SKEWED SVARs: TRACKING THE STRUCTURAL SOURCES OF MACROECONOMIC TAIL RISKS

Carlos Montes-Galdón and Eva Ortega

Documento de Trabajo
N.º 2208

Carlos Montes-Galdón and Eva Ortega
SKEWED SVARs: TRACKING THE STRUCTURAL SOURCES OF MACROECONOMIC TAIL RISKS
SKEWED SVARs: TRACKING THE STRUCTURAL SOURCES OF MACROECONOMIC TAIL RISKS (*)

Carlos Montes-Galdón
EUROPEAN CENTRAL BANK

Eva Ortega
BANCO DE ESPAÑA

(*) Carlos Montes-Galdón: Forecasting and Policy Modelling Division, Directorate General Economics, European Central Bank, Sonnenmünzstrasse 20, 60314 Frankfurt am Main, Germany, e-mail: carlos.montes-galdon@ecb.europa.eu. Eva Ortega: Directorate General Economics, Statistics and Research, Banco de España, e-mail: eortega@bde.es. This paper reflects the views of the authors and not necessarily those of the ECB, Banco de España or the Eurosystem. We are very grateful to an anonymous referee and to participants at seminars in Banco de España, the European Central Bank and at the conference in honor of Fabio Canova, “Advances in Business Cycle Analysis, Structural Modeling and VAR Estimation”.

Documentos de Trabajo. N.º 2208
March 2022
The Working Paper Series seeks to disseminate original research in economics and finance. All papers have been anonymously refereed. By publishing these papers, the Banco de España aims to contribute to economic analysis and, in particular, to knowledge of the Spanish economy and its international environment.

The opinions and analyses in the Working Paper Series are the responsibility of the authors and, therefore, do not necessarily coincide with those of the Banco de España or the Eurosystem.

The Banco de España disseminates its main reports and most of its publications via the Internet at the following website: http://www.bde.es.

Reproduction for educational and non-commercial purposes is permitted provided that the source is acknowledged.

© BANCO DE ESPAÑA, Madrid, 2022

ISSN: 1579-8666 (on line)
Abstract

This paper proposes a vector autoregressive model with structural shocks (SVAR) that are identified using sign restrictions and whose distribution is subject to time-varying skewness. It also presents an efficient Bayesian algorithm to estimate the model. The model allows for the joint tracking of asymmetric risks to macroeconomic variables included in the SVAR. It also provides a narrative about the structural reasons for the changes over time in those risks. Using euro area data, our estimation suggests that there has been a significant variation in the skewness of demand, supply and monetary policy shocks between 1999 and 2019. This variation lies behind a significant proportion of the joint dynamics of real GDP growth and inflation in the euro area over this period, and also generates important asymmetric tail risks in these macroeconomic variables. Finally, compared to the literature on growth- and inflation-at-risk, we found that financial stress indicators do not suffice to explain all the macroeconomic tail risks.

Keywords: Bayesian SVAR, skewness, growth-at-risk, inflation-at-risk.

JEL classification: C11, C32, C51, E31, E32.
Resumen

Este trabajo propone un modelo vectorial autorregresivo con perturbaciones estructurales (SVAR), identificadas mediante restricciones de signo, cuya función de distribución se caracteriza por una asimetría que varía a lo largo del tiempo. Se propone además un algoritmo bayesiano eficiente para su estimación. Este modelo tiene la ventaja de permitir el seguimiento de forma conjunta de los posibles riesgos asimétricos en torno a las principales variables macroeconómicas, incluidas en el SVAR. Además, provee una narrativa acerca de las razones estructurales que subyacen a la evolución en el tiempo de dichos riesgos. La estimación del modelo con datos para el área del euro encuentra que las perturbaciones de demanda, oferta y política monetaria han exhibido una asimetría significativa y variante en el tiempo entre 1999 y 2019. Esta asimetría explica una proporción significativa de la dinámica conjunta del PIB real y la inflación en el área del euro en ese período, y ha generado importantes riesgos asimétricos de cola en estas variables macroeconómicas. Finalmente, relacionado con la literatura sobre los riesgos del crecimiento (growth-at-risk) y la inflación, encontramos que los indicadores de estrés financiero no son suficientes para explicar todos los riesgos de cola macroeconómicos.

Palabras clave: SVAR bayesiano, asimetría, riesgos del crecimiento, riesgos de la inflación.

Códigos JEL: C11, C32, C51, E31, E32.
1 Introduction

In recent years, a new strand of the economic literature has emerged, trying to understand the conditional distribution of GDP growth (GDP-at-risk), possibly as a function of other financial and economic indicators. Starting with the seminal paper of Adrian, Boyarchenko, and Giannone (2019), several authors have explored empirically how different quantiles of the GDP growth distribution have evolved over time, with the findings consistently showing that upside risks are generally stable, but downside risks vary together with deteriorating financial conditions. For example, in the case of the euro area, Figueres and Jarociński (2020) also conclude that financial indicators such as the CISS in Hollo, Kremer, and Lo Duca (2012) can explain the time varying downside risks to GDP growth. More recently, Lopez-Salido and Loria (2020) have additionally explored asymmetric predictive distributions of inflation, and also find that financial indicators are important to explain downside risks to it, both in the United States and in the euro area.

In order to explore those issues, most of the literature has focused on one-equation quantile regression models of GDP growth or inflation on several potential explanatory variables. Afterwards, a skewed distribution is used to fit the forecasts produced for each GDP growth quantile. This methodology, however, abstracts from identifying potential structural sources of downside risks to GDP growth, and from exploring risks on the joint conditional distributions between GDP growth and other variables such as inflation. As an exception to the latter, Chavleishvili and Manganelli (2019) extend the quantile regression methodology to VAR models, with Cholesky identification. Caldara, Cascaldi-Garcia, Cuba-Borda, and Loria (2020) also explore growth-at-risk in a multivariate model, in which they introduce a markov switching VAR model and show that can replicate results in Adrian, Boyarchenko, and Giannone (2019).

In this paper, we contribute to the literature by using a multivariate model that can account for possibly time varying skewness in the joint distribution of different macroeconomic variables, as well as for a structural identification of the drivers of that skewness. This approach allows to directly estimate joint asymmetric distributions, and to account for the time dependence of those distributions, rather than relying on a two-step procedure. Specifically, we extend a standard Structural VAR (SVAR) model so that the structural shocks, identified using sign restrictions, follow a skewed distribution rather than a normal one. We allow for the parameter that determines the skewness of the distribution to be time varying and possibly to depend on other economic factors. We then develop a Gibbs-sampler that allows to estimate efficiently the skewed SVAR model. Finally, we estimate a medium-scale monthly skewed SVAR model of the euro area economy in which we identify skewed demand, supply and monetary policy shocks.

Our findings suggest that demand shocks in the euro area have experienced a significant variation in their skewness, mainly in periods of financial distress, such
as during the financial crisis of 2008. We find that an indicator of financial stress in
the region (CISS) can actually predict the time variation in the skewness of demand
shocks. However, we also find that there is variation in the skewness of supply
shocks and monetary policy shocks that impact both GDP and inflation and which
cannot be fully explained by financial conditions. Our estimated model indicates
that the transmission of financial factors to the negative tails in the distribution
of real GDP and inflation, as found in the literature, occurs mainly via a demand
channel. But we also find that downside risks in the conditional distribution of
GDP growth and inflation go beyond financial factors: richer models such as the
one proposed in this paper are of the utmost importance to understand them.

Our estimations also highlight a possible source of macroeconomic policy er-
ror: disregarding time variation in the skewness of the shocks distributions can
significantly underestimate the macroeconomic impact of structural shocks. In
fact, we find that the euro area GDP response to a one standard deviation demand
shock can be 10 times larger (twice as large for monetary policy or supply shocks)
when the shock distribution is as skewed as during the Great Recession than at
times when the estimated shock distribution is more symmetric.

