AGGREGATE INVESTMENT IN A GROWTH MODEL WITH ADJUSTMENT COSTS

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1 INTRODUCTION

Most of investment literature is concerned with the problem faced by individual firms. There are few models that address the issue of aggregate investment. Individual decision theory predicts upward sloping individual supply curves. At the aggregate level, however, we observe that non-residential investment is negatively correlated with the price of investment over time. This negative correlation can only be reconciled with an upward sloping supply curve for investment goods if supply side fluctuations are large relative to the fluctuations in demand. In this paper, I simulate the general equilibrium structure trying to mimic the observed variability of aggregate investment and the statistic properties of investment with its price and other real variables.

Finding good estimates for the price of investment goods and for the rental price of capital has been a hard problem for economists. The model of aggregate investment presented here generates equilibrium prices at each period of time. The dynamics of the movements of those prices and capital accumulation is still an open question. In this paper I study the co-movements of the price of investment, of rental price of capital and of aggregate investment when exogenous perturbations affect the optimal paths of the variables that solve the assumed structural model.

Finally, I use the data generated by the model to test the performance of alternative partial equilibrium analyses of aggregate investment. Some of them fit real data better than others. I want to know whether those results are maintained when the partial equilibrium models are estimated using data generated by the general equilibrium model .

In a general equilibrium model all the prices are assumed to be market clearing. In particular the price of new capital results from the market clearing condition of the capital goods market. This price determines the rate of investment each period. When an adjustment cost technology is assumed, the rate of investment is an increasing function of that price. Cross section studies of the U.S. economy show a negative correlation between investment and its price (Kydland and Prescott (82)). I have observed, in time series using fixed nonresidential investment and the price deflator of investment goods, that negative correlation is also quite large, -.30. A model economy, ideally should display these two properties: the optimal supply rule for new capital goods as an increasing function of the corresponding prices and, simulations of the model economy should display the negative correlation observed in U.S time series data.

The model that I study in this paper includes, besides the usual productivity shock, a disturbance in the preferences and in the adjustment cost parameter. The model time series reproduces the above two properties of the data. They also display different long and short run elasticities of investment supply. This last property captures the different rates of adjustment of new capital

As is the case in the real business cycle literature, the stochastic fluctuations on productivity represent unpredictable technological change. On the other hand the adjustment cost shock can be view as representing changes in productivity embodied in new capital goods or changes in the taxing of capital and consumption. The shock in preferences represents exogenous factors affecting the agent's willingness to distribute their time between market and nonmarket activities.

The model is therefore a version of the neoclassical growth model with a stochastic adjustment cost technology. The technology on the production side uses only one type of capital good and on the output side it is costly to transform one unit of consumption good into one unit of investment good. Once investment has been committed to production the transformation is irreversible and it implies a fixed capital-labor ratio. Finally, capital goods depreciate at a constant rate.

In a nonlinear environment like the one studied here, the firm's and household's policy functions do not have closed form solutions. Simulations of the model with selected parameters are run using Sims' backsolving method (89). These simulations allow us to address questions about the dynamic effects between the endogenous variables that solve the social planner problem and the prices that come from solving a competitive decentralization.

The remainder of this paper is organized as follows: Section 2 describes the stochastic growth model with adjustments. Section 3 develops the simulation procedure and simulates the model. The discussions of the dynamic relation between variables and the importance of adjustments shocks are also included. Finally, results from simulated investment equations are compared with those from real data. Findings and conclusions are presented in Section 4.

2 A STOCHASTIC GROWTH MODEL WITH ADJUSTMENT COSTS

There is an infinitely lived representative consumer in the economy that maximizes its expected discounted utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left([C_t^{(1-\phi_t)} (1-L_t)^{\phi_t}]^{(1-\gamma)} / (1-\gamma) \right), \quad \gamma \geq 0 \ , 0 \leq \phi \leq 1, \quad \phi_t = \phi \, \theta_{1t}, \quad 0 < \beta < 1 \tag{2.1}$$

The utility function displays unit intratemporal elasticity of substitution between consumption and leisure. This fact is consistent with the U.S. economy observation that the per capita series of labor have shown no significant trend. ϕ is the leisure share parameter and is affected by random shocks θ_{1t}. The parameter γ is the coefficient of risk aversion. β is the discount factor.

A constant returns to scale technology is assumed and the inputs are labor (L) and capital (K). Output (Y) can be allocated to either current consumption or to gross investment (I). Once in place, one good can be transformed into the other, paying a certain cost. There are two exogenous shocks that affect the technology. θ_{2t} is a shock to the production of output. θ_{3t} represents a shock to the adjustment cost. The technology is then written as :

$$\left(A \, C_t^{\eta} + B \, \theta_{3t} \, I_t^{\eta}\right)^{1/\eta} \leq \theta_{2t} \, K_{t-1}^{\alpha} L_t^{(1-\alpha)} \ , \quad 0 < \alpha < 1, \quad A > 0 \ , \ B > 0 \ , \ \eta > 1 \tag{2.2}$$

The production function is of the Cobb-Douglas form with α as the capital's share of output. This corresponds with the observation of a constant capital and labor share of output in the United States since 1955. The random variable θ_{2t} represents technological random fluctuations. This production function, jointly with the utility function, have been widely used in the business cycle literature to study fluctuations on aggregate variables.

The adjustment cost parameter η measures the elasticity of substitution between consumption and investment goods. At the steady state a parameter value for η equal to one with the parameter values A and B equal to one implies that is not cost to transform one unit of investment into one unit of consumption.

Capital has a constant depreciation rate δ :

$$K_{t+1} = I_t + (1-\delta) K_t \qquad , 0 < \delta < 1 \tag{2.3}$$

The technology is homogeneous of degree one. Therefore the distribution of capital between firms is irrelevant.

The random vector $\theta_t = (\theta_{1t}, \theta_{2t}, \theta_{3t})$ is stationary and identically distributed over time. The vector of random shocks follow a lognormal first order autoregressive distribution, i.e.:

$$\log \theta_{t+1} = P \log \theta_t + \varepsilon_{t+1} \quad , \varepsilon_t \sim N(0,\Sigma) \tag{2.4}$$

Usually a highly persistent shock is necessary to match the optimal paths of a neoclassical growth model with real data. This model allows for serial correlation in each of the shocks. Each element of P is denoted by ρ_{ij}. If contemporaneous correlation between the shocks are not allow the matrix P is diagonal.

The social planner's problem is :

$$\text{Max} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left([C_t^{(1-\phi_t)} (1-L_t)^{\phi_t}]^{(1-\gamma)} / (1-\gamma) \right) \tag{2.5}$$

s.t.
$$\left(A\, C_t^{\eta} + B\, \theta_{3t}\, I_t^{\eta} \right)^{1/\eta} \le \theta_{2t}\, K_{t-1}^{\alpha} L_t^{(1-\alpha)} \qquad \forall\, t$$

$$K_{t+1} = I_t + (1-\delta)K_t \qquad \forall\, t$$

$$\log \theta_{t+1} = P \log \theta_t + \varepsilon_{t+1} \quad , \; \varepsilon_t \sim N(0,\Sigma) \qquad \forall\, t$$

$$K_0,\ \theta_0 \ \text{given}$$

C_t, L_t, K_{t+1} and I_t are the decision variables in each period t.

The state variables of that problem are the stock of capital K_t and the vector of shocks θ_t. There exists an optimum for the social planner's problem (2.5). The solution is a vector of stationary stochastic processes: $K_{t+1} = g(K_t, \theta_t)$, $C_t = c(K_t, \theta_t)$, $L_t = l(K_t, \theta_t)$.

The first order conditions of the optimality problem (2.5) once the shadow price of output is substituted out are three: The marginal rate of substitution between consumption and labor equal to its cost, the discounted value of the return in an additional unit of investment next period must be equal to its current cost in terms of consumption goods and the last equation is the transversality condition. This three equations plus the budget constraint form a set of four equations with four unknowns.

Given the functional forms,the above conditions are necessary and sufficient for an interior solution. The stationarity solution for is obtained if the transversality condition is satisfied and the serial correlation matrix P has all its eigen values inside the unit circle. If P is diagonal this means that ρ_{ii} must be between 0 and 1.

The social optimum solution is also the solution for a sequence of market equilibrium allocations. Consequently, there exists time invariant functions for the wages $\omega_t = \omega(K_t, \theta_t)$, rental prices of capital $u_t = u(K_t, \theta_t)$ and prices of investment $p_{It} = p_I(K_t, \theta_t)$ where all prices are relative to the date t consumption good.

3. SIMULATION EXPERIMENTS

3.1 SIMULATION METHOD

The non-existence of closed form solutions for the optimal allocation policies make it necessary to search for a numerical solution. Optimal prices are derived from the optimality conditions for the competitive economy. This section describes the "backsolving" method (Sims,(89)) to find the solution for problem in Section 2. Other simulation methods can be found in Taylor and Uhlig (90).

