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Abstract

Recursive and sequential estimation procedures are applied to each of two long annual data series on the logarithm of real per capital U.S. GNP, each of which includes the Great Depression. The recursively-calculated Augmented Dickey-Fuller statistic for a unit root, and several sequentially-calculated F-type statistics for structural breaks, are compared with critical values appropriate to such sets of statistics. The results for the long data series are consistent with those found on shorter (post-war) data sets; we do not find statistically significant evidence of a trend break.

1. Introduction

A number of recent papers, in particular Stock and Watson (1986) and Perron and Phillips (1987), test the hypothesis that the logarithm of real per capital gross national product (GNP) in the United States follows a stochastic trend by testing for a unit root in deviations from a deterministic trend. Both papers employ non-parametric variants, developed by Phillips (1987), Perron and Phillips (1988) and Perron (1988, 1989), of the Dickey-Fuller (1979) test for a unit root.

Stock and Watson (1986) extract a 1.5% annual trend growth to induce an approximately driftless series, and use a form of the test that does not allow for a fitted trend. Perron and Phillips (1987) use a more general version of the test which accomodates a non-zero drift. Non-parametric tests have been proposed as way of taking account of the presence of autocorrelations in the first-differenced representation of the process. However, recent work by Schwert (1989) suggests that such non-parametric corrections do not perform well even in large samples. The simulations in Schwert (1989) show that for some (especially MA) error processes, the non-parametric variants of the Dickey-Fuller tests are characterised by low power and incorrect sizes, in samples as large as 1000, and that the use of the Augmented Dickey-Fuller (ADF) test is to be preferred to the use of non-parametrically corrected Dickey-Fuller test statistics.

Stock and Watson (1986) consider three separate real per capital GNP series and test for the existence of a unit root in each[1]. The tests are conducted in different sub-samples and the results are varied. They conclude that there is little or no evidence against a unit root in the post-1919 sub-samples.

Perron and Phillips (1987), using the more general version of the test, conclude that using the N-P series does not allow rejection of the null hypothesis of a unit root either before or after World War II. The F-S series permits rejection of the null on pre-World War II data but not on the post-World War II data (rejection is possible for the full sample).

The purpose of this paper is to re-examine the previous evidence in the detail allowed by recursive estimation procedures. A line of argument initiated by Perron (1989), states that tests for a unit root which use the full sample are biased in favour of accepting the unit root hypothesis if the series has a structural break at some intermediate date. Such structural breaks may take the form of changes in the mean level of a series or changes in trend growth rates. Thus while a series may be stationary around a broken mean or a broken trend, (that is, the mean or slope coefficient takes on two or more different values in different parts of the sample, and the deviation of the series from this changing trend is stationary in each of the sub-samples), standard unit root test which do not take account of these breaks occuring in sub-samples of the series will be dogged by low power. By standard unit root tests we mean those proposed by Dickey and Fuller (1979) and Fuller (1976).

Perron (1989) proposed modifying the standard Dickey-Fuller test by including dummy variables in the Dickey-Fuller regression in order to allow for a break in the trend and mean. He computed critical values appropriate to this modified regression and, using the new critical values, found in favour of a structural break in a majority of the time series investigated earlier by Nelson and Plosser (1974).

An important criticism of the Perron approach, identified originally by Christiano (1988) was that the break date in the series was assumed by Perron to be known so that thre is a pre-test bias in the testing procedure which should be accounted for. In general, it is more reasonable not to assume a priori knowledge of the break date but rather to allow its estimation to be part of the empirical exercise. This is the view taken by Christiano (1988) and is also the premise of the present paper. To endogeneise the choice of the break date it seems natural, following Christiano, to pay attention to maximum o minimum values in a sequence of test statistics constructed through the two procedures described below.

Recursive procedures for testing for a unit root or a structural break have their most natural uses in this setting. Such tests date, in their modern form, to Brown, Durbin and Evans (1975) and their use of the cumulative sums of squares statistic, and Quandt (1960). Recent papers by Kramer, Ploberger and Alt (1988) and Ploberger, Kramer and Kontrus (1989) have extended the Brown, Durbin and Evans analysis to dynamic models. Christiano (1988) and Banerjee, Lumsdaine and Stock (1989) (BLS henceforth) deal with the case where, under the null hypothesis, the data generation process is assumed to have a unit root.

Two different clases of statistics are computed to test for the presence of a unit root in the N-P and F-S series. The first class, commonly termed "recursive statistics", consists of a sequence of statistics which is constructed by incrementing the sample, starting from some minimum size, by one data point at each stage of the recursive procedure. The recursive algorithm estimates the same regression using each of these nested samples, given the minimum sample size required to estimate the first regression in the sequence. The critical values of the test statistics used for this class of tests are given in BLS (1989).

The second class, termed "sequential statistics", uses a sequence of statistics computed by using the full sample at each stage. The regressions in this second sequence differ from each other in postulating varying dates for the break. The first regression in the sequence uses a postulated break date of k_0, where $1 < k_0 < T$, with sucessive regressions in the sequence postulating k_0, $k_0 + 1, \ldots,$ $T - k_0$ as dates for the break. Some of the critical values used for these tests appear in Christiano (1988) and in BLS (1989). We provide additional critical values in section 3 of this paper.

Section 2 describes the recursive and sequential tests in some detail. Section 3 describes this asymptotic properties and provides appropiate critical values[2]. Section 4 presents the results of the tests when applied to the N–P and F–S series, and section 5 concludes. In view of the criticisms in Schwert (1989) we limit the discussion to extended versions of the Augmented Dickey–Fuller tests and do not compute any non–parametric versions.

A word on the notation is necessary. The symbol "–>" denotes weak convergence of the associated probability measures, and "$\stackrel{d}{=}$" signifies equality in distribution. Stochastic processes such as Brownian motion with unit variance on [0,1] and on its sub–intervals indexed by δ are written as B and p–dim Brownian motions as B_p. Similarly, integrals such as $\int_0^1 B(r)\,dr$, $\int_0^1 r(B)\,dr$, $\int_\delta^1 B(r)\,dr$, etc. are written simply as $\int_0^1 B$, $\int_0^1 r\,B$, $\int_\delta^1 B$, etc. All limits given are as the sample size $T \to \infty$.

2. Unit Roots and Recursive Estimation

Consider the case where, under the null hypothesis, a univariate process is postulated to have a non–zero drift and a unit root; one possible alternative hypothesis is that the series is a stationary autoregressive process of order p + 1 with a non–zero drift and a linear trend. Other alternative hypotheses considered in this paper are those of stationarity of the autoregressive process about a broken trend or a broken drift or of the series being driven by a unit root in one part of the sample and by a trend–stationary process in the other[3].

The null hypothesis can be tested using either the point estimate of the first autoregressive coefficient or its regression t-statistic. As it is well known the distributions of both statistics, for a given finite sample size and asymptotically, are non-normal and have been tabulated by Fuller (1976) and Dickey and Fuller (1979). Specifically, the model underlying the test for the unit root is[4]

$$\Delta y_t = \mu + \beta t + \alpha_1 y_{t-1} + \sum_{j=2}^{p} \alpha_j \Delta y_{t-j} + \varepsilon_t \qquad (1)$$

with y_t, in this paper, representing the logarithm of real per capital GNP. If there is a unit root then $\alpha_1 = 0$, and this is the null hypothesis. We take ε_t to be iid with variance σ_ε^2.

Se now turn to a discussion of the recursive and sequential testing procedures which we shall use to test for a unit root in the N-P and F-S series.