Closely related to our paper is also the recent paper from Delle Monache,
De Polis, and Petrella (2021) who consider a univariate model of GDP growth
with time varying skewness. However, they abstract from joint risks with other
variables as well as from structural identification of the drivers of the conditional
skewness in the distribution of real GDP growth. With respect to structural iden-
tification, Loria, Matthes, and Zhang (2019) use quantile regressions as in Adrian,
Boyarchenko, and Giannone (2019) to recover the quantiles of the conditional dis-
tribution of real GDP growth and explore how other shocks identified in the lit-
erature might impact those quantiles. Adrian, Duarte, Liang, and Zabczyk (2020)
introduce a small New Keynesian model in which the demand shock distribution
is subject to endogenous risk vulnerabilities and generates time varying skewness.
Ruge-Murcia (2017) also extends a small DSGE model to incorporate (constant)
skewness in productivity and aggregate inflation shocks. Fernández-Villaverde,
Hurtado, and Nuno (2019) also show that a heterogeneous agent model with finan-
cial frictions can generate endogenous asymmetric risks. However, the method-
ologies in those papers are difficult to implement, especially in larger models with
several shocks. The methodology in our paper, while still relying on time series
methods, is easy to implement, to estimate and also to scale up to a larger amount
of variables, while still providing insights about structural developments. Finally,
we should also mention that our methodology, while still providing time vary-
ing tail risks, avoids problems with quantile regressions as documented in Cherno-

This paper is structured as follows. We first present the skewed SVAR model
and estimation strategy. Then, we report an empirical application using monthly
euro area data up to the outburst of the COVID-19 pandemic and we finally pro-
vide some conclusions.
2 A SVAR model with skewed shocks

Structural Vector Autoregressive (SVAR) models have become popular in the literature due to their flexibility as well as their forecasting properties. SVAR models have recently been augmented along several dimensions that make them more realistic and useful for policy analysis. Among other things, new methods to identify structural shocks have been explored, as well as deviations from non-Gaussian disturbances that allow either for fat tails in the distribution of the shocks or time varying heteroskedasticity. For example, Arias, Rubio-Ramírez, and Waggner (2018), Baumeister and Hamilton (2015) and Korobilis (2020) have explored different ways to identify structural shocks in VAR models, and Cogley and Sargent (2005) and Primiceri (2005) are earlier examples on how to introduce and estimate models with stochastic volatility.

SVAR models with fat tails (e.g. \( t \)-distributed errors) and stochastic volatility models have been extensively used to account for changes in the variance of different macroeconomic variables, which is important in order to consider episodes such as the Great Moderation (McConnell and Perez-Quiros (2000)), or large recessions and outliers such as the Great Financial Crisis, which might bias parameter estimates if the covariance matrix of the SVAR model is assumed to be constant. However, in those models, changes in the variance of the disturbances of the model are mean preserving and increase both negative and positive risks symmetrically. Arguably, in a recessionary period like the Great Financial Crisis, negative risks would have increased more than positive ones, as the literature on growth-at-risk suggests. These facts would not be captured by that class of models.

However, the literature has not yet explored SVAR models in which the structural shocks follow a time varying asymmetric distribution. This paper aims at filling this gap. We introduce a SVAR model in which we assume that the structural shocks follow a multivariate skewed distribution. Moreover, the parameter that drives the skewness of the distribution is allowed to change over time, similar to models that incorporate stochastic volatility. The main difference is that with time varying skewness, the change in the distribution of the shocks is no longer symmetric and their mean also changes over time. This has important consequences for the transmission of the shocks (such as in the analysis of impulse response functions) and, therefore for forecasting, as the model could produce asymmetric predictive distributions. Finally, we also consider how to identify structural shocks using sign restrictions in the context of the SVAR with asymmetric shocks. Sign restrictions are a powerful tool used in the macroeconometrics literature to identify orthogonal disturbances in time series that have a meaningful economic interpretation. Using sign restrictions, we can provide a structural narrative on the causes of increasing tail risks in the different macroeconomic variables that may be included in a skewed SVAR model.

In our approach, we follow Gorodnichenko (2005) and Korobilis (2020) who consider a factor representation of an s-lag \( M \)-variable SVAR model. The latter
also introduces an algorithm to impose sign restrictions in the model in order to identify the factors and provide them with a structural narrative. In the factor representation of the SVAR, the structural shocks are treated as latent variables that need to be estimated. To do so, the literature mostly assumes that they follow a standard normal distribution. Nonetheless, this representation of a SVAR model also proves to be the most tractable one for our purposes, in which we deviate from the previous literature by assuming that the latent factors follow a multivariate skewed normal distribution. Thus, the model that we propose can be written as follows,

\[
y_t = C + B_1 y_{t-1} + \ldots + B_s y_{t-s} + G \epsilon_t + \eta_t
\]

\[
\epsilon_t \sim \text{MSN}(0, I_p, \Lambda_t)
\]

\[
\eta_t \sim N(0, \Sigma)
\]

where \(y_t\) is a \(M \times 1\) vector of endogenous variables in the model and \(\eta_t\) is a \(M \times 1\) vector of i.i.d. measurement errors, with diagonal covariance matrix \(\Sigma\). In our case, we consider \(P\) identified structural shocks, so that \(\epsilon_t\) is a \(P \times 1\), \(P \leq M\) vector of unobserved latent factors that represent those shocks.

Following Arellano-Valle, Bolfarine, and Lachos (2007), we assume that the \(P\) structural shocks, \(\epsilon_t\), follow a multivariate skewed normal (MSN) distribution, where the diagonal matrix \(\Lambda_t\) contains the parameters that determine the skewness of the distribution. In Arellano-Valle, Bolfarine, and Lachos (2007), those parameters are constant. As our aim is to account for time variation in the skewness of macroeconomic shocks, in order to obtain distributions that change over time, we allow each element in the diagonal of \(\Lambda_t\) to be time varying.

Finally, \(G\) is a \(M \times P\) loading matrix that determines the structural identification of the skewed factors or shocks. As it is well known from the factor model literature (Anderson and Rubin (1956)), not all the elements of \(G\) can be estimated without imposing further restrictions. If the top \(P \times P\) block of \(G\) is a lower triangular matrix, this is tantamount to assume a Cholesky identification scheme. This identification scheme has been extensively used in the factor model literature (see, for example, Bai and Wang (2015)). However, Korobilis (2020) deviates from that assumption and imposes sign restrictions on each element of \(G\) in order to achieve identification. This approach works as long as there is a sufficient amount of variables to recover the \(P\) shocks.\(^1\)

Had the shocks, \(\epsilon_t\), followed a standard symmetric Gaussian distribution, the model would be relatively straightforward to estimate, and the latent factors

\(^1\)The approach in Korobilis (2020) is a novel methodology that allows a fast estimation of VAR models under sign restrictions, and deviates from other approaches that first look for the covariance matrix of the shocks, and then rotate it until the desired sign restrictions are satisfied, such as in the case of Arias, Rubio-Ramírez, and Waggoner (2018). Given that we deviate from Gaussian disturbances, and we have time varying skewness, the latter approach is impractical, and thus, we impose directly the restrictions on each element in \(G\).
could have been recovered using a linear filter (as in Carter and Kohn (1994) or Durbin and Koopman (2002)), or integrated likelihood methods as in Chan and Jeliazkov (2009). However, under the MSN distribution, the model is linear but non-Gaussian, and therefore those methodologies cannot be used to estimate them. Introducing time variation in the parameter that determines the skewness of the distribution also introduces a non-linearity in the model. Hence, we need to find a representation of the model that allows recovering both the shocks and the time varying process for the parameters that determine the skewness of the distribution in an efficient way.  

Following Proposition 1 in Arellano-Valle, Bolfarine, and Lachos (2007), a shock that follows a skewed normal distribution can be represented as the sum of an i.i.d. normally distributed shock ($\xi_t$) and the product of the skewness parameters $\Lambda$ and a new auxiliary random variable $\tau_t$, where $\tau_t$ is always positive. In our case, considering that we introduce time variation in the skewness parameter, we can then write,

$$\epsilon_t = \Lambda_t |T_{0t}| + \xi_t = \Lambda_t \tau_t + \xi_t,$$

(2)

where $T_0 \sim N(0, I_p)$ and $\xi_t \sim N(0, I_p)$. Therefore $\tau_t = |T_{0t}|$ is a $P \times 1$ vector whose entries are the absolute value of a normal distribution. For a general normal distribution, its absolute value follows a so-called Folded normal distribution (see for example Leone, Nelson, and Nottingham (1961)). However, when the mean of the normal distribution is 0, the folded distribution collapses to a truncated normal distribution which is truncated from below at zero. This result is important for the estimation procedure that we will develop later, as $\tau_t$ is an additional latent variable that will need to be recovered from the data. Under this representation, it becomes more clear that if $\Lambda_t$ takes value 0, then $\epsilon_t = \xi_t$ is a vector of pure i.i.d. normal shocks, as there is no skewness in the distribution. Instead, when $\Lambda_t$ takes negative (positive) values, the $\epsilon_t$ shocks will be negatively (positively) skewed.

Moreover, it should be emphasized to understand some of the results in this paper that, whenever $\Lambda_t \neq 0$, the expectation of $\epsilon_t \neq 0$, as $\tau_t$ is always positive and its mean is given by,

$$\bar{\tau} = \left( \frac{\phi(0) - \phi(\infty)}{\Phi(\infty) - \Phi(0)} \right) I_p = \left( \sqrt{\frac{2}{\pi}} \right) I_p$$

(3)

where $\phi(.)$ is the p.d.f. of a standard normal distribution and $\Phi(.)$ represents its cumulative density function. Thus, the conditional expectation of the disturbances for a given value of the skewness parameter is,

\[2\) Of course, the model could be estimated also using non-linear filters, such as a particle filter. However, particle filters are difficult to implement in larger models, especially when there is also a significant amount of parameters to be estimated, which is usually the case in SVAR models.
\[
E(\varepsilon_t | \Lambda_t) = \sqrt{\frac{2}{\pi}} \text{diag}(\Lambda_t)
\] (4)

This implies that, whenever there is a change in \( \Lambda_t \), the mean of the disturbances of the model will also change, and thus, shocks to the skewness of their distribution will also have an impact on the first moment of the variables in the SVAR model.