The first order conditions, the technology constraint and the law of motion for the vector of shocks give a set of first order stochastic difference equations. The only dynamic equation in the first order conditions corresponds to the derivative with respect to the state variable K_{t+1}:

$$-(Uc_t/Gc_t) \quad +\beta E_t(Uc_{t+1}/Gc_{t+1})((1-\delta)GI_{t+1}+ \; FK_t)= \; 0 \qquad (3.1)$$

Introducing a error term ("expectation error") η_{t+1} s.t. $E_t\eta_{t+1}=0$ in (3.1), we get a system of difference equations in the following way:

$$H(z_{t+1},z_t) = \xi_{t+1} \qquad (3.2)$$

$$\text{where} \quad z_t= (X_t,\theta_t) = (C_t,L_t,K_{t+1},\theta_{1t},\theta_{2t},\theta_{3t})$$
$$\xi'_{t+1} = (0,0,\eta_{t+1},\varepsilon_{1t+1},\varepsilon_{2t+1},\varepsilon_{3t+1})$$

All the solutions for the problem in Section 2 must satisfy (3.2).

An additional equation is required in (3.2) to get a solution path for X_{t+1}. There is a restriction in the distribution of ξ_{t+1}. The restriction guarantees a stable solution for X_{t+1} and uniqueness for given θ_0 and Z_0. The steps to find it are:

1. Linearize (3.2) around the steady state.

$$\Gamma_0 \tilde{z}_{t+1} = \Gamma_1 \tilde{z}_t + \xi_{t+1} \tag{3.3}$$

where $\tilde{z}_t = z_t - z_{ss}$

$z_{ss} = (C_{ss}, L_{ss}, K_{ss}, 1,1,1)$ is the steady state vector.

2. Obtain the generalized eigen values and eigen vectors for (Γ_0, Γ_1).

   Vector of eigen values λ:

$$\begin{aligned}\lambda(\Gamma_0,\Gamma_1) &= \{\lambda \in \mathbf{C} \mid \det(\Gamma_0,\Gamma_1)=0\} \\ &= \lambda(H_0,H_1) = \{\lambda = H_{1ii} / H_{0ii}\}\end{aligned} \tag{3.4}$$

   The corresponding matrix of right eigen vectors Z:

Q , Z non singular $\in \mathbf{C}$ s.t.

$$\begin{aligned}\Gamma_0 &= Q\, H_0 Z \\ \Gamma_1 &= Q\, H_1 Z\end{aligned} \tag{3.5}$$

   Therefore (3.3) can be expressed as :

$$Q\, H_0 Z\, \tilde{z}_{t+1} = Q\, H_1 Z\, \tilde{z}_t + \xi_{t+1}$$

$$Z\, \tilde{z}_{t+1} = H_0^{-1} H_1 Z_t \tilde{z} + {}_0H^{-1}{}_1H\, Q^{-1}{}_{t+1}\xi$$

$$Z\, \tilde{z}_{t+1} = [H_0^{-1} H_1]^t\, Z\, z_0 + \sum_{S=0}^{t} [H_0^{-1} H_1]^S\, \xi_{t+1-S} \tag{3.6}$$

3. Since (3.6) has one eigen value $\lambda_j\ (\Gamma_0,\Gamma_1)$ greater than $1/\beta$, to suppress solutions of (3.2) which grow at rate faster than $1/\beta$, we require the corresponding eigen values to be zero, i.e.:

$$Z_{j.}\tilde{z}_t = 0 \qquad \forall\ t \tag{3.7}$$

If there were no eigen values greater than $1/\beta$ it would imply that there exists many possible solutions for the linearized system since the mapping between η and ε is not unique. If more than one eigen value is greater than $1/\beta$ the original problem has no solution.

As long as we are interested in solutions for the non-linear system around the steady state (i.e. for a small variance of the exogenous processes) we can use (3.7) as an additional equation to solve (3.2). We can treat one of the elements of ξ_{t+1} as unknown, say ξ_{1t+1}, draw values for the remaining ξ_{t+1} elements and use (3.2) and (3.7) to get solution paths for X_{t+1}.

The stochastic simulation algorithm draws values from the distribution of the expectation error and two of the innovations, then uses the stability condition to find the third innovation, and finally the first order conditions for the decision variables are solved. This simulation method is an approximation solution because it uses a restriction on the joint distribution of the exogenous shocks and the expectation error that comes from a linearized version of the model.

The simulation algorithm does not guarantee a feasible solution in a given period for any initial capital, parameter vector and distribution of exogenous perturbations. In fact, given the large amount of uncertainty in the model there are combinations of parameters that generate either nonstationary paths or run into non-feasible solutions at some period of time.

The covariance matrix of the shocks used to simulated the model corresponds to the lower ones obtained in the estimation of the model (Vallés (91)). This facilitates the search for dynamic solutions from different deterministic steady states. The standard deviation' matrix of the shocks innovation is:

$$\Sigma = \begin{bmatrix} .002 & 0 & 0 \\ 0 & .008 & 0 \\ 0 & 0 & .055 \end{bmatrix}$$

All the shocks are assumed to be very persistent. Their serial correlation parameter is .99 and there is no cross-correlation (P is diagonal).

3.2 PROPERTIES OF THE SIMULATED DATA

Here, I compare the vector autoregressive representation of real and simulated data and display some of the statistics from both types of data.

Because of the large standard errors of the parameters for the utility function and the technology found in the estimation of the model, I investigate different values of those parameters to determine whether the properties of the VAR for simulated data approximate those properties of the VAR for real data within one standard error of their estimated value.

Drawing the time series for the simulated data we observe that neither the sample series of consumption, investment nor labor are stationary random variables. The endogenous variables of the model are a function of current and past exogenous shocks. Although the optimal paths of the observed variables are stationary, the reason for the nonstationarity of the sample is the high serial correlation of all the shocks that have been transmitted through the model mechanisms. All subsequent experiments are made with the first differences of the data.

As a criteria to observe the effects of small changes to some of the structural parameters on the dynamics of the data from the model, the impulse response graphs of the simulated data must follow those of the real data. The estimated VARs correspond to a system with the same number of variables as perturbations, i.e. consumption C_t , labor L_t and investment I_t. The system includes a constant and 120 observations. The response, up to 12 periods ahead, corresponds to an unitary shock to the orthogonalized innovations of the system.

To obtain data from the model I fix the discount factor β to .99 so that the model displays an annual 4 per cent real interest rate at the stationary level. The leisure share parameter of the utility function ϕ is 2/3. This is the value that emerges from the literature. The results presented have assumed a parameter of risk aversion γ equal to zero, i.e. agents are risk neutral. The minimized objective function in the estimation gets higher values when γ is greater than zero. The parameters of the adjustment cost function, A and B, are set equal to one and δ is .025.

I experiment with different values around the two remaining estimated structural parameters, the capital share α and the adjustment cost η. Graphs 3.1 and 3.2 show the similarity of simulated data and real data responses to different innovations in the VAR when α is .36 and η is 1.8. When the parameter η is lower, the simulated data shows a first period negative response of investment to consumption that the real data does not display. If the parameter α is below .36 the responses of each variable to its own innovation in the system are low relative to the reaction of the other variables to the consumption innovation.

In the same way, the decomposition of variance for the forecast error of each variable in the VAR shows that greater combinations of α and

η, other than the ones estimated, give a better approximation of the model to the observed data. A value of η equal to 1.2 overestimates the explanatory power of labor to forecast the investment error. If α is lower than .36, then labor relative to the other variables explains a lower percentage of its forecast error compared with the real data.

The estimation criteria has been set up such that the estimated parameter values succeed in satisfying the unconditional moment restrictions of the innovations of the model. Therefore, I do not match the statistics from the simulated data to those observed in the U.S. economy. Table 3.1 reports some of these statistics for the chosen parameters in the VAR analysis with the observable variables. Table 3.2 shows the same statistics for other values of η, α, and γ. To calculate these statistics series of simulations with 120 observations each were repeated.

In the simulations, output Y_t is considered to be the outcome from the production function. If output were measured like in the National Income Accounts, it would be $(C_t + Pinv_t\ I_t)$. Output in this case, either measure in real terms or in nominal terms, shows a close picture to output measured by the production function.

The large amount of uncertainty in the model leads to statistics with large standard deviations. The first differences of consumption and labor are too volatile with respect to output and their correlations with output are too high compared with those observed in the real data. The fact that the preference shock affects both consumption and labor makes them move together but in real data they follow different patterns. The first differences of investment look too volatile, although its correlation with output is similar to that observed in real data.

Some of the statistics, after filtering the simulated series with the Hodrick-Prescott filter, are close to the observed. The consumption series has the same standard deviation than output when in the real data it is half this value. Investment series have the same volatility as the real data.

The empirical labor elasticity of output can be measured as the estimated coefficient when output is regressed against labor. That elasticity is .67 for the simulated data detrended with the H-P filter and it is equal to the true value that corresponds to the labor share parameter. This result is opposed to the overestimated empirical elasticity found by fluctuation models with only a shock to the technology (Prescott, 86).

When other parameters are introduced in the model (Table 3.2) the statistics of the first differences show no improvement. As expected, an increment in the risk aversion parameter brings less volatility to all the series of the model but the changes are small.

The model is able to reproduce the negative correlation between investment goods and its own price. The correlation of prices with respect to output is also negative in its average for data filtered with the first differences or the H-P filter. That correlation, for the first differences, is -.28 in the real data and -.47 with the above chosen parameters (table 3.1). This counter cyclical relation between prices and the level of activity is not reduced as the value of η is approximated to 1, i.e. with no adjustment costs.