2.1. Recursive Tests

The sequence of recursive statistics $\{\hat{\alpha}_1\}_{t_0}^{T}$, $\{\tau_1\}_{t_0}^{T}$ and $\{\hat{\sigma}_\varepsilon^2\}_{t_0}^{T}$ is computed according to a standard algorithm (see, e.g., Harvey (1981)); $\{\hat{\alpha}_1\}$ is the sequence of OLS estimates of the coefficients, $\{\tau_1\}$ is the t-statistic on α_1, $\{\hat{\sigma}_\varepsilon^2\}$ is the sequence of an estimator of the standard error of the ε_t's, and t_0 is the smallest sample used for estimation (i.e., the sample size used for initialization of the recursive procedures). Examination of $\{\tau_1\}$ will give us evidence about the possibility that the time series is governed by a unit root in some part of the sample while it follows a stationary process in another part of the sample.

We will show that although, for any given sample size, the t-statistics have the usual Dickey-Fuller distribution[5], there will be a different critical value for the maximum of minimum of the t-statistics over the sample; in any given sequence of t-ratios the probability that the minimum of this sequence will lie below the standard Dickey-Fuller critical values is greater than the nominal size of the test. Thus the use of the Dickey-Fuller critical values is prone to lead to excessive rejection of the unit root hypothesis. BLS (1989) tabulate some critical values by Monte Carlo simulation, which are reproduced in section 3 below.

2.2. Sequential Tests

A. This test uses the following $T-2k_0$ regressions[6]

$$\Delta y_t = \mu + \beta t + \alpha_1 y_{t-1} + \gamma_1 \Gamma_{1t}(k) + \gamma_2 \Gamma_{2t}(k) + \sum_{j=2}^{p} \alpha_j \Delta y_{t-j} + \varepsilon_t \quad (2)$$

where $\Gamma_{1t}(k) = t\mathbb{1}(t \geq k)$, $\Gamma_{2t}(k) = \mathbb{1}(t \geq k)$, $\mathbb{1}(.)$ being an indicator function, for $t = k, k+1, \ldots, T$ and $k = k_0, k_0+1, \ldots, T-k_0$.

Model (2) represents and trend and mean discontinuous break at time k_0. Under the null hypothesis, $\alpha_1=\gamma_1=\gamma_2=0$, and $\beta=0$, since under H_0, the limiting distributions are invariant to the value of β. The sequential test therefore consists of computing the F-statistics $F_{\gamma 1=\gamma 2=0}$ and/or $F_{\alpha 1=\beta=\gamma 1=\gamma 2=0}$ for each regression, where each regression differs from any other only in the postulated break date. Non-zero values of γ_1 and/or γ_2 are an indication of a structural break in the data series (γ_2 for a mean break, γ_1 for a break in the trend). Again, while for any given break at date k (postulated independently of the data) the F-statistic has the standard F-density, in any given sequence of F's the probability of F^{max} exceeding the standard 5% (10%) critical values is in excess of 5% (10%).

B. Sequential Test for Shift in Trend (BLS (1989))

This test considers the model as in (2) but with $\Gamma_{1t}(k)=(t-k)\mathbb{1}(t\geq k)$ and $\Gamma_{2t}(k)=0$, representing a continous trend with a kink at date k.

In the spirit of Christiano's test, we estimate the $T-2k_0$ regressions given by moving the trend-shift sequentially for $k=k_0$ to $k=T-k_0$. Under the null hypothesis, $\alpha_1=\gamma_1=\gamma_2=0$. Thus for each of the $T-2k_0$ regressions the F-statistic $F_{\gamma 1=\gamma 2=\alpha 1=0}$ is computed and F^{max}, the maximum of this sequence of F-statistics is compared with the appropiate critical values given below.

C. Sequential Test for Jump in Trend (BLS (1989))

This final class of tests is computed as in B above but with $\Gamma_{1t}(k)=\mathbb{1}(t\geq k)$ and $\Gamma_{2t}(k)=0$, since we are testing for a shift in the mean of the series without change of slope in the trend.

3. Asymptotic Distributions and Critical Values

3.1. Recursive Tests

The analysis focuses on the properties of the estimators and test statistics in model (1) under the null $\alpha_1=\beta=0$. Because of the unit root under H_0, it is very convenient to use the transformation suggested in Sims, Stock and Watson (1990), so that under H_0, (2) can be rewritten as

$$\Delta y_t = \Theta' Z_{t-1} + \varepsilon_t \qquad (3)$$

where $Z_t = [Z_t^1, Z_t^2, Z_t^3, Z_t^4]$; $\Theta = [\alpha', \Theta_2, \alpha_1, \Theta_4]$ with $\alpha = [\alpha_2 \ldots \alpha_p]$

where $Z_t^1 = (\Delta y_{t-1} - \bar{\mu}, \ldots, \Delta y_{t-p} - \bar{\mu})'$, $Z_t^2=1$, $Z_t^3=(y_t-\bar{\mu}t)$ and $Z_t^4 = t$

such that $\bar{\mu} = \mu b$ with $b = (1 - \sum_{2}^{p} \alpha_i)^{-1}$, the unconditional mean under H_0.

Also let $E\, Z_t^1 Z_t^{1\prime} = \Omega_p$. The transformed regressors are linear combinations of the original regressors with different stochastic orders of convergence. So define the scaling matrix $\Upsilon_T = \text{diag}\,(T^{1/2} I_p, T^{1/2}, T, T^{3/2})$ partitioned conformably with Z_t and Θ. Since the recursive regressions take place over the samples going from 1 to $k(= k_0, \ldots, T)$, define $\delta = k/T$, and the recursive OLS estimator of the coefficient vector is

$$\hat{\Theta}(\delta) = [\sum_{1}^{[T\delta]} Z_{t-1} Z_{t-1}']^{-1} [\sum_{1}^{[T\delta]} Z_{t-1} \Delta y_t] \tag{4}$$

where $0 < \delta_0 \leqslant \delta \leqslant 1$. Thus

$$\Upsilon_T (\hat{\Theta}(\delta) - \Theta) = V_T(\delta)^{-1} \Phi_T(\delta) \tag{5}$$

where $V_T(\delta) = \Upsilon_T^{-1} \sum_{1}^{[T\delta]} Z_{t-1} Z_{t-1}' \Upsilon_T^{-1}$ and $\Phi_T(\delta) = \Upsilon_T^{-1} \sum_{1}^{[T\delta]} Z_{t-1} \varepsilon_t$

There are analogous expressions for a general recursively computed Wald statistics and for the Dickey-Fuller t-statistic testing the hypothesis that $\alpha_1 = 0$. Suppose that the Wald statistic tests the q hypothesis $R\Theta = r$ where, without loss of generality, the hypothesis are ordered so that the first restrictions involve coefficients on Z_t^1 (and perhaps Z_t^2, Z_t^3, Z_t^4), the next restriccions involve coefficients on Z_t^2 (and perhaps Z_t^3 and Z_t^4), and so forth. The test statistics that we will consider are

$$F_T(\delta) = (R\hat{\Theta}(\delta) - r)' \, [R(\sum_{1}^{[T\delta]} Z_{t-1} Z_{t-1}')^{-1} R']^{-1} (R\hat{\Theta}(\delta) - r) \tag{6}$$

$$t_{DF}(\delta) = T\, \hat{\alpha}_1(\delta) / [V_T(\delta)^{33}\, \hat{\sigma}_\varepsilon^2]^{1/2} \tag{7}$$

where $\hat{\sigma}^2_\varepsilon(\delta)$ is the estimator of σ^2_ε computed using the residuals estimated through the $[T\delta]$–th observation and $V_T(\delta)^{ij}$ denotes the (i,j) element of $V_T(\delta)^{-1}$.

