

**ADVERTISING, INNOVATION
AND ECONOMIC GROWTH**

2019

Laurent Cavenaile and Pau Roldan

**Documentos de Trabajo
N.º 1902**

BANCO DE ESPAÑA
Eurosistema



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Laurent Cavenaile ^(**)

UNIVERSITY OF TORONTO

Pau Roldan ^(***)

BANCO DE ESPAÑA

(*) We are indebted to Jess Benhabib for his invaluable advice and support, and to Gian Luca Clementi, Boyan Jovanovic, and Edouard Schaal for continued discussions and suggestions. For helpful comments, we thank Salomé Baslandze, Alberto Bisin, Jaroslav Borovička, Murat Celik, Diego Daruich, Tülin Erdem, Jeremy Greenwood, Seher Gupta, Siddharth Hari, Masakazu Ishihara, Olga Itenberg, Julian Kozlowski, Pedro Mendi, Simon Mongey, Ilari Paasivirta, Michael Peters, Xavier Ragot, Tom Schmitz and Gianluca Violante. We also received valuable feedback from seminar participants at Aarhus University, Atlanta Fed, Bank of Canada, Banco de España, KU Leuven, New York University, Rotman School of Management, Royal Holloway University of London, Southern Methodist University, Temple University, University of Melbourne, University of Montreal, UQAM, Western University, and from participants at the EEA Congress (Cologne), the Midwest Macroeconomics Meetings (Madison), the North American Summer Meeting of the Econometric Society (Philadelphia), SED (Toulouse), XIII REDg (Madrid), and the XXXI Jornadas de Economía Industrial (Palma de Mallorca). Any remaining errors are our own. The views expressed in this paper are those of the authors and do not necessarily coincide with those of the Banco de España or the Eurosystem.

(**) University of Toronto, 105 Saint George Street, Toronto, Ontario, M5S 3E6, Canada. Email: laurent.cavenaile@utoronto.ca

(***) Banco de España, Alcalá, 48, Madrid, 28014, Spain. Email: pau.roldan@bde.es

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ISSN: 1579-8666 (on line)

Abstract

This paper analyzes the implications of advertising for firm dynamics and economic growth through its interaction with R&D investment at the firm level. We develop a model of endogenous growth with firm heterogeneity that incorporates advertising decisions. We calibrate the model to match several empirical regularities across firm size using U.S. data. Through a novel interaction between R&D and advertising, our model provides microfoundations for the empirically observed negative relationship between both firm R&D intensity and growth, and firm size. Our model predicts substitutability between R&D and advertising at the firm level. Lower advertising costs are associated with lower R&D investment and slower economic growth. We provide empirical evidence supporting substitution between R&D and advertising using exogenous changes in the tax treatment of R&D expenditures across U.S. states. Finally, we find that R&D subsidies are more effective under an economy that includes advertising relative to one with no advertising.

Keywords: endogenous growth, advertising, innovation, research and development, firm dynamics, policy.

JEL classification: E20, L10, M30, O31, O32, O33, O41.

Resumen

Este artículo analiza las consecuencias que los gastos publicitarios tienen para la dinámica empresarial y el crecimiento económico, a través de la interacción entre la publicidad y los gastos en investigación y desarrollo (I+D) en las empresas. Presentamos un modelo de crecimiento económico endógeno con heterogeneidad entre empresas que incorpora decisiones sobre gastos en publicidad. Calibramos el modelo usando microdatos para Estados Unidos, con el fin de estudiar patrones empíricos existentes entre empresas de distinto tamaño. Mediante la interacción entre publicidad e I+D, nuestro modelo propone una nueva explicación para la correlación negativa observada en los datos entre la intensidad innovadora y el tamaño empresarial. Nuestro modelo predice que los gastos en publicidad y en I+D son sustitutos en las empresas. Reducir costes publicitarios conlleva una reducción de inversión en I+D, así como una ralentización del crecimiento económico. A continuación, presentamos la evidencia empírica que demuestra la existencia de dicha sustitución mediante el uso de variación regional exógena en la política impositiva estadounidense referente a gastos en I+D. Finalmente, demostramos que los subsidios a la innovación son más efectivos en una economía que permita a las empresas publicitar sus productos respecto a una en la que la publicidad no esté permitida.

Palabras clave: crecimiento endógeno, publicidad, innovación, investigación y desarrollo, dinámicas empresariales, políticas públicas.

Códigos JEL: E20, L10, M30, O31, O32, O33, O41.

1 Introduction

Since the seminal contributions of Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992), the economic literature has widely emphasized the role of innovation in the process of economic growth. Research and Development (R&D) introduces goods of higher quality and enhanced production technologies which raise living standards. From a firm's perspective, innovation is used strategically to increase profits: by performing R&D, firms can increase their market share by selling higher quality goods and diverting demand from lower-quality products. Yet, while this process and its implications for economic growth are relatively well understood in the literature, there exists a variety of other tools that firms may use to direct demand towards their products.¹ One non-negligible example is advertising.

By advertising their goods, firms can alter consumer preferences and ultimately increase product-specific profits. This suggests that innovation and advertising may be substitutable tools in firms' quest for higher profits. Additionally, advertising and innovation investments may also be complementary, as the former type of investment raises the return to innovation by increasing market shares of new products. In either view, advertising decisions are not neutral in terms of global economic growth. In fact, both these types of investment represent sizable expenditures in aggregate terms. Over the 1980-2013 period in the U.S., the share of R&D expenditures over GDP fluctuated between 2.27% and 2.82%.² Over the same time period, firms in the U.S. spent on average around 2.2% of GDP on advertising each year.³ Yet, the growth literature is relatively silent on the potential interaction between R&D and advertising expenditures and its potential impact on firm growth, firm dynamics and long run economic development.

In this paper, we ask how advertising affects R&D investment decisions at the firm level and study the implications for economic growth and firm dynamics. We build a flexible model that captures different dimensions of complementarity and substitutability between R&D and advertising. Our calibrated model shows that a decrease in the relative cost of advertising can have significant detrimental effects on long run economic growth as the substitution effect dominates. We provide supportive empirical evidence for substitution between R&D and advertising using U.S. firm-level data. We also find that the interaction between R&D and advertising can generate some observed empirical regularities related to firm dynamics, such as decreasing firm growth rates with firm size (a deviation from the so-called Gibrat's law) as well as decreasing R&D intensity with firm size.

Theoretically, we build on the Akcigit and Kerr (2018) model of endogenous growth through R&D, which we extend to incorporate explicit advertising decisions. Firms are heterogeneous and own different portfolios of goods that they monopolistically supply. Product

¹A recent trend in the macroeconomic literature has started investigating several types of intangible investment and their potential implications for the overall economy. See for instance, McGrattan and Prescott (2014), McGrattan (2017), Gourio and Rudanko (2014a), Hall (2014), Molinari and Turino (2015), Gourio and Rudanko (2014b), Arkolakis (2010, 2016) and Atkeson and Kehoe (2005).

²Source: OECD data, available at <http://data.oecd.org/rd/gross-domestic-spending-on-r-d.htm>.

³Source: Coen Structured Advertising Expenditure Dataset, extracted from the McCann Erikson advertising agency (available at <http://www.purplemotes.net/2008/09/14/us-advertising-expenditure-data/>).

quality grows on a ladder through innovation arising from investment in R&D, which can take two forms in our model. Through internal R&D, firms can increase the quality of their own goods. External R&D, on the other hand, enables incumbent firms and potential entrants to improve on the quality of a good that they do not own and displace the former producer through creative destruction. Besides R&D, firms can use advertising to expand their market shares and profit. Firms use advertising to influence the perception that consumers have of their goods, thereby altering preferences and ultimately shifting demand toward those products.

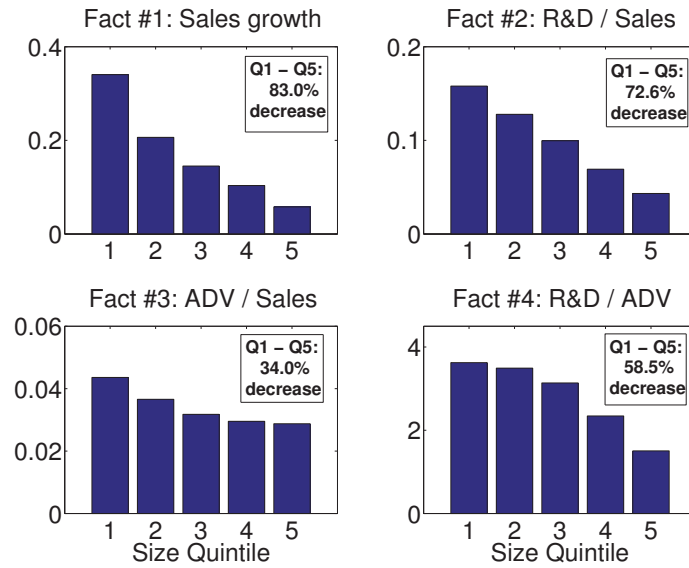
We calibrate the model via indirect inference using both aggregate and micro-level data. Our model is calibrated to match four empirical facts related to firm dynamics and investment across firm size. Figure 1 shows these four facts. The upper-left panel displays the average growth rate of publicly-listed firms in the U.S., by size quintiles.⁴ Growth rates are higher for smaller firms and are consistently decreasing with firm size. The upper-right panel shows that small firms also tend to be relatively more R&D intensive. This suggests that smaller firms experience higher growth through relatively higher R&D investments, a conclusion that has sometimes been interpreted in the existing literature as evidence of differences in R&D technology across firm size. This can have important implications in terms of policy recommendations (e.g. Akcigit (2009), Acemoglu *et al.* (2018) or Akcigit and Kerr (2018)). If small firms are more efficient at innovating, R&D subsidies targeted at small firms may be optimal. An important question is then to understand the source of the deviation from constant R&D intensity across firm size. In this context, our paper provides microfoundations for this deviation. We show that the observation that R&D intensity diminishes with firm size can arise as the result of the optimal allocation of resources between R&D and advertising at the firm level.

Our calibration is also disciplined by two new facts about advertising. First, we find a negative relationship between advertising intensity (i.e., advertising expenditures normalized by size) and firm size, which can be observed in the lower-left panel of Figure 1. Second, we find that larger firms rely relatively more on advertising compared to R&D, as the ratio of R&D to advertising expenditures decreases with firm size (lower-right panel in Figure 1).

Our calibration delivers two main implications regarding the interaction between R&D and advertising. First, our results show the existence of a substitution effect between R&D and advertising at the firm level. A decrease in the cost of advertising leads to an increase in the entry rate and in creative destruction. This, in turn, decreases the incentive for incumbent firms to invest in R&D and shifts the firm size distribution to the left (larger mass of small firms). Overall, investment in R&D decreases which leads to a decrease in economic growth. In particular, we predict that the growth rate of the economy would be 0.64 percentage points higher if advertising is shut down from the model. Second, we find that the interaction between R&D and advertising can quantitatively deliver the four facts reported in Figure 1, which suggests that the deviation from constant R&D intensity across firm size should not necessarily be interpreted as evidence for differences in R&D efficiency across firms.

⁴Data include U.S. listed companies performing R&D and advertising between 1980 and 2015 from the Compustat database. More details on the sample selection in Section 3.

Figure 1: Firm growth, R&D intensity, advertising intensity, and R&D-advertising ratio, across firm size quintiles.



Notes: Firms are ranked in size quintiles according to their normalized level of sales (sales as a ratio of average sales in the same year). R&D and advertising intensities are measured as the ratio of total R&D and advertising expenditures to total sales within each group.

The main new mechanisms in our model are based on well-established empirical observations from the marketing literature. Crucially, there is ample evidence in this literature that larger firms have a cost advantage in terms of advertising compared to firms with fewer products. One reason for this size advantage comes from a spillover effect through different goods under the same brand name (umbrella branding). By advertising one product, a firm can influence not only the perception of the quality of the advertised good but also of other goods sharing the same brand name. The existence of this spillover effect alters firms' dynamic incentives to engage in R&D. As smaller firms gain relatively more in terms of advertising spillover from acquiring an additional product, they optimally choose to perform relatively more (external) R&D in order to expand into new product markets. As a result, these firms grow relatively faster than large ones.

To validate our estimation, we show that the calibrated model also matches several untargeted moments among which the observed decrease in the variance of firm growth, as well as of R&D and advertising intensity, with firm size. Moreover, using exogenous variation in the cost of R&D arising from changes in the tax treatment of R&D expenditures across U.S. states and over time, we provide empirical evidence supporting the substitution between R&D and advertising which is at the core of the predictions of our calibrated model. Interestingly, we also find that R&D is complementary to other types of expenditures.

In the last part of the paper, we study the policy implications of our model. First, we show that taxes on advertising, while desirable from the angle of growth, might have a relatively small impact because (i) the elasticity of advertising to sales is low, and (ii) the tax deters the entry of small and innovative firms. Second, we show that identifying the source of the observed decrease in R&D intensity with firm size is relevant for R&D policy. R&D

subsidies are more effective at promoting economic growth if the deviation from constant R&D intensity comes from the interaction between R&D and advertising rather than from technological differences in terms of R&D efficiency across firm size (for example, the increase in growth is 0.12 percentage point higher in the former case for a 50% subsidy). We further show that the model with advertising performs better in terms of matching moments related to firm growth and R&D intensity across firm size.

The rest of the paper is organized as follows. Section 2 presents a comprehensive review of the literature with a special focus on endogenous growth, firm dynamics and the marketing literature used to motivate our advertising modeling approach. Section 3 presents key empirical facts related to Gibrat's law, R&D and advertising intensity at the firm level, as well as the relative use of R&D and advertising across firm size. We then describe the model in Section 4. Section 5 presents the calibration and shows that our novel advertising mechanism is able to quantitatively replicate the main empirical regularities described in Section 3. We also utilize the calibrated model to ask how advertising affects the firm size distribution, firm entry rates and economic growth. In Section 6 we present a set of validation exercises, including the model's performance in terms of untargeted moments (in Section 6.1), and empirical evidence supporting the main mechanism at work behind the calibrated model (in Section 6.2). Section 7 investigates policy implications regarding advertising taxes and R&D subsidies. We also compare the quantitative performance of our baseline model with a calibration without advertising in matching moments. Section 8 concludes. The Appendix includes proofs of all the results, model extensions mentioned in the main text, and additional tables and figures.

2 Related Literature

Our work is related to several strands of the literature. First and foremost, we build upon models of endogenous firm growth through product innovation. This area was pioneered by Klette and Kortum (2004), who built a stylized version of the Schumpeterian creative-destruction models of Grossman and Helpman (1991) and Aghion and Howitt (1992) into a model of multi-product firm dynamics. Their work is able to exhibit several patterns that are consistent with empirical finding coming from micro-level data, especially regarding the right-skewness in the firm size distribution, the persistence in firms' R&D investments, and the volatility of innovation in the cross-section of firms.⁵

However, by assuming that firm productivity scales perfectly with size, this framework fails to explain why firms of different sizes may grow at different rates. Indeed, empirical studies have argued that there exist deviations from Gibrat's law among innovative firms: small establishments grow faster than large ones, and net exit rates are a decreasing function of size.⁶

⁵The applied work by Lentz and Mortensen (2008) further unveiled the predictive power of the Klette and Kortum (2004) model by putting it to a rigorous empirical test.

⁶See Hall (1987), Sutton (1997), Caves (1998), and Geroski (1998). For a survey of this empirical literature, see Santarelli *et al.* (2006).

One contributor to such phenomenon is that smaller firms tend to be more innovation-intensive.⁷ Accordingly, a wave of second-generation models of innovation-driven firm growth has emerged (Acemoglu *et al.* (2018), Acemoglu and Cao (2015) and Akcigit and Kerr (2018), among others). These papers extend the Klette and Kortum (2004) framework to include heterogeneity in innovation technologies in order to incorporate a more meaningful interaction between different types of research (e.g., product versus process, radical versus incremental), as well as between entrants and incumbent firms. These models are able to replicate the observed size-dependence of growth and R&D through size-dependent R&D technological efficiency.⁸ In contrast, we propose a microfounded theory that is able to deliver size-dependent growth and R&D intensity even in the absence of size-dependent R&D technologies. Our novel trade-off between innovation and advertising decisions introduces the required non-homogeneities in equilibrium. Specifically, we rely on two main strands of the marketing literature in order to build a motive for advertising into our endogenous growth framework.

First, in our model, larger firms experience higher returns to advertising expenditures. A large body of the marketing literature identifies this phenomenon as the equity value of so-called “umbrella-branding”. These studies suggest that there exist spillovers between goods within the firm, in the sense that increasing advertising expenditures on one good not only increases sales for that good but also indirectly for all other goods under the same brand. Lane and Jacobson (1995) and Tauber (1981, 1988) argue that brand developments can decrease marketing costs, and Rangaswamy *et al.* (1993) suggest that they can enhance marketing productivity. Moreover, Smith and Park (1992) show that they can help capture greater market share. Morrin (1999) provides evidence that exposure to advertising of brand extension facilitates parent brand recall. Using household scanner panel data for the U.S., Balachander and Ghose (2003) find a positive and significant spillover effect of advertising for multiple product categories and geographic markets. Dacin and Smith (1994) show, by means of controlled experiments, a positive relationship between the number of products affiliated with a brand and consumers’ confidence in the quality extension of the brand. Erdem (1998) argues empirically that the quality perceptions of a brand in a product category are positively affected by the consumer’s experience with the same brand in a different category, because branding allows consumers to learn faster about quality through use experience. More specifically for our purposes, Erdem and Sun (2002) show that the effects identified in Erdem (1998) further translate to positive spillovers between advertising and sales for different goods within the same brand. Particularly, such effects are present in innovation-driven industries, e.g. the car industry (Sullivan (1990) and Büschken (2007)) or the pharmaceutical industry (see Suppliet (2016)).

The second observation that we draw from the marketing literature is that higher-quality goods benefit relatively more from advertising: everything else equal, a good that compares better to other goods within the brand in terms of quality experiences higher increases in

⁷Cohen and Klepper (1996) show that small innovating firms generate more innovations per dollar spent in R&D, and Akcigit and Kerr (2018) show that small firms spend more in R&D per dollar of sales.

