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Abstract

The role of multilateral lenders in sovereign default has been traditionally overlooked by the literature. However, these creditors represent a significant share of lending to emerging markets and feature very distinct characteristics, such as lower interest rates and seniority. By including these creditors in a traditional DSGE model of sovereign default, I reproduce the high debt levels found in the data while maintaining default probabilities within realistic values. Additionally, I am able to analyze the role of multilateral debt in emerging economies. Multilateral loans complement private financing and reduce the incompleteness of international financial markets. Also, multilateral funding acts as an insurance mechanism in bad times, providing countries with some degree of consumption smoothing, opposite to the role of front-loading consumption fulfilled by private financing.

Keywords: sovereign debt and default, IFIs, multilateral institutions, seniority, consumption smoothing, emerging markets.

JEL classification: F34, F35, G15.

Resumen

Tradicionalmente, la literatura sobre *default* soberano ha pasado por alto el papel de los prestamistas multilaterales. Sin embargo, estos acreedores suponen un porcentaje significativo de los préstamos a los países emergentes y presentan una serie de características que los hacen muy diferentes, como tasas de interés más bajas o ser acreedores preferentes, entre otras. Al incluir a estos prestamistas en un modelo de equilibrio general dinámico estocástico tradicional de *default* soberano, puedo reproducir los altos niveles de deuda encontrados en los datos y mantener las probabilidades de *default* dentro de valores realistas. Además, analizo el papel de la deuda multilateral en las economías emergentes. Los préstamos multilaterales complementan la financiación privada y reducen la falta de completitud de los mercados financieros internacionales. Asimismo, la financiación multilateral actúa como un mecanismo de seguro en tiempos difíciles, lo que brinda a los países un cierto grado de suavización del consumo, en contraposición al papel de anticipación del consumo que cumple el financiamiento privado.

Palabras clave: deuda y *default* soberanos, instituciones financieras internacionales, instituciones multilaterales, orden de prelación de pago, suavización del consumo, mercados emergentes.

Códigos JEL: F34, F35, G15.

1 Introduction

The presence of heterogeneous lenders in sovereign borrowing and default, and particularly, the role of multilateral institutions, has been generally overlooked by sovereign default models. Nevertheless, official lenders,[1] which comprise bilateral and multilateral creditors, are the main source of funding for developing economies. These lenders tend to offer loans at lower interest rates and higher maturities than private lenders, which are mainly banks and bondholders. In spite of the importance of official lenders in general, and multilateral lenders in particular, the sovereign default literature on non-private creditors has primarily focused on the International Monetary Fund (IMF). This may be related to the role of the IMF as a bailout agency, together with the conditionality associated to its loans, and this predominance has occurred despite the relatively small share that IMF debt represents in total lending[2] (see Table 1). In fact, most papers that approach non-private lending do so from the bailout perspective, not considering the overall effect that official financing has on borrowing and default. However, in developing countries, official loans —bilateral and multilateral— are used not only in severe crises, but also as part of their regular funding.

Multilateral development banks (MDBs), which include the World Bank and other regional development banks such as the European Investment Bank (EIB) or the Inter-American Development Bank (IADB), are a significant source of funding for developing economies. In general, these institutions aim at promoting economic development and social progress through the funding of projects in areas such as infrastructure, education, health, etc., and also through budget support, mainly in low and middle-income countries.[3]

Multilateral lenders feature very distinct characteristics: they generally impose no conditionality; they keep financing countries after they default to private lenders, i.e.

[1] Official financing, according to International Debt Statistics database (IDS) by the World Bank are "loans from international organizations (multilateral loans) and loans from governments (bilateral loans)"

[2] Considering as total lending the sum of total public and publicly guaranteed debt and use of IMF credit as defined in IDS.

[3] According to IDS, multilateral loans include "loans and credits from the World Bank, regional development banks, and other multilateral and intergovernmental agencies. Excluded are loans from funds administered by an international organization on behalf of a single donor government; these are classified as loans from governments."

they do not impose financial exclusion after a default to private lenders; they are the only senior creditors together with the IMF; and they offer lower interest rates.[4] Additionally, multilateral lenders are usually repaid in full after they experience a default, being this type of default, indeed, an infrequent event. Then, what are the consequences of a cheap and senior flow of funds on interest rate spreads, debt levels and default probabilities? How do private creditors react to the fact that multilateral creditors will be repaid first in case of default?

In order to analyze this issue, I develop a DSGE model of sovereign default with two different lenders from which countries may borrow simultaneously: private lenders and multilateral institutions. The multilateral lender features the aforementioned characteristics, that is, seniority, lower interest rates, no conditionality, lack of financial exclusion after a private default and full repayment after a default to multilateral institutions themselves. To the best of my knowledge, a lender that combines these characteristics has not been fully portrayed in DSGE sovereign default models. Some of these attributes have been included in sovereign default models that feature an official lender that offers bailout loans, typically with conditionality. However, all these elements have not been portrayed together within a lender that is a regular creditor of the country.

By introducing a multilateral lender with the aforementioned characteristics, the model is able to generate high levels of public debt, 50 percent in the benchmark model —similar to what is found in the data—, with very reasonable levels of default, and with a degree of patience that is higher than many used in this strand of the literature and, therefore, closer to the microeconomic evidence. Furthermore, the combination between seniority and recovery rates that I develop in this model gives rise to a novel private debt price function, which is an important difference with respect to the existing literature and a contribution to it. Also, thanks to this setting, I am able to disentangle the role of multilateral lending vis-à-vis private financing, which is another important contribution of this article. Multilateral funding acts as an insurance mechanism for countries and fulfills a consumption-smoothing role —opposite to the front-loading use of private funds—. As a result, multilateral lending becomes a complement to private financing.

[4] Also, multilateral loans tend to offer longer maturities, but I abstract from this dimension in this paper.

1.1 Facts

In what follows, I will discuss the empirical facts that motivate the inclusion of a lender with the aforementioned characteristics in a version of Arellano (2008) sovereign default model.

Fact 1: Multilateral lending is one of the main sources of funding in developing countries. Multilateral loans, which exclude IMF loans, represent a significant share of total lending, while IMF loans, a typical focus in the literature, account for a much more modest share.[5] Multilateral loans account for almost a third of total lending as shown in Table 1 and are approximately as high as bilateral lending, which is the remaining share of official debt. In this regard, Horn et al. (2021) underscore[6] that "During the 1970s, multilateral lending first overtook bilateral lending and has remained dominant since then" (Horn et al., 2021, p. 11).

Table 1: Debt by Creditor (%)

Lender	**As Share of Total Debt and Use of IMF Credit**	**As Share of GDP**
Private Debt	27.8	11.2
Official Debt	66.3	31.9
of which Multilateral Debt	32.4	14.3
IMF Debt	5.9	2.6

Multilateral loans are also important in terms of GDP. Indeed multilateral funding represents on average 14 percent of GDP in developing countries, a higher share of GDP than bank loans and bonds altogether, which account for roughly 11 percent of GDP, and well above the IMF share.

Fact 2: Conditionality is not present in most multilateral loans. Conditionality is an important element of IMF loans, but it is not necessarily a widespread feature of multilateral loans. Conditionality is defined in Babb and Carruthers (2008) as "making

[5] In this paper I use data that comes mainly from Beers et al. (2020b) for default data, and World Bank's International Debt Statistics (IDS) and World Development Indicators for other economic indicators. Using the aforementioned databases, I create an unbalanced panel of 60 lower-middle and upper-middle income countries with data on sovereign debt and default from 1970 to 2015. For more details on the data used, please see Section A.3.

[6] Horn et al. (2021) include IMF loans as part of multilateral lending and data are scaled by the US GDP rather than by each country's GDP.

the disbursement of resources to national governments contingent on the performance of certain policies" (Babb and Carruthers, 2008, p. 13) and in Koeberle and Malesa (2005) as "the specific set of conditions attached to the disbursement of policy-based lending or budget support" (Koeberle and Malesa, 2005, p. 6). Before 1980 multilateral institutions had almost no loans with conditionality. As explained in Babb and Carruthers (2008), before 1980 the World Bank and other multilateral institutions offered almost only the so-called investment lending, which is typically not associated to conditionality. After 1980 conditionality was part of some loans, but in a small share of the total multilateral lending. For instance —and using the World Bank as an example—, Koeberle and Malesa (2005) show that only around 10-20 percent of operations and 30 percent of volumes from 1980 to 2003 in the World Bank were adjustment operations, which may involve conditionality, while the highest share of loans was still that of investment lending.

Fact 3: Multilateral creditors keep lending to countries that are in default with private creditors. The lack of financial exclusion from multilateral financing after defaulting to private creditors is key, since it casts doubt on one of the main assumptions of sovereign default models: financial exclusion (also called financial autarky). As a matter of fact, quantitative sovereign default models assume that countries repay their debts in order to avoid the penalties that a default involves, namely output losses and financial exclusion. Financial exclusion is usually defined as the inability of obtaining financing in international markets. However, multilateral banks keep offering funds to countries after they default to private lenders.

The empirical evidence in this regard includes Levy Yeyati (2009) who shows that private lending is negatively correlated with default, while official lending is not significantly affected by it. Also Avellán et al. (2021) find empirical evidence supporting this lack of financial exclusion, since they show how during fiscal crises, which include sovereign default, multilateral development banks do not decrease their funding to countries, opposite to private creditors.[7] Furthermore, in Section A.4, I estimate financial exclusion from multilateral lending when a default to private creditors takes place finding similar results.

[7] In Bru Muñoz (2022) I also find that official lenders do not impose financial exclusion to countries that are in default to private lenders. Nevertheless, Flogstad and Nordtveit (2014) find the opposite for concessional official lending.

Fact 4: Multilateral creditors are senior lenders as they enjoy the so-called preferred creditor status. As shown empirically by Schlegl et al. (2019) multilateral institutions and the IMF are the only senior creditors, given their preferred creditor status. According to Schlegl et al. (2019) the basis for their seniority is that it is acknowledged by the main creditor governments and important institutions in financial markets, like Moody's. Similarly, Cordella and Powell (2021) highlight that the preferred creditor status "is not strongly backed in international law"(Cordella and Powell, 2021, p. 2). In the same vein, Perraudin et al. (2016) highlight that the preferred creditor status is not a contractual feature, but "a market practice attributable to the incentives faced by distressed sovereign borrowers" (Perraudin et al., 2016, p. 9). According to Perraudin et al. (2016), the preferred creditor status is the result of countries trying to avoid defaulting to multilateral institutions since these keep funding countries when private lenders do not, which in turn is in line with the lack of financial exclusion after a default to private lenders. Moreover, their preferred creditor status contributes to the high credit standing that multilateral institutions enjoy, as it is also shown by Perraudin et al. (2016).[8]

Fact 5: Multilateral lenders tend to offer lower interest rates than private creditors. Multilateral lenders tend to offer better financial terms than private creditors. In this regard, Cordella and Powell (2021) highlight that international financial institutions can "lend limited amounts at close to the risk-free rate under most circumstances"(Cordella and Powell, 2021, p. 2). In particular, as explained in Nelson (2020) "Due to the financial backing of their member country governments, the MDBs are able to borrow money in world capital markets at the lowest available market rates, generally the same rates at which developed country governments borrow funds inside their own borders. The banks are able to relend this money to their borrowers at much lower interest rates than the borrowers would generally have to pay for commercial loans, if, indeed, such loans were available to them. As such, the MDBs' non-concessional lending windows are self-financing and even generate net income."(Nelson, 2020, p. 7).

Thus, the fact that multilateral lenders can offer relatively lower interest rates is linked to their ability to obtain funds at very favorable rates, which in turn is related to

[8]Other authors, such as Bolton and Jeanne (2009) suggest that seniority may be related to how difficult it is to renegotiate a given debt. They consider that countries may default on debt that is easier to renegotiate and repay debt that is harder to renegotiate, giving rise to some kind of *de facto* seniority.

the multilateral institutions' preferred creditor status. In this regard, Cordella and Powell (2021) underscore for multilateral institutions that "IBRD and the four main regional MDBs (ADB, AfDB, EBRD and IDB) maintain AAA ratings. Moody's and Standard and Poor's both suggest these five organizations enjoy preferred creditor status"[9] (Cordella and Powell, 2021, p. 3). Therefore, the aforementioned high credit standing, linked to the preferred creditor status, is a key part of the multilateral institutions' business: it is a factor that allows them to raise funds at low interest rates, which permits them in turn to offer relatively cheaper financing to countries.

Additionally, according to Dellas and Niepelt (2016) the low interest rate that official lenders offer is the result of the stronger penalties that these creditors can impose to defaulting countries, since these penalties reduce the probability of default.[10]

Fact 6: Default to multilateral lenders is an infrequent event. As a result of these very different and specific characteristics, default to multilateral lenders, represented by the IBRD and the International Development Association (IDA)[11] in panel (a) of Figure 1, is a rare event. Default to multilateral institutions tends to occur in periods of high debt as share of GDP, as shown in panel (b) of Figure 1. Therefore, countries default to multilateral lenders when either GDP is very low, or total debt is very high.

Fact 7: Multilateral lenders are repaid in full after they experience a default. As Perraudin et al. (2016) highlight, in the several defaults analyzed in Cruces and Trebesch (2013), those to multilateral development banks were never accompanied by a decrease in debt's face value and also, the few defaults to the Inter-American Development Bank never involved debt write-downs. Likewise, Schlegl et al. (2019) underscore

[9] The multilateral development banks (MDBs) listed above are the International Bank for Reconstruction and Development (IBRD), the Asian Development Bank (ADB), the African Development Bank (AfDB), the European Bank for Reconstruction and Development (EBRD) and the the Inter-American Development Bank (IDB). The IBRD is one of the two branches of the World Bank that offer loans to governments in developing countries.

[10] An example of one of these penalties is presented by Lang et al. (2021) who underscore how countries that were in arrears to the World Bank or to the IMF were excluded from the Debt Service Suspension Initiative, which, in the context of the Covid-19 pandemic, provided temporary debt relief in the means of a temporary suspension of debt service to official bilateral creditors.

[11] IBRD and IDA are the two branches of the World Bank that offer loans to governments in developing countries. These agencies are not the only multilateral lenders, but these are the only for which disaggregated data on defaulted debt is available in Beers et al. (2020b), therefore I use them in this paper as a proxy for all defaults to multilateral lenders.