Note that the conditional variance also depends on the skewness matrix, and has a closed form solution as follows,

\[
\mathbf{V}(\varepsilon_t | \Lambda_t) = \mathbf{I}_p + \left( 1 - \frac{2}{\pi} \right) \Lambda_t^2
\] (5)

where the variance of the distribution of the shocks increases with the square of the skewness matrix. Thus, in situations with higher asymmetric risks, the volatility in the model, and consequently, of the forecasts that can be produced with it, will increase.

Further, in our model not only we allow the skewness process to vary over time, we also explore whether some exogenous explanatory variables \( z_t \), e.g. indicators of financial stress, can explain the evolution of skewness over time. This would help understand whether skewness can be predicted. In particular, we assume an autoregressive process for each of the diagonal elements in \( \Lambda_t \), \( \lambda_{i,t} \), of the following form,

\[
\lambda_{i,t} = \phi_i \lambda_{i,t-1} + f_i z_t + \omega_i \nu_{i,t}.
\] (6)

Stacking all \( \lambda_t \), we can write,

\[
\lambda_t = \Phi \lambda_{t-1} + F z_t + \Omega \nu_t,
\] (7)

where \( z_t \) is a \( N \times 1 \) vector of exogenous explanatory variables and \( \nu_t \sim N(0, \mathbf{I}_p) \) captures exogenous movements in the variation of the skewness parameters.

We can then re-write the VAR model in equation 1 as follows,

\[
y_t = C + B_1 y_{t-1} + ... + B_s y_{t-s} + G \Lambda_t \tau_t + G \xi_t + \eta_t,
\] (8)
with three latent states to be estimated, \( \tau_t, \xi_t \) and each diagonal element in \( \Lambda_t \). Given that \( \Lambda_t \) is a diagonal matrix and \( \tau_t \) a vector, we can write again the SVAR model as

\[
y_t = C + B_1 y_{t-1} + \ldots + B_s y_{t-s} + G \tau_t \lambda_t + G \xi_t + \eta_t, \tag{9}\]

with \( T_t = \text{diag}(\tau_{1t}, \ldots, \tau_{P_t}) \). The latter representation, together with equation 7, is still a non-linear model, as there are two states that appear multiplicatively (\( \tau_t \) and \( \lambda_t \)). However, conditional on \( \tau_t \), the model is linear and the latent states \( \lambda_t \) and \( \xi_t \) can be recovered using any linear filter. This result is key to develop an efficient Gibbs sampling algorithm to estimate models of the form 7-9.

Finally, for the purposes of estimation, the model can be written as,

\[
y_t = X_t \beta + G \tau_t \lambda_t + G \xi_t + \eta_t \tag{10}\]

where \( X_t = (I_M \otimes [1, y_{t-1}', \ldots, y_{t-s}']) \) and \( \beta = \text{vec}([C, B_1, \ldots, B_s]') \).

### 2.1 Bayesian estimation of the Skewed SVAR model

We develop now a Bayesian inference procedure to estimate the model in equations 7-9. While the model is still non-linear, it can be effectively estimated using a Gibbs sampler that iterates sequentially over a sequence of conditional distributions whose posterior is known and tractable. Our Gibbs sampler borrows from Korobilis (2020) in order to recover the posterior distributions of the parameters that determine the sign restrictions in \( G \), and also from Arellano-Valle, Bolfarine, and Lachos (2007) who develop a Bayesian algorithm for linear models that incorporate constant skewness. As a summary, our Gibbs sampler algorithm iterates over the following steps,

1. **Step 1**: Draw \( \Sigma \) from \( p(\Sigma|\beta, G, \Phi, F, \Omega, \tau_t, \lambda_t, \xi_t) \)
2. **Step 2**: Draw \( \beta \) from \( p(\beta|\Sigma, G, \Phi, F, \Omega, \tau_t, \lambda_t, \xi_t) \)
3. **Step 3**: Draw \( G \) from \( p(G|\Sigma, \beta, \Phi, F, \Omega, \tau_t, \lambda_t, \xi_t) \)
4. **Step 4**: Draw \( \lambda_t \) and \( \xi_t \) from \( p(\lambda_t, \xi_t|\Sigma, \beta, G, \Phi, F, \Omega, \tau_t) \)
5. **Step 5**: Draw \( \Omega \) from \( p(\Omega|\Sigma, \beta, G, \Phi, F, \tau_t, \lambda_t, \xi_t) \)
6. **Step 6**: Draw \( \Phi, F \) from \( p(\Phi, F|\Sigma, \beta, G, \tau_t, \lambda_t, \xi_t) \)
7. **Step 7**: Draw \( \tau_t \) from \( p(\tau_t|\Sigma, \beta, G, \Phi, F, \Omega, \lambda_t, \xi_t) \)
Except for the last distribution, the other steps are relatively standard and the posterior distributions of those parameters and latent states can be derived under common prior distributions used in the literature. Nonetheless we provide below a short summary of each step for a better comprehension of the full algorithm.

First, note that conditional on the latent states ($\tau_t$, $\lambda_t$ and $\zeta_t$), the model becomes a standard SVAR model, with an exogenous term given by the skewed factors $G T_t \lambda_t + G \zeta_t$. Moreover, $\Sigma$ is a diagonal matrix, and thus, we can treat each equation in the SVAR as independent for estimation purposes.

We assume independent priors for $\Sigma$, $\beta$ and $G$ and consider first an inverse-gamma prior distribution over each parameter $\sigma_m^2$, $m = 1, ..., M$ in the diagonal of $\Sigma$,

$$\sigma_m^2 \sim IG(\sigma_0, \Sigma_0).$$  \hspace{1cm} (11)

Then, the posterior distribution also follows an inverse-gamma distribution with parameters

$$\delta_0 = (\sigma_0 + T - 1)/2$$
$$\hat{\Sigma}_0 = \left( \Sigma_0 + \frac{T}{\sum_{t=1}^{T} y_{m,t}^2} \right) / 2$$

where

$$\hat{y}_t = y_t - (X_t \beta + GT_t \lambda_t + G \zeta_t).$$

With a now given draw of $\Sigma$, we consider a normal prior for $\beta$, $\beta \sim N(\beta_0, V_\beta)$. Under this prior, the posterior distribution also has a convenient form, and is given by a normal distribution $N(\hat{\beta}, \hat{V}_\beta)$ with parameters,

$$\hat{V}_\beta = (V_\beta^{-1} + X (I_T \otimes \Sigma^{-1}) X')^{-1}$$
$$\hat{\beta} = \hat{V}_\beta (V_\beta^{-1} \beta_0 + X (I_T \otimes \Sigma^{-1}) \hat{y})$$

where in this case, $\hat{y}_t = y_t - (GT_t \lambda_t + G \zeta_t)$.

Then, with a draw of $\Sigma$ and $\beta$ in hand, we can draw each element in the matrix $G$. For this step, we closely follow Korobilis (2020). Since we identify our structural shocks using sign restrictions, we introduce a truncated normal distribution prior on each element of $G$. The truncation interval is set according to each sign restriction. For example, for a positive restriction, the truncation is set over the interval $(0, \infty)$. As Gelfand, Smith, and Lee (1992) shows, and then Korobilis (2020), under a truncated normal prior for those parameters, the posterior distribution is also a truncated normal distribution where the mean and variance are those that would be obtained if the distribution of the parameters was not truncated. Define $w_t = T_t \lambda_t + \zeta_t$. Then, we can write the model as,
\[ y_t = X_t \beta + G w_t + \eta_t \]  

(12)

\[ y_t = X_t \beta + W_t \gamma + \eta_t \]  

(13)

where \( W_t = I_M \otimes w_t \) and \( \gamma = \text{vec}(G) \). Under the truncated normal distribution prior for each element of \( \gamma \sim N(\gamma_0, V_\gamma) \), then the posterior is a truncated normal distribution with parameters,

\[
\hat{V}_\gamma = (V_\gamma^{-1} + W(\mathbf{I}_T \otimes \Sigma^{-1})W')^{-1}
\]

\[
\hat{\gamma} = \hat{V}_\gamma (V_\gamma^{-1} \gamma_0 + W(\mathbf{I}_T \otimes \Sigma^{-1}) (y - X\beta))
\]

and the truncation interval is the same one as in the prior distribution.

Then, conditional on the previous draws, as well as on a draw of \( \tau_t \), we can now effectively sample from the posterior distribution of \( \lambda_t \) and \( \xi_t \), which we sample jointly. Conditional on \( \tau_t \), the model is linear, and the latent states can be sampled using a linear simulation smoother. For our purposes, we use the algorithm in Durbin and Koopman (2002).

