BANCO DE ESPAÑA

INVESTMENT INCENTIVES IN ENDOGENOUSLY GROWING ECONOMIES

Fernando Restoy and Georg Michael Rockinger

SERVICIO DE ESTUDIOS
Documento de Trabajo nº 9208

BANCO DE ESPAÑA

INVESTMENT INCENTIVES IN ENDOGENOUSLY GROWING ECONOMIES

Fernando Restoy and Georg Michael Rockinger (*)

(*) Bank of Spain and Harvard University. We are grateful to Gary Chamberlain, Dale Jorgenson, Greg Mankiw and Philippe Weil for very helpful comments on an earlier draft. We also thank Jennifer Hunt, Michael Munson, and James Thomson for valuable suggestions. Restoy is thankful for financial support from Spain´s Ministry of Education.

SERVICIO DE ESTUDIOS
Documento de Trabajo nº 9208

In publishing this series the Bank of Spain seeks to disseminate
studies of interest that will help acquaint readers better
with the Spanish economy.

ISBN: 84-7793-147-X

Depósito legal: M-6250-1992

Imprenta del Banco de España

Abstract

In this paper, we present and simulate a stochastic endogenous growth model with adjustment costs. We show that the inclusion of adjustment costs in a stochastic framework constitutes a relevant generalization of the existing endogenous growth models for, at least, two reasons: First, the presence of moderate to small adjustment costs is relevant to explaining the growth rate of the economy. Second, it allows us to relate growth with asset prices in a general equilibrium endogenous growth generalization of the Q-theory of investment. The model is used to analyze the incidence of changes in the corporate tax rate and the investment tax credit. We show that adjustment costs significantly moderate the effects of distorting taxes on the economy's growth rate. However, the responsiveness of the growth rate to changes of taxation significantly increases the influence of the corporate tax rate on a firm's market value per unit of capital and reduces the effect of the investment tax credit. This result in conjunction with the tax-effects on the growth rate of firm's value challenges the traditional analysis of the long-run effect of taxes on the market price of firms in models with exogenous growth.

KEYWORDS: Endogenous growth, Q-Theory, taxation, asset pricing.

1 Introduction

Recently, there has been an emergence of models of economic growth which are able to explain the long-run growth rate of an economy as the result of the optimizing behavior of agents in a particular technological and institutional environment. Those *endogenous* growth models [e.g., Romer (1986, 1989), Lucas (1988), Barro and Becker (1989), and Rebelo (1991)] have challenged the traditional neoclassical models where long-run growth is essentially an exogenous phenomenon related to the growth rate of the population or some predetermined form of technical change. An interesting issue to address within this new framework is the role of government intervention in economic development and, more specifically, the effect of distorting taxes on the growth rate of the economy. This question has been initially addressed by Rebelo (1991) and Barro (1990).

So far the endogenous growth literature has not dealt with adjustment costs of investment. There are strong reasons to believe that adjustment costs are an important element in the determination of the growth rate of the economy and the analysis of the effects of distorting taxation. They characterize the relative price of installed capital with respect to the consumption good and, therefore, modify the opportunity cost of investment. On the other hand, distorting taxes and credits will affect the contribution of adjustment costs to that opportunity cost. For example, the corporate income tax affects the relative price of installed capital through the tax deductibility of adjustment costs and an investment tax credit reduces the acquisition price of new capital goods relative to that of the already installed capital.

The endogeneity of the growth rate of the capital stock has strong implications for the analysis of tax effects on asset prices. The value of a firm in equilibrium is a discounted sum of future expected cash flows. In principle, those after-tax cash flows are affected not only by changes in the tax-rates but also by changes in the optimal investment-dividend policy. The traditional neoclassical Q models [e.g., Abel (1982), Summers (1982), Salinger and Summers (1983), Goulder and Summers (1986), as well as Auerbach and Kotlikoff (1987)] obtain long-run growth rates of the capital stock that are independent of the tax-code. Therefore, taxes do not affect the pre-tax value of future expected cash-flows since the investment policy does not change with modifications of taxes or credits. The evaluation of changes on the market valuation of a firm's capital stock is different in models where the investment policy, and therefore the growth rate of the economy is endogenously determined. Changes in taxes modify the optimal investment policy and this effect adds on to the direct effect on after-tax cash flows. Furthermore, since the change in the tax-incentive scheme affects the growth rate of a firm's capital stock, it also affects the growth rate of the market value of that capital stock. This dynamic effect is also absent in the exogenous growth models.

In this paper we develop a general equilibrium endogenous growth model which incorporates adjustment costs and use it to analyze tax effects on economic growth and on market prices of the productive units. We obtain the endogenous growth feature by introducing the Arrow-Romer *externality* effect into the model. Technical change is a public good related to the aggregate capital stock, but perceived as exogenous by individuals. However, our model incorporates uncertainty by adding random shocks to the endogenous technical shock. This approach is in the spirit of the King-Plosser-Rebelo (1988) attempt to relate the real business cycle and the endogenous growth literatures. More modestly, the uncertainty element provides realism to our description of the relation between optimal investment decisions and asset prices. In addition, the empirical distribution of the technical shock obtained from the data provides confidence intervals for the tax-incidence simulation exercises we perform to evaluate the model. The model provides a one to one relation between the investment-capital ratio and the market value of the firm per unit of capital that constitutes a general equilibrium stochastic endogenous growth version of the Q-theory of investment.

We show that adjustment costs significantly moderate the effects of distorting taxes on the economy's growth rate. However, the responsiveness of the growth rate to changes in taxation significantly increases the influence of the corporate tax rate on a firm's market value per unit of capital and reduces the effect of the investment tax credit. This result in conjunction with the tax-effects on the growth rate of firm's value challenges the traditional analysis of the long-run effect of taxes on the market price of firms in models with exogenous growth.

This paper is organized as follows. Section 2 presents a simple one-input, deterministic, constant returns to scale model. This simple model yields approximate closed form solutions which are used to illustrate the role of adjustment costs in the analysis of the effects of distorting taxation on growth. Section 3 generalizes the economy by introducing labor, technical change, and uncertainty. We then show how the model can be solved using standard numerical techniques. Section 4 establishes a relation between the optimal investment policies and the market valuation of the firm. Next we analyze the differences between the neoclassical and the endogenous growth approaches to tax-incidence, and relate the firm's value with the consumer's welfare. Section 5 performs calibration of the model focusing on the evaluation of the equilibrium growth rates for different combinations of preference and technological parameters. Section 6 analyzes different tax-incidence exercises. Section 7 concludes.

2 Taxes in a Simple Endogenous Growth Model with Adjustment Costs

In this section we present a stylized deterministic model which introduces adjustment costs and distorting taxes in an endogenous growth model with a single input technology. A sufficiently simple technology is chosen so that we can obtain an approximate closed form solution for the growth rate of the economy. The model is particularly useful for understanding the effects of taxes on the growth rate of economies with adjustment costs.

2.1 The Economy

Assume a discrete time[1], infinitely-lived, representative agent economy. The agent receives utility from the consumption of a single storable good. The utility function is time additive. A single input technology allows the agent to transfer the consumption good over time. Technology consists of a production function, an adjustment cost function, and a capital accumulation rule. The production function relates units of output to a single input (capital). The adjustment cost function expresses the loss of output per unit of investment due to installation or adjustment of capital, and is a function of the investment-capital ratio. Finally, the capital accumulation rule establishes the relation between the stock of capital and the investment expenditure. For simplicity, we assume in this section that there is no depreciation of the capital stock.

There exists an exogenous tax system which subtracts resources from the economy in an irreversible fashion[2]. The tax system is composed of a capital income tax and an investment tax credit.

The optimization problem of the representative agent at period t can be characterized by the dynamic program

$$\max_{\{I_t\}} \left\{ \sum_{s=0}^{+\infty} \beta^s U(C_{t+s}) \right\} \tag{1}$$

$$\text{s.t.} \quad C_t = (1-u)Y_t - (1-k+(1-u)\phi(\frac{I_t}{K_t}))I_t \tag{2}$$

[1] The growth literature tends to prefer to set models in continuous rather than in discrete time. The discrete time assumption is used here to facilitate the inclusion of uncertainty in the next section.

[2] An alternative is to assume that the government budget is balanced and the utility function is separable in consumption and government expenditures.

$$Y_t = F(K_t) \tag{3}$$

$$K_{t+1} = K_t + I_t, \tag{4}$$

where Y, C, K, and I denote output, consumption, net capital stock, and gross investment. u and k are the capital income tax rate and the investment tax credit per unit of investment. The utility function $U(C_t)$ is assumed to have the isoelastic form $U(C_t) = C_t^{1-\gamma}/(1-\gamma)$ where γ is the inverse of the elasticity of intertemporal substitution. $\phi(\cdot)$ is the adjustment cost function. This function is multiplied by $(1-u)$ to reflect the tax-deductibility of installation expenses. We assume in this section that the adjustment cost function has the linear form $\phi(i) = \theta i/2$ where i ($\equiv I/K$) is the investment-capital ratio.

The single-input production function is assumed to have the standard constant returns to scale form: $F(K_t) = \lambda K_t$ where λ is a constant.

2.2 The Equilibrium Growth Rate

The first order conditions of the optimization program (1)-(4) are

$$\frac{\lambda(1-u) + (1-u)\theta i_{t+1}^2/2 + 1 - k + (1-u)\theta i_{t+1}}{1 - k + (1-u)\theta i_t} = \frac{1}{\beta}\left(\frac{(1-u)\lambda - (1-k+(1-u)\theta i_{t+1}/2)i_{t+1}}{(1-u)\lambda - (1-k+(1-u)\theta i_t/2)i_t}(1+i_t)\right)^{\gamma}. \tag{5}$$

Notice that the right hand side of (5) represents the inverse of the marginal rate of substitution expressed in terms of the investment-capital ratio. The term in the denominator on the left hand side of (5) is the after tax cost of buying and installing one unit of capital at period t. The numerator on the left hand side is the payoff at period $t+1$ from investing one unit at period t. To see this, notice that $\lambda(1-u)+ (1-u)\theta i_{t+1}^2/2$ expresses the after tax increase in the resources available for consumption at period $t+1$ when one marginal unit of the consumption good has been invested at period t. Similarly, $1-k+(1-u)\theta i_{t+1}$ is the price of one unit of installed capital at period $t+1$. Thus, equation (5) states that, in equilibrium, the return on investing one unit of the consumption good must be equal to some required rate of return equal to the inverse of the marginal rate of substitution.

Equation (5) is a difference equation in the investment-capital ratio that admits a steady state solution under some regularity conditions[3]. That solution has to satisfy

$$1+\frac{\lambda(1-u)+(1-u)\theta i^2/2}{1-k+(1-u)\theta i}=\frac{(1+i)^\gamma}{\beta}. \tag{6}$$

Performing a linear approximation of both sides of (6) around $i=0$ we get

$$\frac{\lambda(1-u)}{(1-k)}-\frac{\lambda(1-u)^2\theta i}{(1-k)^2}=\rho+(1+\rho)\gamma i, \tag{7}$$

where ρ is defined as the time preference parameter (i.e., $1/\beta=1+\rho$).

To analyze expression (7), notice that the term $\lambda(1-u)/(1-k)$ is the effective productivity of one marginal unit of the consumption good invested in expanding the capital stock. Similarly, $\theta i(1-u)/(1-k)$ is the effective increase of adjustment costs due to the investment of one marginal unit of the consumption good. Therefore, the rate of return on investment is approximately the effective marginal productivity of capital minus the effective marginal loss on output due to adjustment costs. The right hand side is the required rate of return on savings which depends positively on the rate of discount of future utility (ρ) and negatively on the elasticity of intertemporal substitution ($1/\gamma$).

Solving (7) for the equilibrium investment-capital ratio we find

$$i=\frac{\lambda\frac{1-u}{1-k}-\rho}{\gamma(1+\rho)+\lambda(\frac{1-u}{1-k})^2\theta}. \tag{8}$$

Since there is no depreciation of the capital stock, equation (8) is an expression for the (endogenously determined) growth rate of this economy. This growth rate depends positively on the effective marginal productivity of capital and the elasticity of intertemporal substitution, and negatively on the time preference rate and the magnitude of the adjustment costs for investment.

In the absence of adjustment costs ($\theta=0$) and taxes ($u=k=0$), equation (8) collapses to an expression very similar to the Barro (1990)-Rebelo (1991) formula for the instantaneous optimal

[3] In particular, we require the following transversality condition to hold

$$(1+\lambda\frac{1-u}{1-k}-\rho)/(\gamma(1+\rho)+\lambda(\frac{1-u}{1-k})^2)\theta)<(1+\rho)^{1/(1-\gamma)}.$$

growth rate of a continuous time-constant returns to scale economy without adjustment costs[4]. In general, the presence of adjustment costs reduces the equilibrium growth rate of the economy by a significant magnitude. To see this, divide the numerator and denominator of (8) by $\gamma(1+\rho)$ and define the growth rate in absence of adjustment costs as $\bar{i}$. Then

$$i = \frac{\bar{i}}{1+\theta\frac{1-u}{1-k}(\bar{i}+\frac{\rho}{\gamma(1+\rho)})}, \tag{9}$$

where

$$\bar{i} = \frac{\lambda\frac{1-u}{1-k}-\rho}{\gamma(1+\rho)}.$$

Take, for example, $\bar{i} = 4\%$, $\theta(1-u)/(1-k) = 2.5$, $\gamma = 1$, and $\rho = 5\%$; we find $i = 3.2\%$. Thus, the presence of adjustment costs, which represent approximately 5% of total after tax investment expenses, implies a growth rate which is 22% lower than the one obtained in the absence of adjustment costs. From equation (9), the importance of adjustment costs would be larger (lower) for higher (lower) elasticities of intertemporal substitution $(1/\gamma)$ and higher (lower) values of the time preference parameter (ρ).