Then the asymptotic behaviour of the recursive estimators and test–statistics are summarized in the following Theorem (proofs in Appendix).

Theorem 1: Suppose that y_t is generated by model (1), then

a) $\Upsilon_T(\hat{\Theta}(\delta)-\Theta) \to \Theta^*(\delta) \equiv V(\delta)^{-1}\ \Phi(\delta)$

b) Under the null $R(\Theta) = r$

$$F_T(\delta) \to [R\ \Theta^*(\delta)]'\ [R^*V(\delta)^{-1}\ R^{*\prime}]^{-1}\ [R\ \Theta^*(\delta)]/\sigma^2 \equiv F^*(\delta)$$

c) Under the null $\alpha_1 = 0$

$$t_{DF}(\delta) \to \Theta^*_3(\delta)/[\sigma^2\ V(\delta)^{33}]^{1/2}$$

where $V(\delta)$ and $\Phi(\delta)$ are partioned conformably with Υ_T and

$$\Phi(\delta) = \sigma_\varepsilon[\Omega_p^{1/2}\ B(\delta)',\ b\ B(\delta),\ b\ \sigma/2\ [B(\delta)^2-\delta],\ \delta\ B(\delta) - \textstyle\int_0^\delta B]'$$

$$V_{11} = \sigma^2_\varepsilon\ \delta\ \Omega_p,\ V_{1j} = 0\ (j = 2,3,4)$$

$$V_{22} = \delta,\ V_{23} = \sigma\ b \textstyle\int_0^\delta Bdr,\ V_{24} = \delta^2/2$$

$$V_{33} = \sigma^2\ b^2 \textstyle\int_0^\delta B^2,\ V_{34} = \sigma\ b \int_0^\delta r\ B,\ V_{44} = \delta^3/3$$

Table 1, taken form BLS (1989), reproduces the 5% (10%9 critical values for four statistics: the full sample Dickey–Fuller test (δ=1), the maximal Dickey–Fuller statistic (t^{max}), the minimal Dickey–Fuller statistic (t^{min}) and the spread between the maximal and minimal statistics (t^{diff}). The Monte Carlo experimentation in BLS show that t^{diff} is more powerful but its size is substantially larger than the level, so that care is needed in the interpretation of the results with this statistic.

3.2 Sequential Tests

The model considered is (2) which under the null that $\delta=\alpha_1=\gamma_1=\gamma_2=0$ can be reparameterised as in (3) with Z_t^1, Z_t^2, Z_t^3 defined as above, and $Z_t^4=\Gamma_{1t+1}(k)$, $Z_t^5=\Gamma_{2t+1}(k)$. The corresponding scaling matrices are, according to each case:

A) $\Upsilon_T^A = \text{diag}\ (T^{1/2}\ I_p,\ T^{1/2},\ T,\ T^{3/2},\ T^{3/2},\ T^{1/2})$

B) $\Upsilon_T^B = \text{diag}\ (T^{1/2}\ I_p,\ T^{1/2},\ T,\ T^{3/2},\ T^{3/2})$

C) $\Upsilon_T^C = \text{diag}\ (T^{1/2}\ I_p,\ T^{1/2},\ T,\ T^{3/2},\ T^{1/2})$

The estimators and test statistics are computed using the full T observations, for $k=k_0,\ldots T-k_0$. For $k/T \rightarrow \delta$, the process of these sequential statistics is given by

$$\hat{\Theta}(\delta) = [\Sigma\ Z_{t-1}[T\delta]\ Z'_{t-1}[T\delta]]^{-1}\ [\Sigma\ Z_{t-1}[T\delta]\ \Delta\ y_t] \qquad (8)$$

$$\Upsilon_T^i(\Theta(\hat{\delta})-\Theta) = V_T(\delta)^{-1}\ \Psi_T(\delta) \quad (i = A,B,C) \qquad (9)$$

$$F_T(\delta) = [R\hat{\Theta}(\delta)-r]'[R(\Sigma\ Z_{t-1}[T\delta]\ Z'_{t-1}[T\delta])^{-1}R']^{-1}[R\hat{\Theta}(\delta)-r]/\hat{\sigma}_\varepsilon^2 \quad (10)$$

where all the sums go from 2 to T and $0<\delta_0\leq\delta\leq 1-\delta_0<1$

As in the recursive statistics case, the asymptotic representation of these processes can be summarized in the following theorem.

Theorem 2: Suppose that y_t is generated by model (2) with $\beta=\alpha_1=\gamma_1=\gamma_2=0$, then

a) In case A, $\Upsilon_T^A(\hat{\Theta}(\delta)-\Theta) \rightarrow U(\delta)^{-1}\ \Psi(\delta)$, where

$$\Psi(\delta) = \sigma[\Omega_p^{1/2}B(1)',\ B(1),\ b\sigma/2[B^2(1)-1],\ B(1)-\int_0^1 B,\ B(\delta)-\int_\delta^1 B,\ B(1)-B(\delta)]$$

$U_{11} = \sigma_\varepsilon^2\ \Omega_p\ ;\ U_{1j} = 0\ (j = 2,3\ldots,6)$

$U_{22} = 1;\ U_{23} = \sigma b\int_0^1;\ U_{24} = 1/2;\ U_{25} = (1-\delta^2)/2;\ U_{26} = 1-\delta$

$U_{33} = b^2\sigma_\varepsilon^2\int_0^1 B^2;\ U^{34} = \sigma b\int_0^1 rB;\ U_{35} = \sigma b\int_\delta^1 rB;\ U_{36} = \sigma b\int_\delta^1 B,$

$U_{44} = 1/3;\ U_{45} = (1-\delta^3)/3;\ U_{46} = (1-\delta^2)/2$

$U_{55} = (1-\delta^3)/3;\ U_{56} = (1-\delta^2)/2;\ U_{66} = 1-\delta$

b) In case B, $\Upsilon_T^B(\hat{\Theta}(\delta)-\Theta) \rightarrow U(\delta)^{-1}\ \Psi(\delta)$, where

$$\Psi(\delta) = \sigma[\Omega_p^{1/2}\ B(1)',\ B(1),\ b\sigma/2[B^2(1)-1],\ B(1)-\int_0^1 B,\ (1-\delta)\ B(1)-\int_\delta^1 B]$$

and U is a(5x5) matrix with elements as in case A except that

$U_{35} = \sigma_\varepsilon[(1-\delta)\ B(1)-\int_\delta^1 B];\ U_{45} = (1-\delta^3)/3 - (\delta-\delta^3)/2$

c) In case C, $\Upsilon_T^C(\hat{\Theta}(\delta)-\Theta) \rightarrow U(\delta)^{-1}\ \Psi(\delta)$, where

$$\Psi(\delta) = \sigma[\Omega_p^{1/2}\ B(1)',\ B(1),\ b\sigma/2[B^2(1)-1],\ B(1)-\int_0^1 B,\ (1) - B(\sigma)]$$

and U is as in case A, except that the 5^{th} column and row of the matrix are eliminated.