3.3 PROPERTIES OF THE ADJUSTMENT COST AND THE DISCOUNT FACTOR PARAMETERS

A relevant question in this type of model is how two parameters, the discount factor and the adjustment cost factor, affect the aggregates of the economy and their variability. For the assumed functional forms, the dynamic effects of these two parameters ,outside the steady state, in the equilibrium variables are analyzed by simulating the model.

Table 3.3a shows that increases in the discount factor β lead to increases in the optimal capital accumulation, i.e. the savings rate of the economy, and the level of output. Also, as in growth models without adjustment costs, increments in the discount factor are accompanied by larger variances on investment and output. At the same time the consumption and labor optimal series show an increase in mean and a decrease in variance as β takes on higher values. The economy, though, fluctuates less with respect to all the variables when it is measured in relative terms by the variance to mean ratio. This result comes from the existence of adjustments between consumption and investment and does not hold in simulated models of standard growth models (e.g., Danthine and Donalson (81)).

At the steady state, the simulations show that increments in the adjustment cost parameter η lead to increments in consumption and capital with labor invariant. The intertemporal optimal paths of the capital accumulation as well as the other variables in the model increase in their average and decrease in their variation as the adjustment cost factor increases (table 3.3b). When this happens for a given distribution of shocks,the price of investment goes down and capital accumulation increases faster than the consumption goods increment.

3.4 RELATIVE EFFECTS OF THE STRUCTURAL SHOCKS

Since the three exogenous perturbations are the driving force of the model we expect movements in investment to be completely explained by the changes in those shocks. As a way to measure the relative importance of each of those structural shocks, I assume a linear relation between the variables. Table 3.4 shows the results of regressing the simulated investment series against the shocks with all variables measured in first differences.

The high correlation coefficient, .99, in the regression shows that the assumed linear relation gives a very good approximation. The standard errors of the coefficients show that all the shocks are important in explaining aggregate investment. The relative importance of each one can be measured by the loss of explanatory power when the shock does not enter in the regression. From this analysis we conclude that changes in the adjustment cost technology account for up to 90 per cent of investment changes. The preference shock follows this perturbation in importance. This preference disturbance, though, explains most of the variation in the other two observed variables, labor and consumption. The presence of an adjustment cost shock in this model offsets the relevance of the productivity shock for investment fluctuations claimed by real business cycles models.

When doing the regression shown in Table 3.4, we assume that second and higher moments of the exogenous shocks distribution are not important explaining investment. The variability of aggregate investment is reduced by four fifths when the adjustment cost shock variance is equal to zero. This shows that the high moments of the exogenous shocks are not important as an approximation.

Altug(89) finds, in a model with only one structural disturbance, 50 per cent of investment variation is explained by the technology shock and the rest by a measurement error. In her model, the technology shock accounts for 90 per cent of output variation and a low percentage of consumption and labor series variation. The remaining movements of those series are explained by specific measurement errors that can not be identified with any disturbance of the economic environment.

The positive effect of a productivity shock in output is reflected by the positive sign of this shock in the regression of investment. A positive shock in preferences negatively affects investment. This is a consequence of the agents' willingness to increase leisure time.

The large variability of the estimated adjustment cost shock gives a supply side explanation for the negative correlation between prices and quantities of investment goods. A positive exogenous shock to the investment good market moves resources of the economy to that sector and the relative value of an extra unit of new capital falls. The shift in the supply of investment goods by the firms, provoked by these exogenous shocks, stimulates an increase in investment and a fall in price.

3.5 THE EFFECT OF PRICES IN THE MODEL

The relation between real variables and prices is interesting and requires further study. Graphs 3.3 and 3.4 show the real and simulated data results for VAR impulse responses in a system with output Y, price of investment Pinv, and investment Inv. The price of investment is obtained from the equilibrium conditions in the competitive problem after solving for the endogenous variables in the social planner's problem. The responses in the VAR are within one standard error of the orthogonalized residuals of the system.

The effects of output innovation in the system are displayed in the first column of Graph 3.4. There is an immediate positive effect in output itself and also in investment, although this is not as big. The negative correlation between Y and Pinv leads to an immediate negative response in the price and this effect remains for the next 12 quarters. These results, when compared with the corresponding results in Graph 3.3, show that the model implies an overreaction of the price response, mainly in the first period, and that the investment path response is less responsive to output innovation than in the real data.

An exogenous increase in Pinv innovation has, like the output innovation, similar responses in real and simulated data. The model, though, overestimates the magnitude of the response in investment. In the first period the negative response is necessary to satisfy the optimal capital first order condition, however its magnitude is too large in comparison to the real data. Following the initial period shock, capital accumulation restarts and the effect of the exogenous increase in Pinv innovation on investment decreases. The simulated responses of output are more important after four periods whereas in real data the most important responses happen in the first quarters. The increase in Pinv innovation may be viewed as an increase in the tax rate between new capital goods and consumption goods.

An increase in current investment has a small effect on future output in both types of data. Nevertheless the response of Pinv has an opposite sign in Graphs 3.3 and 3.4. The response of Inv to its own innovation, relative to the response of output to its own shock, is also overestimated in the model.

I tried a different orthogonalization of the VAR residuals to see if they were the cause of some of the different results between real and simulated data. The structural orthogonalization (see Sims (86)) imposed intends to capture the immediate effect of relative prices of the real variables of the model.

In the assumed orthogonalization of the residuals of the VAR system with Y, Pinv and Inv, output innovation is not affected by the other innovations. That is, residuals of output respond with a delay, to other variable changes in the system and only react to autonomous fluctuations. The Pinv residual responds quickly to new information in the real markets. Investment is last in the order of the system and responds instantaneously to output innovations since investment is a proportion of total output. The estimation results for real and simulated data are:

REAL DATA

$$\varepsilon_{Yt} = u_{1t}$$

$$\varepsilon_{P_{It}} = \underset{(.040)}{.004}\ \varepsilon_{Yt} + \underset{(.176)}{.319}\ \varepsilon_{It} + u_{2t}$$

$$\varepsilon_{It} = \underset{(.021)}{-.133}\ \varepsilon_{Yt} + u_{3t}$$

SIMULATED DATA

$$\varepsilon_{Yt} = u_{1t}$$

$$\varepsilon_{P_{It}} = \underset{(.021)}{.137}\ \varepsilon_{Yt} + \underset{(.019)}{.076}\ \varepsilon_{It} + u_{2t}$$

$$\varepsilon_{It} = \underset{(.101)}{-.279}\ \varepsilon_{Yt} + u_{3t}$$

The above decomposition shows that in the model the price of investment goods is more responsive to output than to investment in the short run when in real data it is the opposite. The investment residual overreacts to output in the simulated data.

Graphs 3.5 and 3.6 show the impulse response for real and simulated data with the structural orthogonalization. This structural decomposition of the simulated data can not reproduce the slow response of Pinv and Inv to output innovation and of Inv to its own innovation. Nevertheless, this latter orthogonalization includes the negative responses of prices to Inv innovations to the simulated data during the first two quarters that it is observed in the real data.

The rental price of capital R_t is now introduced in the VAR system instead of output. From the optimality conditions of a competitive solution, R_t must be equal to the marginal productivity of capital, i.e. $F\kappa_t$ / Gc_t. In Graph 3.7 the impulse response effects in that system are evident.

The Pinv and Inv still have the same negative response to each others impulse. More interesting are the movements of R_t with the other two variables in the system. In a general equilibrium model with adjustment costs the changes in investment no longer respond negatively to an increment in the change of the rental price of capital as partial equilibrium models predict. The effect of an exogenous increase in the rental price of capital, caused for example, by a increase in capital income tax, has instantaneous effects in the market for investment goods and. therefore. in the long run for capital accumulation. The increase in R_t creates more income for the consumers and an upward shift in the demand of investment goods. The firms increase their costs but as new investment goods are purchased the relative price of investment goods declines.

Although in the steady state R_t and $Pinv_t$ move together with exogenous changes, outside of the stationary solution both prices have opposite response in the first two periods to a exogenous impulse in investment. This result is more clear when Inv_t leads the system and R_t is the last variable in the VAR estimation.

An interpretation of the positive response of rental prices to investment innovations is that those exogenous innovations may create an upward pressure in the interest rates that push up the rental price. As a consequence of that shift in investment both investment and rental price will increase. This explains the large positive contemporaneous correlation between interest rates and investment observed in real data .

So far, the experiments have been made using the estimated variance of the innovations to the shocks. Changes in any one of those variances will produce changes in the results of the experiments. Because the shocks are lognormally distributed, changes in the variances of the shocks' innovations affect not only the variance of the shocks but also their mean.

As a result of partial equilibrium models, increments in the uncertainty about future prices increase the current optimal rate of investment (Hartman (72)). These models assume that the uncertainty changes are mean preserving. In the equilibrium model of this paper, though, an increase in the variance in the adjustment cost shock innovation affects the current and future distribution of investment good prices. As a consequence of an increment of uncertainty in the relative prices, the optimal paths of the real variables increase in their variance but their average may increase or decrease.

3.6 Simulated Data and Investment Equations

In this section I compare the fit of different investment equations to data generated by the equilibrium economy with those obtained with actual data. These investment equations are derived from partial equilibrium models. I test if the investment equations used in the literature are able to identify the properties of the technology used to simulate the data. I also study how the effects of tax policies are captured by the investment equations in the simulated data.