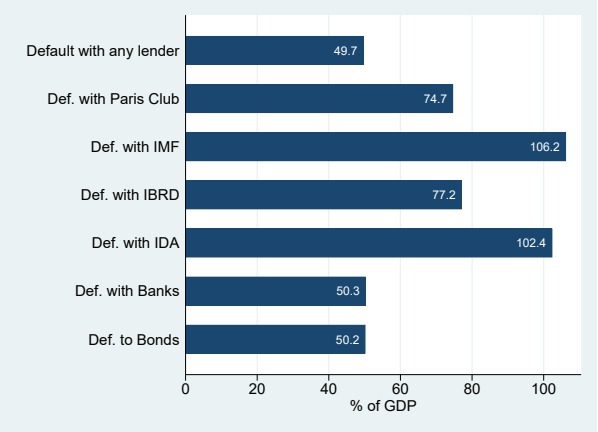
Figure 1: Probability of default and debt in default by lender

(a) Probability of default to each creditor

By Creditor	Probability of default
Paris Club	6.0
IMF	1.1
IBRD	1.4
IDA	0.8
Banks	2.9
Bonds	3.5

Note: The probability of default is computed by dividing the estimated number of default episodes with a specific lender by the number of years with positive debt stock with that lender. For more information on this estimation, please see Bru Muñoz (2022).

(b) Total debt as % of GDP in the first year in default to each of the lenders



that the IMF and the World Bank have only granted debt write-downs exceptionally under the Multilateral Debt Relief Initiative from 2005, but, as highlighted by Cordella and Powell (2021), who also show similar findings, the countries that benefited from this program did not have access to international private financial markets.

Fact 8: Multilateral funding is part of the regular funding of countries rather than an occasional bailout. Multilateral institutions tend to act as long-term lenders that fund either specific projects or provide budget support, while the IMF tends to act as a bailout agency. In this regard, as highlighted in Horn et al. (2021) "In 1944, the IMF was founded with the aim of providing short-term official funds to countries with temporary balance-of-payments problems, alongside with the World Bank that was intended to provide long-term development and reconstruction funds" (Horn et al., 2021, p. 8). Thus, even though some multilateral agencies may have bailout programs as well, it is not the core of their lending. In this regard, Horn et al. (2021) distinguish among several types of official financing according to their objective. These include "economic development" and "financial rescue" loans.[12] The first category —closer to the activity of multilateral developments institutions— tends to be substantially higher than the latter —closer to the IMF role— from the fifties, and except in the years of the Great Financial Crisis when they become very similar.

[12]Horn et al. (2021) distinguish among several types of official financing according to their objective, such as "economic development" and "financial rescue" loans which they define as follows: "The category economic development includes loans and grants extended for the financing of projects in developing countries ranging from infrastructure investments to state-building activities. [...] financial rescue loans covers loans, grants and guarantees during currency, debt and banking crises, including balance-of-payment crises, as well as general budget support" (Horn et al., 2021, p. 14).

In order to reproduce the facts shown in this section, I include within a DSGE sovereign default model a multilateral lender that offers loans with typically lower interest rates (as long as private debt is not very low), that imposes no conditionality[13] and that is senior. Also, this multilateral creditor does not penalize countries after they default to private lenders and continues offering funding to them in the event of a default to private creditors. However, if countries default to the multilateral institution, they face full financial autarky. The fact that countries can obtain multilateral funds after defaulting to private lenders reduces the cost of default, as highlighted by Hatchondo et al. (2017).

Additionally, in order to make seniority relevant in the model, I introduce recovery rates for both types of debt, which is a novelty in this type of models. When defaults to both lenders occur, countries must repay multilateral lenders in full before re-accessing, first multilateral, and second private financial markets. This assumption of full repayment to multilateral lenders is supported by the empirical evidence, as shown above. Furthermore, I also include positive recovery rates for private lenders to replicate what is found in empirical data and in order to maintain consistency between the characteristics of multilateral and private lenders.

Thanks to this new framework, I show that the inclusion of multilateral lenders produces higher levels of public debt and realistic default probabilities, with a discount factor that is relatively high in sovereign default models. Furthermore, this paper contributes to the existing literature on the role of multilateral debt in emerging economies. It is well established in the literature that private debt tends to be procyclical. However, to the best of my knowledge, the cyclicality of multilateral banks' lending (not of IMF lending, which has been widely covered) and how it relates with the cyclicality of private debt has only been approached in the empirical literature, but not in a DSGE sovereign default model. But with this setting, I show that multilateral lending tends to be acyclical or countercyclical, acting as an insurance mechanism for countries that allows them to maintain higher levels of total debt.

[13] Many authors, such as Boz (2011) or Fink and Scholl (2016) among others, include conditionality in their models for bailout loans, and even though conditionality is not present in most multilateral loans, it might be included as a feature of this model. In principle, conditionality would make multilateral funds relatively less attractive. However, the overall general equilibrium effects of conditionality would depend on the different modeling choices of such conditionality and its calibration.

This paper is organized as follows: Section 2 provides a general overview of the literature; Section 3 presents the model; Section 4 shows the main results of the model as well as the calibration and the empirical evidence supporting the findings; and Section 5 concludes.

2 A Review of the Literature

The canonical sovereign default models of Arellano (2008) and Aguiar and Gopinath (2006)[14] do not tackle the different types of creditors that a country may have, namely official and private creditors.[15] Nevertheless, the literature on the effect of official lending on sovereign default has quite developed in recent years, even though it has mainly focused on bailout loans from international institutions.

One of the most influential papers in this strand of the literature, Boz (2011), models an economy that may borrow from private lenders and from an International Financial Institution (IFI) which represents the IMF. IFI's debt is non-defaultable as a way of capturing seniority. To account for the conditionality imposed by the IMF, if countries decide to borrow from the IFI, they switch to a higher discount factor, since this involves lower debt levels. This conditionality explains why countries do not always borrow from the IMF, resulting in a countercyclical financing as countries would only resort to the IFI in bad times.

Similarly, Fink and Scholl (2016) characterize an official lender that offers bailouts and whose debt is non-defaultable in a model where government spending is financed through consumption taxes and loans from private lenders and IFIs. Debt from IFIs is not available in case of default to private lenders and if the government takes the bailout, it faces conditionality in the form of lower spending or higher taxes. With this setting, they are able to show how conditionality reduces default probability in the short-run, although it may increase it in the long-run due to the rise in private debt.

[14] Even though Aguiar and Gopinath (2006) incorporate bailouts from an unmodeled agent, these take the form of transfers rather than loans.

[15] An important attempt to address heterogeneous lending through the lens of seniority, but abstracting from official versus private creditors, is Bolton and Jeanne (2009). They consider two types of lenders, one with whom debt renegotiation is possible and another with whom it is not. Bolton and Jeanne (2009) somehow represent banks as creditors with whom debt renegotiation is possible, and bondholders as disperse lenders for whom the coordination of a renegotiation process is difficult.

Several authors such as Dellas and Niepelt (2016), Hatchondo et al. (2017), Kirsch and Rühmkorf (2017) or Roch and Uhlig (2018) have tried to explain the role of official lenders in sovereign default within the European Union. Hatchondo et al. (2017) also model non-defaultable official debt, in this case as Eurobonds, and find that it is able to reduce interest rate spreads in the short-run, but not in the long-run. They find that the effect of this debt on sovereign debt and default disappears in the medium term unless it is associated with fiscal rules. Thus, again only non-defaultable official debt is considered.

Among the aforementioned authors, only Dellas and Niepelt (2016) and Kirsch and Rühmkorf (2017) consider official defaultable debt. Dellas and Niepelt (2016) focus on the recent European debt crisis in a two-period model with private and official creditors. In this case, debt to official lenders may be defaulted jointly with private creditors under the pari passu clause. However, official lenders are able to impose stricter sanctions after default, sanctions that depend on the size of the official debt. As a result, higher official debt levels decrease the probability of default despite its effect on the increase in overall debt levels.

Also Kirsch and Rühmkorf (2017) analyze the impact of financial assistance on debt and default in a model with a sunspot variable that replicates investors' runs in order to generate defaults that are caused by these runs, and not only by economic fundamentals. They incorporate seniority of official debt but without recovery, so countries may default either to private lenders or to both, private and official. They also define official lending as bailouts that involve conditionality. In Kirsch and Rühmkorf (2017), financial assistance leads to lower spreads and less defaults caused by runs, while it increases debt levels and the overall default probability.

Similarly, Roch and Uhlig (2018) derive the implications of the inclusion of a bailout agency in a default model with two types of default, one driven by fundamentals and another caused by investors' runs. The bailout agency, whose debt is non-defaultable, decides in advance the maximum amount of debt that it is going to buy from the country in case of a crisis and it does not impose conditionality. Similarly to Kirsch and Rühmkorf (2017), but incorporating game-theoretic elements to the equilibrium, they show that these agencies prevent investors' runs, although they do not substantially reduce the default probability.

Likewise, Corsetti et al. (2018) build a model with two types of default, one caused by fundamentals and another by a debt rollover crisis, and two types of official creditors. In this model, countries may resort to both the IMF and the European Stability Mechanism, each of which feature different characteristics in terms of maturity and interest rate. In a setup where the country cannot perform selective defaults, official lending may allow countries to sustain higher levels of total debt, with longer maturities having a more significant effect than lower spreads. Also, Pancrazi et al. (2020) evaluate the welfare effects of a series of IFI's bailouts, which involve conditionality. They find that these welfare effects are not linear in the bailouts' size since there are tradeoffs between the size of the bailout program and the conditionality and duration of these programs.

In order to approach some of the implications of bilateral loans, Alfaro and Kanczuk (2019) study heterogeneity in lenders by introducing a creditor whose loan amounts are unknown by international lenders, what they call NPC (non-Paris Club) lenders. The authors want to analyze the impact of the undisclosed borrowing from China on overall debt sustainability. Taking into account the incomplete information feature, they find that having higher NPC debt involves higher default on international investors. The main reason behind this result is that the cost of default becomes lower as long as countries are able to continue borrowing from the NPC creditor. Similarly, Prein and Scholl (2021) construct a model with official defaultable debt without recovery and political turnover, where the official creditor represents bilateral lenders that offer bailout loans as in Greece's debt crisis. They find that this official financing increases overall debt levels and may cause political turnover, which in turn affects the default probability.

Recently, Cordella and Powell (2021) have approached multilateral development banks' emergency financing —rather than the consumption-smoothing funding used in normal times— analytically, which is uncommon in this strand of the literature. In their paper, Cordella and Powell (2021) show how the seniority of IFIs may be an equilibrium outcome, instead of an assumption, and provide analytical solutions to the problem of the interaction between IFI's and private lending. In their model, the IFI lends at the risk-free rate but manages risk by limiting credit volumes.

Additionally, this paper also benefits from the existing literature on debt recovery after default. One of the most prominent articles in this strand of the literature is Yue (2010).

However, in this article I tackle recovery as a merely random issue, which is actually a simplification since recovery rates in most papers are the result of a debt negotiation process.

Therefore, to the best of my knowledge, the novelties of this paper, i.e. an official lender that offers cheaper senior defaultable loans with positive recovery rates, without conditionality, and that only imposes financial exclusion if the defaulted creditor is the official creditor herself, have not been portrayed in a DSGE model yet. Thus, this paper contributes to better characterize the impact of one of the most important creditors in emerging countries on debt prices, debt dynamics and sovereign default.

3 The Model

This model follows the canonical models of Arellano (2008) and Aguiar and Gopinath (2006). As in Arellano (2008), the model features a benevolent sovereign in a small open economy that maximizes the utility of a representative agent by borrowing in international financial markets. Debt is contracted without commitment, therefore, the sovereign may default on its debt.

3.1 Income Process

As in Aguiar and Gopinath (2006), I assume an income process that consists of a permanent or trend shock, g_t, and a transitory shock, z_t. Then, the stochastic income process is as follows:

$$y_t = e^{z_t}\Gamma_t \tag{1}$$

The transitory shock follows an AR(1) process:

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z \tag{2}$$

with $\varepsilon_t^z \sim \mathcal{N}(0, \sigma_z^2)$ and $|\rho_z| < 1$, while the trend shock is:

$$\Gamma_t = g_t\Gamma_{t-1} \tag{3}$$

where g_t is the growth rate of the trend shock and its logarithm also follows an AR(1) process:

$$
\begin{aligned}
log(g_t) &= (1-\rho_g)(log(\mu_g) - c) + \rho_g log(g_{t-1}) + \varepsilon_t^g \\
c &= \frac{\sigma_g^2}{2(1-\rho_g^2)}
\end{aligned}
\tag{4}
$$

with $\varepsilon_t^g \sim \mathcal{N}(0, \sigma_g^2)$ and, $|\rho_g| < 1$.

The model is solved numerically in detrended form as shown in Section A.2 although in this Section it is presented before any detrending.

3.2 Sovereign's Problem

The model considers two types of lenders with different characteristics and from whom the sovereign may borrow simultaneously. I assume, following the literature, that there is a continuum of risk neutral lenders who buy bonds in private international financial markets. I will denote these bonds as b_t. The other lending instrument that countries can tap is debt contracted with multilateral institutions, m_t. Both private bonds and multilateral loans have a maturity of one-period: b_t and m_t are the private bonds and multilateral loans, respectively, issued or contracted in t-1 that mature in t, and b_{t+1} and m_{t+1} is the debt issued or contracted in t that matures in t+1. Also, $b_t < 0$ and $m_t < 0$, representing that the sovereign is borrowing from either private or multilateral creditors.

Multilateral lending is capped to a maximum that evolves with the growth of the economy: $m_{t+1} \geq \underline{m}_t = \underline{m}\Gamma_t$. The limit in the maximum amount of multilateral debt is in line with empirical evidence on the behavior of international financial institutions. These creditors and the countries usually sign a general portfolio agreement where the country and the multilateral institution settle on the maximum loan volume and the projects to be financed over a period of a few years. Also, multilateral institutions should avoid a high exposure to a single country to maintain a healthy financial situation in order to keep their high credit standing, which is key in their business. Furthermore, several papers addressing official financing also limit its maximum amount, for example Fink and Scholl (2016), Hatchondo et al. (2017), Kirsch and Rühmkorf (2017) or Cordella and Powell (2021). In this regard, Kirsch and Rühmkorf (2017) explain the need of limiting official debt because otherwise "a country in good credit-standing could always borrow arbitrarily large amounts before declaring default" (Kirsch and Rühmkorf, 2017, p. 783).

Multilateral debt features a lower interest rate compared to private financing. Following the literature on official creditors, I define its price, which I denote as $q_t^m(m_{t+1}, \Gamma_t)$, as an extension of the price in Boz (2011) adapted to a model with trend shocks:

$$q^m(m_{t+1}, \Gamma_t) = \frac{1}{1 + r - \phi \frac{m_{t+1}}{\Gamma_t}} \tag{5}$$

where $\phi > 0$ is the premium on the multilateral debt amount. As a result, the higher the amount of multilateral debt, the lower the multilateral debt price becomes, and the higher the interest rate. Given that these institutions are very few, they do not face a perfect competition environment and may be able to extract some profit over the risk free rate from their lending activity, as seen in Nelson (2020). Despite Boz (2011) modeled an institution like the IMF, this pricing function is adequate to model a multilateral institution as well given that MDBs do not maximize profits since their goal is to support development, and as a result, they only charge a small mark-up to borrowing countries, as highlighted by Avellán et al. (2021).

