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Abstract

In this paper we estimate the multilateral target zone model of Serrat (1994) using a simulated method of moments methodology. In contrast to the widely reported poor performance of bilateral target zone models and other nonlinear models of exchange rates, the multilateral model fits European Monetary System data very well. We also conduct Monte-Carlo simulation exercises to evaluate the power of our tests against competing alternative hypotheses. In addition, we can explain the negative results of the previous empircal literature in the context of the model. Thus, the additional insights provided by the multilateral model, turn out to be of extreme empirical relevance. They are driven by the parameters reflecting the degree of cooperation amóng monetary authorities in maintaining the regime, which have been neglected by previous theoretical and empirical literature.

1 Introduction

The goal of the target zone literature has been to characterize the behavior of exchange rates in the context of a two-country monetary model where the range of variation of the exchange rate is bounded by a currency band agreement among monetary authorities. The original model of Krugman (1991) was constructed under the assumptions that the target zone is perfectly credible (the commitment of monetary authorities to keep the exchange rate within the band is complete, and they have the ability to do so) and that marginal interventions are the only intervention tool (monetary authorities intervene by manipulating the relative money supplies if and only if the exchange rate hits the limits of the band). The empirical performance of the model has been, however, very poor (see for example Flood, Rose and Mathieson (1991), Lindberg and Söderlin (1991), DeJong (1994) and Smith and Spencer (1992)). The reasons for the model's rejection lie in the clearcut but counterfactual predictions of the bilateral model. First, the exchange rate should exhibit an unconditional U-shaped distribution within the band; second, the conditional volatility of interest rates should approach zero when the exchange rate approaches the limits of the band. Extensions of the basic bilateral model, which relax the assumptions of perfect credibility and marginal interventions as the leading intervention tool, reconcile the predictions of the model with the evidence, although we are not aware of formal tests of such models.

There is a feature, however, whose theoretical and econometric analysis has been completely neglected by the literature and is actually relevant in real-world target zones: currency bands agreements usually involve more than two currencies. In Serrat (1994), a multilateral target zone with an arbitrary number of currencies is modeled in the spirit of Krugman

(1991). This model is reviewed below. He obtains closed form solutions expressing the exchange rate as a function of vector of underlying aggregate macroeconomic state variables, denominated fundamentals. The predictions of the multilateral model seem to be much more in accordance with the empirical evidence than those of the bilateral model. In particular, a hump-shaped steady-state distribution of the exchange rate can be recovered and the amount of curvature in the conditional volatility of the exchange rate as it approaches the limits of the band (the size of "honeymoon effect") is allowed to vary randomly. Following Krugman's original assumptions, the model is constructed maintaining the assumptions of perfect credibility and marginal interventions, in the space of the fundamentals, as the unique intervention tool.

There are two main features that make the multilateral target zone model different from the bilateral model. First, the existence of cross-currency constraints make the range of variation of one currency versus another tighter than the official bilateral one because the bilateral band with a third currency becomes binding while the exchange rate between the first two currencies is in the interior of its official band. Second, recall from the bilateral model of Krugman (1991), that the fundamentals process is constructed as a linear combination of macroeconomic processes from the *two* countries involved in the target zone agreement. An intervention designed to manipulate the fundamentals between two currencies involved in the system will inevitably affect the fundamentals processes of other currency pairs. In the model it can be shown that even if we choose the parameters such that the first feature above is nowhere binding (such a case corresponds to a *currency influence area*), the second feature alone makes the predictions of the model differ substantially from those of the bilateral model. They actually provide a rationale for a certain type of intramarginal interventions in the space of the exchange rates, not in the space of the fundamentals. In particular, the predictions that change are the problematic ones (unconditional U-shaped distribution and an always strong "honeymoon effect") while the appealing ones are kept (namely the stabilizing effect on the exchange rate of the currency agreement). The purpose of this paper is to conduct an econometric evaluation of the multilateral model using a simulated GMM technique, which is the predominant approach for parameter estimation and hypothesis testing in latent variable models (see, for instance, Ferson and Foerster (1994)). In Section 1.1, we review the empirical evidence regarding Krugman's target zone model. In Section 2, we outline the multilateral target zone model of Serrat (1994). In Section 3 we present the data to be used in the analysis. Section 4 discusses the econometric methods used. Section 5 presents the results for the versions of the model estimated, and the results of the Monte-Carlo analysis to approximate the power of our test. In Section 5, we also present and discuss the results of performing an out-of-sample comparison of the predictive power of the model versus a random walk. Section 6 concludes.

1.1 Empirical Evidence on the Bilateral Model

Empirical work on Krugman's model can be classified into papers that restrict themselves to testing the specific nonlinear specification of the model and papers that use more general methods, including nonparametric methods, to test the qualitative implications of the model.

Flood, Rose and Mathieson (1991) conducted the first extensive empirical analysis of Krugman's model. Assuming uncovered interst rate parity, they construct an instrument for the fundamentals which is equivalent to a weighted sum of the forward and spot rates. They conclude that evidence for the presence of non-linearities in exchange rates within target zones is weak and that their signs are not those predicted by the basic model. In addition, evidence regarding the stabilization effect predicted by Krugman's model (the "honeymoon effect") is ambiguous. Gourinchas (1994) finds evidence of nonlinearities using a semiparametric approach with a sensible instrumentation of the fundamentals process. Moreover, he uncovers the nature of the nonlinearities as possibly due to a phenomenom of asymmetric credibility, not captured by the bilateral model.

A significant degree of mean-reversion in exchange rates within the band was found in Rose and Svensson (1991) (estimating the Bertola and Svensson (1993) model). Pesaran and Samiei (1992), in a discrete time rational expectations model, found that explicitly incorporating beliefs about stabilizing marginal interventions helps in fitting data for the German Mark/French Franc exchange rate. However, contrary to the predictions of the bilateral model (and consistent with those of the multilateral model) they find that the $\mathcal{S}$-shaped relationship between exchange rates and fundamentals is stochastic rather than deterministic. Several studies have applied simulated method of moments estimation, with different variations, to test Krugman's model: Lindberg and Söderlin (1992) propose a mean-reverting specification for the fundamentals process that matches Swedish data better than Krugman's model. Smith and Spencer (1992) estimate Krugman's model for the Italian Lira/German Mark exchange rate series, although they report convergence problems and many relevant statistics are not reported in their paper. De Jong (1994) applies both maximum likelihood and simulated moments methods to study the fit of the bilateral model for several EMS currencies for the same time period as ours. He finds that the parameters estimates differ substantially across countries. In addition, precise parameter estimates are not, in general, obtained. He rejects Krugman's model for the Dutch Guilder, the French Franc and the Italian Lira against the German Mark using an overidentifying restrictions test. Overall he concludes that Krugman's model is misspecified and suggests extensions of the model in terms of the underlying dynamics of the state variable process.

As mentioned in the previous section, the multilateral target zone model can potentially explain several of the empirical regularities at odds with Krugman's model, in the context of a fully credible model with marginal interventions. The fact that notional bands can differ from nominal bands could account for reversing one of the most problematic predictions of Krugman's model, namely, an unconditional steady-state density for the exchange rate within the band. This point has been noted elsewhere in the literature (e.g. Pill (1994)). In Serrat (1994) it is shown that not only cross-currency restrictions but also spillover effects from foreign exchange interventions on third currency fundamentals can reverse Krugman's theoretical predictions. If we accept the multilateral model as a good description of reality, then we will have to conclude that the previous empirical literature has been flawed in attempting to test Krugman's model. The poor empirical results found in the literature may not be due to the inappropriateness of Krugman's assumptions of perfect credibility and marginal interventions as the exclusive intervention tool, but rather to the fact that the tests have been performed on an overly restrictive model on data from multilateral target zones. The multilateral nature of real-world target zones makes Krugman's bilateral model inappropriate even under the most generous assumptions (i.e. that cross currency constraints do not matter and thus a multilateral target zone is, in fact, a currency influence area).

In this paper we apply a simulated method of moments technique (MSM) to estimate the model. One of the advantages of this technique is that instrumentation of the fundamentals process is not needed in deriving testable hypotheses from the model. This is important since no particular construction of the fundamentals process is neither assumed nor implied by the model. Naturally, we choose a time span for which Krugman's assumption of perfect credibility may not be far from reasonable. The results strongly support the multilateral model, not only against Krugman's bilateral model, but also against a reasonable alternative hypothesis, as explicit power computations show. We find that the multilateral feature of real-world target zones is crucial in understanding the results of target zone models and that the pessimistic opinion about the poor performance of target zone models must be reconsidered.

This paper presents an ambitious implementation of the MSM technique, in terms of computational demands. In the next section we outline the model and the equation that we estimate. We first estimate a trilateral version of the model for ten groups of currencies with each group consisting of the German Mark and other two EMS currencies. We then generalize further and estimate a five-currency version of the multilateral model.

We find that both the three-currency (for all groups of currencies) and the five-currency versions of the model are not rejected at the usual levels of significance. In addition, when the parameters of the multilateral target zone model are restricted so as to obtain a multilateral target zone as a combination of simple bilateral (Krugman) versions of the model, we find that these restrictions are strongly rejected by the data. We also conduct an experiment to approximate the power of the overidentfying restrictions test associated with the MSM technique under reasonable specifications for the data generating process under the alternative hypothesis. The model is correctly rejected in most of the cases. It turns out that the estimates of the reflection matrix drive the good performance of the model which is precisely the aspect of reality neglected by the previous literature.

2 The Multilateral Target Zone Model

The multilateral model has different theoretical implications than the bilateral model due to the existence of an additional set of parameters that capture the degree of cooperation among central banks in sharing the intervention burden. In the multilateral model, the state variable consists of an n-dimensional vector of fundamentals that is reflected at each side of the fundamentals domain, (an n-dimensional polyhedron). From the $n+1$ countries involved in the target zone, set a reference (or anchor) country 0.

The underlying theory of target zone modelling is usually obtained from a minimalist monetary model in the following way. Let the money demand equations satisfy $\frac{M_t^i}{P_t^i} = y_{it}^{\alpha^i} \exp\left(-\gamma r_t^i\right)$ for each country i $(i = 1, ..., n)$ in the target zone, where M_t^i, P_t^i, y_{it}, and r_t^i denote the money supply, price level, aggregate endowment and nominal instantaneous interest rate processes for country i at time t. α^i and γ (the semi-elasticity of money demand with respect to the interest rate) are constants. Let purchasing power parity and a logarithmic version of uncovered interest rate parity hold; thus, $P_t^i = S_t^{ij} P_t^j$ and $r_t^i - r_t^j = \frac{E_t(ds_t^{ij})}{dt}$ where $S_t^{ij} = \exp(s_t^{ij})$ is the nominal exchange rate of country i with respect to country j. Taking logarithms of both sides of the money demand equation for countries i and j, subtracting them and using the last two equations, we obtain:

$$s^i = k^i + \gamma \frac{E_t(ds_t^i)}{dt} \tag{1}$$

where k^i, the i^{th} component of the fundamentals process, is constructed as a function of underlying macroeconomic variables:

$$k^i = m^i - m^0 + \alpha^i y^i - \alpha^0 y^0 \tag{2}$$

where m^l and y^l are respectively the logarithms of the money supply and aggregate endowment of country l. α^l, α^l and γ are constants, for $l = i, 0$.

