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Abstract

This paper proposes a statistical model and a conceptual framework to estimate inflation volatility assuming rational inattention, where the decay in the level of attention reflects the arrival of news in the market. We estimate trend inflation and the conditional inflation volatility for Germany, Spain, the euro area and the United States using monthly data from January 2002 to March 2022 and test whether inflation was equal to or below 2% in this period in these regions. We decompose inflation volatility into positive and negative surprise components and characterise different inflation volatility scenarios during the Great Financial Crisis, the Sovereign Debt Crisis, and the post-COVID period. Our volatility measure outperforms the GARCH(1,1) model and the rolling standard deviation in one-step ahead volatility forecasts both in-sample and out-of-sample. The methodology proposed in this article is appropriate for estimating the conditional volatility of macro-financial variables. We recommend the inclusion of this measure in inflation dynamics monitoring and forecasting exercises.

Keywords: inflation, inflation trend, inflation volatility, rational inattention, positive and negative surprises.

JEL classification: C22, C32, E3, E4, E5.

Resumen

Este documento propone un modelo estadístico y un marco conceptual para estimar la volatilidad de la inflación suponiendo *rational inattention,* donde la caída del nivel de atención responde a la llegada de noticias al mercado. Estimamos la tendencia y la volatilidad condicional de la inflación en Alemania, España, la Unión Económica y Monetaria y Estados Unidos empleando datos mensuales desde enero de 2002 hasta marzo de 2022, y contrastamos si la inflación fue igual o inferior al 2 % en ese período y esas regiones. Descomponemos la volatilidad de la inflación en sus componentes de sorpresas «negativas» y «positivas», y caracterizamos los diferentes escenarios de volatilidad de inflación durante la gran crisis financiera de 2008, la crisis de deuda soberana y el período pos-COVID. Nuestra medida de volatilidad supera a una modelización GARCH(1,1) y a la desviación típica móvil de la inflación en ejercicios de previsión un período hacia delante, tanto dentro como fuera de la muestra. La metodología propuesta en este artículo es apropiada para estimar la volatilidad condicional de variables macrofinancieras. Recomendamos incluir esta medida en tareas de seguimiento y previsión de la dinámica de la inflación.

Palabras clave: inflación, tendencia de la inflación, volatilidad de la inflación, *rational inattention,* sorpresas positivas y negativas.

Códigos JEL: C22, C32, E3, E4, E5.

1 Introduction

Inflation and price-level changes are not strictly the same concept. However, from now on, for inflation, we refer to the price-level changes. Strictly speaking, inflation is always a long-term concept but only measurable in a steady state, and the long-run inflation measure can not include short-term price level movements. By contrast, a price-level change is a simple rate of change measurable with only two points in time, including short-term and long-run movements. We present a definition for trend inflation to distinguish between long-term movements and short-term price-level movements.

Short-term movements and volatility clusters relate to the inflation volatility concept. These volatility movements usually relate to market-clearing shocks, mainly supply, and demand shocks. Following this reasoning, inflation volatility can be seen as a conditional standard deviation with an upward movement in the presence of any shock type. On the other hand, unconditional variance does not relate to this notion of inflation volatility because it is a long-term concept. Conditional volatility is a closer concept to volatility. But it is no longer an arithmetic average of squared shocks equally weighted. Instead, we propose to estimate the (inflation) volatility as a weighted average of squared forecast errors, where the weights decay exponentially into the past, such that the realized volatility is more sensitive to recent shocks. Our approach differs from considering a standard GARCH(1,1) process. In this article, we claim that there is a relationship between the weight decay process and the probability of news arriving at the market that affects the investor's "attention."

The scarcity of attention influences the decisions of economic agents and matters for supply and demand decisions and economic outcomes, affecting the price dynamics (e.g., sticky-prices). Agents can choose their "optimal inattention" to form beliefs and actions, so we can model limited attention as a bound on information flow, understanding this as a reduction in uncertainty. Unexpected shocks to prices (uncertainty shocks) modify optimal inattention, see Sims (2003). Indeed, estimate the inflation volatility prioritizing the magnitude of the most recent inflation innovations (responding to the information flow rate to prices) is compatible with the sticky-prices theory hypothesis, which claims that prices react with some delay to demand-supply shocks. Under this hypothesis, prioritizing the last-to-arrive information flow into prices seems desirable to reduce realized volatility estimation bias. The literature already finds evidence of the conditional variance of returns (at a given interval) being proportional to the rate of information arrival to the market, see Clark (1973), Andersen (1996), Janssen (2004), Kalev et al. (2004), among many others. Unlike current developments in the conditional volatility literature, our methodology incorporates the rate of information arrival in the weighting function of one-period-ahead squared forecast errors that generates the inflation volatility expectation under the $\mathbb{P}$ measure. One significant contribution of this paper is offering a statistical model and a conceptual framework to estimate inflation volatility that relies on the historical inflation probability distribution function, and the shock arrival probability.

This paper estimates inflation volatility for the USA, EMU, Germany, and Spain using monthly price data from January 2002 to March 2022. It compares this measure with standard approaches reaching evidence in favor of the good in-sample and out-of-sample forecast performance of the proposed measure. We use this volatility measure to test whether inflation was equal or lower target in this period and provide inflation volatility measures related to negative and positive inflation surprises or shocks. The statistical and conceptual framework proposed in this article to estimate inflation volatility can be easily applied to other inflation series and macroeconomic variables, helping to characterize the global macro-finance uncertainty map in quiet and turmoil times.

The analysis organizes as follows. The next section explores the related literature. Section 3 defines inflation and inflation volatility. Section 4 comprises the notion of inflation volatility and proposes an inflation volatility MLE. Finally, section 5 delivers a volatility estimator using the HCPI monthly prices.

2 Literature review

Our paper relates, at least, to three research lines in the inflation literature. The first one studies how to measure inflation volatility and related definitions. The second research line analyzes the relationship between inflation level, volatility, and uncertainty and the relationship between these and economic growth. The third discusses the relationship between the "rational inattention hypothesis" and inflation volatility.

Traditional approaches to estimating inflation volatility are (i) the (log of the) standard deviation of monthly inflation rates (i.e., Aisen and Veiga, 2008), and (ii) the conditional variance of the inflation shocks based on different specifications of GARCH (i.e., Rother, 2004) or (iii) SV models for the inflation dynamics. Relate to this approach, Elder (2004) uses the one-month-ahead conditional inflation forecast. In contrast, other authors such as Primiceri (2005), Stock and Watson (2007), and Balatti (2020) decompose inflation into a trend and an inflation gap component and model each component by an independent stochastic volatility process. The inflation decomposition may differ slightly, such as in Mumtaz and Surico (2000), who study global inflation dynamics decomposing the last into two factors, a country-specific and a world-specific component, and defines a stochastic volatility model for each of them. Regardless of the inflation decomposition and volatility model, all inflation volatility approaches either weigh past price variation equally or rely on the inflation innovation that emerges from a particular model specification that may have changed significantly with the inflation process and main drivers over time. Additionally, critics of conditional volatility models may arise if we assume inflation dynamics follows an I(1) instead of an I(0) process. The methodology we propose to estimate inflation volatility does not depend on the integration order assumption.

There has been a growing interest in exploring the relationship between inflation, inflation uncertainty, and economic growth, see Al-Marhubi (1998), Elder (2004), Hayford (2000), which reinforces our inquisitiveness in delivering a methodology to estimate inflation volatility that counts for investor conduct. The literature agrees on the relationship between inflation and inflation volatility but not on the causality direction. Thus, Friedman-Ball's (FB) hypothesis states that higher inflation leads to higher inflation uncertainty and lower economic growth, see Friedman (1977) and Ball (1992). Conversely, Cukierman Meltzer's hypothesis (CM) states that higher inflation uncertainty leads to higher inflation, see Cukierman and Meltzer (1986) and that the central bank tends to create inflation surprises in the presence of high inflation uncertainty. Bredin and Fountas (2018) examine the annual US inflation rate from 1801 to 2013 and find that the banking and stock market crisis impacted inflation uncertainty, lowering inflation, in line with Holland's hypothesis ("an increase in uncertainty is viewed by policymakers as costly, inducing them to reduce inflation in the future"). Fountas (2001) finds UK inflationary periods associated with high inflation uncertainty from 1885 to 1987, in line with the Friedman-Ball hypothesis.[1] Related research in the Eurozone provides different results (see Barnett et al., 2020) for a review, who find a significant time-varying relationship between inflation and inflation uncertainty in the Eurozone that varies if stable or turmoil periods). Fountas et al. (2004) develop a similar exercise for six European Union countries from 1960 to 1999 and finds the same positive relationship between

[1]The Friedman-Ball hypothesis states that "a rise in average inflation creates uncertainty about future monetary policy to counter it, leading to wide variations in actual and anticipated inflation, thus resulting in economic inefficiency and lower output growth." See further details in Friedman (1977).

inflation level and volatility for all countries except for Germany. These articles fit a GARCH model to annual inflation rate innovations to recover inflation uncertainty. We advise including the inflation volatility measure proposed in this article in the inflation monitoring toolbox to scrutinize current and future deviations from the inflation target and characterize stable and turmoil inflation scenarios.

The empirical evidence about the relationship between inflation volatility and interest rates (in the Fisher equation framework) is mixed. Some find a positive relationship between the two, understanding that risk-averse investors try to avoid higher uncertainty about future inflation by adding risk premiums to interest rates. Others see a negative relationship and explain their results in the framework of the IS-LM model (inflation uncertainty simultaneously affects investment and savings negatively but reduces investment on the demand side of investment funds more significantly than on the supply side). Finally, some authors find no relationship between the two. Further research using the inflation volatility measure proposed in this article suggests estimating the interest rate volatility using the same methodology and examining the contemporary and lead-lag relationship between inflation and interest rate volatility. Using our inflation volatility measure will bring additional light to the sign and significance of this relationship. We are currently developing this research.