In Boz (2011) the price of official debt only depends on the amount borrowed from these institutions, and the same happens in this model once output is detrended as shown in Section A.2. Therefore, in this model, in a sense, the price of multilateral debt does not depend on the absolute value of multilateral debt[16] but on its relative value compared to the country's income, somehow a back-of-the-envelope estimation of the multilateral debt as share of GDP.

Furthermore, although countries may default on both types of debt, multilateral debt features seniority, that is, defaulting to multilateral lenders means defaulting to private lenders. Additionally, in order to borrow from private lenders after a default to multilateral creditors, countries must repay first the total amount of defaulted debt to the multilateral lender.

Time is infinite and discrete, and the timing of the model is as follows. Every period, first, the sovereign learns the realized income. Second, the sovereign decides whether to repay both debts, default on private debt —default—, or default on private and multi-

[16] If this price depended on the absolute value of multilateral debt, the price of multilateral debt would decrease, and interest rate would increase, throughout the country's history.

lateral debt —double default—. If the sovereign doesn't default, it chooses next period's private and multilateral debt, b_{t+1} and m_{t+1}. If the country defaults on its private debt, it still can borrow from multilateral lenders and has to choose m_{t+1}, but it cannot borrow from private lenders for a random number of periods, set by the parameter θ. However, if the country defaults on all its outstanding debt, it goes to financial autarky —isolation— and cannot borrow from any lender. Similarly, the sovereign will be able to borrow again from multilateral lenders in the next period with probability ξ, where $\xi > \theta$ and both ξ and θ are iid. In order to leave financial autarky, the country must repay its debt to multilateral institutions in full. Once the sovereign has repayed multilateral lenders and reaccessed multilateral financial markets, it still cannot borrow from private lenders for a random number of periods, depending again on θ. Re-access to private financial markets, both after a default and after a double default, requires repaying the outstanding private defaulted debt, although with a haircut.

Additionally, under both types of default, the government faces output losses. These output losses follow Arellano (2008) and depend on the parameter δ. Thus, the default cost function is different from Aguiar and Gopinath (2006) who assume a proportional default cost. Given that the income process in the model contains a trend shock, the economy grows at a rate g_t, and so should the threshold at which countries suffer output losses. Therefore, I construct a moving default threshold for period t that grows at the same rate as the economy in the following way:[17]

$$h(y_t) = \begin{cases} y_t & \text{if } y_t < \delta \mu_{e^z} \mu_g \Gamma_{t-1} \\ \delta \mu_{e^z} \mu_g \Gamma_{t-1} & \text{otherwise} \end{cases}$$

Once the model is detrended, the output losses scheme resembles the usual output cost in Arellano (2008), as shown in Section A.2. With this type of output losses, the country that defaults with relatively lower levels of income does not suffer output losses at all; however, these losses are higher for countries with higher levels of income.

The utility function of the representative agent is a CRRA function as in Aguiar and Gopinath (2006), with a discount factor $\beta \in (0, 1)$:

[17] μ_{e^z} is the mean of the log-normal distribution.

$$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma} \tag{6}$$

The setting in this model is very similar to Kirsch and Rühmkorf (2017). However, compared to their paper, there are several important differences. First, I do not include a sunspot variable that represents investors' runs. Second, the official lender in my model does not impose conditionality, since the focus of this paper is not on bailout loans but on the regular funds provided by multilateral institutions. Finally, even though in Kirsch and Rühmkorf (2017) official lenders are also senior, the lack of debt recovery in their paper compared to this article may reduce the effect that seniority has on debt prices and amounts, and also on default decisions, compared to this paper.

The value function of the sovereign when it has not defaulted to any of its lenders is the highest among the value of compliance, the value of default and the value of isolation:

$$V(b_t, m_t, z_t, \Gamma_t) = \max\{V^c(b_t, m_t, z_t, \Gamma_t), V^d(b_t, m_t, z_t, \Gamma_t), V^i(b_t, m_t, z_t, \Gamma_t)\} \tag{7}$$

where the value of compliance is:

$$V^c(b_t, m_t, z_t, \Gamma_t) = \max_{c_t} \Big\{ u(c_t) + \beta \mathrm{E_t}(V(b_{t+1}, m_{t+1}, z_{t+1}, \Gamma_{t+1}) \Big\} \tag{8}$$

subject to

$$\begin{aligned} c_t &= y_t - q_t(b_{t+1}, m_{t+1}, z_t, \Gamma_t) b_{t+1} + b_t - q_t^m(m_{t+1}, \Gamma_t) m_{t+1} + m_t \\ m_{t+1} &\geq \underline{\mathrm{m}} \Gamma_t \end{aligned} \tag{9}$$

The value under default is:

$$\begin{aligned} V^d(b_t, m_t, z_t, \Gamma_t) = \max_{c_t} \Big\{ u(c_t) + \beta \mathrm{E_t} \Big\{ \theta V^c(b_{t+1}, m_{t+1}, z_{t+1}, \Gamma_{t+1}) + \\ (1-\theta) V^d(b_{t+1}, m_{t+1}, z_{t+1}, \Gamma_{t+1}) \Big\} \Big\} \end{aligned} \tag{10}$$

subject to

$$c_t = h(y_t) - q_t^m(m_{t+1}, \Gamma_t) m_{t+1} + m_t$$

$$b_{t+1} = \lambda b_t g_t \text{ in case of re-access, otherwise } b_{t+1} = b_t g_t \tag{11}$$
$$m_{t+1} \geq \underline{\mathrm{m}} \Gamma_t$$

In order to keep the model tractable, countries that re-access private financial markets are forced to repay the recovery rate of the defaulted debt, $1 - haircut$, which equals λ. This is somehow in line with Yue (2010) who only allows for one round of renegotiation.[18] This way of re-accessing private financial markets is another difference from Kirsch and Rühmkorf (2017), where countries that re-access randomly financial markets after default may default again to private lenders or to official and private lenders simultaneously.

Additionally, the equality $b_{t+1} = b_t g_t$ is used to keep track of the amount of debt that the country must repay in case of re-access. It is not an amount of debt that the country can use while in default, on the contrary, this restriction may even involve an increase in debt levels for the country during default. This equation is needed for practical reasons because in the detrended model, detrended private debt (detrended variables are shown with a hat) remains constant in case of default, that is, $\hat{b}_{t+1} = \hat{b}_t$ which translates into $b_{t+1} = b_t g_t$. On top of the tractability reasons, this equation also fulfills the role of avoiding a debt dilution problem after default. Otherwise, in an economy that grows at a positive rate, the amount of defaulted debt would substantially decrease in a long default, which is not consistent with the empirical evidence. In a way, this equation tries to capture the fact that in default debt tends accumulate additional amounts due to interests in arrears, etc. The reason why m_{t+1} equals $m_t g_t$ in the problem below is analogous to the explanation for private debt.

The value under isolation is:

$$\begin{aligned} V^i(b_t, m_t, z_t, \Gamma_t) = u(h(y_t)) + \beta \mathrm{E_t} \Big\{ & \xi V^d(b_{t+1}, m_{t+1}, z_{t+1}, \Gamma_{t+1}) \\ & + (1-\xi) V^i(b_{t+1}, m_{t+1}, z_{t+1}, \Gamma_{t+1}) \Big\} \end{aligned} \tag{12}$$

[18] Also, the maximum available amount of private debt —$\underline{\mathrm{b}}\Gamma_t$— is sufficiently large not to be binding, while its magnitude is such that, when a country is forced to repay the share of outstanding debt, there are combinations of assets that would not violate the non-negativity of consumption. Furthermore, the non-negativity of consumption should also be granted by the utility functional form. From this follows that the same occurs for the repayment of multilateral debt after a double default given that $\underline{\mathrm{b}} < \underline{\mathrm{m}}$.

subject to

$$b_{t+1} = b_t g_t$$

$$m_{t+1} = m_t g_t$$

Compared to Kirsch and Rühmkorf (2017),another difference with respect to this paper is that in their model countries may go directly from the situation of default to both lenders to a compliance situation, although the probability of re-access after defaulting to both lenders is lower than after defaulting to just private lenders. In a sense, that is similar to what I construct, since the addition of both re-access probabilities, ξ and θ, means that it would take more time for a country in double default to go back to the compliance situation than for a country that only defaults to private creditors. However, in my model, countries also have to repay all or a share of the defaulted debt, a key difference with Kirsch and Rühmkorf (2017).

With this setting, a country will default to private lenders, that is, the dichotomous control variable for the default decision will be $d(b_t, m_t, z_t, \Gamma_t) = 1$, if $V^d(b_t, m_t, z_t, \Gamma_t) > V^c(b_t, z_t, \Gamma_t)$ and $V^d(b_t, m_t, z_t, \Gamma_t) \geq V^i(b_t, m_t, z_t, \Gamma_t)$, and it will equal 0 otherwise. Similarly, a country will perform a double default, $Dd(b_t, m_t, z_t, \Gamma_t) = 1$, only when it is more profitable than just defaulting, that is, when $V^i(b_t, m_t, z_t, \Gamma_t) > V^c(b_t, z_t, \Gamma_t)$ and $V^i(b_t, m_t, z_t, \Gamma_t) > V^d(b_t, m_t, z_t, \Gamma_t)$. This dichotomous decision variable will equal 0 if none of these happen. Thus, the total probability of default, $Totald(b_t, m_t, z_t, \Gamma_t)$ will be equal to the sum of these two decisions, that is, $Totald(b_t, m_t, z_t, \Gamma_t) = d(b_t, m_t, z_t, \Gamma_t) + Dd(b_t, m_t, z_t, \Gamma_t)$, given that $d(b_t, m_t, z_t, \Gamma_t)$ and $Dd(b_t, m_t, z_t, \Gamma_t)$ are two mutually exclusive events.

3.3 Lenders' Problem

With this setting, the price of private debt, $q_t(b_{t+1}, m_{t+1}, z_t, \Gamma_t)$ depends on the different probabilities of default and re-access. Given the international risk-free interest rate, r, the recovery rate, λ, the probabilities of market re-access, θ and ξ, and the different default probabilities, these lenders face a zero-profit condition, which gives rise to the private debt price, as explained in Section A.1:

$$q(b_{t+1}, m_{t+1}, z_t, \Gamma_t) = \mathrm{E_t}\left\{ \frac{1 - Totald(b_{t+1}, m_{t+1}, z_{t+1}, \Gamma_{t+1})}{1+r} \right\} + \mathrm{E_t}\left\{ \frac{\lambda\ \theta\ d(b_{t+1}, m_{t+1}, z_{t+1}, \Gamma_{t+1})}{(r+\theta)(1+r)} \right\} + \mathrm{E_t}\left\{ \frac{\lambda\ \theta\ \xi\ Dd(b_{t+1}, m_{t+1}, z_{t+1}, \Gamma_{t+1})}{(r+\theta)(r+\xi)(1+r)} \right\} \tag{13}$$

3.4 Recursive Equilibrium

Therefore, the equilibrium definition in this small open economy is a set of policy functions for the debt holdings $b_{t+1}(b_t, m_t, z_t, \Gamma_t)$, $m_{t+1}(b_t, m_t, z_t, \Gamma_t)$, default decisions $d(b_t, m_t, z_t, \Gamma_t)$, $Dd(b_t, m_t, z_t, \Gamma_t)$ and the private debt price function $q(b_{t+1}, m_{t+1}, z_t, \Gamma_t)$ such that:

- Taking as given the private debt function $q(b_{t+1}, m_{t+1}, z_t, \Gamma_t)$ and the multilateral debt price $q^m(m_{t+1}, \Gamma_t)$, the policy functions solve the sovereign's maximization problem.
- The private debt function $q(b_{t+1}, m_{t+1}, z_t, \Gamma_t)$ satisfies the zero-profit condition for private lenders.

4 Quantitative Analysis

4.1 Calibration

For the model calibration, I borrow some parameters from the literature, I estimate others to match historical data, and I calibrate two of the parameters, the discount factor β and the output cost δ, to match the model's statistics to the data. Periods in this model are defined as years. First of all, I estimate the income process to match Argentina's GDP using both, parameters from the literature and Argentinian data. In this regard, first, I take the trend component of the income process from Seoane and Yurdagul (2019) and second, I obtain the cycle component by matching some descriptive statistics of Argentinian GDP. These statistics are the standard deviation and the autocorrelation of the GDP growth rate,[19] which are 5.36 percent and 0.1273, respectively, over the period

[19] Given that this is a detrended model, output volatility and correlation of output cannot be computed in the model simulation. This is the reason why I match them for the output growth rate instead.

1970-2015. In order to estimate the cycle components, after an initial guess, I minimize the quadratic differences between the estimated descriptive statistics and the actual statistics of Argentinian GDP and keep iterating until the difference is sufficiently small. The coefficients describing the income process coming from Seoane and Yurdagul (2019), ρ_g and σ_g, and from the estimation, ρ_z and σ_z, are shown in Table 2.

I calibrate the income process to Argentina because it is a typical choice in the sovereign default literature; however, in order to have more observations, especially for multilateral debt and default, I try to match debt and default to a wider sample of countries. For that, I use the panel of 60 countries specified in Section A.3 for which there

Table 2: Descriptive moments of the income process

Parameter	**Mean**
ρ_g	0.5499
ρ_z	0.0
σ_g	0.0353
σ_z	0.0294

is data on sovereign debt and default, including default to multilateral lenders, covering from 1970 to 2015.[20]

The rest of the parameters in the model are calibrated again either using data, or parameters from the literature as shown in Table 3. For instance, I calibrate r to match the annual risk-free rate in international markets and γ as the standard risk aversion parameter in the literature. The private sector exclusion period, θ, comes from two of the most important empirical papers in sovereign default, Gelos et al. (2011) and Cruces and Trebesch (2013). On the other hand, the official sector exclusion period, ξ, is directly estimated from the data as shown in Section A.4. θ equals 0.25 and ξ equals 0.5, which represent 4 and 2 years of private and multilateral financial exclusion, respectively. For the recovery rate, λ, which is equal to $1 - haircut$, I take the average haircut estimate measured in terms of face value reduction calculated by Cruces and Trebesch (2013).

[20]For more information on the sample please see Bru Muñoz (2022).