2.3 Tax Incidence on Growth

In this economy without depreciation allowances, the distorting effect of the tax system is perfectly summarized by the ratio $\tau = (1-u)/(1-k)$. The effect of changes in τ on the growth rate of the economy is given by

$$\frac{di}{d\tau} = \frac{\lambda[\gamma(1+\rho)-\theta\tau(\lambda\tau-2\rho)]}{[\gamma(1+\rho)+\theta\tau^2\lambda]^2}. \tag{10}$$

Changes in τ have ambiguous implications if adjustment costs are positive. The reason is that an increase in τ has two opposite effects. On one hand, it increases the after tax value of the marginal productivity of capital, providing incentives to invest. On the other hand, it reduces the tax-deductibility of adjustment costs and increases the total amount of those costs by increasing $\tau\lambda$.

[4]The exact formula is $i = \frac{\lambda-\rho}{\gamma}$. This expression could have been obtained by imposing $\theta = 0$ in (6) and linearizing the log of the left hand side around $\lambda = 0$ and the log of the right hand side around $i = 0$ and $\rho = 0$.

From equation (10), the prevailing effect on growth of an increase in τ depends upon the relation between γ and the after tax value of the adjustment costs $\theta\tau^2\lambda$. Notice that, in equation (7), a higher investment-capital ratio lowers the left hand side through an increase in the adjustment costs and raises the right hand side through an increase in the required rate of return. If γ is high in relation to θ, the increase in the after tax productivity of capital requires an increase in the investment-capital ratio to restore the equilibrium through an increase in the required rate of return on savings. Conversely, if γ is low in relation to θ, the increase in $\subset$ increases the adjustment costs more than the required rate of return. The equilibrium condition then requires a lowering of the adjustment costs by decreasing the equilibrium value of the investment-capital ratio. Thus, in this endogenous growth model with adjustment costs, the interest elasticity of savings must not be too small if investment incentive policies are not to have perverse effects.

3 A More General Model

The stylized model of the previous section has several limitations. First, it considers a single input technology. With more inputs the marginal productivity of capital will be constant only for increasing returns to scale technologies. This creates problems in guaranteeing existence of a competitive equilibrium allocation. Second, the model does not include the physical depreciation of the capital stock in the capital accumulation rule and the tax code. Third, there is no element of uncertainty in the return of investment. This element prevents a realistic description of the prices that support a competitive equilibrium allocation. In particular, the relation between investment decisions and the market value of the technology cannot be satisfactorily modeled in this set up. In this section we will extend the basic model to accommodate those features.

3.1 The Stochastic Economy

The economy is now composed of many identical, infinitely-lived households that receive utility from the consumption of a single storable good. Each agent owns a single technology which is subject to uncertain technical shocks, and the agent maximizes his expected lifetime utility subject to the available resources.

Technology is represented, as before, by a production function, an adjustment cost function, and a capital accumulation rule. Instead of the previous single input technology, we now assume that the production function relates output with capital and inelastically supplied labor, and incorporates a random technical change, the only element of uncertainty in this economy. As before, the adjustment cost function is homogeneous of degree zero with respect to investment and capital. Finally, the capital accumulation rule incorporates exponential depreciation of the capital stock.

The tax system contains as in section 2 a capital income tax, and an investment tax credit, but it also includes allowances for depreciation of the capital stock.

The optimization problem of a representative agent in this stochastic economy can be characterized by the dynamic program

$$\max E_t \left\{ \sum_{s=0}^{+\infty} \beta^s U(C_{t+s}) \right\} \tag{11}$$

$$\text{s.t.} \qquad C_t = (1-u)Y_t - (1-k+(1-u)\phi(\frac{I_t}{K_t}))I_t + uDA_t \tag{12}$$

$$Y_t = F(K_t, L_t, \omega_t) \tag{13}$$

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{14}$$

where δ and ω_t are respectively the depreciation rate and the random technical change variable. DA_t is the depreciation allowance for the capital stock at period t. For simplicity, we assume that depreciation allowances follow replacement rather than acquisition price criteria. Therefore $DA_t = \delta^T K_t$ where δ^T is the depreciation rate for tax purposes[5].

We will assume from now on that the production function has the Cobb-Douglas form

$$F(K_t, 1, \omega_t) = K_t^{\alpha}\omega_t, \tag{15}$$

where the inelastically supplied labor has been normalized to unity.

[5]The introduction of acquisition price based depreciation could be modelled by assuming $DA_t = \sum_{s=1}^{S} d(s)I_{t-s}$ where S is the depreciation horizon and $d(s)$ is the depreciation allowance for s-year old equipment. However this assumption introduces substantial notational and computational difficulties without altering the theoretical and empirical conclusions of this paper.

3.2 Technical Change

In order to completely characterize the economy, we need to specify the distribution of the technological variable ω_t. If $\omega_t = 1$ the model is very similar to the one suggested by Auerbach and Kotlikoff (1987) to analyze effects of the 1986 tax reform on investment. This model, in the neoclassical tradition, assumes decreasing marginal productivity of capital: $F_K(K, 1, 1) = \alpha K^{\alpha-1}$. Therefore, the economy is constrained to have a predetermined growth rate in the long run as the capital stock approaches a steady state level. This exogeneity of the long run rate of growth remains essentially unaltered if ω_t follows a prespecified deterministic or stochastic process expressing some form of exogenous technical change as in the Real Business Cycle literature (See King, Plosser and Rebelo (1988)).

The endogenous growth approach requires ω_t to be an offsetting force to the decline in the aggregate marginal productivity of capital as the capital stock grows. Then, technical change has to depend positively on the aggregate capital stock. If the production function is linear homogenous in capital and labor for given ω_t, technical change creates increasing returns to scale.

In order to make increasing returns to scale of the aggregate production technology compatible with the existence of a competitive equilibrium, we follow the Arrow-Romer approach and assume that the effects of technical change are external from individuals' investment decisions. Technical progress is related to the aggregate capital stock but is perceived as exogenous by individual agents. Calling $\tilde{K}$ the aggregate (per capita) capital stock, technical change is defined by

$$\omega_t \equiv \omega(\tilde{K}_t, \lambda_t) = \frac{\tilde{K}_t^{1-\alpha}}{\alpha}\lambda_t, \tag{16}$$

where λ_t is a stationary exogenous random variable. Therefore, the production function (15) can be written as

$$F(K_t, \tilde{K}_t, \lambda_t) = \frac{\lambda_t}{\alpha} K_t^{\alpha} \tilde{K}_t^{1-\alpha}. \tag{17}$$

From equation (17) it is clear that the external effect expressed in equation (16) establishes a wedge between the *private* marginal productivity of capital and the *social* one. The former is $F_K(K_t, 1, \tilde{K}_t^{1-\alpha}\lambda_t/\alpha) = \lambda_t K_t^{\alpha-1} \tilde{K}_t^{1-\alpha}$ and the latter is $\frac{\lambda_t}{\alpha}$. Thus, individual agents do not internalize all the social benefits associated with their investment decisions. This source of inefficiency of the competitive equilibrium allocation adds to the one created by the presence of distorting taxes.

The introduction of a stochastic term in the Arrow-Romer relation (16) reflects the existence of short-lived shocks to the marginal productivity of capital. This element will allow us to calibrate the model in a relatively precise way, and to provide confidence intervals in the tax-incidence exercises. In addition, it permits us to deal realistically with the market valuation of technology (a risky asset pricing problem).

3.3 Equilibrium Investment Policies

The inefficiency of the competitive allocation prevents us from using Bellman equation solution techniques to obtain the equilibrium investment policies. This procedure requires Pareto optimality of the competitive equilibrium allocation. (See Stockey and Lucas (1989).) Instead, we are compelled to impose the market clearing equilibrium conditions over the first order conditions of the agent's optimization problem.

Using standard dynamic programming techniques and elementary algebra, one can express the first order conditions of the maximization problem in the following Euler equation form:

$$E_t\left\{\beta\frac{U'(C_{t+1})}{U'(C_t)}R_t^I(K_{t+1},\omega_{t+1},i_{t+1},i_t)\right\}=1, \tag{18}$$

where C_t satisfies (12), (13), and

$$\begin{aligned}R_t^I(K_{t+1},\omega_{t+1},i_{t+1},i_t)=&\Big[F_K(K_{t+1},1,\omega_{t+1})(1-u)+u\delta^T+\\ &\Phi'(i_{t+1})i_{t+1}^2+(1-\delta)(1-k+\Phi'(i_{t+1})i_{t+1}+\Phi(i_{t+1}))\Big]\,/\,[1-k+\Phi'(i_t)i_t+\Phi(i_t)]\,,\end{aligned} \tag{19}$$

where $\Phi(\cdot)=(1-u)\phi(\cdot)$.

The denominator in (19) is the after-tax cost of buying and installing one unit of capital at period t. On the other hand, from equation (12), the first three terms in the numerator express the after-tax increase in the resources available for consumption at period $t+1$ due to the investment of one marginal unit of the consumption good. Therefore, $R_t^I(t+1)$ is the return, net of depreciation and adjustment costs, on investing one unit of the consumption good in expanding the capital stock of the firm. Hence, equation (18) constitutes a set of equilibrium conditions for the return on real investment analogous to the standard arbitrage conditions for the returns on financial claims used in the asset pricing literature[6].

Using equations (12) and (17), and assuming an isoelastic utility function, the intertemporal marginal rate of substitution can be written as

$$\beta \frac{U'(C_{t+1})}{U'(C_t)} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \tag{20}$$

where

$$C_t = (1-u)K_t^{\alpha} \bar{K}_t^{1-\alpha} \lambda_t / \alpha - (1 - k + (1-u)\phi(i_t)) i_t K_t + u\delta^T K_t. \tag{21}$$

Investment returns have the form

$$R_t^I(K_{t+1}, \bar{K}_{t+1}, i_{t+1}, i_t, \lambda_{t+1}) = \frac{K_{t+1}^{\alpha-1} \bar{K}_{t+1}^{1-\alpha} \lambda_{t+1} + u\delta^T + \Phi'(i_{t+1}) i_{t+1}^2 + (1-\delta)(1 - k + \Phi'(i_{t+1}) i_{t+1} + \Phi(i_{t+1}))}{1 - k + \Phi'(i_t) i_t + \Phi(i_t)}. \tag{22}$$

Now, imposing the general equilibrium condition $K_t = \bar{K}_t$ on (21) and (22), we find that the equilibrium value of the marginal rate of substitution and the investment return are both independent of the capital stock in levels, depending only on the investment-capital ratio i and the stationary exogenous random variable λ, for given technological and tax parameters. From the externality assumption, in equilibrium, the *private* average and marginal productivity of capital are respectively λ_t/α and λ_t. Therefore, the productivity of capital does not depend on the level of the capital stock as it does in the standard neoclassical models. Since adjustment costs and the capital growth rate only depend on the investment-capital ratio, according to (21) and (22), consumption growth (and therefore, the marginal rate of substitution) and the investment return at period t+1 are completely characterized by i_t, i_{t+1}, λ_t and λ_{t+1}. Consequently, imposing the general equilibrium condition $K_t = \bar{K}_t$ on (19) yields the following stochastic difference equation

[6]The asset pricing implications of that condition have been derived and empirically exploited in a partial equilibrium framework by Cochrane (1991), Braun (1990) and Restoy and Rockinger (1991).

for the equilibrium investment-capital ratio

$$E_t\left\{\beta\left\{(1-u)\lambda_{t+1}/\alpha + u\delta^T - (1-k+\Phi(i_{t+1}))i_{t+1}\right\}^{-\gamma}\right.$$
$$\left.\left\{(1-u)\lambda_{t+1} + u\delta^T + \Phi'(i_{t+1})i_{t+1}^2 + (1-\delta)(1-k+\Phi'(i_{t+1})i_{t+1} + \Phi(i_{t+1})\right\}\right\}$$
$$= \left\{\frac{(1-u)\lambda_t(1-u)/\alpha + u\delta^T - (1-k+\Phi(i_t))i_t}{1-\delta+i_t}\right\}^{-\gamma}\{1-k+\Phi'(i_t)i_t+\Phi(i_t)\}.$$

This expression is the stochastic counterpart of equation (5) for economies subject to constant *aggregate* marginal productivity of capital.