The fundamentals vector is a regulated n-dimensional reflected arithmetic brownian motion that takes values in an n-dimensional polyhedron, $G \subset \mathcal{R}^n$, with a $n \times 2n$ reflection matrix called R whose column vectors are the reflection vectors at each side of the polyhedron. The concept of reflection vector is explained below. These dynamics are described by the stochastic differential equation:

$$\begin{aligned} dk_t &= \mu dt + A dW_t + \sum_{i=1}^{2n} R^i d\Lambda_t^i && (3) \\ & k_0 \in \overset{o}{G} \subset \mathcal{R}^n && (4) \end{aligned}$$

where μ is a $n \times 1$ drift vector, A is a $n \times n$ diffusion matrix and W_t is a $n \times 1$ Wiener process. The regulator process $\Lambda_t = \{\Lambda_t^i\}_{i=1}^{2n}$ is a nondecreasing $2n$-dimensional process that increases if and only if the fundamentals hits the m^{th} side of the polyhedron that constitutes their domain, for $m = 1, ..., 2n$. It can be proved that the process in (3) is Markov.

The matrix composed of the vectors that indicate the direction of reflection at each side of the fundamentals domain, namely $R = [R^1 : ... : R^{2n}]$ where R^i is a n-dimensional vector, can be interpreted as a measure of the degree of cooperation among central banks in maintaining the currency band agreement (see Serrat (1994) for details). Thus, when the fundamentals process hits any of the $2n$ sides of its domain, the relative money supplies $m^i - m^0$ are adjusted by the monetary authorities to keep the fundamentals process within its domain. The direction in which the fundamentals process is reflected back is related to the relative intensity with which the relative money supplies (or, in this context, foreign exchange interventions) vary. This is exactly what is captured by the reflection matrix R. By assumption, the intervention rules are symmetric, in the sense that $R^i = -R^{n+i}$ for $i = 1, ..., n$. Thus, if two central banks share the intervention burden in a certain way when the fundamentals hits a certain face of its domain, the roles are switched when the fundamentals hit the opposite face of their domain.[1] It will be convenient to write $R = [M : -M]$ where $M = [R^1 : ... : R^n]$ is a $n \times n$ matrix. Hereafter, we denote M the reflection matrix. It is also important to note that the size of the reflection vector does not matter, only its direction. Thus, we can normalize the column vectors of M to have unit norm.[2] This is important from an econometric point of view; otherwise, the model would not be identified.

The model collapses naturally to Krugman's model if we impose that cross-currency

[1] Note that this is a reasonable assumption, the formal design of EMS intervention rules is entirely symmetrical by the 'Belgian Compromise', see Vehrkamp (1994, page 28).

[2] With this normalization, if the reflection matrix is diagonal it is normalized to be the identity matrix.

constraints are not binding and that the reflection matrix is the identity matrix.[3] This corresponds to a currency influence area in which the anchor currency never collaborates in the foreign exchange interventions by manipulating its own money supply.

Note from equation (1) that the fundamentals of country i are modified by the montetary authorities of country i and/or those of country 0. Thus, if the monetary authority of the anchor currency never intervenes and the burden of intervention falls on non-anchor currencies, then there are no spillover effects on third country fundamentals. This corresponds to a diagonal M matrix. In general, if country i intervenes by (say) decreasing its money supply at some point on the boundary of the fundamentals and country 0 increases its money supply to help decrease country i fundamentals, then the fundamentals of country j will also decrease, although by a smaller amount than country i's fundamentals. Thus, the further any particular column vector of M is from being parallel to any of the axes (i.e. M diagonal), the higher is the involvement of the anchor currency in the intervention operations and thus the higher is the degree of real symmetry in the system. Note also that each column vector of M can be associated with a particular currency.

We always expect that the elements of each column vector of the reflection matrix have the same sign: central bank interventions should work towards the same goal. We also expect the diagonal elements of the M matrix to be larger than the off-diagonal elements, because they indicate a more pronounced direction of reflection for the fundamentals process of the country whose currency is the weakest (or strongest) in that particular region of the domain for the fundamentals. This arises because in our application, we order the data such that the

[3] I.e. $R = [I_{n \times n} : -I_{n \times n}]$.

elements of the diagonal of the matrix M correspond to the fundamentals of the currency with the weakest (or strongest for the complementary reflection matrix, $-M$) position. For example, suppose that M is a 2×2 matrix (corresponding to a three country target zone). If the interventions of the anchor currency are of smaller magnitude than the interventions of the weakest (or strongest) currency at some point on one of the sides of the domain for the fundamentals, then the ratio that we should observe between the largest and smallest component of each column vector of M should be larger than 2. This is precisely what we obtain in section 5 for most of the cases (i.e. different combinations of trilateral target zones).

The expression for the logarithm of the exchange rate of country i, as a function of the fundamentals in a n-lateral target zone, when cross currency constraints are nowhere binding, is obtained in Serrat (1994) using equations (1) and (2) and is given by:

$$s^i(k^1, .., k^n) = \gamma\mu_i + k^i + \sum_{j=1}^{n} C^i_{1j} \exp\left(\lambda^j_+ \left(\sum_{l=1}^{n} v^{(j)}_l k^l\right)\right) + \sum_{j=1}^{n} C^i_{2j} \exp\left(\lambda^j_- \left(\sum_{l=1}^{n} v^{(j)}_l k^l\right)\right) \quad (5)$$

where $v^{(j)}$ is the j^{th} row vector of M^{-1}. The constants λ^j_+, λ^j_- $j = 1, .., n$, are given by

$$\lambda^j_+ = \frac{-\theta_j + \sqrt{\theta_j^2 + 2\frac{\phi_j^2}{\gamma}}}{\phi_j^2} \qquad \lambda^j_- = \frac{-\theta_j - \sqrt{\theta_j^2 + 2\frac{\phi_j^2}{\gamma}}}{\phi_j^2} \quad (6)$$

where $\theta = M^{-1}\mu$ and $\phi \odot \phi = diag\,(M^{-1}AA'M^{-1})$ are $n \times 1$ vectors, $[v_{ij}] = [M^{-1}]_{ij}$ and where $\odot$ indicates member-wise multiplication. The constants, C^i_{1j}, C^i_{2j} are obtained as the solution of a $2n$-dimensional system:

$$m_{ij} + \lambda_+^j C_{1j}^i \exp\left(\lambda_{1j}\left(\sum_{l=1}^n v_l^{(j)} \bar{k}^l\right)\right) + \lambda_-^j C_{2j}^i \exp\left(\lambda_{2j}\left(\sum_{l=1}^n v_l^{(j)} \bar{k}^l\right)\right) = 0 \tag{7}$$

$$m_{ij} + \lambda_+^j C_{1j}^i \exp\left(\lambda_{1j}\left(\sum_{l=1}^n v_l^{(j)} \underline{k}^l\right)\right) + \lambda_-^j C_{2j}^i \exp\left(\lambda_{2j}\left(\sum_{l=1}^n v_l^{(j)} \underline{k}^l\right)\right) = 0. \tag{8}$$

Finally, $\bar{K} = (\bar{k}_1, ..., \bar{k}_n)$, $\underline{K} = (\underline{k}_1, ..., \underline{k}_n)$, together with the auxiliary variables vectors $(\bar{X}_1, ..., \bar{X}_n)$ and $(\underline{X}_1, ..., \underline{X}_n)$, solve the $4 \times n$ system of equations:[4]

$$s^i\left(M\left[\bar{X}_1, ..., \bar{X}_n,\right]\right) = log(\bar{S}^i), i = 1, ..., n \tag{9}$$

$$s^i\left(M\left[\underline{X}_1, ..., \underline{X}_n,\right]\right) = log(\underline{S}^i), i = 1, ..., n \tag{10}$$

$$(\bar{k}_1, ..., \bar{k}_n)' = M\left(\bar{X}_1, ..., \bar{X}_n\right)' \tag{11}$$

$$(\underline{k}_1, ..., \underline{k}_n)' = M\left(\underline{X}_1, ..., \underline{X}_n\right)'. \tag{12}$$

It is important to note that if we impose that M be a diagonal matrix (and thus normalize it to be the identity matrix) then the solution in (5) collapses to Krugman's solution. The matrix M controls for the amount of spillover effects of foreign exchange interventions on third-currencies fundamentals. Therefore, we insist on this point: it is not the fact that the components of the fundamentals vector are correlated, but rather that M is non-diagonal (i.e. there is collaboration among monetary authorities for intervention purposes) that makes the multilateral solution differ in an essential way from the bilateral solution of Krugman.

[4]Note that we are implicitly assuming that each element of M is greater than or equal to zero.

Let $S^{12} = \frac{S^1}{S^2}$ be the cross-exchange rate between currencies 1 and 2 (quantity of currency 1 per unit of currency 2). The equation (5) is obtained in Serrat (1994), theorem 1, by assuming that the nominal target zone for the cross exchange rate, namely, $[\underline{S}^{12}, \bar{S}^{12}]$ is wide enough so that the cross-currency constraint is never binding. In other words, (5) is valid as long as:

$$\underline{S}^{12} \leq \inf_{(k^1,k^2)\in G} \frac{s^1(k^1,k^2)}{s^2(k^1,k^2)} \leq \sup_{(k^1,k^2)\in G} \frac{s^1(k^1,k^2)}{s^2(k^1,k^2)} \leq \bar{S}^{12} \tag{13}$$

is satisfied. In Serrat (1994) theorem 3, a closed form solution is provided when (13) is not satisfied. However, such solution is much more difficult to implement econometrically. Thus, if the parameters of the problem happen to satisfy (13), (5) provides a valid solution to the trilateral target zone modelling problem even when $[\underline{S}^{12}, \bar{S}^{12}] \subset [\frac{\underline{S}^1}{\bar{S}^2}, \frac{\bar{S}^1}{\underline{S}^2}]$. In other words, there are limits to the range of variation of the exchange rate between currency 1 and currency 0 and between currency 2 and currency 0 that are due not to the limits of the two respective bands, but rather to the fact that the implied exchange rate between currency 1 and currency 2 (i.e. $\frac{S^1}{S^2}$) hits its own band before either S^1 or S^2 reach theirs.

Our objective in this paper is to estimate (5) using the simulated method of moments on data from the European Monetary System (EMS). Given the dimensionality of the problem, we will limit ourselves to the estimation of (5) for the three-currency case (the trilateral model) and the five-currency case (the five-lateral model). Additional tests will be conducted to test Krugman's model as a special case. In addition, the power of our model specification test (the test overidentyfing restrictions associated with minimum distance estimators) will be tested against reasonable specifications for the alternative hypothesis. Regarding the cross-currency constraints restrictions, in all estimations we will assume that (13) is satisfied. Once we obtain our estimates, we will check whether this is indeed the case.

3 Data

We use weekly observations of exchange rates data from the EMS, in particular, the Belgian Franc (BFr), Dutch Guilder (DFl), Danish Krone (DKr), French Franc (FFr) and Irish Pound (IP) versus the German Mark (DM). The sample consists of 189 observations from January 14, 1987 to August 22, 1990. We choose this sample period for two reasons. First, the multilateral model assumes perfect credibility which means that realignments are not allowed. Thus, in an effort to isolate the ideal credibility conditions of the model, we draw our data from the longest period in which the EMS did not experience any realignments, namely the so-called "hard-EMS" period (January 1987-September 1992)[5]. Within that time period, we choose the period from the beginning until the collapse of the eastern bloc (September 1990), because we cannot control for the effect of such events on the overall credibility of the system. Note that this exercise is not evidence of a sample selection bias, but rather the selection of a valid sample -one that does not boldly contradict the basic assumptions, i.e. no jumps, of the model-. Second, to facilitate comparison with previous results for the bilateral model, our time span coincides with that used in the latest empirical study of Krugman's model (De Jong, 1994) and partially coincides with the sample of Flood, Rose and Mathieson (1991)[6]. In addition, given the high computational costs of estimating

[5]With the exception of a realignment of the central parity against DM for the Italian Lira of 3.7% on the January 5, 1990. However, we do not use the Lira in this paper.

[6]Namely their regimes 12 and 13.

the model, we are forced to economize in choosing the currencies in our study. For ease of comparison, we choose the same currencies as in the study of De Jong (1994) (except for the Italian Lira).