⁸For instance, weak scalability in the innovation technology (Akcigit and Kerr (2018)); larger step sizes of technical advance for small firms (Acemoglu and Cao (2015)); or lower productive capacity as a highly absorbing state (Acemoglu *et al.* (2018)).

sales per dollar spent into advertising. Archibald *et al.* (1983) and Caves and Greene (1996) find a positive correlation between advertising and quality for innovative goods. Tellis and Fornell (1988) examine the relationship over the product life cycle, and find that advertising and profitability are both positively correlated to quality. Finally, Homer (1995), Kirmani and Wright (1989) and Kirmani (1990, 1997) offer experimental evidence suggesting that the positive relationship exists because consumers perceive higher advertising expenditures indicating that the good is of high quality.⁹

Regarding advertising in Economics, our paper relates to a long tradition studying advertising as an explicit factor affecting consumer tastes, as in Dorfman and Steiner (1954), Dixit and Norman (1978), Becker and Murphy (1993) and Benhabib and Bisin (2002). Our approach is similar to those in that firms' advertising effectively acts as a demand shifter in equilibrium. A parallel body of literature introduces a role for valuable customer capital into macro models, viewing marketing as a tool to build continuing buyer-seller relationships for firms due to either the existence of frictions in product markets (e.g. Gourio and Rudanko (2014b), Hall (2008), Roldan and Gilbukh (2018)) or because of costs to market penetration (e.g. Arkolakis (2010, 2016) and Eaton *et al.* (2014)). Dinlersoz and Yorukoglu (2012) and Perla (2017) view advertising as a way of signalling firm efficiency and raising product awareness among customers, respectively, and focus on the implications for industry dynamics and the degree of market competition. Fitzgerald and Priolo (2018) find empirical support for the idea that marketing and advertising activities are important for firm growth. Fishman and Rob (2003) study the implications of customer accumulation for firm size and R&D choices, but they also do not focus on how marketing and innovation interact. To our knowledge, only Grossmann (2008) has considered this potential interaction in a general equilibrium model. However, while in that paper advertising and R&D are assumed to be direct complements, we opt to be agnostic about the degree of complementarity between the two, and instead discipline their substitutability by matching directly firm-level cross-sectional data. In addition, we incorporate firm heterogeneity which allows us to study the implications of advertising for firm dynamics. Moreover, our policy analysis puts a special emphasis on how advertising alters both the R&D-vs-advertising composition of firm investment on the intensive margin, and the size distribution of firms on the extensive margin.

3 Empirical Findings

In this section, we present empirical results on firm growth, R&D and advertising expenditures. Some of the coefficients that we obtain will then be used to calibrate the model that is presented in Section 4 where we show that these facts can be explained as the result of an optimal allocation of resources between R&D and advertising at the firm level.

Data We use annual data on U.S. listed companies from Compustat over the period 1980-2015. Firms reporting nonpositive sales or nonpositive employment are excluded from the sample. We also exclude non-innovative (i.e., no R&D expenditures) and non-advertising (i.e.,

⁹See Kirmani and Rao (2000) and Bagwell (2007) for a survey of the literature.

no advertising expenditures) firms. To exclude outliers, we ignore firms experiencing year-on-year sales growth rates of more than 1,000%, as well as R&D-to-sales and advertising-to-sales ratios of more than 100%.¹⁰ Moreover, we exclude mergers and acquisitions, and in order to correct for a possible survival bias in the growth regressions, we make the conservative assumption that exiting firms have a growth rate of -100% in their last period of operation. Basic descriptive statistics of the resulting sample are reported in Table A.1 of Appendix D.

Empirical Facts and Relation to the Literature Our main focus is on the regression results for the four empirical facts that we presented in Figure 1. We use firm sales as our baseline measure of firm size.¹¹ All four regressions are of the form:

$$y_{ij,t} = \alpha_0 + \beta_1 \log(\text{Sales}_{ij,t}) + \beta_2 \text{FirmAge}_{ij,t} + \beta_3 \text{FinConst}_{ij,t} + \alpha_j + \alpha_t + u_{ij,t}$$

for firm i in industry j in year t , where α_t controls for time fixed effects and α_j controls for industry fixed effects. The dependent variable is:

$$y_{ij,t} \in \left\{ \frac{\Delta \text{Sales}_{ij,t}}{\text{Sales}_{ij,t}}, \log\left(\frac{\text{R\&D}_{ij,t}}{\text{Sales}_{ij,t}}\right), \log\left(\frac{\text{Adv}_{ij,t}}{\text{Sales}_{ij,t}}\right), \log\left(\frac{\text{R\&D}_{ij,t}}{\text{Adv}_{ij,t}}\right) \right\}$$

for each of the four facts, respectively. To control for firm characteristics, we include firm age, which we measure as the number of years elapsed since the firm first appeared in the sample,¹² and a measure of financial constraints, which is the difference between sales and purchases of common and preferred stock as a share of firm sales.

In the results that follow, Facts #1 and #2 (related to firm growth and R&D intensity, respectively) confirm results that have already been discussed in the literature. We then introduce two new facts that relate to advertising and R&D (Facts #3 and #4). These new facts highlight that firms of different sizes have different advertising intensities and that they use R&D and advertising in different proportions.

Fact #1: *Smaller innovative firms grow faster on average.*

Column (1) in Table 1 shows that there exists a significant deviation from Gibrat's law among innovative firms in our sample. The results show that larger innovative firms tend to experience lower growth rates in terms of sales. A 1% increase in sales translates into a 0.0325% decrease in sale growth.

These results provide evidence that is similar to that reported in Akcigit (2009) and Akcigit and Kerr (2018).¹³ The phenomenon has also been studied in the literature within different theoretical frameworks. Models of firm dynamics following Hopenhayn (1992), based

¹⁰Using different thresholds does not change the sign nor the significance of the coefficients.

¹¹Table A.2 shows similar results for all four facts when size is measured by assets and employment.

¹²This measure of firm age in the Compustat database is standard and has been used among others by Shumway (2001), Lubos and Veronesi (2003) and Fama and French (2004).

¹³Haltiwanger *et al.* (2013) argue, using data about the whole universe of firms in the U.S., that the observed deviation from Gibrat's law does not systematically hold when one controls for firm age. In column (2) of Table 1, we find that the significant deviation from Gibrat's law still holds after controlling for firm age in our sample of innovative firms.

Table 1: Firm level regressions (Fact #1 - #4).

	(1)	(2)	(3)	(4)
	$\frac{\Delta Sales}{Sales}$	$\log\left(\frac{R\&D}{Sales}\right)$	$\log\left(\frac{Adv}{Sales}\right)$	$\log\left(\frac{R\&D}{Adv}\right)$
$\log(Sales)$	-0.0325 (0.00288)	-0.1035 (0.00892)	-0.0317 (0.01000)	-0.0719 (0.0120)
<i>Firm Age</i>	-0.00441 (0.000367)	0.00296 (0.00161)	0.000688 (0.00190)	0.00227 (0.00219)
<i>Fin. Const.</i>	0.00270 (0.00172)	0.00538 (0.00304)	0.00745 (0.00432)	-0.00207 (0.00498)
Time FE	✓	✓	✓	✓
Industry FE	✓	✓	✓	✓
Observations	24856	24856	24856	24856
R^2	0.09	0.50	0.28	0.28

Notes: Compustat data (1980-2015). The sample is restricted to firms reporting strictly positive sales, strictly positive employment, strictly positive R&D and advertising expenditures, with year-on-year sales growth less than 1,000%, and R&D-to-sales and advertising-to-sales ratios of less than 100%. Mergers and acquisitions have been excluded from the sample. Age is measured as the elapsed time since the first observation in the data. Our measure of financial constraints is sale minus purchases of common and preferred stock, divided by sales. Sales and advertising expenditures are in thousands of U.S.D. Standard errors are clustered by firm (in parentheses).

on stochastic (Markov) productivity shocks, can replicate the negative correlation between firm size and firm growth. If the set of productivity shocks is finite and mean-reverting, larger firms (i.e., firms with higher productivity) eventually face a limit to growth and experience lower average growth rates. Jovanovic (1982), in contrast, proposes a model of learning in which firms receive noisy signals about their productivity. In his model, younger firms are more uncertain about their productivity and hence learn more than older firms, leading to higher growth rates. The correlation between firm size and firm growth in such a setting thus comes through an age effect and the positive correlation between age and size. Consequently, in this type of models, the observed association between firm size and firm growth should vanish as one controls for firm age.¹⁴ Yet, we find that the size effect is still significant after controlling for age.

Cooley and Quadrini (2001) link the dependence of firm growth on size and age to financial market frictions. In a model of firm dynamics similar to that of Hopenhayn (1992), Cooley and Quadrini (2001) show that the interaction of persistent productivity shocks with costly equity issuance and default generates a higher financial leverage for small and young firms and qualitatively replicates the decreasing relationship between firm size, firm age and expected growth.¹⁵ In column (1) of Table 1, we find that the deviation from constant growth holds after controlling for financial constraints.¹⁶

¹⁴Clementi and Palazzo (2016) propose an extension of the Hopenhayn (1992) model with capital accumulation in which, conditional on size, growth rates are on average decreasing in age.

¹⁵See also Clementi and Hopenhayn (2006). Itenberg (2015) links the high innovation intensity of small firms to their extensive use of external equity financing. For a related approach, see Schmitz (2017).

¹⁶This result is consistent with the findings of Dinlersoz *et al.* (2018) for their sub-sample of public firms.

Fact #2: *Smaller innovative firms have higher R&D intensity on average.*

An emerging trend in the economic growth literature has investigated the departure from constant growth across firm size by linking it to another cross-sectional empirical fact related to firm size: the higher R&D intensity of smaller firms (see, for example, Akcigit (2009), Acemoglu and Cao (2015), and Akcigit and Kerr (2018)). As larger firms invest relatively less in innovation, they experience relatively lower growth rates.¹⁷

Column (2) in Table 1 shows that this pattern holds in our sample as well: larger firms are less R&D intensive. The effect of age on R&D intensity is positive though not strongly significant. A 1% increase in sales translates into a 0.1035% decrease in the R&D expenditure to sales ratio.

In sum, smaller innovative firms grow relatively faster, and these firms spend relatively more in R&D. While we are not the first ones to make these observations using Compustat data (see, for instance, Akcigit (2009) and Itenberg (2015)), we show in addition that size remains significant after controlling for age and financial constraint both in growth and R&D intensity regressions, which suggests that explanations based on the firm's product life-cycle and financial constraints cannot entirely account for the deviation from Gibrat's law observed in the data. We propose an alternative explanation for this deviation based on the interaction between R&D and advertising in firms' optimal decisions. For this, we present two new facts which are used to calibrate our model.

Fact #3: *Smaller innovative firms have higher advertising intensity on average.*

Column (3) in Table 1 shows that advertising intensity is decreasing in firm size: smaller firms spend more in advertising per dollar of sales. Once again, this correlation remains significant after controlling for age and financial constraints. We find that a 1% increase in sales translates into a 0.0317% decrease in the advertising expenditure to sales ratio.

Fact #4: *Smaller innovative firms have higher R&D to advertising ratios on average.*

Finally, in column (4) of Table 1, we find that larger firms tend to use relatively more advertising than R&D, as the ratio of R&D to advertising expenditure is significantly decreasing in size. In particular, a 1% increase in sales leads to a 0.0719% decrease in the R&D to advertising expenditure ratio. This suggests that as innovative firms grow larger, they tend to substitute R&D for advertising. We also find that the effect of age on the relative use of R&D and advertising is not robustly significant.

Overall, these results suggest that the interaction between R&D and advertising can potentially affect firm growth through a size effect, consistent with the advertising spillover that our model captures. In addition, our calibrated model will deliver all four of these predictions in equilibrium.

¹⁷Acemoglu *et al.* (2018) also propose a model in which older and larger firms invest relatively less in R&D and grow more slowly as a result, while Acemoglu *et al.* (2014) link innovation to the age of managers.

4 An Endogenous Growth Model with Advertising

Inspired by the empirical findings of the previous section, we build a theory of advertising into a model of endogenous growth. The basic structure follows the framework set up by Klette and Kortum (2004), and specifically allows for heterogeneous innovations as in the recent work by Akcigit and Kerr (2018). The advertising theory, on the other hand, is based on empirical regularities that we take from the marketing literature. Our broader goal is to illustrate a novel interplay between innovation and advertising, which critically shapes firms' dynamic incentives for growth and growth in the economy as a whole.

4.1 Environment

Preferences Time is continuous, infinite, and indexed by $t \in \mathbb{R}_+$. The economy is populated by a measure-one of identical, infinitely-lived individuals with discount rate $\rho > 0$. A representative household has preferences given by:

$$U = \int_0^{+\infty} e^{-\rho t} \ln(C_t) dt \quad (1)$$

where C_t is consumption of the single final good, whose price is normalized to $P_t = 1, \forall t$. The household is endowed with one unit of time every instant, which is supplied inelastically to the productive sector of the economy in the form of labor. The wage rate is denoted by w_t and is determined endogenously to clear the labor market. The household owns all the firms in the economy and carries each period a stock of wealth A_t , equal to the total value of corporate assets. Wealth earns an instantaneous and time-varying rate of return r_t . The flow budget constraint is $\dot{A}_t = r_t A_t + w_t - C_t$ for a given $A_0 \geq 0$.

Final Good Sector The final good is produced by a representative final good firm using a measure-one continuum of input varieties, indexed by $j \in [0, 1]$, which are supplied by an intermediate good sector. Technology is given by the Cobb-Douglas production function:

$$Y_t = \frac{1}{1-\beta} \int_0^1 \tilde{q}_{jt}^\beta y_{jt}^{1-\beta} dj$$

where $\beta \in (0, 1)$. y_{jt} is the quantity of intermediate good j that is used at time t . Quantities are weighted by the term \tilde{q}_{jt} , which stands for the *quality* of the good that is being *perceived* by agents in the economy.

Product Qualities The total perceived quality \tilde{q}_{jt} of a certain good j at time t is defined by:

$$\tilde{q}_{jt} \equiv q_{jt}(1 + d_{jt})$$

Total perceived quality includes two components. The first component, referred to as the *intrinsic* quality of the product and denoted by $q_{jt} > 0$, stands for the currently leading-edge technological efficiency of the product. This component of quality is built over time through

a process of technical innovation. In particular, it advances on a ladder, as in the models of Grossman and Helpman (1991) and Aghion and Howitt (1992). This phenomenon is induced by expenditures into R&D at the productive sector level. The second component in total perceived quality is the so-called *extrinsic* quality of the product, given by $\phi_{jt} \equiv q_{jt}d_{jt}$, where $d_{jt} \geq 0$ denotes advertising-induced quality.¹⁸ This component of total quality refers to the part of total perceived quality of a good that is induced by the producer’s advertising efforts on that specific product. Therefore, we may also refer to ϕ_{jt} as the *effectiveness of advertising* product j in period t .

This functional form implies that intrinsic (q_j) and extrinsic (ϕ_j) quality are substitutes at the good level, as they enter additively into total quality. However, note that we also allow for a degree of complementarity coming from the fact that ϕ_{jt} is itself an increasing function of intrinsic quality, q_j . Namely, advertising is effective in raising perceived quality only if the good has nonzero intrinsic quality, and it is more effective the higher the intrinsic quality of the good. This dual substitution and complementarity is consistent with the marketing literature reviewed in Section 2. It will also allow us flexibility when calibrating the model, as in equilibrium the two technologies may be substitutes or complements at the firm level, depending on parameter values.¹⁹

In our model, advertising acts as a preference shifter. This follows Dixit and Norman (1978), Becker and Murphy (1993), Benhabib and Bisin (2002) and Molinari and Turino (2015), among others, who model advertising through product-specific taste parameters or as an explicit argument in the utility function.²⁰ In Appendix G.2, we present various alternative ways to model advertising and show that, in all cases, advertising appears as a demand shifter.

We further adopt the view that advertising is *persuasive*, and not purely *informative*. By this we mean that consumers cannot choose what information to be exposed to, but rather behave according to the shifts in tastes induced by advertising, which they take as given.²¹ Yet, as we show in Appendix G.2.4, a version of our model in which advertising is informative about the product’s quality would also feature the type of demand shift in equilibrium that our baseline specification exhibits.

Finally, although we model advertising as a static component of firm profits, its choice embodies a general-equilibrium effect that critically changes the dynamic incentives of acquiring new product lines through R&D, and it thus has an indirect effect on firm and economic growth. For completeness, in Appendix G.2.1, we introduce an extension with accumulation of a stock of advertising (or so-called “goodwill” accumulation) over time, and show that this would not affect the qualitative results that we obtain in our baseline model.

Production Sector At any instant, there is an endogenously determined set \mathcal{F} (of measure $F > 0$) of active incumbent producers operating in a monopolistically competitive

¹⁸We borrow the nomenclature “intrinsic”, “extrinsic” and “perceived” from the marketing literature (e.g., Zeithaml (1988)).

¹⁹See the comparative statics in Appendix F.

²⁰In the baseline model, advertising appears in the final good sector’s production function. In Appendix G.2.2, we show that a model in which advertising features in the utility function is isomorphic to our baseline model.

²¹For an extensive discussion of the two views on advertising, see Becker and Murphy (1993).

product market. Each good $j \in [0, 1]$ is produced by a single firm $f \in \mathcal{F}$, and a single firm may own multiple goods. A firm owns a product if it can produce it at a higher intrinsic quality than any other firm. A firm f is summarized by the countable set of products for which it has the leading-edge technology, denoted $\mathcal{J}_f \subseteq [0, 1]$. The number of active product lines owned by firm f is $n \equiv |\mathcal{J}_f| \in \mathbb{Z}_+$. Finally, the product portfolio quality of firm f is given by $\mathbf{q}_f \equiv \{q_j : j \in \mathcal{J}_f\} \in \mathbb{R}_+^n$.

Each good variety j is produced with linear technology, $y_{jt} = \bar{Q}_t l_{jt}$, where l_{jt} is labor input and $\bar{Q}_t \equiv \int_0^1 q_{jt} dj$ is the average intrinsic quality in the economy. This linear formulation implies that all good producers face the same marginal cost, w_t/\bar{Q}_t . Moreover, by making production scale with aggregate quality, we can ensure that output grows at the same rate as productivity, which is necessary for the existence of a balanced growth path equilibrium.

4.2 Quality Improvements

Even though R&D-induced intrinsic quality innovations in the production sector are the only engine of long run growth in the economy, firms can choose to advertise on their currently owned product lines in order to increase demand through the extrinsic quality margin. In this section, we introduce both of these quality enhancement margins.