The result provides joint uniform convergence of all estimators and test statistics, including Wald tests as in Theorem 1. Accordindly we use simulations to compute the critical values of the distribution of F^{max} in the sequence of regressions given by model (2). The critical values, in case A, are given in the second and third columns in Table 2, for sample sizes of 50, 100 and 150 in their 2 an 4 d.f versions and are consistent with those reported by Christiano (1988) for a sample size of 152, using a bootstrap method. This table again reflects the relative insensibility of the critical values to changes in the sample size. The critical values, in cases B and C, are presented in the third and fourth columns in Table 2 respectively and again we rely upon the insensitivity of these critical values in applying them to both F-S and N-P series.

Note finally, that although the results in both theorems are stated for the null model in which $\alpha_1=0$ and $\beta=0$ (also $\gamma_1=\gamma_2=0$ in Theorem 2, the results are sufficiently general to handle the case where $|\alpha_1|<1$, $\beta\neq0$ ($\gamma_1\neq0$, $\gamma_2\neq0$) as noted by BLS (1989). In this case naturally the result on $t_{DF}(\delta)$ does no longer hold, since its distribution is standard.

4. Empirical Results

Figures 1 and 2 plot the Nelson-Plosser and Friedman-Schwartz real per capital GNP series. As is clear from inspection of these figures, the two series are well, but not perfectly, correlated over the periods in which they overlap. We begin our analysis of these series by reporting the full-sample estimates of the Augmented Dickey-Fuller regression applied to each (with one lag):

Nelson-Plosser (1911-1970)

$$\underset{}{\Delta y_t} = \underset{(3.05)}{1.278} + \underset{(3.02)}{0.0035t} - \underset{(3.05)}{0.182y_{t-1}} + \underset{(3.39)}{0.410\,\Delta y_{t-1}} \qquad (11)$$

s = 0.059; LM(5) = 2.52; N(2) = 3.61; ARCH(5) = 1.94; H(6) = 10.81

Friedman–Schwartz (1871–1975)

$$\Delta y_t = \underset{(3.93)}{1.474} + \underset{(3.81)}{0.0037t} - \underset{(3.89)}{0.226y_{t-1}} + \underset{(2.55)}{0.247\Delta y_{t-1}} \tag{12}$$

s = 0.061; LM(5) = 8.50; N(2) = 24.35; ARCH(5) = 6.79; H(6) = 12.15

Absolute values of t–ratios are in parentheses; s denotes the standard error of the regression; LM(q) is the Lagrange Multiplier test against an AR(q) or MA(q) in the disturbance (see Godfrey (1978)); N(2) is the Jarque–Bera (1980) normality test; ARCH(g) is Engle's (1982) test against 8^{th} order ARCH distrubances; H(.) is White's (1980) heteroskedasticity test. In each case the statistic is asymptotically distributed χ^2 with the number of degrees of freedom bracketed.

Using Fuller's critical values of –3.50 and –3.45 for the t–ratios on $\hat{\alpha}_1$ in (4) and (5) respectively, we can reject the null hypothesis of a unit root on the full sample for the F–S series, but not for the shorter N–P series. There are, however, several features of these results which lead us to consider a further examination of the data.

First, as Figures 3 and 4 show, the t–ratios change substantially, particularly in the vicinity of the Great Depression, so that the inference with respect to the null of a unit root would change over the sample. One could possibly argue, following Perron, that a structural break could account for the low absolute values of these statistics in the vicinity of 1930 in each series. Second, in the case of the F–S series the null hypothesis of normality of the residuals is rejected. This suggests that the Dickey–Fuller critical values will not be precisely appropriate, these values having been computed using a data generation process which incorporated normal errors. Since the realised statistic for the F–S series does not exceed the 5% critical value by a large margin, the test cannot be deemed conclusive. Moreover, as this skewness–kurtosis normality test is very sensitive to a few especially large deviations, it is conceivable that a structural break or trend shift in the series could also account for the normality test result. For these reasons, we test for structural breaks below.

From BLS we take the critical values for the maximum and minimum Dickey and Fuller statistics observed over a set of successive samples. The sample size of 100 is close to the size of the F-S data set and may also realiably be used to conduct inference in the shorter N-P series, since the critical values are not very sensitive to changes in the sample size.

On the N-P sample, the minimum of the t-statistics is -3.80 in 1924; on the F-S series the minimum is -3.89 in 1975, with a local minimum in 1930 of -3.76. From the values in Table 1, it is clear that these are very close to the 10% critical values. Bearing in mind the non-normality in the F-S series that makes literal application of these critical values hazardous, we may have some suggestive evidence against the hypothesis of a unit root, but not a firm rejection. However, the maximum values of the t-statistics are -1.36 and -1.68 whereas the difference t-statistics are 2.44 and 2.21 respectively, in both cases non rejecting the null hypothesis.

Figures 5 and 6 plot Christiano-type F-statistics with 2 d.f. for the trend and mean break. The maxima are 8.19 in 1939 for the N-P series, and 6.07 in 1930 for the F-S series. Neither of these is significant at even the 10% level, under the null of a random walk model with no break, and for a joint alternative of trend and mean break.

In order to be confident in the result of this structural break test (since it is not possible to be certain that the stochastic trend is a better characterisation of the data than a deterministic trend), we might also want to compute the critical values for the 2 d.f. test under the null hypothesis of stationary deviations from a deterministic trend (again following Christiano). We do so in Table 3. The form of null hypothesis chosen is based upon the regressions (3) and (4): $y_t = 1 + 0.003t + 0.8y_{t-1} + \varepsilon_t$ is a fair approximation to these results and to those of models which drop the term in Δy_{t-1}, on both data sets. The initial observation is again chosen to be a fixed constant.

Again, the test statistics just given are lower than the critical values. For neither of the null models, then, can we conclude that there is a structural break. The t-statistics in the unit root tests reported earlier can therefore be interpreted in the usual way; a structural break is not a persuasive explanation for any failures to reject the unit root model.

Similarly, Christiano-type F-statistics with 4 d.f. are computed, with maximal values of 13.75 and 14.10 for the N-P and F-S series, both values being highly non significant.

Sequential statistics for the alternative of a trend break against the null of no break in each of the series are plotted in Figures 7 and 8. The maxima of the F-statistics are 5.73 in 1932 for N-P, and 6.61, also in 1932, for F-S. Again, neither is significant at even the 10% level, using the BLS critical values (reproduced in Table 2 above).

Finally, sequential statistics for the alternative hypotyhesis of a mean break are reported in Figure 9 and 10. The maxima are 6.87 in 1938 (N-P) and 6.63 in 1938 (F-S). Once again, the statistics offer no statistically significant evidence of such a break.

5. Conclusion

In summary, we are able to reject the null hypothesis of a unit root only for the full F-S sample, although we give reasons for interpreting this rejection cautiously; the recursive test statistics come close to rejecting this null as well. There is much less evidence against the null hypothesis of no structural break, whether or not the null also incorporates a difference-stationary or trend-stationary process for GNP; although the statistics used for testing the break have their largest values in the neighbourhood of 1930's, they do not reach even our 10% Monte Carlo critical values for these tests. This finding allows us to interpret the recursive t-statistics at face value, putting aside the possibility of a break.

In classical statistics, the investigator is viewed as using data to test an hypothesis formed independently of the data on which it is to be tested. When the hypothesis under test is chosen based on the data, it is clearly relatively easy to find one which will not be rejected on the given sample.

In the case of at least some hypotheses relating to real per capita GNP, particularly those involving structural breaks, it is a little difficult to imagine an investigator in the purely classical position; the general features of the data are well known. An hypothesis such as H_0: a structural break in GNP took place c. 1930 seems very likely to have been influenced by knowledge of the data. It is with this in mind that sequential tests are applied here.