Finally, a growing research line on behavioral economics and finance focuses on studying "rational inattention" as opposed to the "rational behavior" hypothesis; see Gabaix (2019), and Mackowiak et al. (2022) for a review. This hypothesis argues that scarcity of attention influences the decisions of economic agents and matters for economic outcomes, even for price dynamics (sticky prices). Under the rational inattention hypothesis, agents can choose their "optimal inattention" to form beliefs and actions, see Sims (2003), so we can model limited attention as a bound on information flow, understanding this as a reduction in uncertainty. Thus, uncertainty shocks modify optimal inattention decisions. Our paper contributes to this literature and claims that agents estimate inflation volatility by attending to the magnitude of the most recent inflation innovations and prioritizing the most recent events (responding to the information flow). We also claim that the arrival of news, surprises, or shocks to the market impacts investor attention and uncertainty.

3 Inflation Definition

This section relates the concepts of price-level changes, inflation, and inflation trend. Let P_t stand for the aggregate price level at time t. The price-level changes are computed as the logarithmic difference in the aggregate price-level, $p_t = \Delta \ln P_t = \ln P_t - \ln P_{t-1}$. We will call this *the inflation*. Therefore, throughout the paper we use indistinctly the terms, p_t, or inflation.

We build on the idea that price-level series needs at least one difference to be stationary. The economic theory suggests that any market-clearing nominal price follows a non-stationary process over time, reflecting that shifts in supply or demand imply price adjustments to clear the market in the long run. The price level index is a weighted average of individual prices, and the aggregation of non-stationary stochastic processes should produce a non-stationary process. Thus, price-level differences are either stationary or non-stationary. We consider that the inflation p_t satisfies:

$$\phi_p(B)(\Delta^d p_t - \mu) = \theta_q(B)a_t \tag{1}$$

where μ is the constant mean, $\phi_p(B) = 1 - \phi_1 B - ... - \phi_p B^p$ is strictly stationary, $\theta_q(B) = 1 - \theta_1 B - ... - \theta_q B^q$ is strictly invertible, B is the backshift (lag) operator such that $Bp_t = p_{t-1}$, and $\Delta^d := (1-B)^d$ is the difference operator with d an integer such that $d \geq 0$. Moreover,

a_t is an innovation or one-step prediction error. We assume that the innovations' unconditional variance is constant, but its conditional variance is time-dependent. Specifically, $a_t \sim (0, \sigma_a^2)$ but $a_t|\mathcal{F}_{t-1} \sim (0, h_{a,t})$, where $\mathcal{F}_{t-1}$ denotes all information available at period $t-1$. For the moment, we express the function for the conditional variance generally as $h_{a,t} = h_a(a_{t-1}, a_{t-2}, a_{t-3}, ...)$.

3.1 Inflation background

Notice that price level changes do not always fit to the inflation definition stated by Friedman (1963): "By inflation, I shall mean a steady and sustained rise in prices."[2] Even though "steady" and "sustained" terms are not formally defined in Friedman (1963), it is common to interpret these terms as related to the inflation permanent component or the long-run statistical equilibrium of the price-level changes. This author proposes decomposing price-level changes into transitory and permanent components based on exponential smoothing. Nevertheless, the Beveridge and Nelson (1981) decomposition offers a more general framework in which Friedman's approach is only a particular case. Thus, Beveridge and Nelson (1981) proposes a decomposition of non-stationary time series between a transitory and a permanent component, where the long-run forecast represents the permanent component. Therefore, we define the inflation trend by combining Beveridge-Nelson's decomposition with Friedman's definition of inflation.

Definition 1 *Trend Inflation, π_t, is the expected change in the (log) price level in the long-run, conditional to all information available at period $t-1$:*

$$\pi_t = \lim_{l \to \infty} \mathbb{E}\big[p_{t+l}|\mathcal{F}_{t-1}\big].$$

Notice that if inflation is (stochastic) non-stationary, so that $d = 1$ in Model (1), the trend inflation follows a time-dependent stochastic trend. From this perspective, following Definition (1), inflation p_t, can be decomposed as a trend, π_t, and transitory changes around this trend, $\eta_t = p_t - \pi_t$. Therefore, we associate Laidler and Parkin (1975) and Friedman (1963) inflation definitions to our trend inflation, π_t, as a purely monetary phenomenon, and we assume that the transitory component, η_t, is driven by non-monetary shocks (as changes in raw materials prices, etc.).

On the other hand, if inflation, p_t, is (stochastic) stationary, the constant unconditional inflation mean exists. Then, the trend inflation is not time-dependent and is equal to this unconditional mean. In model (1) with $d = 0$, replacing price-level changes for its components in the same model and taking rational expectation yields:

$$\pi = \lim_{l \to \infty} (\mu + \mathbb{E}\big[a_{t+l}|\mathcal{F}_{t-1}\big]) = \mu.$$

In such a case, the inflation trend is constant, and the long-run rise in prices would be "steady" and "sustained". Therefore, Definition 1 coincides with Friedman's when inflation follows an I(0) process with unconditional mean, μ. The inflation can be decomposed in a transitory component, $\eta_t = p_t - \pi$, and a constant long term inflation, component $\pi = \mu$.

If p_t follows a mean stationary process, the trend inflation can be estimated using a sample of price-level changes. Under the Central Limit Theorem, the inflation sample mean $\overline{p}$ is a random variable that follows a normal distribution with $\overline{p} \sim N(\pi, \sigma_{\overline{p}})$, where $\sigma_{\overline{p}}$ is the standard error of the sample mean, $\sigma_{\overline{p}} = (\sigma_p^2/n)^{1/2}$, being σ_p^2 the inflation unconditional variance and n the sample

[2]See Friedman (1963) p. 39.

size. Alternatively, if inflation unconditional variance is unknown, the sample mean $\overline{p}$ follows a t-student distribution with $\overline{p} \sim t_{n-1}(\pi, S_{\overline{p}})$, where $S_{\overline{p}}$ is the standard error of the sample mean, $S_{\overline{p}} = (S_p^2/n)^{1/2}$, being S_p^2 the inflation sample variance. In both cases, standard statistical inference about the value of π applies.

Several authors propose a similar inflation decomposition in inflation trend, and a transitory component, e.g., Ireland (2007), Stock and Watson (2007), Cogley and Sbordone (2008), and Cogley et al. (2010). However, their works differ from ours in the assumptions about the permanent and transitory components. A driftless random walk is usually used to approach the inflation trend, while a stationary serially un- or correlated noise is used to measure the inflation gap. We will make no assumptions about these components as these will depend on the data, and we will extract this information later in our empirical exercise from the sample.

4 Inflation Unconditional and Conditional Variance

Following Definition (1), inflation volatility is not the unconditional standard deviation of the price-level changes nor the conditional standard deviation. Instead, inflation volatility relates to the short-run dynamics. Unconditional inflation variance and inflation trend variance are two concepts related to long-run expected values and should be evaluated in the long run. The forecast origin and the forecast horizon would be the same. Thus, we define the unconditional variance of inflation as the statistical second moment of long-term inflation. Formally

Definition 2 *Inflation unconditional variance, σ_p^2, is the variance of the fluctuations of the inflation around its trend:*

$$\sigma_p^2 = \mathbb{E}\left[(p_t - \pi_t)^2\right],$$

and, therefore, σ_p is the inflation unconditional standard deviation. Two remarks can be added to this definition. First, under stationarity of p_t, it is easy to show that $\sigma_p^2 = k\sigma_a^2$, where k is a constant that depends on the inflation's ARMA process (1).[3] This occurs because the trend inflation, π_t, coincides with the inflation unconditional mean when p_t is stationary. Second, under the non-stationarity of p_t, the trend inflation is not constant (or even not finite, in the unlikely case of inflation with drift), and so, $\sigma_p^2 = \infty$.

In contrast, inflation volatility is related to the short-term movements of the price-level changes and is strictly time and sample-dependent. Our definition of inflation volatility relies on the one-period-ahead estimated forecast error. From Model (1) the one-period-ahead forecast error, hereinafter *forecast error*, can be written as follows:

$$a_t = p_t - p_{t|t-1}, \tag{2}$$

where $p_{t|t-1} = \mathbb{E}\left[p_t|\mathcal{F}_{t-1}\right]$. The forecast error can be split into a strong white noise process, z_t, and the time-dependent standard deviation $h_{p,t}^{1/2}$, as $a_t = z_t h_{p,t}^{1/2}$. Therefore, the conditional variance of inflation refers to a time-dependent variance in a_t. Formally:

Definition 3 *Inflation conditional variance, $h_{p,t}$, is the variance of the inflation innovations, conditional to all information available at period t-1:*

[3] For instance, when p_t follows a stationary AR(1) or invertible MA(1) process, then k equals, respectively, $1/(1-\phi^2)$ or $(1+\theta^2)$.

$$h_{p,t} = h_{a,t} = \mathbb{E}\left[a_t^2|\mathcal{F}_{t-1}\right] = \sum_{i=0}^{\infty} \omega_i a_{t-i-1}^2.$$

We will denote by V_t the *inflation conditional volatility*, i.e., $V_t = h_{p,t}^{1/2}$.

