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# ESTIMATION OF A GROWTH MODEL WITH ADJUSTMENT COSTS IN PRESENCE OF UNOBSERVABLE SHOCKS

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# **ESTIMATION OF A GROWTH MODEL WITH ADJUSTMENT COSTS IN PRESENCE OF UNOBSERVABLE SHOCKS**

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## 1 INTRODUCTION

The purpose of this paper is to examine the origin and to measure the size of aggregate investment fluctuations in a stochastic general equilibrium model which I estimate using U.S. data. Business cycle theory views fluctuations in productivity as the main source of variation in output and other real variables including capital accumulation. The general equilibrium structure that I study includes disturbances to preferences and to the adjustment costs parameter as two additional sources of fluctuations. This enables me to identify what the relative size of these sources of fluctuations for the model economy must be to mimic the observed variability of aggregate investment.

Traditionally, the empirical literature for investment has used two partial equilibrium models to study its fluctuations. On the one hand, studies based on the neoclassical theory, as originally stated, resulted in good fits but it is difficult to find a structural interpretation for the ad hoc estimated distributed lags. These studies include the so called accelerator models in which the distributed lag structure is just a function of past output. On the other hand, studies based on adjustment cost models did not fit the real data as well as the other models.

Studies based on the adjustment cost technology estimate the positive slope between investment and relative price of new capital,  $q$ , that represents the firm's supply of capital goods. Hayashi (82) assumes a homogeneous technology of degree one and measures  $q$  with asset market prices. Abel and Blanchard (86) approximate the expectation of the discounted present value of a stream of marginal profits to obtain  $q$ . Both studies find some of the positive correlation predicted by the theory, even though the high serial correlation of the residuals of the estimated equations raises some doubts about the specification of the model.

Every partial equilibrium model discussed above presents a simultaneity problem. The consumption and saving decisions of individual

agents are represented through changes of the exogenous variables of the model. In the adjustment cost model, for example, the relative price of capital  $q$  is assumed to capture the expected changes of the cost of capital and of the demand for the firm's output. This specification does not take into account that changes on those variables will also affect the demand for investment goods via the savings decisions.

Kydland and Prescott (82) and Sargent (89) have addressed the issue of aggregate investment in general equilibrium setups. They both assume a single source of aggregate fluctuations. Kydland and Prescott (82) use a version of Solow's neoclassical growth model to study the comovements of real variables resulting from random productivity fluctuations. They do not address the question of the aggregate investment fluctuations and prices. On the other hand, Sargent (89) rationalizes the use of the accelerator model introducing measurement errors on the optimal decision rules of the real variables in a similar structure.

When theory focuses on investment fluctuations, however, it is still a open question as to whether the reported results are robust to changes on the number and the nature of the sources of fluctuations. Several papers have shown the importance of different sources of real shocks to explain economic fluctuations. Blanchard and Quah (88) identify a demand shock and a supply shock in a VAR that includes unemployment and output. Christiano and Eichenbaum (90) consider the effect of two types of shocks, one in government demand and the other in productivity, in a business cycle model to account for the correlation between aggregate hours and productivity.

Here, I try to measure the relative importance of the demand shock, in preferences, and two supply shocks, one to the adjustment costs and one to the production technology. I also study if each disturbance effect is permanent or not. As is the case in the real business cycle literature, the stochastic fluctuations on productivity represent unpredictable technological change. On the other hand the adjustment cost

shock can be view as representing changes in productivity embodied in new capital goods or changes in the taxing of capital and consumption. The shock in preferences represents exogenous factors affecting the agent's willingness to distribute their time between market and nonmarket activities.

The model I study to estimate the sources of investment fluctuations is a version of the neoclassical growth model with a stochastic adjustment cost technology. The technology on the production side uses only one type of capital good and on the output side it is costly to transform one unit of consumption good into one unit of investment good. Once investment has been committed to production the transformation is irreversible and it implies a fixed capital-labor ratio. Finally, capital goods depreciate at a constant rate.

The agents of this economy own an initial stock of capital and receive a time endowment of labor each period. They choose between saving and consumption. Firms choose their own level of output, taking as given the demand for its product. The goods and the input markets are perfectly competitive.

The innovations in preferences, technology and adjustment that drive the cycle, create stochastically varying investment opportunities. All the shocks are allowed to be serially correlated. The rational expectations hypothesis closes the model.

I estimate the structural parameters of the model as a way to test the model for the U.S. quarterly data from 1959 to 1988. The estimation procedure uses the first order conditions of the social planner problem. The fact that the innovations of the shocks at each period must be uncorrelated with the variables in the information set of the agent allows me to create a set of orthogonality conditions. The methods developed by Hansen (82) are then used to minimize an objective function with respect

to the parameters of the model.

The remainder of this paper is organized as follows: Section 2 describes the stochastic growth model with adjustments. Section 3 develops the estimation procedure, describes the data used and discusses the results for the estimated parameters. The last section shows the main findings.

## 2 A STOCHASTIC GROWTH MODEL WITH ADJUSTMENT COSTS

There is an infinitely lived representative consumer in the economy that maximizes its expected discounted utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, (1-L_t), \theta_{1t}), \quad 0 < \beta < 1 \quad (2.1)$$

I chose the functional form of U to be:

$$U = \left( [C_t^{(1-\phi_t)} (1-L_t)\phi_t]^{(1-\gamma)} / (1-\gamma) \right), \quad \gamma \geq 0, 0 \leq \phi \leq 1$$

$$\phi_t = \phi \theta_{1t}$$

The utility function displays unit intratemporal elasticity of substitution between consumption and leisure. This fact is consistent with the U.S. economy observation that the per capita series of labor have shown no significant trend.  $\phi$  is the leisure share parameter and is affected by random shocks  $\theta_{1t}$ . The parameter  $\gamma$  is the coefficient of risk aversion.  $\beta$  is the discount factor.  $\theta_{1t}$  represents a shock to preferences.