Assume now that λ_t follows a m-state homogeneous Markov process with transition probabilities $\{\pi_{rs}\}_{r,s=1}^{m}$. Then, by expression (18), any stationary investment policy must satisfy the system of equations

$$H(\lambda_r, i_r) = \sum_{s=1}^{m}\pi_{rs}G(\lambda_s, i_s) \qquad r, s = 1, \cdots, m, \tag{23}$$

where

$$H(\lambda_r, i_r) = \left\{\frac{(1-u)\lambda_r/\alpha + u\delta^T - (1-k+\Phi(i_r))i_r}{1-\delta+i_r}\right\}^{-\gamma}\left\{1-k+\Phi'(i_r)i_r+\Phi(i_r)\right\}$$

and

$$G(\lambda_r, i_r) = \beta\left\{(1-u)\lambda_r/\alpha + u\delta^T - (1-k+\Phi(i_r))i_r\right\}^{-\gamma}$$
$$\{(1-u)\lambda_r + u\delta^T + \Phi'(i_r)i_r^2 + (1-\delta)(1-k+\Phi'(i_r)i_r+\Phi(i_r))\}.$$

Then, the optimal investment policy and therefore the growth rate of the economy $(i-\delta)$ are completely characterized by the realization of the stationary technical shock λ.

To ensure that the system (23) has a solution, we require the following

[A.1] There exists a constant $g < +\infty$ s.t. $\phi(i) \geq 0$, $\phi'(i) \geq 0$ and $\phi''(i) \geq 0$ for all $i \geq g$.

[A.2] $H(\lambda_r, g) \geq \sum_{s=1}^{m}\pi_{rs}G(\lambda_s, g)$.

The first assumption just requires the positiveness, monotonicity and convexity of the adjustment cost function for an economically meaningful set of values of the investment-capital ratio. The second assumption establishes a lower bound for the expected risk adjusted return on investment in that set.

PROPOSITION 1: If the vector $\lambda = \{\lambda_r\}_{r=1}^{m}$, the matrix $\Pi = \{\pi_{rs}\}_{r,s=1}^{m}$ and the function $\phi(\cdot)$ are such that assumptions [A.1] and [A.2] hold, then there exists a vector $i = \{i_r\}_{r=1}^{m}$ with $i \geq g$ that is a solution of the system (23).

Proof: See Appendix A.

This proposition shows that the equilibrium investment-capital ratio is a variable that follows a time homogenous Markov process which is the solution of a system of nonlinear equations. Unfortunately this system does not admit closed-form solutions, unlike the simple deterministic model presented in the second section. The model is solved numerically in Section 6.

4 Value of the Firm, Contingent Claim Prices, and Consumer Welfare

So far we have formulated the model in terms of the maximization problem of consumers who own the capital stock of the economy. This approach has been useful in solving for the equilibrium investment policies. Analogously we can obtain the competititive equilibrium allocation by further decentralizing the model and considering identical price taking value maximizing firms and expected utility maximizing consumers in a complete markets framework. This approach will allow us to relate the equilibrium investment policies with the market value of the firm, consumer's welfare, and to obtain a wider picture of the effects of taxation in this economy.

4.1 Competitive Firm's Problem

Assume there exists a sufficient number of long-lived securities to dynamically complete the markets (in the sense of Duffie and Huang (1985)). Denote by F_t the information set at period t and by a_t a typical element of F_t. Now call D_{a_t} the firm's cash flows in state a_t and $p_t(a_s)$ the price at period t of one unit of consumption in state a_s at period s. Given a complete set of contingent claim prices, the firm's problem consists of obtaining the life-time investment schedule that maximizes total cash flows evaluated at the contingent claim prices subject to the technological constraints. The maximization problem of the firm can be written as

$$
\begin{aligned}
V(K_t,\lambda_t) &\equiv \max_{\{I_{a_s},L_{a_s}\}} \left\{ \sum_{s=t+1}^{+\infty} \sum_{a_s \in F_s} p_t(a_s) D_{a_s} \right\} \qquad (24)\\
&= \sum_{s=t+1}^{+\infty} \sum_{a_s \in F_s} p_t(a_s)\{(1-u)[Y_{a_s} - X_{a_s} L_{a_s}] - (1-k+\Phi(i_{a_s}))I_{a_s} + u\delta^T K_s\}\\
\text{s.t.} \quad Y_{a_s} &= F(K_s, L_{a_s}, \omega_{a_s}) = K_s^{\alpha} L_{a_s}^{1-\alpha} \omega_{a_s} \qquad (25)\\
\omega_{a_s} &= \tilde{K}_s^{1-\alpha} \lambda_{a_s}/\alpha \qquad (26)\\
K_{s+1} &= (1-\delta)K_s + I_{a_s}, \qquad (27)
\end{aligned}
$$

where X_t is the wage rate.

In the problem (24)-(27) uncertainty plays no role as long as the relative prices of the single good in the different states of nature are known *ex ante* to the firm.

The solution to the program will give the firm's optimal investment and labor demand in each state as a function of the set of contingent claim prices and the wage rate. That solution has to satisfy the set of first order conditions

$$
X_{a_t} = F_{L_{a_t}} = (1-\alpha)(Y_{a_t}/L_{a_t}) \qquad \text{For all} \quad t, a_t \in F_t \qquad (28)
$$

and

$$
\sum_{a_{t+1}} p_t(a_{t+1}) R_t^I(a_{t+1}) = 1, \qquad (29)
$$

where

$$
\begin{aligned}
R_t^I(a_{t+1}) = \Big[\tilde{F}_K(a_{t+1}) + u\delta^T + \Phi'(i_{a_{t+1}}) i_{a_{t+1}}^2 + \\
(1-\delta)(1-k+\Phi'(i_{a_{t+1}}) i_{a_{t+1}} + \Phi(i_{a_{t+1}}))\Big] \,/\, [1-k+\Phi'(i_t)i_t + \Phi(i_t)] \qquad (30)
\end{aligned}
$$

and

$$
\tilde{F}_K(a_{t+1}) = (1-u) K_{t+1}^{\alpha-1} \tilde{K}_t^{1-\alpha} \lambda_{a_{t+1}}.
$$

Notice that (29) is a restatement of the arbitrage equation (18) in terms of contingent claim prices instead of preferences. In a world of complete markets, the return of all investment strategies evaluated at the contingent claim prices have to be equal to prevent arbitrage opportunities.

In order to obtain an expression for the value of the firm, impose the equilibrium conditions

$L_t = 1$ and $K_t = \bar{K}_t$ in the investment equation (30). Then $\bar{F}_K(a_{t+1}) = (1-u)\lambda_{a_{t+1}}$ and the equilibrium investment-capital ratio depends only on the variable λ for given contingent prices. Furthermore, the equilibrium value of the aggregate profits of the firm in state a_s in period s is

$$\begin{aligned} D_{a_s} &= (1-u)[Y_{a_s} - X_{a_s}] - (1-k+\Phi(i^*{}_{a_s}))I^*_{a_s} + u\delta^T K_s = \\ &= K_s[(1-u)\lambda_{a_s} - (1-k+\Phi(i^*_{a_s}))i^*_{a_s} + u\delta^T], \end{aligned} \tag{31}$$

where $*$ refers to equilibrium values.

Now, define the after tax net value of the firm as the sum of future cash flows evaluated at the contingent prices minus the cost of initial investment. Then, along the optimal investment path, this variable satisfies the recursive expression

$$J(K_t, \lambda_t) = -\left(1 - k + \Phi(i^*_t)\right)i^*_t K_t + \sum_{a_{t+1} \in F_{t+1}} p_t(a_{t+1})\{\lambda_{a_{t+1}}(1-u) + J(K_{t+1}, \lambda_{a_{t+1}})\}, \tag{32}$$

where $J(\cdot,\cdot)$ is the after tax net value function. Since the equilibrium investment-capital ratio is a function of the technical shock variable, and λ_t follows a Markov process, we can write the equilibrium investment-capital ratio as $i^*_t = i(\lambda_t)$. A natural guess for the functional form of the net value function is $J(K_t, \lambda_t) = \Psi(\lambda_t)K_t$. Using the first order conditions (28)-(29) we obtain a first order stochastic difference equation for $\Psi(\lambda_t)$ with solution

$$\Psi(i(\lambda_t)) = \Phi'(i(\lambda_t))i(\lambda_t)^2 + (1-\delta)\Big([1 - k + \Phi'(i(\lambda_t))]i(\lambda_t) + \Phi(i(\lambda_t))\Big). \tag{33}$$

The ex-dividend market value of the firm at period t, V_t, is simply the net value plus the investment expenses at period t. Thus

$$\begin{aligned} V(K_t, \lambda_t) &= J(K_t, \lambda_t) + (1 - k + \Phi(i(\lambda_t)))i(\lambda_t))\, K_t \\ &= (1 - \delta + i(\lambda_t))\,[(1 - k + \Phi'(i(\lambda_t))i(\lambda_t) + \Phi(i(\lambda_t))]\, K_t \\ &= Q(\lambda_t)K_{t+1}, \end{aligned} \tag{34}$$

where

$$Q(\lambda_t) = (1 - k + \Phi'(i(\lambda_t))i(\lambda_t) + \Phi(i(\lambda_t)). \tag{35}$$

This expression states that in an economy with a linear homogeneous adjustment cost function, constant *private* returns to scale and increasing *aggregate* returns to scale, the market value of the firm is proportional to the replacement cost of its capital stock. The factor of proportionality just depends on the realization of the stochastic marginal productivity of capital for given tax and technological parameters. Therefore, as in the neoclassical deterministic model of Hayashi (1982), marginal Q is equal to average Q. Furthermore, assuming linear adjustment costs per unit of investment ($\Phi(i) = (1-u)\theta i/2$), we obtain the popular linear relation between $Q(\lambda_t)$ and the investment-capital ratio

$$Q(\lambda_t) = 1 - k + \theta(1-u)i(\lambda_t). \tag{36}$$

Therefore, the model we have presented, with endogenous growth, uncertainty and risk aversion, provides a relation between the investment-capital ratio and the market value of the firm per unit of capital similar to the standard Q model. In this general equilibrium set-up, the endogeneity of both investment and the firm's value becomes apparent since both variables are simultaneously determined by the realization of the technical shock (λ_t). This simultaneity is one of the econometric difficulties in estimating variants of equation (36).

In the tax incidence analysis, the endogeneity of the growth rate of the economy establishes serious differences with respect to the neoclassical Q models [e.g., Able (1982), Summers (1982), Salinger and Summers (1983), Goulder and Summers (1983), and Auerbach and Kotlikoff (1988)]. In standard Q models, the economy grows in the long-run at a rate which is independent of individuals' decisions. Therefore the investment-capital ratio is fixed. The exogeneity of the growth rate has two implications in the tax incidence analysis. First, according to Q-theory (summarized in expressions (35)-(36)), the value of the firm grows at the same prespecified rate as capital, regardless of changes in the tax-code. Second, in the long-run, taxes only affect value through their direct effects on the relative price of installed capital. In our model, the equilibrium investment-capital ratios are affected by changes in taxes. Since Q is a stationary variable, changes in tax rates that affect the equilibrium growth rate of capital affect the growth rate of the firm's value in the same amount. In addition to that, from equation (35), those changes in the equilibrium investment-capital ratio affect the equilibrium stationary value of Q in conjunction with the direct effect produced by the changes in taxation.

In order to get a better understanding of the difference between the two approaches, consider a change in the investment tax credit. The value of the firm per unit of installed capital is decreasing with the investment tax credit for given investment-capital ratios since a higher investment tax credit lowers the relative price of installed capital with respect to consumption. This effect is the only one considered in the neoclassical analysis of the long-run incidence of changes in the investment tax-credit on the market valuation of the firm. However, an increase in k will affect positively the equilibrium investment-capital ratio for reasonable parameter values[7]. From equations (35) and (36), this change in the investment policy will offset the direct *neoclassical* effect of the change in k by an amount that depends positively on the size of the adjustment costs. Furthermore, the increase in the investment-capital ratio will imply an increase in the growth rate of the value of the firm. As a consequence, a decrease in the equilibrium value for Q when k rises implies a short term market devaluation for the firm that eventually disappears as a consequence of the higher rates of growth.

On the other hand, an increase in the corporate tax rate affects the value of the firm through the decrease in the tax deductibility of adjustment costs ($\Phi(\cdot) = (1-u)\phi(\cdot)$) as in the standard model, but also through the modification of the equilibrium investment-capital ratio. Thus, if the latter is negative, the total effect is unambiguously larger than the one obtained under the standard Q approach.

The rest of the technological and tax-parameters, and the contingent claim prices, only affect the value of the firm through their influence on the determination of the optimal investment-capital ratio. An increase in the contingent claim prices for future consumption will typically increase the equilibrium investment-capital ratio and, therefore, the value of the firm per unit of capital. (The future and uncertainty are less heavily discounted.) This effect is absent in the neoclassical approach even when the model includes time-varying endogenous discount factors (as in Auerbach-Kotlikoff (1987))[8], as long as the long-run growth rate of the capital stock is exogenously determined.

[7] As we saw in section 2, this typically requires that adjustment costs are sufficiently small in relation to the inverse of the elasticity of intertemporal substitution.

[8] Notice from equations (35) and (36) that the Q-relation is independent of the contingent claim prices for a given equilibrium investment-capital ratio.