Therefore, we define inflation volatility as a time-dependent standard deviation that follows an ARCH(∞) process, a general case of a GARCH(p,q) process. Under this definition, inflation volatility can be estimated as a weighted average of squared one-step-ahead forecast errors. However, in the short term, agents form expectations weighting more recent one-period-ahead errors the most ($\partial\omega_i/\partial i < 0$), such that the more recent contribute the most to the inflation uncertainty and volatility. The conceptual framework we propose in this paper links this empirical evidence to current behavioral finance and volatility literature results. There is evidence of a significant positive relationship between the rate of news arrival and volatility. At the same time, this rate relates to the agent's "optimal inattention" choice (Sims, 2003) when estimating the volatility under the $\mathbb{P}$ measure. Thus, ceteris paribus, volatility is more affected by recent shocks at higher rates of news arrival. This time-varying attention mechanism sets in our conceptual framework and explains the motivation to incorporate the rational inattention hypothesis in the model. Thus, it makes no sense to assume equal weight for historical surprises regardless of the time since realization. We propose a restricted structure that carries growing relative importance for more recent shocks (weights) such that the last forecast error always has the highest weight:

$$\omega_i = \varphi(1-\varphi)^i, \quad \forall i \geq 0, \tag{3}$$

where $\sum_{i=0}^{\infty} \varphi(1-\varphi)^i = 1$ and $0 < \varphi < 1$. The weight decreases with the time elapsed in a geometrical structure, and the last forecast error keeps the highest weight. Thus, the sensitivity of volatility to unexpected forecasting errors is proportional and decreases with elapsed time. The main idea is that an outlier or news arrival affecting the innovation process a_t follows a multiplicative binomial white noise process. Thus, understanding that in our model specification the one-period-ahead forecasting error $e_t(l=1) = p_t - p_{t|t-1} = a_t$, from (2), $a_t \sim N(0, \sigma_a^2)$, and $a_t|\mathcal{F}_{t-1} \sim N(0, h_{a,t})$. Then, we define the Bin(n, φ) random variable $u = \sum_{i=0}^{n} I_{|a_{t-i}/\sigma_a| > z_{\alpha/2}}$, being $z \sim N(0,1)$, to approach the probability of unexpected one-period ahead inflation forecasting error. Thus,

$$f(u|\varphi) = \binom{n}{u} \varphi^u (1-\varphi)^{n-u}, \tag{4}$$

where n is the sample size and $\varphi = P(|a_t/\sigma_a| > z_{\alpha/2})$ for all t. Notice that the researcher may choose a different distribution function and α as far as these choices serve the research question to address.[4] This feature constitutes an additional advantage of this methodology. Notice that the MLE for φ is $\hat{u}/n$. Replacing (3) in Definition (3), and using $\hat{\varphi}$, we obtain the MLE for the conditional variance of inflation as:

$$\hat{h}_{p,t} = \hat{h}_{a,t} = \sum_{i=0}^{n} \hat{\varphi}(1-\hat{\varphi})^i \; \hat{a}_{t-i}^2, \tag{5}$$

where $\hat{a}_t$ is the residuals from the estimation of Model (1), n is the sample size (chosen by the user), and $\sum_{i=0}^{n} \hat{\varphi}(1-\hat{\varphi})^i$ is, by construction, smaller than one for finite n. As an example, for an estimated value $\hat{\varphi} = 10\%$ with $n = 50$, then $\sum_{i=0}^{50} \hat{\varphi}(1-\hat{\varphi})^i = 99.5\%$, meaning that the last

[4]Here, without loss of generality, we set $\alpha = 0.05$, and so $z_{\alpha/2} = 1.96$.

fifty observations almost cover 100% of total weights. Although other weights could be used, in the Appendix we argue in favor of the weights proposed; see Appendix A.

From (5), one can notice that the estimated impact of historical innovations on volatility, $\hat{\omega}_i = \hat{\varphi}(1-\hat{\varphi})^i$, depends on two factors: (1) the time gap between the innovation and current time, i, and (2) the estimated probability of news arrival, $\hat{\varphi}$. From Eq (6) below, we observe that the impact of innovations in the conditional volatility exponentially decreases with i, in line with the rational inattention theory (*individual people have limited capacity for processing information*). On the other hand, Eq (7) shows that the conditional and unconditional inflation volatility tend to be equal when the news arrival rate is zero. This claim aligns with the general belief in the literature that the news arrival impact conditional volatility.

$$\lim_{i\to\infty} \varphi(1-\varphi)^i = 0, \text{ where: } 0 < \varphi < 1 \tag{6}$$

$$\lim_{\varphi\to 0} \varphi(1-\varphi)^i = \lim_{\varphi\to 1} \varphi(1-\varphi)^i = 0, \text{ where: } i \geq 0 \tag{7}$$

If $\varphi = 0$, no news arrives in the market, so we cannot define a criterion to differentiate between past and new information. If $\varphi = 1$, all past innovations are extreme, so the information arrival content is "similar" across them. In both cases, we would end up with the same result: equal unconditional and conditional volatility. Therefore, as Kalev et al. (2004), we use news intensity as a proxy for information flows. In contrast, rather than identifying news intensity with "the number of news about a company at day t", as the later authors do, we use a binomial distribution to signal the news arrival and the information flow at period t.

Inflation variance responds to unexpected price movements independently of the movement sign that generates stress. However, estimating inflation variance (volatility) components responsible for likely unexpected upward and downward price movements is critical. The methodology proposed in this article enables us to implement this decomposition. Using the weighted scheme determined by φ in (5), we flag negative and positive forecasting errors and average these as follows:

$$h_{t,p} = \sum_{i=1}^{\infty} \omega_i a_{t-1}^2 = \underbrace{\sum_{i=1}^{\infty} \omega_i I_{t-i}^- a_{t-i}^2}_{h_{t,p}^-} + \underbrace{\sum_{i=1}^{\infty} \omega_i I_{t-i}^+ a_{t-i}^2}_{h_{t,p}^+} = h_{t,p}^- + h_{t,p}^+ \tag{8}$$

where:

$$I_{t-i}^- = 1, \text{ if } a_{t-i} \leq 0, \text{ and zero, otherwise.} \tag{9}$$

$$I_{t-i}^+ = 1, \text{ if } a_{t-i} > 0, \text{ and zero, otherwise.} \tag{10}$$

Broadly speaking, as it is known, volatility is a latent variable, which makes the volatility dynamics dependent on the investor/consumer expectations, yet under the $\mathbb{P}$ measure. When economic agents proxy (inflation) volatility, they mostly pay attention to the news (recent shocks), as the GARCH models do. However, we: (1) claim that the attention to exponential decay only depends on the probability of news arrival on the market (as in IGARCH models) and (2) obtain this probability by MLE applied on a sample with a window size n. Our conceptual framework involves rational inattention because we identify the rate of news arrival to the agent's optimal inattention choice, as Sims (2003) describes. Thus, if the frequency of news arrival (unexpected forecasting error) is high (e.g., 10%), the agent reduces the backward time horizon to create her volatility expectation under the $\mathbb{P}$ measure, focusing on the more recent events. If the rate of news arrival is low (e.g., 1%), the agent will spread the attention to the far past more smoothly to create volatility expectations under the $\mathbb{P}$ measure. This mechanism supports our conceptual framework and explains the motivation to incorporate the rational inattention hypothesis into the model.

Appendix B demonstrates that our model is equivalent to an IGARCH(1,1), which is the volatility model used in RiskMetrics (RM). However, our volatility model differs significantly from the former. First, the RM model sets the $(1-\varphi)$ parameter to minimize the model forecasting error.[5] Instead, we do not use a forward-looking criterion to set φ, but an MLE with historical data. A forward-looking estimation is related to the $\mathbb{Q}$, while our proposal is instead based on the $\mathbb{P}$ measure. Therefore, our conceptual framework to estimate φ differs significantly from RM's. The innovation weights suggested depend on a statistical-economical criterion: the historical news arrival rate to the market that explains the agent's rational inattention in elaborating the second-moment expectation under the $\mathbb{P}$ measure. Second, our approach to estimating inflation volatility implies estimating the innovations; hence, it requires assessing the filter that better fits the inflation dynamics, while the RM volatility approach only relies on financial returns. Inflation volatility is not the weighted average of squared log-price changes, which also sets a difference between our approach and RM's. Last, the most common practice in the literature to estimate conditional inflation volatility is estimating a GARCH(1,1) model, which also sets a difference between our paper and the standard approach.

Finally, if inflation follows an I(0) process and volatility clusters, we can perform inference on the trend inflation relying on the inflation sample mean and volatility. In this case, the estimated time-dependent inflation volatility $\hat{h}_{p,t}^{1/2}$, obtained in (5), can be used to compute the standard error of the sample mean, instead of the inflation conditional or unconditional volatility. This estimator is more robust to heteroscedasticity than the previous measures. Given a sequence of price-level changes, $p_1, ..., p_n$, inflation sample mean, $\overline{p}$, follows a t-student distribution with $\overline{p} \sim t_{n-1}(\pi, \hat{V}_{\overline{p},t})$, where $\hat{V}_{\overline{p},t}$ is the standard error of the sample mean using the inflation volatility measure, $\hat{V}_{\overline{p}} = (\hat{h}_{p,t}/n)^{1/2}$. We will perform this analysis in the empirical section.

5 Empirical Exercise

In this section, based on the methodology explained in sections 3 and 4: 1) we present the estimates of trend inflation and the inflation conditional volatility for Germany, Spain, the EMU, and the US using monthly data from January 2002 to March 2022, 2) we test if inflation was equal to or below 2% in this period, and 3) we estimate the inflation volatility due to positive and negative surprises and examine its dynamics. Finally, we compare the in-sample and out-of-sample performance of our inflation volatility measure, V_t, relative to alternative standard approaches such as the GARCH(1,1) and the rolling sample deviation of the inflation series.