Given a draw of those latent states, each element in equation 7 is a linear regression model, and therefore, the posterior distribution of \( \Omega \), \( \Phi \), and \( F \) is also standard under normal-inverse gamma priors.

Finally, to obtain a draw of \( \tau_t \), the nonstandard distribution, we use equation 10. Then, we use the fact that \( \tau_t \) follows a truncated standard normal distribution as its prior. To find the posterior distribution of \( \tau_t \), define first \( H \) as a matrix that vectorizes a diagonal matrix, \( H : R^n \rightarrow R^{n^2} \). Let \( e_i, i = 1, \ldots, n \) be the canonical basis vectors of \( R^n \), and let \( h_{ij}, j = 1, \ldots, n^2 \). Then,

\[
H = \sum_{i=1}^{n} h_{ni+n+i} \otimes e_i^T.
\]

(14)

We can then rewrite equation 10 as,

\[
y_t = X_t \beta + (\lambda'_t \otimes G) \text{vec}(T_t) + G \xi_t + \eta_t
\]

(15)

\[
y_t = X_t \beta + (\lambda'_t \otimes G) H \xi_t + G \xi_t + \eta_t,
\]

(16)

and stack the latter equation over time to obtain,

\[
y = X \beta + \tilde{\Lambda}_t T + (I_T \otimes G) \xi + \eta.
\]

(17)

Then, the posterior distribution of \( \tau \) can be obtained as follows. First, conditional on \( \tau \), \( y \) follows a multivariate normal distribution. The latent states, \( \tau \), have a truncated normal distribution prior. We follow again the results in Gelfand, Smith, and Lee (1992), so that the resulting posterior is a truncated normal distribution also with \( \tau \in [0, \infty) \) in which the mean and the variance are those that would be obtained if the constraints were ignored. Thus, the posterior distribution for \( \tau \) will have mean \( \tilde{\tau} \) and variance \( \tilde{V}_\tau \).
\[ \hat{\tau} = (I_{TM} + \tilde{A}_T(I_T \otimes \Sigma^{-1})\tilde{A}_t)^{-1} \]
\[ \hat{\tau} = \hat{\tau}_t(\tilde{A}_t'(I_T \otimes \Sigma^{-1})(y - X\beta - (I_T \otimes G)\xi)). \]

Since each latent state is independent of each other, a random vector \( \tau \sim TN(0, \infty, \hat{\tau}, \hat{\tau}) \) can be easily simulated as in, for example, Botev (2016).

3 Tracking macroeconomic tail risks in the euro area

The above model, together with the estimation algorithm, is most suitable to understand the evolution of macroeconomic tail risks. Tracking time variation in the skewness of the shocks helps to identify and understand downside and upside risks to macroeconomic variables such as real GDP and inflation. For forward looking policy makers, this is critical for the design of different policies.

This section estimates the aforementioned model for the euro area. We subsequently analyse how the skewness of three different structural shocks identified using sign restrictions (demand, supply and monetary policy) have evolved over time as well as their contribution to the dynamics of different variables. Finally, we also explore the estimated transmission of the different shocks when their distribution might be skewed.

3.1 Data and model specification

In order to estimate the model in the previous section for the euro area, we consider the following data up until the COVID-19 pandemic, that is, spanning from January 1999 until December 2019: real GDP growth, HICP inflation, HICP inflation excluding energy and food, short-term nominal interest rate, 10-year government bond yields, the nominal effective exchange rate of the euro, industrial production index, and real stock prices. While in this paper we want to focus on the role of skewness on real GDP growth and inflation, we need a larger model to be able to identify three structural shocks or factors (demand, supply and monetary policy). An 8-variable VAR allows to identify three shocks using the methodology in Korobilis (2020). We have selected the additional variables so that they can provide useful information for the identification of the shocks.

First, introducing HICP inflation excluding energy and food together with the aggregate HICP, provides information to disentangle the evolution of more supply side driven changes in prices. The long-term rate also helps with the identification of the impact of monetary policy as many financial decisions, such as mortgages, are based on longer term rates rather than the short-term nominal interest rate. It also incorporates information about financial markets, together with the real stock
prices. The nominal effective exchange rate incorporates foreign information, and also helps to disentangle the different shocks. The short-term nominal interest rate has suffered in our sample from the presence of the effective lower bound. In principle, this could be a challenge for time series models. In order to overcome this problem, we use the shadow interest rate developed by Krippner (2013). Finally, six lags are used for the estimation of the skewed SVAR.

We consider that changes in tail risks might change rapidly and, therefore, a monthly model will be more suitable than a quarterly one. Moreover, we need to estimate a significant amount of parameters and latent states and, for the euro area, the sample size in a quarterly model might be small to properly identify all of them. However, in our dataset, real GDP is only available at quarterly frequency. In order to avoid this problem, we use results from the mixed-frequency modelling literature, and we treat monthly real GDP growth in the model as an additional latent state that needs to be estimated. Thus, we add a final step to the estimation algorithm that we developed above in which we get a draw of monthly real GDP growth using a Kalman filter as in Schorfheide and Song (2015). For the filter, we link observable quarterly GDP growth to the unobserved monthly growth rates as,

\[ GDP^q_t = GDP^m_t + GDP^m_{t-1} + GDP^m_{t-2}, \]  

(20)

where \( GDP^q_t \) is the quarterly growth rate of GDP and \( GDP^m_t \) the monthly one.\(^3\)

We identify three structural shocks, demand, supply and monetary policy shocks, using the following sign restrictions in Table 1. They are standard restrictions on the impact effect of each shock on model variables in the SVAR literature, and are also in line with the predictions of more complex estimated structural mod-

<table>
<thead>
<tr>
<th></th>
<th>Demand</th>
<th>Supply</th>
<th>Monetary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real GDP Growth</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>HICP inflation</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>HICP inflation excl. energy and food</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Short-term nominal interest rate</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Long-term rate</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Exchange rate</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Industrial Production</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Real stock prices</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 1: Sign restrictions used to identify the structural shocks

\(^3\)Figure A.1 in the Appendix shows the posterior estimation of our estimated monthly real GDP growth time series. The series is estimated with low uncertainty as suggested by the credible intervals of the posterior distribution.

\[ 3 \]
related to monetary and financial conditions. In the case of supply side shocks, real variables move in opposite directions to the nominal ones, capturing events such as productivity or cost push shocks. In this case, we let both the short term and the long term rates unrestricted, as the response of monetary policy to supply shocks might not be clear a priori. Finally, we label our third shock as a monetary one, in which we observe a deterioration of real and nominal variables which is associated with an increase in nominal rates and financial conditions as well as a drop in the stock market. Note that the three structural shocks are well identified even without imposing all of the signs on impact. However, when using sign restrictions in SVAR models, the latent shocks are only set-identified. That is, they are not uniquely pinned down. Adding more variables and restrictions help to shrink the identification set, and thus to estimate the shocks more accurately. Variation in the data not captured by the three shocks that we identified will be absorbed by the measurement errors, \( \eta_t \). As we will show later, these shocks play a very small role in our estimation.

As most of the literature on growth-at-risk has focused on the role of financial factors in explaining macroeconomic tail risks, we investigate whether financial stress can explain the skewness found in the distribution of the three structural shocks. For that purpose we use the ECB’s Composite Indicator of Systemic Stress (CISS) in the euro area financial system of Hollo, Kremer, and Lo Duca (2012), as it is the indicator that Figueres and Jarociński (2020) find to be the most informative about tail risks in the euro area. The CISS is then used as an explanatory variable in the equation that determines the law of motion of each time varying skewness process.

### 3.2 The evolution of skewness in the euro area

First, we discuss the estimated evolution of the skewness of the distributions of the three structural shocks that we identify within the model. Figure 1 presents a plot of the posterior median and the 16th and 84th percentiles of the posterior distribution of the time varying skewness of the shocks. That is, the estimated evolution of \( \lambda_t \) defined in equation 7. The figure suggests several interesting characteristics of the distribution of the shocks in the euro area.

Our results suggest that there has been a significant variation in the skewness process of the three different structural shocks that we identify. The skewness of the demand shock, which captures exogenous shifts in consumption and investment expenditures, is the most volatile one, displaying the largest swings, although it is not very persistent. It also shows large negative spikes in the periods that are associated with recessions in the euro area, especially between 2008 and 2012.

With respect to the skewness of the supply shocks, our estimation also picks up some variation which coincides with periods in which inflation was above the
ing that the size of the monetary stimulus at the onset of the financial crisis might not have been as found in the literature, occurs mainly via a demand channel.

of financial factors to the tails of the real GDP growth and inflation distributions, less significantly, financial conditions can also explain negative skewness found in the demand shock driving downwards GDP growth.

persistent, while in the case of monetary policy shocks they are highly persistent. Our estimation suggests that changes in the skewness of the distribution of demand shocks is the least components as well as the impact of the financial indicator (CISS). Our estimation suggests that other factors determining tail risks in GDP and possibly other variables should not be ignored.