I assume that the econometrician observes series of output Y_t, investment I_t, capital K_t, price of investment goods $Pinv_t$ and rental price of capital R_t that solve the equilibrium model with adjustment costs. The rental price is the expression that is equal to the marginal productivity of capital ($F\kappa_t/Gc_t$) in the capital accumulation Euler Equation. The relative price of investment goods in equilibrium is B/A $(I_t/C_t)^{\eta-1}$. It is further assumed that the econometrician knows the true depreciation rate δ.

A neoclassical investment equation is obtained from assuming a Cobb-Douglas production function and embodying a distributed lag response of actual net investment to changes in the desired stock of capital K^*_t i.e.:

$$\sum_{l=0}^{s} \omega_l (I_{t-l} - \delta K_{t-l-1}) = \sum_{l=0}^{n} \gamma_l (K^*_{t-l} - K^*_{t-l-1})$$

$$\text{s.t.} \qquad K^*_t = \alpha Y_t / R_t$$

The parameter α is the elasticity of output with respect to capital. The neoclassical model implies a unitary elasticity of investment with respect to output and the rental price of capital (with opposite sign) because of the unitary elasticity of substitution between labor and capital. This production technology is the same as the one used to simulate the data.

At the other extreme of the neoclassical specification is the accelerator investment equation. It is obtained assuming zero substitutability between labor and capital in a CES production function. Net investment is taken to be a function of the first difference of lagged real output and the rental price does not enter in the specification. The second investment equation considered in this section includes lagged values of changes in real output, lagged values of changes in the rental price of capital and lagged values of the dependent variable.

$$\sum_{l=0}^{s} \omega_l (I_{t-l} - \delta K_{t-l-1}) = \sum_{l=m}^{n} \gamma_{Yl} \Delta Y_{t-l} + \sum_{l=m}^{n} \gamma_{Rl} \Delta R_{t-l}$$

This equation can be used to test the role of rental prices when the unitary elasticity of substitution is believed to be uncertain.

The third investment equation is based on the q-theory. Contrary to the other two specifications, this equation represents the importance of adjustment costs that are transmitted via prices. Furthermore, the q-theory would consider the current price of an additional unit of investment relative to its replacement cost $Pinv_t$, as the only explanatory variable.

Table 3.5 shows different estimations of the three investment equations when the generated random numbers for the given distribution of the exogenous shocks imply, by the Akaike criteria, a lag length of the accelerator equation similar to that observed in real data. The simulated data contains 120 observations.

The choice of the distributed lag function in the estimated neoclassical equation was made restricting the polynomial in the lag operator $\omega(L)$ to a second order, and then choosing the lag length for $\gamma(L)$ between one and eight. For the three regressions in Table 3.6 corresponding to the neoclassical specification the implied elasticity of net investment with respect to the ratio of output and rental price of capital ($\Sigma\ \gamma\ \hat{\alpha}$ / $\Sigma\hat{\omega}$) is very small, around 1.E-4. Eisner and Nadiri (67), for quarterly data between 1947 and 1962, found that elasticity to have values between .4 and .08 depending on the lag specification.

In general, the Hybrid accelerator equation fits the data better than a neoclassical equation. The accelerator specification that includes eight lagged values of output and four of rental price of capital and of net investment gives the highest R^2 but the wrong sign for the coefficients of the rental price. When the dependent variables start at lag one with the dependent variable following a second order lag polynomial, the coefficients of the rental price have the right sign but in this case only the lagged values of the dependent variable are significant.

Both specifications of the hybrid accelerator equation imply an elasticity of investment with respect to output ($\Sigma\gamma_Y$ / $(I-\Sigma\omega)$) smaller than the one with respect to the rental price ($\Sigma\gamma_R$ / $(I-\Sigma\omega)$). Eisner and Nadiri (67) found contrary results. Gordon and Veitch (86), with quarterly data from 1949 to 1983, found that the accelerator and the price of investment become insignificant and the lagged values of the dependent variable have high explanatory power for different types of investment.

The estimated neoclassical equation, with the imposition of the constraint on the elasticity of net investment with respect to desired capital equal to one, permits estimates of α (Jorgenson and Stephenson (67)), i.e.

$$\hat{\alpha} \quad = \quad \sum_s (\gamma_s \hat{\alpha}) \;/\; \sum_l \hat{\omega}_l$$

The corresponding estimated elasticity of output with respect to capital in Table 3.5 varies between 2.8E-4 and .78E-4. The estimated value of α is far from the true one, .36, with which the data has been generated. Because the variables observed by the econometrician are the true ones, the difference between $\hat{\alpha}$ and α can not be attributed to errors in measuring the variables. Also, the accelerator model is not able to obtain the unitary elasticity of net investment with respect to output and with respect to rental capital implied by the generated data.

The price of investment equation explains 50 per cent of net investment variation. The Durbin-Watson statistic shows there exists serial correlation in the error term. The same regression with real data, using the ratio of the price deflator of investment goods to the price deflator of consumption goods as a proxy for the relative price of investment goods, shows the same serial correlation problem and almost zero explanatory power for the variation of net investment.

Changes in the capital tax policy will affect the rate of return on capital. In Table 3.6 the effects on net investment of a hypothetical change in tax policy that doubles the rental price of capital in the simulated period 110 are represented. This analysis assumes that the level of output will not be affected by such a policy change.

Net investment depends on the parameters of the investment function. In the calculations with one specification of the neoclassical equation and one of the hybrid accelerator, these parameters are replaced by the estimates given in Table 3.5. When the new tax policy affects the rental price of capital at period 110 the resulting changes in net investment after that period are calculated from the fitted investment equations.

The investment equations with simulated data, as with actual data, predict a negative response of investment to an increase in taxes. This response has a lag that depends on the specified equation. Net investment in the accelerator model responds more strongly than in the neoclassical equation. Contrary to the findings with the same equations in real data (see ,e.g., Hall and Jorgenson (67)) this tax effect lasts one period and net investment increases the period after the tax increase effect has appeared. For both specifications in Table 4.4 the accumulative change in net investment is almost zero five periods after the tax policy has taken effect.

We have found that the simulated elasticities of investment with respect to prices and output are quite different from those found in the literature of the 60's with real data. Nevertheless, as with real data, the lagged values of net investment have high explanatory power compared with output and rental price. Those partial equilibrium investment equations obtain misleading structural parameters of the technology. Also, the tax policy effects on investment appear to have a different pattern than in real data when the data has been generated with a technology with adjustment costs.

4. Conclusions

To reproduce the observed volatility of aggregate investment and the negative correlation with its price in the U.S. data a growth model with large variability in the adjustment costs technology has been constructed. That model gets the observed labor elasticity of output but overestimates the variability of consumption.

The policy effect of changes in the discount factor and the adjustment cost parameters have been studied for the specification of the model. Increments in any of those parameters cause an increase in the average savings rate of the economy and a reduction in the variability of all the real variables.

Partial equilibrium adjustment cost models use market asset pricing data as a proxy for the price of an additional unit of capital relative to its replacement cost. These models explain a low percentage of the observed investment variability and show large serial correlation in the error term of the investment equation. An equilibrium price of investment goods was obtained that explains a large part of aggregate investment variation in the model. This equilibrium price contains more information in explaining the simulated paths of investment than the investment price deflator does in explaining the fixed investment in the U.S. economy.

The neoclassical and the accelerator investment equations are not able to identify the elasticity of investment with respect to output and with respect to the price of capital that has been used to generate the simulated data. These equations display elasticities and responses to tax policy changes different to those found with actual data.

The obtained simulations indicate that the estimated model gives a good structural interpretation for the vector autoregressive impulse response of the observed variables. Nevertheless, the model overestimates the response of investment to its own price innovation. In the dynamic analysis of the equilibrium model, contrary to the results from the steady state analysis, an increase in the rental price of capital creates an upward shift in the demand for investment goods.

Table 3.1 STATISTICS OF THE ADJUSTMENT COST ECONOMY

	U.S. Economy 59:1,88:4		Simulated Economy	
	H-P FILTER	FIRST DIFF.	H-P FILTER	FIRST DIFF.
Standard Deviations				
Y	.018	.017	.023 (.022)	.026 (.019)
C	.009	.004	.023 (.035)	.023 (.024)
L	.015	.002	.030 (.032)	.006 (.005)
I	.054	.005	.076 (.053)	.041 (.027)
Corr. with Y				
C	.74	.42	.93 (.24)	.94 (.18)
L	.87	.54	.88 (.29)	.88 (.18)
I	.80	.58	.60 (1.37)	.50 (.88)
$C(I,P_I)$	-.47	-.32	-.54 (.78)	-.52 (.71)

The parameters used in simulated data are A=B=1, δ=.025, ϕ =2/3, α=.36, β=.99, η=1.8 and γ=0. The statistics correspond to the means for 20 simulations, each with 120 observations. the standard deviations of the statistics are in parenthesis. The H-P filter refers to the Hodrick-Prescott filter.