Improving Intrinsic Quality (q) Firms can invest in R&D to either improve their own product lines (so-called *internal* innovation) or to build upon the intrinsic quality of goods that are currently produced by other incumbent monopolists (so-called *external* innovation). Firms expand or contract in the product space on the basis of these innovations.²²

Particularly, to create a Poisson flow rate $z_j \geq 0$ of improving the intrinsic quality of its own product $j \in \mathcal{J}_f$, a firm f must spend a cost of $R_z(z_j)$ units of the final good. This cost function is assumed to be convex in the innovation rate and linear in quality, so that

$$R_z(z_j) = \hat{\chi} q_j z_j^{\hat{\psi}}$$

where $\hat{\chi} > 0$ and $\hat{\psi} > 1$. If successful, internal R&D improves intrinsic quality by a factor of $(1 + \lambda^I) > 1$, so that $q_{j,t+\Delta t} = (1 + \lambda^I)q_{jt}$ if there is only one innovation within an arbitrarily small time interval of size $\Delta t > 0$. The fact that R_z increases with q_j captures that more advanced technologies face higher R&D costs.

External R&D is undertaken by incumbents and potential entrants. For simplicity, external R&D is assumed to be undirected, so that the successful innovator uniformly draws a good from the set $[0, 1]$.²³ Incumbents and entrants face, however, different innovation technologies.

²²Garcia-Macia *et al.* (2016) show that both internal and external innovations by incumbent firms contribute significantly to aggregate growth. In our model, advertising affects the firm R&D-size relationship through external innovation intensity only. Our calibration delivers a share of growth coming from internal innovation that is in line with Garcia-Macia *et al.* (2016).

²³By the law of large numbers, this draws a good that is almost-surely not in the innovator's current portfolio of products.

On the one hand, entrants (firms with $n = 0$) must incur an expenditure of

$$R_e(x_e) = \nu \bar{Q} x_e$$

units of the final good in order to generate a Poisson flow rate of x_e for acquiring their first product, where $\nu > 0$ is a constant parameter. We assume that there is a measure-one mass of potential entrants, and determine x_e by a free-entry condition.²⁴

On the other hand, in order to create a flow Poisson rate of X_n , an incumbent firm with $n \geq 1$ product lines must incur an expenditure of $R_x(X_n, n)$ units of the final good. In particular, the cost function is assumed to be convex in the rate of innovation, X_n , such that

$$R_x(X_n, n) = \tilde{\chi} \bar{Q} X_n^{\tilde{\psi}} n^\sigma$$

where $\tilde{\chi} > 0$, $\tilde{\psi} > 1$, and $\sigma \leq 0$. If successful, external R&D improves intrinsic quality by a factor of $(1 + \lambda^E) > 1$, so that $q_{j,t+\Delta t} = (1 + \lambda^E)q_{jt}$ for a randomly selected product line $j \in [0, 1]$, if there is only one innovation within an arbitrarily small time interval of size $\Delta t > 0$.

We define total R&D expenditure by a firm of size n choosing innovation rates $(\{z_j\}, X_n)$ by:

$$R_n \equiv \sum_{j \in \mathcal{J}} R_z(z_j) + R_x(X_n, n) \quad (2)$$

Slightly abusing terminology, we say that there exist decreasing, constant, or increasing returns to scale in R&D if an n -product firms finds it respectively more expensive, as expensive, or less expensive to grow through innovation by a given rate than n firms of one product each.

On the one hand, as we shall see, the specification for R_z ensures that $z_j = z, \forall j$, because both costs and benefits of internal innovations are linear in quality. This is equivalent as internal R&D scaling proportionally with firm size, which we label as constant returns to scale. Therefore, the degree of returns to scale in the R&D technology operates solely through external innovations.

Absent our advertising margin, it is this degree of scalability which determines whether the model can deliver the empirically observed deviations from Gibrat's law. With constant returns to scale in innovation costs (i.e., $\tilde{\psi} + \sigma = 1$), ex-ante technological advantages do not exist with respect to differences in firm size. In other words, in a model without advertising, external innovation investment would in this case scale up one-for-one with added product lines. This naturally delivers a theoretical version of Gibrat's law, as in the model of Klette and Kortum (2004): firm growth is independent of firm size, n .

Because this specialization of the model is ill-suited to analyze why smaller firms are more innovation-intensive in the data, subsequent work has relaxed the assumption of homogeneity of degree one on the returns to innovation. Akcigit and Kerr (2018) extend the Klette and Kortum (2004) framework to incorporate decreasing returns to external R&D (i.e., $\tilde{\psi} + \sigma > 1$).

²⁴In equilibrium, x_e denotes both the optimal Poisson rate of entry and the mass of entrants.

In contrast, we remain agnostic regarding the degree of returns to scale of R&D. In our model with advertising, we are able to replicate empirical patterns in the data (e.g., deviations from Gibrat's law, and decreasing R&D and advertising intensities across firm sizes) when there exist constant, and even increasing, returns to scale in R&D. Our formulation of advertising, which relies on empirical evidence from the marketing literature, delivers as an equilibrium outcome that smaller firms are more concerned with innovation even when technological advantages in innovation might not be particularly tailored toward them. This is because those firms are the ones that marginally benefit the most in terms of increased advertising efficiency from an increase in the size of their portfolio of products. This is a novel explanation that highlights the potential effects of firms' advertising on firm growth, and it can have significant implications in terms of aggregate growth as well as policy as discussed in Section 7.²⁵

Improving Extrinsic Quality (ϕ) Besides building upon the intrinsic quality component of their products, firms can undertake advertising expenditures in order to enhance quality on the extrinsic dimension. Specifically, for a firm of size $n \geq 1$ with a portfolio of intrinsic qualities \mathbf{q} , we assume the following function for advertising effectiveness on good $j \in \mathcal{J}_f$:

$$\phi_j = \tilde{\theta}_j m_j^\zeta n^\eta \quad (3)$$

where m_j is the expenditure into advertising good j , with $(\zeta, \eta) \in \mathbb{R}_+^2$, and $\tilde{\theta}_j$ is a good- and firm-specific efficiency component, defined by $\tilde{\theta}_j \equiv \theta \frac{q_j}{Q_{\mathbf{q}}} \bar{Q}^{1-\zeta}$. The object $Q_{\mathbf{q}} \equiv \left(\sum_{q_{j'} \in \mathbf{q}} q_{j'}^{1/\alpha} \right)^\alpha$ is a within-firm measure of aggregate intrinsic quality, and $\theta \geq 0$ is the component of advertising efficiency that is constant across time, goods and firms.

Equation (3) embodies some of the main effects of advertising identified by the marketing literature reviewed in Section 2 that are necessary within our framework to confirm the four main empirical facts listed in Section 3.

First, everything else equal, the return to advertising a good j is increasing in advertising expenditures on that same good (m_j), where the parameter $\zeta > 0$ controls the elasticity of advertising returns to advertising expenditure. To discipline this parameter, we rely on ample evidence in Marketing establishing diminishing returns to advertising expenditures (Simon and Arndt (1980), Sutton (1991), Jones (1995) and Bagwell (2007)), and thus assume that $\zeta < 1$. This concavity assumption in the advertising function is also standard in economic models with marketing, e.g. Stigler (1961), Arkolakis (2010) and Dinlersoz and Yorukoglu (2012).²⁶

Second, the marketing return increases in the object $q_j/Q_{\mathbf{q}}$, which is a measure of the relative quality of the good within the firm. That is, advertising is more effective for goods

²⁵In Appendix G.1, we extend our baseline model with a theory of patent citations following Akcigit and Kerr (2018). Our model can deliver two other empirical regularities i.e. smaller firms tend to produce relatively more patents and those patents tend to be of higher quality.

²⁶In addition, effectiveness is also concave in aggregate quality \bar{Q} to ensure the existence of a balanced growth path.

that are intrinsically of relatively higher quality compared to other products in the firm. This implies that, in equilibrium, firms spend relatively more on advertising higher intrinsic quality products, a result that has been identified empirically by the marketing literature (e.g. Archibald *et al.* (1983), Caves and Greene (1996), Marquardt and McGann (1975), Rotfeld and Rotzoll (1976), Bagwell (2007) and Kirmani and Rao (2000)).

Third, and critically for our mechanism, larger firms have an absolute marketing advantage over smaller firms in that, everything else equal, the return to advertising is greater for higher n ($\eta > 0$). We interpret this as there being spillover effects within the firm. This effect is meant to capture the value of firm branding, as it is reminiscent of the “umbrella branding” effect discussed in the empirical marketing literature (e.g. Lane and Jacobson (1995), Tauber (1981, 1988), Rangaswamy *et al.* (1993), Smith and Park (1992), Balachander and Ghose (2003), Erdem (1998) and Erdem and Sun (2002)).

4.3 Entry and Exit, Pricing and Resource Constraints

There is no exogenous exit, and firms move endogenously on the size distribution in a step-wise fashion via the external innovation margin. Creative destruction occurs whenever a good is taken over by a successful external innovator, and a firm exits when it loses its last remaining product.

We assume that a firm produces a good if it possesses a technological leadership on it. The last innovator in each product line then owns the leading patent and has monopolistic power until it is displaced by another firm. Once a firm makes an innovation, it acquires a perpetual patent on it. However, this patent does not preclude other firms from investing into research to improve the intrinsic quality of the product. As it is standard in the literature, we assume that the new innovator is able to price the old incumbent out of the market and charge the unconstrained monopoly price.

The economy is closed so GDP equals aggregate consumption plus aggregate investment. The latter is split between aggregate R&D expenditure by entrants and incumbents, denoted Z_t , and aggregate advertising expenditure by incumbents, denoted M_t . The resource constraint at time t is

$$C_t + Z_t + M_t \leq Y_t \tag{4}$$

Finally, the labor market must clear. Labor demand comes from intermediate goods producers only, and therefore feasibility requires that $\int_0^1 l_{jt} dj \leq 1$ for all $t \in \mathbb{R}_+$.

4.4 Equilibrium

In this section we derive the Markov Perfect Equilibrium of the economy. Later on, we specialize the equilibrium to a balanced-growth path (BGP) in which all aggregates grow at a constant and positive rate, g .

Consumer’s Problem Taking initial wealth A_0 as given, the representative consumer chooses a path for consumption to maximize utility subject to the flow budget constraint and

a no-Ponzi condition, $\lim_{t \rightarrow +\infty} e^{-\int_0^t r_s ds} A_t \geq 0$. The optimality condition yields the standard Euler equation for consumption:

$$\frac{\dot{C}_t}{C_t} = r_t - \rho \quad (5)$$

Intermediate-good firms are owned by the household, so the value of household wealth is equal to the value of corporate assets. Namely, $A_t = \int_{\mathcal{F}} V(\mathbf{q}_f) df$, where $V(\mathbf{q}_f)$ denotes the net present value of the whole future expected stream of profits for a monopolist f who owns the (intrinsic) quality portfolio \mathbf{q}_f at time t . In equilibrium, the following transversality condition holds:

$$\lim_{t \rightarrow +\infty} \left[e^{-\int_0^t r_s ds} \int_{\mathcal{F}} V_t(\mathbf{q}_f) df \right] = 0 \quad (6)$$

Final Good Firm's Problem The representative final good producer takes qualities $\{\tilde{q}_j : j \in [0, 1]\}$ and input prices $\{p_j : j \in [0, 1]\}$ as given and chooses intermediate goods $\{y_{jt} : j \in [0, 1]\}$ to maximize profits. This leads to:

$$p_{jt} = \left(\frac{\tilde{q}_{jt}}{y_{jt}} \right)^\beta \quad (7)$$

Recalling that $\tilde{q}_{jt} = q_{jt}(1 + d_{jt})$, Equation (7) says that the inverse demand function for goods is iso-elastic, and β is the price-elasticity. Importantly, this demand function makes apparent that advertising, by increasing total quality ($\tilde{q}_{jt} = q_{jt} + \phi_{jt}$) through its extrinsic component ($\phi_{jt} = q_{jt}d_{jt}$), effectively impacts consumer preferences, and works as a demand shifter that alters consumption decisions.²⁷

Incumbent Firm's Problem A monopolist chooses labor, quantities, prices, R&D and advertising expenditures over each good in its portfolio in order to maximize the present discounted value of the total future stream of profits. Our set-up allows us to break this problem into a static part, in which the firm sets price, quantity and advertising expenditures over the goods that it currently owns, and a dynamic part, in which R&D decisions are made.

Before the R&D and advertising choices, the static maximization problem of monopolist $j \in [0, 1]$ holding the patent for the leading-edge intrinsic quality of good j is:

$$\pi(\tilde{q}_{jt}) = \max_{y_{jt}} \left\{ p_{jt} y_{jt} - w_t l_{jt} \right\} \quad \text{s.t. } y_{jt} = \bar{Q}_t l_{jt} \quad \text{and} \quad p_{jt} = \tilde{q}_{jt}^\beta y_{jt}^{-\beta}$$

where π stands for operating profits before advertising and R&D expenditures. The optimality condition implies $p_{jt} = \left(\frac{1}{1-\beta} \right) \frac{w_t}{\bar{Q}_t}$, meaning that all monopolists set the same price every period, or $p_{jt} = p_t, \forall j$. This price is the optimal unconstrained monopoly solution: a constant markup over the marginal cost. Using labor market clearing, $\int_0^1 l_{jt} dj = 1$, we can find the market-clearing wage $w_t = (1 - \beta) [\bar{Q}_t + \bar{\Phi}_t]^\beta \bar{Q}_t^{1-\beta}$, where $\bar{\Phi}_t \equiv \int_0^1 \phi_{jt} dj$ denotes aggregate extrinsic quality. The monopoly price can thus be written as:

²⁷In Appendix G.2.5, we present an extension of the model in which advertising also affect the price-elasticity of demand.

$$p_t = \left(\frac{\bar{Q}_t + \bar{\Phi}_t}{\bar{Q}_t} \right)^\beta \quad (8)$$

Aggregate output is:

$$Y_t = \frac{1-\beta}{\beta} [\bar{Q}_t + \bar{\Phi}_t]^\beta \bar{Q}_t^{1-\beta} \quad (9)$$

and flow operating profits before advertising costs can be written as:

$$\pi_{jt} = \tilde{\pi}_t \tilde{q}_{jt} \quad (10)$$

where

$$\tilde{\pi}_t = \beta \left(\frac{\bar{Q}_t}{\bar{Q}_t + \bar{\Phi}_t} \right)^{1-\beta} \quad (11)$$

is the component of profits that is constant across goods. Equation (10) shows that any difference in flow profits across goods must be due to differences in either intrinsic or extrinsic quality, or both. Therefore, R&D and advertising represent two non-mutually exclusive ways to increase profits. From Equation (11), we can notice that the profit of a firm is a decreasing function of the overall investment in advertising in the economy through $\bar{\Phi}_t$.

Advertising Choices Henceforth, we drop time subscripts unless otherwise needed. When choosing advertising expenditures (m_j), a firm of size n and portfolio of (intrinsic) qualities $\mathbf{q} \in \mathbb{R}_+^n$ solves the static problem:

$$\begin{aligned} \pi^{adv} &\equiv \max_{\{m_j: j \in \mathcal{J}\}} \sum_{q_j \in \mathbf{q}} \left[\tilde{\pi}(q_j + \phi(m_j, n)) - m_j \right] \\ &\text{s.t. } \phi(m_j, n) = \theta \frac{q_j}{\left(\sum_{q_{j'} \in \mathbf{q}} q_{j'}^{1/\alpha} \right)^\alpha} \bar{Q}^{1-\zeta} m_j^\zeta n^\eta \end{aligned} \quad (12)$$

where π^{adv} denotes flow profits after the advertising decision. We make the assumption that $\alpha = 1 - \zeta$ in order to reduce the dimensionality of the parameter space and be able to find closed-form expressions.²⁸ The optimality condition gives good-specific advertising expenditures of

$$m_j = (\zeta \theta \tilde{\pi})^{\frac{1}{1-\zeta}} \frac{q_j^{1/(1-\zeta)}}{\sum_{j'} q_{j'}^{1/(1-\zeta)}} \bar{Q} n^{\frac{\eta}{1-\zeta}} \quad (13)$$

and thus firm-level advertising expenditures of

$$M_n \equiv \sum_{q_j \in \mathbf{q}_f} m_j = (\zeta \theta \tilde{\pi})^{\frac{1}{1-\zeta}} \bar{Q} n^{\frac{\eta}{1-\zeta}} \quad (14)$$

²⁸Our results would not be qualitatively affected by the choice of a different value of α because the main mechanism regarding advertising and R&D intensity works through firm size n . In addition, this assumption implies a positive relationship between advertising expenditure and quality at the product level but not at the firm level. This is in line with empirical results in Archibald *et al.* (1983) who study this relationship at both the product and brand level. A similar result is obtained in Caves and Greene (1996) at the brand level.

for a firm of size n . Recall from our empirical analysis that (i) advertising expenditures are increasing in size, and (ii) advertising intensity is decreasing in size (Fact #3). The first observation is delivered directly by the fact that $\eta > 0$. Defining advertising intensity by M_n/n , in order to satisfy the second requirement it must be that M_n is concave in n . This leads to the following parametric restriction: $\frac{\eta}{1-\zeta} < 1$.

Because we imposed diminishing returns to advertising ($\zeta < 1$), the empirically-informed parameter restriction directly implies that $\eta < 1$. Since this parameter controls for the degree of size spillovers coming from advertising (recall Equation (3)), we obtain that, in order to replicate the decreasing advertising intensity, the spillover effect from size must be marginally stronger for smaller firms. Namely, the gain in advertising return for an n -to- $(n+1)$ transition is higher when n is smaller. As a consequence, smaller firms are relatively more concerned with expanding to new product markets, and they choose a relatively higher external innovation intensity in equilibrium.

In sum, the mechanism implies that the interaction between R&D and advertising can generate a decreasing R&D intensity with firm size (Fact #2) and hence a deviation from Gibrat's law (Fact #1). Our model can then qualitatively generate Facts 1 to 3 even in the presence of non-decreasing returns to scale in R&D.