During our sample period, each of the currencies in our sample were restricted to lie within 2.2753% of their central parities with respect to the DM and each other currency within our sample of currencies.[7] Our data have been transformed such that our exchange rate variable for country i is the logarithm of the ratio of the exchange rate divided by the central parity of currency i with respect to the DM. Table 2 presents the descriptive statistics for the logarithm (net of central parity) of the exchange rates in our sample. In Figures 1, 2, 3, 4 and 5, we plot the exchange rates in our sample.

4 Econometric Methodology

In this section, we outline the simulated method of moments (MSM) technique for a time series estimation problem. The spirit of this technique is to ask the model to come as close as possible to predicting the observed moments of the exchange rate series. Briefly, given a candidate parameter vector, we draw a random path, much longer than the sample size, from the distribution of fundamentals paths using the dynamics (3) evaluated at our candidate parameter vector. We then compute the exchange rate series that corresponds to the simulated fundamentals series using our candidate parameter vector and (5) and then we compute the value of certain loss function. The procedure is repeated for many

[7] See Grabbe (1991) for a full explanation of the formula for the upper and lower limits of the target zone.

different candidate parameter vectors.[8] The simulated method of moments estimator is the parameter vector that minimizes the loss function. Under certain conditions, reviewed below, the MSM estimator is consistent and asymptotically normally distributed and a diagnostic test of the overall fit of the model can be constructed. The essential difference between this method and Hansen's GMM (1982) lies in the fact that an analytical solution for the relevant Euler equations as a function of the parameter vector does not exist. On the other hand, the implementation of MSM requires a specific assumption of the economic environment, i.e. the data generating process, which is not required in a GMM investigation of Euler equations. This allows a direct examination of the implications of the model for several moments of exchange rates. In addition, the ability of the model to fit these empirical moments is easy to interpret. In our case, MSM is particularly well suited because an analytical form for the transition density of the exchange rates is unknown (and thus maximum likelihood is not implementable) while we have a full description of the data generating mechanism imposed by the model. The properties of MSM estimators in a time series context have been studied by Duffie and Singleton (1993) and Lee and Ingram (1991). The difference between our problem and McFadden's (1989) lies in the parameter dependency of the simulated series for the fundamentals.

More precisely, suppose we are given a method of moments problem

$$E\left[g(s,\theta_0)\right] = 0 \tag{14}$$

[8] Although the noise used in the Monte-Carlo generation of the fundamentals path is the same across candidate parameter vectors.

where s is a $T \times 1$ vector of data, g expresses the moment conditions generated by a distributional theory on the data parametrized by the $p \times 1$ vector θ_0 belonging to some compact parameter set $\Theta \subset \mathcal{R}^p$.[9] Depending upon the model, it may be impossible to obtain a closed form expression for the $m \times 1$ vector g. This is our case, since we do not know the ergodic distribution or transition densities of the diffusion (3) except for very special cases.

Suppose we can identify a measurable transition function $T : \mathcal{R}^n \times \mathcal{R}^n \times \Theta \rightarrow \mathcal{R}^n$ that describes the dynamics of the state variable $k_{i+1} = T(k_i, \varepsilon_{i+1}, \theta)$, where $\{\varepsilon_i\}_{i=1}^{T\times l}$ is an *i.i.d.* sequence of random variables on some probability space and l is the length of the simulation divided by the length of the time series of data, T. Suppose also that we also have a measurable observation function $f : \mathcal{R}^n \times \Theta \rightarrow \mathcal{R}^n$ mapping the range of the state vector to the moments constructed from the dependent variable. In our case, the transition function T corresponds to a discretization of the integral representation of (3), which is shown to converge weakly to the true dynamics in Appendix A. Also, the observation function corresponds to moments constructed from observations of the exchange rate. The MSM estimator circumvents the difficulty of obtaining analytical expressions for the moments by assuming that we have an $\mathcal{R}^n$-valued sequence of random variables $\{\hat{\varepsilon}_i\}_{i=1}^{T\times l}$, identically distributed but independent from $\{\varepsilon_i\}_{i=1}^{T\times l}$. Then, for any initial point k_0 and parameter vector $\theta \in \Theta$, we can construct inductively a simulated state variable (fundamentals) process by letting $k_0^\theta = k_0$ and

$$k_{j+1}^\theta = T(k_j^\theta, \hat{\varepsilon}_{j+1}, \theta) \tag{15}$$

[9]In our application s is an $T \times n$ matrix. The method of moments problem we outline here generalizes easily to this case.

while the simulated $\mathcal{R}^n$-valued observation process (the moments of the exchange rates) is constructed as $h_j^\theta = h\left(s(k_j^\theta, \theta)\right)$. Denote by $\{h\}_{t=1}^T$ the $m \times 1$ real valued sequence of moments constructed from the data.

It is convenient to produce a replication of size $T \times l$, where l is an integer, and thus we obtain the series $\left\{h^\theta\right\}_{j=1}^{T\times l}$. In this case we can match l replications to each data observation and reorder the above series as $\left\{\left\{h_{ti}^\theta\right\}_{i=1}^l\right\}_{t=1}^T$. Now, for any parameter vector $\theta \in \Theta$, we can construct the sequences:

$$\hat{g}_i(s_i, \theta) = \frac{1}{l}\sum_{i=1}^{l}(h_t - h_{ti}^\theta) \tag{16}$$

$$\hat{g}_T(\theta) = \frac{1}{T}\sum_{i=1}^{T}\hat{g}_i(h_i, \theta). \tag{17}$$

The MSM estimator is then a Generalized Method of Moments (GMM) estimator using $\hat{g}_T(\theta)$ as moment conditions. An estimate, $\hat{\theta}$, for the parameter vector θ is obtained by solving:

$$\min_{\theta\in\Theta} \hat{g}_T(\theta)'\hat{W}\hat{g}_T(\theta) \tag{18}$$

where $\hat{W}$ is a weighting matrix with rank of at least p. It is useful to note that if the model is correct, and under the assumptions stated and checked below, the statistic

$$\mathcal{X}^{\epsilon}{}_{m-p} = T\hat{g}_T(\hat{\theta})'\hat{W}\hat{g}_T(\hat{\theta}) \tag{19}$$

is distributed asymptotically as a Chi-squared variate with $m - p$ degrees of freedom.

We now check a list of regularity conditions needed to ensure consistency and asymptotic normality of the MSM estimator. In addition, we will obtain its asymptotic distribution.

4.1 Regularity Conditions

In addition to the common regularity conditions assumed to obtain consistency and asymptotic normality of the GMM estimator, there are two additional problems in the MSM estimation. We need to ensure that the dependence of the estimator of an initial arbitrary state used to initialize the simulated series (15) fades away as the simulation size increases and that the simulated state process converges to its stationary distribution[10]. Also, a perturbation in the parameter vector affects the whole history of transitions, not only the current observation. This is unlike the usual GMM problem or even McFadden's (1989) MSM problem. Thus we need conditions to insure that this effect is damping rather than exploding and we have some kind of uniform continuity necessary to obtain asymptotic results. In what follows we present a list to insure that in our case, the proper convergence is attained. We follow Duffie and Singleton (DS) (1993).

It is easy to check that Assumptions 1 and 7 of DS (1993) involving Lipschitz conditions uniformly in probability are satisfied in our case because the state space is compact and the observation function is continuously differentiable. In addition, the state vector (the fundamentals) is geometrically ergodic (DS, 4.1) because it has full support and is aperiodic while the transition function is bounded above and below. Now, if the minimizer of (18) is unique, the MSM estimator $\hat{\theta}$ converges to θ_0 in probability as $T \to \infty$ given that the above assumptions are satisfied in our case (DS theorem 1).

[10] Of course, a stationary distribution for the fundamentals vector process exists because it is bounded.

4.2 The Asymptotic Distribution

4 To obtain asymptotic normality, it is necessary that the estimator $\hat{\theta}$ belongs to the interior of Θ. In addition, $h_j^\theta = h\left(s(k_j^\theta, \theta)\right)$ must be continuously differentiable (which is our case from (5)) and $E\left(\lim_{j\to\infty} \partial h_j^{\theta_0}/\partial\theta\right)$, where ∂ is shorthand notation for the Jacobian matrix, must exist and be finite with full rank. But, since h is continuously differentiable and k ergodic, this follows from Fatôu's lemma in our case.

It follows from the geometric ergodicity assumptions that the simulated series is independent of the data and that $\sqrt{T}(\hat{\theta} - \theta_0)$ converges in distribution to a normal random vector with mean zero and covariance matrix:

$$avar1(\hat{\theta}) = (\tilde{G}'W\tilde{G})^{-1}\tilde{G}'W\tilde{\Omega}W\tilde{G}(\tilde{G}'W\tilde{G}) \tag{20}$$

$$\tilde{G} = E\left[\partial\hat{g}_i(s_i, \theta_0)/\partial\theta\right] \tag{21}$$

$$W = \text{plim } \hat{W} \text{ as } T \to \infty \tag{22}$$

$$\tilde{\Omega} = var\left(\hat{g}_i(s_i, \theta_0)\right) \tag{23}$$

where the expectations are taken with respect to the density of the stationary distribution of the variate $\partial h_\infty^{\theta_0}/\partial\theta$, which, again exists because s is continuously differentiable with a bounded derivative and k is bounded by construction. Thus we may also have written $\tilde{G} = E\left(\lim_{j\to\infty} \partial h_j^{\theta_0}/\partial\theta\right)$.

We have two approaches to estimate (20). First, we can estimate $\tilde{\Omega}$ using simulated data. This alternative may prove useful since one has control over the size of the simulations. In this case we can estimate the functions in (20) by replacing $\tilde{G}$ by $\hat{G} = \frac{\partial\hat{g}(s_i,\hat{\theta})}{\partial\theta}$, W by $\hat{W}$ and

$\tilde{\Omega}$ by $\hat{\Omega} = \frac{1}{T}\sum_{i=1}^{T} \hat{g}_i(s_i, \hat{\theta})\hat{g}_i(s_i, \hat{\theta})'$.

Second, Hansen (1982) showed that the choice of $W = \Omega_0^{-1}$, where Ω is given below in (24), leads to the most efficient GMM estimator among those with a positive definite distance matrix. By using this weighting matrix, the model is asked to come as close as possible to predicting the observed moments, but to weight more heavily those moments that are estimated more accurately. In particular, we may assume that $\hat{W} \rightarrow \Omega_0^{-1}$ $a.s.$ where:

$$\Omega_0 = \sum_{j=-\infty}^{\infty} E\left([h_t - E(h_t)]\,[h_{t-j} - E(h_{t-j})]'\right) \tag{24}$$

and where Ω_0 is a function of the moments calculated from the data alone (see Lee and Ingram, 1991). Thus Ω_0 can be estimated using the Newey-West (1987) approach. In this case we would use as weighting matrix an estimate of Ω_0 and the covariance formula (20) simplifies and thus $\sqrt{T}(\hat{\theta} - \theta_0)$ converges in distribution as $T \rightarrow \infty$ to a normal random vector with mean zero and covariance matrix[11]

$$avar2(\hat{\theta}) = (1 + \tau)\left(\bar{G}'\Omega_0^{-1}\bar{G}\right)^{-1} \tag{25}$$

where $\tau = 1/l$ (this is corollary 3.1 of DS (1993)). We can implement this formula replacing $\bar{G}$ by $\tilde{G} = \frac{\partial \hat{g}(s_i, \hat{\theta})}{\partial \theta}$ and Ω_0 by a consistent estimator $\hat{\Omega}$ of (24). Only in this case, where we have an asymptotically efficient estimator, the diagnostic statistic in (3.19) has the stated asymptotic distribution.

[11]Since our data are not i.i.d over time, for this result to hold it is necessary that the simulated series have the same frequency as the true series. See Appendix A.