Table 3.2 STATISTICS OF THE ADJUSTMENT COST ECONOMY

Simulated Data, First Differences

	β=.99, γ=0.0				β=.99, γ=1.0 α=.36, η=1.8
	α=.22, η=1.2	α=.22, η=1.8	α=.36, η=1.2	α=.36, η=1.8	
Standard Deviations					
Y	.021 (.018)	.018 (.021)	.026 (.034)	.029 (.008)	.027 (.031)
C	.018 (.022)	.016 (.020)	.023 (.047)	.024 (.014)	.023 (.028)
L	.014 (.015)	.010 (.012)	.008 (.013)	.007 (.002)	.007 (.008)
I	.017 (.007)	.014 (.017)	.045 (.051)	.044 (.012)	.038 (.042)
Corr. with Y					
C	.92 (.16)	.87 (.93)	.46 (.89)	.95 (.07)	.85 (.91)
L	.97 (.04)	.84 (.90)	.80 (.86)	.90 (.05)	.80 (.85)
I	.60 (.57)	.60 (.75)	.50 (1.09)	.66 (.92)	.59 (1.05)
Cor(I,P_I)	.78 (.32)	.66 (.75)	.74 (.86)	-.68 (.82)	- .72 (.84)

The parameters used in simulated data are A=B=1, δ=.025, ϕ =2/3, and γ = 0. The statistics correspond to the means for 8 simulations each with 120 observations.
the standard deviations of the statistics are in parenthesis.
The H-P filter refers to the Hodrick-Prescott filter.

Table 3.3a

EFFECTS OF CHANGES IN THE DICOUNT FACTOR PARAMETER

levels of the variables

	β=.985	β=.99	β=.995	β=.999
Output				
E(y)	1.42	1.55	1.73	1.92
σ(y)	.225	.226	.231	.242
σ(y)/E(y)	.15	.14	.13	.12
Investment				
E(I)	.594	.707	.865	1.04
σ(I)	.243	.274	.318	.370
σ(I)/E(I)	.40	.38	.36	.35
Consumption				
E(C)	1.26	1.35	1.46	1.56
σ(C)	.181	.177	.175	.177
σ(C)/E(C)	.15	.13	.12	.11
Labor				
E(L)	.264	.278	.296	.315
σ(L)	.044	.042	.039	.038
σ(L)/E(L)	.16	.15	.13	.12

The parameters used in simulated data are A=B=1, δ=.025, ϕ =2/3, α=.36, η=1.8 and γ=0. The number of observations is 120.

Table 3.3b

EFFECTS OF CHANGES IN THE ADJUSTEMENT COST PARAMETER

levels of the variables

	η= 1.2	η= 1.6	η= 1.8	η=2.0
Output				
E(y)	1.2	1.46	1.55	1.63
σ(y)	.279	.236	.226	.218
σ(y)/E(y)	.23	.16	.14	.13
Investment				
E(I)	.348	.599	.707	.804
σ(I)	.277	.276	.274	.271
σ(I)/E(I)	.81	.46	.38	.33
Consumption				
E(C)	1.0	1.25	1.35	1.43
σ(C)	.201	.179	.177	.176
σ(C)/E(C)	.20	.15	.13	.12
Labor				
E(L)	.263	.275	.278	.280
σ(L)	.065	.046	.042	.039
σ(L)/E(L)	.25	.17	.15	.14

The parameters used in simulated data are A=B=1, δ=.025, ϕ =2/3, α=.36, η=1.8 and γ=0. The number of observations is 120.

Table 3.4

Effects of the structural shocks on investment

Dependent Variable: first differences in investment

Independent variables (first differences)				
Preference shock (θ_1)	Productivity shock (θ_2)	Adjustment cost shock (θ_3)	R^2	D-W
-1.59 (.017)	1.22 (.026)	-.76 (.004)	.995	1.21
.068 (.209)	1.3 (.390)	——	.08	1.68
-1.65 (.077)	——	-.77 (.021)	.91	1.97
——	1.42 (.222)	-.50 (.032)	.70	1.49

The parameters used in simulated data are A=B=1, δ=.025, ϕ =2/3, α=.36, η=1.8 and γ=0. The number of observations is 120.

Table 3.5 Fit of Investment Equations to Simulated Data

Neoclassical Investment Equation

$$\sum_{i=0}^{2} \omega_i \quad (I_{t-i} - \delta K_{t-1-i}) = \sum_{i=m}^{n} \gamma_i \alpha \quad \Delta(Y_{t-i} / R_{t-i}) + \varepsilon_t$$

	$\sum_{i=m}^{n} \gamma \alpha$	$\sum_{i=1}^{2} \omega$	R^2
m=3,n=4	.08E-4 (.28E-4)	.9364 (.1875)	.882
m=3,n=7	.045E-4 (.86E-4)	.9435 (.1948)	.886
m=n=6	.17E-4 (.12E-4)	.9420 (.1911)	.882

Price of Investment Equation

$$(I_t - \delta K_{t-1}) = \omega_0 + \omega_1 \text{ Pinv}_t + \varepsilon_t$$

ω_0	ω_1	D-W	R^2
2.0316 (.1791)	-3.5108 (.3109)	.126	.52

Table 3.5 (continued) Fit of Investment Equations to Simulated Data

Hybrid Accelerator Model

$$\sum_{i=0}^{s} \omega_i \left(I_{t-i} - \delta K_{t-1-i} \right) = \sum_{i=m}^{8} \gamma_{Yi} \Delta Y_{t-i} + \sum_{i=m}^{4} \gamma_{Ri} \Delta R_{t-i} + \varepsilon_t$$

	$\Sigma \gamma_{Yi}$	$\Sigma \gamma_{Ri}$	$\Sigma_{i=1} \omega_i$	R^2
s=4, m=0	.704 (.711)	5.266 (.938)	.951 (.519)	.991
s=2, m=1	1.996 (1.860)	-2.802 (2.092)	.739 (.526)	.901

Notes: The dependent variable is the level of net investment for all the equations. For the neoclassical and the accelerator equations the sum of the estimated standard error coefficents are in parenthesis. The simulated data comes from the parameters β=99, η=1.8, A=B=1, ϕ=2/3, δ=.025 and α=.36.

Table 3.6 Change In Net Investment Resulting From Change In Rental Price In Period 110.

Neoclassical Investment Equation

$$\sum_{i=0}^{2} \hat{\omega}_i \left(I_{t-i} - \delta K_{t-1-i} \right) = \sum_{i=3}^{4} \gamma_i \hat{\alpha} \;\Delta (Y_{t-i} / R_{t-i})$$

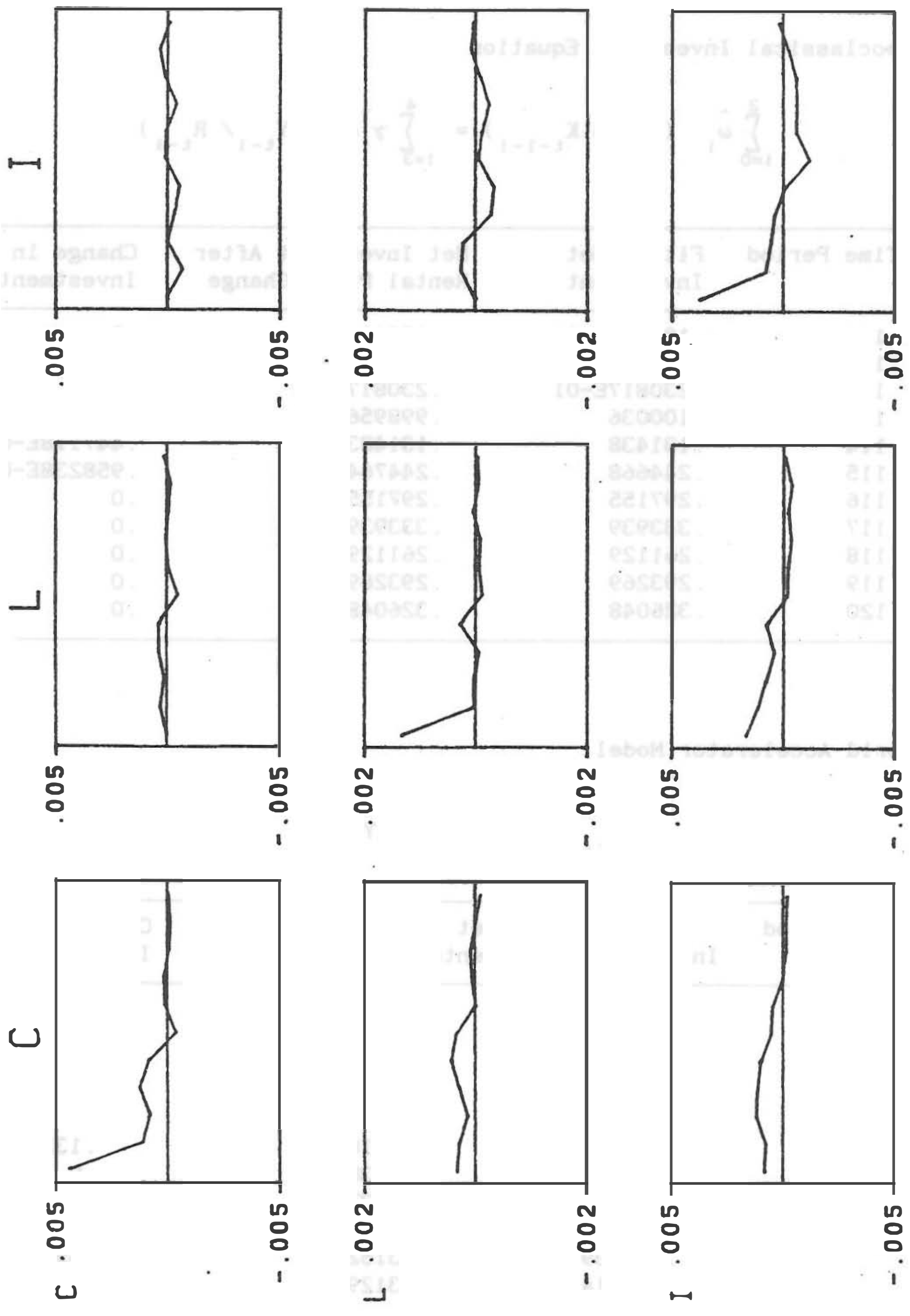
Time Period	Fitted Net Investment	Net Investment After Rental Price Change	Change in Net Investment
110	-.129113E-01	-.129113E-01	.0
111	.120337E-01	.120337E-01	.0
112	.230817E-01	.230817E-01	.0
113	.100036	.998956E-01	-.140536E-03
114	.131438	.131483	.447118E-04
115	.244668	.244764	.958238E-04
116	.297155	.297155	.0
117	.333939	.333939	.0
118	.261129	.261129	.0
119	.293269	.293269	.0
120	.326048	.326048	.0