Combining returns and costs, we have that the overall static profits of the firm are

$$\pi^{adv} = \underbrace{\tilde{\pi} \sum_{q_j \in \mathbf{q}} q_j}_{\text{Intrinsic profits}} + \underbrace{\gamma \bar{Q} n^{\frac{\eta}{1-\zeta}}}_{\text{Extrinsic profits}} \quad (15)$$

where $\gamma \equiv \tilde{\pi} \tilde{\gamma} - (\theta \tilde{\pi} \zeta)^{\frac{1}{1-\zeta}}$ is constant across goods and firms. Equation (15) is the static value of firm-level flow operating profits net of advertising expenditures. It has two components. The first one grows linearly with the aggregate intrinsic quality within the firm, with a factor of proportionality that does not vary across firms (although it typically may grow with the economy). The second one is invariant to the firm's intrinsic quality portfolio, but is increasing and concave in firm size. This component comes from investment in advertising and the extrinsic margin of product quality. While the first term is standard in models of endogenous firm growth with scalable returns, the second term is new and critically alters the dynamic incentives to conduct research vis-à-vis advertising in the way that we have described above.

BGP Equilibrium Characterization A Balanced Growth Path equilibrium is defined as an equilibrium allocation in which aggregate variables grow at a constant rate, denoted by $g > 0$. To show the existence of such an equilibrium, we make use of the fact that economy-wide extrinsic quality $\bar{\Phi}$ in fact grows at the same rate as that of aggregate intrinsic quality. Formally, we have that $\bar{\Phi} = \Phi^* \bar{Q}$ where Φ^* is a constant, both across time and firms. Under this result, it is clear from Equation (9) that, if a BGP exists where output grows at the rate g , then $g = \dot{\bar{Q}}/\bar{Q}$. Moreover, we can note from Equation (11) that $\tilde{\pi}$ is time-invariant, and therefore flow operating profits before advertising costs can be written as $\pi_{jt} = \pi \tilde{q}_{jt}$ in BGP, where $\pi = \beta(1 + \Phi^*)^{\beta-1}$ is a constant. In turn, γ is then constant, and $\bar{\Phi}$ becomes a linear function of \bar{Q} .

By construction, we can express total aggregate extrinsic quality on the BGP as $\bar{\Phi} = \sum_{n=1}^{+\infty} F\mu_n\bar{\Phi}_n$, where μ_n is the invariant share of size- n firms (which we derive explicitly below), such that $\mu_n \in [0, 1]$ and $\sum_{n=1}^{+\infty} \mu_n = 1$. Combining $\bar{\Phi} = \Phi^*\bar{Q}$ with the formula for $\bar{\Phi}$ and the equilibrium firm-level extrinsic quality, we then obtain that Φ^* is the solution to the following expression:

$$\Phi^* = \theta^{\frac{1}{1-\zeta}} \left(\frac{\zeta\beta}{(1+\Phi^*)^{1-\beta}} \right)^{\frac{\zeta}{1-\zeta}} \sum_{n=1}^{+\infty} F\mu_n n^{\frac{\eta}{1-\zeta}} \quad (16)$$

Equation (16) delivers a unique solution $\Phi^* > 0$. A direct consequence of the result is that aggregate marketing expenditures, M , grow at the rate g , since $M = \sum_{n=1}^{+\infty} F\mu_n M_n$, and M_n has been shown to be linear in \bar{Q} (Equation (14)). In particular, we obtain

$$M = \sum_{n=1}^{+\infty} F\mu_n (\zeta\theta\tilde{\pi})^{\frac{1}{1-\zeta}} n^{\frac{\eta}{1-\zeta}} \bar{Q} \quad (17)$$

and thus $\dot{M}/M = g$. From the resource constraint in (4), if total R&D expenditures Z grow at the rate of \bar{Q} (a result whose proof we relegate to the end of this section), so does aggregate consumption, C . From Equation (5), we then have that $g = r_t - \rho$, and therefore $r_t = r = g + \rho$, $\forall t$.

Value Functions We now describe the R&D choices of firms. Denote by τ the endogenous rate of creative destruction along the BGP, and by x_n the external R&D intensity of a firm of size n , i.e. $x_n \equiv X_n/n$, where X_n is the Poisson rate of external innovation. Taking (r, τ, g) as given, a firm f with a product portfolio with $1 \leq n = |\mathbf{q}_f|$ goods chooses external R&D intensity x_n , and internal R&D intensities $\{z_j : j \in \mathcal{J}_f\}$, to maximize firm value $V_n(\mathbf{q}_f)$, written in the Hamilton-Jacobi-Bellman form (see Appendix A.1 for a derivation):

$$\begin{aligned} rV_n(\mathbf{q}_f) = \max_{x_n, \{z_j\}} \left\{ \sum_{q_j \in \mathbf{q}_f} \left[\tilde{\pi}q_j - \tilde{\chi}z_j^{\tilde{\psi}}q_j + z_j \left(V_n(\mathbf{q}_f \setminus \{q_j\} \cup_+ \{q_j(1 + \lambda^I)\}) - V_n(\mathbf{q}_f) \right) \right. \right. \\ \left. \left. + \tau \left(V_{n-1}(\mathbf{q}_f \setminus \{q_j\}) - V_n(\mathbf{q}_f) \right) \right] \right. \\ \left. + nx_n \left(\int_0^1 V_{n+1}(\mathbf{q}_f \cup_+ \{q_j(1 + \lambda^E)\}) dj - V_n(\mathbf{q}_f) \right) \right. \\ \left. - \tilde{\chi}n^{\sigma+\tilde{\psi}}x^{\tilde{\psi}}\bar{Q} + \gamma\bar{Q}n^{\frac{\eta}{1-\zeta}} \right\} + \dot{V}_n(\mathbf{q}_f) \quad (18) \end{aligned}$$

subject to Equation (3), where \cup_+ and \setminus_- are multiset union and difference operators.²⁹ The first two terms in the first line of Equation (18) are good-specific intrinsic flow operating profits net of internal R&D costs. The third term in the first line is the change in value due to the internal improvement over a currently held good. This event occurs at a Poisson flow rate of z_j , and it increases the intrinsic quality of the good by a factor of $(1 + \lambda^I) > 1$. The first term in the second line is the change in the firm's value when losing a good j to another

²⁹These operators are defined by $\{a, b\} \cup_+ \{b\} = \{a, b, b\}$, and $\{a, b, b\} \setminus_- \{b\} = \{a, b\}$, and they are needed because the set \mathbf{q}_f may include more than one instance of the same element.

incumbent or an outside entrant through creative destruction, an event that occurs at the equilibrium Poisson rate τ . In this case, the firm becomes of size $n - 1$. The second term on this line is the change in value for the firm when it successfully acquires a new product line through external innovation and transitions to $n + 1$, which occurs at the Poisson rate nx_n (recall that here x_n denotes the intensity rate). The third line includes the flow resources spent in external R&D, extrinsic flow profits (recall equation (15)), and the instantaneous change in firm value due to economic growth.

As for entrants, their value function is given by the following equation (see Appendix A for a derivation):

$$rV_0 = \max_{x_e > 0} \left\{ x_e \left[\int_0^1 V_1(\{q_j(1 + \lambda^E)\}) dj - V_0 \right] - \nu x_e \bar{Q} \right\} + \dot{V}_0 \quad (19)$$

where V_0 denotes firm value at $(n, \mathbf{q}_f) = (0, \emptyset)$. Entrants are choosing the rate of entry x_e . If there is a positive mass of entrants in equilibrium, the free-entry condition reads $V_0 = 0$.

Our first proposition shows that the growth rate of the economy is measured as the combined contributions of internal and external innovators, advancing aggregate productivity by λ^I and λ^E , respectively.

Proposition 1 (Growth rate) *The growth rate of the economy in the BGP equilibrium is given by*

$$g = \tau \lambda^E + z \lambda^I \quad (20)$$

Proof. See Appendix A.2.

Moreover, since innovation rates are independent, the rate of creative destruction τ is given by the aggregate Poisson rate of external innovation coming from entrants and incumbents. Therefore,

$$\tau = x_e + \sum_{n=1}^{+\infty} F \mu_n n x_n \quad (21)$$

In turn, we can find a closed-form expression the invariant firm size distribution along the BGP:

Proposition 2 (Invariant firm-size distribution) *The invariant firm-size distribution is given by*

$$\mu_n = \frac{x_e}{F} \frac{\prod_{i=1}^{n-1} x_i}{n \tau^n}; \quad \forall n \geq 1 \quad (22)$$

with $\mu_n \in [0, 1], \forall n \geq 1$, and $\sum_{n=1}^{+\infty} \mu_n = 1$.

Proof. See Appendix A.3.

We are now ready to solve for the value function and find optimal R&D intensities. To this purpose, we guess a functional form of the type $V_n(\mathbf{q}_f) = \Gamma \sum_{q_j \in \mathbf{q}_f} q_j + \Upsilon_n \bar{Q}$ for which

we must verify the value of the constant coefficient $\Gamma \in \mathbb{R}_+$ and the sequence $\{\Upsilon_n\}_{n=1}^{+\infty}$. This guess is informed by the expression for the flow value of holding a given product, seen in (15). Namely, a firm derives value from the intrinsic quality of the products it is selling, independently of its size (the first term, whose coefficient Γ is constant across firm types), but also from the option value of marketing its goods in order to increase demand and revenue (the second term, whose coefficient Υ_n depends on firm type).³⁰

The following proposition summarizes the solution:

Proposition 3 (Value functions) *Assuming that there is positive entry in equilibrium ($x_e > 0$), the value for a firm of size $n \geq 1$ is $V_n(\mathbf{q}_f) = \Gamma \sum_{q_j \in \mathbf{q}_f} q_j + \Upsilon_n \bar{Q}$, where*

$$\Gamma = \frac{\nu - \Upsilon_1}{1 + \lambda^E} \quad (23)$$

and Υ_n , for $n \geq 1$, is the solution to the second-order difference equation

$$\Upsilon_{n+1} - \Upsilon_n + \Gamma(1 + \lambda^E) = \tilde{\vartheta} \left(\rho \Upsilon_n n^{\frac{\sigma}{\tilde{\psi}-1}} - (\Upsilon_{n-1} - \Upsilon_n) \tau n^{\frac{\sigma + \tilde{\psi} - 1}{\tilde{\psi} - 1}} - \gamma n^{\frac{\eta}{1-\zeta} + \frac{\sigma}{\tilde{\psi} - 1}} \right)^{\frac{\tilde{\psi}-1}{\tilde{\psi}}} \quad (24)$$

where $\tilde{\vartheta} \equiv \tilde{\psi} \left(\frac{\tilde{\chi}}{(\tilde{\psi}-1)^{\tilde{\psi}-1}} \right)^{\frac{1}{\tilde{\psi}}}$ is a parameter, with boundary condition $\Upsilon_0 = 0$.

Proof. See Appendix A.4.

In the appendix we show that we obtain the following optimal R&D intensities:

$$z_j = \left(\frac{\lambda^I (\nu - \Upsilon_1)}{\widehat{\psi} \widehat{\chi} (1 + \lambda^E)} \right)^{\frac{1}{\tilde{\psi}-1}} \quad (25)$$

$$x_n = n^{\frac{1-\sigma-\tilde{\psi}}{\tilde{\psi}-1}} \left(\frac{\nu - \Upsilon_1 + \Upsilon_{n+1} - \Upsilon_n}{\tilde{\psi} \tilde{\chi}} \right)^{\frac{1}{\tilde{\psi}-1}} \quad (26)$$

for internal and external innovations, respectively, and all $n \geq 1$. First, as announced before, internal R&D intensity is constant across goods ($z_j = z$). Second, n affects external innovation intensities through two distinct channels: (i) the degree of scalability in external R&D technology ($\sigma + \tilde{\psi}$); (ii) the degree of decreasing returns in advertising through the branding effect (i.e, the values for ζ and $\eta < 1 - \zeta$). The former channel features both explicitly in the exponent of the first multiplicative term of (26), and implicitly within the Υ_n sequence in (24). The advertising channel, however, only features through Υ_n . Thus, we can obtain the desired inverse dependence of innovation expenditures to size as long as the advertising channel scales weakly enough to compensate any strong scalability coming from the R&D technology itself. This allows for the possibility of balanced growth in equilibrium even when there exist increasing returns to scale in external R&D.

For the sake of intuition, let us focus on the case in which there are constant returns in external R&D ($\sigma + \tilde{\psi} = 1$), which shall be the case in our baseline calibration. In the

³⁰When $\tilde{\psi} + \sigma \neq 1$, $\Upsilon_{n+1} - \Upsilon_n$ also captures the dependence of external R&D on firm size.

absence of the advertising channel ($\theta = 0$), the model is equivalent to that of Klette and Kortum (2004). In this case, $\Upsilon_n = n\Upsilon$ for some constant $\Upsilon > 0$. This implies that the gain in value from acquiring one additional good (i.e., $\Upsilon_{n+1} - \Upsilon_n$) is constant across firm size and we obtain the theoretical results that (i) external R&D investment scales perfectly with size, and consequently (ii) firm growth is independent of size (Gibrat's law). When $\theta > 0$, however, deviations from Gibrat's law can occur even when $\sigma + \tilde{\psi} = 1$: small firms might still be more innovation-intensive than large firms because they benefit marginally more in terms of advertising efficiency gains from the new product lines.

Equilibrium Definition To close the optimality characterization and define an equilibrium, it remains to show two conditions that have been stated without proof in the above discussion.

First, as shown in Appendix A.4, the BGP equilibrium exhibits positive entry ($x_e > 0$) if the equilibrium condition $\nu < \Upsilon_1$ is satisfied (so that $\Gamma > 0$). Intuitively, this condition says that the cost of entry ν is lower than the gain, coming from the profits accrued for the acquisition of the first product line of the firm. In practice, this condition is always checked ex-post in our computation.

Second, from (4) and the result that $g = \dot{Q}/\bar{Q} = \dot{C}/C = \dot{M}/M$, we need to check that aggregate R&D expenditure Z grows at the rate g , as well. Using the cost functions for R&D, we have that:

$$Z = \left(\hat{\chi} z^{\hat{\psi}} + \sum_{n=1}^{\infty} F \mu_n \tilde{\chi} n x_n^{\tilde{\psi}} + \nu x_e \right) \bar{Q} \quad (27)$$

The right-hand side includes aggregate R&D expenditures by incumbents attempting internal innovations, incumbents attempting external innovations, and outside firms attempting to enter into the market through external R&D, respectively. Indeed, we get that Z is linear in \bar{Q} , and hence $\dot{Z}/Z = g$. Aggregate consumption is then computed as the residual $C = Y - Z - M$.

We are now ready to define a BGP equilibrium:

Definition 1 (BGP Equilibrium) *A Balanced Growth Path Equilibrium is, for all $q_j \geq 0$, $j \in [0, 1]$, $n \in \mathbb{Z}_+$, and $\bar{Q} > 0$, allocations y_j , l_j and m_j ; extrinsic quality ϕ_j ; aggregates Y , C , Z , M , A ; a mass of incumbents F ; a constant Γ and a sequence $\{\Upsilon_n\}_{n=1}^{+\infty}$; a firm size distribution μ_n ; prices w , p , and r ; and rates g , z , x_n , x_e , and τ ; such that: (i) given prices, final good producers maximize profit; (ii) y_j (input quantity) and p (input price) solve the intermediate sector problem and satisfy (8); (iii) m_j (expenditure) and ϕ_j (effectiveness) solve the advertising problem and satisfy (13); (iv) innovation flows z (internal for incumbents) and x_n (external for incumbents) solve the innovation problem and satisfy (25) and (26); (v) μ_n (the invariant measure of firms) satisfies (22); (vi) x_e (entry flow) and F (mass of incumbent firms) solve the entry problem and satisfy the free-entry condition $V_0 = 0$ and the restriction $\sum_n \mu_n = 1$; (vii) Γ (the value of a firm's intrinsic-quality portfolio) and Υ_n (the value of owning n products) solve the value function in (18), and satisfy (23)*

and (24), respectively; (viii) g (growth rate) and τ (creative destruction rate) satisfy (20) and (21), respectively; (ix) aggregates Y (output), M (advertising expenditure), and Z (R&D investment) satisfy (9), (17), and (27), respectively, and C (consumption) is found residually via (4); (x) A (household wealth) satisfies $A = \sum_n F \mu_n V_n$ and the transversality condition (6); (xi) r (interest rate) satisfies $r = g + \rho$, and w (wage) clears the labor market.

5 Quantitative Analysis

In this section, we calibrate the model combining the micro data used in Section 3 with aggregate moments for the U.S. in the period 1980-2015. We use the same sample from Compustat described in Section 3 and seek to match empirical regression coefficients with model-implied slopes from simulation-based regressions (an indirect inference approach). This calibration strategy is fairly standard in the endogenous growth and firm dynamics literature. The novelties are that we use coefficients on advertising regressions to calibrate our theory of firm growth and, most importantly, that we are able to match the slopes of *both* firm growth and R&D intensity regressions without relying on decreasing returns to scale in the R&D technology. The BGP algorithm is described in Appendix E.

5.1 Calibration Strategy

There are 13 parameters to identify: $(\rho, \lambda^E, \lambda^I, \tilde{\chi}, \tilde{\psi}, \hat{\chi}, \hat{\psi}, \sigma, \nu, \beta, \eta, \zeta, \theta)$. Some of these parameters are externally identified, with the rest being internally calibrated via our indirect inference approach.

In our baseline calibration, we impose constant returns to scale in external innovation ($\sigma = 1 - \tilde{\psi}$) and assume that no innovation is more radical than the other ($\lambda^E = \lambda^I \equiv \lambda$). Once again, we do this to isolate the effects of our advertising-R&D interaction on firm growth in order to show that the spillover effect on advertising is not only able to quantitatively match advertising intensity slopes, but also the observed deviations from Gibrat's law and R&D intensity across sizes, suggesting that those facts do not necessarily constitute evidence in favor of decreasing returns to scale in R&D. Later on, in the context of our policy analysis of Section 7, we will compare our baseline calibration with a second calibration without advertising and with decreasing returns to scale in R&D.

The parameters $(\rho, \hat{\psi}, \tilde{\psi}, \beta, \zeta)$ are externally calibrated. We set $\rho = 0.02$, which approximately corresponds to a discount factor of 97% annually. We impose $\hat{\psi} = \tilde{\psi} = 2$, following Akcigit and Kerr (2018) and prior empirical literature estimating the cost curvature of different types of R&D. To find a value for β , we use (7), (8), and (10) to obtain $\beta = \frac{\int_0^1 \pi_{jt} dj}{\int_0^1 p_{jt} y_{jt} dj}$. Using our Compustat sample described in Section 3, we calculate the corresponding ratio of average operating income before depreciation to average sales, and set $\beta = 0.1645$. Finally, we set the elasticity of sales to advertising expenditures to $\zeta = 0.1$, which is in line with estimations from the empirical marketing literature (see Tellis (2009)).