In this paper we will report asymptotic standard errors computed from the second covariance matrix, (25). Standard error calculations using the first covariance matrix were essentially the same. We use as our weighting matrix the Newey-West estimate of (24) with truncation lag length equal to three. This reduces the computational demands of our approach as we need not recalculate the matrix for each candidate parameter vector. We choose moments which are most informative about the parameters that we estimate. We use moments of both the level and first differences of the exchange rates in general using moments (generalized appropriately for the greater dimensionality of our estimation problem) used in the other papers which apply MSM to target zone models.

4.3 Implementation

Since it is not possible to simulate a continuous record of observations for the fundamentals path, we must discretize the model. An issue arises as to whether the discretization chosen converges weakly to the true continuous time process in the limit as the time interval approaches zero. In addition, there is another issue arising from the approximation of multidimensional reflected processes. It is difficult to approximate a diffusion process with an arbitrary reflection matrix on the boundaries of its domain directly (i.e. on the state variable space) through a discrete time processes, i.e. accommodate oblique reflections. This is because one does not know the properties of the graph of the stochastic process with oblique reflections. However, this is quite tractable if the reflection matrix is diagonal. In this case, the regulator is a buffer stock and can be written explicitely as a functional of the Wiener sample paths [12] and thus the discrete time simulator which converges weakly to the true process can be constructed easily. Thus, to solve this problem we perform a linear transformation of the state space, i.e. from the space of the fundamentals to an auxiliary space of identical dimension. Under this transformation, the auxiliary variables exhibit, by construction, a diagonal reflection matrix.[13] We will actually perform the simulations and estimation on the transformed space and then we will back up the results to read them in terms of the fundamentals processes. This transformation is the same one performed in Serrat (1994) to obtain (5). Details about our simulation of the exchange rate process are explained in Appendix A. The specific moments that we used, for both the three country and five country models, are listed in Appendix B. In Appendix C, we explain the numerical procedures that we used in the paper.

5 Empirical Results

5.1 Results for Three Currency Case

The trilateral model according to (5) can be applied to an arbitrary number of currencies. Given the computational demands of the MSM method we first estimate the model as a trilateral target zone applied to subsets of two elements of the set of five currencies that we work with (the third currency is always the DM).[14] Thus we first estimate the model for a total of ten trilateral target zones formed from our data. Each trilateral target zone

[12] See for instance chapter 2 of Harrison's 1985 book.

[13] Note that this reflection matrix is not the matrix M, which is the reflection matrix in the space of the untransformed fundamentals.

[14] This is the simplest generalization of the bilateral model.

model requires the estimation of eight parameters. This number incorporates the restriction that the elements of the column vectors of the reflection matrix sum to one.[15] In addition, the vertices for the domain of the fundamentals are calculated directly from the parameters and thus do not constitute parameters to be estimated in its pure sense. We then take the estimated parameters and use them to construct an initial guess for the estimation of a five-lateral target zone involving the currencies: Dutch Guilder (DFl), Danish Krone (DKr), French Franc (FFr), Irish Pound (IP) along with the German Mark (DM). This involves the estimation of a vector of twenty-seven parameters. Of these parameters, twelve are associated with estimation of the 4×4 M matrix as for the five currency case.

Tables 3, 4, 5, 6 and 7 report the estimates for the trilateral target zones. Each table corresponds to a country matched with each of the other four currencies as well as the DM. To help with the reading of the tables, we explain the results contained in the first column of Table 3 which contains the results of the estimation of the trilateral target zone involving Germany, Belgium and the Netherlands. The parameters μ_i and σ_i are the estimates of the drift and diffusion terms for the fundamentals of countries $i = 1$ (Belgium) and $i = 2$ (Netherlands). ρ is the estimated correlation coefficient of the fundamentals processes for the two countries, while γ is the semi-elasticity of money demand with respect to the interest rate. m_{11} and m_{22} are the two estimated diagonal elements of the M. Recall that our normalization is such that the sum of each column of M is one; thus, the two column vectors of M are $[m_{11}, 1 - m_{11}]'$ and $[1 - m_{22}, m_{22}]'$, respectively.

The overidentifying restrictions test (19) indicates that the trilateral model is not rejected

[15] The choice of normalization is unimportant, alternatively we could have imposed the norm of each column vector to be one.

for any of the ten trilateral target zone models. Although some parameters are estimated imprecisely,[16] the diagonal elements of the reflection matrix M are always significant. We performed Wald tests of the restriction that M is the identity matrix, which is a test whether Krugman's model is not rejected as a restriction on the trilateral model, for each of the ten models.[17] In eight of the ten cases these restrictions were rejected. Only for the target zone formed by the FFr, DKr and DM and for the target zone formed by the IP, DFl and DM, do we not reject that each of these trilateral combinations can be explained by two bilateral Krugman models. Unlike previous applications of MSM to bilateral target zone data (see subsection 3.1.1), our parameter point estimates are of the same order of magnitude across currencies and trilateral target zone combinations. The range for the estimated components of the drift vector for the fundamentals process oscillate between $-.021$ and $.0053$, while the estimated diffusion coefficients vary between $.0032$ and $.024$ across all trilateral combinations.[18] The point estimates for the interest rate semielasticity of money γ varies between .02 and .5 (with one outlier, .8), although this parameter is estimated imprecisely. In Figures 6 and 7 we present the simulated steady-state densities of the exchange rates using the parameter estimates. We can see that the multilateral model is able to generate a variety of shapes for the unconditional density, and is not restricted

[16]However, recall that our model is a nonlinear model and GMM standard errors peform poorly in small samples. It is the global overidentyfing restrictions test that is important, even if the point estimates are imprecise. It is important to note that our results do not conform to the overrejection phenomenom derived from the performance of the overidentifying restrictions test in small samples (see Hansen, Heaton and Yaron, 1994).

[17]Recall that if the M matrix is the identity matrix the multilateral model collapses to two bilateral models.

[18]Our estimates of the standard deviation of the diffusion process look reasonable when compared with the actual data, with the Netherlands (with its highly stable exchange rate) with the lowest estimated standard deviation and with France and Denmark with the highest estimated standard deviations. See Figures 1, 2, 3, 4 and 5.

to hump-shaped distribution like the bilateral model of Krugman (1991). This is one of the reasons why the empirical fit of the multilateral features improve dramatically the empirical fit of the target zone model.

In Table 8 we show the results of the examination of whether the cross-currency restrictions (13) are violated in the estimated models. The cross currency constraints are not violated in six of the ten models, while in two of the models (DM-BFr-IP) and (DM-DFl-DKr) the cross currency constraints are violated trivially. We will see, however, in the results for the five-currency case that this problem is strongly mitigated. Only for the zones comprised of DM-DFl-IP and DM-DKr-FFr are the constraints rejected significantly. This set of trilateral target zones are characterized by reflection matrices that are very close to being diagonal. Thus the result is not surprising: when the reflection matrix is diagonal, the trilateral target zone collapses to two bilateral target zones. Because, in this case the range of variation of the exchange rate is independent they can indeed reach their maximum distance equal to twice the size of the bilateral zones. However, in the EMS, the official bilateral bands between currencies other than the DM are of the same size as the individual bands with respect to the DM. Thus we see another effect of a non-diagonal reflection matrix, namely, it places restrictions on the range of variation of the cross-exchange rate, *independent* of whether the fundamentals processes are correlated or not.

Table 1: Estimated M matrix in the Five Country Case.

$$M = \begin{bmatrix} .970 & .370 & .121 & .353 \\ .001 & .838 & .121 & .148 \\ .001 & .295 & .881 & .225 \\ .244 & .271 & .442 & .896 \end{bmatrix}$$

As noted in the text, the F test of the restriction that $M = I(4)$ is soundly rejected by the model. Due to the ordering of the countries in our data set, the columns of M correspond to the Netherlands, Denmark, France and Ireland respectively.

5.2 Results for the Five Currency Case

Table 9 reports results for the multilateral target zone involving five currencies (DFl, DKr, FFr, IP and DM).[19] The point estimates of the parameters characterizing the dynamics of the fundamental process are similar to the corresponding estimates for the set of trilateral target zones except for the correlation coefficient among the components of the fundamentals vector. The overidentifying restrictions test does not reject the model at a 96% confidence level.[20] The correlation coefficients exhibit less variation than in the previous cases and they range between −0.39 and 0.63. However, many parameters continue to be estimated quite imprecisely. The point estimate for the interest rate semielasticity of money demand is 0.167. This is consistent with previous estimates in the literature (Diebold, 1986) and is also very close to the estimate used by Flood, Rose and Mathieson (1991).

We present in Table 1 the point estimate for the reflection matrix M in the five currency case.[21] The restriction that M is diagonal is strongly rejected by a conventional Wald

[19]The information in this table is presented as in the tables for the trilateral models (generalized appropriately for increased dimensionality). ρ_{ij} is the estimated correlation coefficient of the fundamentals of countries i and j.

[20]The calculated test statistic is 17.03 which is slightly above the critical value of the overidentifying restrictions test at the 95% confidence level: $\chi^2_{.05} = 16.8$.

[21]In the five currency case, we estimated M imposing that the norm of each column vector be exactly one. In practice, we thus did not estimate the 16 elements of the M matrix directly. Rather, we estimated 12

Test.[22] In addition, the cross-currency variation restrictions, if violated, are violated by trivial amounts. In Table 10 we report the maximum and minimum values of each cross-currency exchange rate implied by the estimated five country model. Note that the maximum violation of the cross-currency constraint, .0252 for the French Franc-Danish Krone exchange rate, exceeds the actual constraint by only 10%. In addition, we conducted a simulation of length 30,000 for each cross-currency pair to assess empirically how frequently the model predicts cross-currency constraint violations. In the simulation run, the cross currency restrictions were violated for only .292% of the simulated sample.[23] In Figures 6 and 7, we show the histogram of the four simulated exchange rates. Note that the histograms, appear to be characterized by the hump shaped distribution not found in simulations of the bilateral model.

5.3 Power Analysis

The satisfactory performance of the model in both the three-currency and five-currency case leads us to question whether our application of the simulated method of moments has statistical power to reject data constructed under a reasonable alternative data generating process. In this subsection, we explore the power of our method for the trilateral case of Section 5.1. As noted above, econometric estimation of the model is very computer intensive; therefore, we examine the power of the trilateral model for one set of countries: Germany,

fundamental parameters from which the M matrix is obtained. The estimates of these parameters are not reported as it is the implied estimate of the M matrix which is relevant.

[22]The F test value for this test was 744.3 which is much larger than the critical value obtained from the F distribution, 2.36, with degrees of freedom, 162 and 12.

[23]In the simulation, the cross currency restrictions for the DKr-FFr, FFr-DKr and DKr-IP cross currency rates were exceeded 327, 193 and 6 times respectively. The other cross-currency rates were not violated.

Belgium and France.[24]

The results of any power calculation are highly sensitive to the specification of the alternative data generating process. We considered generating exchange rate data with Ornstein-Uhlenbeck process; however, a such mean reverting process is not adequate for our data since, as is reported in Table 2, for four of the five countries the first autocorrelation coefficient of the first difference of the exchange rate in logarithms is positive.[25] Instead, we chose the most natural alternative data generating process: unregulated arithmetic Brownian motion which corresponds to the 'free float' solution of the model. We choose the drift and variance parameters as (roughly) an average of the fundamental parameter estimates for Belgium and France for the trilateral model.[26] Note that the data generating process under the alternative hypothesis is not regulated at the boundaries. However, it is constructed with parameters such that the exchange rate sample path remains within the band in practically all our simulations. This alternative data generating process can also be interpreted as a test of the hypothesis that gamma is zero, and therefore the model collapses to a free-float solution of equation (3.1). This is particularly useful since the confidence interval for gamma is big due to a small sample phenomenom. Thus, since the model exhibits high power against this alternative, we conclude that if in the true data generating process gamma is zero, we would most probably reject the model using the overidentyfing restrictions test.