5.1 Inflation Trend and Inflation Conditional Volatility

In our study, we analyze the Consumer Price Index (CPI) for Germany, Spain, and the USA and the Harmonized Consumer Price Index (HCPI) for the EMU between January 2002 and March 2022.[6] The original price series are monthly and not seasonally treated. In each case, inflation, p_t is the first logarithmic difference of each nominal price series, CPI, and HCPI, respectively. We measure

[5] RM technical documentation claims that "RiskMetrics currently processes 480 time series, and associated with each series is an optimal decay factor that minimizes the root mean squared error of the variance forecast."

[6] The data source is Eurostat for the EMU area (HCPI) and the respective national statistical institute (CPI), and it is available from the authors upon request.

inflation volatility for each country in this period in two steps. First, we use data from 2002 to 2020 to identify and estimate the univariate model for each inflation p_t. Second, we calculate the inflation variance $h_{t,p}$ and annualized volatility V_t for the whole sample using the one-step-ahead forecast errors. All nominal prices show similar statistical properties. They (i) are integrated of order one, so inflation series are integrated of order zero; (ii) have an autoregressive structure for the stochastic part, (iii) have a constant mean, μ, and a deterministic seasonal component for the deterministic part, and, iv) have a seasonal component. [7] These results are summarized in Table 1. The estimated parameters and some diagnostic tools are also reported, except those related to seasonal modeling.

In the cases of Germany and Spain, an AR(2) and AR(1) are fitted, respectively. In the case of Germany, a simple random walk also can be fitted. The EMU prices need an additional $AR(1)_{12}$. The USA prices include AR(2) structures, regular and seasonal, with two imaginary conjugate roots, leading to damped oscillations.

In the residuals (one-step ahead forecasting error), Q statistics by Ljung and Box (1978) show no signs of poor fit, except in the EMU case. The Jarque-Bera statistics reject normality only in the case of the USA. In this case, some volatility clusters around 2009 caused the high value in the Jarque-Bera statistics. There is no evidence of inflation volatility clusters in the 2000-2020 sample for Germany, Spain, and the EMU area. The unconditional standard deviation is very similar in all the cases, around 0.25%, and a little lower in the EMU area, around 0.17%.

The last column in Table 1 includes the estimated $\hat{\varphi}$ parameter. In all cases, this is not statistically different from 5% (the strictly Gaussian case), with point estimations lying between 6.1% and 4.2%. Figure 1 shows the inflation volatility series for each case with $n = 50$. This figure also depicts the sample standard deviation for the inflation with a rolling window of size $n = 50$. Germany, Spain, and the EMU have no high volatility states until 2021. However, there is a noticeable inflation volatility episode in these areas starting in 2021. On the other hand, we find a high volatility episode in the USA around 2009 that spans between 2008 and 2011. There was also a volatility episode in the USA in 2021, but this was not as big as the one witnessed in the EMU, Germany, and Spain. These results suggest that the high inflation episode in the euro area in 2021-2022 was not the same as the one observed in the USA. The figures also illustrate the slow response of the conditional standard deviation to shocks relative to the proposed inflation volatility measure.

[Include here Table 1 and Figures 1 and 2]

Figures (3) and (4) include the 90% confidence intervals for the inflation in the four cases. The inflation sample mean, $\bar{p}$, is estimated monthly using a rolling window with $n = 50$. The figures include confidence intervals using the unconditional volatility for the sample mean, $\sigma_a/\sqrt{n}$ (shadow-area), the rolling sample standard deviation of p_t, S_t (dashed line), and the conditional volatility under rational inattention, V_t (solid line). We use the same rolling window size ($n = 50$) to compute the conditional volatility and confidence interval estimates for the inflation trend. The straight horizontal line is the unconditional inflation estimation in Table 1. We can test two hypotheses with the information in the figures: i) if the inflation is higher than 2%, and ii) if the trend inflation is higher than the long-term estimate (π), i.e., is there empirical evidence in favor of an increase in the inflation trend?

[Include here Figure 3 and Figure 4]

[7] The initial specification for the stochastic part is according to *pacf* values, AIC, and H-Q criteria. The three criteria agree with the same initial specification.

Inflation volatility is low in Spain, Germany, and the EMU area until 2021, with conditional and unconditional volatility measures providing similar statistical results.

The hypothesis that the inflation trend is less or equal to 2% and that it is less or equal to its historical inflation trend ($\pi = 1.80$) are rejected for Spain from 2006 to 2009. We cannot reject the hypothesis that the inflation trend in Germany and EMU was less or equal to 2% using conditional volatility. However, there is some evidence that the EMU inflation trend was higher than its historical inflation trend ($\pi = 1.56\%$) at the beginning and end of the sample. We reject the hypothesis that the EMU inflation trend was lower or equal to 2% when we use the unconditional volatility since approximately March 2022. The higher volatility in this period increases the amplitude of the conditional confidence intervals, which makes that the hypothesis that long-term inflation is lower or equal to 2% is not rejected.

By contrast, inflation data in the USA reveals different conclusions. At the beginning of the sample, the hypothesis that long-term inflation is less or equal to 2% is mostly rejected until 2009. We obtain the same result when the hypothesis is relative to the long-term inflation estimation, 2.04%. Notoriously, the volatility cluster between 2009 and 2013 is visible in the form of higher conditional confidence intervals, especially for S_t, but there is not a high inflation episode in this case. From 2009 until 2021, the hypothesis that long-term inflation is less or equal to 2%, or 2.04%, cannot be rejected. However, both hypotheses are rejected from the last part of 2021 onwards. In March 2022, there is some evidence that the long-term inflation in the USA has increased, exceeding the 2% target, but not the 2.04% historical long-term inflation estimation.

5.2 Forecasting inflation volatility: a horse-race

In this section, we study the ability of conditional volatility measures to approach in-sample and out-of-sample inflation volatility. We divide the sample into two parts; the first goes from 01/2002 to 12/2019, and the second from 01/2020 to 03/2022, both included. Given the relationship between σ_p^2 and σ_a^2, we use the first subsample to study the ability of three conditional volatility measures to inform about changes in squared innovations when available. We implement this exercise in two parts. First, we estimate the conditional volatility using monthly data from 01/2002 to 12/2019 and classify volatility measures due to their in-sample (inflation volatility) forecasting capability in this period. Second, we evaluate the out-of-sample forecasting ability for the volatility measures using model estimation until 12/2019 to forecast the inflation innovation dynamics from 01/2020 to 03/2022.

We compare our conditional variance measure (h_t) with the 50-month rolling-window p_t sample variance (h_t^{RW}) and GARCH(1,1) conditional variance (h_t^G), a model commonly used in the literature to approach inflation volatility. Table 3 includes estimated univariate models assuming a GARCH(1,1) model for conditional volatility, using monthly data from 01/2002 to 12/2019. We only find evidence of volatility clustering for the US and some in Spain, while conditional volatility for Germany and the EMU converges to the unconditional variance until 2019.

Volatility is a latent variable, and it is impossible to get a "true" volatility measure to bring to light the forecasting capability of a volatility measure. The literature usually compares the conditional variance estimated to the squared innovations and employs the R-squared and loss functions, such as RMSE or MAPE, among others, to rank the volatility measures forecasting ability. We proceed similarly and estimate the following model using the first subsample:

$$\hat{a}_t^2 = \beta_0 + \beta_1 \hat{X}_t^2 + \varepsilon_t, \text{ where: } X_t^2 : h_t, h_t^G, h_t^R, \tag{11}$$

where a_t is the ARIMA model innovation, h_t is our conditional variance, h_t^G is the GARCH(1,1) conditional variance, and h_t^{RW} is the rolling variance for $n = 50$. Conditional volatility under rational inattention reports the best in-sample results, see Table 3, and increases its explanatory power after 2019, as volatility clusters emerged. The delay in the h^{RW} changes explains the high negative estimated β_1 and the low R^2 for all the countries. More importantly, conditional volatility under rational inattention (h_t) seems useful to forecast inflation volatility in 2020/2022 when other measures do not seem capable of it. This result is significant since it reports evidence of conditional volatility under rational inattention beating GARCH(1,1) in forecasting inflation volatility before (in-sample) and after (out-of-sample) 2020, a period of particular need for informative risk measures.[8]

[Include here Table 3]

5.3 Positive and negative inflation surprises and volatility

Figure 5 includes inflation variance, h_t, decomposition for the US and EMU following (8), and Figure 6 for Germany and Spain. The negative-to-positive surprises inflation variance decomposition differs significantly at current times relative to the Sovereign Debt Crisis and the Great Financial Crisis. While most of the inflation uncertainty discounted back on these turmoil episodes came from negative inflation surprises (inflation less than expected), current inflation volatility responds primarily to positive surprises, between 80-90% of positive surprises contribution in the EMU, the US, and Germany. Although Spain's inflation volatility reports higher participation of negative shocks, positive shocks still prevail at the end of the sample. During the beginning of the GFC, inflation surprises were primarily positive in EMU and the USA. However, less than one year ahead, they mainly became negative, indicating an inflation level lower than expected, likely related to the fear of slow economic growth onwards. During the Sovereign Debt crisis, we find a high contribution of positive surprises to inflation variance in the EMU that were not in the USA inflation. The fears about slow economic growth during the Covid-19 shock explained the higher contribution of negative surprises to inflation uncertainty in 2020. Finally, most of the inflation uncertainty in 2022 relates to positive surprises. One significant advantage of using this methodology and representation to estimate inflation volatility is the feasible interpretable representation of positive-to-negative shocks' contribution to the resulting inflation volatility measure. This exercise helps to characterize inflation volatility scenarios for the monetary policy design. This exercise is also helpful in assessing policies' impact on markets and macroeconomic variables dynamics, among other uses.