A constant returns to scale technology is assumed and the inputs are labor (L) and capital (K). Output (Y) can be allocated to either current consumption or to gross investment (I). Once in place, one good can be transformed into the other, paying a certain cost. There are two exogenous shocks that affect the technology.  $\theta_{2t}$  is a shock to the production of output.  $\theta_{3t}$  represents a shock to the adjustment cost. The technology is then written as :

$$G(C_t, I_t, \theta_{3t}) \leq F(K_{t-1}, L_t, \theta_{2t}) \quad (2.2)$$



The function F is

$$F = \theta_{2t} K_{t-1}^{\alpha} L_t^{(1-\alpha)}, \quad 0 < \alpha < 1$$

The production function is of the Cobb-Douglas form with  $\alpha$  as the capital's share of output. This corresponds with the observation of a constant capital and labor share of output in the United States since 1955. The random variable  $\theta_{2t}$  represents technological random fluctuations.

These functional forms for U and F have been widely used in the business cycle literature to study fluctuations on aggregate variables.

The Adjustment cost function G has the following form:

$$G = \left( A C_t^{\eta} + B \theta_{3t} I_t^{\eta} \right)^{1/\eta}, \quad A > 0, B > 0, \eta > 1$$

The adjustment cost parameter  $\eta$  measures the elasticity of substitution between consumption and investment goods. At the steady state a parameter value for  $\eta$  equal to one with the parameter values A and B equal to one implies that is not cost to transform one unit of investment into one unit of consumption.

Capital has a constant depreciation rate  $\delta$  :

$$K_{t+1} = I_t + (1-\delta) K_t, \quad 0 < \delta < 1 \quad (2.3)$$

The functions F and G are homogeneous of degree one. Therefore the distribution of capital between firms is irrelevant.

The random vector  $\theta_t = (\theta_{1t}, \theta_{2t}, \theta_{3t})$  is stationary and identically distributed over time. The vector of random shocks follow a lognormal first order autoregressive distribution, i.e.:

$$\log \theta_{t+1} = P \log \theta_t + \varepsilon_{t+1}, \quad \varepsilon_t \sim N(0, \Sigma) \quad (2.4)$$

Usually a highly persistent shock is necessary to match the optimal paths of a neoclassical growth model with real data. This model allows for serial correlation in each of the shocks. Each element of  $P$  is denoted by  $\rho_{ij}$ . If contemporaneous correlation between the shocks are not allowed the matrix  $P$  is diagonal.

The social planner's problem is :

$$\text{Max } E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, (1-L_t), \theta_{1t}) \quad (2.5)$$

$$\text{s.t. } G(C_t, I_t, \theta_{3t}) \leq F(K_{t-1}, L_t, \theta_{2t}) \quad \forall t$$

$$K_{t+1} = I_t + (1-\delta)K_t \quad \forall t$$

$$\log \theta_{t+1} = P \log \theta_t + \varepsilon_{t+1}, \quad \varepsilon_t \sim N(0, \Sigma) \quad \forall t$$

$$K_0, \theta_0 \text{ given}$$

$C_t, L_t, K_{t+1}$  and  $I_t$  are the decision variables in each period  $t$ .

The state variables of that problem are the stock of capital  $K_t$  and the vector of shocks  $\theta_t$ . Given the functional forms for  $F, G$ , and  $U$  there exists an optimum for the social planner's problem. The solution is a vector of stationary stochastic processes:  $K_{t+1} = g(K_t, \theta_t)$ ,  $C_t = c(K_t, \theta_t)$ ,  $L_t = l(K_t, \theta_t)$ .

The first order conditions of the optimality problem (2.5) and the technology constraint once the shadow price of output is substituted out are:

$$\left( A C_t^\eta + B \theta_{3t} I_t^\eta \right)^{1/\eta} - \theta_{2t} K_{t-1}^\alpha L_t^{(1-\alpha)} = 0$$

$$\frac{\theta_{1t} \phi C_t}{(1-\theta_{1t} \phi) (1-L_t)} - \frac{\left( A C_t^\eta + B \theta_{3t} I_t^\eta \right)^{(1-1/\eta)}}{A_t C_t^{(\eta-1)}} (1-\alpha) \theta_{2t} K_{t-1}^\alpha L_t^{-\alpha} = 0$$

$$- \frac{B \theta_{3t} I_t^{(\eta-1)} (1-\theta_{1t} \phi)}{A C_t^{-(1-\phi \theta_{1t}) (1-\gamma) + \eta} (1-L_t)^{-\phi \theta_{1t} (1-\gamma)}} + \tag{2.6}$$

$$\beta E_t \frac{(1-\phi \theta_{1t+1}) \left( B (1-\delta) \theta_{3t+1} I_{t+1}^{(\eta-1)} + \alpha \theta_{2t+1} [A C_{t+1}^\eta + B \theta_{3t+1} I_{t+1}^\eta]^{(1-1/\eta)} K_t^{(\alpha-1)} L_{t+1}^{(1-\alpha)} \right)}{A C_{t+1}^{-(1-\phi \theta_{1t+1}) (1-\gamma) + \eta} (1-L_{t+1})^{-\phi \theta_{1t+1} (1-\gamma)}} = 0$$

$$\lim_{t \rightarrow \infty} \beta^t \lambda_t K_t = 0$$

The first equation in (2.6) is the budget constraint. The second optimality condition express the marginal rate of substitution between consumption and labor equal to its cost. The third equation states that the discounted value of the return in an additional unit of investment next

period must be equal to its current cost in terms of consumption goods. The last equation is the transversality condition.  $\lambda$  is the shadow price with respect to the output.

Given the functional forms for  $U$ ,  $F$ , and  $G$  the above conditions are necessary and sufficient for an interior solution. The stationarity solution for (2.6) is obtained if the transversality condition is satisfied and the serial correlation matrix  $P$  has all its eigen values inside the unit circle. If  $P$  is diagonal this means that  $\rho_{ii}$  must be between 0 and 1.

The social optimum solution is also the solution for a sequence of market equilibrium allocations. Prescott and Mehra (1980) show this for a class of economies that includes this one. Consequently, there exists time invariant functions for the wages  $\omega_t = \omega(K_t, \theta_t)$ , rental prices of capital  $u_t = u(K_t, \theta_t)$  and prices of investment  $p_{It} = p_I(K_t, \theta_t)$  where all prices are relative to the date  $t$  consumption good. In this competitive economy the representative consumer sells its labor and capital stock at the competitive prices. The representative firm maximizes profits for each period. Given the sequence of prices for labor and capital the firm produces consumption and investment goods.