[24]We chose the Belgian Franc and the French Franc because those currencies have a larger share in world markets than are the Danish Krone and Irish Pound. The Dutch Guilder was not chosen because it is atypical of other currencies in our sample and exhibits a relatively low variance with respect to the German Mark.

[25]one can check that for an Ornstein-Uhlenbeck process, the first order autocorrelation coefficient must lie between -1/2 and 0.

[26]The exact parameters (for Belgium and France respectively) were $-.007$ and $-.007$ for the drift terms and .008 and .014 for the variance terms. We assumed zero correlation between the Wiener processes driving each exchange rate.

Our results strongly suggest that the model has much power to reject data generated under a reasonable alternative hypothesis. We simulated 100 random exchange rate paths. In only 5 cases did the target zone model incorrectly not reject the data generated under the alternative hypothesis. In most cases, the generated data strongly rejected the target zone model: in 61 simulations out of 100, the model converged to an overidentifying restrictions test statistic of over 75 which is well above the critical value $\chi^2_{.05} = 14.1$.

5.4 Meese-Rogoff Horseraces

In an influential paper, Meese and Rogoff (1983) compared the out-of-sample forecasts produced by various exchange rate models with forecasts produced by a random walk model of the exchange rate. Even though actual future values of the righthandside variables were allowed in the dynamic forecasts (thus bestowing an informational advantage upon the exchange rate models), the random walk performed as least as well as the other nonlinear models tested, particularly in short-run predictions. Since Meese and Rogoff (1983), forecasting better than the random walk has become a standard metric by which one can judge models of the exchange rates. However we will argue that, for the reasons outlined below (mainly related to the power of random walk tests) these horserace exercises have little relevance in our context (and perhaps in other contexts as well). As it has become standard in the literature to report the results of such an exercise, below we report results of our own horserace calculations even though the multilateral target zone model has not been constructed for purposes of prediction.

In the "races" performed, we used a trilateral version of the model applied to the FF/DM, BF/DM and FF/BF exchange rates.[27] We reestimated the trilateral model on a subset of our data leaving out the last τ data points where $\tau = \{5, 10, 15, 20, 25, 30, 35\}$. For each value of τ, we compute the implied fundamentals at the last point of the subsample by numerically inverting the closed form solution for the exchange rate. Then, using the estimates for the dynamics of the fundamentals process, we forecast the fundamentals τ periods ahead. We then evaluate the nonlinear functional form expressing the exchange rate vector as a function of the fundamentals vector at the predicted fundamentals vector, using the estimates obtained in the subsample. To this we add an extra term correcting for Jensen's inequality to form the forecasted exchange rate. The error made by this forecast is then compared to the error made by using a random walk with drift model directly estimated on the exchange rate series. In this way we eliminate the informational advantage given to the model by the previous studies, i.e., by not using future information in the forecasts.

Table 11, columns 1 and 2, exhibit the ratio of the mean squared error (MSE) of the forecast obtained with the multilateral model over the MSE of the forecast obtained with the random walk, for each size of the out-of-sample data set. Two conclusions arise from our analysis: first, both forecasts are very poor. Second, the random walk model does roughly as well as the multilateral model in predicting future exchange rates. Thus, it seems that the results widely reported in the literature about the robustness of the random walk model are reproduced here. However, in Table 11, columns 3 and 4, we report the results of the following experiment: we generate data using the multilateral model evaluated at the estimated parameters for the trilateral case of FF-BF-DM and then perform the same

[27] This is the same set of currencies that we used for the power calculations reported above.

prediction exercise with the simulated data as we did with the true data. Surprisingly enough, the random walk model again does as well as the multilateral model, even when the multilateral model is true by construction.[28] The above result led us to examine the power of the random walk tests against the multilateral model. Toward this end we chose the variance ratio test of Lo and MacKinlay (1988) whose finite sample power advantages (against a wide range of alternatives) over other random walk tests such as Dickey-Fuller or Box-Pierce are well known (Hausman (1988) and Lo and MacKinlay (1989)).[29] We performed the variance ratio test on 10,000 simulations of the trilateral model previously estimated for the FF-BF-DM case. In Figures 8 and 9 we plot the histograms of the test statistic which under the null that the simulated series is a random walk is distributed as a Chi-Squared variate with two degrees of freedom.[30] There are four histograms plotted (corresponding to four values of q ($q = \{2, 4, 8, 16\}$) where q is the window used in the variance ratio test (see Lo and MacKinlay (1988))). We find that the maximum power of the test to correctly reject the random walk hypothesis is less than 8%.

We also performed the random walk test on the true data. Similar to the simulated data, the null hypothesis that the true data is a random walk again is not rejected. This is not surprising as our simulation results show that random walk tests have low power. This is consistent with the fact that the random walk predicts quite well even when it is false by construction. With this evidence we conclude that conducting Meese-Rogoff horseraces

[28]We do not obtain a perfect forecast with the multilateral model because the fundamentals have to be predicted.

[29]This test is essentially a Hausman-type specification test built around the idea that, under the random walk hypothesis, the variance of the increments of the process are linear in the length of the observation interval (we direct the reader's attention to the papers cited in the text for details).

[30]At the 5% level of significance the critical value is 5.99.

does not make sense in our context, and that the lack of clear forecasting superiority of the trilateral model versus a random walk model is not evidence against the multilateral model. Our Monte-Carlo evidence against "horse-race" tests to evaluate nonlinear models of exchange rates in our context likely has implications for the empirical literature outside the scope of this paper.

6 Conclusions

In this paper we apply a simulated method of moments technique to estimate a multilateral target zone model for which we have closed form solutions. This model is presented in Serrat (1994) and is based on full credibility assumptions and marginal interventions on the space of the fundamentals (and thus endogenous intramarginal interventions in the space of the exchange rates). The theoretical model has Krugman's (1991) bilateral model as a special case, although, in general, its implications are very different. In this paper, we investigated whether taking into account the multilateral feature of multilateral target zones is important for empirical purposes.

Although the econometric problem is computationally demanding, we estimate the model for a three-currency version (for ten different sets of data) and a five-currency version of the model. We use data from the so-called "hard-EMS" period and we compare the model to Krugman's (1991) bilateral model and to other non-target zone alternatives. The model performs very well when measured with conventional goodness-of-fit criteria and also when explicit power computations are carried out with a reasonable data generating process as an alternative hypothesis. Moreover, the parameter restrictions which make the multilateral model collapse to a superposition of bilateral models of the Krugman-type are strongly rejected. Our positive results are a sharp contrast not only to the previous empirical target zone literature but also to most literature on empirical nonlinear models of exchange rates.

We claim that the good empirical performance of the multilateral model is driven by the parameters that capture the degree of cooperation in maintaining the exchange rate regime, the reflection matrix, that have been neglected by previous literature. We can explain the success of the model and the well-documented empirical failure of the bilateral model as follows. First, allowing the reflection matrix to be non-diagonal permits spillover effects of the foreign exchange interventions of the monetary authorities of one country on all the other's fundamentals. This is why the interventions "look like" intramarginal interventions in the space of the exchange rates (but not in the space of the fundamentals). If we impose that the reflection matrix be diagonal (Krugman's restriction) then these spillover effects are lost and interventions are marginal both in the space of the fundamentals and the space of the exchange rates. In this way, we can explain not only the negative results of the empirical literature on the bilateral model reported in Section 1.1, but also the relative success in reconciling the model with the data that some authors have achieved by introducing ad-hoc intramarginal interventions into Krugman's model (e.g. Lindberg and Soderlin, 1991). Second, the nature of the relationships among exchange rates and fundamentals in the versions of the multilateral model estimated here imply that the cross-currency restrictions derived from differing nominal and effective exchange rate bands are rarely violated and that such violations are of a small size. We insist that this is an empirical result that we have achieved without imposing the cross-currency restrictions explicitely, unlike in Serrat (1994). However, direct estimation of Krugman's model applied to several exchange rates would imply, by cross-arbitrage restrictions, more frequent violations of the cross-currency restrictions and of larger size. Since the estimated reflection matrix is far from diagonal, we detect a high degree of cooperationy in the maintenance of the exchange rate system during the period studied.

Thus we conclude that the profession perhaps has discarded full credibility target zone models of the Krugman-type too quickly. Even though realignments exist, our results indicate that a full credibility model of exchange rate dynamics that explicitly takes into account the multilateral nature of real world target zones can perform well during "calm" periods on EMS data.

A Simulation of the Model in Discrete Time

In this appendix we explain our discrete-time approximation of the regulated diffusion process. We prove that the discrete-time approximation converges weakly to the continuous time regulated diffusion process. We present our results for the trilateral model; the generalization to a five-currency case is straightforward.

The state variables consist of the fundamentals process $k_t = (k^1, k^2)$ taking values on a quadrilateral $G \subset \mathcal{R}^2$. On this domain, k follows the dynamics:

$$dk_t = \mu dt + AdW_t + \sum_{i=1}^{4} R^i d\Lambda_t^i \tag{26}$$

where $k_t = (k_t^1, k_t^2)'$, μ is a 2×1 vector and A is a 2×2 matrix. W_t is a bivariate Wiener process on some probability space, and the initial conditions k_0^1 and k_0^2 are given. The process $\Lambda_t = [\Lambda_t^1, \Lambda_t^2, \Lambda_t^3, \Lambda_t^4]$ is a 2×4 dimensional *regulator* process which is a continuous process, whose increment set is singular with respect to the Lebesgue measure, and whose components Λ_t^i are bidimensional nondecreasing processes with $\Lambda_0^i = 0 (i = 1, 2, 3, 4)$ that increase if and only if (k_t^1, k_t^2) hits side i of the fundamentals domain.[31] Let $R^i (i = 1, 2, 3, 4)$ be 2×1 reflection vectors. We will decompose the instantaneous covariance matrix of k^1 and k^2, AA' as the product of two triangular matrices with $A = \begin{pmatrix} \sigma^1 & 0 \\ \sigma^2\rho & \sigma^2\sqrt{1-\rho^2} \end{pmatrix}$. With this decomposition, we are able to identify the instantaneous correlation coefficient between k^1 and k^2 as ρ and the conditional variances of k^1 and k^2, as σ^1 and σ^2, respectively.

In the solution method outlined in the theoretical paper, two auxiliary processes are

[31] The processes Λ^i coincide with the local time process of the fundamentals at each side the quadrilateral.

defined by changing the axes of the state variable space. This simplifies the partial differential equation to be solved. It is also convenient to perform the same rotation for empirical purposes. In our simulations we work with the exchange rate as a function of the transformed state variables rather than the fundamentals, for reasons that will become clear.

Define two auxiliary stochastic processes, X and Y, as follows:

$$\begin{pmatrix} X_t \\ Y_t \end{pmatrix} = \begin{pmatrix} \mu_X \\ \mu_Y \end{pmatrix} t + B \begin{pmatrix} W_1(t) \\ W_2(t) \end{pmatrix} + \begin{pmatrix} \Lambda_t^1 - \Lambda_t^3 \\ \Lambda_t^2 - \Lambda_t^4 \end{pmatrix} \tag{27}$$

where (W_1, W_2) is the same Wiener process as before, and $(\mu_X, \mu_Y)' = M^{-1}\mu$, $B = M^{-1}A$. Obviously $[k_t^1, k_t^2]' = M[X_t, Y_t]'$ *a.s.* for $(k^1, k^2) \in G$.

Given the dynamics (27), we can restate the properties of this process as:

1. Λ_t^1, Λ_t^2, Λ_t^3 and Λ_t^4 are increasing and continuous with $\Lambda_0^1 = \Lambda_0^3 = \Lambda_0^2 = \Lambda_0^4 = 0$

2. $X_t = X_t' + (\Lambda_t^1 - \Lambda_t^3) \in [\underline{X}, \bar{X}]$ and $Y_t = Y_t' + (\Lambda_t^2 - \Lambda_t^4) \in [\underline{Y}, \bar{Y}]$ for all $t \geq 0$, where $X_t' = X_0 + \mu_X t + [1, 0]BW_t$ and $Y_t' = Y_0 + \mu_Y t + [0, 1]BW_t$.