[Include here Figures 5 and 6]

6 Conclusion

This paper proposes a statistical model and a conceptual framework to estimate inflation volatility under the $\mathbb{P}$ measure, assuming rational inattention and time-diminishing sensitivity to shocks.[9] We use it to estimate the monthly inflation volatility for Germany, Spain, the USA, and the Eurozone from January 2002 to March 2022. We test whether the inflation in these countries reached

[8] A horse race between h_t and 50-month rolling-window sample variance of squared innovations also results in better results for our measure. This exercise is available upon request.

[9] Rabin (1998) describes "diminishing sensitivity" as another significant reference-level effect in the investor utility function in addition to loss aversion as follows: "the marginal effect in perceived well-being are greater for changes close to one's reference level than for changes further away."

values above 2% in this period using our inflation volatility approach and find significant differences between the USA and the Eurozone inflation volatility processes in recent and past turmoil episodes such as the Great Financial Crisis, and the European Sovereign debt crisis. Regarding the most recent episode, the Eurozone inflation volatility reached in March 2022 maximum values since 2006, while the USA inflation volatility kept around the maximum value reached during the GFC. While we cannot reject that EMU inflation was 2% in March 2022, we find statistical evidence of the contrary for the USA in the same period. We decompose the conditional inflation variance in negative and positive surprise components and study its dynamics, which helps identify different inflation volatility scenarios useful in monetary policy design exercises. Finally, we compare our volatility measure's in-sample and out-of-sample forecast performance with standard approaches in the literature, such as a GARCH(1,1) model and the inflation rolling sample standard deviation. Our volatility measure outperforms the traditional approaches. The volatility estimation methodology and conceptual framework we propose are flexible enough to apply to all AEs and EMEs inflation series and other macroeconomic variables.

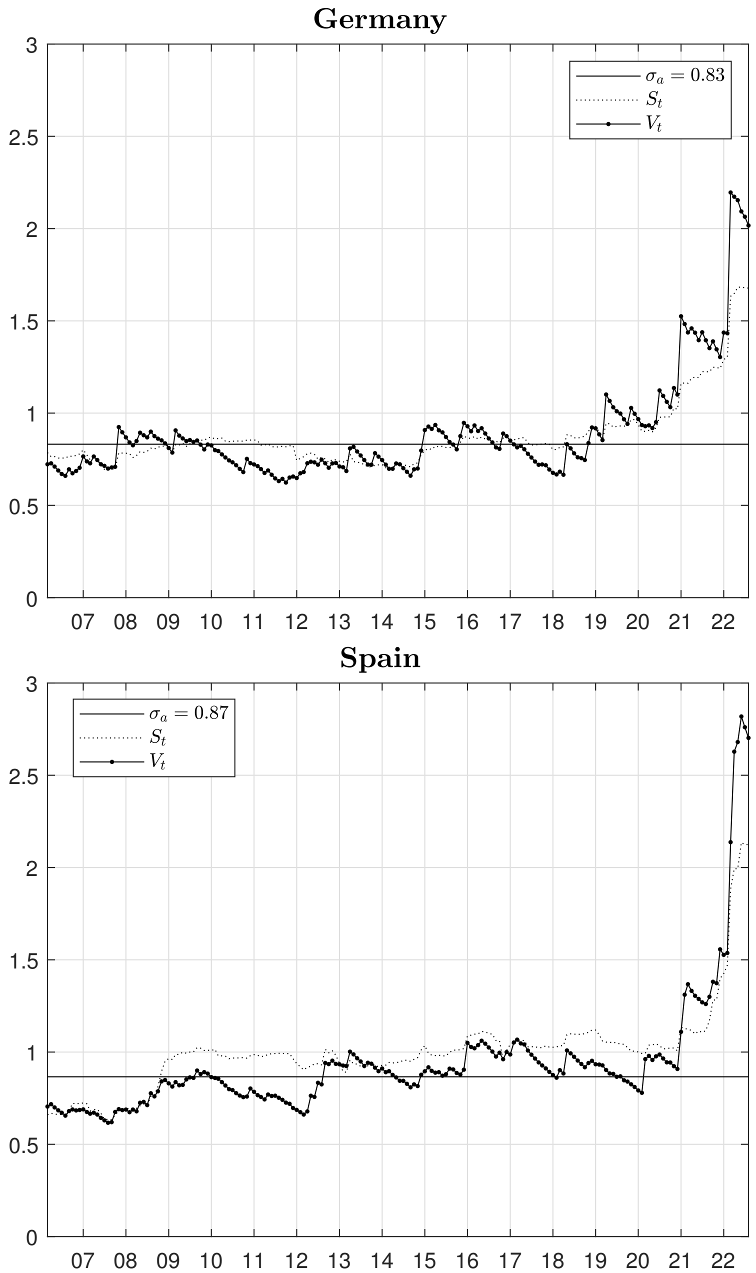


Figure 1: Germany and Spain inflation volatility: unconditional inflation volatility (σ_a), conditional volatility under rational inattention ($V_t = h_{p,t}^{1/2}$), and the inflation sample standard deviation (S_t) with a rolling window of size n = 50. Sample: January 2002 to March 2022.

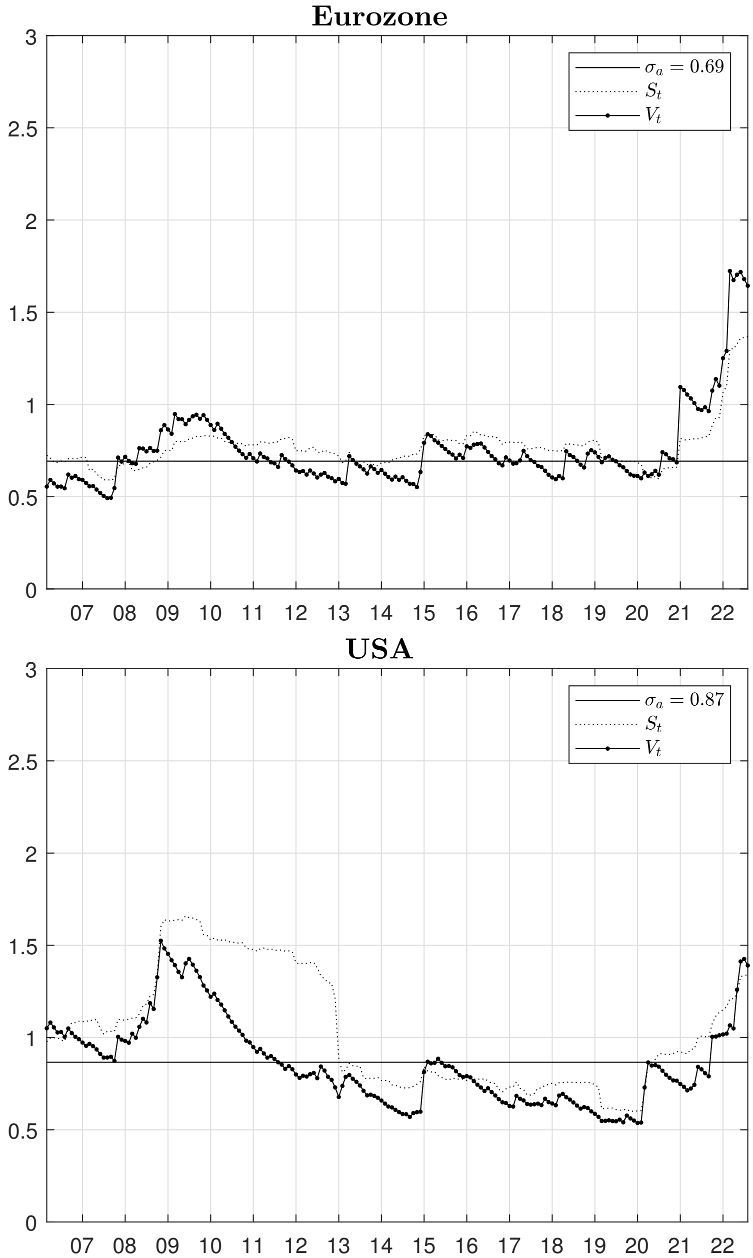
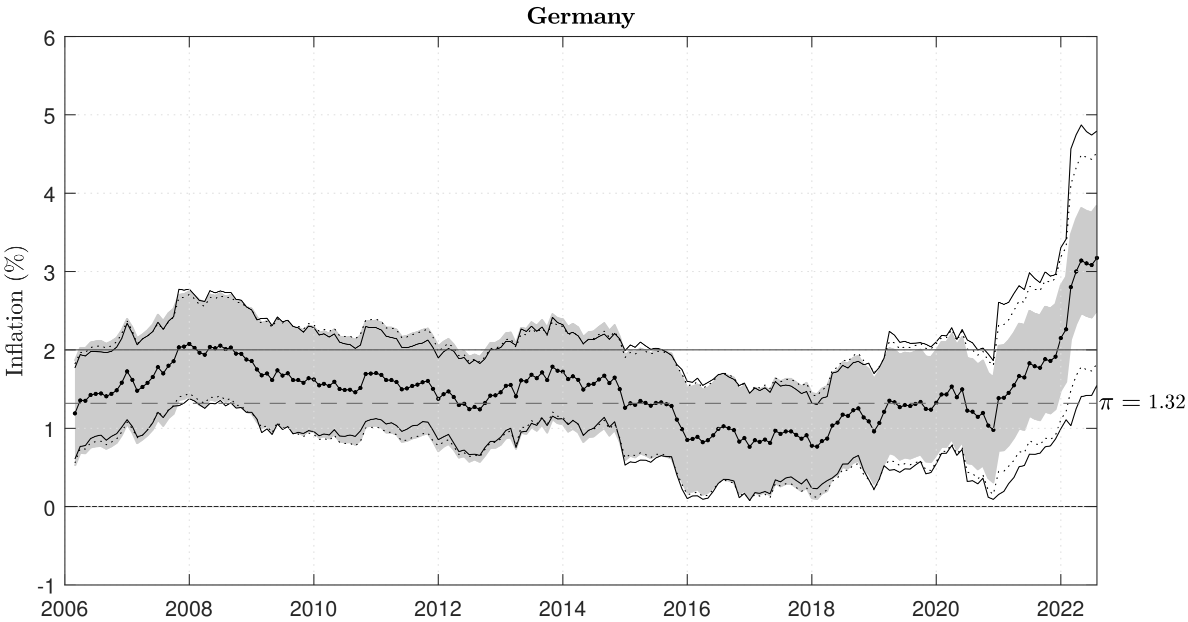