This is an interesting result, as most of the literature has emphasized the role of financial conditions in shaping the possibly asymmetric time varying distribution of real GDP. Our estimation suggests that other factors determining tail risks in GDP and possibly other variables should not be ignored.

A strong positive skewness is found for monetary policy shocks starting in 2008. If we consider that the monetary policy shocks identified capture shifts in monetary policy that go beyond what a systematic policy rule might dictate, this implies that the ECB, during this period, was setting the short-term nominal interest rate above what a historical monetary policy rule would imply, one of the reasons being that at the moment the short-term nominal interest rate was constrained by its effective lower bound. Of course, this implies contractionary monetary policy that in this case will also generate negative tail risks in both real GDP and inflation. Except for that moment, the skewness of the monetary policy shocks is rather stable and close to zero, suggesting that the shocks are closer to a normal distribution. Still, some downward bias of monetary policy shocks can be found in the later period of mostly expansionary monetary policy. Given that we are using a shadow short term nominal interest rate measure, it also captures the impact of the different asset purchase programs from the ECB.

Note that, even if we are using a shadow rate in our estimation that incorporates information about unconventional monetary policy, it does not go into negative territory until 2011, suggesting that the size of the monetary stimulus at the onset of the financial crisis might not have been sufficient.
To understand the dynamics of the estimated time varying skewness processes, $\lambda_t$, figure 2 shows the posterior distribution of their autoregressive components as well as the impact of the financial indicator (CISS). Our estimation suggests that changes in the skewness of the distribution of demand shocks is the least persistent, while in the case of monetary policy shocks they are highly persistent.

The second row of figure 2 shows that financial stress certainly explains the negative skewness found in the demand shock driving downwards GDP growth and inflation during some periods of time, in line with other results found in the growth-at-risk literature. Less significantly, financial conditions can also explain some small positive skweness in monetary policy shocks, pushing upwards interest rates and downwards GDP and inflation. However, financial stress has zero impact on supply shocks skewness. Thus, our model indicates that the transmission of financial factors to the tails of the real GDP growth and inflation distributions, as found in the literature, occurs mainly via a demand channel.

Figure 2: Parameters in the evolution of skewness

![Figure 2](image.png)

Note: The figure shows the prior and posterior distributions of the parameters that determine the time varying skewness process ($\lambda_t$). The first row shows the autoregressive coefficient of the $\lambda_t$ process, and the second one the impact of the CISS indicator of financial stress on it, as estimated in equation 7.

3.3 The macroeconomic impact of time varying skewness

The previous section provided a descriptive analysis of the estimated evolution of time varying skewness in the euro area. However, it is important to understand how that evolution has impacted the dynamics of different macroeconomic variables, and to what extent that impact might be sizeable.
3.3.1 Historical shock decomposition

To understand the impact of time variation in the skewness of the three shocks, we first focus on a historical shock decomposition of real GDP growth and inflation in the euro area. Note that in our model, each shock has an effect on macroeconomic variables via two terms, one of them related to the variation in skewness. The total effect of the identified structural shocks is given by,

\[ S_t = GT_t \lambda_t + G \xi_t. \]  

(21)

It order to compute the posterior distribution of the contribution of each structural shock, we proceed as follows. First, we obtain a draw from the joint posterior distribution of the parameters \( \beta, G \) and the latent states \( \tau_t, \lambda_t, \xi_t \) that we obtained in our estimation. Then, for each of the draws, we recover the estimated measurement error, \( \eta_t \). Finally, for each structural shock we compute a counterfactual time series, such that for \( p = 1, ..., P \), we set \( G^p \) as a loading matrix that is equal to \( G \), except that the \( p - 1 \) row is set to 0. Then, we define the counterfactual as,

\[ y_t^f = C + B_1 y_{t-1}^f + ... + B_s y_{t-s}^f + G^p T_t \lambda_t + G^p \xi_t + \eta_t, \]  

(22)

and the contribution of the structural shock \( p \) is given by \( y_t - y_t^f \).

Figure 3 shows the estimated median of the posterior distribution of the contribution of each structural shock to the historical evolution of real GDP growth and HICP inflation in the euro area.\(^5\) The figure offers several insights on the narrative about the structural determinants of the euro area economy. First, demand shocks have a higher weight on the dynamics of real GDP growth than on inflation. This goes in line with the literature that shows that the structural slope of the Phillips curve might be small, and therefore, the transmission of real activity to prices is more muted than expected. Second, supply shocks are an important determinant of HICP inflation. This is also not surprising given the dynamics of especially energy prices in the euro area. The contribution of supply shocks goes actually in line with them. For example, 2008 was a period of abnormally high HICP inflation, mostly due to oil prices being close to their historical maximum value (close to 140 USD per barrel). This translates into higher costs and therefore adverse supply shocks that our model captures properly. On the other hand, between 2013 and 2016, energy prices declined rapidly, and stayed at low values, which implied a period of low inflation.\(^6\) Our model captures this as positive

---

\(^5\)Figure 3 also shows that the impact of the measurement errors \( \eta_t \), represented by the blank spaces, is minimal in our model. These errors can be thought of as the divergence between the data and the contributions of the structural shocks. The fact that the measurement errors are small implies therefore that our three identified structural shocks are indeed a good explanation for the euro area macroeconomic dynamics.

\(^6\)Ciccarelli and Osbat (2017) provide a thorough explanation of the causes of low inflation in this period. They also find that low inflation in this period can be related to global factors which include lower energy prices.
supply shocks that pushed down inflation, but helped to keep positive real GDP growth rates.

Our estimation also provides a structural narrative to the two major recessions in the euro area until 2019. The first recession, in 2008, which is characterised by a sharp decline in real GDP growth and inflation, is explained by demand shocks,\(^7\) and there is some role for a delayed response of the supply shocks that hit the economy at the beginning of the recession. However, there is a major role for monetary policy shocks. This goes also in line with the estimated time varying skewness process of monetary policy. As we discussed above, this period was characterised for policy rates which were significantly above what a historical rule might have determined, due to the effective lower bound constraint, and therefore, the monetary policy stance was contractionary.

The subsequent recovery in real GDP growth and inflation after 2010 is then explained by smaller negative demand shocks, positive supply developments, but mostly by more accommodating monetary policy, which also continues even during the recession of 2011Q3-2013Q1, according to the dating of the euro area business cycle dating committee\(^8\). This reflects also the change in the ECB’s monetary policy conduct. That is, the ECB implemented different non-standard monetary policies that might be captured by the shadow rate measure in the model, and translate into a positive impact of monetary policy shocks. The 2011-2013 recession is however mostly explained by supply-side structural shocks. Compared to the 2008 recession, this one is distinguished by higher HICP inflation rates. Thus, this implies a slower adjustment of prices and wages which, together with higher energy prices that were again close to their maximum in 2008, dragged down real GDP growth. Note also that the results go in line with the narrative that arises in more complicated estimated structural models, such as Coenen, Karadi, Schmidt, and Warne (2018), in which productivity shocks have a large role explaining the 2011-2013 recession.

The decomposition also reveals that the three shocks that we have identified can explain most of the variation in the data. Thus, even if we have more variables than shocks in the model, there is a very marginal role for the measurement errors which would capture other factors that we have not identified in the model.

How much of the contribution of each structural shock is due to changes in the skewness of their distribution or to the i.i.d. shocks? While the previous explanation considers the total impact of each structural shock, we focus now on the impact of their time varying skewness component. As we discussed in section 2, if the structural shocks follow a skewed distribution, a change in the parameter that determines the asymmetry also implies a change in the conditional mean of

\(^7\)Note that these could be of either domestic or foreign origin, as in our model specification we do not identify separately foreign demand shocks.

\(^8\)See https://eabcn.org/dc/chronology-euro-area-business-cycles for the full chronology of peaks and troughs provided by the the CEPR-EABCN euro area dating committee.
the different variables in the model. If changes in skewness have a strong impact on the different variables in the SVAR, this implies that there are also significant macroeconomic tail risks that vary over time.

**Figure 3: Historical shock decomposition, total impact**

![Graph showing historical shock decomposition for Real GDP growth and HICP inflation.](image)

Note: The figure shows the median contribution of each structural shock to the dynamics of monthly year on year real GDP growth (top panel) and monthly year on year HICP inflation (bottom panel). The decomposition considers the impact of both the skewness shock, $\lambda_t$, and the normally distributed one, $\xi_t$. Both variables are expressed as annual growth percentage, in deviation from the steady state of the model.