Clearly an investigator in the classical position, with the a priori hypothesis that there was a structural break in GNP c. 1930, would reject a null of no break; the corresponding F-statistics exceed 5% critical values by a comfortable margin. However, for the perhaps more realistic test, that of the hypothesis that a break took place somewhere on the sample — so that we do not use our knowledge of the data ("pre-test" examination) to choose the break point -- rejection of the null of no break is not possible at conventional levels, in spite of the appearance of some fairly large statistics. Thus, these relatively long data sets confirm the results found by Christiano (1988) and BLS (1989) for post-war data sets.

Appendix

For convenience it is assumed that $y_0=0$ and that ε_t is $iid(0,\sigma_\varepsilon^2)$. Both assumptions can be generalised, to y_0 being drawn from a stationary distribution and to ε_t being a martingale difference sequence (MSD) with at least bounded fourth moments, at the cost of complicating the algebra.

Proof of Theorem 1

a) Without loss of generality it is assumed that $\mu=0$. Let $r=[T\delta]$ and define the sequence of partial seems $S_T(\delta)=\sum_1^r \varepsilon_i$. The functional Central Limit Theorem implies that $S_T(\delta) \rightarrow \sigma\, B(\delta)$. Then let $C(L) = (1 - \sum_2^p \alpha_i\, L^i)^{-1}$ and write $y_t = C(1)\, S_t + C^*(L)\, \varepsilon_t$ where $C^*(L)$ has all its roots outside the unit circle. It follow that $T^{-1/2}$ $y_r \rightarrow \sigma b B(\delta)$ with $b = C(1)$. Because the sample size is 1 to k $(k=k_0 \ldots T)$, there is uniform convergence of each element of Φ_T and V_T in $(0,\delta)$. Then apply the following lemma in Stock (1987):

Lemma: Let y_t be an I(1) time series with a Wold representation given by $y_t=C(1)\, S_t + C^*(L)\, \varepsilon_t$, then

i) $T^{-2} \sum y_t^2 \rightarrow \sigma_\varepsilon^2\, C(1)^2 \int B^2$

ii) $T^{-1} \sum y_{t-1}\, \varepsilon_t \rightarrow C(1)\, \sigma_\varepsilon^2/2\, [B^2(1)-1]$

iii) $T^{-3/2} \sum t\, \varepsilon_t \rightarrow \sigma_\varepsilon\, [B(1) - \int_0^1 B]$

to obtain all the stochastic limiting distributions in the text, except Φ_1 and V_{11}, interchanging the (0,1) integral limits by

$(0,\delta)$. Because $_{t-1}\varepsilon_t$ is a MSD with $\sup(z^1_{it-1}\varepsilon_t)^4 < \infty$, it follows that $\Phi_{1T}(\delta) \rightarrow \sigma_\varepsilon \Omega_p^{1/2} B_p(\delta)$, i.e a p–dim. Brownian motion with covariance $\sigma^2_\varepsilon E z^1_t z^{1\prime}_t = \sigma^2_\varepsilon \Omega_p$. Finally the deterministic components such as V_{22}, V_{24} and V_{44} are obtained as $\int ds$, $\int sds$ and $\int s^2 ds$ with integral limits between 0 and δ.

b & c) Given the convergence results in (a) and that $\hat{\sigma}^2(\delta) \rightarrow \sigma^2$, both limiting distributions follow directly from Theorem 2 of Sims, Stock and Watson (1990).

<u>Proof of Theorem 2</u>

a) The proof is similar to the proof of Theorem 1 except that now the sample size goes from 1 to T. The terms involving $\Gamma_{1t}(k)$ and $\Gamma_{2t}(k)$ such as Ψ_5 and Ψ_6 follow directly. For example $T^{-3/2} \sum_1^T t\, \mathbf{1}(t \geq k)\, \varepsilon_t = T^{-3/2} \sum_k^T t\, \varepsilon_t$ wich implies integral limits going from δ to 1. The deterministic components such as $U_{55T}(\delta) = T^{-3} \sum_1^r [t\mathbf{1}(t \geq k)]^2$ tend in limit to $\int_\delta^1 s^2\, ds$ etc.

b & c) The argument for both cases is similar to that for case 1.

Footnotes

1. These are the Friedman and Schwartz (1982) annual series from (1869–1975) (a modified version of the Kuznets (1961) series, the Nelson and Plosser (1982) annual series from 1909 to 1970 (constructed by the US Dpt. of Commerce) and the National Income and Product Account series. Only the first two, which we refer to as F–S and N–P, are used here; the NIPA series starts only in 1929, and is therefore not long enough for examination of the Great Depession when allowance is made for initialization of the recursive procedures.

2. This section relies upon results in BLS (1989) where weaker conditions are assumed for the disturbance ε_t.

3. This is, of course the alternative considered by Christiano (1988) and Perron (1989).

4. Which may differ from the data generation process itself.

5. Banerjee et al. (1990) provide an approximate algorithm to compute critical values for the Dickey–Fuller statistic in recursive samples.

6. Where T is the total sample minus the two observations lost in estimating two lags. This sample is further trimmed by two observations at each end.

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TABLE 1

Recursive Unit Root Tests: Critical Values

(2.000 Replications)

5% (10%) Critical Values

T	$t_{DF}(1)$	t^{min}	t^{max}	t^{diff}
100	−3.45(−3.15)	−4.13(−3.88)	−1.93(−2 21)	3.37(2.95)
250	−3.43(−3.13)	−4.07(−3.80)	−1.88(−2.14)	3.36(2.98)
500	−3.42(−3.13)	−4.10(−3.82)	−1.88(−2.14)	3.45(3.01)

Note: Critical values for $t_{DF}(1)$ taken from Table 8.5.2 in Fuller (1976); the critical values for the remaining t-statistics taken from Table 1 in BLS (1989), generated by the null model $\Delta y_t = \varepsilon_t$, with $0.25 \leqslant \delta \leqslant 1$.

TABLE 2

Sequential Tests for Trend and Mean Break (Cases A, B, C):

Estimated Critical values (Null incorporates a stochastic trend)

(2.000 Replications)

5% (10%) Critical Values

T	A(2)	A(4)	B(3)	C(3)
50	12.11(10.81)	20 38(18.13)	17.63(15.16)	18 73(16.02)
100	12.32(10.91)	20.31(18.11)	16.74(14.30)	18.40(15.92)
150	12.15(10.90)	20.29(18.12)	16.39(13.85)	18.50(16.06)

Note: The entries we computed using data generated by the null model $\Delta y_t = \varepsilon_t$ for $0.10 \leqslant \delta \leqslant 0.90$, A(2) and A(4) refer to tests with 2 and 4 d.f. respectively, whereas B(.) and C(.) have 3 d.f.