### 3. ESTIMATION

#### 3.1 ESTIMATION METHOD

Until very recently, the difficulties in finding equilibrium solutions of rich dynamic rational expectations models have limited the econometric analysis. Two estimation methods have been used for these models, maximum likelihood and Euler equations. In the last few years several methods to solve discrete time dynamic rational expectation models have been proposed. These methods allow the estimation of richer model economies. One of those methods is estimation by simulation. Here I propose a different estimation method for the economy of section 2. I approximate a solution to the optimal problem and use the first order conditions to estimate the structural parameters with instrumental variable methods.

Maximum likelihood methods are used when closed form solutions for the optimal paths of the stochastic models can be obtained. Hansen and Sargent (80) study different strategies with a linear quadratic optimization problem. Outside the linear quadratic problems the optimal solutions to dynamic rational expectations models have no closed form solutions. Altug (89), in a growth model with a source of uncertainty, obtains a linear quadratic approximation of the social planner problem that enables her to solve the equilibrium model and then use maximum likelihood estimations. Other maximum likelihood estimation of a dynamic rational expectations setup also need to solve the stochastic model first incorporating a certainty equivalence assumption.

The generalized method of moments estimation method (Hansen, 82) is based on the conditional moment restriction of the expectation error that comes from the Euler conditions in rational expectations models. In general, the set of first order conditions can be written as:

$$H_1(X_t, b_0) + E_t H_2(X_{t+1}, b_0) = 0 \quad (3.1)$$

And the expectation error  $u_{t+1}$  in (3.1) is defined as:

$$u_{t+1} = H_1(X_t, b_0) + H_2(X_{t+1}, b_0) \quad (3.2)$$

In those equations  $X_t$  is a vector of observable variables, either endogenous or exogenous in the economic model. For example, Hansen and Singleton (82), estimate an asset pricing model with this method. In their model the vector  $X_t$  is formed by consumption and return of different assets. The first component is a decision variable and the others are a function of asset prices that are given.

The presence of unobservable variables in (3.1) does not allow the use of the instrumental variable estimation method developed by Hansen using the expectation error term  $u_{t+1}$ . Suppose that the economic model of interest has a vector of unobservable variables  $\theta_{t+1}$  (different perturbations or shocks, for example) and it has a distribution function that follows a Markov process. Then the first order conditions, budget constraint and distribution of the exogenous process can be expressed as:

$$H_1(X_t, \theta_t, b_0) + E_t H_2(X_{t+1}, \theta_{t+1}, b_0) = 0 \quad (3.3)$$

$$\theta_{t+1} = H_3(\theta_t, \varepsilon_{t+1}, \rho_0), \quad \{\varepsilon_{t+1}\} \text{ i.i.d. process}$$

The estimation by simulation solves the problem of finding estimation parameters for  $\beta_0 = (b_0, \rho_0)$  in a problem like (3.3). Ingram and Lee (89) and Duffie and Singleton (90) study this method of estimation. First, it is necessary to find simulated equilibrium paths of the economy. The different methods of simulation in the literature involve approximations to either the distribution of the exogenous variables or the model itself.

The obtained paths for the economy, for any vector of parameters  $\beta$ , imply an equilibrium transition function for the state variables of the economy  $S_t$  and optimal decision functions for the control variables  $C_t$ , i.e. :

$$S_{t+1} = H_S ( S_t, \varepsilon_{t+1}, \beta )$$

$$C_{t+1} = H_C ( S_{t+1}, \varepsilon_{t+1}, \beta )$$

The second step in the estimation by simulation is to define a function of current and past state variables. The estimator of  $\beta_0$  is chosen to minimize the discrepancy between these functions of observed variables and the corresponding simulated values.

When the number of equations and unobservables variables in the optimality conditions (3.3) are the same we have an alternative estimation method to the estimation by simulation method. The goal is to solve the system of equations (3.3) for the innovations of the unobservable shocks.  $\{ \varepsilon_{t+1} \}$ , i.e.,

$$\varepsilon_{t+1} = G(X_{t+1}, X_t, X_{t-1}, \beta_0) \quad (3.4)$$

These innovations, by construction, are uncorrelated with current and lagged values of the observed variables.

The first step in this estimation procedure involves, as in the estimation by simulation method, an approximation around the system of equations (3.3) that allows one to find the mapping between innovations of the unobservable and observable variables (3.4). Depending on the simulation method we may obtain an expectation error  $u_t$ . This term, like the innovations, is uncorrelated with current and lagged values of observed variables  $X_t$  at each period  $t$ .

Therefore we have found a set of conditions in a rational expectations model with an equal number of observable variables and unobservable variables that the vector of parameters  $\beta_0$  must satisfy. These conditions have been obtained using the system of equations formed by the optimality conditions and the distribution of exogenous variables.

The following is a description of how the above estimation method works for a specific approximation method. The economy structure in Section 2, with the first order conditions given by (15) and the lognormal distribution for the exogenous shocks (14) can be written in the general form of (3.3). In this case,  $X_t = (C_t, L_t, K_t)$  and  $H_i$  is a vector with three components. In this economy the vector of observables,  $X_t$ , contains only endogenous variables.

The expectation term in the first order conditions (15) appears only in the third component of  $H_2$  referring to the optimality condition respect to capital in the next period. This expectation term can be substituted by a random variable  $u_{t+1}$  as shown in (3.2). Then (3.3) becomes:

$$H_1(X_t, \theta_t, b_0) + H_2(X_{t+1}, \theta_{t+1}, b_0) = u_{t+1} \quad \forall t \quad (3.5)$$

$$\log \theta_{t+1} = P \log \theta_t + \varepsilon_{t+1}, \quad \varepsilon_t \sim N(0, \Sigma)$$

$$\text{s.t. } u_t = (0, 0, u_{3t})$$

A transversality condition must be added to system (3.5). Also to satisfy the rational expectations assumption the expectation error must satisfy  $E_t u_{t+1} = 0$ .

Given the observed variables  $\{X_t\}$ , the expression (3.5) can be viewed as a system of three equations and four unknowns  $\{\varepsilon_t, u_t\}$ . The approximation method adds an additional equation to the system of first order conditions on (3.5) that allows one to solve for the vector of



innovations and the expectation error  $\{\varepsilon_t, u_t\}$ .