Figure 2: Eurozone and the USA inflation volatility: unconditional inflation volatility (σ_a), conditional volatility under rational inattention ($V_t = h_{p,t}^{1/2}$), and the inflation sample standard deviation (S_t) with a rolling window of size n = 50. Sample: January 2002 to March 2022.



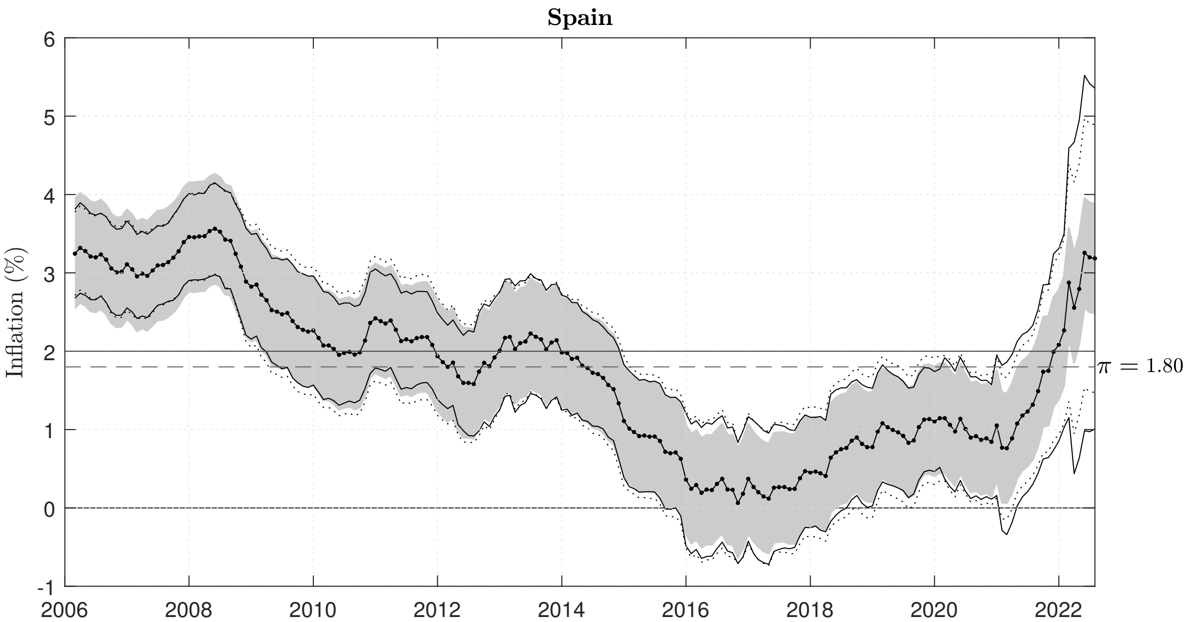
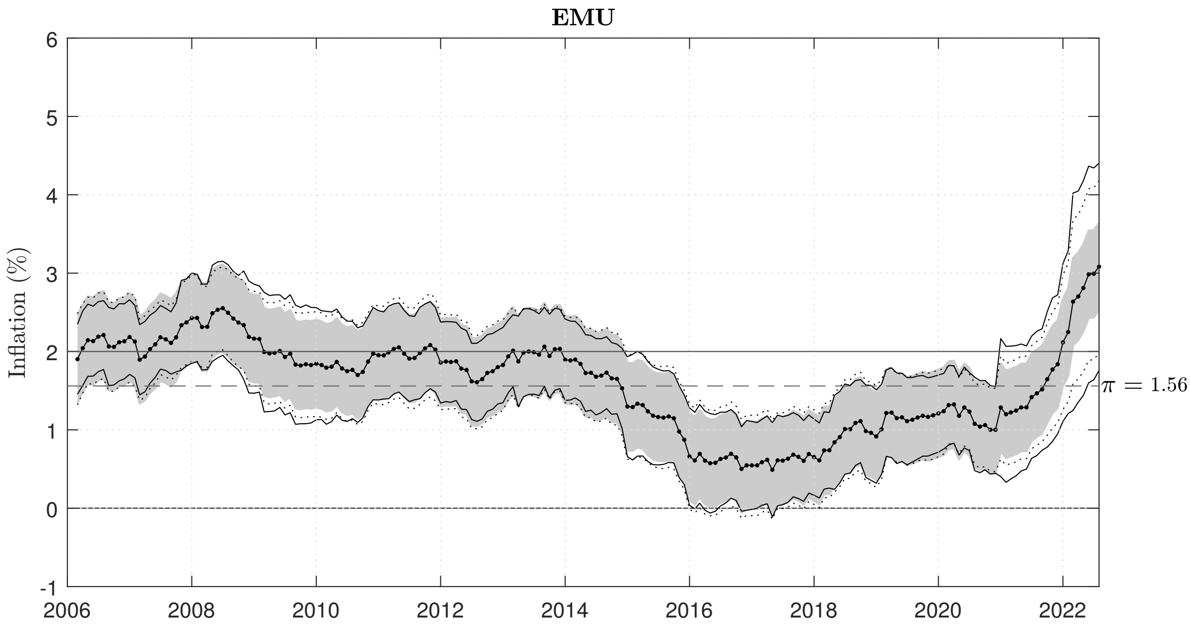


Figure 3: Germany and Spain. Estimated inflation using inflation sample mean ($\bar{p}$) using a rolling window of size $n = 50$, and 90% confidence interval using the unconditional inflation volatility ($\sigma_{\bar{p}}$), the conditional inflation volatility under rational inattention ($V_t/\sqrt{n}$), and the rolling inflation standard deviation ($S_t/\sqrt{n}$) with n = 50. Sample: January 2002 to August 2022. Unconditional inflation volatility is estimated using data from January 2002 to December 2019.



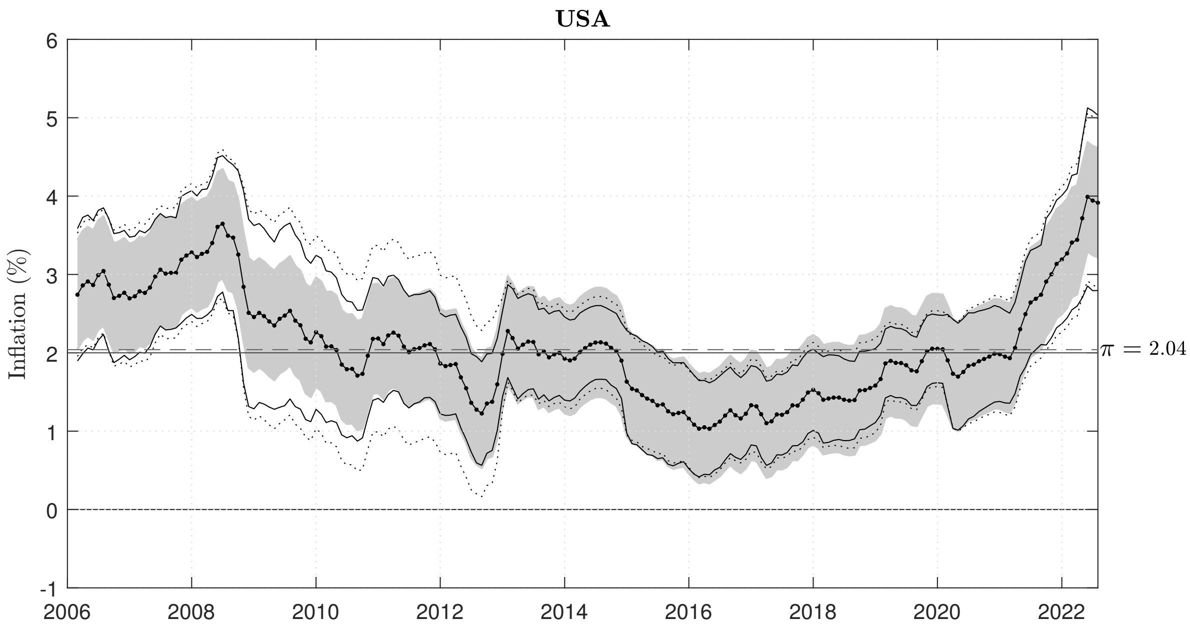


Figure 4: EMU and USA. Estimated inflation using inflation sample mean ($\bar{p}$) using a rolling window of size $n = 50$, and 90% confidence interval using the unconditional inflation volatility ($\sigma_{\bar{p}}$), the conditional inflation volatility under rational inattention ($V_t/\sqrt{n}$), and the rolling inflation standard deviation ($S_t/\sqrt{n}$) with n = 50. Sample: January 2002 to August 2022. Unconditional inflation volatility is estimated using data from January 2002 to December 2019.