To compute the macroeconomic impact of time varying skewness, we proceed as in the previous exercise, but we keep the impact of the normally distributed shock, $\xi_t$, as originally estimated. That is, the counterfactual is defined as follows:

$$y_t = C + B_1y_{t-1} + \ldots + B_8y_{t-s} + G^P T_t \lambda_t + G^C \xi_t + \eta_t.$$  (23)
Figure 4 shows the estimated median of the posterior distribution of the contribution of each skewness process on real GDP growth and HICP inflation. We focus first on the impact of changes in the distribution of monetary policy shocks. Until the 2008 financial crisis, the impact of changes in the skewness of monetary policy shocks is minimal, reflecting the fact that the distribution was stable as also suggested by figure 1. This changed in 2008. A sizeable proportion of the contribution of monetary policy shocks is attributed to changes in their distribution. The model finds that the systematic contractionary deviations from the model’s historical rule translated into a perceived change in the skewness of the shocks which, in turn, contributed to the sharp and protracted contraction in real GDP growth, as well as a decline in inflation. This result also suggests that the effective lower bound at the moment had a major impact. After the crisis, and given the change in monetary policy conduct, we see that indeed the monetary policy shocks become now slightly skewed to the more expansionary area, which contributed to the recovery of real GDP growth and partially to HICP inflation, too.

Changes in the skewness of demand shocks are also a major determinant in the dynamics of real GDP growth, especially during the financial crisis. As in the case of monetary policy shocks, a sequence of negative demand shocks, together with the impact of the CISS on their skewness, implies that the model understands that the distribution of the shocks became skewed to the left, with fatter negative tails, during the crisis. This change also had an impact on HICP inflation, which was a long-lasting one until approximately 2013. Finally, while the impact of changes in the skewness of the supply shocks distribution is smaller than in the other two shocks, it cannot be disregarded. In the case of inflation, that variation helps explain the build up of inflation before the financial crisis (again, in a period with higher than usual energy prices) as well as part of the low inflation period starting in 2014. Time variation in the skewness of supply shocks can also explain some of the variation in real GDP growth, especially at the beginning of the 2008 recession (a consequence of the previous high inflation period), as well as during the second major recession in 2012.

Therefore, we find again a structural narrative in terms of changes in skewness. Several important points should be mentioned. First, tail risks can be an important driver of the dynamics of real GDP growth, but also inflation, which has not been analysed as extensively as real GDP growth (an exception is Lopez-Salido and Loria (2020), who find that financial conditions can also generate downside risks in inflation). Second, tail risks in real GDP growth and inflation are not only driven by financial factors. Changes in the distribution of real GDP and inflation arise also due to changes in the distribution of monetary policy shocks as well as of supply side shocks (technology, mark-ups or energy prices). We find that strong deviations from a normal distribution of the various sources of structural macroeconomic fluctuations could indeed have strong negative effects on different macroeconomic variables. Thus, shortcuts using financial stress indicators to account for time varying tail risks may not be sufficient.
3.3.2 Impulse response function analysis

We center now our attention on the transmission of the different shocks. To do so, we compute impulse response functions (IRFS) to the different structural shocks. Since our model is non-linear, the impact of the different shocks is therefore dependent on their underlying skewed distribution. In order to show that the IRFS in the model are state dependent, we show two different cases, in which we assume that there is no skewness or that the skewness of each shock is set at the maximum value that we recover in our estimation.⁹

⁹A different comparison would be to estimate the model with and without skewness. In our case we want to emphasize the role of the changing distributions of the different shocks within the same estimated model and show how the IRFS change under different values of the parameters that determine the skewness of those distributions.
First, we compute the IRFS in the case in which the distribution of the shocks is symmetric, so that \( \lambda_t, \tau_t = 0 \). We first get a draw from the posterior distribution of the estimated parameters in the model, and then, for each structural shock, we set \( \xi^p_t = 1 \) (one standard deviation) and compute the dynamics of the model as is standard in the literature. Figure 5 shows the estimated impact of the different shocks in this scenario, with the 90%, 68% and 50% credible intervals, which incorporate parameter uncertainty. We find first that a demand shock has a persistent negative response on real GDP growth and HICP inflation, although the impact is not sizeable when there is no skewness. Moreover, the response of inflation is small in relative terms compared to that of real GDP. This again confirms that the Phillips curve might be significantly flat in the euro area. Supply shocks exhibit a less persistent behavior, although after three years the impact on real GDP growth is still negative while inflation has already stabilised. Supply shocks capture both technology, energy prices and possibly mark-up shocks. A larger model with more structural shocks identified would be required to disentangle all the mechanisms. Still, our results point out to the fact that supply side shocks in the euro area can have long-lasting effects on real economic activity. Finally, monetary policy shocks are the least persistent ones and their impact vanishes after one year, especially in the case of HICP inflation.\(^{10}\)

Let us consider now an impulse response function analysis in the presence of skewness. In order to compute the response in this case, we first fix the skewness parameter for each shock to a value \( \bar{\lambda} \). We set these values to correspond to the largest ones in figure 1, in absolute terms. For the case of demand shocks, we set \( \bar{\lambda}^d = -10 \), for supply shocks, \( \bar{\lambda}^s = 1 \), and for the monetary policy shocks, \( \bar{\lambda}^n = 2 \). This allows us to understand the transmission of the different shocks in periods of stress for the euro area economy. Note again that, as the model is non-Gaussian, the responses are actually highly dependent on the value of the skewness parameter.

Note however that in this case, skewness introduces a change in the steady state value of the SVAR model, compared to a standard SVAR. This modification should be taken into account when calculating the impulse response functions. The steady state value of the model with a permanent change in \( \lambda_t \) becomes,\(^{11}\)

\(^{10}\)Note that the distribution of the different IRFS is not fully symmetric. This is because the posterior distribution of the loading matrix \( G \) is a truncated normal distribution (see figure B.1 in the Appendix). Thus, the contemporaneous distribution of the impact of the shock might be asymmetric. In the appendix we also show the same IRFS analysis when we consider only the response at the mean of the posterior distribution of \( G \). In this case, the responses become symmetric and allow for a better comparison with the case in which we incorporate skewness in the analysis (see figures B.2 and B.3).

\(^{11}\)Note that if we consider the evolution of \( \lambda_t \) in equation 7, the steady state value of the model would not need to be modified, as \( \lambda_t \) would converge to 0 in the long-run. However, we want to explore the impact of skewness in specific periods of time and consider that a permanent change in \( \lambda_t \) is more appropriate to understand the transmission mechanisms. By fixing \( \lambda_t \) and considering the impact in the steady state, we can isolate the impact of the iid shock. If we consider a change in \( \lambda_t \), but we consider that it is 0 in steady state, we would have the joint impact of the i.i.d. shock as well as the skewed one.
would not need to be modified, as we explore the impact of skewness in specific periods of time and consider that a permanent change in λ would not need to be modified, as

\[ \lambda \]

shock when \( h \) in the previous case.

The steady state value of the model with a permanent change in \( \lambda \) should be taken into account when calculating the impulse response functions. This modification should be taken into account when calculating the impulse response functions. This modification should be taken into account when calculating the impulse response functions. This modification should be taken into account when calculating the impulse response functions.

Well as the skewed one.

HICP inflation, confirming also the results from the historical shock decomposition case of the demand shock, which tries to mimic the impact of skewness during tail risks arise and can change the distribution of the IRFS significantly. In the impact is almost ten times as large as in the non-skewed IRFS, while in the case considering a symmetric distribution. In the case of the demand shock, the median asymmetries in the distribution do not affect it. However, we observe first that

\[ \text{Note that if we consider the evolution of } \tau \]

is a vector of zeros for

\[ h \]

is more appropriate to understand the transmission mechanisms. By fixing \( t \)

is the mean of a truncated standard normal distribution in the interval

\[ (0, \infty) \]

, as discussed in Section 2.

Then, to compute the response in this case, in which we incorporate skewness risks, we draw first from the posterior distribution of the parameters of the model, and compute the response at horizon \( h \) for each structural shock as,

\[ IR_h = C + B_1 IR_{h-1} + \ldots + B_s IR_{h-s} + G \tau_h \bar{\lambda} + G \xi_h - y^{ss}, \]  

(25)

where in each simulation, \( \tau_h \) is a vector with a random draw from a truncated standard normal distribution in the position of the structural shock of interest, and \( \xi_h \) is a vector of zeros for \( h > 1 \), and a vector with a 1 in the position of the structural shock when \( h = 1 \). Thus, the size of the normally distributed shock is the same as in the previous case.
Figure 6 shows the estimated impact of the different structural shocks in this scenario in which we introduce skewness, again with the 90%, 68% and 50% credible intervals. Note that compared to the previous IRFS, we have now also uncertainty about $\tau_h$.