4.2 Competitive Consumer's Problem

Consider now the optimization problem of an infinitely-lived competitive consumer who owns a claim on the stock of a representative firm in the economy. The cash flows of that firm are represented by the sequence $D_{a_s}, a_s \in F_s$. In addition, the agent receives from the firm at the beginning of period t a wage income equal to X_t. Since markets are complete, a claim on the stock of the firm is marketed. Define $R_t(a_{t+1})$, the market return between t and state a_{t+1} in period $t+1$, as

$$R_t(a_{t+1}) = \frac{V_{a_{t+1}} + D_{a_{t+1}}}{V_t}, \tag{37}$$

where V_t is the ex-dividend market value of the firm.

Define as $\pi_t(a_s)$ the probability of reaching state a_s in F_s $(s > t)$, conditional on F_t. Then, the optimization problem for the price-taking consumer who maximizes his expected life-time utility is

$$\max_{\{C_{t+s}\}} \sum_{s=0}^{+\infty} \sum_{a_{t+s} \in F_{t+s}} \pi_t(a_{t+s}) \beta^s \frac{C_{a_{t+s}}^{1-\gamma}}{1-\gamma} \tag{38}$$

$$\text{s.t.} \qquad V_{a_{s+1}} = V_{a_s} R_{a_s}(a_{s+1}) + X_{a_{s+1}} - C_{a_{s+1}}. \tag{39}$$

Since the labor income (X_t) is exogenous to the individual, the first order conditions have the form

$$\sum_{a_{t+1} \in F_{t+1}} \beta \pi_t(a_{t+1}) \left(\frac{C_{a_{t+1}}}{C_t}\right)^{-\gamma} R_t(a_{t+1}) = 1. \tag{40}$$

Notice that expression (40) has an Euler equation form similar to the competitive firm's first order conditions (29). Equation (40) is a *no arbitrage* condition for the market return of the firms as equation (29) is a *no arbitrage* condition for investment returns. It turns out that both returns are equal state by state in this economy with constant *private* returns to scale and linear homogenous adjustment cost function.

PROPOSITION 2: Consider a firm whose optimization problem is represented by the program

(25) to (27). Then

$$R_t^I(a_{t+1}) = R_t(a_{t+1}) \qquad \text{for all } t, a_{t+1} \in F_{t+1}. \tag{41}$$

Proof: See Appendix A.

This result extends to this economy, with distorting taxes and endogenous growth, the results of Cochrane (1990) and Restoy and Rockinger (1991) for neoclassical economies without taxes[9]. From proposition 2 and the equilibrium conditions of competitive firms and consumers, we obtain that the equilibrium allocation represented by the solution to equation (18) is supported by a complete set of contingent claim prices $\{p_t(a_s), s > t, a_s \in F_s\}$ which satisfy

$$p_t(a_{t+1}) = \beta\pi_t(a_{t+1})\left(\frac{C_{a_{t+1}}}{C_t}\right)^{-\gamma} = \beta\pi_t(a_{a_{t+1}})\left(\frac{c(\lambda_{a_{t+1}})(1-\delta+i(\lambda_{a_{t+1}}))}{c(\lambda_t)}\right)^{-\gamma}, \tag{42}$$

where

$$c(\lambda_t) = \frac{C_t}{K_t} = \frac{\lambda_t(1-u)}{\alpha} + u\delta^T - (1-k+\Phi(i(\lambda_t)))i(\lambda_t). \tag{43}$$

The contingent claim prices express the evaluation of the tradeoff between current and future consumption by the economic agents. Since the returns on savings are stochastic, those prices involve both the agents' willingness to delay consumption, and their risk aversion. From equations (42) and (43) the contingent prices are negatively related to the investment-capital ratio. This introduces a factor of stability in the system. Incentives to invest decrease the tradeoff between present and future consumption and, therefore, reduce the evaluation of future cash flows by the firm. Consequently, further investment becomes less attractive.

Finally, we are interested in relating the value of the firms with the consumer's welfare in the competitive equilibrium allocation. Denote by $W(\lambda_t, K_t)$ the value of the consumer's utility function in the competitive equilibrium allocation. We then have

PROPOSITION 3: Assume that firms solve the optimization problem (24)-(27) and consumers

[9]Cochrane (1990) obtains the equivalence betwen market returns and investment returns in an economy with quadratic adjustment costs and a linear production function. Restoy and Rockinger (1991) show that this reult holds in general stochastic economies that satisfy the Hayashi (1982) conditions: price-taking firms and linear homogenous adjustment cost and production functions.

solve (38)-(39). Then the consumer's indirect utility function in the competitive equilibrium allocation is given by

$$W(\lambda_t, K_t) = m(\lambda_t) K_t^{1-\gamma}, \tag{44}$$

where

$$m(\lambda_t) = \frac{c(\lambda_t)^{-\gamma}}{1-\gamma}[q(\lambda_t) + c(\lambda_t)] + E_t \left\{ \sum_{s=1}^{+\infty} \left(\frac{c_{t+s}}{c_t}\right)^{-\gamma} \left(\frac{K_{t+s}}{K_t}\right)^{1-\gamma} \frac{(1-\alpha)}{\alpha} \lambda_{t+s} \right\}$$

and $q(\lambda_t) = V(K_t, \lambda_t)/K_t$.

Proof: See Appendix A.

Not surprisingly, the presence of labor makes it impossible to obtain an analytically closed form solution for the indirect utility function. However, the indirect utility function normalized by a power of the capital stock is still a stationary variable that follows a Markov distribution given our assumption about the distribution of λ_t. The value of $m(\lambda_t)$ can be easily computed numerically up to arbitrary accuracy by taking a sufficient number of terms in the sum under the expected value operator. (see Appendix D).

Expression (44) shows that the rate of growth of welfare is different from the rate of growth of the capital stock. In particular, the closer the coefficient of relative risk aversion γ is to 1, the lower the expected rate of growth of the indirect utility function is in relation to that of the capital stock.

5 Model Calibration

In this section we solve numerically the optimal investment policy problem presented in Section 3. To do so we need to calibrate the stochastic technical shock variable. We assume that (λ_t) follows a lognormal first order autoregressive process whose parameters are estimated using U.S. data. Since the model predicts that in equilibrium $Y_t - X_t = \alpha Y_t = \lambda_t K_t$ we estimate $\Lambda_t \equiv \ln(\lambda_t)$ by using data on output (Y_t), wages (X_t), and capital (K_t), taking λ_t equal to $(Y_t - X_t)/K_t$. For output we use Gross Domestic Product in private industries. Wages are measured as total labor compensation in private industries and capital is defined as fixed private domestic capital plus the stock of inventories[10].

The Markov process for λ_t is obtained by estimating a first order autocorrelated process for annual observations[11] of $\ln(\lambda_t)$. We then approximate the continuous distribution of this variable by a discrete set of values and their associated transition probabilities. We took the number of states to be equal to 10 as a compromise between maximization of accuracy in the approximation and minimization of time required by the numerical routine[12]. The ten states were distributed to capture 99% of the empirical unconditional distribution of $(Y_t - X_t)/K_t$. Technical details are given in Appendix B.

For the numerical analysis, we assume the following quadratic adjustment cost function

$$\phi(i) = \frac{\eta}{2} i^2. \tag{45}$$

Notice that this adjustment cost function is monotonically increasing only for positive values of i. Therefore, in order to obtain the equilibrium investment policies by the fixed point argument of Proposition 1, we require the minimum admissible i (g in [A.1] and [A.2]) to be zero. Furthermore, since the equilibrium investment–capital ratios are obtained as a fixed point of a monotonic operator, a natural choice for the starting values of the routine is $i_0 = 0$. Appendix C contains a description of the numerical routine.

We set the technical and tax parameters as follows: $\delta = 0.05$, $\beta = 0.96$, $\delta^T = 0.1$, $u = 0.5$, $k = 0.1$, and solved the model for several values of the relative risk aversion coefficient (γ) and the adjustment cost parameter (η)[13]. The application of a fixed point search routine yields the equilibrium distribution of the investment–capital ratios for each pair of γ and η and the assumed technological and tax parameters. In order to be able to evaluate the performance

---

[10]Gross Domestic Product in private industries, wages and inventories were obtained from DRI. Fixed private domestic capital comes from the *Survey of Current Business* (1990) p. 31.

[11]We find

$$\log(\lambda_t) = \underset{(0.101)}{-0.169415} + \underset{(0.054)}{0.911641} \log(\lambda_{t-1})$$

where standard deviations are in parenthesis. $\sigma_\epsilon = 0.03346789$, $\bar{R}^2 = 0.88$ and $DW = 1.9$.

[12]Data simulated by using the estimated transition probability matrix indicated a high degree of accuracy of the approximation.

[13]We set the capital share parameter α to match the estimated mean of $1 - X_t/Y_t$. The depreciation rate is approximately equal to the implicit depreciation rate obtained from regressing the capital stock series from *Department of Commerce (1987)* on itself lagged one period and the corresponding investment series.

of our calibration exercise, we also calculated some moments of the investment-capital ratios obtained from the data. Those statistics are given in Table 1.

Mean	STD	corr(i_t,i_{t-1})	min	max
0.0863	0.0103	0.4389	0.0615	0.1102

Table 1: Statistics of actual i_t

In Table 3 we present the equilibrium investment-capital ratios associated with the mean, minimum and maximum value of the productivity shock (λ_t) for each pair (γ, η). Similarly, in Table 4 we report the standard deviation of i_t. Notice that for sensible levels of risk aversion and adjustment costs it is possible to replicate the empirical mean value of the investment–capital ratio. Table 3 suggests that a value of γ between 0.7 and 0.9 can be considered as consistent with the data on investment and capital provided the adjustment cost parameter η is equal to 15. Smaller γ's would be acceptable for η between 25 and 35.

As Table 2 documents, the investment-capital ratio associated with a value for γ of 0.7 and values for η between 15 and 35 implies before tax adjustment costs of investment in a range between 5% and 10% of the total investment expenses when evaluated at the observed mean of the investment–capital ratio. This magnitude is below the ones found in many econometric estimations [e.g., Hayashi (1982), Salinger and Summers (1983) and, more recently, Schaller (1990)]. However, there is a general consensus that the estimations in those studies imply implausibly large losses of GNP.

Less satisfactory is the ability of our model to match the standard deviation of the observed data for reasonable parameter values. The model generates investment-capital ratios which are systematically less volatile than the observed data. This fact comes from the relative smoothness

$i_t \vert \eta$	15	25	35
0.07	3.7	4.9	6.1
0.08	4.8	6.4	8.0
0.09	6.1	8.1	10.1
0.10	7.5	10.0	12.5
0.11	9.1	12.1	15.1

Table 2: Adjustment costs in percent: $100 \cdot (\eta/2) \cdot i_t^2$

of the marginal productivity in relation to the investment series. As a consequence, our 10-state Markov process is able to collect only 85% of the observed rates of growth of capital in the last 40 years instead of the theoretical 99%. However 100% of the investment–capital ratios empirically observed in the last 10 years are within the minimum and maximum of the model's equilibrium values for those combinations of η and γ that best fit the mean value.

Table 5 shows that the equilibrium investment--capital ratio (i_t), and therefore the growth rate of the economy ($i_t - \delta$), is negatively related to the parameter γ. This effect is due to both the riskiness of the real investment and the agent's unwillingness to save large amounts when his elasticity of intertemporal substitution ($1/\gamma$) is low[14]. Notice that small changes in γ cause significant changes in the growth rate of the economy. For instance when $\eta = 25$ the mean of the equilibrium stationary distribution of the growth rate of capital goes from 4.9% if $\gamma = 0.5$ to 2.2% if $\gamma = 1.3$.

The adjustment cost parameter also affects negatively the equilibrium investment-capital ratios. Confirming the analysis made in section 2 for the simple deterministic model with linear adjustment costs, the magnitude of the negative effect of the adjustment costs is positively related to the elasticity of intertemporal substitution ($1/\gamma$). Changing the adjustment cost parameter (η) from 15 to 35 implies changes in the expected equilibrium growth rates that range from 2 points if $\gamma = 0.5$ to 0.7 points[15] if $\gamma = 1.3$. Thus, by modifying the relative price of investment with respect to consumption, the presence of adjustment costs of investment constitutes an important element in the determination of the rate of growth of the economy.

6 Tax Incidence Analysis

The general equilibrium model presented in this paper provides a general framework in which one can analyze tax effects. In the previous section we saw that the model predicts equilibrium investment-capital ratios which imply positive growth rates for the economy. Therefore, the value

[14] The equivalence of the coefficient of risk aversion and the inverse of the elasticity of intertemporal substitution for the assumed preference structure makes it impossible to distinguish between those two effects. However, work with a generalized preference structure suggests that unlike the portfolio choice, the consumption-saving decision is mainly characterized by the elasticity of intertemporal substitution for empirically sensible specifications of asset returns. (See Essay 1.)

[15] According to Table 2 this change implies changes in the relative size of the adjustment costs over the total before-tax investment expenses of 6 points if $\gamma = 0.5$ and 1.4 points if $\gamma = 1.3$.

of the firm, welfare and taxes are non-stationary variables. However, we have seen in Section 3 that the ratio of those variables with respect to capital (or a power of capital) is stationary in equilibrium. Thus, we can distinguish between static effects characterized by the change in the normalized variables and dynamic effects produced by the variations in the growth rate of the economy.