Hybrid Accelerator Model

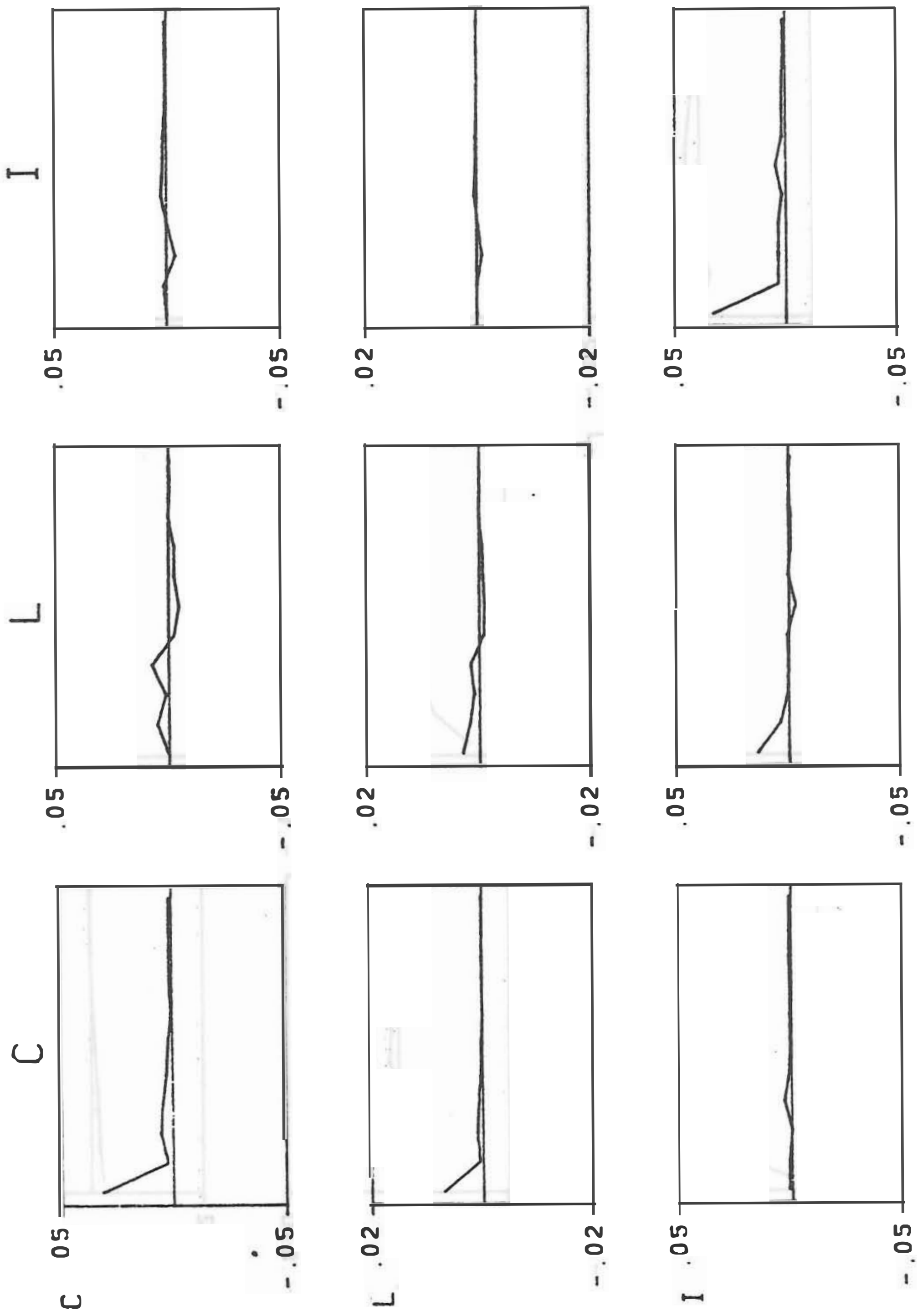
$$\sum_{i=0}^{2} \hat{\omega}_i \left(I_{t-i} - \delta K_{t-1-i} \right) = \sum_{i=1}^{8} \hat{\gamma}_{Yi} \;\Delta Y_{t-i} + \sum_{i=1}^{4} \hat{\gamma}_{Ri} \;\Delta R_{t-i}$$

Time Period	Fitted Net Investment	Net Investment After Rental Price Change	Change in Net Investment
110	-.117273E-01	-.117273E-01	.0
111	.210530E-01	.118610E-04	-.210412E-01
112	.300881E-02	.325462E-02	.245816E-03
113	.101750	.105913	.416314E-02
114	.130332	.133698	.336555E-02
115	.257733	.271000	.132667E-01
116	.313881	.313881	.0
117	.363708	.363708	-.111022E-15
118	.273942	.273942	.0
119	.315239	.315239	-.555112E-16
120	.312914	.312914	.0

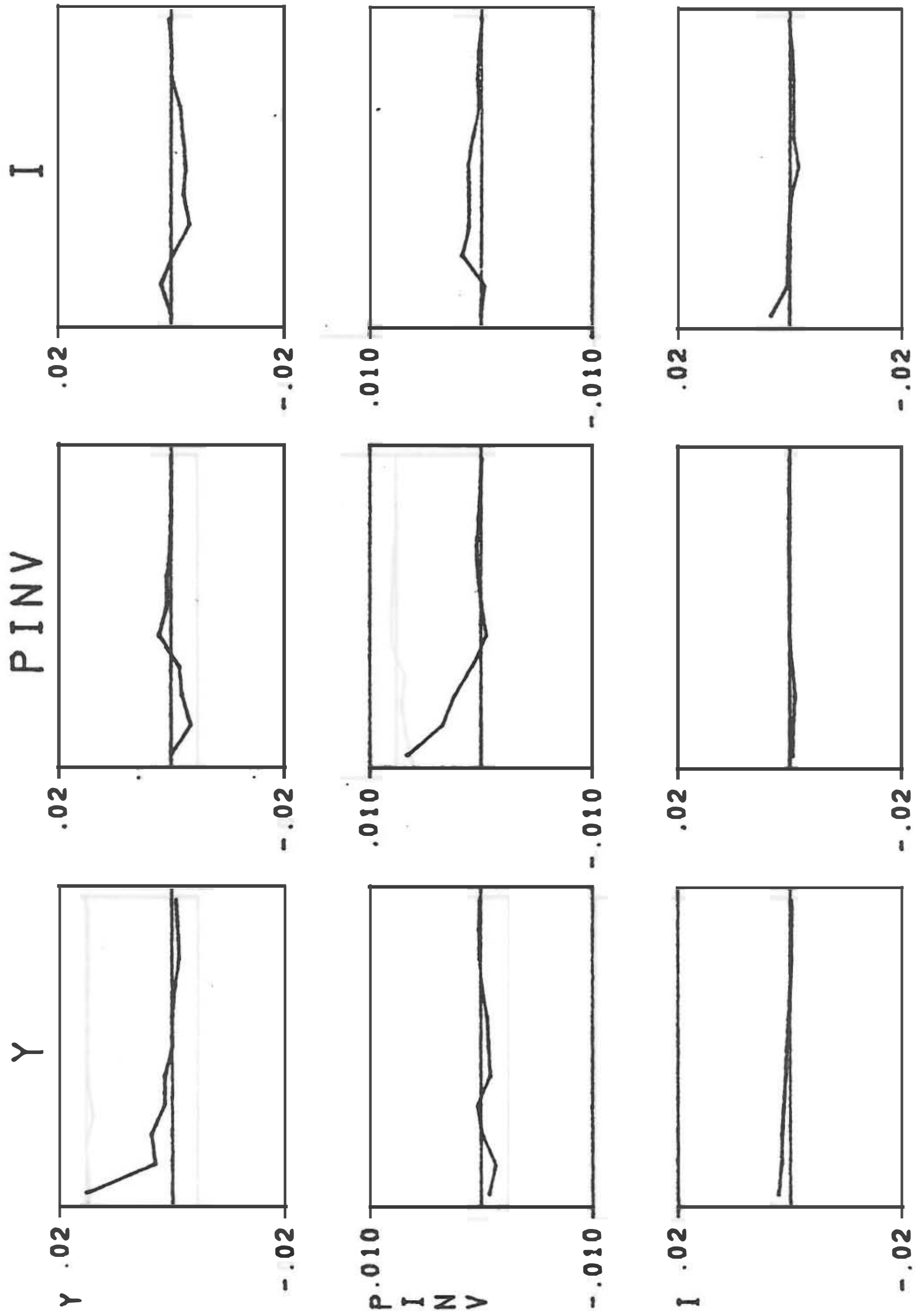
GRAPH 3.1 INPULSE-RESPONSE, REAL DATA



GRAPH 3.2 INPULSE-RESPONSE, SIMULATED DATA (β=.99, α=.36, η=1.8)