I consider a first order approximation to the set of first order conditions (15) in Section 2. An eigen value decomposition on that approximation shows that the linearized system of equations around its nonstochastic steady state has solution paths for  $X_t$  that grow at a rate greater than  $1/\beta$  for a given distribution of  $\theta_t$ . This property of the system occurs for all sets of parameters  $b$  considered. Therefore, the transversality condition in that equation system is not satisfied. I force all decision variables to follow stable paths in the linearized system imposing a relation between the innovations and the expectation error.

The condition that guarantees stable solutions for the linear approximation of the optimality problem is added to the nonlinear system (3.5). This condition limits the distribution for the joint random vector  $(\varepsilon_t, u_t)$ . The extra condition is the same one that Sims (89) uses to simulate equilibrium models. It can be written in terms of a linear relation between observed and unobserved variables derived from imposing the restriction on the innovations and the expectation error in the linearized system as:

$$D(X_t, \theta_t; u, \beta_0) = 0 \quad \forall t \quad (3.6)$$

The relationship found for the unobserved vector  $(\varepsilon_t, u_t)$  does not guarantee a stable solution for the stochastic problem (3.5). It provides stable solutions for the optimal problem in a neighborhood of the nonstochastic steady state. Equation (3.6) holds also locally for the nonlinear system of equations (3.5) as long as the stochastic vector has a small variance.

Consider a new system of equations that includes the optimal equations (3.5) and the stability condition from the linearized system (3.6). Those four equations, given a sample for  $X_t$ , can be used to generate solutions for innovations to the shocks  $\{\varepsilon_t\}$  and the expectation error  $\{u_t\}$ .

Consider the random vector  $d_{t+1}$ . The components for  $d_{t+1}$  are the vector of innovations  $\varepsilon_{t+1}$  that correspond to the serially correlated shocks  $\theta_{t+1}$  plus the expectation error  $u_{t+1}$ . The vector  $d_{t+1}$  solves the systems (3.5) and (3.6). By construction, the expectation at  $t$  of  $d_{t+1}$  is zero:

$$E_t d_{t+1}(X_{t+1}, X_t, X_{t-1}, \beta_0) = 0 \quad \forall t \quad (3.7)$$

$$\text{s.t. } d_t = (\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}, u_{3t})'$$

This implies that the expectation at  $t$  of the product of variables in the information set at  $t$  and  $d_{t+1}$  must be zero. Taking unconditional expectations :

$$E d_{t+1} \circ Z_t = E[g(X_{t+1}, X_t, X_{t-1}, \beta)] = 0 \quad (3.8)$$

$$\text{where } Z_t = \{X_{t-s}, \theta_{t-s}, s \geq 0\}$$

Now, following Hansen (82) it is possible to characterize an optimal estimator for  $\beta_0$ . A consistent estimate of the parameter  $\beta$  will satisfy the orthogonality conditions (3.8) for large values of the sample length  $T$ . The estimator  $\hat{\beta}$  is chosen such that it minimizes the quadratic form  $J_T$ :

$$J_T = g_T(\beta)' W_T g_T(\beta) \quad (3.9)$$

$$\text{where } g_T(\beta) = 1/T \sum_{t=1}^T g(X_{t+1}, X_t, X_{t-1}, \beta)$$

The weighting matrix  $W_T$  is chosen to be symmetric and positive definite. Hansen (82) shows the matrix  $W_T$  that implies the smallest asymptotic covariance matrix for an estimator of  $\beta$  that minimizes (3.9) is  $S_0^{-1}$ , the inverse of the autocovariance matrix. The vector  $d_t$  is serially uncorrelated for the adjustment cost problem, therefore, a consistent estimate of  $S_0$  is just the sample autocovariance, i.e. :

$$S_T = 1/T \sum \left[ (d_{t+1} \circ Z_t)' (d_{t+1} \circ Z_t) \right] \quad (3.10)$$

The estimated parameter vector  $\beta_0$  has an asymptotic normal distribution, with covariance matrix :

$$(d_0' S_0 d_0)^{-1}$$

$$\text{where } d_0 = E \left[ (\partial g(X_{t+1}, X_t, X_{t-1}, b_0) / \partial b) \circ Z_t \right] \quad (3.11)$$

The fact that the system of efficiency conditions (3.5) is not linear in the variables requires the use of numerical solutions to get the values for the vector  $d_{t+1}$  at each period of time. The described instrumental variable estimation has some computational advantages over maximum likelihood methods. After employing a simulation method that eliminates the expectation term and using the distribution for the unobservable variables, the recursive structure of the model can be written as:

$$X_{is} = g ( X_{s-1}, \varepsilon_1, \varepsilon_2, \dots, \varepsilon_s, \beta_0 ) \quad \forall s \leq T, i = 1, 2, 3$$

or

$$X_s = G ( X_{s-1}, \varepsilon_1, \varepsilon_2, \dots, \varepsilon_s, \beta_0 ) \quad \forall s \leq T$$

The log conditional likelihood of the sample is the sum of marginals, .e.,

$$\begin{aligned} \log f(X_1, X_2, \dots, X_T; \beta_0) &= \sum \log f_x (X_s | X_1, X_2, \dots, X_{s-1}; \beta_0) = \\ &= \sum_{s=1}^T \log f_{\varepsilon} (\varepsilon_s | \varepsilon_1, \varepsilon_2, \dots, \varepsilon_{s-1}; \beta_0) \quad \left| \frac{\partial \varepsilon_s}{\partial X_s} \right| \end{aligned}$$

where  $f_{\varepsilon}$  is the normal probability distribution function that can be written as function of the sample  $X_1 \dots X_T$ .

Most likely the cost of finding the Jacobian of the transformation between observed variables and the vector of unobservable variables at each  $t$  will be higher than the the instrumental variable procedure.

Garber and King (1983) show how different parameter identification can be achieved when the restrictions on the behavior of the unobservable variables change. In this estimation method the vector  $\beta_0$  is estimated for given distribution of the unobservable shocks. Furthermore the estimation method proposed for the model in Section 2, like other Euler equation methods, is subject to the Garber and King's critique.

### 3.2 DATA

The data used to estimate the adjustment cost model that appears in Section 2 corresponds to the period 1959:1 to 1988:4 for the U.S. economy. The observed series are from Citibank Economic Database. They are expressed in 1982 dollars, at quarterly rates and seasonally adjusted.