Table 3: Calibrated parameters

Parameter	Value	Basis
r	0.04	Data average
γ	2	Standard value in the literature
θ	0.25	4 years of private financial exclusion from literature
ξ	0.5	2 years of multilateral financial exclusion (data)
λ	0.83	Recovery rate: 1 minus the haircut in Cruces and Trebesch (2013)
ϕ	0.05	Data
μ_g	1.01	Average growth of per capita GDP
$\underline{\mathrm{m}}$	0.15	Data
δ	0.92	Output loss of 8.0%
β	0.915	Default frequency target of 3%

In the benchmark model I set $\underline{\mathrm{m}} = 15\%$ which is approximately the average multilateral debt as share of GDP in my sample. Given how multilateral debt is granted, it makes sense to match the maximum level of multilateral debt to the average level, since it is unlikely that a country would use an amount of m_t far below $\underline{\mathrm{m}}$ given that multilateral debt tends to offer better financial terms than private debt. Nevertheless, in Section 4.3 I consider other values for $\underline{\mathrm{m}}$ going from the absence of multilateral debt to $\underline{\mathrm{m}} = 20\%$ in order to further study the effect of multilateral debt, and analyze the implications of changes in the maximum available amount of multilateral debt. Regarding ϕ, as shown in Faure et al. (2015) there is substantial heterogeneity across interest rates in multilateral development banks although the spread tends to be below 2 percent. Then, in order to calibrate ϕ, I take the World Bank's average spread of 0.5 percent to the 6-month Libor in Faure et al. (2015), which becomes a 1 percent annual spread when the detrended multilateral debt equals its maximum threshold in Section 4.3, that is, $\underline{\mathrm{m}} = \hat{m}_{t+1} = 20\%$. Finally, the discount factor β and the output cost δ are set to match the model's average default frequency to the data.

As explained in Section A.5, I solve the model in detrended form through value function iteration, discretizing the grid for the endogenous states (private and multilateral debt) and exogenous states (cycle and trend shocks). The figures in Section 4.2 correspond to the detrended model.[21]

[21] Detrended variables are presented with a hat.

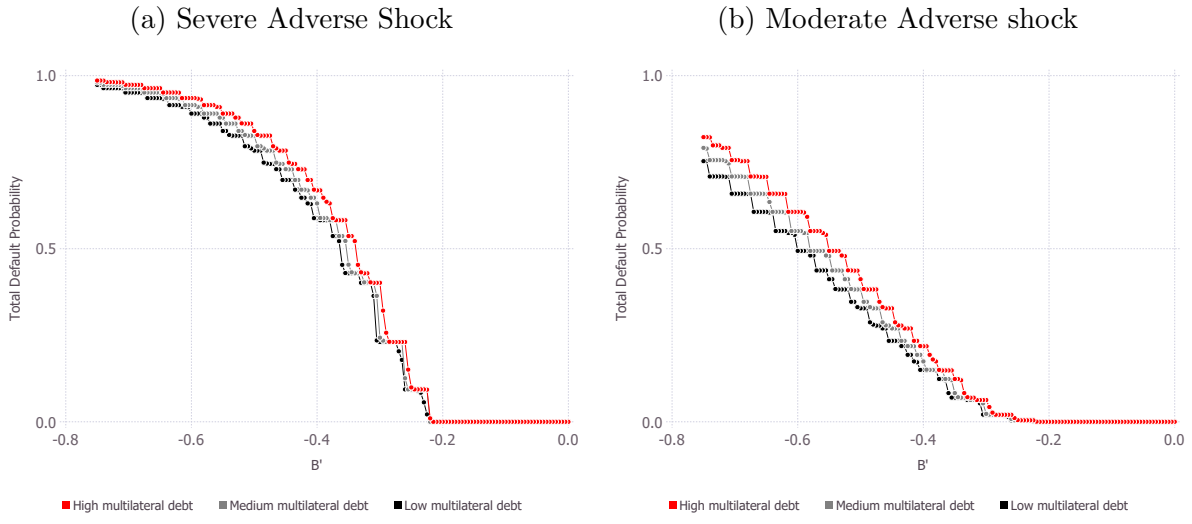
4.2 Results

4.2.1 Policy Functions

By including multilateral lenders in the model, I get to improve some of the traditional features of the sovereign default model. In particular, I am able to generate high sustainable debt levels and realistic default probabilities, with a value for β that is relatively high in sovereign default models. Also, I can disentangle the role of multilateral debt as a consumption-smoothing mechanism versus a front-loading system, as I will show in Section 4.3.

Total default probability increases with the amount of multilateral debt, as presented in Figure 2. Higher levels of multilateral debt have a negative effect on private and multilateral debt sustainability, rising the overall default probability compared to situations when the country has not used the available multilateral funds, which is represented by the black line in Figure 2. In that case, total default probability is lower since the sovereign still has at her disposal multilateral funds to be used if they are needed.

Figure 2: Total default probability for different levels of multilateral and private debt



In Figure 3 I find that the probability of default to only private lenders decreases for high levels of private debt. The reason for this decrease is that for those high debt levels,

countries do not only default to private lenders, but they also default to multilateral creditors, as shown in Figure 4. Therefore, it is not an actual fall in the default probability to private lenders, on the contrary, that decline reflects a positive probability of a double default. Thus, total default probability never decreases with private debt levels.

As in the standard sovereign default model, the default probabilities are substantially higher for severe adverse shocks, compared to less adverse situations. In particular, the increase in the probability of a double default is substantially higher for severe adverse shocks than for moderate adverse shocks, where this probability is only positive for high private debt levels, as shown in Figure 4.

The fact that default to multilateral lenders (together with private creditors) occurs mainly when shocks are very adverse and for high debt levels is in line with the evidence presented in panel (b) of Figure 1. Thus, the model is able to reproduce the empirical evidence: default to multilateral lenders happens in periods of high overall debt as share of GDP, due to debt being relatively high, GDP relatively low, or both.

Figure 3: Probability of default to private lenders only for different levels of multilateral and private debt

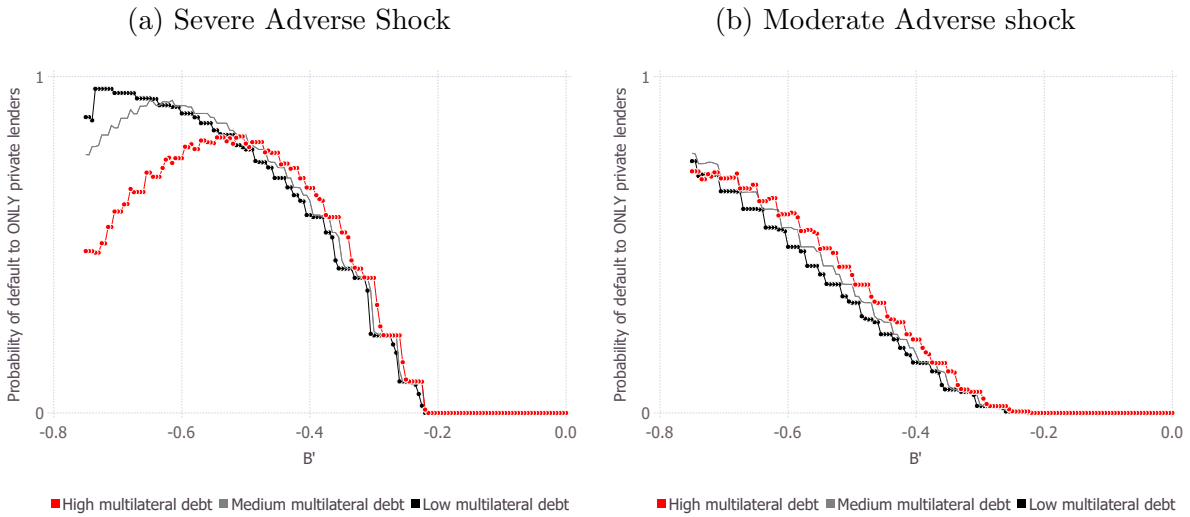
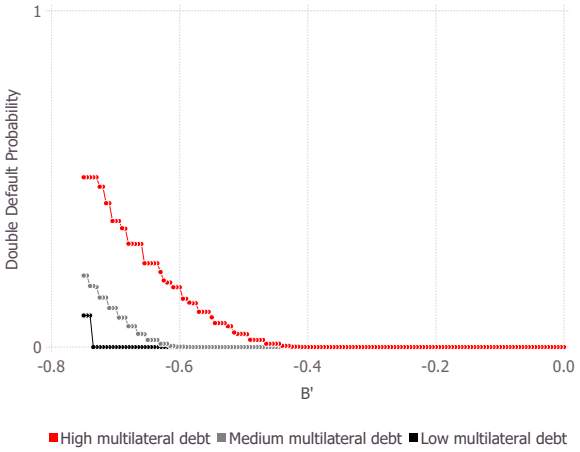
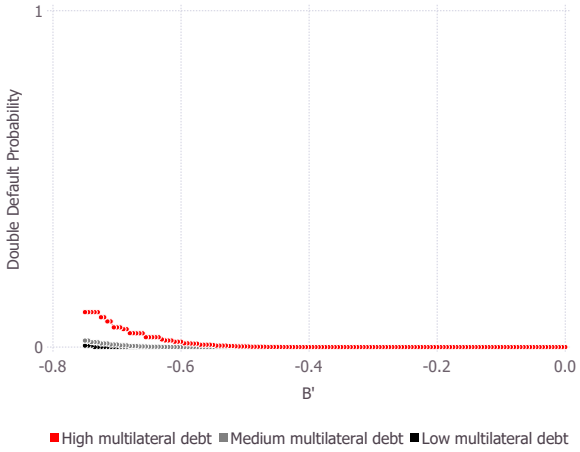


Figure 4: Probability of default to private and multilateral lenders (double default) for different levels of multilateral and private debt

(a) Severe Adverse Shock



(b) Moderate Adverse shock



Private lenders internalize the higher risk of default that comes with the higher multilateral debt levels. As a result, private debt prices decrease as multilateral debt increases, as shown in panel (a) of Figure 5.

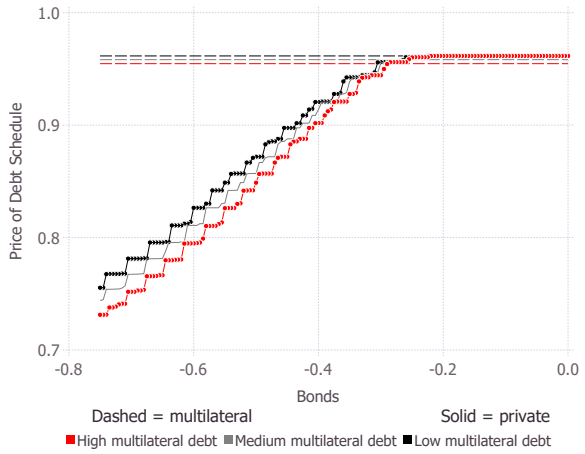
Similarly to Boz (2011), the probability of default rises with official debt, but at a slower pace than it does with private debt. And likewise, the changes in the private debt price schedule are higher when private debt increases than when multilateral debt does. Figure 5 provides evidence in this regard. In panel (a) of Figure 5 the black line, i.e. low multilateral debt, represents no multilateral debt at all, and the red line, i.e. high multilateral debt, represents the maximum total amount of multilateral debt in the benchmark model, $\underline{m} = 15\%$. Therefore, it is easy to see that for the same level of total debt,[22] the price of private debt is lower if that debt is comprised of private debt only.

On the one hand, an increase in multilateral debt reduces private debt prices. The reduction in debt prices is related to the fact that high multilateral debt means that debt service to these lenders will be high in the next period, which rises the default probability since multilateral lenders are senior creditors. Furthermore, high levels of multilateral debt reduce the risk that a country faces in the case of a private default given the lack of financial exclusion provided by multilateral creditors if the country defaults to private lenders only. Also, defaulting to multilateral lenders implies a higher cost, since it involves total financial exclusion (rather than partial exclusion as in the default to only

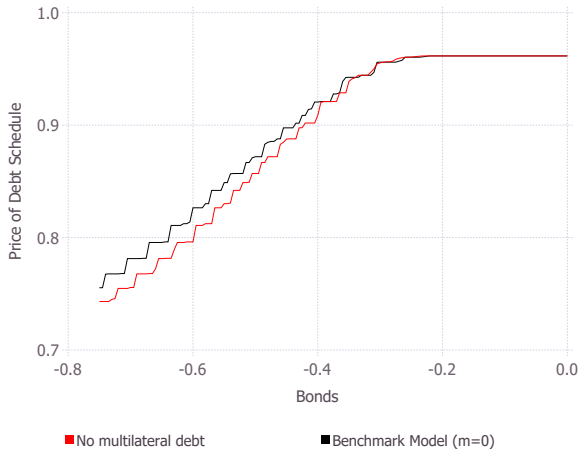
[22] Being total debt the sum of multilateral and private debt.

Figure 5: Price of private debt schedule

(a) For different levels of multilateral and private debt



(b) In a model without multilateral debt and in the benchmark model with $m_t = 0$



private lenders). Therefore, all these factors contribute to the reduction of private debt prices when multilateral debt grows.

However, on the other hand, there is a counteracting effect of multilateral debt that increases private debt prices. Multilateral financing rises private debt prices since a government with positive multilateral debt is less constrained in terms of borrowing. In this regard, Dellas and Niepelt (2016) underscore that "private loans may also become safer when official credit serves to enhance the debtor country's repayment capacity" (Dellas and Niepelt, 2016, p. S17). In my model the country may, in a sense, use multilateral debt, which is cheaper, to repay more expensive private debt. This is also in line with what may happen in developing economies. Given that multilateral loans tend to be earmarked, they may free up public resources that can be used for other purposes, such as repaying private debt.

As a result, I find that a debt portfolio that includes private and multilateral debt involves a higher debt price than a portfolio with the same total debt amount that includes only private debt, as shown in panel (a) of Figure 5. Additionally, I compare the price in the benchmark model when $\hat{m}_t = 0$ with a model like the benchmark model but without any available multilateral debt. The results are shown in panel (b) of Figure 5: for almost every debt level, private debt price tends to be lower when multilateral debt is not available than when multilateral debt is available, even though it is not used.

In sum, the availability of multilateral funds increases private debt prices. Nevertheless, as those funds are used, private debt price falls, as shown in Figure 5. For a given level of debt, the availability of multilateral debt reduces private debt spreads as the sovereign is less constrained in terms of borrowing, but as multilateral debt is used, the country becomes more constrained and private debt spreads increase.

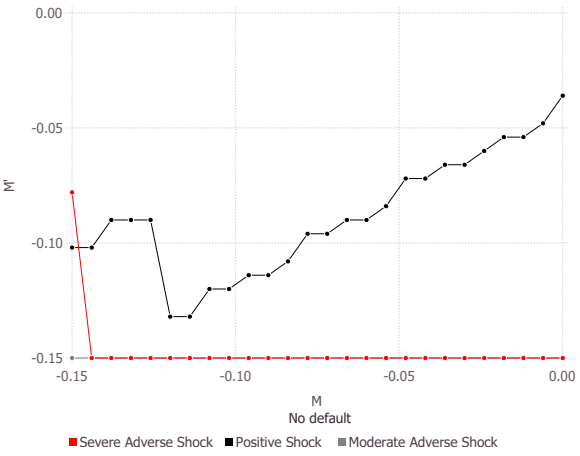
In this regard, despite a country with access to multilateral debt is less constrained in terms of borrowing, if the sovereign has chosen levels of $\hat{m}_t$ close to $\underline{m}$, it may also become constrained. I conjecture that this is the reason why countries do not always choose $\hat{m}_t = \underline{m}$, as shown in Figure 6. In Figure 6 the horizontal axis represents the existing level of multilateral debt and the vertical axis the level of multilateral debt chosen for the next period. Therefore, when the lines are above the 45 degree line, it involves a reduction in multilateral detrended debt compared to the existing levels, and the opposite happens when it is below the 45 degree line.