3. Λ_t^1 (Λ_t^3) increases if and only if $X_t = \underline{X}$ $(X_t = \bar{X})$. Λ_t^2 (Λ_t^4) increases if and only if $Y_t = \underline{Y}$ $(Y_t = \bar{Y})$.

Lemma: There exist a unique set of processes $\{\Lambda_t^1, \Lambda_t^2, \Lambda_t^3, \Lambda_t^4\}$ which satisfy 1-3. These processes are given implicitly by:

$$\Lambda_t^3 = \sup_{0 \leq s \leq t} (X_s' - \underline{X} - \Lambda_s^1)^- \tag{28}$$

$$\Lambda_t^1 = \sup_{0 \leq s \leq t} (\bar{X} - X_s' - \Lambda_s^3)^- \tag{29}$$

$$\Lambda_t^4 = \sup_{0\le s\le t} (Y_s' - \underline{Y} - \Lambda_s^2)^- \tag{30}$$

$$\Lambda_t^2 = \sup_{0\le s\le t} (\bar{Y} - X_s' - \Lambda_s^4)^- \tag{31}$$

where $f^- = -\min(f, 0)$.

Proof: available upon request.

For simulation purposes, we fix t¿0 and construct the discrete time process:

$$\begin{aligned} X_n' &= X_0' + \mu_X nt + (1,0)B\left[\textstyle\sum_{i=1}^{nt} a_i, \sum_{i=1}^{nt} b_i,\right] \quad ; \quad X_0' \text{ given} \\ Y_n' &= Y_0' + \mu_Y nt + (0,1)B\left[\textstyle\sum_{i=1}^{nt} a_i, \sum_{i=1}^{nt} b_i,\right] \quad ; \quad Y_0' \text{ given} \end{aligned} \tag{32}$$

where $\{a_i\}$ and $\{b_i\}$ are two independent sequences of independent standard normal random variables and $[x]$ denotes the integer part of a real number x. For now we consider $t \geq 0$ to be fixed. From the processes (32) we construct the following bounded processes recursively:

$$\begin{aligned} X_n &= \min\left\{\bar{X}, \max\{\underline{X},\ X_{n-1} + \ (X_n' - X_{n-1}')\}\right\}, \\ Y_n &= \min\left\{\bar{Y}, \max\{\underline{Y},\ Y_{n-1} + \ (Y_n' - Y_{n-1}')\}\right\}. \end{aligned} \tag{33}$$

Now from (33) by induction on n we obtain an expression for X_n, Y_n in terms of the path of X' and Y' :

$$X_n = X_n' + Z_n^1 - Z_n^2$$

$$Y_n = Y_n' + V_n^1 - V_n^2$$

where Z_n^i, V_n^i solve

$$Z_n^2 = \max_{0 \le i \le n} -(X_i' - \underline{X} - Z_i^1) \tag{34}$$

$$Z_n^1 = \max_{0 \le i \le n} -(\bar{X} - X_i' - Z_i^2) \tag{35}$$

$$V_n^2 = \max_{0 \le i \le n} -(Y_i' - \underline{Y} - V_i^1) \tag{36}$$

$$V_n^1 = \max_{0 \le i \le n} -(\bar{Y} - Y_i' - V_i^2) \tag{37}$$

Now for each n, let $X_{t,n}, X_{t,n}'$ and $Y_{t,n}, Y_{t,n}'$ be the processes defined by

$$\begin{aligned} X_{t,n}' &= (\sqrt{n})^{-1} X_{[nt]}' \qquad & X_{t,n} &= (\sqrt{n})^{-1} X_{[nt]} \\ Y_{t,n}' &= (\sqrt{n})^{-1} Y_{[nt]}' \qquad & Y_{t,n} &= (\sqrt{n})^{-1} Y_{[nt]} \end{aligned} \tag{38}$$

for each $t \geq 0$.

Theorem: For each t, $(X_{t,n}, Y_{t,n})$ converges in distribution to (X_t, Y_t).[32]

Proof: By the functional central limit theorem, as $n \to \infty$:

$$\sqrt{\frac{[nt]}{n}} \left(\sqrt{[nt]}\right)^{-1} \sum_{i=1}^{nt} a_i \xrightarrow{\mathcal{L}} W_t^1 \tag{39}$$

$$\sqrt{\frac{[nt]}{n}} \left(\sqrt{[nt]}\right)^{-1} \sum_{i=1}^{nt} b_i \xrightarrow{\mathcal{L}} W_t^2 \tag{40}$$

where W_t^1 and W_t^2 are independent Wiener processes. It follows from (32) that, for each $t \geq 0$, $(X_{t,n}', Y_{t,n}') \xrightarrow{\mathcal{L}} (X_t', Y_t')$. In fact, $\{(X_{t,n}', Y_{t,n}') \; ; \; 0 \leq t \leq k\}$ converges weakly on

[32] A sequence (x_n, y_n) of pairs of random variables converges in distribution (or in law) to a pair of random variables (x, y) if $\lim_{n \to \infty} P(x_n \leq a, y_n \leq b) = P(x \leq a, y \leq b)$ for each $(a, b) \in \mathcal{R}^2$ which is a point of continuity of the function $f(a, b) = P(x \leq a, y \leq b)$.

$D[0,k] \times D[0,k]$ to $\{(X_t', Y_t')\ ; 0 \leq t \leq k\}$ for all $k \in \mathcal{R}^+$. By the continuous mapping theorem applied to (34)-(37) and (28)-(31) $(Z_{t,n}^1, Z_{t,n}^2, V_{t,n}^1, V_{t,n}^2)$ converges weakly to $\{\Lambda_t^1, \Lambda_t^3, \Lambda_t^2, \Lambda_t^4\}$ and the same theorem applied to (34), (28) and (27) indicates that $\{(X_{t,n}, Y_{t,n})\ ; 0 \leq t \leq k\}$ converges weakly on $D[0,k] \times D[0,k]$ to $\{(X_t, Y_t)\ ; 0 \leq t \leq k\}$ for all $k \in \mathcal{R}^+$. Letting $k \to \infty$ yields the desired result.

In view of the above theorem, our approximation scheme (34) is justified as it converges weakly to the continuous stochastic processes upon which the model is built.
Now it is also clear the reason why we simulate the model in the transformed space. The regulator process in (34)-(37) only adjusts along one dimension at a time, without using the reflection matrix.

In our simulations, we set $\Delta t = 1/(52x5)$. We thus interpret $[e^1(X,Y), e^2(X,Y)]$ as a simulated series of daily exchange rates from which we sample one out of five observations to obtain a series of simulated exchange rates comparable to our weekly exchange rate data. An issue also arises as to how to generate the first observation of the simulated series. For the trilateral models estimated in Section 5.1 and for the power calculations reported in Section 5.3, we began each simulated series by matching the first observation of the simulated series with the first observation in the actual data. For the five country model estimated in Section 5.2, we set the first observation of the vector $X(resp.Y)$ to be $(\underline{X} + \bar{X})/2$ $(resp.(\underline{Y} + \bar{Y})/2)$. This is roughly similar to fitting the first simulated observation to that of the actual data (the first observations of each of the series are roughly in the middle of the bands).

B Moments

In this appendix we list the moments that we used during estimation. Let the logarithm of the level of currency i in German Marks divided by central parity at time t be $e_{i,t}$. In addition, denote $\Delta e_{i,t} \equiv e_{i,t} - e_{i,t-1}$ and $\rho_k(x)$ the k^{th} autocorrelation coefficient of x.

In the trilateral case we match the following fifteen moments:

$mean(e_{i,t}),\ i = 1,2$
$var(e_{i,t}),\ i = 1,2$
$mean(\Delta e_{i,t}),\ i = 1,2$
$var(\Delta e_{i,t}),\ i = 1,2$
$\rho_k(\Delta e_{i,t}),\ i = 1,2; k = 1,2$
$cov(\Delta e_{1,t}, \Delta e_{2,t})$
$cov(\Delta e_{1,t-1}, \Delta e_{2,t})$
$cov(\Delta e_{1,t}, \Delta e_{2,t-1})$.

In the case with five country case we match the following thirty six moments:

$mean(e_{i,t}),\ i = 1,2,3,4$
$var(e_{i,t}),\ i = 1,2,3,4$
$mean(\Delta e_{i,t}),\ i = 1,2,3,4$
$var(\Delta e_{i,t}),\ i = 1,2,3,4$
$\rho_k(\Delta e_{i,t}),\ i = 1,2,3,4; k = 1,2$
$cov(\Delta e_{i,t}, \Delta e_{j,t}),\ i = 2,3,4; j = 1,2,3; i > j$
$cov(\Delta e_{i,t-1}, \Delta e_{j,t}),\ i = 2,3,4; j = 1,2,3; i > j$
$cov(\Delta e_{i,t}, \Delta e_{j,t-1}),\ i = 2,3,4; j = 1,2,3; i > j$.

C Numerical Methods

In this appendix, we summarize the numerical procedures used in Chapter 3. Given a candidate parameter, we first must solve for the vectors $\bar{X} = [\bar{X}_1, ..., \bar{X}_n]$ and $\underline{X} = [\underline{X}_1, ..., \underline{X}_n]$. This necessitates solving a system of $2n$ non-linear equations. We then simulate the transformed fundamentals as discussed in Appendix A and calculate the criterion function given by (19). The simulated method of moments estimator is the parameter vector, θ^*, which minimizes (19). We first discuss the methods we used to estimate the trilateral models. Then, we discuss minimization of the criterion function in the five currency case. Finally, we conclude with a description of the power calculations reported in Section 5.3.

For the trilateral model, for those parameters which are in theory unbounded we conducted an initial exploratory search of a relatively wide parameter space to discover the empirically relevant parameter space. We then discretized the parameter space (as is done in the simulated annealing algorithm for example) so that along each dimension the parameter space contained 50 equally spaced points. For each trilateral model, we then searched randomly 500 times over this restricted parameter space in search of parameter vectors yielding the three lowest levels of the criterion function. We used the parameter vectors associated with the three lowest levels of the criterion function as starting values for our minimization algorithm.

Our algorithm is a variant of the gradient descent method of numerical optimization. In gradient descent, the gradient at a candidate vector is first calculated. The next candidate vector is chosen by moving in the direction of the gradient with the minimization routine ending when a local minimum is reached. Our algorithm is similar except that we do not use information about the gradient to calculate the next candidate vector. (In our problem, calculation of the gradient is costly while use of the information contained by the gradient is in any case problematic when the state space is discrete.) Instead, we calculate the next candidate parameter by randomly choosing a direction in which to move. If search in the direction is successful then the program descends along that dimension until the objective function increases again. If search is unsuccessful another direction (randomly chosen) is searched. In eight dimensions, there are of course a multiplicity of directions to search amongst neighbouring points. We defined convergence to a local minimum to be reached when each of the sixteen directions defined by perturbing each of the eight parameters up and down within the grid had been searched and found to yield higher objective functions around the objective function minimizing parameter vector. For each of the trilateral models (and for each of the three starting values at which we began the algorithm) at convergence the test statistic (proportional to the value of the criterion function) was always found to be under 35 and in most cases was below 20. In general, the parameter estimates across local minima were comparable to each other and of the same magnitude. From the parameter vector which yielded the lowest criterion function, we undertook a further local search (with a more fine grid) to obtain a better fit of the model and more precise parameter estimates. The final estimates for each of the trilateral models are the resulting parameter estimates obtained from this search.[33]

Our method to estimate the five currency case is similar to that of the three currency case. One important difference is the much larger number of parameters to estimate and

[33]This second stage typically reduced the criterion function by a further 10-15%.

greater number of exchange rate series to simulate. This increased dimensionality restricted us to estimating the five currency model from only one set of starting parameters. We chose our starting values with guidance from the results from the three currency estimation. We also in our minimization algorithm we allowed the candidate parameter vector to be different along more than one dimension from the previous best candidate vector. (We adopted this strategy in order to obtain faster convergence.) Our strategy of choosing starting values for our algorithm (for both the three and five country cases) in the region of the parameter space most likely to yield the lowest value of the criterion function maximizes the probability that our algorithm reaches a global minimum. It is important to point out that in any case if we did not reach a global minimum this only hurts us as the probability of rejecting the overidentifying restrictions is correspondingly higher.