All the series have been used in per capita values to interpret the behavior of the representative agent in the economy. Capital, employment and consumption series have been divided by the population age 16 and over.

Because of measurement problems, capital stock series are constructed from the gross non-residential investment series (GIN82). That measure of investment includes all fixed investment on equipment and structures. The initial capital on the sample period is the sum of the value of structures (GOC82), non-residential equipment (GIPNR8) and inventories in durable goods (GLN82) in 1959:1.

Aggregate real consumption is the result of adding the consumption series of nondurable goods (GCN82) and service goods (GCI82).

Labor is constrained in the model to be between zero and one. Therefore, the labor series is the ratio of hours worked and time endowment. Hours worked are measured by manhours of the employed labor force (LHOURS) The representative consumer has a time endowment of 112 hours per week and 4.25 weeks a month which gives a quarterly time endowment of 1428 hours. The obtained average of the labor ratio is around 0.2. This labor series measures hours worked by the employed and unemployed persons.

The observed data for consumption and investment is not stationary, but prefiltering of the data to take out the trend is avoided. This implies an arbitrary restriction. Besides, it could violate the expectation condition that the innovations of the shocks and the expectation error must satisfy in the estimation, i.e.,  $E_t d_{t+1} = 0$ . Therefore, the estimation methods are carried out with the levels of the variables. The cyclical and trend behavior of the endogenous variables are then determined by the behavior of the exogenous variables transmitted via the model mechanisms.

### 3.3 ESTIMATION RESULTS

The parameters of the adjustment cost economy have been estimated following the procedure explained in section 3.1. The estimations are made for two different models. The first model considers the levels of the variables and unitary elasticity of substitution in the utility function at the steady state. The shock in preferences affects the allocation of consumption and leisure. The second model has a constant elasticity of substitution between consumption and leisure. In this case, the shock in preferences affects only the leisure exponent. The technology and the distribution of exogenous shock is the same in both models. The estimations in this second model correspond to optimality conditions with the levels of the observed variables or assuming there is a common trend that affects the series of consumption and capital. this is a way to deal with the existence of real growth in the data.

Several parameters of the model were held constant. I fixed the depreciation rate of capital to .025. It allows me to use the law of motion of capital (3) to construct series of capital from observed investment series. Hall and Jorgenson (1967) who study tax effects on capital accumulation pin down annual depreciation rates for the manufacturing sector. It varies from 15 per cent for equipment to 6 per cent for structures. I use an average annual rate of depreciation of 10 per cent for fixed investment or .025 each quarter.

Also in the technology, the parameters of the G function A and B are equal to one. There is no information available that could help me to fix them differently. In equilibrium, the price of new capital must be equal to the marginal rate of transformation between consumption and investment goods. Then at the steady state  $p_I = (B/A) (\delta K / C)^{\mu-1}$ . If there is no adjustment cost in the economy the equilibrium price of investment goods each period is constant and equal to one as in the standard growth model.

At each period  $t$  the shadow price of output in the optimality conditions (15) depends on four parameters:  $\rho_{11}$ ,  $\eta$ ,  $\phi$ ,  $\gamma$ . At least one is fixed to avoid a possible problem of identification. Of these four, a priori, I expect the labor share parameter  $\phi$  and the risk aversion parameter  $\gamma$  to have less effect over the fluctuations on investment and are pinned to a certain value.

The parameters of the exogenous shocks distribution that make reference to the serial correlation properties,  $\rho_{11}$ ,  $\rho_{22}$  and  $\rho_{33}$  are estimated jointly with three structural parameters  $\beta$ ,  $\alpha$  and  $\eta$  and the trend parameter  $\omega$ . During the estimation procedure all the parameters were restricted to vary within the feasible set that the functions U, F, G and H impose.

Tables 3.1 to 3.4 show the estimated parameters for four different sets of instruments. All include a vector of ones to guarantee that the innovations of the shocks and the expectations error have unconditional mean equal to zero. The other instruments are first and second order lags of the observable set of variables, i.e. C, L and K. The vector of instruments contains three or four elements, therefore the system of equations that form the objective function is overidentified.

A test for the overidentifying restrictions equal to zero is included in the tables. The reported value  $k$  is the number of instruments times four (the number of errors or innovations) minus the number of estimated parameters.  $\chi^2$  is the minimized value of the objective function times the number of observations 116. The probability of accepting the overidentifying restrictions as zero is also reported.

Table 3.1 and 3.2 show the results of estimating the parameters of the model with unitary elasticity of substitution on the utility and the levels of the variables. The labor share parameter  $\phi$  is equal to .67 and



the risk aversion parameter  $\gamma$  is equal to 0.

Although the estimated values do not change much when different sets of instruments are used, the best objective values are obtained when lagged values of labor are considered as instruments. As the number of instruments increases the standard deviation of the estimated parameters are reduced and the objective function increases. Therefore we will reject the model more strongly. Neither the minimized objective function nor the estimated parameters change their values when different starting points are used in the procedure.

The three estimated structural parameters have large standard deviations. Specially the capital share,  $\alpha$ , and the adjustment cost,  $\eta$ , parameters which have standard deviations larger than their point estimates. The discount factor  $\beta$  is always greater than .9 and never hits 1, its upper bound. The parameter  $\alpha$  has estimated values around .12 but its standard deviation indicates that the model can fit the data with values from 0 to .5. The parameter that measures the adjustment cost elasticity  $\eta$  always has an estimated value above one.

The parameters that measure the persistence of different sources of shocks in the model have been estimated precisely. All the shocks are very persistent. The serial correlation parameters of the shocks are close to their maximum value, 1, for all the sets of instruments used. The small variation of the observed series between quarters causes the small estimated changes in the shocks between periods. That persistence implies that the logarithm of the productivity shock follows a random walk in a model in which adjustment costs and other sources of fluctuations are present.

The standard deviation for the innovation of the structural shock innovation for the sample are reported on Table 3.2. For the model with unitary elasticity of substitution the innovation with largest variance is

the adjustment cost innovation. The obtained properties of serial correlation and the sample variance for the logarithm of the technology shock makes the variance of the productivity shock higher than the other shocks.