The relation between the choice of multilateral debt and productivity shocks is not linear. As shown in panel (a) of Figure 6 in case of no default, for adverse negative shocks countries choose the maximum available multilateral debt, $\underline{m}$, regardless the existing level of $\hat{m}_t$ except for severe adverse shocks and very high multilateral debt stocks. In that case, countries cut their existing multilateral debt stock roughly in half from one period to the next. Meanwhile, when the shocks are positive countries avoid hitting $\underline{m}$. Also, for positive shocks countries increase their levels of multilateral debt unless the existing debt levels are at the higher end of the distribution. In that case, countries reduce their stock of debt with multilateral creditors. This behavior suggests that when countries enjoy good times, they try to avoid the higher risks associated with high multilateral debt stocks, since a default to these lenders would leave a country in isolation. Additionally, private debt becomes cheaper in good times and, on top of that, for low $\hat{b}_{t+1}$ and high $\hat{m}_{t+1}$, the interest rate of the former may be lower than the one of the latter, as shown in Figure 5. Nevertheless, in bad times countries use multilateral funding since it tends to be cheaper.

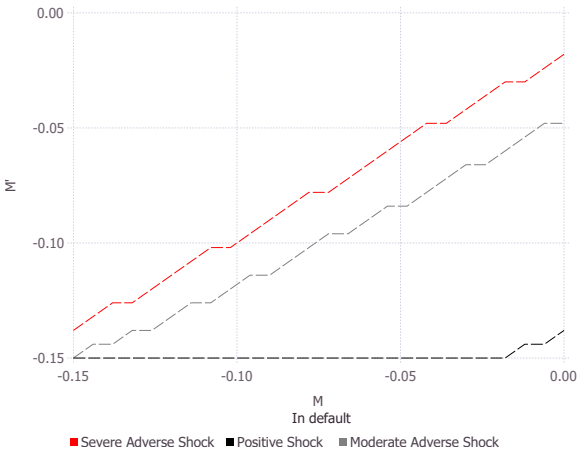
Conversely, when countries are in default with private lenders, they only reach $\underline{m}$ if the shock is positive. For moderate adverse shocks, countries increase their multilateral

Figure 6: Multilateral debt choice under different shocks

(a) No default



(b) Under default to private lenders



debt levels without reaching the maximum available debt stock (unless their existing level was already $\underline{m}$). If the shock is severely adverse, countries keep their multilateral debt stock relatively stable. I conjecture that the reason behind this different behavior under default is related to the way countries re-access financial markets after default. When countries re-access private financial markets randomly after default, they must repay the corresponding share of the stock of private debt defaulted, $\lambda \hat{b}_t$. Then, by not taking high $\hat{m}_{t+1}$ in bad times, countries try to avoid a substantial fall in consumption in case they had to repay $\lambda \hat{b}_t$ together with high stocks of $\hat{m}_{t+1}$ in the next period. Furthermore, it is related to the findings of Kirsch and Rühmkorf (2017) who explain that the larger the amount of official loans that is still available, the more relevant the insurance role of official lending is.

Therefore, the role of multilateral debt is not always the same, and it changes depending on the income and the existing level of multilateral debt. In a sense, cheap multilateral debt can act as an insurance mechanism. It is a factor that makes a country less constrained in terms of borrowing, which in turn, increases debt prices. However, there exists a risk that comes with multilateral lending, which is total financial exclusion. In very adverse economic situations, when reaching the maximum threshold $\underline{m}$ is more attractive, the country may end up being constrained. Therefore countries balance the positive effect of multilateral debt as a cheap funding resource with the higher risk

associated with it in case of adverse economic shocks. This is also related to the role of multilateral debt as a consumption-smoothing mechanism, as I show in Section 4.3.

4.2.2 Private Debt Price Dynamics in Related Studies

Boz (2011) highlights the counteracting effect of official debt on private debt prices. She points out that the reduction in private debt prices coming from official funds is caused by the high debt service to official lenders, which in her model increases the default probability given the non-defaultability of official debt. However, on the other hand, in Boz (2011) the availability of official loans increases private debt prices through the discount factor. As a result, in her model a debt portfolio with private and official debt involves a lower spread than a portfolio with only private debt, as in this paper. Likewise Fink and Scholl (2016) highlight how high levels of official debt reduce the risk of a private default. In this regard, Fink and Scholl (2016) and Kirsch and Rühmkorf (2017) also find that official financing rises private debt prices because, as in this model, a government with positive official debt is less constrained.

Additionally, the results of the comparison of the private debt price in the benchmark model when $\hat{m}_t = 0$ with a model without any available multilateral debt are the same as in Fink and Scholl (2016), Kirsch and Rühmkorf (2017) and Pancrazi et al. (2020), that is, private debt spreads tend to be higher when official debt is not available compared when it is available, even if it is not used.

4.2.3 Debt Dynamics around Default

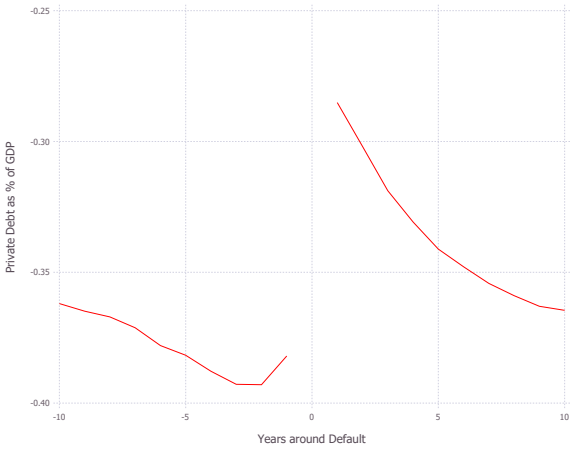
Both private and multilateral debt tend to increase before default. However, when private default happens, multilateral debt decreases.[23] This suggests that countries need lower levels of multilateral debt when they do not have to repay private debt in the next period, since a significant amount of resources are freed up by not repaying private debt.

Despite the benchmark model can only produce very small rates of double default (0.002 percent probability of double default), a model with a lower $\underline{m}$, i.e. a model that allows higher multilateral debt, can produce higher rates of double default. For

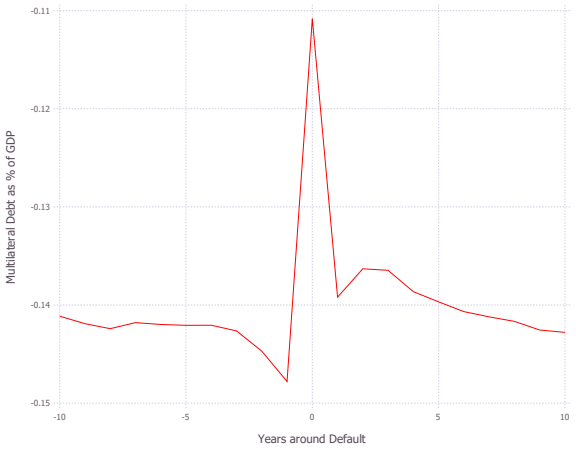
[23] Private debt disappears by construction, despite it will be repaid with a haircut.

Figure 7: Debt dynamics around default

(a) Private Debt



(b) Multilateral Debt

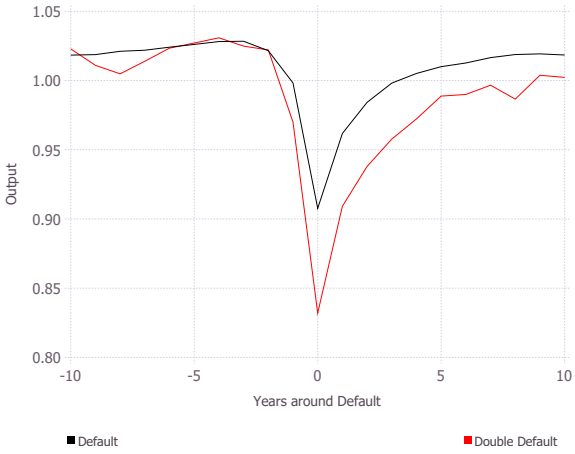


example, with $\underline{m} = 20\%$ I find a double default probability of 0.01 percent, which allows me to analyze the different dynamics around default episodes and double default episodes.

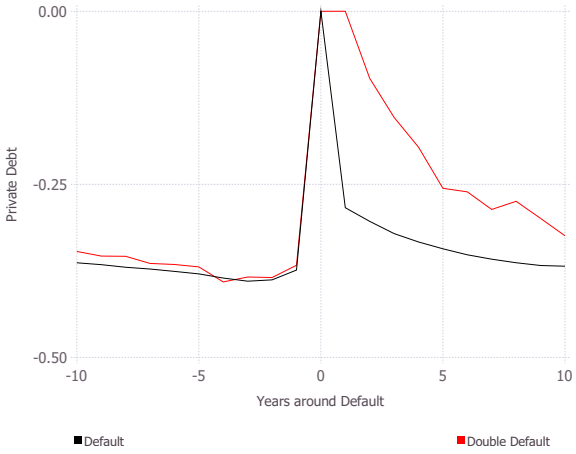
Double default tends to occur in periods of substantially deeper recessions compared to defaults, as shown in panel (a) of Figure 8. Then, even though private debt levels are quite similar, and even slightly higher in periods when the sovereign undergoes default compared to periods in double default, the stronger fall in output is able to compensate for that lower debt level driving the country to a double default episode. This result is also in line with the empirical findings in panel (b) of Figure 1. Thus, the model is able to reproduce the fact that default to multilateral lenders happens in periods of high overall debt as share of GDP, mainly due to a stronger fall in GDP.

Figure 8: Differences in output and debt dynamics between default and double default

(a) Output Dynamics



(b) Private Debt Dynamics



4.2.4 Cyclical Properties

In order to analyze the cyclical properties of the model and compare them with the data, I take annual data for Argentina from 1970 to 2015, except for the spreads that cover the period from 1997 to 2015.[24] However, and in line with the calibration of the model shown in Section 4.1, for private and multilateral debt levels and default and double default probabilities I use the sample shown in Section 1.[25] The comparison between the empirical data and the results of the model simulation is shown in Table 4.

The benchmark model performs well in matching most of the important moments in the data, except the spread. The model captures very well the level of dispersion and also the direction of the correlations of the main variables. Likewise, it approximates accurately the sample means.

Table 4: Business cycle statistics

	Data	**Model**
Sample standard deviations (%)		
$\sigma(\gamma_t)$	5.36	6.49
$\sigma(\Delta c_t)/\sigma(\gamma_t)$	1.27	1.37
$\sigma(s_t)$	2.66	0.83
$\sigma(tb_t/y_t)$	4.05	5.18
Sample correlations		
$\rho(\Delta c_t, \gamma_t)$	0.97	0.69
$\rho(s_t, \gamma_t)$	-0.10	-0.41
$\rho(tb_t/y_t, \gamma_t)$	-0.19	-0.38
Sample means (%)		
$E(b_{t+1}/y_t)$	28.8	36.3
$E(m_{t+1}/y_t)$	14.3	14.0
$E(s_t)$	7.1	1.0
$E(tb_t/y_t)$	2.2	1.3
$E(d)$	2.9 - 3.5	2.8
$E(Dd)$	0.8 - 1.4	0.0

Note: γ_t represents GDP growth. In the computation of γ_t standard deviation from the model, I disregard the observations with output losses. s_t represents spreads charged by private lenders to Argentina measured by the J.P. Morgan Emerging Markets Bond Spread (EMBI+). Simulation is run for one million periods.

[24] Data on spreads come from World Bank's Global Economic Monitor. In order to measure the spread and compare it with model results, I only consider those years in which Argentina was not in default, since the model, by construction, is not able to produce interest rates for the periods in default. Therefore, I exclude spreads data from 2001 to 2005 following the database in Asonuma and Trebesch (2016), and for 2014.

[25] In order to compare the amount of private debt in the model, $\frac{b_{t+1}}{y_t}$, with that from the data, I take into account all debt except multilateral and IMF debt. Therefore, I consider total public and publicly guaranteed debt minus the amount corresponding to public and publicly guaranteed multilateral debt.

The dispersion coefficients in the model are similar to those in the data, although they tend to be somewhat higher. Also, as in the data, consumption is more volatile than output. Regarding the spread, the model delivers interest rates with substantially lower dispersion than in the data.

With respect to sample correlations, the signs are the ones expected, with countercyclical spreads and trade balance. Also, the model is able to reproduce the strong procyclical relation between consumption growth and income growth, even though it is not as strong as in the data. Additional statistics for correlation coefficients can be found in Table 5 in Section 4.3.

Regarding sample means, the model is able to produce high levels of public debt while maintaining the default probability within realistic values by introducing cheap and senior multilateral debt and recovery rates. Therefore, this is an additional approach that allows sovereign default models to reproduce the high debt levels found in the data without the need to use very low β values.[26] Actually, in the sample specified in Section 1, the average public debt stock reaches 43.1 percent of GDP, while in the benchmark model it amounts to 50.3 percent. The positive recovery rates also contribute to this result because, as underscored by Yue (2010), the insurance role of recovery rates allows countries to support higher debt levels.

The model also performs well in terms of default probabilities. Even though the benchmark model is able to produce only very small double default probabilities, models with lower $\underline{\mathrm{m}}$ can produce higher rates of double default as shown in Section 4.3. In this regard, the low double default probability in the benchmark model, together with the pricing schedule for multilateral debt, implies that multilateral creditors in the benchmark model would make, on average, profits. This fact already gives an intuition about the implications of the existence of multilateral debt in terms of welfare, given that countries ask for multilateral loans voluntarily and multilateral lenders obtain profits from this activity.

Finally, the low sample mean of the spread and its substantial difference with the data are very similar to the results in Yue (2010). The low spreads in the simulation are

[26] In recent years, many sovereign default papers have been able to achieve higher debt levels, partially overcoming the issue in this strand of the literature that required low values of β in order to reach high debt levels. There are several approaches that have achieved higher debt levels, such as restructuring processes, partial defaults, recovery rates, multiple maturities, etc.

related to recovery rates after default, similarly to Yue (2010). These recovery rates are incorporated in the debt pricing schedule, substantially reducing the overall interest rate spread. Also, as she highlights, "the bond spreads in the model do not include a risk premium, which may be an important component of bond spreads in the data" (Yue, 2010, p. 182).