We are left with the parameters $(\lambda, \tilde{\chi}, \hat{\chi}, \nu, \eta, \theta)$, which are determined internally. In particular, we solve and simulate the economy and find parameter values that match model moments and simulation-implied regression coefficients to those observed in the data and

presented in tables in Section 3. The simulation of the model uses 1,000 firms and discretizes time to $T = 100$ periods of length $\Delta t = 0.01$.³¹

We find values for the 6 internally identified parameters by targeting 7 moments for our period of interest. We target the aggregate firm entry rate from the Business Dynamics

Table 2: Full set of calibrated parameters in baseline estimation (model with advertising and constant returns to scale in R&D).

Parameter	Value	Description	Source
<i>External</i>			
ρ	0.02	Time discount rate	Standard
ψ	2	Elasticity of R&D	Akcigit and Kerr (2018)
β	0.1645	Profitability ratio	Compustat
ζ	0.1	Sales-ADV elasticity	Tellis (2009)
σ	$1 - \psi$	Constant RTS in R&D	Assumption
<i>Internal</i>			
λ	0.0143	Innovation step	.
$\hat{\chi}$	0.0017	Internal R&D scale param.	.
$\tilde{\chi}$	0.6256	External R&D scale param.	.
ν	0.7206	Entrant's R&D scale param.	.
η	0.8527	ADV spillover effect	.
θ	1.1022	ADV efficiency	.

Statistics (BDS) data of the U.S. Census Bureau,³² the long-run average growth rate of per capita GDP in the U.S., and the average ratios of R&D-to-sales and R&D-to-advertising expenditures from our sample of Compustat firms.³³ In addition, we run simulation-based regressions to target the 4 empirical coefficients of interest that relate to the empirical facts presented in the full specifications of Section 3: the Gibrat's law coefficient (column (1) in Table 1), the R&D intensity and advertising intensity coefficients (column (2) and (3) in Table 1, respectively), and the R&D-to-advertising coefficient (column (4) in Table 1).³⁴ To obtain these simulation-based results, we compute the corresponding coefficients via OLS and take the time-series average of the corresponding coefficients.³⁵ Throughout, we use sales as our measure of size.³⁶ Our selection criterion is a simple unweighted minimum absolute-distance estimator.³⁷

³¹All firms are assumed to start the same, with qualities $q_0 = 1$. In the event of ties, namely multiple successful firms drawing the same good over which to perform an external innovation, we assign equal probabilities to each firm and draw the new monopolist from the corresponding discrete uniform distribution of tied firms.

³²Data available at <http://www.census.gov/ces/dataproducts/bds/data.html>.

³³For comparison, the average advertising to sales ratio in the data is about 5%.

³⁴Note that the coefficient on the R&D-to-advertising substitution across sizes can be obtained as the difference between the coefficients on R&D and advertising intensities.

³⁵To allow for convergence of the simulation, we ignore the first 20% of simulations in our calculations.

³⁶In the model, size is measured by the number of products. Since we don't have data about the number of products, we rely on sales in our calibration. A number of studies find a positive correlation between sales and the number of products (see for instance, Scherer (1983), Katila and Ahuja (2002) and Plehn-Dujowich (2013)).

³⁷In particular, we find the minimum of $\sum_{m=1}^k \left| \frac{Model(m) - Data(m)}{Data(m)} \right|$, where k is the total number of targeted moments.

Table 2 shows the full set of calibrated parameter values. Table 3 shows the results for our targeted moments, compared to the values in the data. We do well on all targeted moments. Particularly, all four simulation-based slopes have the correct sign and magnitude. Even though there are constant returns to scale in the R&D cost function, the spillover effect in advertising, which is set to target the decreasing advertising intensity with size observed in the data, is also able to correctly predict both the decreasing R&D intensity and the deviation from constant firm growth. This implies that the observed decreasing R&D intensity in firm size observed in the data is not necessarily evidence for the existence of decreasing returns to scale in R&D. Our calibration shows that it could simply be the result of the interaction between R&D and advertising even when there exist constant returns to scale in R&D. As we will argue in Section 7, this aspect can have important policy implications regarding the optimal design of R&D subsidies.

Importantly, the R&D size dependence comes from the interaction between advertising and external R&D only. Our model predicts that 64.45% of growth comes from internal R&D, which is in line with estimates in Garcia-Macia *et al.* (2016).

Table 3: Targeted moments: model vs. data. Model with advertising and constant returns to scale in R&D.

Moment		Model	Data	Data Source
<i>Aggregate moments</i>				
Average growth rate	g	0.02	0.02	Standard
Firm entry rate	x_e/F	0.101	0.098	BDS
Average R&D-Sales ratio	$\sum_n F\mu_n R_n/(py)$	0.153	0.102	Compustat
Average R&D-ADV ratio	$\sum_n F\mu_n R_n/M_n$	24.15	26.40	Compustat
<i>Regression coefficients</i>				
Gibrat's coefficient	$\beta_{\Delta sales/sales}$	-0.0326	-0.0325	Table 1
R&D intensity coefficient	$\beta_{rd/sales}$	-0.1030	-0.1035	Table 1
ADV intensity coefficient	$\beta_{adv/sales}$	-0.0353	-0.0317	Table 1
R&D/ADV coefficient	$\beta_{rd/adv}$	-0.0677	-0.0719	Table 1

Notes: Sales are used as the measure of firm size. R_n stands for the total R&D expenditures of a firm of size n , defined in Equation (2). M_n is total advertising expenditures at the firm level, defined in Equation (14).

5.2 Growth Effects of Advertising

In this section, we study the effect of a decrease in advertising cost (equivalently, an increase in θ) on the growth rate of the economy. (We relegate the implications for welfare to Appendix B.) Specifically, we show that in our calibrated model R&D and advertising act as substitutes at the firm level. This implies that more efficient advertising can have a detrimental effect on growth by decreasing the rate of innovation in the economy. This is ultimately a quantitative result. Indeed, in Appendix F we show that R&D and advertising can be either complements or substitutes at both the firm and aggregate level depending on parameter values.³⁸

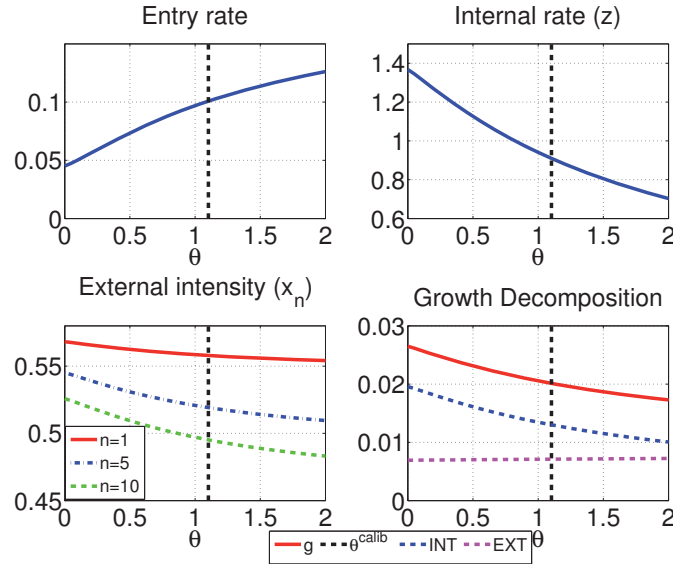
³⁸We further show in Appendix F that our model can generate decreasing R&D intensity with firm size even in the presence of increasing returns to scale in R&D. The possible existence of an equilibrium when there exist increasing returns in R&D is an interesting feature of our model.

Interestingly, even when R&D and advertising are substitutes, the effects of advertising efficiency on growth is ambiguous. To see this, we can decompose the growth rate of the economy in the model by combining (20) and (21) to obtain:

$$g = \underbrace{\lambda^E x_e}_{\text{Entrants}} + \underbrace{z\lambda^I}_{\text{Incumbents doing internal}} + \underbrace{\lambda^E \sum_{n=1}^{+\infty} F\mu_n n x_n}_{\text{Incumbents doing external}} \quad (28)$$

The three additive terms in g correspond to the contribution to growth by entrants, incumbents performing internal improvements, and incumbents performing external innovations, respectively. In addition, there is a compositional effect coming from shifts in the distribution of firm, $F\mu_n$. Since firms of different size have different R&D investment behavior, this compositional channel might also affect the rate of growth of the economy.

Figure 2: Growth decomposition, for different values of advertising efficiency (θ), in the calibrated economy. The calibrated value of θ is marked by the dashed vertical line. The growth rate is $g = \tau\lambda^E + z\lambda^I$, where $\tau = x_e + \sum_{n=1}^{+\infty} F\mu_n n x_n$ is the creative destruction rate. The entry rate is measured by x_e/F .

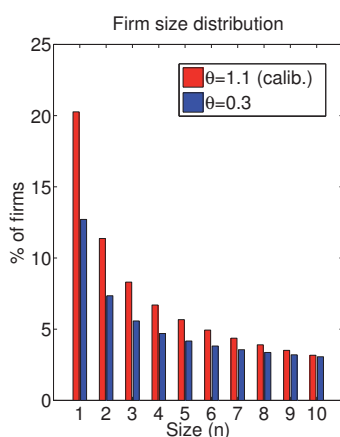


We plot these different effects against advertising efficiency (θ) in Figures 2 and 3. On the one hand, when advertising becomes more efficient (higher θ), the value of becoming an incumbent firm increases, which fosters new entry of new firms. The increased R&D investment by entrants has a positive effect on innovation and economic growth. It also increases the rate of creative destruction which modifies the incentive to perform internal R&D. On the other hand, a higher creative destruction rate implies that firms are now more likely to lose one of their products by being displaced by another firm. As a result, firms have smaller incentives to perform internal R&D. This decrease in internal R&D investment depresses economic growth. Moreover, incumbent firms also decrease their external R&D intensity when advertising becomes more efficient. This decrease is even more pronounced for larger firms. Finally, there is a compositional effect operating through changes in the firm size distribution. As a result of higher entry rate and lower external innovation by incumbent, the firm size distribution shifts to the left (Figure 3). This places more mass on the part of

the firm distribution that is most R&D intensive. This compositional channel thus tends to foster growth.

Overall, the decrease in internal and external R&D investment more than offset the increase in the entry rate and the compositional shift in the firm size distribution, and the growth rate ends up decreasing when advertising efficiency goes up. Within our calibrated economy, we find that these detrimental effects can be substantial. In particular, growth would increase by 0.64 percentage points (from its calibrated value of 2% to 2.64%) if the advertising sector was to be shut down completely (namely, if θ changed from its calibrated value of $\theta = 1.1022$ to $\theta = 0$).

Figure 3: Change in firm size distribution as a function of advertising efficiency (θ).



6 Validation Exercises

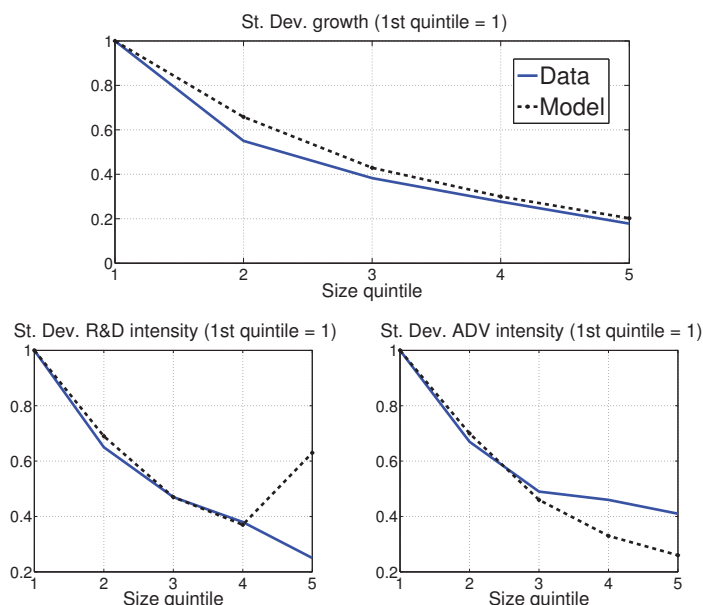
Our set of validation exercises are made of two parts. First, we compare untargeted moments in our model and in the data. Second, we test empirically the R&D-advertising substitution predicted by our calibrated model.

6.1 Untargeted Moments

The first set of untargeted moments that we consider is the variance of firm growth, R&D and advertising intensities across firm size. It is a well-known stylized fact in the literature that not only the average but also the variance of firm growth rates decreases with firm size (see among others Hymer and Pashigian (1962), Evans (1987) and Klette and Kortum (2004)). In Figure 4, we compare the evolution of these variances in the model and in the data.

Our model fits the relationship between the variance of firm growth and firm size well. In addition, we also report the standard deviation of R&D and advertising intensity with firm size. Figure 4 shows that these standard deviations are also decreasing with firm size in the data. Our model predicts a decrease in the variance in R&D and advertising intensity which is both qualitatively and quantitatively (broadly) in line with the data even though these moments were not targeted in our estimation.

Figure 4: Standard deviations (relative to 1st quintile): Model vs. Data.



Notes: Firms are ranked in size quintiles according to their normalized level of sales (sales as a ratio of average sales in the same year). R&D and advertising intensities are measured as the ratio of total R&D and advertising expenditures to total sales within each group. The standard deviation in the first quintile is normalized to one.

In Table 4, we compare additional untargeted moments. The correlation coefficients between R&D intensity and firm (sale) growth as well as between advertising intensity and firm growth predicted by the model are broadly in line with those in the data. Our model also predicts high levels of serial correlation in R&D and advertising intensity within firms which are comparable to their empirical counterparts. The ratio of R&D to advertising expenditures is also very persistent both in the model and in the data.

6.2 Empirical Test of the R&D-Advertising Substitution

While our theoretical model could potentially generate complementarity between R&D and advertising (see Appendix F), the calibrated version of the model predicts substitutability. In this section, we provide empirical evidence for the crowding out between advertising and R&D.

Table 4: Correlation and Autocorrelation Coefficients: Model vs. Data.

	DATA	MODEL
$corr(\text{R\&D intensity, firm growth})$	0.15	0.25
$corr(\text{ADV intensity, firm growth})$	0.10	0.22
$autocorr(\text{R\&D intensity})$	0.92	0.89
$autocorr(\text{ADV intensity})$	0.88	0.76
$autocorr(\text{R\&D/ADV ratio})$	0.92	0.92

Notes: R&D and advertising intensity in logarithm.

To test for the existence of substitution between R&D and advertising expenditure, we use exogenous variation in the cost of R&D over time and across U.S. states. In the early 1980s, U.S. states started implementing tax incentives for innovation. Starting with the state of Minnesota in 1982, several U.S. states subsequently introduced R&D tax incentives to stimulate innovation. These tax incentives are typically in the form of tax credits, corresponding to a certain share of R&D expenditures, which can be deducted from state corporate income tax. The inception date of the tax varies across states. The tax credit rate also varies across states and over time within each state. This allows us to obtain exogenous variation in the relative price of R&D over time and across states. In 2009, 32 states offered such credits with rates varying between 2% and 20%.³⁹

We use annual data from Compustat on R&D, advertising, sales, and our measures of firm age and financial constraints from Section 3, for the period 1950 - 2009. We also retrieve data on the relevant state for tax credit from Compustat. We focus our analysis on firms reporting positive advertising and R&D expenditures. Data on tax credit rates are obtained from Wilson (2009) and Falato and Sim (2014).⁴⁰

To provide evidence for substitution between R&D and advertising, we show that a decrease in the relative price of R&D (through an increase in R&D tax credits) leads to a significant decrease in advertising intensity. We perform the following firm-level regressions:

$$\log \left(\frac{Adv_{iszt}}{Sales_{iszt}} \right) = \alpha_0 + \beta_1 Tax\ Credit_{st} + \beta_2 \mathbf{X}_{ist} + \alpha_t + \alpha_s + \alpha_z + u_{iszt} \quad (29)$$

for firm i in state s and industry z at time t , where α_t , α_z and α_s control for time, industry and state fixed effects respectively, $Tax\ Credit_{jt}$ is a measure of R&D tax credit and \mathbf{X}_{ist} is a vector of control variables including sales, the age of the firm, the level of equity financing, state corporate tax rates, and selling, general and administrative expense.⁴¹

Regression results are reported in Table 5. Column (1) reports the results from regressing advertising intensity on a dummy variable which takes value one if the state has a tax credit implemented at time t . The results show a negative correlation between the existence of an R&D tax credit and the level of advertising intensity in the state, conforming with our mechanism.

Using this specification, however, ignores differences in the tax credit rate between states and over time. To amend this issue, columns (2) to (5) use different measures of tax credit rate and R&D relative cost in order to take into account the magnitude of the credit. Column (2) shows a significantly negative relationship between advertising intensity and the relative price of R&D using changes in the statutory tax credit across states and time. In particular, a one-percentage-point increase in the statutory state R&D tax credit is associated with a 1.75% decrease in advertising expenditures. In column (3), we use a tax-adjusted credit

³⁹Since 1981, an R&D tax credit has also been implemented at the federal level (with the exception of 1995). We use it for computing a measure of the R&D user costs.

⁴⁰We use data for all 50 U.S. states as well as for the District of Columbia. When states have several credit brackets, we use the top marginal rate.

⁴¹This includes operation expenses not directly related to production such as accounting, advertising, delivery, directors' remuneration, engineering, marketing, legal expenses or R&D.

Table 5: Effect of R&D subsidy on advertising intensity at the firm level

	(1)	(2)	(3)	(4)	(5)
	$\log\left(\frac{Adv}{Sales}\right)$	$\log\left(\frac{Adv}{Sales}\right)$	$\log\left(\frac{Adv}{Sales}\right)$	$\log\left(\frac{Adv}{Sales}\right)$	$\log\left(\frac{Adv}{Sales}\right)$
<i>Tax credit dummy</i>	-0.0629 (0.0369)				
<i>State credit rate</i>		-1.748 (0.582)			
<i>Tax-adjusted state rate</i>			-1.753 (0.627)		
<i>Effective state rate</i>				-1.901 (0.608)	
<i>R&D user cost</i>					1.823 (0.678)
Controls	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓
Industry FE	✓	✓	✓	✓	✓
State FE	✓	✓	✓	✓	✓
Observations	28743	27455	27455	25333	25333
R^2	0.42	0.42	0.42	0.43	0.43

Notes: Data from Compustat from 1950 to 2009, Wilson (2009) and Falato and Sim (2014). Controls include sales (logarithm), age, financial constraint, state tax and selling, general and administrative expense (logarithm). Firm age is measured as the elapsed time since the first observation in the data. Our measure of financial constraints is sale minus purchases of common and preferred stock, divided by sales. Standard errors are clustered at the state level (in parentheses).