The bottom of Table 3.1 reports the test for the overidentified restrictions of the objective function. At the five per cent significance level the estimated model is not rejected when less than sixteen orthogonality conditions are used. The lags of labor seem to be the best variables in the information set of the agents to fit the model with the real data.

The technology parameters A and B that have been fixed to a value equal to one are scale factors in the budget constraint. Different values for either one would be equivalent to adding a constant term in the lognormal distribution of the technology shocks  $\theta_{2t}$  and  $\theta_{3t}$ .

When these two parameters are freed up in the estimation procedure for only two of the instruments that appear in Table 3.1 with the estimation results, the program converged to a minimum. When the instruments are  $\{1, l(t-1), l(t-2)\}$  the estimated value for A is 2.2 and for B is 3.0 with standard errors 30 and 135 respectively. For this set of instruments the minimum of the objective function times the number of observations ( $T=116$ ) is reduced by 7 with respect to the restricted model. Therefore the hypothesis that A and B are equal to one is accepted at a 4% critical value with a chi-square with two degrees of freedom.

When the instruments used are  $\{1, c(t-1), c(t-2), l(t-1)\}$  the estimated values are 3.5 and 5.6. Nevertheless, the estimated standard errors have four digits. The difference of the minimized objective function times the number of observations between the restricted and unrestricted model is 23.4 which means that the hypothesis of A and B equal to one is not accepted.

The set of orthogonality conditions have problems trying to identify the estimated values for A and B. With the last set of instruments a different starting point gave very different point estimates for A and B. The inclusion of those scale parameters in the estimation increased the standard error of the other parameters of the model and did not change their point estimates or the sample standard deviations of the shocks innovations with respect to the restricted model with  $A=B=1$ .

The leisure share parameter of the utility function  $\phi$  was fixed at  $2/3$ . This is the value that emerges from the literature. When  $\phi$  is treated as a free parameter in the estimation the standard deviations of the estimated parameters go up. The correlation between the estimated  $\phi$  and  $\eta$  are always above .8 for all the instruments used.

The results presented have assumed a parameter of risk aversion,  $\gamma$ , equal to zero. that is, the agents are risk neutral. The justification is that when the parameter  $\gamma$  is estimated jointly with the other parameters of the model the estimated risk aversion parameter goes to zero for all the instruments used.

Most growth models without adjustment costs consider very modest values of  $\gamma$  for their simulation exercises. The objective function obtains higher values when the utility function is logarithmic (i.e. risk aversion parameter equal to one) rather than when the utility function has a risk parameter equal to 0. The standard error of the estimates are also larger for  $\gamma$  equal to one.

The above result is not new. Recent estimation studies that include the risk aversion parameter and use aggregate data obtain similar conclusions. Hansen and Singleton (82) estimate an asset pricing model using the generalized method of moments. The estimated parameter  $\gamma$  in their model ranges from 1.59 to -1.26 with very large standard deviation. Eichenbaum, Hansen and Singleton (88) using the same methodology in a model

of intertemporal consumption and leisure decisions find low parameter values, from .84 to .15, although always on the concave region for the utility function.

Tables 3.3 and 3.4 present the estimation results for the model that has a utility function with constant elasticity of substitution at the stationary level. Depending on the instruments, the minimized objective function values in Table 3.3, for the observed variables on levels, are lower or higher than the values for the model in Table 3.1. Neither the point estimates nor the standard errors of the estimates vary by much between estimated models. The main difference is that to match the same data, the variance of the shock in preferences for the model with the shock affecting the exponent to leisure has to be 5 times bigger than in the model with the preference shock affecting the leisure share parameter.

When I introduce a common trend on some variables in the model with constant elasticity of substitution the main findings of the estimation do not change. The objective function is reduced because a new parameter,  $\omega$ , is left free. The point estimate for the trend parameter  $\omega$  is very low. Again the large standard deviation of this estimated parameter is non conclusive in regard to the existence of a common trend.

The large standard errors found may be caused by a problem of identification of the estimated parameters. An identification problem occurs when there is an exact relation between the set of parameters that are tried to be estimated. These relations depend on the set of optimal equations. A sufficient condition for identification is that the matrix  $(\begin{smallmatrix} d' & S_0 & d_0 \end{smallmatrix})$  is not singular in expression (3.11).

Table 3.5 has the estimated covariance matrix of the estimated parameters (3.11). This corresponds to the model with unitary elasticity of substitution and lags on labor and capital as instruments. If there exist a linear relation between the parameters, their estimated correlation will be close to one. The highest correlation is between  $\alpha$  and  $\eta$ ,  $-.78$ . Table 3.6

has the eigenvalues and eigenvectors for that estimated covariance matrix. Although none of the eigenvalues are zero, the large difference between the largest and the smallest,  $1.E+5$ , could imply that the covariance matrix is close to a non full rank.

The low estimated discount factor  $\beta$  means a high stationary interest rate in the model. If  $\beta$  is .973 as in Table 3.1, the implied annual interest rate is 11 per cent. This per centage is too high to reproduce the average interest rates in the U.S for the past thirty years. This result is also in contrast to studies that estimate the discount factor from equilibrium asset price equations using Treasury Bill returns. Hansen and Singleton (82), for example, find  $\beta$  above .99. The model that is studied here, though, obtains the return of the asset with a non trivial production sector and does not use information on the behavior of equilibrium prices.

The estimated capital share parameter  $\alpha$  is around .12. It is lower than that estimated from National Income Accounts, .36. Altug(89), in a growth model with time to build technology and Cobb-Douglas production function, obtains a value with large standard error. Her model is consistent with capital share parameter values between .64 and .07. The estimated  $\alpha$  suggests a production sector very intensive in capital and with large elasticity of labor with respect to output. The observed series of hours worked show little variation over the cycle. This fact is more accentuated in this economy where the representative agent is the aggregate of employed and unemployed. The average time worked is one fourth of the total endowment. Capital instead varies a lot with output. Therefore, labor has to be more productive relative to capital .

The estimated adjustment cost parameter implies the existence of some degree of curvature in the product transformation frontier. When the parameter  $\eta$  is fixed to one, the overidentified restrictions of the model with a linear transformation between consumption and investment goods are rejected at the 95 per cent of probability. For that value of  $\eta$  the

estimated discount factor and capital share parameter have still large standard errors and the standard deviation of the adjustment cost shock innovation varies between 1 and 3 per cent.

