1 + \sigma + 2\theta + 2\beta \rho')^{-1} \), it follows that:

\[
m_t^* | c = \frac{(1 + \delta \theta)}{\sigma + (1 + 2\theta)^2} [(u_i - u_2) - (1 - 2\tau)x_i] = \frac{(1 + \beta \rho') \sqrt{u_i - u_2} - (1 - 2\tau) \rho \sqrt{u_i - u_2}}{1 + \sigma + 2\theta + 2\beta \rho'}
\]

Considering each shock separately and equating both terms to solve for \( \beta' \), for a given value of \( \rho \):

is the general expression for the optimal degree of commitment to a given exchange rate target. Given the values of the parameters, it is immediate to check that \( \beta' \) is positive for demand shocks.
\[ \forall u_1, u_2, \quad \beta^* = \frac{\sigma 2\theta \sqrt{\eta} \xi}{\sqrt{\rho} \left[(1+2\theta)^2 + \sigma - 2\theta \sqrt{\eta} (1+2\theta) \sqrt{\rho} - \rho \right]} \]

\[ \forall x_1 = -x_2, \quad \beta^* = \frac{(1-2\tau)\sigma 2\theta \sqrt{\eta}}{2\sqrt{\rho} \left[(1-2\tau)\xi + \frac{1}{2} (1+2\theta)^2 + \sigma - \sqrt{\eta} (1+2\theta)(1-2\tau) \sqrt{\rho} - \rho \right]} \]

in the considered range of \( \rho \); on the contrary, for opposite supply shocks the function presents a discontinuity at \( \rho^* = 2\theta(1 + 2\theta\sigma/(1 + 2\theta) - \xi \sqrt{\eta} f') > 0 \). Values lower than \( \rho^* \) yield negative \( \beta^* \).

Taking the derivative of \( \beta^* \) with respect to \( \rho \) reveals that \( \partial \beta^*/\partial \rho > 0 \) for demand shocks and \( \partial \beta^*/\partial \rho < 0 \) for opposite supply shocks.

**Proof of proposition 9 (Positive externality).** Substituting the value of the shocks in the Nash solution \([VI]\) we get

\[ \forall x_i > 0, x_j \notin \left[ -\frac{1+\sigma-(1+\sigma-2\theta)\tau}{2\theta + (1+\sigma-2\theta)\tau} x_i, -\frac{2\theta + (1+\sigma-2\theta)\tau}{1+\sigma-(1+\sigma-2\theta)\tau} x_i \right], i = 1, 2, i \neq j \Rightarrow \text{sign}(m_i^N) = \text{sign}(m_j^N) \]

\[ \forall x_i > 0, \quad x_j \leq \frac{1+\sigma-(1+\sigma-2\theta)\tau}{2\theta + (1+\sigma-2\theta)\tau} x_i, x_j < \frac{2\theta + (1+\sigma-2\theta)\tau}{1+\sigma-(1+\sigma-2\theta)\tau} x_i, i = 1, 2, i \neq j \Rightarrow \text{sign}(m_i^N) \neq \text{sign}(m_j^N) \]

\[ \forall u_1, u_2 \Rightarrow m_i^N = -m_j^N \]

Note that the first case covers just the range of values for which neither optimal nor incentive compatible exchange targeting strategies can be devised and other two cases conveyed the range of shocks for which propositions 2-7 apply. Thus, the proposition is proved.
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