In Table 5 we present the expected equilibrium values for the investment capital ratio (i), value per capital (V/K), normalized welfare ($W/K^{1-\gamma}$) and government revenue per unit of capital (T/K) associated with the base case. T/K is computed by evaluating tax revenues at the equilibrium investment-capital ratios

$$\frac{T}{K}(\lambda_r) = \frac{\lambda_r u}{\alpha} - u\delta^T - (k + u\phi(i(\lambda_r))i(\lambda_r) \qquad r = 1, \cdots, m. \tag{46}$$

6.1 Change in the Corporate Tax Rate

In order to evaluate the sensitivity of the equilibrium values of the variables with respect to changes in the corporate tax rate, we obtained the equilibrium investment-capital ratios keeping all parameters as before but decreasing u from 0.5 to 0.4.

The results of a ten point decrease in the corporate tax rate (reported in Tables 6 and 7) show a moderate increase of the equilibrium investment--capital ratio in a range between 3.5% and 6.5%. Therefore, the effect of the tax-change on the after tax-marginal productivity of capital is slightly stronger than the effect of the increase in the after tax value of the adjustment costs on investment. The change is proportionally higher with low risk aversion and higher values of the adjustment cost parameter (η). Overall, the results imply an increase of the expected rate of growth of the economy of about one half of a percentage point.

The value of the firm per unit of capital increases also moderately. However, in this case, the relative change is larger for low risk aversion and high adjustment costs. The estimated mean effect is now more sensitive to the size of η and γ and ranges from 2.3% to 7.2%.

We saw in Section 4 that changes in the investment-capital ratio affect both the rateof growth of the value of the firm and the equilibrium value of the market value per unit of capital. Neither effect is considered in the traditional neoclassical Q models. Since the expected rate of growth of the economy increases by approximately 0.5 points, this is exactly the expected increase in the rate of growth of the market valuation of the firm produced by the change in the tax code. Yet, as Table 7 shows, keeping the investment-capital ratio at its initial level, the effect of the change in the corporate tax rate on value per unit of capital is about 50% smaller than the one obtained taking account of changes in the optimal investment policies. This constitutes a measure of the underestimation of the long-run incidence of changes in u on the market value of the firms per unit of capital under the standard neoclassical Q-models.

Finally, we study the effect of a decrease in u on total tax revenues. The effect on government's revenue is substantial, but relatively homogeneous, across values of risk aversion and adjustment cost parameters. The expected tax revenues per unit of capital decreases between 21% and 23%. Since tax revenue normalized by capital is a stationary variable in equilibrium, this variable grows over time at the same expected rate as capital. One can wonder how much time the new economy needs to catch up with the path of government revenues associated with the old economy characterized by larger taxes but smaller growth (Laffer effect). According to Table 7 and Figure 5, the Laffer effect is only perceivable after a period not smaller than 39 years for the 10 point decrease in the corporate tax rate. This striking result is due to both the important loss of revenues per unit of capital, and the moderate impulse in the economy produced by the tax change.

6.2 Change in the Investment Tax Credit

In Tables 8 and 9 we report the value, and percentage changes of the different variables as the tax parameter k decreases from 0.1 to 0.

As expected, we observe a decrease in the equilibrium investment–capital ratios. The effect is important in relative terms, and is negatively related to both the adjustment cost parameter (η) and the relative risk aversion (γ). The relative change in investment-capital ratios varies widely with the choice of parameters in a range between 8.5% and 15.9%. Thus, the implied effect on the expected rate of growth of the economy is between 0.6 and 1.7 points. In absolute terms, for the cases closest to the empirically observed investment-capital ratio, the depressing effect on the rate of growth of capital implies slightly over a 1 point reduction in the economy's growth rate.

(For the ($\gamma = .7$, $\eta = 25$) case, the economy's average growth rate changes from 3.7% to 2.6%.)

The changes in value per unit of capital display the effects we described in Section 4. A decrease in the investment tax credit increases the market value per unit of existing capital due to the increase in the relative price of installed capital. However, since the equilibrium investment-capital ratio decreases as a consequence of the elimination of the tax incentive, according to (35), the initial positive change of value is partially offset by this endogenous growth effect. Thus, the net expected percentage change in value per unit of capital varies from 2.9% to 8.3%. Table 7 shows the mean percentage effect on value we would have obtained without considering the offsetting endogenous growth effect. This ranges from 8.9% to 10.3%. That data implies that the standard Q-model would have overestimated the absolute effect of the change of the investment tax credit on the market value of the firm per unit of capital by a magnitude between 17% and 73%.

The dynamic effect on value of changes in the investment tax credit is also very striking. In a standard analysis with neoclassical Q models, the long run situation is characterized by the steady state corresponding to the new tax code. After a decrease in k, the new steady state unambiguously implies higher levels of the market value per unit of capital. The only way to interpret this result in the neoclassical framework is that the tax change permanently depresses the steady state level of both the capital stock of the economy and the market value of the firms. In our model, the change in the market value per unit of capital does not require that unattractive interpretation and is compatible with positive growth in both value and capital. Thus, the elimination of the investment tax credit immediately increases the market valuation of the already installed capital. However, the long-run effect on value is negative as long as the tax change reduces the growth rate of that variable. As Table 9 shows, the mean path of the value of the firm intersects the one associated with the new tax code after a period of 4 to 14 years, depending on the values of γ and η. In order to illustrate this effect graphically we present in Figure 2 the mean, min and max path of the value of the firm before and after the tax reform for the empirically relevant case of $\gamma = 0.7$ and $\eta = 25$. The expanding effect of the elimination of the investment tax credit on value lasts from 6.5 to 7.5 years with a probability of 99%. Thus, the elimination of the investment tax credit will negatively affect the market valuation of the firm in the long-run by reducing its expected growth rate. This negative long-run effect on value is also obtained in the neoclassical models, but only associated with a decrease in the level of the capital stock of the economy.

A similar exercise can be done with the changes in total tax revenues. In Table 9 we find that the reduction in the investment tax credit creates an expected increase in tax revenues between 6.3% and 11.4%. We also find that this expanding effect lasts only between 7.5 and 11.6 years due to the reduced rate of growth of the economy and the moderate initial increase in government revenues. In Figure 6, we show for the $(\gamma,\eta) = (0.7, 25)$ case the expanding effect lasts between 8.4 and 8.5 years with a probability of 99%.

Therefore, compared to the endogenous growth approach, the analysis of changes in the investment tax credit under exogenous growth models implies a substantial overvaluation of the effects on the market value of the firm per unit of capital and a much less empirically sensible interpretation of the long-run effects on the market value of the firm and total tax revenues.

6.3 Comparison of Tax Instruments

We have demonstrated the effects on growth, market value of the firm and total tax revenue of changes in u and k that imitated the ones of the 1986 Tax Reform Act. Since the utility function chosen has only private consumption as an argument, welfare analysis of different tax reforms will always be biased against a reform that increases the fiscal burden. This problem is eliminated in the following exercise where we set tax parameters in such a way that tax revenues remain the same.

In order to compare in a more homogeneous way the distorting effects of both tax instruments, we recalculate the equilibrium value of the variables for the case where the corporate tax rate is set to match T/K obtained from decreasing k from 0.1 to 0. Since under both tax reforms we will be subtracting the same amount of resources from the private sector, we can make comparisons in terms not only of value or growth, but also of welfare[16].

First, comparison of Table 5 with Tables 8 and 10 shows that the increase in the corporate tax rate provokes less slow-down of the economy than the reduction in the investment tax credit.

[16] Notice, however, that total tax revenues are only equal as a percentage of the economy's output (or capital stock).

The difference is of the order of 1 percentage point. Second, the effect on market value per unit of capital is dramatically different. While the decrease in the investment tax credit increases the value of the firm for up to 8 years, the increase of the corporate tax rate reduces it permanently. However, as a consequence of the different effects on the equilibrium expected growth rates of the economy, in the long run, the change in k would be more negative for the market value of the firm. Finally, the welfare loss of the reduction of the investment tax credit is, in general, significantly higher than the welfare effect of an increase in the corporate tax rate. The total magnitude of this difference is however very sensitive to the value of the coefficient of relative risk aversion, being almost negligible for γ sufficiently close to 1.

Those results suggest that the investment tax credit is a much more powerful instrument than the corporate tax rate for providing an incentive to invest. This incentive has positive short-term effects on welfare and long-term effects on the market value of the firms that are stronger than the ones associated with an increase in the corporate tax rate. Conversely, an increase in the corporate tax rate has a lower impact on the economy's growth rate than a reduction in the investment tax credit and provides a higher increase in government revenues. Thus, the corporate tax has less distorting effects on the economy and is a more powerful tool for raising funds from the private sector.

7 Conclusions

In this paper, we present a stochastic endogenous growth model with adjustment costs. We show that the inclusion of adjustment costs in a stochastic framework constitutes a relevant generalization of the existing endogenous growth models for, at least, two reasons. First, the presence of small adjustment costs is empirically relevant for explaining the growth rate of the economy. Second, it allows us to relate growth to asset prices in a general equilibrium endogenous growth generalization of the Q-theory of investment. In particular, it is possible to obtain substantial asset pricing implications from the analysis of growth incentives in market economies. The model is used to analyze the incidence of taxes in the economy from a double perspective. On one side, we evaluate the role of adjustment costs in analyzing tax effects on growth. On the other side, we investigate how asset prices respond, in the long-run, to changes in taxes when the optimal investment policy is endogenously determined.

The results show that moderate to small adjustment costs play a role in the determination of the growth rate of the economies comparable to the role played by attitudes toward intertemporal substitution. Moreover, adjustment costs constitute a force that moderates and eventually reverses the effects of traditional investment incentive policies. In the text, we considered two policy instruments: the corporate tax and the investment tax credit. A change in those fiscal instruments that necessarily has a positive effect on the profitability of investment in an economy without adjustment costs increases the magnitude of the effective adjustment costs, offsetting the expanding effect on growth. This effect is produced either by a decrease in the tax-deductibility of adjustment costs or by the reduction in the value of installed capital in terms of non-installed capital. Empirical simulations show that the magnitude of the offsetting effect is significant and positively related to the elasticity of intertemporal substitution.

The theoretical and numerical analysis of tax-incidence on the market price of the firm underlies the important role played by the endogeneity of the growth rate in dealing with long-run effects on market value. The responsiveness of the optimal investment policies to changes in taxation provokes two effects. On one side it modifies the equilibrium level of the market value per unit of capital. This effect significantly increases the effect of the corporate tax rate on the market value and reduces the influence of the investment tax credit. Due to this effect, the standard neoclassical models produce an underestimation of the effects of changes in the corporate tax rate and a severe overestimation of the effects of changes in the investment tax-credit compared to the endogenous growth approach. On the other side, unlike in the neoclassical Q-models, the change in the optimal investment policy produced by the changes in taxation alters the growth rate of the market price of the firms. Therefore, by ignoring the endogeneity of the economy's growth rate, the standard neoclassical long-run policy analysis produces an inaccurate evaluation of the effects of changes in taxation on the market value of the firms and a misleading description of the dynamic long-run effects of the tax-policy.

APPENDIX A

Proof of Proposition 1

First, define $B(\lambda,i) = (1-u)\lambda/\alpha + u\delta^T - (1-k+\Phi(i))i$. Since B is strictly decreasing in i for each λ, there exists a function $i(\lambda) = b[B(i(\lambda),\lambda),\lambda]$ implicitly defined.

Define the family of intervals $S_r = [g, b(0,\lambda_r)]$ for $r = 1,\cdots,m$. Notice that $H(\lambda_r,i_r)$ is continuous and monotonically increasing in i_r for a given λ_r and $i_r \in S_r$. Therefore, we can define a function $i(\lambda_r) = M[H(\lambda_r, i(\lambda_r)), \lambda_r]$.

Now consider the set $S = S_1 \times S_2 \times \cdots \times S_m$ and the operator T on S where

$$Ti_r = M\left[\sum_{s=1}^{m} \Pi_{rs} G(\lambda_s, i_s), \lambda_r\right]. \tag{47}$$

It may be seen immediately that there exists a solution to (23) if and only if T has a fixed point in S. By Brouwer's theorem it suffices to show that T maps S onto itself, and that S is closed, bounded and convex.