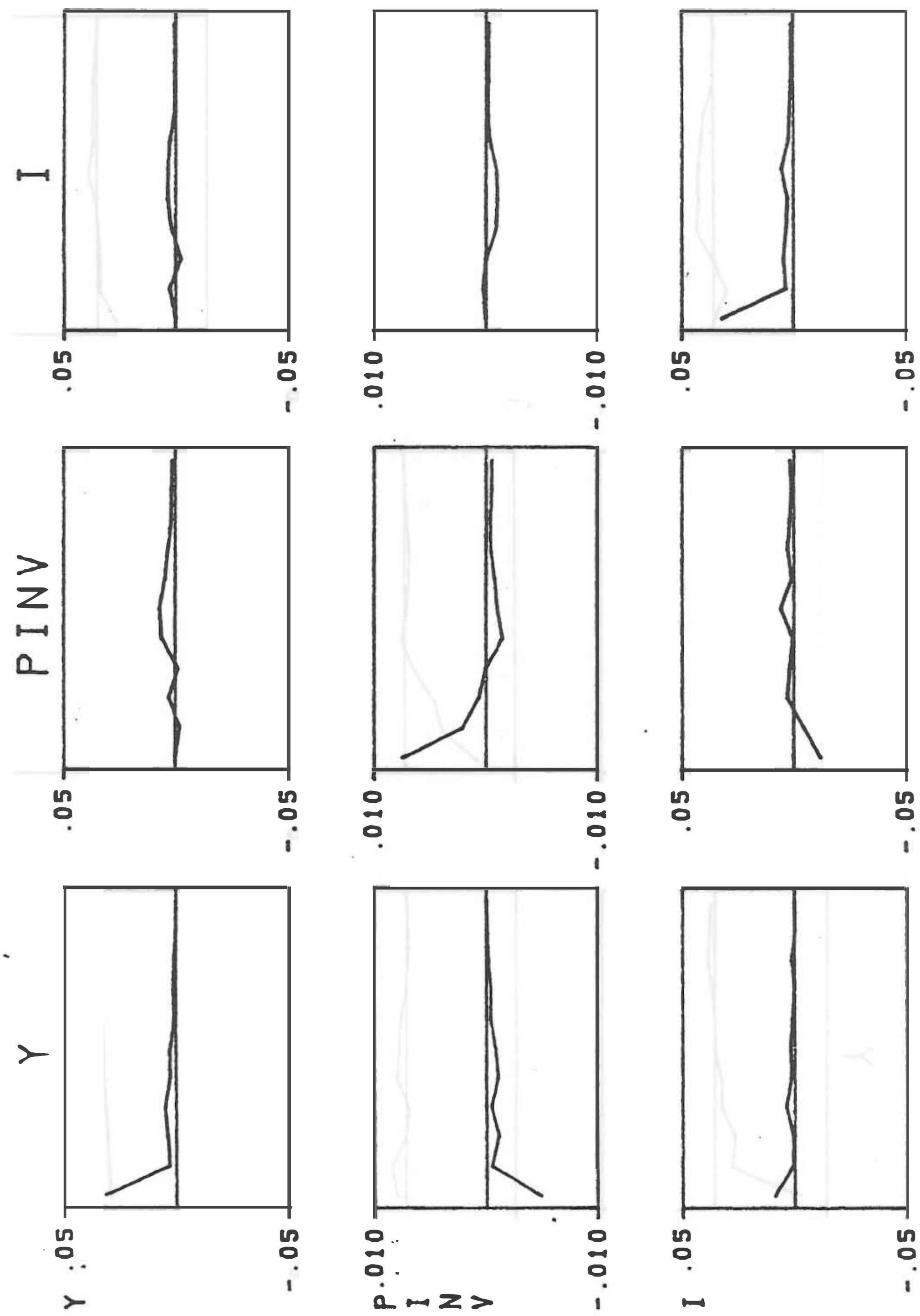


GRAPH 3.3 INPULSE-RESPONSE, REAL DATA



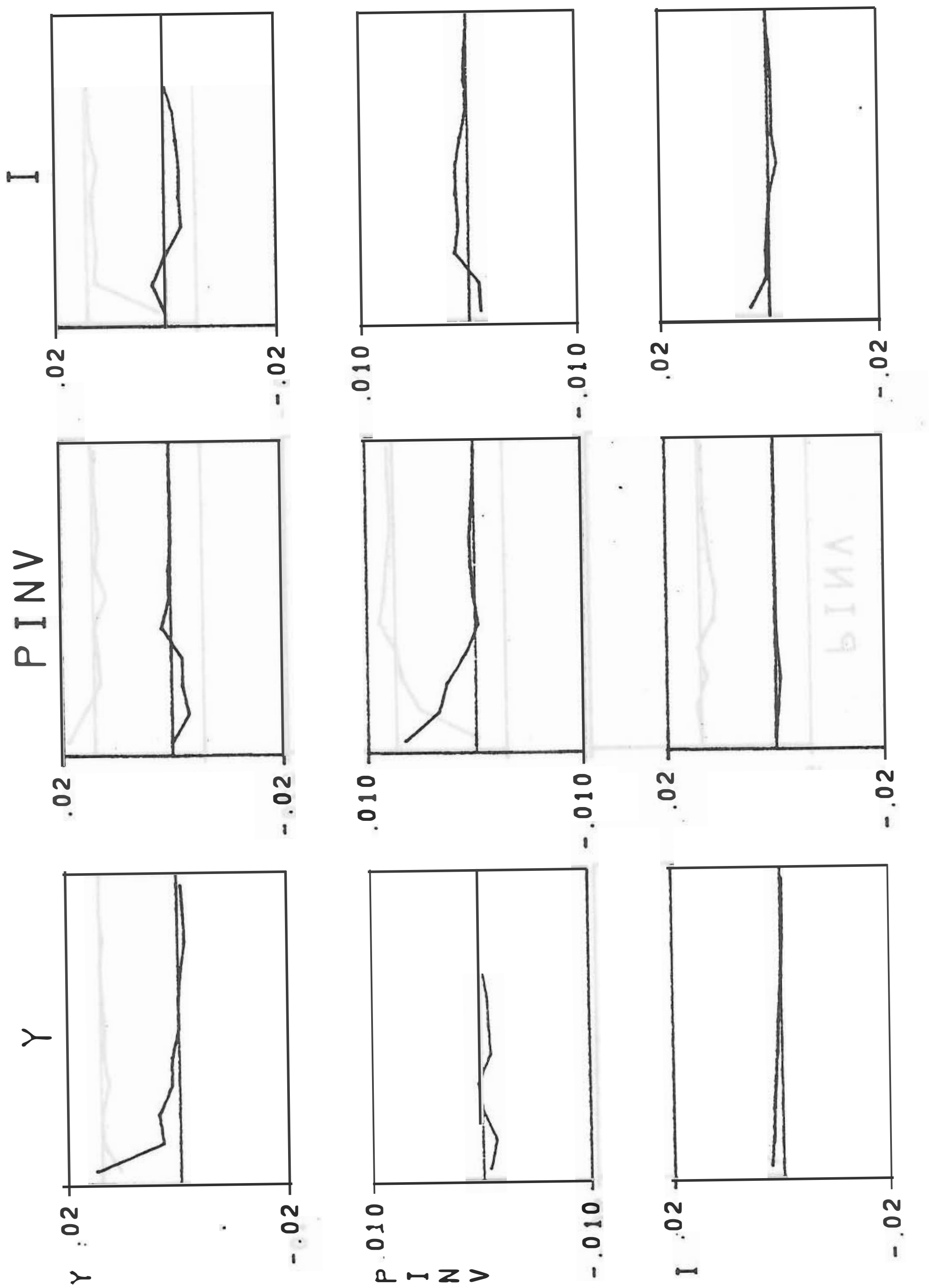
Notes: The graphs, in columns, are responses to the variables innovation heading the column. All variables are in first differences.