1.75% decrease in advertising expenditures. In column (3), we use a tax-adjusted credit rate.⁴² Column (4), in turn, uses an alternative measure of the marginal effective R&D tax credit which acknowledges the different definitions of the R&D expenditures which are eligible for tax credit as well as the horizon over which the tax credit is calculated.

As shown in columns (3) and (4) of Table 5, using the tax-adjusted and marginal effective subsidy rate confirms the negative relationship between advertising intensity and R&D subsidy. Finally, we use one additional state-level measure of the cost of R&D: R&D user cost. Column (5) of Table 5 shows that higher R&D user costs are also associated with higher levels of advertising intensity, as expected if R&D and advertising are substitutes.⁴³

Interestingly, R&D seems to be a complement to other types of expenditures at the firm level. In Appendix C, we show that a lower cost of R&D is associated with higher levels of selling, general and administrative expenses (excluding R&D and Advertising).

7 Policy Implications

We now present potential policy implications of our model. First, since we have showed that advertising can be detrimental to growth and welfare, we analyze the possibility of a tax on advertising.⁴⁴ Second, we have proposed a new explanation for the observed decreasing

⁴²A full description of the measures of R&D costs is reported in Appendix H.1.

⁴³In Appendix H.2, we provide state-level evidence for substitutability between R&D and advertising.

⁴⁴The idea of taxing advertising has been discussed before, for instance in Solow (1968).

R&D intensity with firm size as the result from the optimal allocation of resources between R&D and advertising at the firm level. Our second policy exercise compares the effectiveness of R&D subsidies under our baseline calibration versus a world with technological differences in R&D production function.

7.1 Advertising Tax

We introduce a tax on advertising expenditure along the BGP. The firm's optimal advertising choice maximizes:

$$\max_{\{m_j\}} \sum_j \left[\tilde{\pi} \cdot (q_j + q_j d_j) - (1 + \xi) m_j \right] \quad \text{s.t.} \quad q_j d_j = \theta_j m_j^\zeta n^n$$

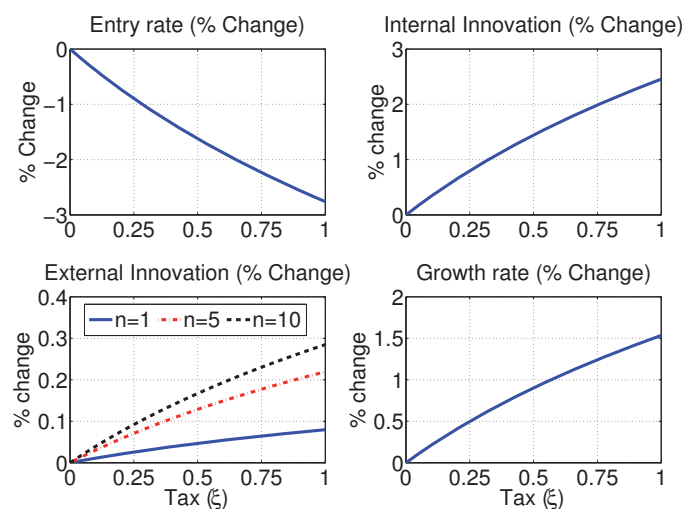
where $\xi \geq 0$ is the tax rate. Namely, for every unit of expenditure in advertising, the firm must pay an additional cost of ξ . Proceeds of the tax are rebated lump-sum to households.

Figure 5 plots, for different levels of the tax, the percentage change (with respect to $\xi = 0$) in the growth, entry, internal and external innovation rates (the latter for firms of different size n) of the calibrated economy. We observe that the advertising tax is equivalent to a decrease in advertising efficiency, which increases the growth rate of the economy.⁴⁵ However, the tax is considerably ineffective in the sense that the change in growth is small.

The elasticity of sales to advertising expenditure is low, which implies that firms' response to the change in the advertising cost from higher tax rates is timid. In particular, they only very modestly substitute advertising spending for R&D spending, and overall the economy's growth rate responds very little. The advertising tax also deters entry which tends to decrease growth.

Overall, we find that even though the advertising sector is detrimental to growth, taxation has limited effective power. For instance, a 0.1 percentage-point increase in g would require a tax rate around 910%.

Figure 5: Percentage change (with respect to $\xi = 0$) in growth, entry, creative destruction and internal innovation rates of the calibrated economy, for different sizes of the advertising tax rate.



⁴⁵In particular, $\theta^{post-tax} \equiv \theta^{pre-tax} (1 + \xi)^{-\zeta}$, inside γ (see Equation (15)).

7.2 R&D Subsidies

Next, we analyze the effect of R&D subsidies with and without the presence of advertising. A study of this type of industrial policy in the context of an endogenous growth model of firm dynamics is not new to our work. Akcigit (2009) and Acemoglu *et al.* (2018), among others, find that R&D subsidies may increase growth, particularly when targeted toward entrants and small incumbent innovative-intensive firms. This policy scheme is typically welfare-improving because it has the potential of correcting for inefficiencies stemming from R&D: namely, in the decentralized equilibrium, firms fail to appropriate the full social value of their innovations, as they do not internalize that their current innovations will be embedded within the goods that will be passed on to other firms in the future through creative destruction.⁴⁶ Unfortunately, analytically solving for the social planner’s allocation in order to study optimal taxation is famously hard within this class of models, and beyond the scope of this paper.

In our calibrated economy, advertising poses a second source of distortions, as it impedes the economy from reaching its full growth potential by altering innovation decisions. Here, we intend to explore the implications of a subsidy on R&D, when the advertising channel provides the necessary spillover effect to explain the deviation from Gibrat’s law seen in the data. We then compare this baseline economy to another one in which there are decreasing returns to scale in R&D and no advertising.

Calibration Without Advertising and DRTS in R&D We compare two subsidized economies. The first economy corresponds to our baseline calibration of Section 5.1, in which the spillover effect on advertising is active and there exist constant returns to scale in R&D. The second economy corresponds to a calibration of the model in which advertising is shut down (e.g. $\theta = 0$) and returns to R&D need not scale proportionally with firm size. In this new specification, we have 2 fewer parameters to estimate, and 5 as opposed to 7 moments to target (the same moments, except those related to advertising).

Table 6: Internally calibrated parameters in the model without advertising and with decreasing returns to scale in R&D.

Parameter	Value	Description
λ	0.0226	Innovation step
$\hat{\chi}$	0.0119	Internal R&D scale param.
$\tilde{\chi}$	3.2280	External R&D scale param.
ν	1.2351	Entrant’s R&D scale param.
σ	-0.9411	Degree of DRTS in R&D
Implied $\tilde{\psi} + \sigma$	1.0589	

Table 6 presents the internally-identified parameter values for the calibrated model without advertising.⁴⁷ As expected, absent the advertising channel, we need decreasing returns

⁴⁶The inefficiency in Schumpeterian models arising from a business-stealing effect is usually shown to be dominated by positive externalities. See for instance Jones and Williams (2000), Alvarez-Pelaez and Groth (2005) and Bloom *et al.* (2013).

⁴⁷As in the baseline calibration, we externally set $\rho = 0.02$, $\psi = 2$ and $\beta = 0.1645$.

to scale in R&D (i.e., $\psi + \sigma > 1$) in order to match the deviations from constant growth and R&D intensity. In particular, we find $\sigma = -0.9411 < 1$ (so that $\tilde{\psi} + \sigma = 1.0589$).⁴⁸

Comparing Calibrations Before comparing the effectiveness of R&D subsidies in the two different calibrations, we compare their performance in terms of calibrated and uncalibrated moments. We show that, on several dimensions, the model with advertising more closely matches empirical features of the data compared to the model with DRTS in R&D.

Table 7 presents the performance of the calibrated model without advertising in terms of moment matching, next to the results from the baseline calibration with advertising that we presented before in Table 3. First, the model without advertising has a harder time quantitatively explaining both deviations from constant R&D intensity and Gibrat's law jointly. In contrast, the baseline calibration of Section 5.1 with advertising spillover effects and constant returns in R&D was not only able to closely match both of these deviations, but also the advertising regression coefficient.

In Figure 6, we report the relationship between the standard deviation of firm growth with R&D intensity and firm size. We can notice that the model with advertising matches the evolution of those standard deviations more closely than the model without advertising. Even though the model without advertising correctly predicts that the standard deviation

Table 7: Targeted moments: Model #1: advertising and constant returns to scale in R&D; Model #2: No advertising and decreasing returns to scale in R&D.

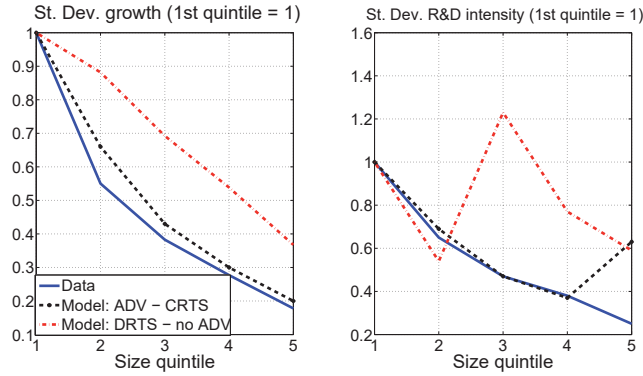
Moment		Model with ADV	Model without ADV	Data	Data Source
<i>Aggregate moments</i>					
Average growth rate	g	0.02	0.02	0.02	Standard
Firm entry rate	x_e/F	0.101	0.092	0.098	BDS
Average R&D/Sales	$\sum_n F\mu_n R_n/(py)$	0.153	0.103	0.102	Compustat
Average R&D/ADV	$\sum_n F\mu_n R_n/M_n$	24.15	.	26.40	Compustat
<i>Regression coefficients</i>					
Gibrat's coefficient	$\beta_{\Delta sales/sales}$	-0.0326	-0.0218	-0.0325	Table 1
R&D intensity coeff.	$\beta_{rd/sales}$	-0.1030	-0.1237	-0.1035	Table 1
ADV intensity coeff.	$\beta_{adv/sales}$	-0.0353	.	-0.0317	Table 1
R&D/ADV coefficient	$\beta_{rd/adv}$	-0.0677	.	-0.0719	Table 1

Notes: Sales are used as the measure of firm size. R_n stands for the total R&D expenditures of a firm of size n , defined in Equation (2). M_n is total advertising expenditures at the firm level, defined in Equation (14).

of firm growth decreases with firm size, the rate at which it does is much lower than both that of the model with advertising and that of the data. Regarding the standard deviation of R&D intensity, the model without advertising does not predict the monotonically decreasing relationship with firm size that we see in the data, while the model with advertising matches this observation (with the exception of the last quintile). From that perspective, the model with advertising does a better job at explaining the evolution of second order moments of firm growth and R&D intensity.

⁴⁸This implies that a 10-product firm would find it 14.5% more expensive to grow by a given rate than it would be for 10 firms of one product each.

Figure 6: Standard deviations (relative to 1st quintile): Models vs Data.



Notes: Firms are ranked in size quintiles according to their normalized level of sales (sales as a ratio of average sales in the same year). R&D intensities are measured as the ratio of total R&D expenditures to total sales within each group. The standard deviation in the first quintile is normalized to one.

Results We assume that a time- and size-invariant subsidy $s \in (0, 1)$ is given to each type of firm conducting R&D. This subsidy is financed by a lump-sum tax on households. The R&D cost functions now read $R_e(x_e) = (1 - s)\nu\bar{Q}x_e$, $R_z(z_j) = (1 - s)\hat{\chi}q_jz_j^{\hat{\psi}}$ and $R_x(X_n, n) = (1 - s)\tilde{\chi}\bar{Q}X_n^{\tilde{\psi}}n^\sigma$.

Figure 7 shows the percentage change, with respect to the no-subsidy allocation ($s = 0$), in growth, entry and internal innovation rates, under both calibrated economies and for different levels of the subsidy. In line with previous studies, we find that the subsidy increases growth. This is because innovation becomes cheaper, which fosters entry and raises R&D expenditure for all firms.

More importantly, we find that the subsidy has a bigger impact on growth in the economy with advertising and constant returns to scale in the R&D technology (red dashed line in Figure 7). For instance, in the economy with advertising, a 50% subsidy increases the growth rate to $g = 2.79\%$, while in the economy without advertising a subsidy of the same size increases the rate to $g = 2.67\%$, i.e. 0.12 percentage points lower. This differential gets bigger with the subsidy. For instance, a subsidy of 75% increases the growth rate by 0.29 percentage points more in the economy with advertising.

In the economy with advertising, the subsidy has a much larger positive impact on entry, as R&D is cheaper and smaller firms benefit more from innovating through the advertising spillover channel. The impact of the subsidy on the internal innovation rate is also stronger. Both of these contribute to growth disproportionately more than they do in the economy without advertising.

Another noteworthy effect lies in Figure 8, which plots the change in the external innovation rate for different firm sizes for a change from $s = 0\%$ to $s = 20\%$. When there are constant returns to scale in R&D and the spillover effect in advertising is active (red dashed line), firms are more responsive to the subsidy in terms of external R&D. This is because in the economy without advertising, larger firms are less efficient in conducting R&D, which dampens their response to the subsidy. This is particularly true for larger firms as the difference in external R&D response to a given change in subsidies gets larger with firm size.

Figure 7: Percentage change, with respect to $s = 0$, in growth (g), entry (x_e/F), and internal innovation (z) rates, in both calibrated economies (dashed red line: economy calibrated with advertising and constant returns in R&D [Table 2]; solid blue line: economy calibrated with no advertising and decreasing returns to scale in R&D [Table 6]).

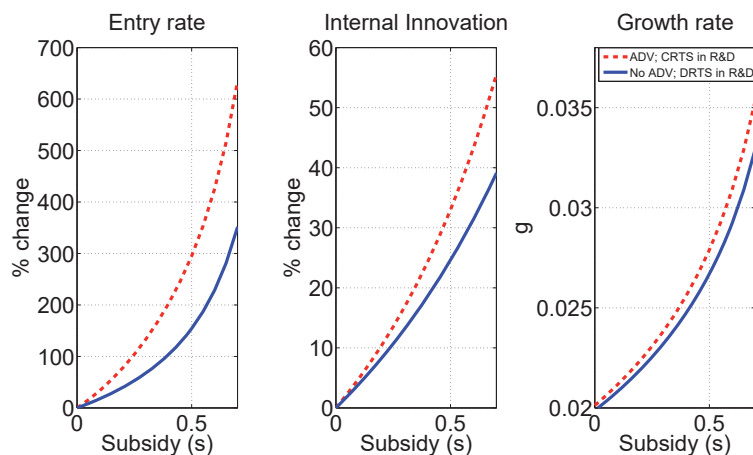
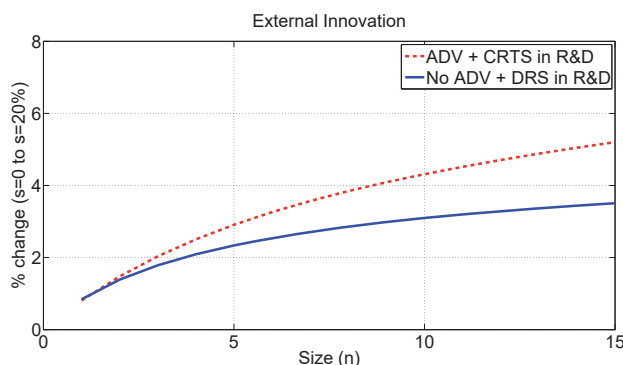


Figure 8: Percentage change in external innovation intensity for an increase from $s = 0\%$ to $s = 20\%$, across firm sizes, under both calibrated economies.



Our results suggest that identifying the source of technical efficiency in producing innovation across firm sizes has very relevant implications for the impact of industrial policies directed to increase growth.

8 Conclusion

In this paper, we have presented a model of firm dynamics and endogenous growth through product innovations that explicitly incorporates advertising decisions by firms. In modeling advertising, we have been inspired by observations from the empirical marketing literature, specifically the existence of advertising spillovers across goods. In our calibrated model, smaller firms benefit more in terms of decreased advertising costs from brand extension. Such extensions are achieved through external R&D, and smaller firms are, therefore, relatively more concerned with conducting innovative activities, as these firms benefit more from

advertising additional product lines due to the spillover effect. This mechanism generates the empirical observation that R&D intensity is decreasing with firm size even in the absence of differences in R&D technology across firm size.

We have shown that the model can both qualitatively and quantitatively match empirical regularities across sizes in sales growth, R&D intensity and advertising intensity. On the basis of our calibration, we have shown that advertising is detrimental to growth (and welfare) because it crowds out innovation investment within the firm. Even though advertising increases aggregate consumption contemporaneously by shifting preferences toward certain goods, it substantially decreases long term growth as it shifts incentives away from productivity-enhancing activities. Key to this argument is that R&D and advertising are strategic substitutes in the equilibrium of the calibrated model. We confirm this substitution empirically by exploiting exogenous cross-state and time variation in the cost of innovation arising from changes in the tax treatment of R&D in the U.S. from the 1980s onwards.

Based on these results, we have proposed a set of growth-enhancing industrial policies. First, we have shown that advertising taxes are relatively ineffective because the elasticity of advertising to sales is low and outside firms are deterred from entry. Both of these mute the effects of the tax on growth. Second, R&D subsidies have a higher impact on growth in an economy with advertising and constant returns to scale in R&D than in one in which there is no advertising and where larger firms are technologically less efficient in conducting innovative activities.

Identifying the source of decreasing R&D intensity with firm size is then an important question. Further understanding the consequences of R&D size dependence and studying the role of other types of intangible investments on innovation and firm dynamics remain interesting avenue for future research.