In terms of persistence, the responses follow the same pattern as before, as the asymmetries in the distribution do not affect it. However, we observe first that the median responses can be of several orders of magnitude larger than when we consider a symmetric distribution. In the case of the demand shock, the median impact is almost ten times as large as in the non-skewed IRFS, while in the case of supply and monetary policy shocks, the response is twice as large. Second, tail risks arise and can change the distribution of the IRFS significantly. In the case of the demand shock, which tries to mimic the impact of skewness during the financial crisis, negative tail risks are important both for real GDP growth and HICP inflation, confirming also the results from the historical shock decomposition that we presented before. This is also the case for the monetary policy shock. They

Figure 6: Impulse response function, with skewness

Note: This figure shows the median (solid line) and the 90%, 68% and 50% credible intervals (shaded areas) of the impulse response functions to the three structural shocks that have been identified in the model for real GDP growth and inflation, under skewness.
can have significant pervasive effects in the risks of both distributions. In the case of the supply side shock, the asymmetric risks are more evident in the case of HICP inflation, although they also change real GDP growth.

Thus, we should conclude first that the structural shocks that we have identified have stronger impact when the distributions are skewed, for the same value of $\xi_t$. Second, tail risks can be important, and should not be disregarded by policy makers.

### 3.3.3 The impact of asymmetric risks on the forecasting distributions

Finally, our results also have strong implications for forecasting. Predictive distributions under skewness will exhibit asymmetric risks and should be taken into consideration when designing macroeconomic policies. To understand how asymmetric risks might impact the forecasting distribution of real GDP and inflation over time, we construct rolling forecasts from the estimated model at each period of time for up to one year. It should be noted, however, that this is not an out-of-sample prediction exercise. That is, to construct our in-sample forecasting distributions, we consider the posterior distribution of the parameters estimated using the full sample. Constructing the forecasting distribution in the skewed model is straightforward. For a given draw of the parameters from their posterior distribution, as well as the estimated starting point at each period in time, recursive forecasts can be derived as usually done in the BVAR literature, using equations 7 and 9. Note that compared to standard forecasts in BVAR models, there are additional sources of uncertainty stemming from drawing $\tau_t$ as well as $v_t$ in the dynamic equation that determines the time variation in the skewness parameters.

To understand the impact of skewness in the forecasting distributions, we focus on the relationship between the mean forecast and its variance for different horizons. Adrian, Boyarchenko, and Giannone (2019) show that, in the case of the United States, this relationship is negative. Thus, lower values of the forecasting mean are associated with higher realisations of volatility. Figueres and Jarociński (2020) show a similar association for the euro area. While they use the results from quantile regressions to generate those results (thus allowing for different means and volatilities in each quantile), our estimated model is also capable of generating this interdependence between mean and volatility. As we mentioned in section 2 a skewed distribution has an implied relationship between the mean and the variance of the distribution. When there is no skewness, the volatility of the distribution is constant and independent of the mean. This result translates to our forecasting distributions.

Figure 7 shows a scatter plot of the estimated relationship between mean and volatility in the forecasting distributions of monthly real GDP growth at each period of time in our sample, for different forecasting horizons (1, 6 and 12 months). The first row in the figure shows this relationship when we consider skewness in...
the three shocks that we estimate. We find indeed a negative relationship between the conditional mean of the forecasting distribution and its volatility, measured as its standard deviation. The relationship is however stronger the longer the forecasting horizon, suggesting higher asymmetric risks at the end of the forecasting distribution. The following rows in the figure, show the breakdown for the three different shocks. To compute the impact of each shock, we keep only skewness risks in one of the shocks at the time, setting the parameter $\lambda$, to zero for the remaining shocks. The chart suggests that the asymmetric risks arise mostly due to skewness in the demand shocks, and to some extent, in the monetary one. Skewness in the supply side shocks only contribute to asymmetric risks in the short run, while in longer horizons, the standard deviation of the forecasting distribution is mostly independent of the mean.

Figure 7: Mean and volatility of monthly real GDP growth at different forecasting horizons

Note: The figure shows the scatter plot between the mean and the standard deviation of the forecasting distributions of annual real GDP growth, for each month in the estimation sample. The first column shows the results for the one-month ahead forecasting distribution, the middle column the six-months ahead and the last column, the one-year ahead distribution. The first row shows the estimated relationship when considering skewness in all shocks, while the remaining rows only consider skewness in one shock.
Figure 8 also shows the mean-volatility barrier in the forecasting distributions for monthly HICP inflation. In this case, there is a more limited correlation between the mean and the standard deviation of the forecasting distributions of inflation. This goes in line with other results in the literature for the euro area, such as Figueres and Jarociński (2020), who find less skewness in the forecasting distribution of inflation than of GDP. As with real GDP, demand and monetary shocks seem the main drivers of the negative relationship between mean and volatility, which in this case is only relevant in longer forecasting horizons.

Finally, we also explore how much skewness the model is able to generate in the forecasting distributions. We compute the skewness of each forecasting dis-

Figure 8: Mean and volatility of monthly HICP inflation at different forecasting horizons

Note: The figure shows the scatter plot between the mean and the standard deviation of the forecasting distributions of annual HICP inflation, for each month in the estimation sample. The first column shows the results for the one-month ahead forecasting distribution, the middle column the six-months ahead and the last column, the one-year ahead distribution. The first row shows the estimated relationship when considering skewness in all shocks, while the remaining rows only consider skewness in one shock.
distribution, also for the distributions in which we incorporate skewness in one of the shocks only. Figure 9 summarizes the results for real GDP growth. The figure shows how the skewness of the different forecasting distributions has been evolving over time, up to a horizon of one year. First, we find that episodes with negative skewness risks are more prevalent than those with positive risks, although these ones are not negligible. The episodes with negative skewness are associated with recessions in the euro area, and they can be explained mostly via negative skewness in the evolution of demand shocks, but also due to the impact of contractionary monetary policy shocks at the time. However, we also find periods in which there are positive asymmetries, like in 2009-2010. These positive asymmetries arise mostly from supply shocks and accommodative monetary policy shocks, and confirm the narrative that arises in the shock decomposition discussed in the previous subsections.

Figure 9: Evolution of skewness in the forecasting distributions, real GDP

![Figure 9](image)

Note: The figure shows the evolution over time of the skewness in the forecasting distributions of annual real GDP growth, for different forecasting horizons. Dark red colors imply negative skewness, while blue colors signify positive skewness in the distribution. Colors closer to green indicate that the forecasting distribution is close to being symmetric. The first row considers skewness in all shocks, while the remaining rows only consider skewness in one shock.
In the appendix, we also show the evolution of skewness in the forecasting distributions of annual HICP inflation. We find that, compared to real GDP growth, the distributions do not become as asymmetric. However, we observe that the model captures properly downside risks to annual inflation during the Great Recession, due to the negative skew in demand and monetary policy shocks, but also risks in the period after the crisis that reflect the evolution of the skewness in the distribution of supply shocks.

4 Conclusion

In this paper we propose a structural VAR model in which we identify structural shocks using sign restrictions while at the same time we relax the assumption of normality in the disturbances. We thus allow for the structural shocks in the model to follow a skewed distribution, whose parameters might be time varying. A skewed distribution with time varying parameters allows to track variation in asymmetric risks to different macroeconomic variables.

We also propose a fast and efficient Gibbs sampling algorithm to estimate the model. In our empirical application, we use euro area data to estimate a model with demand, supply and monetary policy shocks, and we show that the distributions of the shocks have been changing over time, showing significant asymmetries over time.

Our findings suggest that demand shocks have experienced a significant variation in their skewness, mainly in periods of financial distress, such as during the financial crisis of 2008, in which the shocks become skewed to their negative area for a significant amount of time. We find that an indicator of financial stress (CISS) can actually predict that time variation in the skewness of the demand shocks. However, we also find significant time variation in the skewness of supply shocks and monetary policy shocks that impact both GDP growth and inflation, which can not be explained by financial conditions. Thus, downside risks in the conditional distribution of GDP growth and inflation in the euro area go beyond financial factors and richer models such as the one proposed in this paper are of the utmost importance to understand them.

We also show that structural shocks have stronger macroeconomic impact during the periods in which their distributions are skewed. As tail risks can be important, models such as the skewed SVAR with time varying asymmetries proposed here should be considered in order to avoid significant macroeconomic policy mistakes.

Future research plans include a cross-country comparison on the estimates of time-varying skewness. Given that skewness arises mostly in periods of macroeconomic distress, it would prove useful to estimate the model for other countries
in which there is more data available, as it is the case for the United States, for example. Moreover, in our estimation, we have only considered financial indicators as a potential explanation for the variation in skewness in order to make a clear connection to the extant literature on growth-at-risk. Given that we find that financial conditions cannot explain all that variation, it will be useful to explore alternative indicators. For example, variables such as global activity or, in the case of the supply shocks, the price of oil, might be relevant to explain variations in the skewness of the distribution of the shocks.
References


A Estimated monthly real GDP growth

In the estimation of our skewed SVAR model, we treated monthly real GDP growth as an unobservable latent factor which we recovered using a linear filter, conditional on the other parameters and states of the model. Figure A.1 shows the estimated time series for monthly GDP growth that we recover from our estimation algorithm. The figure shows the median estimate, as well as the 64% credible interval.