For the power calculations reported in Section 5.3 we adopted the same minimization routine as in estimation of the other trilateral models. There were two important differences to note. First, as we were generating the data under a known alternative hypothesis we used this information to provide starting estimates for the minimization algorithm. This led to relatively fast convergence. Second, we did not undertake a local search of a finer grid around the best parameter estimates found for each sample path of generated data. In practice, however, we found in the estimation of the original trilateral models that the local search of a finer grid reduced the test statistics by roughly 10-15%, a number which is relatively small. Given the level at which convergence was usually achieved, this local search would prove immaterial in most cases.

Table 2: Descriptive Statistics of the Exchange Rate Data

	Belgian Franc	Dutch Guilder	Danish Krone	French Franc	Irish Pound
maximum	.00513	.00594	.0147	.00816	.00562
minimum	−.0198	−.00313	−.0224	−0.0194	−.0180
mean	−.0107	−.000250	−.00647	−.00682	−.00302
variance	.0000367	.00000362	.000114	.0000644	.0000193
mean of first difference	.0000444	.00000503	−.0000805	−.0000366	.0000694
variance of first difference	.00000119	.000000326	.00000470	.00000414	.00000287
autocorrelation coefficient of first difference	.0918	.0366	.0614	.0990	−.0759

Exchange rate variable is the logarithm of the ratio of the exchange rate divided by the central parity.

Table 3: Estimates of the Trilateral Model: Belgium

	Netherlands	Denmark	France	Ireland
μ_1	−.00408	−.0211	−.00736	−.0126
	(.00234)	(.00680)	(.00170)	(.00661)
μ_2	.00148	−.0186	−.00728	−.00496
	(.00167)	(.00969)	(.00458)	(.00246)
m_{11}	.651	0.704	.640	.714
	(.210)	(.0419)	(.0661)	(.134)
m_{22}	.881	.645	.782	.8364
	(.324)	(.0274)	(.196)	(.213)
σ_1	.00863	.00321	.00706	.0112
	(.00227)	(.00755)	(.00165)	(.00377)
σ_2	.00401	.0189	.0151	.0145
	(.00136)	(.0113)	(.00736)	(.0142)
ρ	.353	−.947	.515	.184
	(.129)	(.147)	(.0819)	(.218)
γ	.0456	.159	.0808	.801
	(.282)	(.126)	(.313)	(1.61)
Test Statistic	6.59	9.88	8.19	10.14
Krugman F Test	82.0	196.7	34.55	4.99
l	5	5	5	5
m	5	5	5	5

l is the number of simulations of the exchange rate. m is the number of simulated observations of the exchange rate per week. Test Statistic is the value of the Overidentifying Restrictions Test. Krugman F test is the Wald Test of the restriction that the M matrix is the 2×2 identity matrix. * indicates that Wald Test does not reject the restriction that $M = I(2)$. Standard errors are in parentheses.

Table 4: Estimates of the Trilateral Model: Netherlands

	Belgium	Denmark	France	Ireland
μ_1	.00148	.00204	.00188	.000400
	(.00167)	(.00491)	(.00246)	(.000554)
μ_2	−.00408	−.00732	−.00736	.0000400
	(.00234)	(.00440)	(.00278)	(.00304)
m_{11}	.881	.989	.792	.772
	(.324)	(.245)	(.169)	(1.89)
m_{22}	.651	.679	.628	1.00
	(.210)	(.159)	(.131)	(.699)
σ_1	.00401	.00395	.00349	.00359
	(.00136)	(.00152)	(.00154)	(.000374)
σ_2	.00863	.0187	.0143	.01128
	(.00223)	(.0114)	(.00476)	(.00825)
ρ	.355	.358	.0796	.0996
	(.129)	(.752)	(.141)	(.252)
γ	.0456	.141	.0328	.390
	(.282)	(.883)	(.167)	(2.38)
Test Statistic	6.59	3.70	5.08	11.83
Krugman F Test	82.01	8.98	361.5	.082*
l	5	5	5	5
m	5	5	5	5

l is the number of simulations of the exchange rate. m is the number of simulated observations of the exchange rate per week. Test Statistic is the value of the Overidentifying Restrictions Test. Krugman F test is the Wald Test of the restriction that the M matrix is the 2×2 identity matrix. * indicates that Wald Test does not reject the restriction that $M = I(2)$. Standard errors are in parentheses.

Table 5: Estimates of the Trilateral Model: Denmark

	Belgium	Netherlands	France	Ireland
μ_1	−.0186	−.00732	−.0108	−.0115
	(.00969)	(.00440)	(.00680)	(.00542)
μ_2	−.0211	.00204	−.00884	.00121
	(.00680)	(.00491)	(.00756)	(.00833)
m_{11}	.645	.679	.9556	.606
	(.0274)	(.159)	(.515)	(.098)
m_{22}	.704	0.989	1.00	.866
	(.0419)	(.245)	(.579)	(.220)
σ_1	.00189	.0187	.0240	.0183
	(.0113)	(.0114)	(.0120)	(.00608)
σ_2	.00321	.00395	.0214	.01369
	(.00755)	(.00152)	(.00613)	(.00593)
ρ	−.947	.358	.569	.0658
	(.147)	(.752)	(.142)	(.212)
γ	.159	.141	.517	.0217
	(.126)	(.883)	(.577)	(.119)
Test Statistic	9.88	3.70	11.66	6.17
Krugman F Test	196.7	8.98	.364*	16.08
l	5	5	5	5
m	5	5	5	5

l is the number of simulations of the exchange rate. m is the number of simulated observations of the exchange rate per week. Test Statistic is the value of the Overidentifying Restrictions Test. Krugman F test is the Wald Test of the restriction that the M matrix is the 2×2 identity matrix. * indicates that Wald Test does not reject the restriction that $M = I(2)$. Standard errors are in parentheses.

Table 6: Estimates of the Trilateral Model: France

	Belgium	Netherlands	Denmark	Ireland
μ_1	−.00728	−.00736	−.00884	−.0122
	(.00458)	(.00278)	(.00756)	(.00355)
μ_2	−.00736	.00188	−.01076	.00532
	(.00170)	(.00246)	(.00680)	(.00471)
m_{11}	.782	.628	1.00	.594
	(.196)	(.131)	(.579)	(.0691)
m_{22}	.640	.792	.956	.694
	(.0661)	(.169)	(.515)	(.138)
σ_1	.0151	.0143	.02141	.0183
	(.00736)	(.00476)	(.00613)	(.00683)
σ_2	.00706	.00349	.0240	.0147
	(.00165)	(.00154)	(.0120)	(.0102)
ρ	.5152	.0796	.569	.143
	(.082)	(.141)	(.142)	(.180)
γ	.0808	.0328	.517	.101
	(.313)	(.167)	(.577)	(.309)
Test Statistic	8.18	5.08	11.66	6.57
Krugman F Test	34.55	361.5	.364*	8.74
l	5	5	5	5
m	5	5	5	5

l is the number of simulations of the exchange rate. m is the number of simulated observations of the exchange rate per week. Test Statistic is the value of the Overidentifying Restrictions Test. Krugman F test is the Wald Test of the restriction that the M matrix is the 2×2 identity matrix. * indicates that Wald Test does not reject the restriction that $M = I(2)$. Standard errors are in parentheses.

Table 7: Estimates of the Trilateral Model: Ireland

	Belgium	Netherlands	Denmark	France
μ_1	−.00496	.0000400	.00121	.00532
	(.00246)	(.00304)	(.00833)	(.00471)
μ_2	−.0126	.0004	−.0115	−.0122
	(.00661)	(.000554)	(.00542)	(.00355)
m_{11}	.836	1.00	.866	.694
	(.213)	(.699)	(.220)	(.138)
m_{22}	.714	0.772	.606	.594
	(.134)	(1.89)	(.0984)	(.0691)
σ_1	.0145	.0113	.0137	.0147
	(.0142)	(.00825)	(.00593)	(.0102)
σ_2	.0112	.00359	.0183	.0183
	(.00377)	(.000373)	(.00608)	(.00683)
ρ	.184	.0996	.0658	.143
	(.218)	(.252)	(.212)	(.180)
γ	.801	.390	.0217	.101
	(1.61)	(2.38)	(.119)	(.309)
Test Statistic	10.14	11.83	6.17	6.57
Krugman F Test	4.99	.0823*	16.1	48.74
n	5	5	5	5
m	5	5	5	5

l is the number of simulations of the exchange rate. m is the number of simulated observations of the exchange rate per week. Test Statistic is the value of the Overidentifying Restrictions Test. Krugman F test is the Wald Test of the restriction that the M matrix is the 2×2 identity matrix. * indicates that Wald Test does not reject the restriction that $M = I(2)$. Standard errors are in pareotheses.

Table 8: Range of Cross-currency Variation in the Trilateral Model

Cross-currency exchange rate	Maximum Value	Minimum Value
Dutch Guilder-Belgian Franc	.0195	−.0195
Danish Krone-Belgian Franc	.0152	−.0152
French Franc-Belgian Franc	.0169	−.0169
Irish Pound-Belgian Franc	.0235	−.0235
Danish Krona-Dutch Guilder	.0236	−.0236
French Franc-Dutch Guilder	.0160	−.0160
Irish Punt-Dutch Guiler	.0317	−.0317
French Franc-Danish Krone	.0429	−.0429
Irish Punt-Danish Krone	.0148	−.0148
Irish Punt-French Franc	.0114	−.0114

The maximum value of the simnlated exchange rate refers to the maximum value taken over the domain of the regulated fundamentals process. Note that the maximum value is the negative of the minimum value only because we have included only three significant digits in the table.

Table 9: Estimates of the Five Currency Model

parameter	estimate	parameter	estimate
μ_1	.00315 (.0109)	ρ_{12}	.642 (.382)
μ_2	−.0176 (.00795)	ρ_{13}	−.362 (.295)
μ_3	−.0134 (.0191)	ρ_{14}	.165 (.336)
μ_4	−.00526 (.0346)	ρ_{23}	.425 (.310)
σ_1	.000681 (.00443)	ρ_{24}	.198 (.253)
σ_2	.0217 (.00868)	ρ_{34}	.307 (.163)
σ_3	.0198 (.0119)	γ	.193 (.446)
σ_4	.0197 (.0130)		
Test Statistic	17.03	l	5
Krugman F Test	744.3	m	5

l is the number of simulations of the exchange rate. m is the number of simulated observations of the exchange rate per week. The estimated M matrix is presented in the text. Krugman F test is the Wald Test of the restriction that the M matrix is the 4×4 identity matrix. * indicates that Wald Test does not reject the restriction that $M = I(4)$. Standard errors are in parentheses.

Table 10: Range of Cross-currency Variation in Five Country Model.