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A Derivations and Proofs

A.1 Value Functions

The value function for a typical incumbent firm with intrinsic quality portfolio \mathbf{q} and n products at time t , denoted by $V_{n,t}(\mathbf{q})$, is given by:

$$\begin{aligned}
 V_{n,t}(\mathbf{q}) = & \max_{x_t, \{z_{j,t}\}} \left\{ \sum_{q_j \in \mathbf{q}} \left(\tilde{\pi}_t q_j - \hat{\chi} z_{j,t}^{\hat{\psi}} q_j \right) \Delta t - \tilde{\chi} n^{\sigma + \tilde{\psi}} x_t^{\tilde{\psi}} \bar{Q}_t \Delta t + \gamma_t \bar{Q}_t n^{\frac{n}{1-\zeta}} \right. \\
 & + e^{-r_{t+\Delta t} \Delta t} \left(\sum_{q_j \in \mathbf{q}} \left[z_{j,t} \Delta t + o(\Delta t) \right] V_{n,t+\Delta t} \left(\mathbf{q} \setminus \{q_j\} \cup_+ \{q_j(1 + \lambda^I)\} \right) \right. \\
 & \quad + \sum_{q_j \in \mathbf{q}} \left[\tau_t \Delta t + o(\Delta t) \right] V_{n-1,t+\Delta t} \left(\mathbf{q} \setminus \{q_j\} \right) \\
 & \quad + \left[n x_t \Delta t + o(\Delta t) \right] \int_0^1 V_{n+1,t+\Delta t} \left(\mathbf{q} \cup_+ \{q_j(1 + \lambda^E)\} \right) dj \\
 & \quad \left. \left. + \left[1 - \sum_{q_j \in \mathbf{q}} z_{j,t} \Delta t - \sum_{q_j \in \mathbf{q}} \tau_t \Delta t - n x_t \Delta t - o(\Delta t) \right] V_{n,t+\Delta t}(\mathbf{q}) \right) \right\} \\
 & + o(\Delta t)
 \end{aligned}$$

where $o(\Delta t)$ has the property $\lim_{\Delta \rightarrow 0} \frac{o(\Delta t)}{\Delta t} = 0$. Here, we use the fact that, for each Poisson arrival rate $k \in \{z, x, \tau\}$, the term $k \Delta t + o(\Delta t)$ (respectively, $1 - k \Delta t - o(\Delta t)$) approximates the probability of exactly one Poisson event (respectively, zero Poisson events) within an interval of short length (i.e., for small $\Delta > 0$). The probability of two or more events is equal to $o(\Delta t)$ in the limit as $\Delta \rightarrow 0$.

The interpretation of the right-hand side of the equation is as follows. The first line includes flow intrinsic profits for each good in the firm's portfolio, net of internal and external R&D costs, with the last term being extrinsic flow profits; the second, third, fourth and fifth lines include the different scenarios that can arise at the next instant, all of which are exponentially discounted by the interest rate $r_{t+\Delta t} \Delta t$: an internal innovation on some good j , with overall probability $\sum_{q_j \in \mathbf{q}} [z_{j,t} \Delta t + o(\Delta t)]$, which advances the quality portfolio by a factor of λ^I on the newly innovated product (second line); external innovations, either by an external firm on some currently held good, which occurs with overall probability $\sum_{q_j \in \mathbf{q}} [\tau_t \Delta t + o(\Delta t)]$ (third line), or by the firm in question over a randomly drawn good from the $[0, 1]$ continuum, which occurs with overall probability $\sum_{q_j \in \mathbf{q}} [n x_t \Delta t + o(\Delta t)]$ (fourth line). With the remaining probability, none of these events occurs, and both the quality portfolio and the number of products of the firm stay the same (fifth line).

To obtain Equation (18), simply subtract $e^{-r_{t+\Delta t} \Delta t} V_{n,t}(\mathbf{q})$ from both sides, divide every term by Δt , and take the continuous-time limit as $\Delta \rightarrow 0$ (using the fact that $\lim_{\Delta \rightarrow 0} \frac{o(\Delta t)}{\Delta t} = 0$ and $\lim_{\Delta \rightarrow 0} \frac{1 - e^{-r_{t+\Delta t} \Delta t}}{\Delta t} = r_t$).

As for entrants, the value function is

$$\begin{aligned}
 V_{0,t} = & \max_{x_{e,t} > 0} \left\{ -\nu x_e \bar{Q} \Delta t + e^{-r_{t+\Delta t} \Delta t} \left(\left[x_{e,t} \Delta t + o(\Delta t) \right] \mathbb{E}_j \left\{ V_{1,t+\Delta t}(\{q_j(1 + \lambda^E)\}) \right\} \right. \right. \\
 & \left. \left. + \left[1 - x_{e,t} \Delta t - o(\Delta t) \right] V_{0,t+\Delta t} \right) \right\} + o(\Delta t)
 \end{aligned}$$

where $V_{0,t}$ denotes firm value at $(n, \mathbf{q}) = (0, \emptyset)$. The first line includes the cost of entry (first term), and the value of the firm in the event of entry at the next instant, which occurs with prob-

ability $\left[x_{e,t}\Delta t + o(\Delta t) \right]$. The second line shows that the firm remains with zero products with the complementary probability. Once again, the probability of two or more such events occurring simultaneously is a higher-order term inside $o(\Delta t)$.

To obtain Equation (19), once again subtract $e^{-r_t+\Delta t}\Delta t V_{0,t}$ from both sides, divide through by Δt , and take the limit as $\Delta \rightarrow 0$.

A.2 Proof of Proposition 1

Assume that $z_{j,t} = z_t, \forall j \in [0, 1]$, a result that we prove independently in Proposition 3. At any time t , aggregate quality is given by:

$$\begin{aligned} \bar{Q}_{t+\Delta t} = & \underbrace{\left[\tau_t \Delta t + o(\Delta t) \right] (1 + \lambda^E) \bar{Q}_t}_{\text{One external innovation}} + \underbrace{\left[z_t \Delta t + o(\Delta t) \right] (1 + \lambda^I) \bar{Q}_t}_{\text{One internal innovation}} \\ & + \underbrace{\left[1 - \tau_t \Delta t - z_t \Delta t - o(\Delta t) \right] \bar{Q}_t}_{\text{No innovations}} + \underbrace{o(\Delta t)}_{\text{2 or more innovations}} \end{aligned}$$

The interpretation of this equation is straightforward: after an instant of length Δt , aggregate intrinsic quality either increases by a fixed factor, due to an external innovation or an internal one (with probability $\tau_t \Delta t + o(\Delta t)$ and $z_t \Delta t + o(\Delta t)$, respectively), or it remains the same (with the complementary probability). The probability of two or more innovations is a second-order event. Subtracting \bar{Q}_t from both sides, dividing through by Δt , taking the limit as $\Delta \rightarrow 0$ and using that $\lim_{\Delta \rightarrow 0} \frac{o(\Delta t)}{\Delta t} = 0$, gives $\dot{\bar{Q}}_t = \tau \lambda^E \bar{Q}_t + z \lambda^I \bar{Q}_t$. Therefore,

$$g = \tau \lambda^E + z \lambda^I$$

as we sought to show. \square

A.3 Proof of Proposition 2

Let μ_n denote the equilibrium share of incumbent firms that own $n \geq 1$ product lines, such that $\mu_n \in [0, 1], \forall n$, and $\sum_{n=1}^{+\infty} \mu_n = 1$. The invariant distribution must satisfy the following flow equations:

<u># products</u>	<u>Inflows</u>	<u>Outflows</u>
$n = 0 :$	$F \mu_1 \tau$	$= x_e$
$n = 1 :$	$F \mu_2 2\tau + x_e$	$= F \mu_1 (x_1 + \tau)$
$n \geq 2 :$	$F \mu_{n+1} (n+1) \tau + F \mu_{n-1} (n-1) x_{n-1}$	$= F \mu_n n (x_n + \tau)$

The left-hand (or right-hand) side of these equalities describes the mass of firms that enters into (or exits out of) the state identified by the first column. For instance, for $n \geq 2$, inflows are given by the mass of size- $(n+1)$ incumbent firms that lost a product through creative destruction, and the mass of size- $(n-1)$ incumbent firms that gained one through external innovation. Similarly, outflows are given by the mass of size- n incumbents that either lost a product to another incumbent, or gained one thanks to external innovation. Because innovation allows firms to gain or lose only one product at a time, potential entrants (entering at rate x_e) only feature as a flow out of $n = 0$, or into $n = 1$.

We prove that Equation (22) is the solution to the above system of flow equations by use of mathematical induction. Specializing the formula to $n = 1$, and using the convention that $\prod_{i=1}^0 x_i = 1$, we have

$$\mu_1 = \frac{x_e}{F\tau}$$

which is true by the first flow equation. For $n = 2$, Equation (22) gives

$$\mu_2 = \frac{x_e x_1}{2F\tau^2}$$

which can be written as $2F\tau^2\mu_2 = x_1x_e$. Adding $x_e\tau$ to both sides and dividing through by $F\tau$ we obtain $\mu_2 2\tau + \frac{x_e}{F} = \frac{x_e}{F} \left(\frac{x_1}{\tau} + 1 \right)$. Rearranging, we obtain exactly the flow equation for $n = 2$.

Having shown that Equation (22) holds for $n = 1$ and $n = 2$, it is enough to show that it holds for $n + 1$, assuming it does for n and $n - 1$, where $n \geq 2$ is arbitrary. Accordingly, suppose $\mu_{n-1} = \frac{x_e}{F} \frac{\prod_{i=1}^{n-2} x_i}{(n-1)\tau^{n-1}}$ and $\mu_n = \frac{x_e}{F} \frac{\prod_{i=1}^{n-1} x_i}{n\tau^n}$. Plugging these into the third flow equation, we have

$$F\mu_{n+1}(n+1)\tau + x_e \frac{\prod_{i=1}^{n-2} x_i}{\tau^{n-1}} x_{n-1} = x_e \frac{\prod_{i=1}^{n-1} x_i}{\tau^n} (x_n + \tau)$$

Simplifying,

$$\begin{aligned} F\mu_{n+1}(n+1)\tau &= x_e \left(\frac{\prod_{i=1}^{n-1} x_i}{\tau^n} (x_n + \tau) - \frac{\prod_{i=1}^{n-2} x_i}{\tau^{n-1}} x_{n-1} \right) \\ &= x_e \left(\frac{\prod_{i=1}^n x_i}{\tau^n} + \tau \frac{\prod_{i=1}^{n-1} x_i}{\tau^n} - \frac{\prod_{i=1}^{n-1} x_i}{\tau^{n-1}} \right) \\ &= x_e \frac{\prod_{i=1}^n x_i}{\tau^n} \end{aligned}$$

which implies that $\mu_{n+1} = \frac{x_e}{F} \frac{\prod_{i=1}^n x_i}{(n+1)\tau^{n+1}}$, what we wanted to show. \square

A.4 Proof of Proposition 3

We find Γ and $\{\Upsilon_n\}_{n=1}^{+\infty}$ using the method of undetermined coefficients. Plugging the guess $V_n(\mathbf{q}_f) = \Gamma \sum_{q_j \in \mathbf{q}_f} q_j + \Upsilon_n \bar{Q}$ into (18), we get that

$$\begin{aligned} r\Gamma \sum_{q_j \in \mathbf{q}_f} q_j + r\Upsilon_n \bar{Q} &= \max_{\substack{x \in [0, \bar{x}] \\ \{z_j \in [0, \bar{z}]\}_{\mathcal{J}_f}}} \left\{ \sum_{q_j \in \mathbf{q}_f} \left[z_j \Gamma \lambda^I q_j + \tilde{\pi} q_j + \tau \left((\Upsilon_{n-1} - \Upsilon_n) \bar{Q} - \Gamma q_j \right) - \widehat{\chi} z_j^{\widehat{\psi}} q_j \right] \right. \\ &\quad \left. + nx \left(\Gamma \bar{Q} (1 + \lambda^E) + (\Upsilon_{n+1} - \Upsilon_n) \bar{Q} \right) - \tilde{\chi} n^{\sigma + \tilde{\psi}} x^{\tilde{\psi}} \bar{Q} + \gamma \bar{Q} n^{\frac{n}{1-\zeta}} \right\} + \Upsilon_n \bar{Q} g \end{aligned}$$

Equating the terms with q_j and \bar{Q} gives the following pair of optimization problems:

$$\begin{aligned} (q_j) : \quad r\Gamma &= \max_{z_j} \left\{ \tilde{\pi} + z_j \Gamma \lambda^I - \tau \Gamma - \widehat{\chi} z_j^{\widehat{\psi}} \right\} \\ (\bar{Q}) : \quad (r-g)\Upsilon_n &= \max_{x_n} \left\{ (\Upsilon_{n-1} - \Upsilon_n) n \tau + nx_n \left(\Gamma (1 + \lambda^E) + \Upsilon_{n+1} - \Upsilon_n \right) \right. \\ &\quad \left. - \tilde{\chi} n^{\sigma + \tilde{\psi}} x_n^{\tilde{\psi}} + \gamma n^{\frac{n}{1-\zeta}} \right\} \end{aligned}$$

The first-order conditions are:

$$\begin{aligned} z_j &= \left(\frac{\Gamma \lambda^I}{\widehat{\psi} \widehat{\chi}} \right)^{\frac{1}{\widehat{\psi}-1}} \\ x_n &= n^{\frac{1-\sigma-\tilde{\psi}}{\widehat{\psi}-1}} \left(\frac{\Gamma (1 + \lambda^E) + \Upsilon_{n+1} - \Upsilon_n}{\widehat{\psi} \tilde{\chi}} \right)^{\frac{1}{\widehat{\psi}-1}} \end{aligned}$$

respectively. Assuming that there is positive entry in equilibrium ($x_e > 0$), we can exploit the free-entry condition $V_0 = 0$ in (19) to get that

$$\Gamma = \frac{\nu - \Upsilon_1}{1 + \lambda^E}$$

This means that the optimal internal R&D investment by incumbents is

$$z_j = \left(\frac{\lambda^I(\nu - \Upsilon_1)}{\widehat{\psi}\widehat{\chi}(1 + \lambda^E)} \right)^{\frac{1}{\widehat{\psi}-1}}$$

so $z_j = z, \forall j \in [0, 1]$. Back into the optimality condition for z , we can obtain the implied rate of creative destruction:

$$\tau = \frac{1}{\nu - \Upsilon_1} \left[\widehat{\pi} - \widehat{\chi} \left(\frac{\lambda^I(\nu - \Upsilon_1)}{\widehat{\chi}\widehat{\psi}(1 + \lambda^E)} \right)^{\frac{\widehat{\psi}}{\widehat{\psi}-1}} \right] - \frac{\rho}{1 + \lambda^E}$$

where we have used that $g = r - \rho$ from the Euler equation, and $g = \tau\lambda^E + z\lambda^I$. It remains to find an expression for Υ_n . Using free-entry, we know

$$x_n = n^{\frac{1-\sigma-\widehat{\psi}}{\widehat{\psi}-1}} \left(\frac{\nu - \Upsilon_1 + \Upsilon_{n+1} - \Upsilon_n}{\widetilde{\psi}\widetilde{\chi}} \right)^{\frac{1}{\widehat{\psi}-1}}$$

Back into the second maximization problem, we get the second-order difference equation

$$(r - g)\Upsilon_n = (\Upsilon_{n-1} - \Upsilon_n)n\tau + \gamma n^{\eta/(1-\zeta)} + \widetilde{\chi}(\widetilde{\psi} - 1)n^{-\frac{\sigma}{\widehat{\psi}-1}} \left[\frac{\nu - \Upsilon_1 + \Upsilon_{n+1} - \Upsilon_n}{\widetilde{\psi}\widetilde{\chi}} \right]^{\frac{\widehat{\psi}}{\widehat{\psi}-1}}$$

Solving for Υ_{n+1} gives (24). Using $V_0 = 0$ and the guess $V_n = \Gamma \sum_j q_j + \Upsilon_n \bar{Q}$, it is clear that the boundary condition for this difference equation must then be $\Upsilon_0 = 0$. \square

B Welfare Effects of Advertising

Having argued that advertising is detrimental for growth within the calibrated economy, we now turn to welfare effects. There exists a long-lasting debate in Economics about the welfare implications of advertising. This literature has studied the effects of advertising on welfare through mainly two potential channels: informative advertising and taste-shifting advertising.

Models of informative advertising are theories in which advertising can be used to remove frictions in markets with imperfect information by providing relevant information about a product quality (or simply about the existence of the product). Such approach can be found for instance in Nelson (1974), Butters (1977), and Grossman and Shapiro (1984). In that context, advertising could be welfare improving. On the other hand, Becker and Murphy (1993) argue that advertising could simply be used to alter consumer preferences by creating favorable associations to the good that is being advertised. Kihlstrom and Riordan (1984) and Milgrom and Roberts (1986) further argue that expenditures in uninformative advertising could still signal a product quality and hence provide ex-post information.

Another strand of the literature considers advertising as a pure taste shifter by manipulating consumer preferences (see among others Dixit and Norman (1978), Becker and Murphy (1993) and Benhabib and Bisin (2002, 2011)). Molinari and Turino (2015) consider the case of purely combative advertising, in which advertising does not affect consumer preferences in equilibrium but firms nev-

ertheless advertise to maintain their market shares. In this case, advertising expenditures are a pure waste of resources. When advertising directly affects tastes, a relevant question is whether welfare should be compared using ex-ante or ex-post preferences. In addition, advertising could distort prices and monopoly power, in which case advertising could be welfare decreasing even when measured by ex-post preferences (Dixit and Norman (1978), Benhabib and Bisin (2002)). In this section, we argue that one should consider yet another side effect of advertising when quantifying its welfare implications, namely its effect on R&D investment, innovation and economic growth.