GRAPH 3.4 INPULSE-RESPONSE, SIMULATED DATA



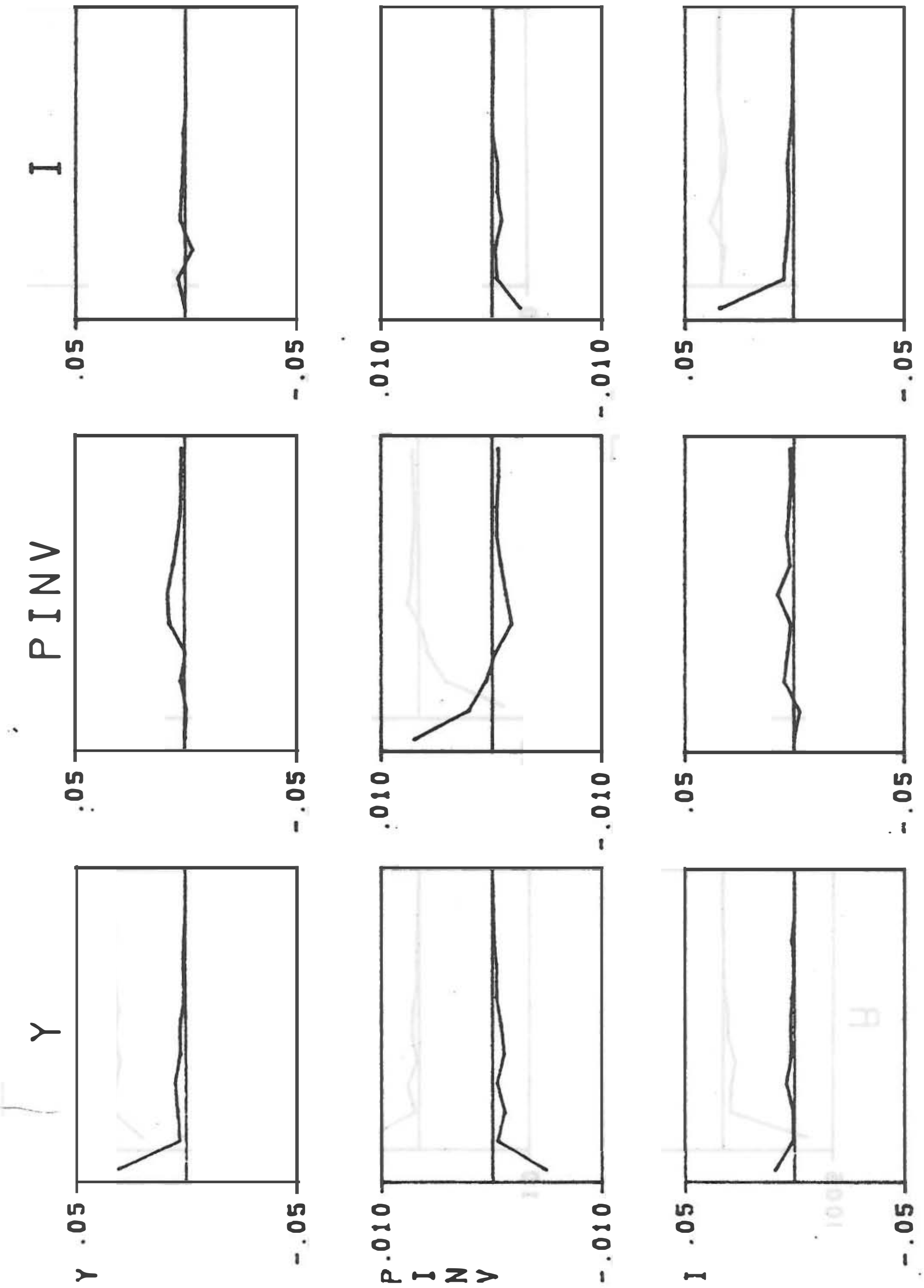
Notes: The graphs, in columns, are responses to the variables innovation heading the column. All variables are in first differences.

GRAPH 3.5 INPULSE-RESPONSE, REAL DATA
Structural Orthogonalization of the residuals



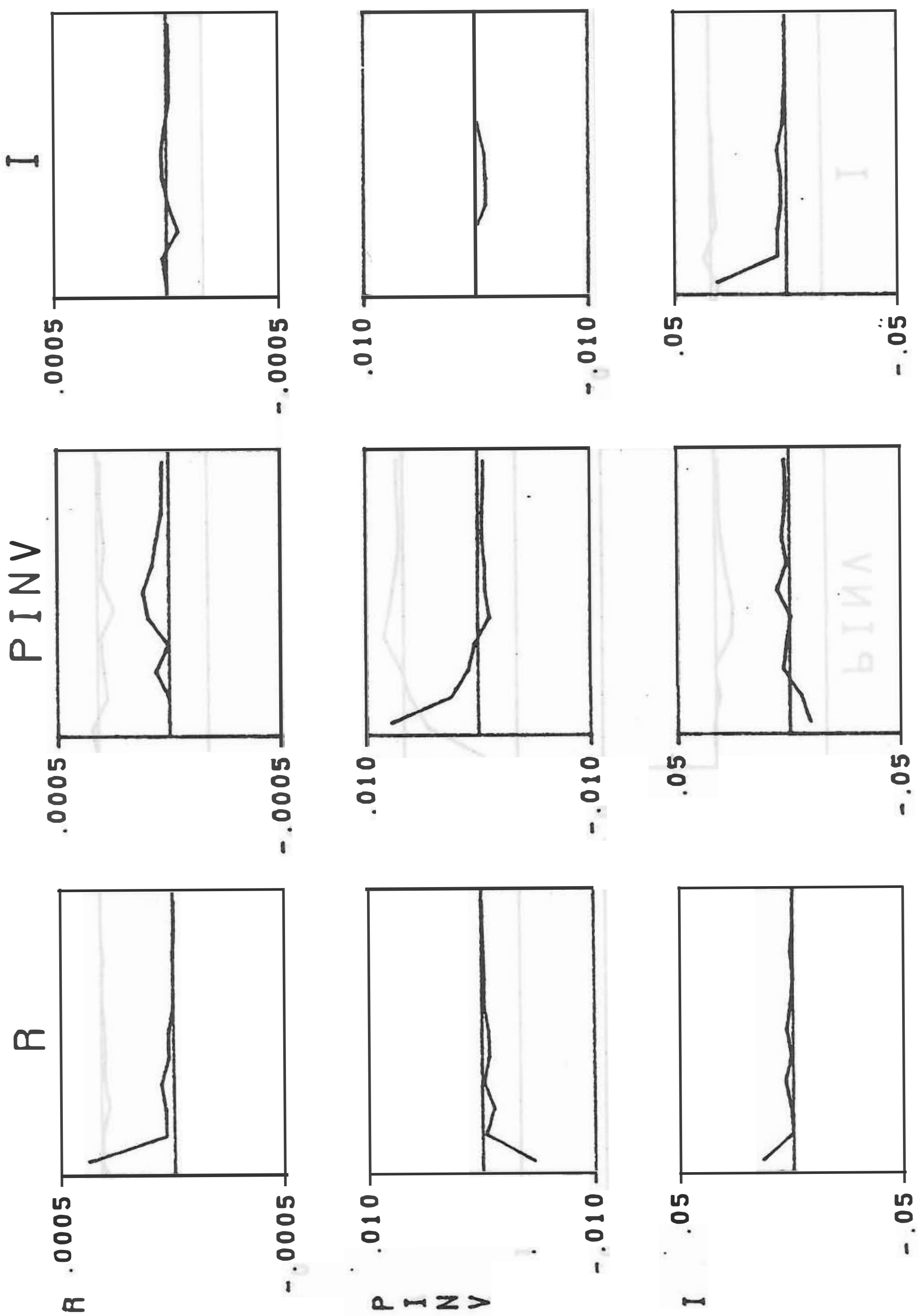
Notes: The graphs, in columns, are responses to the variables innovation heading the column. All variables are in first differences.

GRAPH 3.6 INPULSE-RESPONSE, SIMULATED DATA
Structural Orthogonalization of the residuals



Notes: The graphs, in columns, are responses to the variables innovation heading the column. All variables are in first differences.

GRAPH 3.7 INPULSE-RESPONSE, SIMULATED DATA



Notes: The graphs, in columns, are responses to the variables innovation heading the column. All variables are in first differences.

REFERENCES

Danthine, J.P. and Donaldson, J.B. 1981. Stochastic properties of fast vs. slow growing economies. Econometrica 49, 1007-1031.

Eisner, R. and Nadiri, M.I. 1968. Investment Behavior and Neo-classical Theory. The Review of Economics and Statistics,50: 369-382.

Hall, R.E. and Jorgenson D.W. 1967. Tax policy and Investment behavior, American Economic Review 57, 391-414.

Hartman, R. 1972. The effects of price and cost uncertainty on investment, Journal of Economic theory 5, 228-266.

Jorgenson, D. W. 1963. Capital theory and investment behavior, American Economic Review, 53, 247-59.

Jorgenson, D. W. and Stephenson, J. A. 1967. Investment Behavior in U.S. Manufacturing, 1947-1960. Econometrica,35, 169-220.

Kydland, F. E. and Prescott, E. C. 1982. Time to build and aggregate fluctuations, Econometrica, 50, 1345-70.

Prescott, E. C. and Mehra, R. 1980. Recursive Competitive Equlibrium: The case of homogeneous households, Econometrica, 48,1365-79.

Precott, E. C. 1986. Theory ahead of business cycle measurement. Quarterly Review, Fall. Federal Reserve Bank of Minneapolis.

Taylor, J.B. and Uhlig, H. 1990. Solving nonlinear stochastic growth models: A comparison of Alternative Solution Methods.

Sims, C. A. 1986. Are forecasting models usable for policy analysis? Quaterly Review, Winter. Federal Reserve Bank of Minneapolis. Journal of Business and Economic Statistics,8, 1-51.

Sims, C. A. 1988. Solving nonlinear stochastic optimization and equilibrium models backwards, Discussion Paper 15, Institute for Empirical Macroeconomics, Federal Reserve Bank of Minneapolis and University of Minnesota.

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