4.3 The Effect of Multilateral Lending

In order to analyze the role of multilateral debt, I compare the results of the benchmark model simulation with the results of models with different maximum amounts of multilateral debt, $\underline{m}$, that keep constant the rest of the parameters in the calibration. Thus, this exercise does not consist of a series of recalibrations for the different values of $\underline{m}$; on the contrary, it is a counterfactual exercise. I follow this approach because I want to test what is the role of multilateral debt on the macroeconomic outcomes in my model, assuming that it is a good description of reality. The main statistics of this exercise are shown in Table 5.

The total amount of debt that is sustainable increases with multilateral debt. The average amount of total debt as share of income, which is the sum of private and multilateral debt, increases from 37.2 percent to 55.1 percent when the availability of multilateral debt increases from 0 to 20 percent. However, private debt tends to decrease when multilateral debt is introduced, although only slightly, implying that the crowding-out effect of its introduction is small.

Given the small decrease in private debt due to the availability of official funds, multilateral debt appears to be a complement rather than a substitute of private debt. Substantial increases in the availability of multilateral debt, from 0 to 20 percent, only reduce private debt by less than one percentage point, leading to an overall substantial rise of public debt. Thus, there seems to be no competition among private and multilateral lenders. This result is in line with the empirical findings of Gelos et al. (2011) who show that official and private debt flows tend to be complements.

The default probability to private lenders tends to increase with $\underline{m}$, and so does the double default probability and the amount of total debt that is sustainable. Default

probability to private lenders goes from 2.8 percent with no multilateral debt to 2.92 percent with $\underline{m}$ equal to 20 percent. Nevertheless, this increase is relatively small, especially considering that it is accompanied by an increase in total debt of almost 18 percentage points. Therefore, the use of multilateral debt would allow countries to increase the total sustainable amount of debt without raising substantially the probability of default, given that it helps countries to be less constrained in terms of borrowing. Multilateral lending acts somehow as an insurance mechanism that provides cheaper funds in bad times.

Additionally, the size of the trade balance surplus as share of income also increases with $\underline{m}$ and becomes closer to the data shown in Table 4. This rise in the trade balance surplus is related to the increase in the overall debt levels.

Even when there is no multilateral default, the average multilateral debt is not equal to $\underline{m}$, as shown in Table 5. This goes in line with the analysis in Section 4.2.1, that is, the higher risk that involves a double default prevents countries to reach the highest available multilateral debt $\underline{m}$. In this regard, there are two effects that go in opposite directions. On the one hand, the lower price of multilateral debt when private debt is relatively high would imply that countries would use all the available multilateral credit before increasing their borrowing from private lenders. But, on the other hand, the already mentioned higher risk of a double default makes countries try to avoid the debt limit $\underline{m}$. The second effect is quantitatively more important, given that countries remain on average above $\underline{m}$ even in specifications where double defaults do not occur.

Table 5: Statistics according to the maximum amount of multilateral debt available $\underline{m}$

Statistics	$\underline{\mathbf{m}} = 0\%$	$\underline{\mathbf{m}} = 7.5\%$	$\underline{\mathbf{m}} = 10\%$	$\underline{\mathbf{m}} = 15\%$	$\underline{\mathbf{m}} = 20\%$
Sample means					
b_{t+1}/y_t (%)	37.2	36.6	36.5	36.3	36.5
m_{t+1}/y_t (%)	0.0	6.9	9.4	14.0	18.6
Private Default Only (%)	2.80	2.88	2.92	2.84	2.91
Double Default (%)	-	0.00	0.00	0.00	0.01
Total Private Default (%)	2.80	2.88	2.92	2.84	2.92
q_t^m	-	0.96	0.96	0.96	0.95
r_t (%)	4.94	4.98	4.99	4.97	5.00
r_t spread (%)	0.94	0.98	0.99	0.97	1.00
tb_t/y_t (%)	0.79	0.99	1.09	1.27	1.48
Sample correlations					
$\rho(\gamma_t, b_{t+1}/y_t)$	-0.42	-0.43	-0.44	-0.45	-0.47
$\rho(\gamma_t, m_{t+1}/y_t)$	-	-0.05	-0.01	0.14	0.24
$\rho(\gamma_t, tb_t/y_t)$	-0.32	-0.35	-0.37	-0.38	-0.40

Note: γ_t represents GDP growth

The larger $\underline{m}$ is, the larger the average multilateral debt rate —m_{t+1}/y_t— becomes. However, for values of $\underline{m}$ higher than 10 percent, the higher the threshold $\underline{m}$ becomes, the further the country is from reaching that threshold, i.e. from being constrained, measured as the average multilateral debt ratio as share of $\underline{m}$. Thus, the larger that limit becomes, the country becomes more cautious and chooses levels of multilateral debt further from the threshold, given the higher penalty associated with multilateral default and the effect it has on private debt prices. As a result, the country is less constrained in terms of borrowing and would be able to use multilateral debt to roll-over private debt in bad times.

Regarding the sample correlations, private debt is procyclical, as expected. This procyclicality implies that the sovereign tends to obtain more funds from private international financial markets during expansions. This is the usual result in sovereign default models and a well-known fact in the empirical literature. Furthermore, the larger $\underline{m}$ and m_t are, private debt becomes more procyclical. Conversely, multilateral funds are found to be sometimes procyclical —virtually acyclical— and sometimes countercyclical. That is to say, multilateral financing sometimes slightly increases during expansions, but it tends to rise in crises. However, the magnitude of the multilateral funds cyclicality is always lower than that of private lending, and depending on the amount of $\underline{m}$ it is virtually acyclical. Then, countries differentiate and tend to use cheap multilateral debt in recessions and private debt, which becomes cheap in expansions and expensive in recessions, in periods of economic growth. Thus, multilateral and private debt complement each other. This result is consistent with the empirical findings of Levy Yeyati (2009).

4.3.1 The Effect of Official Debt in Related Studies

In this model, private debt tends to decrease with multilateral debt, although only slightly. This result differs to previous studies. Bailout loans in Fink and Scholl (2016) and Kirsch and Rühmkorf (2017) rise private debt, while in Boz (2011) private debt decreases substantially with official lending.

Also, default probability to private lenders increases only slightly with multilateral debt. This result is very similar to the one in Kirsch and Rühmkorf (2017) who also find a small increase in the default probability when bailout loans are present thanks to the

ability of these loans to avoid defaults caused by runs, and despite the general equilibrium effect that increases the overall debt levels. In this regard, Kirsch and Rühmkorf (2017) underscores that "the general equilibrium effect outweighs the insurance effect and the default probability is higher in the presence of the official lending facility" (Kirsch and Rühmkorf, 2017, p. 779). Conversely, Boz (2011) and Fink and Scholl (2016) find that official lending rises the default probability substantially.

The relationship between $\underline{m}$ and m_t, that is, the fact that the larger $\underline{m}$ becomes, the country becomes more cautious, is again in line with the findings of Kirsch and Rühmkorf (2017) about the insurance role of official debt. Conversely, in Hatchondo et al. (2017) countries choose the maximum available official debt limit because their debt is riskless since it is non-defaultable. As a result, given that sovereigns cannot roll-over private debt by issuing more non-defaultable debt, neither the default probability nor the private debt increase with official debt, not even by rising its threshold. Nevertheless, Hatchondo et al. (2017) also highlight that increasing the debt limit could also have implications in terms of moral hazard, from which I abstract in this paper.

Finally, regarding the cyclicality of official debt, Boz (2011) and Kirsch and Rühmkorf (2017) find that official lending is countercyclical, while Fink and Scholl (2016) find that it is procyclical. Nevertheless, the three studies tackle bailout loans rather than multilateral support.

4.3.2 Cyclicality of Multilateral Debt

Alfaro and Kanczuk (2017) explain that when fiscal policy is countercyclical, public debt fulfills the role of consumption smoothing, while when it is procyclical, it is used to front-load consumption. Empirical evidence shows that private debt is procyclical, meaning that the benefits of front-loading consumption are higher than those coming from consumption smoothing, as it is the case in Boz (2011), Fink and Scholl (2016), Kirsch and Rühmkorf (2017) and Alfaro and Kanczuk (2017), among others. But multilateral debt can be either procyclical (almost acyclical) or countercyclical. Thus, multilateral debt may act as a consumption-smoothing or a front-loading mechanism, and this role depends on the maximum available amount of multilateral debt. When this amount is relatively low, such as $\underline{m} = 7.5\%$, multilateral debt is slightly procyclical, that is, it is used as a

front-loading mechanism, similar to private debt. However, as the threshold for multilateral debt increases, it becomes countercyclical and the magnitude of the countercyclicality increases with this threshold. Thus, multilateral debt becomes a consumption-smoothing mechanism.

While the literature has established the procyclicality of private lending in developing countries, the cyclical properties of multilateral lending have been much more debated. In this regard, Levy Yeyati (2009) finds that multilateral lending decreases in good times, but does not change in bad times. Conversely, Perry (2009) finds that multilateral lenders tend to be procyclical, except in very adverse economic situations when they become part of a joint program with the IMF. On the contrary, Galindo and Panizza (2018) show that lending from the World Bank is countercyclical, while regional development banks' financing is acyclical.[27] Recently, Avellán et al. (2020) approach this issue from the demand side and find that multilateral lending is countercyclical but due to a confounding factor issue. They find that once expenditure is taken into account, multilateral flows are acyclical and follow public spending. Therefore, according to Avellán et al. (2020), they may be procyclical or countercyclical depending on whether the country's fiscal policy is procyclical or countercyclical. Similarly, Avellán et al. (2021) analyze how multilateral funding changes during fiscal crises (which include sovereign default crises) and find that multilateral development banks do not decrease their financing to countries in these situations, opposite to private creditors.

Within this context and in order to go more in depth into the analysis of the multilateral debt cyclicality, I use the sample in Section A.3[28] to evaluate the cyclical properties of multilateral funds and their relation with the multilateral debt stock. In this regard, Figure 9 presents for each country in the sample the correlation coefficient between multilateral debt and GDP growth[29] against the average multilateral debt stock as share of GDP. The different graphs in Figure 9 are presented by quartile of private debt as share of GDP. In other words, in panel (a) only countries in the top 25 percent of the private

[27] However, regional development banks' data is obtained as the difference between total multilateral lending and World Bank's lending so the conclusions for them are not as strong as those for the World Bank.

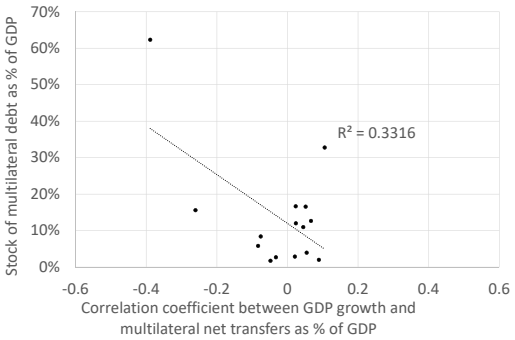
[28] However, I exclude Iran due to its low overall debt levels.

[29] To compute the cyclicality of multilateral debt I compute the correlation coefficient between multilateral net transfers as share of GDP and GDP growth.

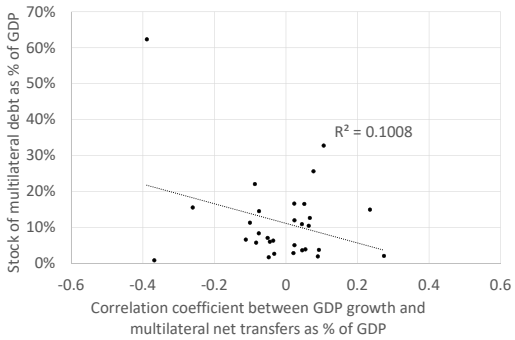
debt distribution (the first quartile) are included, in panel (b) only countries in the top 50 percent, and so on. Given that debt is negative in the model, but positive in data, the correlation coefficients in the empirical analysis below have the opposite sign to those in the model.

Figure 9: Multilateral Debt Cyclicality

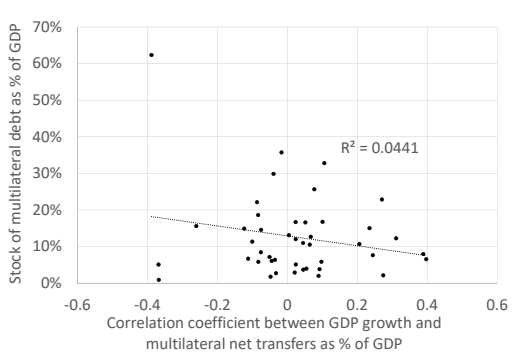
(a) Countries in the top 25% in terms of private debt as % of GDP



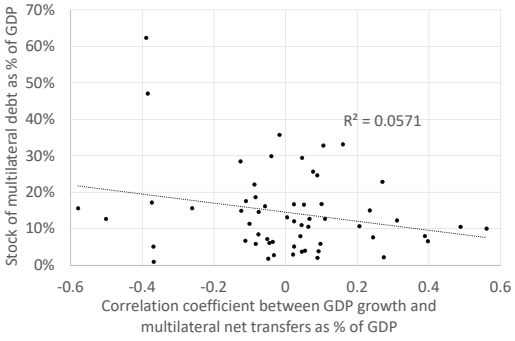
(b) Countries in the top 50% in terms of private debt as % of GDP



(c) Countries in the top 75% in terms of private debt as % of GDP



(d) All Countries



Panel (a) of Figure 9 shows how multilateral debt tends to be more countercyclical when multilateral debt is higher for those countries with relatively higher private debt, i.e. in the highest quartile of the distribution of private debt as share of GDP. However, this relationship between the stock of multilateral debt and the degree of cyclicality gradually weakens when countries with lower levels of private debt as share of GDP are included in the sample. Thus, the model performs well for those countries where private debt is relatively high and, consequently, multilateral debt may act as a complement of private financing. However, the model does not accurately reproduce the characteristics of those countries with relatively low levels of private debt, where multilateral debt tends to be

the main source of funding. These results are consistent with the model's design where multilateral debt is capped to a level that is lower than that of private debt, whose limit is not binding.

In sum, one of the main contributions of this paper is to disentangle the role of multilateral lending as means of consumption smoothing versus a front-loading mechanism like private financing. Under financial market completeness, countries would be able to insure consumption regardless of their income level, achieving in turn full consumption smoothing. But in a model with incomplete financial markets and without commitment, private funds become more expensive in recessions and cheaper in expansions. In this case, the availability of multilateral funding, whose price does not depend directly on the output realizations,[30] contributes to completing the market. Cordella and Powell (2021) obtain similar results since "having two lenders that offer different conditions is a way of "completing the market." The repayments of market lenders are "state contingent," but they come at the cost of market exclusion [...]; IFI repayments are not state contingent but ensure access even if there is default on the market." (Cordella and Powell, 2021, p. 18). Therefore, multilateral financing provides countries with some degree of insurance, which is reflected in the countercyclicality (or acyclicality) of multilateral funds. Also, the increase in the size of the magnitude of the countercyclicality as $\underline{m}$ rises reflects how a higher $\underline{m}$ increases the size of multilateral debt as an insurance mechanism.