#### 4 Conclusions

We have been able to estimate the structural parameters of a nonlinear general equilibrium setup that contains unobservable shocks. The estimation procedure uses instrumental variables contemporaneously uncorrelated with the disturbances innovation and it has been based on the fact that the number of shocks is equal to the number of endogenous variables.

The estimation procedure uses a linear approximation around the non-stochastic steady state jointly with the non-linear optimality conditions. It has not been analyzed, though, how the linear approximation affects the results. A more accurate procedure would be to estimate the parameters of a nonlinear approximation (e.g. a polynomial form) of the expectation term jointly with the set of structural parameters.

We have found that the model is not statistically rejected, although the estimated structural parameters have large standard error. When the economy has not adjustment costs the efficiency conditions are rejected by the data.

All the disturbances in this adjustment cost model are close to a random walk. The shock innovation to the production technology has a standard deviation very similar to that used in the business cycle literature, around .8 per cent. The variability of the adjustment cost parameter must be high (5.5 per cent of standard deviation) to fit the data. The estimated disturbance in preferences, smaller than the supply disturbances, is around .2 per cent.

The estimated parameters of the preferences and the technology present large standard errors. This result is common to other estimation studies. Nevertheless an analysis of the estimated covariance matrix shows a possible problem of identification between the parameters of the model.

Table 3.1  
Estimated parameters of the adjustment cost economy \*

$$U_t = \left( C_t^{(1-\phi_t)} (1 - L_t) \phi_t \right)^{(1-\gamma)} / (1-\gamma), \quad \phi_t = \phi \theta_{1t}, \quad \gamma = 0.0, \quad \phi = .67$$

|             | Sets of instruments |                    |                    |                    |                    |
|-------------|---------------------|--------------------|--------------------|--------------------|--------------------|
|             | A                   | B                  | C                  | D                  | E                  |
| $\beta$     | .9352<br>(.8284)    | .9738<br>(.1303)   | .9546<br>(.4203)   | .9725<br>(.1144)   | .9753<br>(.1216)   |
| $\alpha$    | .1233<br>(.8521)    | .1262<br>(.3964)   | .1357<br>(.6536)   | .1141<br>(.2961)   | .1011<br>(.2971)   |
| $\mu$       | 2.0227<br>(3.1778)  | 1.6164<br>(1.6554) | 1.7776<br>(1.5510) | 1.6868<br>(1.3195) | 1.7456<br>(1.5356) |
| $\rho_{11}$ | .9968<br>(.0148)    | .9969<br>(.0122)   | .9971<br>(.0134)   | .9966<br>(.0109)   | .9961<br>(.0111)   |
| $\rho_{22}$ | .9999<br>(.0137)    | .9999<br>(.0067)   | .9999<br>(.0108)   | .9999<br>(.0057)   | .9999<br>(.0057)   |
| $\rho_{33}$ | .9840<br>(.1240)    | .983<br>(.0560)    | .9816<br>(.1307)   | .9787<br>(.0714)   | .9795<br>(.0714)   |
| k           | 6                   | 6                  | 10                 | 10                 | 10                 |
| $\chi^2$    | 7.88                | 12.06              | 17.98              | 19.14              | 24.00              |
| Prob        | .7529               | .939               | .944               | .961               | .992               |

\* Sets of instruments: A=[1, l(t-1), l(t-2)]; B=[1, l(t-1), k(t-1)];  
C=[1, l(t-1), l(t-2), l(t-3)]; D=[1, C(t-1), l(t-1), l(t-2)];  
E=[1, c(t-1), c(t-2), l(t-1)];

Numerical standard deviations are in parentheses.

$\chi^2$  is T times the minimized value for expression (3.9). T=116.

Prob refers to  $\Pr[\chi^2(k) < c]$ , It is the probability that  $\chi^2$  random variate is less than the computed value of the test statistic under the hypothesis that a restriction (3.7) is satisfied.



Table 3.2

Sample standard deviations for the innovation in the exogenous shocks

$$U_t = \left( C_t^{(1-\phi_t)} (1 - L_t) \phi_t \right)^{(1-\gamma)} / (1-\gamma), \quad \phi_t = \phi \theta_{1t}, \quad \gamma = 0.0, \quad \phi = .67$$

| Set of instruments*   | A     | B     | C     | D     | E     |
|-----------------------|-------|-------|-------|-------|-------|
| $\sigma_{\epsilon_1}$ | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 |
| $\sigma_{\epsilon_2}$ | 0.008 | 0.008 | 0.008 | 0.008 | 0.008 |
| $\sigma_{\epsilon_3}$ | 0.072 | 0.055 | 0.064 | 0.067 | 0.070 |

\* Set of instruments: A=[1, l(t-1), l(t-2)]; B=[1, l(t-1), k(t-1)];  
 C=[1, l(t-1), l(t-2), l(t-3)]; D=[1, c(t-1), l(t-1), l(t-2)];  
 E=[1, c(t-1), c(t-2), l(t-1)];

Table 3.3

Estimated parameters of the adjustment cost economy \*

$$U_t = \left( C_t^{\phi_1} (1-L_t)^{\phi_2} \right)^{(1-\gamma)} / (1-\gamma), \phi_{2t} = \phi_2 \theta_{1t}, \gamma = 0.0, \phi_1 = .33, \phi_2 = .67$$