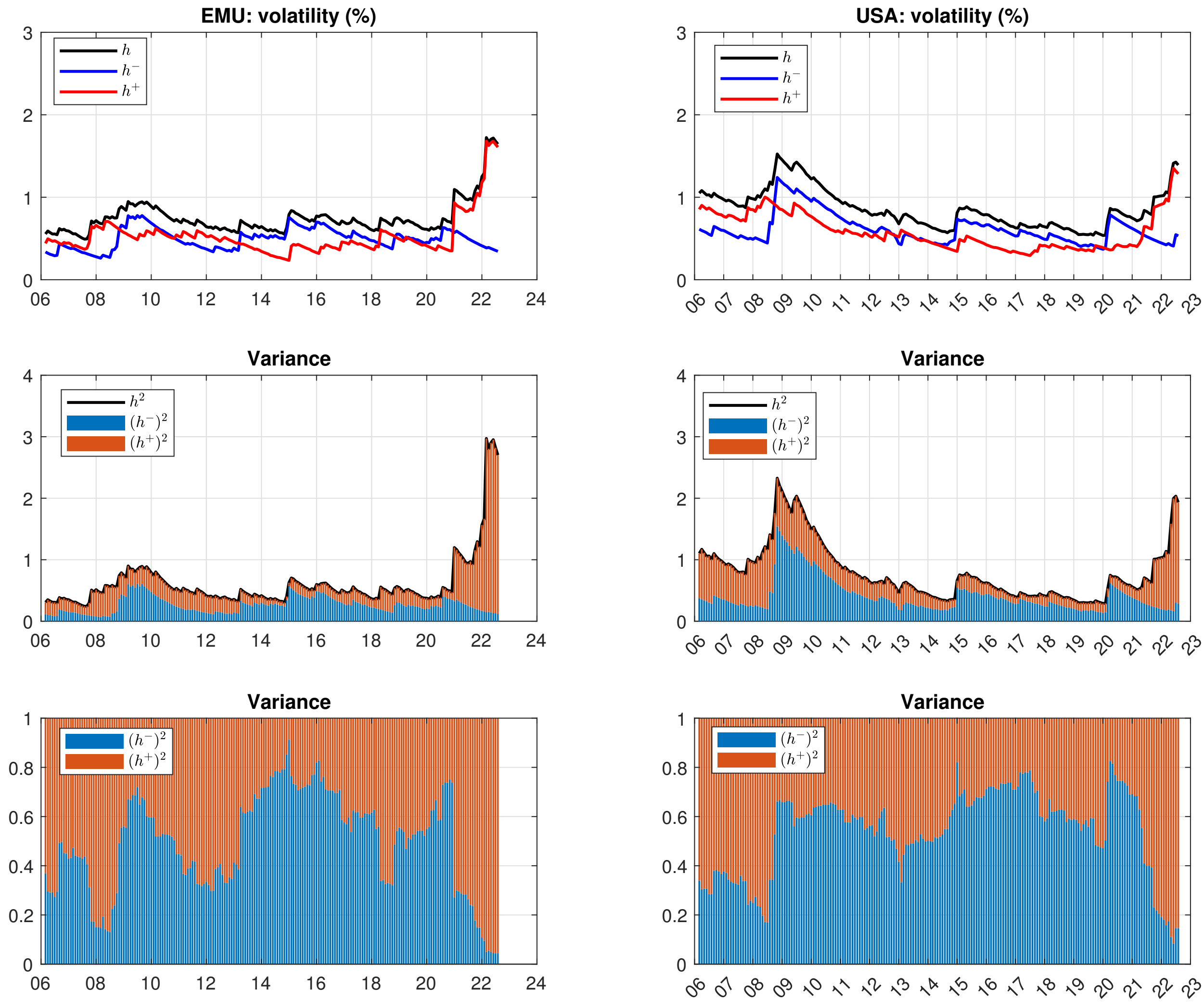


Figure 5: EMU and US inflation volatility ($h^{1/2}$) and variance (h) decomposition: unexpected upward (h^+) and downward (h^-) movements in prices (p). The upper panel includes annualized volatility series (%). Middle panel includes the conditional variance and each component. Downward panel is the proportion of variance due to positive and negative surprises.

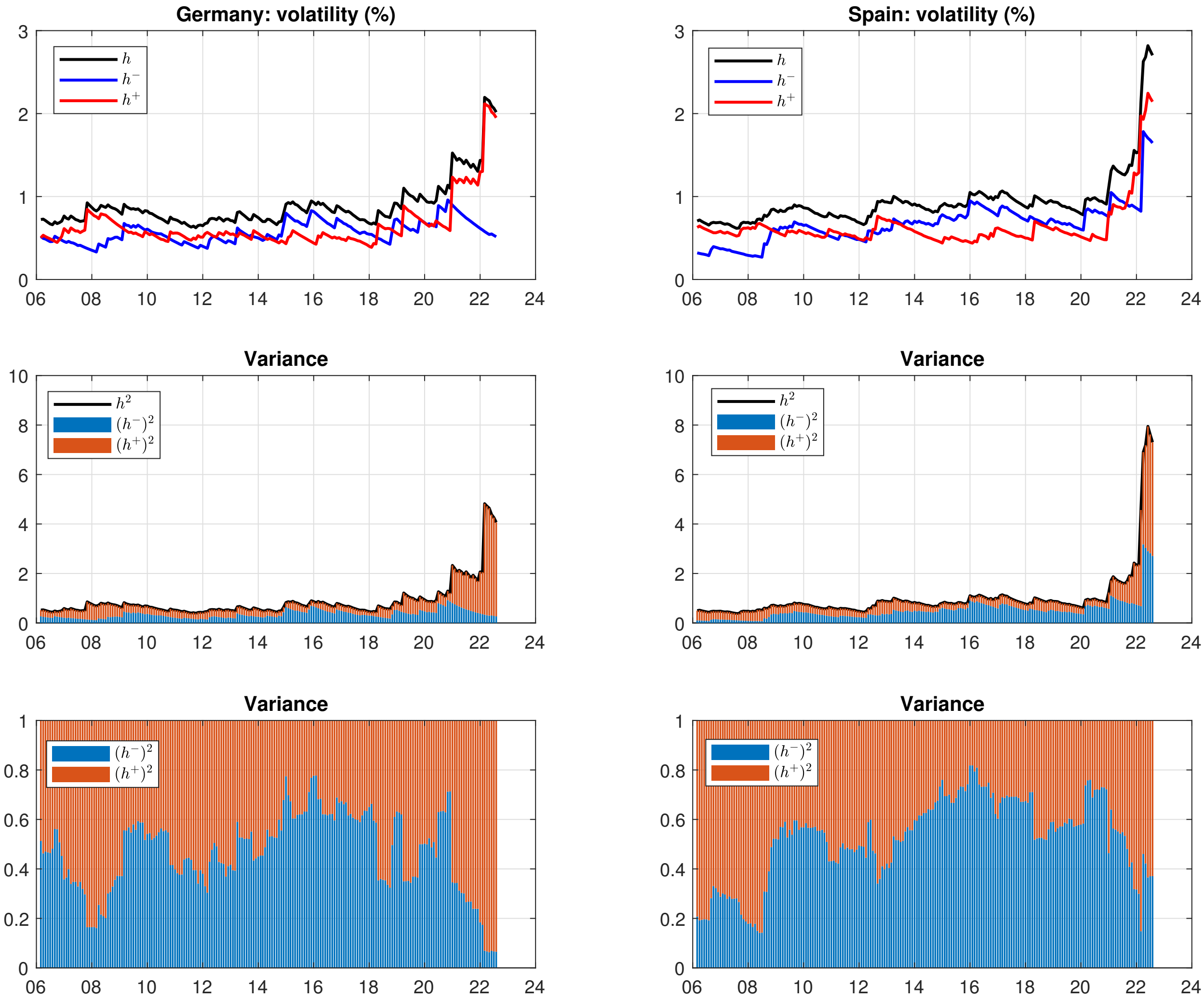


Figure 6: Germany and Spain inflation volatility ($h^{1/2}$) and variance (h) decomposition: unexpected upward (h^+) and downward (h^-) movements in prices (p). The upper panel includes annualized volatility series (%). Middle panel includes the conditional variance and each component. Downward panel is the proportion of variance due to positive and negative surprises.

Table 1: Estimated univariate models for inflation.

Variable	AR(2)		$AR_{12}(2)$		Mean	$SF^{(1)}$	Forecast Error			
	$\hat{\phi}_1$ (s.e.)	$\hat{\phi}_2$ (s.e.)	$\hat{\phi}_{12}$ (s.e)	$\hat{\phi}_{24}$ (s.e)	(s.e.) (%)		$ACF^{(2)}$ $Q(\nu)$	$JB^{(3)}$	$\hat{\sigma}_a$ (%)	$\hat{\varphi}$
Germany	0.043	0.11	–		0.11	67.4**	51.0*	4.2	0.24	0.061
	(0.067)	(0.07)			(0.02)					
Spain	0.40	–	–		0.15	75.1**	21.3	0.8	0.25	0.042
	(0.06)				(0.03)					
EMU	0.17	–	0.29	–	0.13	91.2**	56.1**	3.9	0.20	0.061
	(0.07)		(0.07)		(0.02)					
USA	0.54	-0.19	-0.18	-0.15	0.17	–	36.6	537**	0.25	0.051
	(0.07)	(0.07)	(0.07)	(0.07)	(0.02)					

Notes:

(1) SF: Shin and Fuller (1998) statistic tests whether an AR(1) operator is nonstationary. (2) Q is the Ljung and Box (1978) statistic for the innovations' autocorrelation function (ACF). H_0 is that there is no autocorrelation in the first 39 lags. H_0 is that there is no autocorrelation in the first 39 lags. (3) The Jarque-Bera statistic rejects normality at 90% (95%) confidence with critical values 4.61 (5.99)

*Rejects the null hypothesis at the 10% level, **Rejects the null hypothesis at the 5% level.