Furthermore, as highlighted by Boz (2011), countercyclicality is a characteristic of enforceable debt contracts. However, multilateral debt in this model is provided without commitment. Then, the countercyclicality of multilateral funds is related to the conclusions of Uribe and Schmitt-Grohé (2016): "In the absence of commitment, consumption smoothing is a direct function of the ability of the lender to punish debtors in the case of default" (Uribe and Schmitt-Grohé, 2016, p. 770-771). And multilateral lenders, as featured in this model, are able to impose stricter penalties than private lenders, namely the total financial exclusion and the full recovery rates that private creditors cannot impose.

Therefore, the cyclicality of multilateral funds in this model is the result of two features of multilateral financing. First, given that the price of multilateral debt does not depend

[30] Given that when the model is detrended $q^m(m_t, \Gamma_t)$ only depends on $\hat{m}_{t+1}$

directly on output realizations, multilateral financing can act as an insurance mechanism whose usefulness depends on the size of that financing. And second, the superior capacity of multilateral lenders compared to private creditors to impose the penalties of financial exclusion and full recovery rates also contributes to the countercyclicality of multilateral funds.

5 Conclusion

Multilateral creditors have been the forgotten lenders in DSGE sovereign default models. However, they are key in the sovereign debt portfolio of emerging economies. Actually, in terms of magnitude, they are as important as private lenders. However, the sovereign default literature on non-private lenders has traditionally focused on the bailout loans from the IMF or other lenders of last resort.

Multilateral lenders feature very different characteristics compared to the rest of the sovereigns' lenders. Their funds are cheaper, senior, generally without conditionality and furthermore, these creditors do not exclude countries from their funding if they default to private lenders. These distinct characteristics have a significant effect on the usual predictions of sovereign default models.

Thus, the main contribution of this model is to show that the inclusion of multilateral creditors in the traditional sovereign default model improves the fit of the results to the data, especially in terms of the overall debt level, at the same time that it provides an explanation for the role of multilateral loans vis-à-vis private funding. Multilateral debt is an instrument for consumption smoothing, opposite to the front-loading role that private lending usually fulfills. This consumption-smoothing role is related to the greater ability of multilateral lenders to impose penalties in case of default.

Multilateral lenders allow countries to obtain cheap funding in times of adverse shocks, becoming, as a result, a source of consumption smoothing and providing them with some degree of insurance against bad times. In this regard, Levy Yeyati (2009) underscores that "the very insurance view that has been used to explain why a sovereign repays its debts with private lenders may rationalize the remarkable efficacy displayed by multilat-

eral financial institutions in enforcing their implicit "preferred creditor status": countries may be playing the reputation game with the only lender that can credibly commit to lend in bad times" (Levy Yeyati, 2009, p. 504). Therefore, multilateral debt is an insurance mechanism and a complement to private financing that allows countries to smooth consumption without significantly increasing the default probability, while allowing them to sustain higher debt levels.

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A Appendix

A.1 Private Debt Price

Risk neutral lenders, in a perfect competition environment, face the zero-profit condition, which gives rise to the private bond price. If the sovereign does not default, the bond is payed back, which happens with probability $1 - Totald(b_t, m_t, z_t, \Gamma_t)$ and the lender receives the value of the bond, which is discounted, that is $b_t/(1+r)$. However, in case of default the lender receives the value of the bond minus a haircut after a random number of periods. And similarly in the case of double default, the lender also receives the value of the bond minus a haircut, but in this case, the number of periods is higher since first the country has to pay back its multilateral debt, also after a random number of periods.

Then, for a bond issued in period t=0, we find that the discounted expected payment for the private lenders is:

- In case of no default at all:

$$t = 1: \ \frac{1 - Totald(b_0, m_0, z_0, \Gamma_0)}{1+r} b_0$$

- In case of default only to private lenders, and considering that θ is the probability of reentering the private financial markets and $(1-\theta)$ is the probability of remaining in private financial exclusion:

$$\begin{aligned}
t = 1: \ & 0 \\
t = 2: \ & \frac{\lambda \ \theta \ d(b_0, m_0, z_0, \Gamma_0)}{(1+r)^2} b_0 \\
t = 3: \ & \frac{\lambda \ (1-\theta) \ \theta \ d(b_0, m_0, z_0, \Gamma_0)}{(1+r)^3} b_0 \\
t = 4: \ & \frac{\lambda \ (1-\theta)^2 \ \theta \ d(b_0, m_0, z_0, \Gamma_0)}{(1+r)^4} b_0
\end{aligned}$$

Being this a geometric series, the result is:

$$\frac{\lambda \ \theta \ d(b_0, m_0, z_0, \Gamma_0)}{(1+r)^2} \ b_0 \sum_{t=0}^{\infty} \left(\frac{1-\theta}{1+r} \right)^t = \frac{\lambda \ \theta \ d(b_0, m_0, z_0, \Gamma_0)}{(1+r)(r+\theta)} \ b_0$$

$$\text{since } \frac{1-\theta}{1+r} < 1$$

- Similarly, in case of a double default, in other words a default to private lenders and to multilateral creditors, we need to take into account that before re-accessing private financial markets countries must repay its multilateral debt, which happens with probability ξ and with probability $(1-\xi)$ countries remain in isolation. After leaving isolation, countries may re-access private financial markets randomly with probability θ, as in the case of a default only to private lenders. With this setting, the expected payment is:

$$t=1:\ 0$$

$$t=2:\ 0$$

$$t=3:\ \frac{1}{(1+r)^3}\ \theta\ Dd(b_0,m_0,z_0,\Gamma_0)\ \xi\ \lambda b_0$$

$$t=4:\ \frac{(1-\theta)+(1-\xi)}{(1+r)^4}\ \theta\ Dd(b_0,m_0,z_0,\Gamma_0)\ \xi\ \lambda b_0$$

$$t=5:\ \frac{(1-\theta)^2+(1-\theta)(1-\xi)+(1-\xi)^2}{(1+r)^5}\ \theta\ Dd(b_0,m_0,z_0,\Gamma_0)\ \xi\ \lambda b_0$$

$$t=6:\ \frac{(1-\theta)^3+(1-\theta)^2(1-\xi)+(1-\theta)(1-\xi)^2+(1-\xi)^3}{(1+r)^6}\ \theta\ Dd(b_0,m_0,z_0,\Gamma_0)\ \xi\ \lambda b_0$$

Given that the expression above boils down to a geometric series, the resulting expected payment is:

$$\frac{\theta\ Dd(b_0,m_0,z_0,\Gamma_0)\ \xi\ \lambda b_0}{(1+r)^3}\sum_{t=0}^{\infty}\sum_{i=0}^{t}\frac{(1-\theta)^{t-i}\ (1-\xi)^i}{(1+r)^t}=\frac{\theta\ Dd(b_0,m_0,z_0,\Gamma_0)\ \xi}{(1+r)(r+\theta)(r+\xi)}\ \lambda b_0$$

The left hand side equation can be rearranged as:

$$\frac{\theta\ Dd(b_0,m_0,z_0,\Gamma_0)\ \xi\ \lambda b_0}{(1+r)^3}\sum_{t=0}^{\infty}\sum_{i=0}^{t}\frac{(1-\theta)^t}{(1+r)^t}\frac{(1-\xi)^i}{(1-\theta)^i}=$$

$$\frac{\theta\ Dd(b_0,m_0,z_0,\Gamma_0)\ \xi\ \lambda b_0}{(1+r)^3}\sum_{t=0}^{\infty}\left(\frac{1-\theta}{1+r}\right)^t\sum_{i=0}^{t}\left(\frac{1-\xi}{1-\theta}\right)^i=$$

$$\frac{\theta\ Dd(b_0,m_0,z_0,\Gamma_0)\ \xi\ \lambda b_0}{(1+r)^3}\sum_{t=0}^{\infty}\left(\frac{1-\theta}{1+r}\right)^t\frac{1-(\frac{1-\xi}{1-\theta})^{t+1}}{1-\frac{1-\xi}{1-\theta}}=$$

$$\frac{\theta\ Dd(b_0,m_0,z_0,\Gamma_0)\ \xi\ \lambda b_0}{(1+r)^3}\frac{1-\theta}{\xi-\theta}\sum_{t=0}^{\infty}\left(\frac{1-\theta}{1+r}\right)^t\left(1-\left(\frac{1-\xi}{1-\theta}\right)^{t+1}\right)=$$

$$\frac{\theta\ Dd(b_0,m_0,z_0,\Gamma_0)\ \xi\ \lambda b_0}{(1+r)^3}\frac{1-\theta}{\xi-\theta}\sum_{t=0}^{\infty}\left[\left(\frac{1-\theta}{1+r}\right)^t-\left(\frac{(1-\theta)(1-\xi)}{(1+r)(1-\theta)}\right)^t\left(\frac{1-\xi}{1-\theta}\right)\right]=$$

$$\frac{\theta\ Dd(b_0,m_0,z_0,\Gamma_0)\ \xi\ \lambda b_0}{(1+r)^3}\frac{1-\theta}{\xi-\theta}\left[\sum_{t=0}^{\infty}\left(\frac{1-\theta}{1+r}\right)^t-\sum_{t=0}^{\infty}\left(\frac{1-\xi}{1+r}\right)^t\left(\frac{1-\xi}{1-\theta}\right)\right]=$$

since $\frac{1-\theta}{1+r}<1$ and $\frac{1-\xi}{1+r}<1$

$$\frac{\theta\ Dd(b_0,m_0,z_0,\Gamma_0)\ \xi\ \lambda b_0}{(1+r)^3}\frac{1-\theta}{\xi-\theta}\left[\frac{1+r}{\theta+r}-\frac{1+r}{\xi+r}\frac{1-\xi}{1-\theta}\right]=$$

$$\frac{\theta\ Dd(b_0,m_0,z_0,\Gamma_0)\ \xi\ \lambda b_0}{(1+r)^2}\frac{1}{\xi-\theta}\left[\frac{1-\theta}{\theta+r}-\frac{1-\xi}{\xi+r}\right]=$$

$$\frac{\theta\ Dd(b_0,m_0,z_0,\Gamma_0)\ \xi\ \lambda b_0}{(1+r)(\theta+r)(\xi+r)}$$

Therefore, the price of debt issued in international private financial markets becomes:

$$q(b_{t+1},m_{t+1},z_t,\Gamma_t)=\mathrm{E_t}\left\{\frac{1-Totald(b_{t+1},m_{t+1},z_{t+1},\Gamma_{t+1})}{1+r}\right\}+$$

$$\mathrm{E_t}\left\{\frac{\lambda\ \theta\ d(b_{t+1},m_{t+1},z_{t+1},\Gamma_{t+1})}{(r+\theta)(1+r)}\right\}+\mathrm{E_t}\left\{\frac{\lambda\ \theta\ \xi\ Dd(b_{t+1},m_{t+1},z_{t+1},\Gamma_{t+1})}{(r+\theta)(1+r)(r+\xi)}\right\}$$

A.2 Detrended Model

The model is solved numerically in detrended form. Given that this problem is analogous to the one in Aguiar and Gopinath (2006), I detrend the model dividing it by Γ_{t-1} in the same way as they do, and present the detrended variables with a hat:

$$\hat{y}_t=e^{z_t}g_t$$

The price of multilateral debt can be detrended directly given that it already depends only on the amount of the detrended multilateral debt:

$$q^m(m_{t+1},\Gamma_t)=\frac{1}{1+r-\phi\frac{m_{t+1}}{\Gamma_t}}=\frac{1}{1+r-\phi\hat{m}_{t+1}}=q^m(\hat{m}_{t+1}) \tag{14}$$

Also, the cap for multilateral debt is detrended as follows:

$$m_{t+1}\geq \underline{\mathrm{m}}_t=\underline{\mathrm{m}}\Gamma_t$$

$$\frac{m_{t+1}}{\Gamma_t}\frac{\Gamma_t}{\Gamma_{t-1}}\geq \underline{\mathrm{m}}\frac{\Gamma_t}{\Gamma_{t-1}}$$

$$\hat{m}_{t+1}\geq \underline{\mathrm{m}}$$

As in Aguiar and Gopinath (2006), since the price of the private debt is homogeneous of degree zero in b, m and Γ, it can be defined as $q_t(b_{t+1}, m_{t+1}, z_t, \Gamma_t) \equiv \hat{q}_t(\hat{b}_{t+1}, \hat{m}_{t+1}, z_t, g_t)$. The detrended default and double default functions are defined analogously.