Figure A.1: Posterior distribution of monthly real GDP growth in the euro area
B Additional figures

Figure B.1: Posterior distributions of the parameters in $G$, euro area estimation

Note: This figure shows the prior and posterior distributions of each element in the matrix $G$, which captures the estimated impact response of the three structural shocks in the model on the eight variables in the SVAR.
Figure B.2: Impulse response function, no skewness. Evaluated at the posterior mean of $G$, euro area case

Note: This figure shows the median of the (solid line) and the 90%, 68% and 50% credible intervals (shaded areas) of the impulse response functions to the three structural shocks that have been identified in the model for real GDP growth and inflation, without skewness and when the parameters in the matrix $G$ are set to their posterior mean.
Figure B.3: Impulse response function, with skewness. Evaluated at the posterior mean of $G$, euro area case

Note: This figure shows the median of the (solid line) and the 90%, 68% and 50% credible intervals (shaded areas) of the impulse response functions to the three structural shocks that have been identified in the model for real GDP growth and inflation, under skewness and when the parameters in the matrix $G$ are set to their posterior mean.
Figure B.4: Evolution of skewness in the forecasting distributions, HICP

Note: The figure shows the evolution over time of the skewness in the forecasting distributions of annual HICP inflation, for different forecasting horizons. Dark red colors imply negative skewness, while blue colors signify positive skewness in the distribution. Colors closer to green indicate that the forecasting distribution is close to being symmetric. The first row considers skewness in all shocks, while the remaining rows only consider skewness in one shock.
BANCO DE ESPAÑA PUBLICATIONS

WORKING PAPERS

2030  BEATRIZ GONZÁLEZ: Macroeconomics, Firm Dynamics and IPOs.
2031  BRINDUSA ANGHEL, NÚRIA RODRÍGUEZ-PLANAS and ANNA SANZ-DE-GALDEANO: Gender Equality and the Math Gender Gap.
2032  ANDRÉS ALONSO and JOSÉ MANUEL CARBÓ: Machine learning in credit risk: measuring the dilemma between prediction and supervisory cost.
2033  PILAR GARCÍA-PEREA, AITOR LACUESTA and PAU ROLDAN-BLANCO: Raising Markups to Survive: Small Spanish Firms during the Great Recession.
2034  MÁXIMO CAMACHO, MATÍAS PACCE and GABRIEL PÉREZ-QUIRÓS: Spillover Effects in International Business Cycles.
2035  BRINDUSA ANGHEL, PILAR CUADRADO and FEDERICO TAGLIATI: Why cognitive test scores of Spanish adults are so low? The role of schooling and socioeconomic background
2036  ÁNGEL IVÁN MORENO and TERESA CAMINERO: Application of text mining to the analysis of climate-related disclosures.
2037  EFFROSYNI ADAMOPOULOU and ERNESTO VILLANUEVA: Wage determination and the bite of collective contracts in Italy and Spain: evidence from the metal working industry.
2038  MIKEL BEDAYO, GABRIEL JIMÉNEZ, JOSÉ-LUIS PEYDRÓ and RAQUEL VEGAS: Screening and Loan Origination Time: Lending Standards, Loan Defaults and Bank Failures.
2039  NEZIH GUNER, JAVIER LÓPEZ-SEGOVIA and ROBERTO RAMOS: Reforming the individual income tax in Spain.
2101  DARÍO SERRANO-PUENTE: Optimal progressivity of personal income tax: a general equilibrium evaluation for Spain.
2102  SANDRA GARCÍA-URIBE, HANNES MUELLER and CARLOS SANZ: Economic uncertainty and divisive politics: evidence from the Dos Españas.
2103  IVÁN KATARYNIUK, VÍCTOR MORA-BAJÉN and JAVIER J. PÉREZ: EMU deepening and sovereign debt spreads: using political space to achieve policy space.
2105  JENNIFER PEÑA and ELVIRA PRADES: Price setting in Chile: micro evidence from consumer on-line prices during the social outbreak and Covid-19.


EDUARDO GUTIÉRREZ and CÉSAR MARTÍN MACHUCA: The effect of tariffs on Spanish goods exports.

JACOPO TIMINI: Revisiting the ‘Cobden-Chevalier network’ trade and welfare effects.


RICARDO GIMENO, MARÍA BRU MUÑOZ: Decoupling? 

ALBERT BANAL-ESTAÑOL, ENRIQUE BENITO, DMITRY KHAMETSHIN, MATÍAS LAMAS, JUAN S. MORA-SANGUINETTI, ROBERTO PASCUAL: Adjustment costs, and capacity utilization.

DIEGO COMIN, JAVIER QUINTANA, TOM SCHMITZ: Uncertainty.

EDUARDO GUTIÉRREZ, AITOR LACUESTA: Reference cycle of the Spanish economy.

MÁXIMO CAMACHO, MARÍA DOLORES GADEA: Inflation forecasts?

MARTA BAŃBURA, DANILO LEIVA-LEÓN: Inequality: Evidence from Spanish administrative records.

ANTONELLA TRIGARI: The role of a green factor in stock prices. When fama & french go green.

ANTONIO VEALENNE, PEDRO SERRANO, ANTONI VAELLO-SEBASTIÀ: The impact of heterogeneous unconventional monetary policies on the expectations of market crashes.

MARÍA T. GONZÁLEZ-PÉREZ: Lessons from estimating the average option-implied volatility term structure for the Spanish banking sector.

SIMÓN A. RELLA, YULIYA A. KULIKOVA, EMMANOUIL T. DERMITZAKIS and FYODOR A. KONDRAKHOV: Rates of SARS-COV-2 transmission and vaccination impact the fate of vaccine-resistant strains.

MATÍAS LAMAS and DAVID MARTÍNEZ-MIERA: Sectoral holdings and stock prices: the household-bank nexus.

ALBERT BANAL-ESTANÇÓL, ENRIQUE BENITO, DMITRY KHANETSHIN and JIANXING WEI: Asset encumbrance and bank risk: theory and first evidence from public disclosures in Europe.

ISABEL ARGIMÓN and MARÍA RODRÍGUEZ-MORENO: Business complexity and geographic expansion in banking.

LUIS GUIROLA: Does political polarization affect economic expectations?: Evidence from three decades of cabinet shifts in Europe.

CHRISTIANE BAUMEISTER, DANILO LEIVA-LEÓN and ERIC SIMS: Tracking weekly state-level economic conditions.

SERGI BASCO, DAVID LÓPEZ-RODRÍGUEZ and ENRIQUE MORAL-BENITO: House prices and misallocation: The impact of the collateral channel on productivity.

MANUEL ARELLANO, STEPHANE BONHOMME, MICOLE DE VERA, LAURA HOSPIDO and SIQI WEI: Income risk inequality: Evidence from Spanish administrative records.

ANGELA ABBATE and DOMINIK THALER: Optimal monetary policy with the risk-taking channel.

MARTA BANÍBURA, DANILO LEIVA-LEÓN and JAN-OlIVER MENZ: Do inflation expectations improve model-based inflation forecasts?

MÁXIMO CAMACHO, MARÍA DOLORES GADEA and ANA GÓMEZ LOSCOS: An automatic algorithm to date the reference cycle of the Spanish economy.

EDUARDO GUTIÉRREZ, AITOR LACUESTA and CÉSAR MARTÍN MACHUCA: Brexit: Trade diversion due to trade policy uncertainty.

JULIO A. CREGO and JULIO GÁLVEZ: Cyclical dependence in market neutral hedge funds.

HERVE LE BIHAN, MAGALI MARX and JULIEN MATHERON: Inflation tolerance ranges in the new keynesian model.

DIEGO COMIN, JAVIER QUINTANA, TOM SCHMITZ and ANTONELLA TRIGARI: Measuring TFP: the role of profits, adjustment costs, and capacity utilization.


BEATRIZ GONZÁLEZ, GALO NUÑO, DOMINIK THALER and SILVIA ABRIZO: Firm heterogeneity, capital misallocation and optimal monetary policy.

RYAN BANERJEE and JOSÉ-MARÍA SERENA: Dampening the financial accelerator? Direct lenders and monetary policy.

JUAN S. MORA-SANGUINETTI and ISABEL SOLER: La regulación sectorial en España. Resultados cuantitativos.

JORGE E. GALÁN, MATÍAS LAMAS and RAQUEL VEGAS: Roots and recourse mortgages: handing back the keys.


CORNINNA GHIRELLI, DANILO LEIVA-LEÓN and ALBERTO URTASUN: Housing prices in Spain: convergence or decoupling?

MARÍA BRU MUÑOZ: Financial exclusion and sovereign default: The role of official lenders.

RICARDO GIMENO and CLARA I. GONZÁLEZ: The role of a green factor in stock prices. When fama & french go green.

CARLOS MONTES-GALDÓN and EVA ORTEGA: Skewed SVARs: tracking the structural sources of macroeconomic tail risks.