TABLE 3

Sequential Tests for Trend and Mean Break (Case A): Estimated Critical Values

(Null incorporates a deterministic trend)

(2,000 Replications)

T/size	10%	5%	1%
50	9.1	10.7	13.7
100	8.4	9.8	11.7

Note: The entries were computed using data generated by the null model $y_t=1+0.003t+0.8 y_{t-1}$ for $0.10\leq\delta\leq0.90$

Figure 1

N-P: 1909-1970

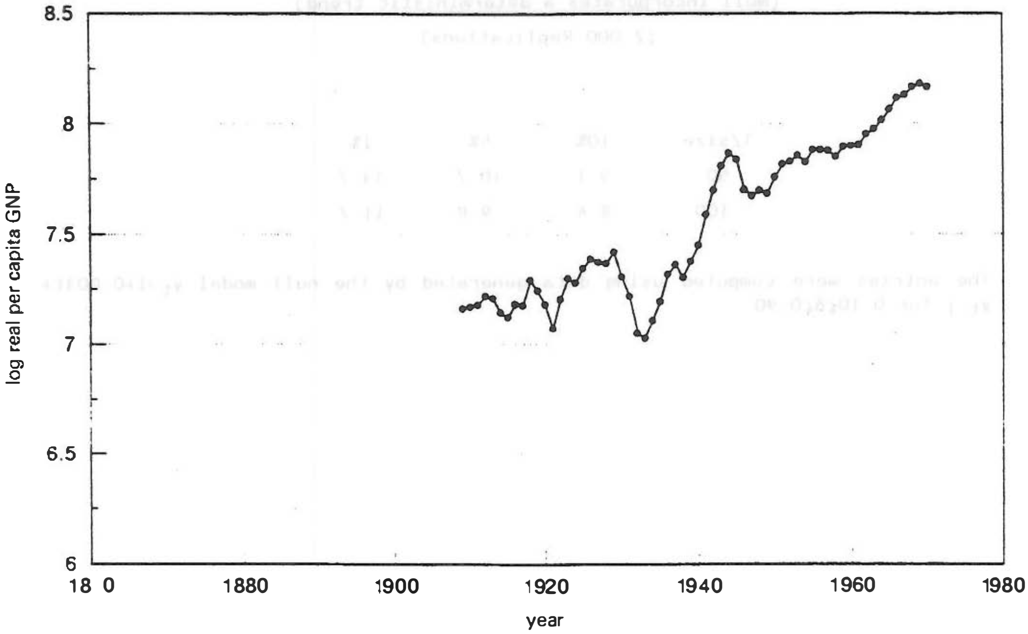


Figure 2

F-S: 1869-1975

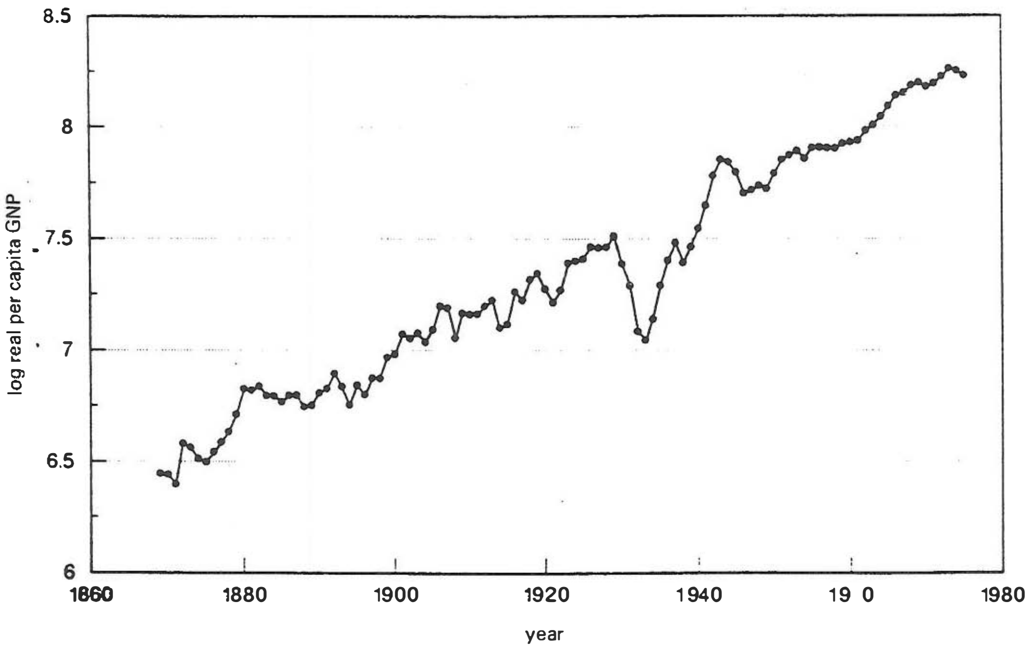


Figure 3

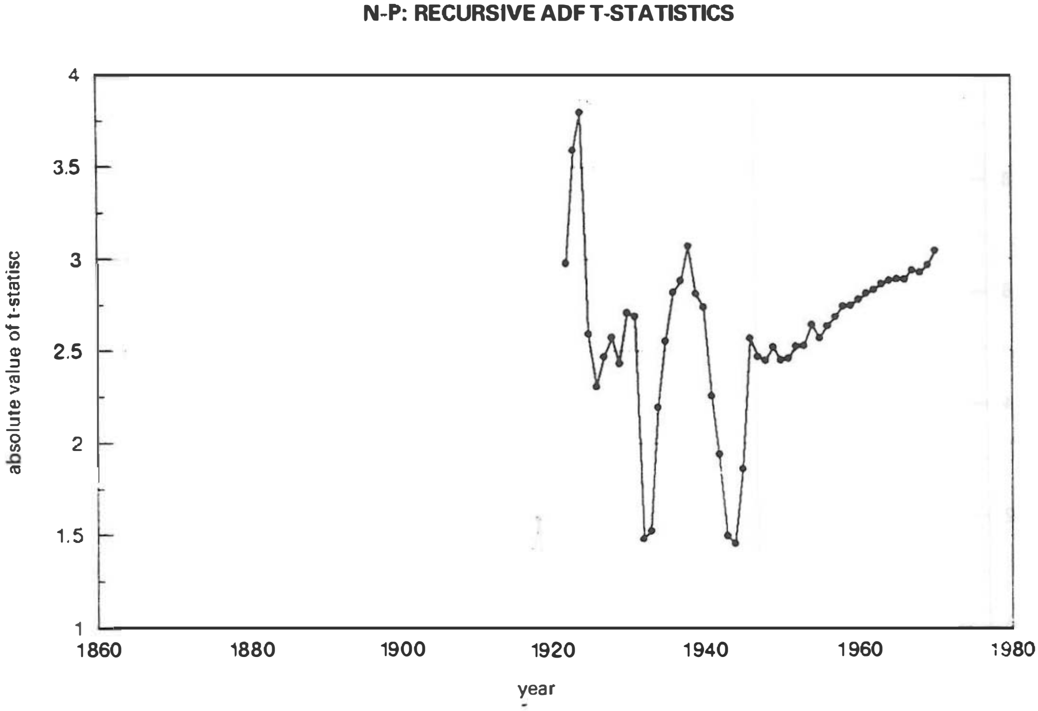


Figure 4

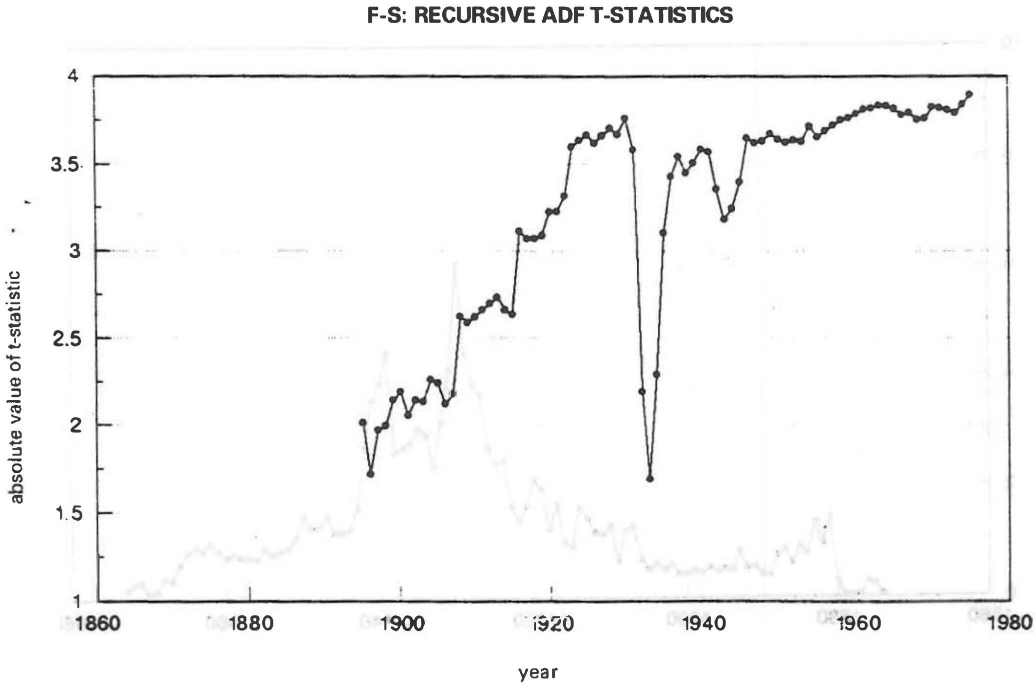


Figure 5

N-P: F-TESTS FOR TREND AND MEAN BREAK

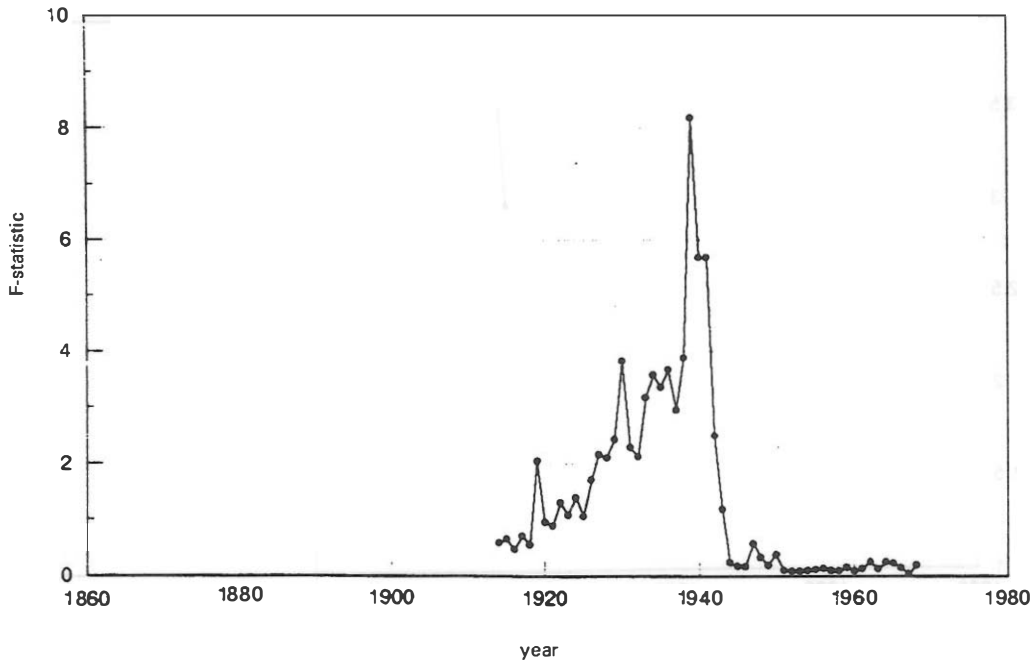


Figure 6

F-S: F-TESTS FOR TREND AND MEAN BREAK