The budget constraint is homogeneous of degree one, and when it is divided by Γ_{t-1} in order to detrend it, it becomes:

$$\hat{c}_t = \frac{c_t}{\Gamma_{t-1}} = \hat{y}_t - \frac{\hat{q}_t(\hat{b}_{t+1}, \hat{m}_{t+1}, z_t, g_t)b_{t+1}}{\Gamma_{t-1}} + \frac{b_t}{\Gamma_{t-1}} - \frac{q_t^m(\hat{m}_{t+1})m_{t+1}}{\Gamma_{t-1}} + \frac{m_t}{\Gamma_{t-1}}$$

$$\hat{c}_t = \hat{y}_t - \frac{\hat{q}_t(\hat{b}_{t+1}, \hat{m}_{t+1}, z_t, g_t)b_{t+1}}{\Gamma_{t-1}}\frac{\Gamma_t}{\Gamma_t} + \hat{b}_t - \frac{q_t^m(\hat{m}_{t+1})m_{t+1}}{\Gamma_{t-1}}\frac{\Gamma_t}{\Gamma_t} + \hat{m}_t$$

$$\hat{c}_t = \hat{y}_t - \hat{q}_t(\hat{b}_{t+1}, \hat{m}_{t+1}, z_t, g_t)\hat{b}_{t+1}g_t + \hat{b}_t - q_t^m(\hat{m}_{t+1})\hat{m}_{t+1}g_t + \hat{m}_t$$

Similarly, the budget constraint when a country is in default (or in isolation) is homogeneous of degree one. Here I show only the constraint when the income of the country is above the moving income threshold for default, that is when $h(y_t) = \delta\mu_{e^{z_t}}\mu_g\Gamma_{t-1}$, since the other case would be the same as the regular budget constraint, but eliminating the different financings that the country cannot access:

$$\hat{c}_t = \delta(\frac{\mu_{e^z}\mu_g\Gamma_{t-1}}{\Gamma_{t-1}}) - q_t^m(\hat{m}_{t+1})\hat{m}_{t+1}g_t + \hat{m}_t$$

$$\hat{c}_t = \delta\mu_{e^z}\mu_g - q_t^m(\hat{m}_{t+1})\hat{m}_{t+1}g_t + \hat{m}_t$$

Following Aguiar and Gopinath (2006), given that the utility function is homogeneous of degree $1-\gamma \leq 1$ and that the budget constraint is homogeneous of degree one, the value function is homogeneous of degree $1-\gamma$. Thus, I need to divide both equation sides by $\Gamma_{t-1}^{1-\gamma}$ in order to detrend the model:

$$(\frac{1}{\Gamma_{t-1}})^{(1-\gamma)}V(b_t, m_t, z_t, \Gamma_t) = (\frac{1}{\Gamma_{t-1}})^{(1-\gamma)}\max_{c_t}\left\{\frac{c_t^{1-\gamma}}{1-\gamma} + \beta \mathrm{E_t}(V(b_{t+1}, m_{t+1}, z_{t+1}, \Gamma_{t+1}))\right\}$$

$$V(\hat{b}_t, \hat{m}_t, z_t, g_t) = \max_{\hat{c}_t}\left\{\frac{\hat{c}_t^{1-\gamma}}{1-\gamma} + \beta \mathrm{E_t}(V(\frac{b_{t+1}}{\Gamma_{t-1}}, \frac{m_{t+1}}{\Gamma_{t-1}}, z_{t+1}, \frac{\Gamma_{t+1}}{\Gamma_{t-1}}))\right\}$$

$$V(\hat{b}_t, \hat{m}_t, z_t, g_t) = \max_{\hat{c}_t}\left\{\frac{\hat{c}_t^{1-\gamma}}{1-\gamma} + \beta \mathrm{E_t}(V(\frac{b_{t+1}}{\Gamma_{t-1}}\frac{\Gamma_t}{\Gamma_t}, \frac{m_{t+1}}{\Gamma_{t-1}}\frac{\Gamma_t}{\Gamma_t}, z_{t+1}, \frac{\Gamma_{t+1}}{\Gamma_{t-1}}\frac{\Gamma_t}{\Gamma_t}))\right\}$$

$$V(\hat{b}_t, \hat{m}_t, z_t, g_t) = \max_{\hat{c}_t}\left\{\frac{\hat{c}_t^{1-\gamma}}{1-\gamma} + \beta \mathrm{E_t}(V(\hat{b}_{t+1}g_t, \hat{m}_{t+1}g_t, z_{t+1}, g_{t+1}g_t))\right\}$$

$$V(\hat{b}_t, \hat{m}_t, z_t, g_t) = \max_{\hat{c}_t}\left\{\frac{\hat{c}_t^{1-\gamma}}{1-\gamma} + \beta \mathrm{E_t}(g_t^{1-\gamma}V(\hat{b}_{t+1}, \hat{m}_{t+1}, z_{t+1}, g_{t+1}))\right\}$$

With this setting, debt ratios can be easily derived from the detrended model as follows:

$$\frac{b_{t+1}}{y_t} = \frac{\hat{b}_{t+1}\Gamma_t}{\hat{y}_t\Gamma_{t-1}} = \frac{\hat{b}_{t+1}}{\hat{y}_t}g_t$$

and similarly for income growth, which I denote as γ_t:

$$\gamma_t = \frac{y_{t+1}}{y_t} = \frac{\hat{y}_{t+1}\Gamma_t}{\hat{y}_t\Gamma_{t-1}} = \frac{\hat{y}_{t+1}}{\hat{y}_t}g_t$$

A.3 Data

The main databases used in this paper, unless otherwise specified, are:

- **Beers et al. (2020b): BoC-BoE Sovereign Default Database: What's New in 2020?** (June 2020) and the information on methodology and assumptions in Beers et al. (2020a). Available at `https://www.bankofcanada.ca/2020/06/staff-analytical-note-2020-13/`.

- **H.15 Selected Interest Rates retrieved from the Board of Governors of the Federal Reserve System**. Database available at: `https://www.federalreserve.gov/releases/h15/`. Indicator used: 6-month Treasury bill secondary market rate - discount basis [$H15/H15/RIFSGFSM06\_N.A$].

- **World Bank. 2021. International Debt Statistics 2021.** Washington, DC: World Bank. doi:10.1596/978-1-4648-1610-9. License: Creative Commons Attribution CC BY 3.0 IGO. Database available at: `https://data.worldbank.org/products/ids`

- **World Bank. 2020. World Development Indicators**. Washington, D.C.: World Bank. License: CC BY-4.0. Database available at: `https://datatopics.worldbank.org/world-development-indicators/`.

- **World Bank. 2020. Global Economic Monitor**. Washington, D.C.: World Bank. License: CC BY-4.0. Database available at: `https://databank.worldbank.org/source/global-economic-monitor-(gem)`. This database is used for the spreads data.

In particular, the main database used in the paper is the same as in Bru Muñoz (2022), but adjusting the sample from 1970 to 2015 in order to match other sections in this paper. I have made use of the database above developed by Beers et al. (2020b), which provides information on annual amounts of sovereign debt in default disaggregated by lender, together with the World Bank's IDS and WDI databases. The sample of countries that I use in both papers, this one and Bru Muñoz (2022), which results from the merge of such databases, is smaller than the one provided by Beers et al. (2020b) and World Bank's databases above. In this regard, in what follows I provide a non-exhaustive list of the adjustments[31] that I have performed to the dataset that contribute to the smaller sample size:

- I include only those countries that were common to IDS and Beers et al. (2020b) databases.
- I take into account in which year countries gained complete independence/sovereignty and adjusted the sample accordingly. In this regard, I follow the BBC country profiles and country timelines available online at: `http://news.bbc.co.uk/1/hi/country_profiles/default.stm`. I further restrict the sample with the information for Bulgaria and Zambia in Gelos et al. (2011) regarding the date of inclusion in the sample of former soviet countries.
- I only include countries classified by the World Bank as lower-middle income and upper-middle income according to the World Bank's 2021 fiscal year classification. I exclude low income countries because these countries tend to have little access to private financial markets, not being a good sample to study private financing vis-à-vis multilateral funding.
- I exclude the countries with no records of default with the main six lenders (Paris Club, IMF, IBRD, IDA, international banks and bondholders) in Beers et al. (2020b) database. Since Beers et al. (2020b) is a database on defaulted debt, when there is no record of debt in default with any of these lenders in the database, I assume there was no default.
- Due to lack of debt data for several years in IDS database, I also exclude the observation of several countries without available debt data in IDS. The excluded

[31] A comprehensive list of the adjustments performed is provided in Bru Muñoz (2022).

periods by country are: Angola from 1975 to 1988, Bosnia Herzegovina in 1992, Cabo Verde from 1975 to 1980, Dominica from 1978 to 1980, Iran from 1970 to 1980, North Macedonia in 1992, St. Lucia from 1978 to 1980 and Vietnam from 1970 to 1978.

- Finally, I exclude: Sao Tome and Principe because of inconsistencies between IDS and default data; South Africa because in the years with information on default, IDS does not provide information on debt; and countries with almost no private commitments (Cambodia and Comoros, with just one year with positive private commitments each).

With these adjustments, I construct an unbalanced panel of 60 countries that defaulted at least to one of their six main lenders —the Paris Club, the IMF, the World Bank's agencies IBRD and IDA, international banks and foreign currency bondholders— from 1970 to 2015. It is important to highlight that in Beers et al. (2020b) there is no disaggregated record of default to other multilateral lenders apart from the two agencies that comprise the World Bank, IBRD and IDA. Therefore, I take the default to these two agencies as a proxy for all the defaults to multilateral institutions, despite the stock of debt with other multilateral creditors is relevant. Nevertheless, being these defaults infrequent events due to the MDBs aforementioned preferred creditor status, I consider that the default to the World Bank is a good proxy for the default to multilateral institutions as a whole.

A.4 Calibration of Multilateral Financial Exclusion After Default

In order to calibrate multilateral financial exclusion after default, I consider that a country has access to multilateral financing in a given year if it has positive multilateral commitments. In order to compute this period, I use the sample and databases in Section A.3. Beers et al. (2020b) provides information on annual amounts of sovereign defaulted debt disaggregated by lender. Nevertheless, it is important to highlight that Beers et al. (2020b) database does not identify default episodes, but the amount of loans in default together with interest arrears by lender every year. Thus, it provides information on years in default by country rather than on default episodes, since several years in default in a row for a given country may involve one or more default episodes. Data on debt

commitments comes from the World Bank's IDS database. As I do in Bru Muñoz (2022), when multilateral commitments are missing, I assume that they are zero.[32]

The estimation of the multilateral financial exclusion period counts the number of years without multilateral debt commitments after a default. Therefore, if there are several years in a row where a country has positive amounts of defaulted debt and it has no multilateral debt commitments, this measure counts the number of years without commitments starting in the first year without them. I estimate this measure for multilateral debt commitments after a default to the World Bank, as a proxy for default to multilateral lenders, and after a default to private lenders (foreign currency bank loans and foreign currency bonds).

According to this measure, data shows that there is virtually no multilateral financial exclusion after a default to private creditors as shown in Table 6. Multilateral creditors almost immediately lend again to countries after defaulting to private lenders (approximately after six months). Conversely, multilateral lenders impose a two-year exclusion period after a default to the World Bank agencies. This number is used to calibrate ξ to 0.5, which means an average exclusion from multilateral lending markets of two years.

Table 6: Years without Multilateral Commitments after Default to the Following Lenders

Defaulted Creditor	**Average Years**
Private lenders	0.55
Multilateral lenders	2.04

In order to compute these measures, I need to make some assumptions. When there are several years in a row with positive amounts of defaulted debt and the lack of multilateral commitments starts at some point that is not the first or second years in default, I consider that there is no financial exclusion at first (so zero years) and then I count a

[32]This decision is due to the fact that there are no years with zero commitments in IDS, which seems unrealistic. Therefore, in order to exploit as much information as possible, I assume that all the years with missing commitments actually represent zero commitments. Furthermore, in earlier versions of IDS many of the years where now the categories of official and private commitments appear as missing used to be zero. Thus, it is highly likely that the previous difference between zero and missing commitments was actually almost non existent since both of them implied the absence of identifiable commitments, and as a result they were recorded as missing in the database update.

new exclusion period starting at the first year without access. Also, if throughout the years in default the country re-accesses and loses access again, I count them as different exclusion periods.

There are assumptions in the construction of this measure that pose some caveats. Given that I lack microdata, within a year I cannot distinguish whether commitments occurred before or after the default (in case it was the first year the country is in default), which may affect the results. I try to mitigate this issue by considering as financial exclusion the lack of commitments the year after a default occurred, even when there was no defaulted debt in the year after. Another caveat is related to the fact that if a country was not receiving multilateral funds and it defaults to a creditor and after the default it continues not receiving multilateral financing, the country is considered to be in exclusion from multilateral lending, even if it was already not being financed by these creditors. This assumption overestimates the exclusion measure. This overestimation should not be of a great magnitude given that lack of multilateral funds is quite unusual. Similarly, if a country was not receiving funds from MDBs after a default, and there is a year in which the country is not in default and then it defaults again, the measure starts counting a new period of exclusion following the second default. This happens even if there was no multilateral access between both default episodes, which may underestimate the period of exclusion.

Despite this descriptive measure has some limitations, it is still useful to estimate ξ. Furthermore, the results for the multilateral financial exclusion after a default to the World Bank are supported by the results for multilateral exclusion after a default to private creditors, given that the lack of exclusion from multilateral financing after a default to private lenders found here has already been shown by Avellán et al. (2021) and Bru Muñoz (2022).

A.5 Solution Method

In this model every period, after observing output, the sovereign must make a default decision (either defaulting to private lenders, to multilateral and private lenders or to none), and in case of no default, two portfolio allocation decisions, the amount of private bonds, b_{t+1}, and of multilateral loans, m_{t+1}. As explained in Boz (2011), to which this

setting is relatively similar, given that the lenders offer different terms for their loans, the problem can be solved with standard techniques.

Therefore, I discretize the state space for output, z_t and g_t, and the state variables $\hat{b}_t$ and $\hat{m}_t$, given that the model is solved in detrended form. Regarding output, I approximate the income process with a Markov chain using Tauchen's method. In order to determine the size of the grid for the shocks, I keep increasing the grid size until the point where the results of the simulation remain almost unchanged if the grid size is increased. This occurs with 13 grid points for each of the shocks, which gives a wide range of income shocks. Also, I establish a grid size for $\hat{b}_t$ that is not binding for the sovereign, that is, [-0.8, 0.0], with 151 grid points, and a grid size for $\hat{m}_t$ [$\underline{m}$, 0]. In the benchmark model the grid size for $\hat{m}_t$ is [-0.15, 0], realistic given the empirical evidence, and it includes 26 grid points. I also use different values for $\underline{m}$ as specified in the article, with a proportional change in the number of grid points.

With these inputs, the following algorithm is completed:

1. Start from the discretization of the state space above.

2. In order to solve the model, use an initial risk-free private debt price: $\hat{q}(\hat{b}_{t+1}, \hat{m}_{t+1}, z_t, g_t) = \frac{1}{1+r}$.

3. Solve for the optimal decisions of default and portfolio allocations $\hat{b}_{t+1}$ and $\hat{m}_{t+1}$ given the private debt price $\hat{q}(\hat{b}_{t+1}, \hat{m}_{t+1}, z_t, g_t)$ and the multilateral debt price, $q^m(\hat{m}_{t+1})$, using value function iteration.

4. Given these optimal decisions, compute the different default probabilities, $\hat{d}(\hat{b}_t, \hat{m}_t, z_t, g_t)$, $\hat{Dd}(\hat{b}_t, \hat{m}_t, z_t, g_t)$ and $\hat{Total}(\hat{b}_t, \hat{m}_t, z_t, g_t)$.

5. With these default probabilities, update the private debt schedule $\hat{q}(\hat{b}_{t+1}, \hat{m}_{t+1}, z_t, g_t) = \mathrm{E_t}\left\{\frac{1-\hat{Total}(\hat{b}_t,\hat{m}_t,z_t,g_t)}{1+r}\right\} + \mathrm{E_t}\left\{\frac{\lambda\ \theta\ \hat{d}(\hat{b}_t,\hat{m}_t,z_t,g_t)}{(r+\theta)(1+r)}\right\} + \mathrm{E_t}\left\{\frac{\lambda\ \theta\ \xi\ \hat{Dd}(\hat{b}_t,\hat{m}_t,z_t,g_t)}{(r+\theta)(1+r)(r+\xi)}\right\}$

6. With this new private debt price, repeat steps 3 (update the optimal decisions of default, $\hat{b}_{t+1}$ and $\hat{m}_{t+1}$), 4 (update default probabilities) and 5 (update the private debt schedule) until convergence.

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