Notice that T is increasing in S by [A.1] and [A.2] and that $Tg \geq g$ by [A.2]. T is also bounded from above by $Tb(0,\lambda)$. Therefore, T maps S onto itself. Finally, since the cartesian product of closed, bounded and convex sets is also closed, bounded and convex, S satisfies trivially that condition. **Q.E.D.**

Proof of Proposition 2

To simplify notation let $\hat{\imath} \equiv i(\lambda_{t+1})$ and let $Q(i) = 1 - k + \Phi'(i)i + \Phi(i)$ so that

$$\begin{aligned} R_t^I(a_{t+1}) &\equiv \left\{F_K(a_{t+1})(1-u) + u\delta^T + \Phi'(\hat{\imath})\hat{\imath}^2 + (1-\delta)Q(\hat{\imath})\right\}/Q(i_t) & (48)\\ &= \left\{F_K(a_{t+1})(1-u)K_{t+1} - (1-k+\Phi(\hat{\imath}))\hat{\imath}K_{t+1} + u\delta^T + \right.\\ &\quad \left.(1-k+\Phi(\hat{\imath}))\hat{\imath}K_{t+1} + \Phi'(\hat{\imath})\hat{\imath}^2K_{t+1} + (1-\delta)Q(\hat{\imath}))K_{t+1}\right\} / [Q(i_t)K_{t+1}], & (49)\end{aligned}$$

where equality (48) is definitional. Equation (49) follows from the addition and subtraction of $(1-k+\Phi(\hat{\imath}))\hat{\imath}$ in the numerator and subsequent multiplication of the numerator and denominator of (48) by K_{t+1}. Now, note in the denominator of (49) that $V_t = Q(i_t)K_{t+1}$ from equation (34)).

Similarly, the first three terms in the numerator verify in equilibrium

$$F_K(a_{t+1})(1-u)K_{t+1} - [(1-k+\Phi(\hat{\imath}))\hat{\imath} + u\delta^T]K_{t+1} =$$

$$= \left[\lambda_{a_{t+1}}(1-u) - (1-k+\Phi(\hat{\imath}))\hat{\imath} + u\delta^T\right] K_{t+1} = D_{a_{t+1}}. \tag{50}$$

Finally, the last three terms verify

$$\begin{aligned}
(1-k+\Phi(\hat{\imath}))\hat{\imath}K_{t+1} + \Phi'(\hat{\imath})\hat{\imath}^2 K_{t+1} + (1-\delta)Q(\hat{\imath})K_{t+1} \\
= [1-k+\Phi(\hat{\imath}) + \Phi'(\hat{\imath})\hat{\imath}]\hat{\imath}K_{t+1} + (1-\delta)Q(\hat{\imath})K_{t+1} \\
= Q(\hat{\imath})\hat{\imath}K_{t+1} + (1-\delta)Q(\hat{\imath})K_{t+1} \\
= Q(\hat{\imath})[I_{t+1} + (1-\delta)K_{t+1}] \\
= Q(\hat{\imath})K_{t+2} \equiv V_{a_{t+1}}
\end{aligned}$$

and, therefore,

$$R_t^I(a_{t+1}) = \frac{D_{a_{t+1}} + V_{a_{t+1}}}{V_t} \equiv R_t(a_{t+1}). \tag{51}$$

Q.E.D.

Proof of Proposition 3

From the definition of contingent claim prices in (42), the value of the firm can be written as

$$\begin{aligned}
V(\lambda_t, K_t) = \sum_{s=1}^{+\infty} \pi_t(a_{t+s})\beta^s \left\{\frac{C_{t+s}}{C_t}\right\}^{-\gamma} D_{a_{t+s}} \\
= (1-\gamma)C_t^{-\gamma}W(\lambda_t, K_t) - C_t - E_t \sum_{s=1}^{+\infty} \left(\frac{C_{t+s}}{C_t}\right)^{-\gamma} X_{t+s},
\end{aligned} \tag{52}$$

where the sequence $\{C_{t+s}\}_{s=0}^{+\infty}$ satisfies the first order conditions of the consumer's maximization problem. Now, replace X_t by the equilibrium value of the marginal productivity of capital $((1-\alpha)\lambda_t/\alpha)$, divide both members of (52) by K_t and rearrange terms to find

$$W(\lambda_t, K_t) \quad = \quad m(\lambda_t)K_t^{1-\gamma}, \tag{53}$$

where

$$m(\lambda_t) = \frac{c(\lambda_t)^{-\gamma}}{1-\gamma}[q(\lambda_t)+c(\lambda_t)] + E_t\left\{\sum_{s=1}^{+\infty}\left(\frac{c_{t+s}}{c_t}\right)^{-\gamma}\left(\frac{K_{t+s}}{K_t}\right)^{1-\gamma}\frac{(1-\alpha)}{\alpha}\lambda_{t+s}\right\}.$$

Q.E.D.

APPENDIX B

Discrete Approximation of the Continuous Distribution of the Exogenous Technical Shock

We essentially follow Tauchen (1986) to obtain a discrete homogenous Markov process for λ. Consider a random variable X_t that follows the stochastic process

$$\begin{aligned} X_t &= \mu + \rho X_{t-1} + \epsilon_t \\ \epsilon_t &\sim \mathrm{N}(0,\sigma_\epsilon^2). \end{aligned} \tag{54}$$

By the assumption of normality it is easy to show that X_t satisfies

$$X_t \sim \mathrm{N}\left(\frac{\mu}{1-\rho},\frac{\sigma_\epsilon^2}{1-\rho^2}\right) \equiv \mathrm{N}(\mu_X,\sigma_X^2)$$

and

$$X_t|X_{t-1} \sim \mathcal{N}(\mu+\rho X_{t-1},\sigma_\epsilon^2).$$

For numerical purposes it is possible to bound the range of X_t by imposing the condition that the grid of states has to contain X_t with a probability of, say $1-\alpha$. If $c_{\frac{\alpha}{2}}$ is the $\frac{\alpha}{2}$ percentile, then a simple computation of confidence intervals yields that

$$\mu_X - c_{\frac{\alpha}{2}}\sigma_X \le X \le \mu_X + c_{\frac{\alpha}{2}}\sigma_X.$$

Define the upper and lower bounds for X as $a_L = \mu_X - c_{\frac{\alpha}{2}}\sigma_X$ and $a_U = \mu_X + c_{\frac{\alpha}{2}}\sigma_X$. It is then possible to construct a grid defining the various states of X. If we wish to have m states then defining $\zeta = (a_U - a_L)/(m-1)$, we get the m states from

$$S_j = a_L + \zeta(j-1) \qquad j = 1, \cdots, m. \tag{55}$$

To introduce transition probabilities later on, it is useful to define

$$\tau_j \;=\; a_L + \frac{\zeta}{2} j \qquad j = 1, \cdots, m-1. \tag{56}$$

Notice that the conditional distribution of X follows from (54) and is given by

$$X|S_j \sim \mathrm{N}(\mu + \rho X_j, \sigma_\epsilon^2). \tag{57}$$

Now, define the transition probability matrix $\{\Pi\}_{r,s=1}^{m}$ where $\Pi_{r,s} = \Pr(X_{t+1} = s|X_t = r)$ is the transition probability of going from state r to state s. By using (55), (56) and (57) we can approximate Π by setting

$$\Pi_{r,s} = \begin{cases} \Pr(X_s \leq \tau_1|S_r) & \text{if } s = 1 \\ \Pr(\tau_{s-1} \leq X_s \leq \tau_s|S_r) & \text{if } s = 2, \cdots, m-1 \\ \Pr(X_s \geq \tau_{s-1}|S_r) & \text{if } s = m. \end{cases} \tag{58}$$

The unconditional probabilities are given by the solution to the system $\mu_s = \sum_{r=1}^{m} \mu_r \Pi_{r,s}$ for $s = 1, \cdots, m$.

APPENDIX C

Numerical Solution of the Equilibrium Investment Problem

Using the assumed functional forms for the consumer's preferences, the adjustment cost function, and the production function we can write the Euler equation (18) as

$$
\sum_{s=1}^{m} \quad \Pi_{r,s}\left\{ \beta \left[\frac{(1-u)\lambda_s/\alpha + u\delta^T - (1-k+(1-u)\frac{\eta}{2}i_s^2)i_s}{(1-u)\lambda_r/\alpha + u\delta^T - (1-k+(1-u)\frac{\eta}{2}i_r^2)i_r} \right] \quad (1-\delta+i_r)^{-\gamma} \cdot \right.
$$
$$
\left. \frac{(1-u)\lambda_s + u\delta^T + \frac{3\eta}{2}i_s^3 + (1-\delta)[1-k+(1-u)\frac{3\eta}{2}i_s^2]}{1-k+(1-u)\frac{3\eta}{2}i_r^2} \right\} = 1,
$$

which has to hold for all states $r = 1, \cdots, m$. By regrouping all variables which depend on s (or r) on the left (right) hand side, we obtain

$$
\sum_{s=1}^{m} \Pi_{rs} \left\{ \beta \left\{ (1-u)\lambda_s/\alpha + u\delta^T - (1-k+(1-u)\frac{\eta}{2}i_s^2)i_s \right\}^{-\gamma} \right.
$$
$$
\left. \left\{ (1-u)\lambda_s + u\delta^T + (1-u)\frac{\eta}{2}i_s^3 + (1-\delta)(1-k+(1-u)\frac{3\eta}{2}i_s^2) \right\} \right\}
$$
$$
= \left\{ \frac{(1-u)\lambda_r/\alpha + u\delta^T - (1-k+(1-u)\frac{\eta}{2}i_r)i_r}{1-\delta+i_r} \right\}^{-\gamma} \{1-k+(1-u)\frac{3\eta}{2}i_r^2\}.
$$

Equation (59) can be written as

$$
\sum_{s=1}^{m} \Pi_{r,s} G(\lambda_s, i(\lambda_s)) = H(\lambda_r, i(\lambda_r)) \quad r = 1, \cdots, m. \tag{59}
$$

A functional fixed point in equation (59) can then be seen as the solution of a system of non–linear equations. The numerically most trouble free method (see Press et al. (1988)) for solving (59) is to use an iterative minimum distance algorithm. Therefore, we solve the problem

$$
\min_{\{i(\cdot)\}} \sum_{r=1}^{m} a(i(\lambda_r))^2,
$$

where

$$
a(i(\lambda_r)) = \sum_{s=1}^{m} \Pi_{rs} \{G(i(\lambda_s))\} - H(i(\lambda_r)) \qquad r = 1, \cdots, m.
$$

To perform the minimization we use the *Optmum* procedure of the Gauss matrix language. Accuracy is set to 10^{-6} and on average computations took 8.66 seconds [17]. Gradients are computed numerically by using available Gauss subroutines.

[17] Performed on a 386/33Mhz IBM-PC clone.

APPENDIX D

Numerical Computation of the Indirect Utility Function

Numerical computations of the welfare function involve estimation of an infinite sum of expected future consumptions. In this section we consider technical aspects of how such evaluations can be performed.

First recall that we have to compute the following expression

$$W_t = E_t\Big\{\sum_{s=0}^{+\infty} \beta^s \frac{C_{t+s}^{1-\gamma}}{1-\gamma}\Big\}. \tag{60}$$

Define $c_s \equiv (C_s/K_s)^{1-\gamma}/(1-\gamma)$ as well as $\kappa_s \equiv \beta(K_{s+1}/K_s)^{1-\gamma} = \beta\ (1-\delta+i_s)^{1-\gamma}$ and notice that (60) can be rewritten as

$$\frac{W_t}{K_t^{1-\gamma}} = \sum_{s=t}^{+\infty} E_t\Big\{c_s \prod_{r=t}^{s-1} \kappa_r\Big\}, \tag{61}$$

where we have interchanged expectations and summations. This normalization is useful because we know that C_s/K_s and K_{s+1}/K_s are stationary variables depending only on the investment-capital ratio (i_s). The latter satisfies, in equilibrium, a one-to-one relation with the stochastic marginal productivity of capital. Taking expectations conditioned on one variable or the other is equivalent.

Even though each summand of (61) involves a complicated product of future realizations of the random variable i_s, each one can be computed by recalling the definition of a multivariate conditional expectation and the properties of a Markov chain. More precisely, we have

$$\begin{aligned} h(\lambda_t,s) &\equiv E_t\Big\{c_s \prod_{r=t}^{s-1} \kappa_r\Big\} = E_t\Big\{c_s\kappa_{s-1}\kappa_{s-2}\cdots\kappa_{t+1}\kappa_t\Big\} \\ &= \sum_{\lambda_s}\cdots\sum_{\lambda_{t+1}} c(\lambda_s)\kappa(\lambda_{s-1})\cdots\kappa(\lambda_{t+1})\kappa(\lambda_t)\Pr\Big[i_s = i(\lambda_s), i_{s-1} = i(\lambda_{s-1}), \\ &\quad \cdots, i_{t+1} = i(\lambda_{t+1}) \;\Big|\; i_t\Big] \qquad \forall s > t,. \end{aligned} \tag{62}$$

Since the underlying process (λ_t) is markovian, one can show (See Shiryayev (1984), page 109—

110) that

$$\Pr(i_s = i(\lambda_s), i_{s-1} = i(\lambda_{s-1}), \cdots, i_{t+1} = i(\lambda_{t+1}) \mid i_t)$$
$$= \Pr(i_s = i(\lambda_s)|i_{s-1} = i(\lambda_{s-1})) \cdots \Pr(i_{t+1} = i(\lambda_{t+1}) \Bigm| i_t), \tag{63}$$

so that the sum in (62) can be broken into products by using (63). Reordering summations we eventually get

$$h(\lambda_t, s) =$$
$$\sum_{\lambda_{t+1}} \cdots \Big(\sum_{\lambda_{s-1}} \Big(\sum_{\lambda_s} c(\lambda_s)\Pr(i_s = i(\lambda_s)|i_{s-1} = i(\lambda_{s-1}))\Big)\Pr(i_{s-1} = i(\lambda_{s-1})|i_{s-2} = i(\lambda_{s-2}))\Big)$$
$$\cdots \Pr(i_{t+1} = i(\lambda_{t+1})|i_t).$$

Such an expression can be computed recursively by starting with the innermost term [18]. From the time homogeneity property of the Markov chain, $h(\lambda_{t+1}, s+1) = h(\lambda_t, s)$. This implies

$$h(\lambda_t, s+1) = \sum_{\lambda_{t+1}} h(\lambda_{t+1}, s+1)\Pr(\lambda_{t+1}|\lambda_t)\kappa_t. \tag{64}$$

Expression (64) is numerically well behaved and converges steadily. Any arbitrary degree of accuracy in the calculation of W/K can be achieved by recursively computing a sufficiently large number of terms using (64).