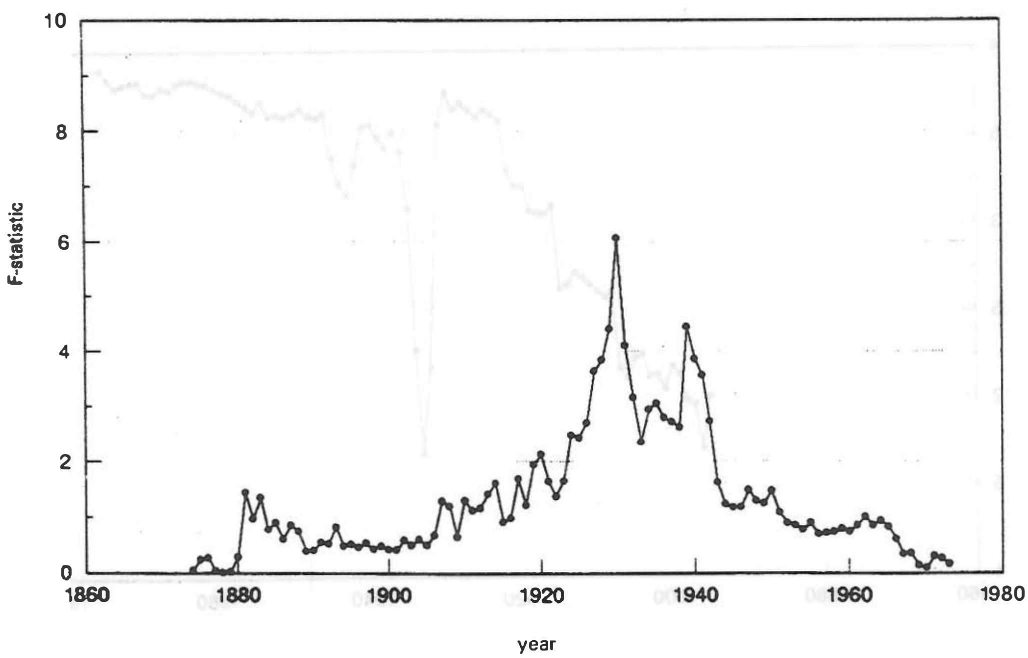


Figure 7

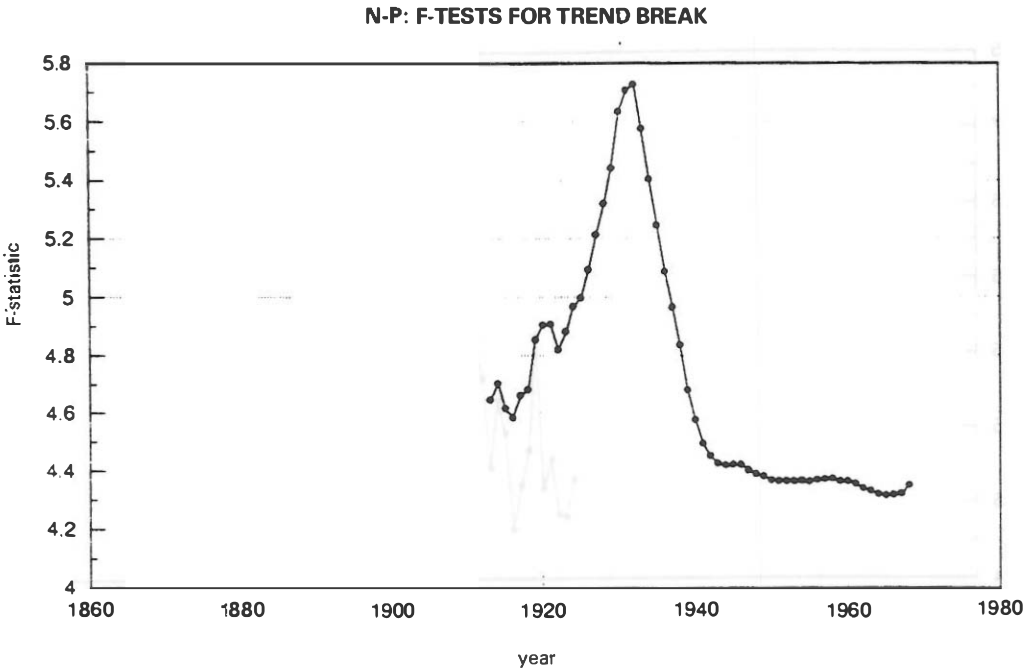


Figure 8

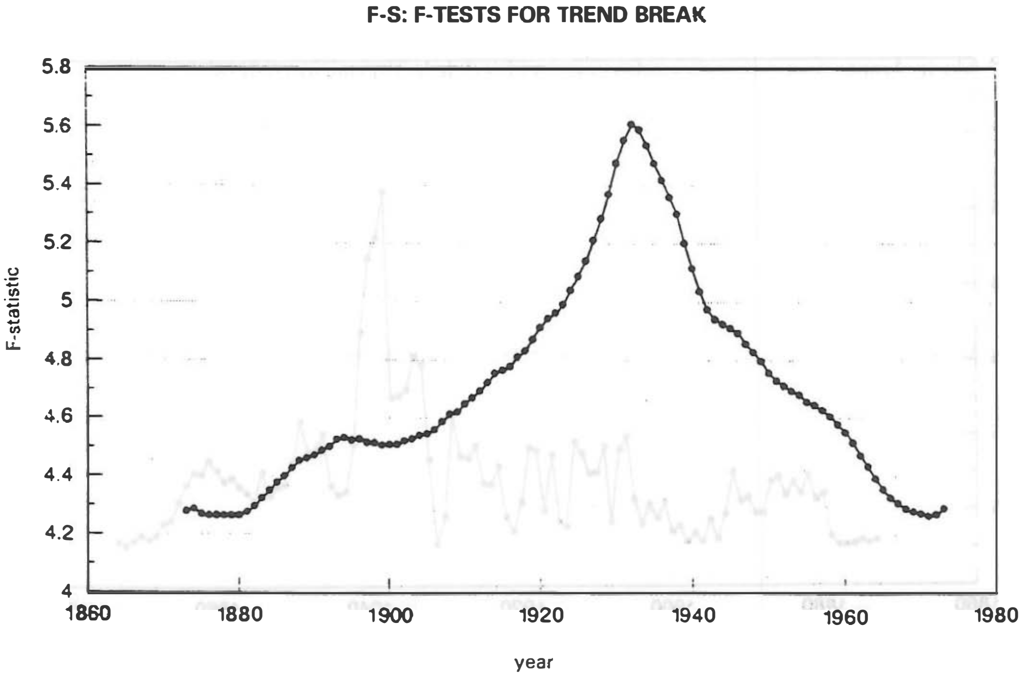


Figure 9

N-P: F-TESTS FOR MEAN BREAK

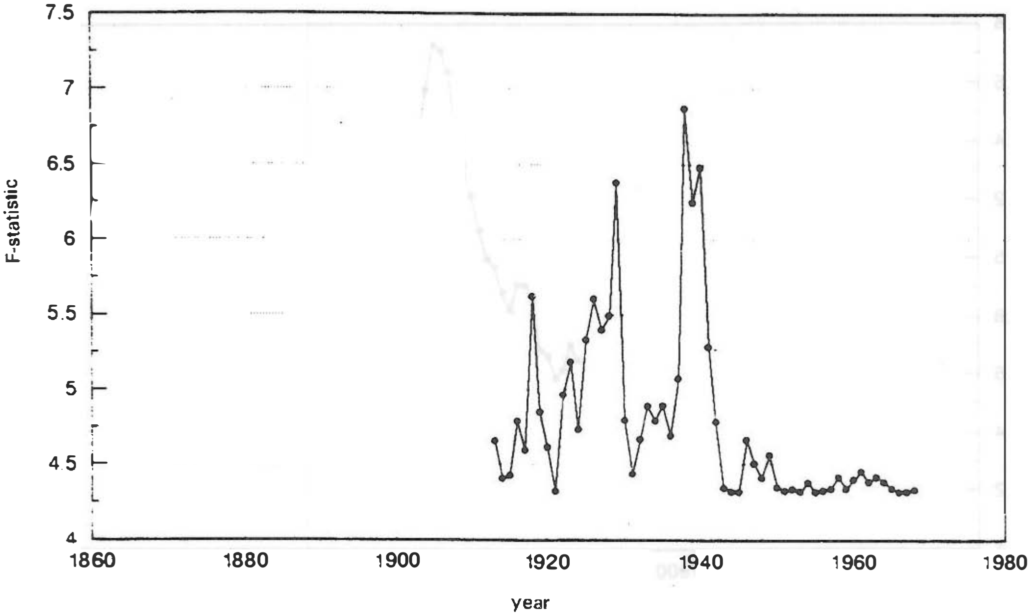
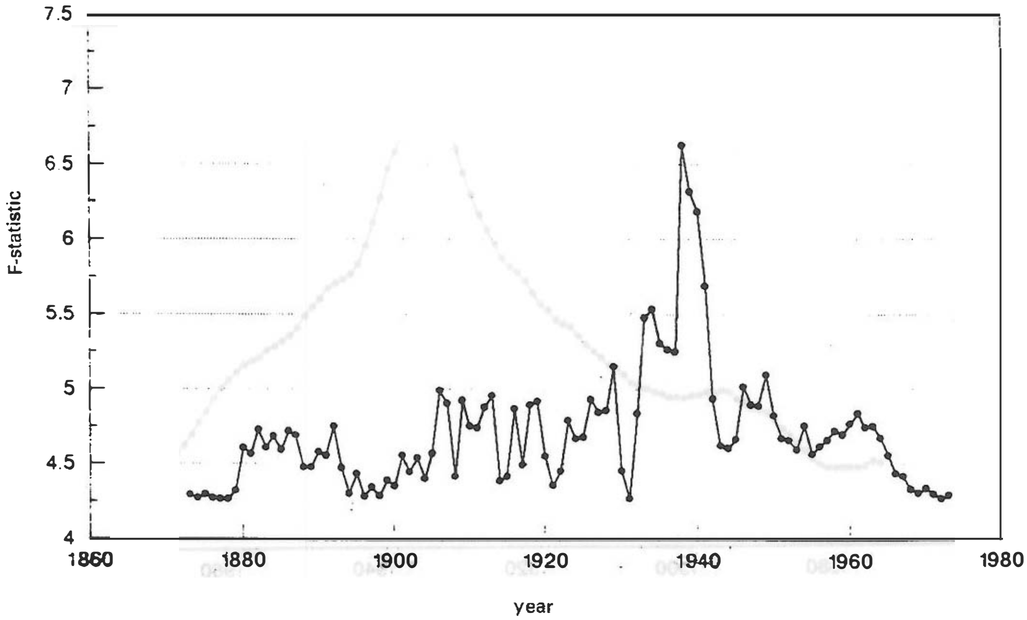


Figure 10

F-S: F-TESTS FOR MEAN BREAK



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