|             | Sets of instruments |                    |                    |                    |                    |                    |
|-------------|---------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
|             | A                   |                    | B                  |                    | C                  |                    |
|             | levels              | detrend.           | levels             | detrend.           | levels             | detrend.           |
| $\beta$     | .9569<br>(.4696)    | .9497<br>(1.4647)  | .3823<br>(2.1596)  | .9688<br>(.5485)   | .9493<br>(.2647)   | .9419<br>(.2822)   |
| $\alpha$    | .1290<br>(.7811)    | .1301<br>(.8879)   | .0932<br>(.3371)   | .4918<br>(.1325)   | .1442<br>(.4911)   | .1454<br>(.5043)   |
| $\mu$       | 1.7883<br>(2.2040)  | 1.8466<br>(3.8380) | 4.5329<br>(7.0543) | 1.6397<br>(6.2461) | 1.7843<br>(1.4174) | 1.8409<br>(1.5204) |
| $\rho_{11}$ | .9963<br>(.0184)    | .9964<br>(.0208)   | .9960<br>(.0161)   | .9963<br>(.9999)   | .9966<br>(.0174)   | .9966<br>(.0179)   |
| $\rho_{22}$ | .9999<br>(.0131)    | .9999<br>(.0291)   | .9999<br>(.0006)   | .9999<br>(.0071)   | .9999<br>(.0093)   | .9999<br>(.0097)   |
| $\rho_{33}$ | .9838<br>(.1315)    | .9841<br>(.1363)   | .9979<br>(.0170)   | .9858<br>(.0838)   | .9884<br>(.1144)   | .9892<br>(.1170)   |
| $\omega$    |                     | .8e-11<br>(.0001)  |                    | .4e-5<br>(3.e-7)   |                    | .4e-11<br>(1.e-5)  |
| k           | 6                   | 5                  | 6                  | 5                  | 10                 | 9                  |
| $\chi^2$    | 6.32                | 5.90               | 13.31              | 10.90              | 15.75              | 15.27              |
| Prob        | .611                | .684               | .961               | .946               | .892               | .916               |

\* Set of instruments: A=[1,1(t-1),1(t-2)]; B=[1,1(t-1),k(t-1)]; C=[1,1(t-1),1(t-2),1(t-3)].

Numerical standard deviations are in parentheses.

$\chi^2$  is T times the minimized value for expression (3.9). T=116.

Prob refers to  $\Pr[\chi^2(k) < c]$ , It is the probability that  $\chi^2$  random variate is less than the computed value of the test statistic under the hypothesis that restriction (3.7) is satisfied

Table 3.4  
Sample standard deviation for the innovation in the exogenous shocks \*

$$U_t = \left( C_t^{\phi_1} (1 - L_t)^{\phi_{2t}} \right)^{(1-\gamma)} / (1-\gamma) , \phi_{2t} = \phi_2 \theta_{1t} , \gamma = 0.0, \phi_1 = .33 , \phi_2 = .67$$

---

| Sets of instruments *    | A      |         | B      |         | C      |         |
|--------------------------|--------|---------|--------|---------|--------|---------|
|                          | Levels | detrend | levels | detrend | levels | detrend |
| $\sigma_{\varepsilon_1}$ | 0.010  | 0.010   | 0.010  | 0.010   | 0.010  | 0.010   |
| $\sigma_{\varepsilon_2}$ | 0.008  | 0.008   | 0.008  | 0.008   | 0.008  | 0.008   |
| $\sigma_{\varepsilon_3}$ | 0.07   | 0.068   | 0.064  | 0.080   | 0.063  | 0.065   |

---

\* Sets of instruments: A=[1,1(t-1),1(t-2)]; B=[1,1(t-1),k(t-1)];  
C=[1,1(t-1),1(t-2),1(t-3)];

TABLE 3.5  
COVARIANCE MATRIX OF ESTIMATED PARAMETERS.\*

| $\beta$                          | $\alpha$                         | $\rho_{11}$    | $\rho_{22}$    |
|----------------------------------|----------------------------------|----------------|----------------|
| $\rho_{33}$                      | $\mu$                            |                |                |
| 0.1759607D-01<br>0.3521847D-02   | -0.3405070D-01<br>0.4123833D-02  | 0.2648459D-03  | 0.3745229D-03  |
| -0.3405070D-01<br>-0.7036255D-02 | 0.1572293D+00<br>-0.5052132D+00  | -0.4973901D-03 | -0.1614639D-02 |
| 0.2648459D-03<br>0.1635865D-04   | -0.4973901D-03<br>-0.2330242D-03 | 0.1464027D-03  | 0.4870443D-04  |
| 0.3745229D-03<br>0.6085550D-04   | -0.1614639D-02<br>0.4870651D-02  | 0.4870443D-04  | 0.4530359D-04  |
| 0.3521847D-02<br>0.3148646D-02   | -0.7036255D-02<br>0.4557636D-02  | 0.1635865D-04  | 0.6085550D-04  |
| 0.4123833D-02<br>0.4557636D-02   | -0.5052132D+00<br>0.2740491D+01  | -0.2330242D-03 | 0.4870651D-02  |

VALUE DERIVATIVE OBJECTIVE FUNCTION AT THE ESTIMATED PARAMETERS

| $\frac{\partial g(b^*)}{\partial b}$ | $W_T$                           | $g_T(b^*)$    |                |
|--------------------------------------|---------------------------------|---------------|----------------|
| -0.4104716D-02<br>-0.1568251D-05     | -0.4132533D-04<br>0.2606831D-01 | 0.1265032D-04 | -0.3188127D+02 |

Notes: The estimated parameters correspond to the ones in Table 3.1 with instruments [1, l(t-1), k(t-1)]. The epsilon for the numerical derivative is .1e-6.

TABLE 3.6 EIGENVALUES AND EIGENVECTORS FOR THE ESTIMATED COVARIANCE MATRIX OF ESTIMATED PARAMETERS.\*

---

EIGENVALUES

---

0.1409453D-04  
0.1533708D-03  
0.2082215D-03  
0.2526286D-02  
0.7991422D-01  
0.2835841D+01

---

EIGENVECTORS

---

0.2329677D-02 -0.1816918D+00 -0.8639825D+00 -0.5382309D-01  
-0.4664830D+00 0.3680619D-02  
  
-0.6889117D-02 -0.1053539D+00 -0.4372182D+00 -0.1286431D+00  
0.8641610D+00 -0.1853917D+00  
  
0.3192951D+00 -0.9264191D+00 0.1986788D+00 0.1642104D-01  
-0.7443787D-02 -0.4784526D-04  
  
-0.9476252D+00 -0.3118518D+00 0.6773065D-01 0.8406253D-02  
-0.9669564D-02 0.1793855D-02  
  
-0.1991982D-02 0.5970290D-02 0.1094470D+00 -0.9898125D+00  
-0.9082307D-01 0.2046150D-02  
  
0.4411472D-03 -0.1868422D-01 -0.7959321D-01 -0.2202226D-01  
0.1649900D+00 0.9826540D+00

---

\* The estimated covariance matrix corresponds to the model with unitary elasticity of substitution estimated with lags of L and K (Table 3.1, column 2).

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