Table 2: Estimated univariate models for inflation. GARCH(1,1) innovations.

Variable	AR(2)		$AR_{12}(2)$		GARCH(1,1)		Forecast Error			
	$\hat{\phi}_1$ (s.e.)	$\hat{\phi}_2$ (s.e.)	$\hat{\phi}_{12}$ (s.e)	$\hat{\phi}_{24}$ (s.e)	h_{t-1} $(s.e.)$	a^2_{t-1} $(s.e.)$	$ACF^{(2)}$ $Q(\nu)$	$JB^{(4)}$	$\hat{\sigma}_a$ (%)	$ACF^{(3)}$ $Q(\nu)$
Germany	0.048	0.12	–		0.53	0.000	48.0	3.5	0.24	46.8
	(0.074)	(0.08)			(175.87)	(0.063)				
Spain	0.41	–	–		0.62	0.12	21.2	0.8	0.25	42.1
	(0.07)				(0.32)	(0.10)				
EMU	0.15	–	0.34	–	0.00	0.07	59.7**	3.6	0.20	47.1
	(0.07)		(0.05)		(1.28)	(0.08)				
USA	0.46	-0.18	-0.11	-0.21	0.73	0.17	41.1	93.0**	0.26	67.8**
	(0.09)	(0.07)	(0.06)	(0.06)	(0.09)	(0.06)				

Notes:

(1) SF: Shin and Fuller (1998) statistic tests whether an AR(1) operator is nonstationary. (2) Q is the Ljung and Box (1978) statistic for the innovations' autocorrelation function (ACF). H_0 is that there is no autocorrelation in the first 39 lags. (3) Q is the Ljung and Box (1978) statistic for the squared innovations' autocorrelation function (ACF). H_0 is that there is no autocorrelation in the first 39 lags. (4) The Jarque-Bera statistic rejects normality at 90% (95%) confidence with critical values 4.61 (5.99)

*Rejects the null hypothesis at the 10% level, **Rejects the null hypothesis at the 5% level.

Table 3: In-sample and out-of-sample goodness of fit for inflation conditional volatility measures.

In-sample	h_t			h_t^G			h_t^{RW}			n
01/2002-12/2019	β_1	RMSE	R^2	β_1	RMSE	R^2	β_1	RMSE	R^2	
Germany	0.23**	0.09	0.14	–	–	–	0.08	0.10	0.00	166
Spain	0.16**	0.10	0.08	0.05	0.06	0.00	0.03	0.10	0.00	166
EMU	0.15**	0.07	0.10	–	–	–	0.05	0.06	0.00	166
USA	0.11**	0.12	0.14	0.06**	0.13	0.07	0.01	0.07	0.00	166
Out-of-sample	h_t			h_t^G			h_t^{RW}			
01/2020-03/2022	β_1	RMSE	R^2	β_1	RMSE	R^2	β_1	RMSE	R^2	n
Germany	0.79**	0.49	0.62	–	–	–	-8.86**	0.76	0.10	27
Spain	0.86**	0.57	0.60	-0.04	0.92	0.00	-2.35**	0.90	0.00	27
EMU	0.57**	0.26	0.61	–	–	–	-3.08**	0.39	0.12	27
USA	0.29**	0.16	0.12	0.00	0.17	0.00	0.42	0.17	0.00	27

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A Appendix: Weights comparison

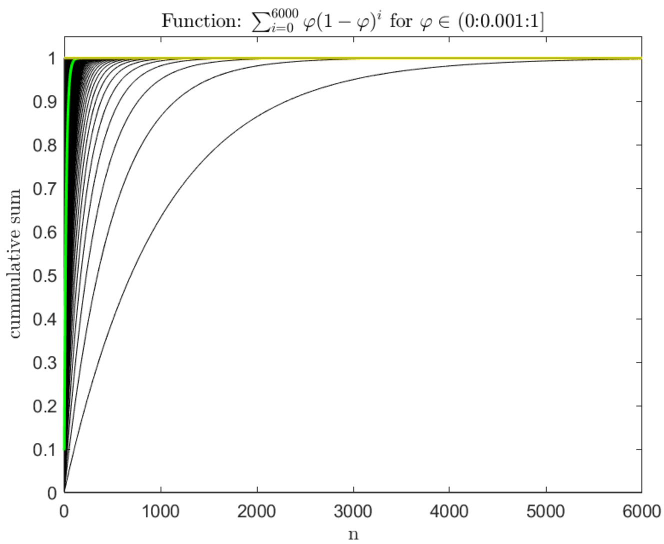
Of course, different weights are valid. For instance, here we consider two: (1) a typical function whose weights sum one, and (2) the restricted ARCH(q) proposed by Engle (1982). However, both definitions depend on the sample size n. Unlike these two approaches, our conceptual framework aligns with ARCH(∞) specifications for GARCH(p, q) type models intensively used in the literature, with $p > 0$ and $q \geq 0$. This specification aligns with our claim and literature intuition that the rate of news arrival drives the attention, and the exponential decay in the variance function instead of a specific truncation point in time n or q.

$$\text{Alternative 1:} \qquad \omega_i = \frac{\lambda^{i-1}(1-\lambda)}{(1-\lambda^n)} \tag{12}$$

$$\text{Alternative 2 (ARCH}(q)\text{):} \qquad \omega_i = \frac{(q+1)-i}{\frac{1}{2}q(q+1)}, \text{ where: } \sum_{i=1}^{q} \omega_i = 1 \tag{13}$$

Let us elaborate a little more this point. In our case, the cumulative weighted function tends to 1 as $i \to \infty$, for $\varphi \in (0, 1]$; see Figure 7, green and yellow lines flag $\varphi = 0.05$ and $\varphi = 1.0$ cases, respectively. When $\varphi = 0.05$, we reached 0.99 at $n = 89$, 0.95 at $n = 58$, and 0.90 at $n = 44$. Table 1 in the paper includes MLE for φ per country, from 0.042 to 0.061. We set $n = 50$ for all the countries to compare the volatility series consistently. This is also the time interval in the rolling standard deviation. In the cases analyzed in this paper, quantitative results do not vary if we set $n > 50$. Our weighted sum tends to one as $i \to \infty$, a standard set up in the GARCH(p,q) literature, with $p \geq 0$.

Figure 7: Sample size needed to cover a specific cumulative weight sum per each $\varphi \in (0, 1]$.



B Appendix: Comparison with GARCH models

We define the (inflation) variance conditional to the information available at period $t-1$, see Definition 3, as:

$$h_t = \sum_{i=0}^{\infty} \omega_i a_{t-1-i}^2 \tag{14}$$

where $\omega_i = \varphi(1-\varphi)^i$, $\forall i \geq 0$. We assume $0 < \varphi < 1$, which implies $\sum_{i=0}^{\infty} \varphi(1-\varphi) = 1$.

Let us, for simplicity, define $\bar{\varphi} = 1 - \varphi$. From (14), by replacing ω by φ and $\bar{\varphi}$, we get:

$$h_t = \sum_{i=0}^{\infty} \varphi \bar{\varphi}^i a_{t-1-i}^2 = \varphi(1 + \bar{\varphi}B + \bar{\varphi}^2 B^2 + \bar{\varphi}^3 B^3 + ...)a_{t-1}^2, \tag{15}$$

being B the backshift operator. Eq (15) is the exponential smoothing model with $\bar{\varphi}$ as discount factor. Note that $(1 + \bar{\varphi}B + \bar{\varphi}^2 B^2 + \bar{\varphi}^3 B^3 + ...)$ is summable, as $0 < \varphi < 1$.

On the other hand, a GARCH(1,1) is usually defined as:

$$h_t = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 h_{t-1}, \tag{16}$$

which can be equivalently written as:

$$h_t = \frac{\alpha_0}{1 - \beta_1 B} + \frac{\alpha_1}{1 - \beta_1 B} a_{t-1}^2. \tag{17}$$

Assuming $\alpha_0 = 0$, which is equivalent to assume that the unconditional variance is equal to zero, i.e., $\mathbb{E}(a_t^2) = 0$, and $|\beta_1| < 1$, allows (17) to be expressed as:

$$h_t = \alpha_1(1 + \beta_1 B + \beta_1^2 B^2 + \beta_1^3 B^3 + ...)a_{t-1}^2, \tag{18}$$

and clearly (18) is equal to (15) when $\alpha_1 = \varphi$ and $\beta_1 = \bar{\varphi}$.

Finally, a stationary GARCH(1,1) requires $|\alpha_1 + \beta_1| < 1$ in (16). However, in our model, by construction, $\varphi + \bar{\varphi} = 1$. Therefore, the model presented in (14) is equivalent to a restricted IGARCH(1,1) with $\alpha_0 = 0$ and $\alpha_1 = \varphi$.[10] The IGARCH(1,1) model with $\alpha_0 = 0$ is of particular interest, as it is the volatility model used in RiskMetrics.

[10] In an IGARCH(1,1) model, $\beta_1 = 1 - \alpha_1$.

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