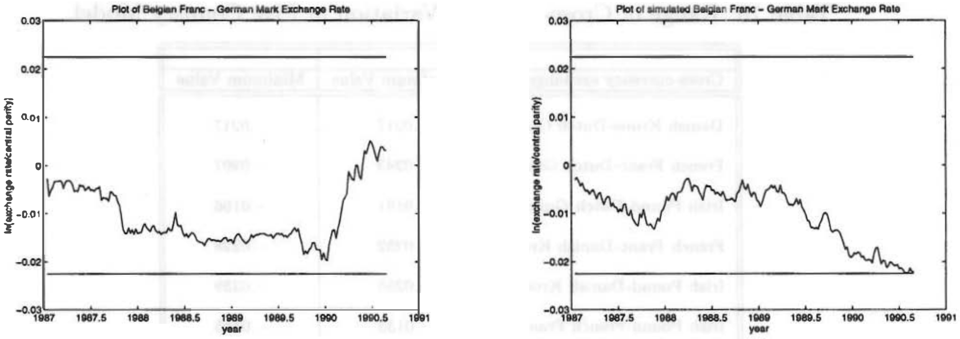
Cross-currency exchange rate	Maximum Value	Minimum Value
Danish Krone-Dutch Guilder	.0217	−.0217
French Franc-Dutch Guilder	.0243	−.0207
Irish Pound-Dutch Guilder	.0181	−.0166
French Franc-Danish Krone	.0252	−.0228
Irish Pound-Danish Krone	.0255	−.0239
Irish Pound-French Franc	.0133	−.0155

The maximum value is defined as the maximum value of the second currency in units of the first currency within the regulated domain of the fundamentals; the minimum is defined in the opposite manner.

Table 11: Horserace Results for DM-BFr-FFr Target Zone

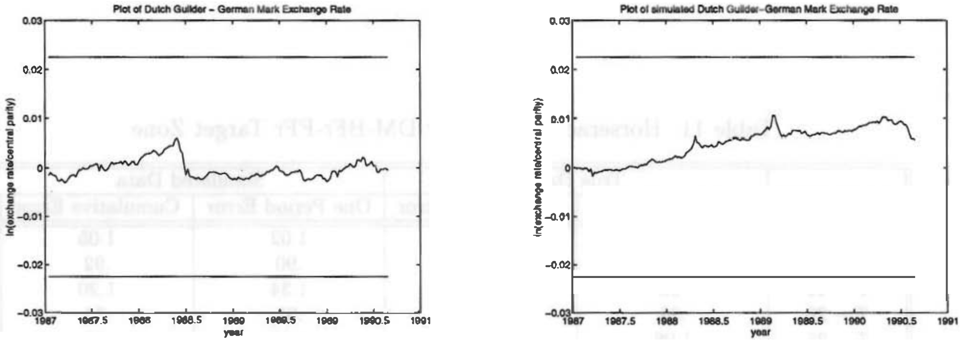
	True Data		Simulated Data	
Sample Size	One Period Error	Cumulative Error	One Period Error	Cumulative Error
$T-5$	1.17	1.19	1.02	1.05
$T-10$	1.30	.93	.90	.92
$T-15$	.98	1.36	1.34	1.20
$T-20$	.88	1.50	.95	.95
$T-25$	1.06	1.19	1.03	.82
$T-30$	.83	1.01	1.20	.78
$T-35$	1.17	.79	.44	.73

With both the true data and simulated data and for sample sizes $T-\tau$ ($\tau = 5, 10, 15, 20, 25, 30, 35$) we estimate the parameters of a random walk model and the target zone model. For each sample size, we then forecast one period and τ periods ahead using both the random walk and target zone models. The numbers reported above are the ratios of the mean squared error of the forecast of the target zone model over the mean squared error of the random walk model. A number greater than one thus implies the target zone model (given the data and the sample size) does not fit as well as a random walk.



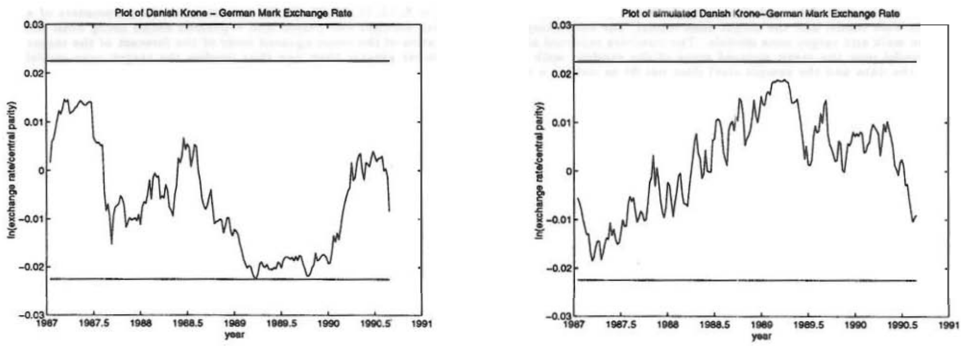
(a) (b)

Figure 1: (a) BFr-DM Exchange Rate (b) Simulated BFr-DM Exchange Rate



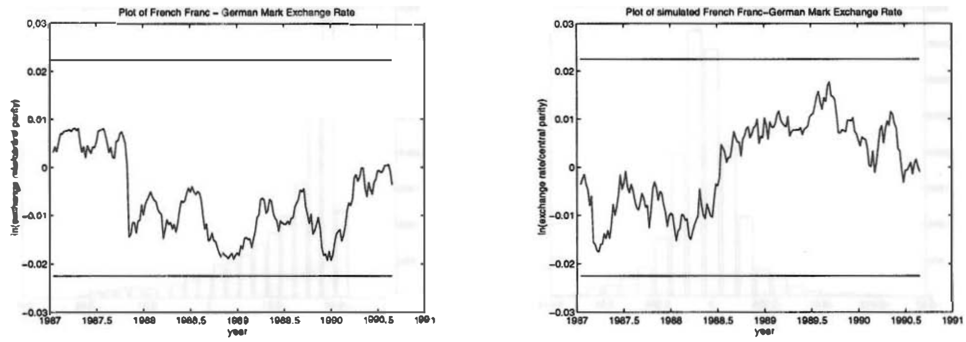
(a) (b)

Figure 2: (a) DFl-DM Exchange Rate (b) Simulated DFl-DM Exchange Rate



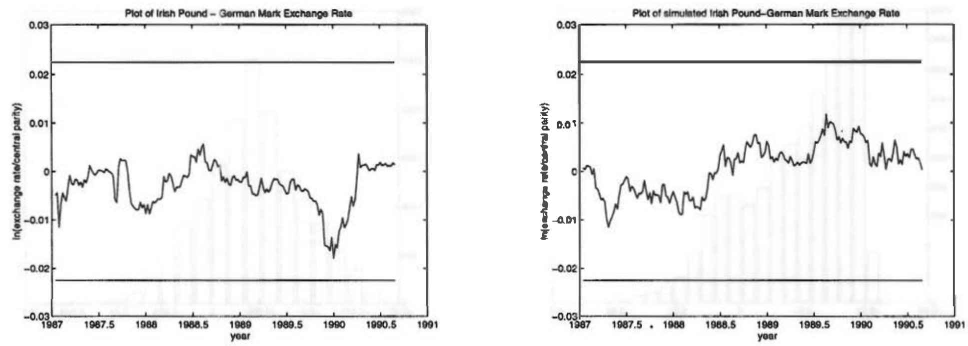
(a) (b)

Figure 3: (a) DKr-DM Exchange Rate (b) Simulated DKr-DM Exchange Rate



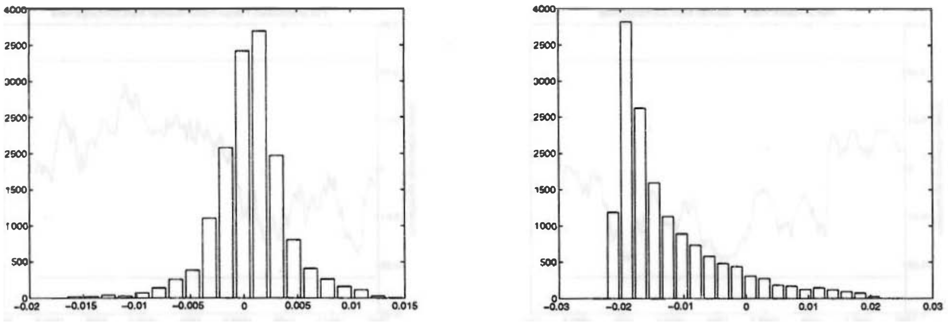
(a) (b)

Figure 4: (a) FFr-DM Exchange Rate (b) Simulated FFr-DM Exchange Rate



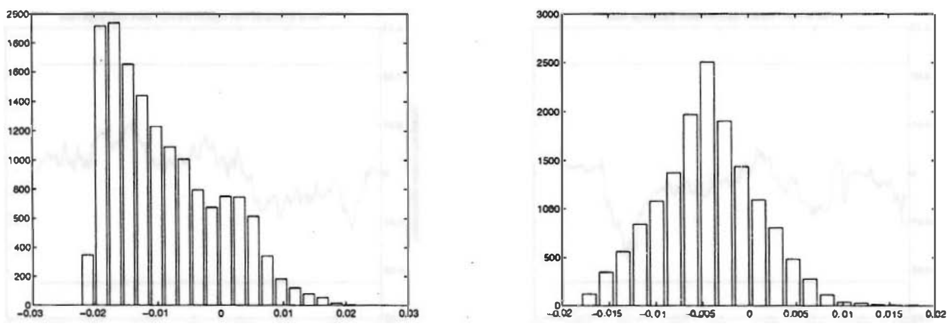
(a) (b)

Figure 5: (a) IP-DM Exchange Rate (b) Simulated IP-DM Exchange Rate



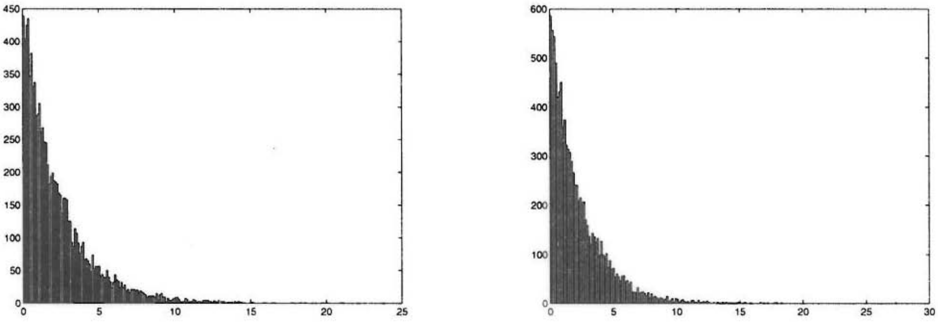
(a) (b)

Figure 6: Histogram of (a) Simulated DFl-DM and (b) Simulated DKr-DM Exchange Rates



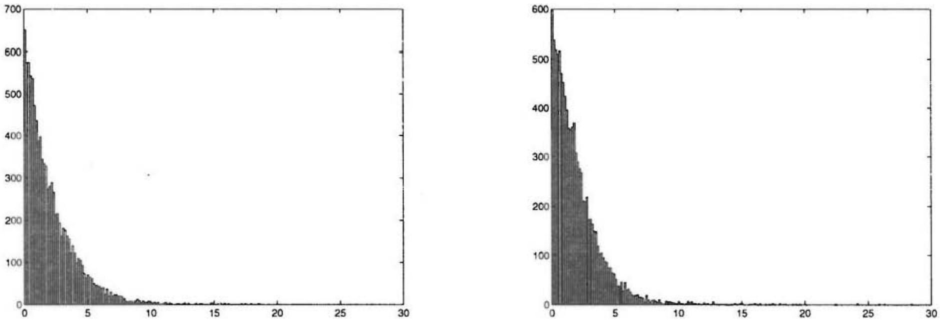
(a) (b)

Figure 7: Histogram of (a) Simulated FFr-DM and (b) Simulated IP-DM Exchange Rates



(a) (b)

Figure 8: Histogram of Test Statistics of Random Walk Tests (a) q=2 (b) q=4



(a) (b)

Figure 9: Histogram of Test Statistics of Random Walk Tests (a) q=8 (b) q=16

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