[18] In a matrix language like Gauss or SAS–IML the evaluation of $\sum_{\lambda_s} c_s\Pr(i_s = i(\lambda_s)|i_{s-1} = i(\lambda_{s-1}))$ is tantamount to a multiplication of a (transition) matrix Π and a vector of consumption to capital ratios c: Πc.

$\gamma \vert \eta$	15	25	35
	0.110	**0.099**	**0.090**
0.5	0.093	0.085	0.079
	0.128	0.113	0.103
	0.094	**0.087**	**0.081**
0.7	0.078	0.073	0.069
	0.112	0.101	0.094
	0.086	**0.080**	**0.076**
0.9	0.070	0.066	0.064
	0.103	0.094	0.088
	0.080	**0.075**	**0.072**
1.1	0.065	0.062	0.060
	0.096	0.090	0.085
	0.076	**0.072**	**0.069**
1.3	0.061	0.059	0.057
	0.092	0.086	0.082

Table 3: Expectation (in boldface), minimum and maximum values of i (in the 99% confidence interval) for different adjustment costs (η) and different coefficients of risk aversion (γ). Other parameters: $u = 0.5$, $k = 0.1$, $\delta = 0.05$, $\delta^T = 0.1$.

$\gamma \vert \eta$	15	25	35
0.5	0.0077	0.0062	0.0053
0.7	0.0073	0.0061	0.0054
0.9	0.0070	0.0060	0.0054
1.1	0.0068	0.0060	0.0054
1.3	0.0067	0.0059	0.0054

Table 4: Standard deviation of i for given pairs of η and γ. Parameters are the same as in Table 3.

η	γ	i_t	V/K	$W/K^{1-\gamma}$	T/K
15	0.5	0.110	1.100	62.13	0.114
	0.7	0.094	1.046	67.81	0.118
	0.9	0.086	1.018	225.90	0.119
	1.1	0.080	1.001	-281.22	0.120
	1.3	0.076	0.990	-122.21	0.121
25	0.5	0.099	1.136	43.75	0.114
	0.7	0.087	1.081	63.68	0.117
	0.9	0.080	1.051	223.31	0.119
	1.1	0.075	1.033	-283.40	0.120
	1.3	0.072	1.020	-124.32	0.121
35	0.5	0.090	1.161	36.38	0.115
	0.7	0.081	1.107	60.93	0.117
	0.9	0.076	1.078	221.35	0.119
	1.1	0.072	1.059	-285.16	0.120
	1.3	0.069	1.046	-126.10	0.120

Table 5: Expectations of investment, value, welfare and taxes in terms of capital for various pairs of η and γ when the corporate tax rate (u) is set to **0.5** and the investment tax credit (k) is set to **0.1**.

η	γ	i_t	V/K	$W/K^{1-\gamma}$	T/K
	0.5	0.117	1.161	94.18	0.088
	0.7	0.100	1.086	74.45	0.091
15	0.9	0.090	1.050	231.32	0.093
	1.1	0.083	1.028	-275.48	0.094
	1.3	0.079	1.013	-115.44	0.095
	0.5	0.105	1.211	56.08	0.088
	0.7	0.091	1.134	69.10	0.091
25	0.9	0.084	1.094	228.29	0.093
	1.1	0.078	1.069	-277.87	0.094
	1.3	0.075	1.052	-117.66	0.095
	0.5	0.096	1.244	43.96	0.088
	0.7	0.085	1.169	65.65	0.091
35	0.9	0.079	1.129	226.04	0.093
	1.1	0.075	1.103	-279.79	0.094
	1.3	0.072	1.085	-119.52	0.094

Table 6: Expectations of investment, value, welfare and taxes in terms of capital for various pairs of η and γ when the *corporate tax rate (u) is* 0.4 and the investment tax credit (k) remains at 0.1.

η	γ	i_t	V/K	$W/K^{1-\gamma}$	T/K	$\dagger V/K$	$t_{V/K}$	$t_{W/K}$	$t_{T/K}$
	0.5	6.55	5.53	51.59	-23.32	2.65	N-C	N-C	39.1
	0.7	5.40	3.88	9.80	-22.42	2.02	N-C	N-C	52.1
15	0.9	4.76	3.10	2.40	-22.02	1.69	N-C	N-C	63.3
	1.1	4.30	2.63	2.04	-21.79	1.49	N-C	N-C	73.9
	1.3	3.93	2.31	5.53	-21.64	1.35	N-C	N-C	84.1
	0.5	6.14	6.59	28.19	-23.10	3.39	N-C	N-C	45.6
	0.7	5.07	4.89	8.52	-22.32	2.73	N-C	N-C	59.5
25	0.9	4.47	4.04	2.23	-21.96	2.37	N-C	N-C	71.6
	1.1	4.03	3.50	1.95	-21.74	2.13	N-C	N-C	82.9
	1.3	3.69	3.14	5.35	-21.60	1.97	N-C	N-C	93.7
	0.5	5.85	7.21	20.86	-22.89	3.86	N-C	N-C	51.3
	0.7	4.85	5.58	7.74	-22.22	3.24	N-C	N-C	65.9
35	0.9	4.27	4.72	2.12	-21.90	2.87	N-C	N-C	78.6
	1.1	3.85	4.17	1.88	-21.70	2.63	N-C	N-C	90.6
	1.3	3.52	3.78	5.22	-21.57	2.46	N-C	N-C	102.0

Table 7: Percentage changes of expectations of economic variables for various pairs of η and γ when the *corporate tax rate decreases from* $u = 0.5$ *to* $u = 0.4$, (k remaining at 0.1). $\dagger V/K$ is the percentage change in V/K keeping I/K constant. The last 3 columns indicate after how many years both tax codes yield the same level of a given variable. If the change is such that levels do not cross this is indicated by N-C.

η	γ	i_t	V/K	$W/K^{1-\gamma}$	T/K
	0.5	0.093	1.144	37.40	0.127
	0.7	0.082	1.110	60.53	0.128
15	0.9	0.075	1.092	220.49	0.129
	1.1	0.071	1.080	-286.41	0.129
	1.3	0.068	1.072	-127.80	0.129
	0.5	0.084	1.174	32.03	0.126
	0.7	0.076	1.139	58.16	0.127
25	0.9	0.071	1.119	218.72	0.128
	1.1	0.068	1.107	-288.05	0.128
	1.3	0.066	1.098	-129.49	0.128
	0.5	0.078	1.194	29.03	0.126
	0.7	0.072	1.161	56.46	0.127
35	0.9	0.068	1.142	217.31	0.127
	1.1	0.065	1.129	-289.42	0.128
	1.3	0.063	1.121	-130.94	0.128

Table 8: Expectations of investment, value, welfare and taxes in terms of capital for various pairs of η and γ when the corporate tax rate (u) is set to **0.5** and the *investment tax credit (k) is* **0.0**.

η	γ	i_t	V/K	$W/K^{1-\gamma}$	T/K	$\dagger V/K$	$t_{V/K}$	$t_{W/K}$	$t_{T/K}$
	0.5	-15.86	4.05	-39.80	11.44	9.64	3.4	N-C	7.5
	0.7	-13.50	6.15	-10.74	8.99	9.99	5.9	N-C	8.0
15	0.9	-12.00	7.20	-2.39	7.81	10.17	8.0	N-C	8.5
	1.1	-10.85	7.85	-1.84	7.11	10.28	9.9	N-C	9.1
	1.3	-9.92	8.30	-4.58	6.63	10.36	11.8	N-C	9.7
	0.5	-14.45	3.28	-26.78	10.62	9.23	3.4	N-C	8.4
	0.7	-12.34	5.35	-8.67	8.54	9.59	6.0	N-C	8.9
25	0.9	-10.98	6.44	-2.06	7.52	9.80	8.3	N-C	9.5
	1.1	-9.94	7.14	-1.64	6.89	9.93	10.4	N-C	10.1
	1.3	-9.09	7.63	-4.16	6.46	10.02	12.4	N-C	10.7
	0.5	-13.45	2.92	-20.20	9.88	8.96	3.4	N-C	9.0
	0.7	-11.52	4.86	-7.34	8.14	9.31	6.2	N-C	9.6
35	0.9	-10.24	5.93	-1.83	7.25	9.51	8.6	N-C	10.2
	1.1	-9.27	6.64	-1.49	6.69	9.65	10.8	N-C	10.9
	1.3	-8.48	7.14	-3.84	6.30	9.74	13.0	N-C	11.6

Table 9: Percentage changes of expectations of economic variables for various pairs of η and γ when the *investment tax credit decreases from k* **=0.1** *to k* **=0.0**, (u remaining at **0.5**). $\dagger V/K$ is the percentage change in V/K keeping I/K constant. The last 3 columns indicate after how many years both tax codes yield the same level of a given variable. If the change is such that levels do not cross this is indicated by N-C.

η	γ	u	i_t	V/K	$W/K^{1-\gamma}$	T/K
	0.5	0.55	0.107	1.074	52.39	0.127
	0.7	0.54	0.092	1.031	65.25	0.128
15	0.9	0.54	0.084	1.008	223.91	0.129
	1.1	0.53	0.079	0.993	-283.25	0.129
	1.3	0.53	0.075	0.984	-124.51	0.129
	0.5	0.55	0.096	1.106	39.32	0.126
	0.7	0.54	0.085	1.062	61.65	0.127
25	0.9	0.53	0.079	1.038	221.54	0.128
	1.1	0.53	0.074	1.022	-285.29	0.128
	1.3	0.53	0.071	1.011	-126.53	0.128
	0.5	0.54	0.088	1.129	33.59	0.126
	0.7	0.54	0.080	1.087	59.23	0.127
35	0.9	0.53	0.075	1.062	219.73	0.127
	1.1	0.53	0.071	1.046	-286.95	0.128
	1.3	0.53	0.068	1.035	-128.23	0.128

Table 10: Column 3 represents the value of u when $k = 0.1$ which would have yielded the same tax income as when $u = 0.5$ and $k = 0.0$. The other columns present expectations of investment, value, welfare and taxes in terms of capital for those u and $k = 0.1$.

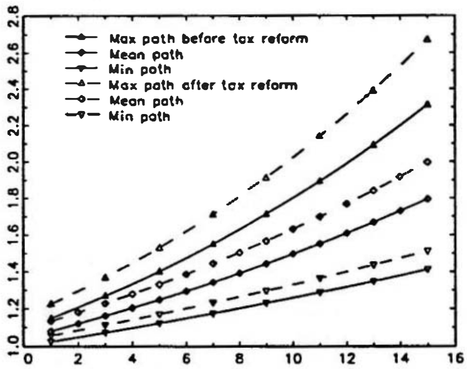

Figure 1: V_t in levels. Corporate tax rate changes from u =0.5 to u =0.4; k=0.1; $\eta = 25$; $\gamma = 0.7$.

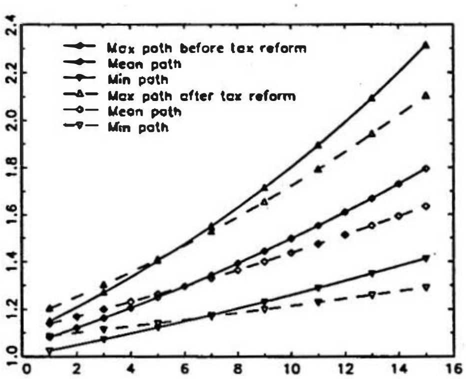

Figure 2: V_t in levels. Investment tax credit changes from k =0.1 to k =0; u=0.5; $\eta = 25$; $\gamma = 0.7$.

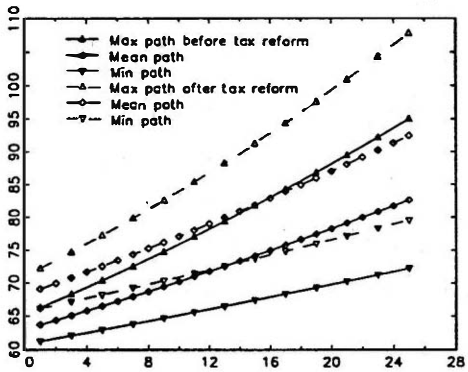

Figure 3: W_t in levels. Corporate tax rate changes from u =0.5 to u =0.4; k=0.1; $\eta = 25$; $\gamma = 0.7$.

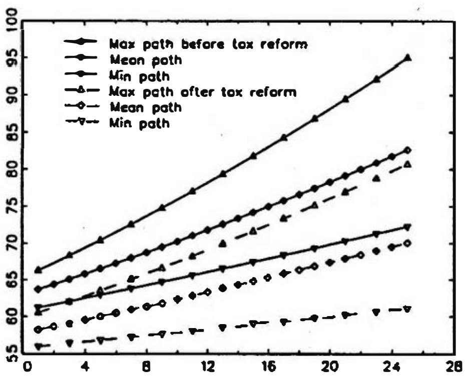

Figure 4: W_t in levels. Investment tax credit changes from k =0.1 to k =0; u=0.5; $\eta = 25$; $\gamma = 0.7$.

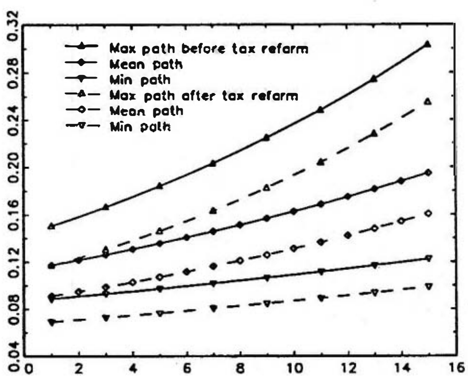

Figure 5: T_t in levels. Corporate tax rate changes from u =0.5 to u =0.4; k=0.1; $\eta = 25$; $\gamma = 0.7$.

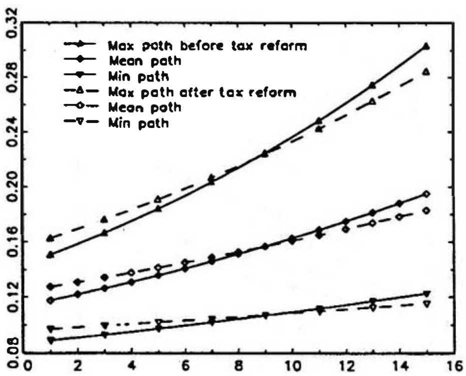

Figure 6: T_t in levels. Investment tax credit changes from k =0.1 to k =0; u=0.5; $\eta = 25$; $\gamma = 0.7$.

REFERENCES

ABEL, A., 1982, "Dynamic Effects of Permanent and Temporary Tax Policies in a q Model of Investment", *Journal of Monetary Economics*, 353-373.

AUERBACH, A. J. AND L. J. KOTLIKOFF, 1987, *Dynamic Fiscal Policy*, Cambridge University Press, Cambridge.

BARRO, R. J., 1990, "Government Spending in a Simple Model of Endogenous Growth", *Journal of Political Economy*.

BARRO, R. J. AND G. BECKER, 1989, "Fertility Choice in a Model of Economic Growth", *Econometrica*, 57, 481–501.

BRAUN, PH. A., 1990, "Asset Pricing and Capital Investment: Theory and Evidence", Manuscript, University of Chicago.

COCHRANE, J. H., 1991, "Production-Based Asset Pricing and the Link Between Stock Returns and Economic Fluctuations", *Journal of Finance*, 46, 209–237.

DUFFIE, D. AND CHI-FU HUANG, 1985, "Implementing Arrow–Debreu Equilibria by Continuous Trading in a Few Long–Lived Securities", *Econometrica*, November, 53, 1337–1356.

GOULDER, AND L. SUMMERS, 1986, "Tax Policy, Asset Prices and Growth: A General Equilibrium Analysis", NBER working paper.

HAYASHI, F., 1982, " Tobin's Marginal q and Average q: A Neoclassical Interpretation", *Econometrica*, 50, 213–224.

KING, R. G. AND CH. I. PLOSSER AND S. T. REBELO, 1988, "Production, Growth and Business Cycles; II. New Directions", *Journal of Monetary Economics*, 21, 309–341.

LUCAS, ROBERT E. JR., 1988, "On the Mechanics of Economic Development", *Journal of Monetary Economics*, 22, 3–42.

MEHRA, R. AND E. C. PRESCOTT, 1985, "The Equity Premium Puzzle", *Journal of Monetary Economics*, 15, 145-161.

PRESS, W. H., B. P. FLANNERY, S. A. TEUKOLSKY AND W. T. VETTERLING, 1988, *Numerical Recipes in C: The Art of Scientific Computing*, Cambridge University Press, Cambridge.

REBELO, S., 1991, "Long-Run Policy Analysis and Long-Run Growth", *Journal of Political Economy*, 99, 500-521.

RESTOY, F. AND M. ROCKINGER, 1991, "On Stock Market Returns and Returns on Investment", Manuscript, Harvard University.

ROMER, P. M., 1986, "Increasing Returns and Long-Run Growth", *Journal of Political Economy*, 94, Oct. 1002-1037.

ROMER, P. M., 1989, "Capital Accumulation in the Theory of Long-Run Growth", in Robert Barro ed., *Modern Business Cycle Theory,* Harvard University Press, Cambridge, MA.

SALINGER, M. A. AND L. SUMMERS, 1983, "Tax Reform and Corporate Investment: A Microeconomic Simulation Study", in Martin Feldstein ed., *Behavioral Simulation Methods in Tax Policy Analysis*, University of Chicago Press, Chicago, IL.

SCHALLER, H., 1990, "A Re-Examination of the Q Theory of Investment using U.S. Firm Data", *Journal of Applied Econometrics*, 5, 309–325.

SHIRYAYEV, A. N., 1984, *Probability*, Springer-Verlag, New York.

STOKEY, N. L. AND R. E., JR. LUCAS, 1989, *Recursive Methods in Economic Dynamics*, Harvard University Press, Cambridge, Massachusetts.

SUMMERS, L. H., 1982, "Taxation and Corporate Investment: A q-Theory Approach", *Brooking Papers on Economic Activity*, 67–140.

TAUCHEN, G., 1986, "Finite State Markov–Chain Approximations to Univariate and Vector Autoregressions", *Economic Letters*, 177–181.

TOBIN, J., 1969, "A General Equilibrium Approach to Monetary Theory", *Journal of Money, Credit, and Banking*, 1, 15–29.

TREADWAY, A. B., 1969, "On Rational Entrepreneurial Behavior and the Demand for Investment", *Review of Economic Studies*, 36, 227–239.

U.S. DEPT OF COMMERCE, 1990, Survey of Current Business.

U.S. DEPARTMENT OF COMMERCE, 1987, "Fixed Reproducible Tangible Wealth in the United States."

DOCUMENTOS DE TRABAJO (1)

8801 **Agustín Maravall:** Two papers on ARIMA signal extraction.

8802 **Juan José Camio y José Rodríguez de Pablo:** El consumo de Alimentos no elaborados en España: Análisis de la información de MERCASA.

8803 **Agustín Maravall and Daniel Peña:** Missing Observations in Time Series and the «DUAL» Autocorrelation Function.

8804 **José Viñals:** El Sistema Monetario Europeo. España y la política macroeconómica. (Publicada una versión en inglés con el mismo número.)

8805 **Antoni Espasa:** Métodos cuantitativos y análisis de la coyuntura económica.

8806 **Antoni Espasa:** El perfil de crecimiento de un fenómeno económico.

8807 **Pablo Martín Aceña:** Una estimación de los principales agregados monetarios en España: 1940-1962.

8808 **Rafael Repullo:** Los efectos económicos de los coeficientes bancarios: un análisis teórico.

8901 **Mª de los Llanos Matea Rosa:** Funciones de transferencia simultáneas del índice de precios al consumo de bienes elaborados no energéticos.

8902 **Juan J. Dolado:** Cointegración: una panorámica.

8903 **Agustín Maravall:** La extracción de señales y el análisis de coyuntura.

8904 **E. Morales, A. Espasa y M. L. Rojo:** Métodos cuantitativos para el análisis de la actividad industrial española. (Publicada una edición en inglés con el mismo número.)

9001 **Jesús Albarracín y Concha Artola:** El crecimiento de los salarios y el deslizamiento salarial en el período 1981 a 1988.

9002 **Antoni Espasa, Rosa Gómez-Churruca y Javier Jareño:** Un análisis econométrico de los ingresos por turismo en la economía española.

9003 **Antoni Espasa:** Metodología para realizar el análisis de la coyuntura de un fenómeno económico. (Publicada una edición en inglés con el mismo número.)

9004 **Paloma Gómez Pastor y José Luis Pellicer Miret:** Información y documentación de las Comunidades Europeas.

9005 **Juan J. Dolado, Tim Jenkinson and Simon Sosvilla-Rivero:** Cointegration and unit roots: A survey.

9006 **Samuel Bentolila and Juan J. Dolado:** Mismatch and Internal Migration in Spain, 1962-1986.

9007 **Juan J. Dolado, John W. Galbraith and Anindya Banerjee:** Estimating euler equations with integrated series.

9008 **Antoni Espasa y Daniel Peña:** Los modelos ARIMA, el estado de equilibrio en variables económicas y su estimación. (Publicada una edición en inglés con el mismo número.)

9009 **Juan J. Dolado and José Viñals:** Macroeconomic policy, external targets and constraints: the case of Spain.

9010 **Anindya Banerjee, Juan J. Dolado and John W. Galbraith:** Recursive and sequential tests for unit roots and stuctural breaks in long annual GNP series.

9011 **Pedro Martínez Méndez:** Nuevos datos sobre la evolución de la peseta entre 1900 y 1936. Información complementaria.

9101 **Javier Valles:** Estimation of a growth model with adjustment costs in presence of unobservable shocks.

9102 **Javier Valles:** Aggregate investment in a growth model with adjustment costs.

9103 **Juan J. Dolado:** Asymptotic distribution theory for econometric estimation with integrated processes: a guide.

9104 **José Luis Escrivá y José Luis Malo de Molina:** La instrumentación de la política monetaria española en el marco de la integración europea. (Publicada una edición en inglés con el mismo número.)

9105 **Isabel Argimón y Jesús Briones:** Un modelo de simulación de la carga de la deuda del Estado.

9106 **Juan Ayuso:** Los efectos de la entrada de la peseta en el SME sobre la volatilidad de las variables financieras españolas. (Publicada una edición en inglés con el mismo número.)

9107 **Juan J. Dolado y José Luis Escrivá:** La demanda de dinero en España: definiciones amplias de liquidez. (Publicada una edición en inglés con el mismo número.)

9108 **Fernando C. Ballabriga:** Instrumentación de la metodología VAR.

9109 **Soledad Núñez:** Los mercados derivados de la deuda pública en España: marco institucional y funcionamiento.

9110 **Isabel Argimón y José Mª Roldán:** Ahorro, inversión y movilidad internacional del capital en los países de la CE. (Publicada una edición en inglés con el mismo número.)

9111 **José Luis Escrivá y Román Santos:** Un estudio del cambio de régimen en la variable instrumental del control monetario en España. (Publicada una edición en inglés con el mismo número.)

9112 **Carlos Chuliá:** El crédito interempresarial. Una manifestación de la desintermediación financiera.

9113 **Ignacio Hernando y Javier Vallés:** Inversión y restricciones financieras: evidencia en las empresas manufactureras españolas.

9114 **Miguel Sebastián:** Un análisis estructural de las exportaciones e importaciones españolas: evaluación del período 1989-91 y perspectivas a medio plazo.

9115 **Pedro Martínez Méndez:** Intereses y resultados en pesetas constantes.

9116 **Ana R. de Lamo y Juan J. Dolado:** Un modelo del mercado de trabajo y la restricción de oferta en la economía española.

9117 **Juan Luis Vega:** Tests de raíces unitarias: aplicación a series de la economía española y al análisis de la velocidad de circulación del dinero (1964-1990).

9118 **Javier Jareño y Juan Carlos Delrieu:** La circulación fiduciaria en España: distorsiones en su evolución.

9119 **Juan Ayuso Huertas:** Intervenciones esterilizadas en el mercado de la peseta: 1978-1991.

9120 **Juan Ayuso, Juan J. Dolado y Simón Sosvilla-Rivero:** Eficiencia en el mercado a plazo de la peseta.

9121 **José M. González-Páramo, José M. Roldán y Miguel Sebastián:** Issues on Fiscal Policy in Spain.

9201 **Pedro Martínez Méndez:** Tipos de interés, impuestos e inflación.

9202 **Víctor García-Vaquero:** Los fondos de inversión en España.

9203 **César Alonso y Samuel Bentolila:** La relación entre la inversión y la «Q de Tobin» en las empresas industriales españolas.

9204 **Cristina Mazón:** Márgenes de beneficio, eficiencia y poder de mercado en las empresas españolas.

9205 **Cristina Mazón:** El margen precio-coste marginal en la encuesta industrial: 1978-1988.

9206 **Fernando Restoy:** Intertemporal substitution, risk aversion and short term interest rates.

9207 **Fernando Restoy:** Optimal portfolio policies under time-dependent returns.

9208 **Fernando Restoy and Georg Michael Rockinger:** Investment incentives in endogenously growing economies.

---

(1) Los Documentos de Trabajo anteriores a 1988 figuran en el catálogo de publicaciones del Banco de España.

Información: Banco de España
Sección de Publicaciones. Negociado de Distribución y Gestión
Teléfono: 338 51 80
Alcalá, 50. 28014 Madrid