To compute ex-post welfare along a BGP, some algebra shows that we can rewrite the household utility in equilibrium as:

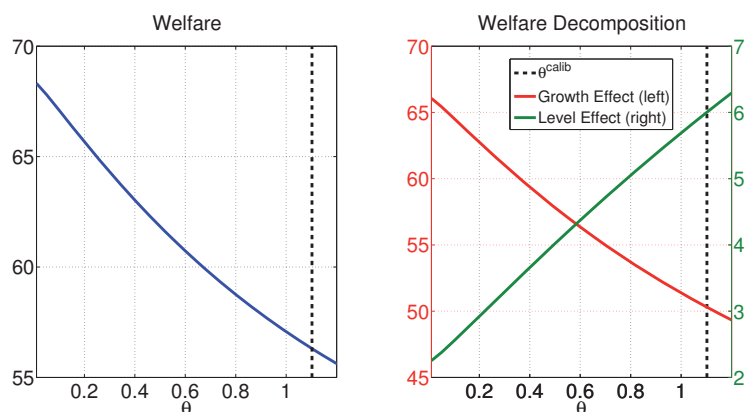
$$U(C_0, g) = \underbrace{\ln(C_0)/\rho}_{\text{Level effect}} + \underbrace{g/\rho^2}_{\text{Growth effect}} \quad (30)$$

where C_0 is proportional to $\bar{Q}_0 = 1$ (a normalization). To study the effect of advertising, we compare ex-post welfare in the calibrated economy across different advertising efficiency levels (θ).

The first term in the ex-post welfare decomposition accounts for a “level effect” of advertising: by conducting advertising, firms expand demand contemporaneously, which increases the perceived utility derived from aggregate consumption. This is because advertising ultimately constitutes a demand shifter for the firm by effectively changing consumer’s preferences, as seen in Equation (7). Additionally, as we have argued above, advertising critically shapes the dynamic incentives to conduct R&D and, therefore, has an indirect effect on growth. This is captured by the second term, which we label the “growth effect” of advertising.⁴⁹

While advertising unambiguously increases welfare through the level effect, its impact through the growth effect is negative in our calibrated model. Figure 9 plots welfare and its level-growth decomposition as a function of θ . Once again, the level effect is unambiguously positive and, as we have just discussed, advertising decreases growth in our calibration, so the final effect is ambiguous. Using our calibrated set of parameters, however, we find that welfare is reduced because the growth effects dominates. According to this result, a policy scheme in which the advertising sector is taxed can be welfare-improving for the economy. We analyze this implication in detail in Section 7.1.

Figure 9: Welfare decomposition, for different values of advertising efficiency (θ). The calibrated value of θ is marked by the dashed vertical line.



⁴⁹In Appendix G.2.3 we present a version of the model in which advertising does not increase the value of consumption in equilibrium. Making welfare comparisons in that case would be equivalent to comparing utility using ex-ante preferences. In that case, welfare is unambiguously decreasing in advertising efficiency (θ).

In sum, in our calibrated economy R&D and advertising crowd each other out, which is relevant for our subsequent policy analysis. Generally, our results add to the existing debate about advertising and welfare by showing that, even in a model in which advertising effectively increases utility (through a “level effect” in our model but potentially from information in alternative formulations), one should consider the potential side effects of cheaper advertising. We show that one such side effect is its impact on economic growth in the long run coming from the substitution between R&D and advertising.

C Effect of R&D Subsidies on Other Expenses

Table 8: Effect of R&D subsidy on selling, general and administrative expenses (excluding R&D and Advertising) at the firm level

	(1)	(2)	(3)	(4)	(5)
	$\log\left(\frac{\text{Other Exp.}}{\text{Sales}}\right)$	$\log\left(\frac{\text{Other Exp.}}{\text{Sales}}\right)$	$\log\left(\frac{\text{Other Exp.}}{\text{Sales}}\right)$	$\log\left(\frac{\text{Other Exp.}}{\text{Sales}}\right)$	$\log\left(\frac{\text{Other Exp.}}{\text{Sales}}\right)$
<i>Tax credit dummy</i>	0.0309 (0.0151)				
<i>State credit rate</i>		0.686 (0.168)			
<i>Tax-adjusted state rate</i>			0.696 (0.184)		
<i>Effective state rate</i>				0.844 (0.236)	
<i>R&D user cost</i>					-0.939 (0.264)
Controls	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓
Industry FE	✓	✓	✓	✓	✓
State FE	✓	✓	✓	✓	✓
Observations	28660	27376	27376	25260	25260
R^2	0.57	0.57	0.57	0.57	0.57

Notes: Data from Compustat from 1950 to 2009, Wilson (2009) and Falato and Sim (2014). Controls include sales (logarithm), age, financial constraint and state tax and selling. Firm age is measured as the elapsed time since the first observation in the data. Our measure of financial constraints is sale minus purchases of common and preferred stock, divided by sales. Standard errors are clustered at the state level (in parentheses).

D Data

Table A.1: Descriptive statistics. Compustat data from 1980 to 2015. Means and standard deviations in our sample of firms performing R&D and advertising vs. all firms in Compustat. Reported shares are computed for firm/year observations with respect to the whole Compustat universe.

	ADV and R&D firms		All firms	
	Mean	St. dev.	Mean	St. dev.
Sales (million U.S.D.)	2539.6	12222.5	1823.4	10014.5
Age (in years)	14.6	12.72	14.44	12.65
Equity Fin./Sales	0.269	6.09	1.38	80.14
R&D (million U.S.D.)	113.5	580.8	63.55	397
ADV (million U.S.D.)	73.2	402.9	46.1	271.1
R&D/Sales	0.102	0.136	0.109	0.157
ADV/Sales	0.037	0.068	0.036	0.067
R&D/ADV	26.4	229.56		
Share of firms doing R&D		33.07%		
Share of sales from firms doing R&D		46.23%		
Share of firms doing ADV		33.86%		
Share of sales from firms doing ADV		36.16%		
Share of ADV by firms doing R&D		58.02%		

Table A.2: Facts 1 to 4 with assets and employment as measures of size.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$\frac{\Delta Assets}{Assets}$	$\log\left(\frac{R\&D}{Assets}\right)$	$\log\left(\frac{Adv}{Assets}\right)$	$\log\left(\frac{R\&D}{Adv}\right)$	$\frac{\Delta Emp}{Emp}$	$\log\left(\frac{R\&D}{Emp}\right)$	$\log\left(\frac{Adv}{Emp}\right)$	$\log\left(\frac{R\&D}{Adv}\right)$
$\log(Assets)$	-0.0869 (0.0121)	-0.116 (0.00830)	-0.0687 (0.00985)	-0.0475 (0.0117)				
$\log(Emp)$					-0.0200 (0.00433)	-0.0657 (0.0112)	-0.00529 (0.0129)	-0.0604 (0.0136)
<i>Firm Age</i>	-0.00464 (0.000933)	0.00618 (0.00152)	0.00590 (0.00186)	0.000281 (0.00216)	-0.00306 (0.000313)	0.00300 (0.00191)	0.00181 (0.00227)	0.00119 (0.00226)
<i>Fin. Const.</i>	-0.00225 (0.000947)	-0.00176 (0.00233)	-0.000601 (0.00284)	-0.00116 (0.00490)	0.000648 (0.00127)	-0.0000700 (0.00214)	0.00142 (0.00330)	-0.00149 (0.00498)
Constant	0.652 (0.126)	-2.979 (0.370)	-4.334 (0.301)	1.355 (0.228)	0.127 (0.0622)	0.696 (0.540)	-0.482 (0.456)	1.177 (0.220)
Time FE	✓	✓	✓	✓	✓	✓	✓	✓
Industry FE	✓	✓	✓	✓	✓	✓	✓	✓
Observations	21954	24856	24856	24856	23145	23427	23427	23427
R^2	0.07	0.47	0.33	0.44	0.05	0.53	0.28	0.45

Notes: Sales, R&D and advertising expenditures are in thousands of U.S.D. Employment is in thousands of employees. See Table 1 for details on the sample selection. Standard errors are clustered by firm (in parentheses). Legend: 10%; 5% ; 1%.

E The BGP Algorithm

To compute the equilibrium variables in the BGP, we loop over the fixed point of $\bar{\Phi}_t$ by using $\bar{\Phi}_t = \Phi^* \bar{Q}_t$, with Φ^* from Equation (16). In light of the above discussion, we check for convergence by imposing that the value of firm size, Υ_n , converges to a line as n grows larger. We truncate the size space at some large $N \in \mathbb{N}$. The following describes the steps of the algorithm:

1. Guess a number $\Phi^* > 0$.
2. Compute Υ_1 by imposing $\Upsilon_2 = 2\Upsilon_1$ in Equation (24) at $n = 1$.
 - (a) Compute g from (20), τ implied by (21), Γ from (23), and z from (25).
 - (b) Compute $\{\Upsilon_n\}_{n=2}^{+\infty}$ using (24) forward from Υ_1 and $\Upsilon_0 = 0$, $\{x_n\}_{n=1}^{+\infty}$ from (26), and F/x_e using that $\sum_{n=1}^N \mu_n = 1$, where μ_n comes from (22).
3. Verify convergence of firm size distribution.⁵⁰ If there is no convergence by iteration $k \in \mathbb{N}$, go back to step 2 with the new guess for Υ_1 to be the solution to (24) at $n = 1$ when $\Upsilon_2 = 2\Upsilon_1 - \varepsilon_k$, for a small $\varepsilon_k > 0$.
4. Compute Φ^* as the solution to (16), and compare it to the initial guess. If it is too far, go back to step 1 using this solution as the new guess.

In step 2, we compute the maximum value for Υ_1 such that Υ_n can be weakly concave (i.e., $(\Upsilon_{n+1} - \Upsilon_n)$ decreasing). In particular, we force Υ_n to be straight line from $n = 1$ to $n = 2$ (note $\Upsilon_2 - \Upsilon_1 = \Upsilon_1 - \Upsilon_0$). If μ_n does not converge, it must be because $(\Upsilon_n - \Upsilon_{n-1})$ has not settled to a flat line as n has approached N , which means that the guess for Υ_1 was incorrect. Then, we iterate on new guesses for Υ_1 to allow for more concavity on the Υ_n sequence (indeed, note that in any iteration $k \geq 1$ we always start the Υ_n sequence at a Υ_1 such that $\Upsilon_2 - \Upsilon_1 < \Upsilon_1 - \Upsilon_0$). In step 3, we in turn bisect the new guess by a factor of ten on each new iteration, i.e. $\varepsilon_{k+1} = \varepsilon_k/10$. Finally, in step 4, we drop the old Φ^* guess in case of no convergence, and use the resulting fixed-point as the new guess.

F Comparative Statics on BGP (Numerical Example)

We present a few comparative statics results in order to illustrate our main advertising-innovation trade-off.

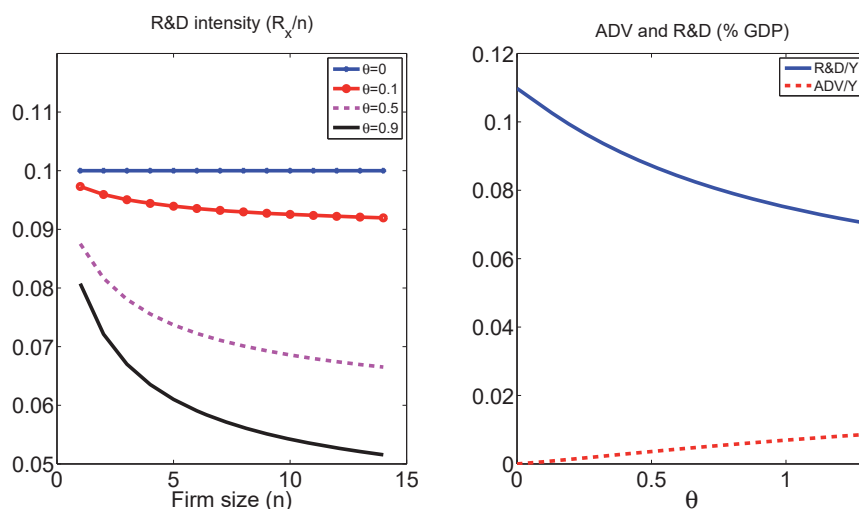
To illustrate the qualitative features of the model, this section presents a set of comparative statics exercises on outcomes of the balanced growth path equilibrium with respect to advertising efficiency (θ) and the degree of returns to scale in R&D ($\tilde{\psi} + \sigma$). The purpose of this section is threefold: (i) to show how our model can generate decreasing R&D intensity with firm size under constant returns to scale in R&D; (ii) to confirm that the model can potentially deliver both substitutability and complementarity between R&D and advertising; and (iii) to show that our model can be solved with increasing returns to scale in R&D technology. All the parameter values are set for expositional purposes.

Advertising Efficiency (θ) Figure 10 shows, as a function of firm size (n), BGP results for the expenditure intensity on external R&D (i.e. R_x/n) on the left panel, and the expenditure intensity on advertising (i.e. M_n/n) on the right panel for different values of advertising efficiency, θ (recall Equation (3)).⁵¹

⁵⁰We further verified that there exists a unique value of Υ_1 for which we obtain convergence for a given Φ^* .

⁵¹The parameter values used in this example are: $\rho = 0.02$, $\hat{\psi} = \tilde{\psi} = 1 - \sigma = 2$, $\lambda^E = \lambda^I = 0.05$, $\hat{\chi} = 0.1$, $\tilde{\chi} = 10$, $\nu = 2$, $\beta = 0.2$, $\eta = 0.5$ and $\zeta = 0.1$.

Figure 10: External R&D expenditure intensity in BGP (left panel), and R&D and advertising expenditures as a share of GDP in BGP, for different values of advertising efficiency, θ (right panel).



Note: External R&D expenditure intensity is normalized by aggregate intrinsic quality, \bar{Q} .

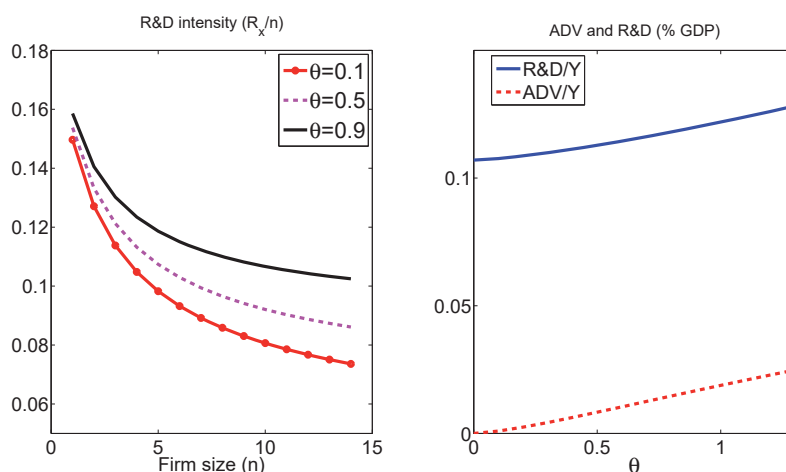
First, in an economy with no role for advertising ($\theta = 0$), we return to a Klette and Kortum (2004) environment in which innovation incentives are constant in size. The gain of acquiring new products is proportional to size, or $\Upsilon_n = n\Upsilon$, and therefore expenditures are constant in the cross-section of firms. When we introduce a motive for advertising ($\theta > 0$), all firms start investing in the latter. Moreover, because smaller firms benefit marginally more from expanding their product portfolios ($\eta < 1$), they invest more in R&D relative to larger firms.

As θ increases further (with all other parameters fixed), advertising becomes more efficient for all levels of firm size, and the aforementioned effects are reinforced. Moreover, since advertising now delivers higher returns, in the margin small firms now benefit even more than they did before, relative to larger firms. As a consequence, expenditures decrease faster with size (i.e. R_x/n and M_n/n become steeper). In other words, the advertising spillover effect tilts dynamic R&D incentives toward smaller firms. In this example, we can notice that R&D and advertising are substitutes since larger values of θ are associated with lower investment in R&D and larger advertising expenditures. Next, we provide an example in which R&D and advertising are complements.

R&D and advertising as complements We now show that our model can also generate complementarity between R&D and advertising. Figure 11 depicts the change in external R&D across firm size for different levels of advertising efficiency as well as total incumbent R&D and advertising as a share of GDP.⁵² In this example, R&D and advertising are complements. An increase in advertising efficiency is associated with larger investment in both advertising and R&D. As we shall see, whether R&D and advertising are complements or substitutes plays a crucial role in determining the growth and welfare effect of advertising. In Section 5.1, we calibrated our model to determine which case is the relevant one in the data.

⁵²The parameter values used in this example are: $\rho = 0.02$, $\hat{\psi} = \tilde{\psi} = 2$, $\sigma = -0.85$, $\lambda^E = \lambda^I = 0.05$, $\hat{\chi} = 1$, $\tilde{\chi} = 5$, $\nu = 2$, $\beta = 0.2$, $\eta = 0.9$ and $\zeta = 0.3$.

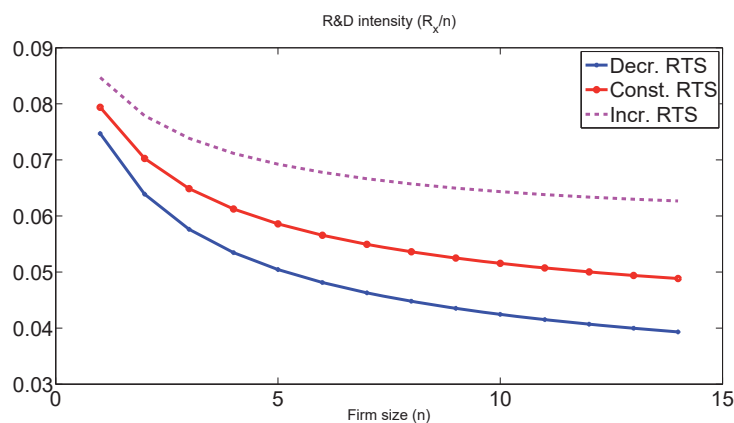
Figure 11: External R&D intensity in BGP (left panel), and R&D and advertising expenditures as a share of GDP in BGP, for different values of advertising efficiency, θ (right panel).



Note: External R&D expenditure intensity is normalized by aggregate intrinsic quality, \bar{Q} .

Returns to Scale in Innovation Finally, we show that the result that small firms are more innovation-intensive does not hinge on the relative degree of scalability in the returns of different types of innovation technologies. In Figure 12 we show three solutions of the advertising model, with decreasing, constant, and increasing returns to scale in external R&D, respectively.

Figure 12: Comparative statics for R&D intensity across n , for $\sigma + \tilde{\psi} = 1.05 > 1$ (decreasing returns), $\sigma + \tilde{\psi} = 1$ (constant returns), and $\sigma + \tilde{\psi} = 0.95 < 1$ (increasing returns).



Decreasing returns lower innovation incentives for all sizes with respect to constant returns, but make it even more profitable for small firms to invest more intensely into R&D (i.e., the R_x/n line steepens). Symmetrically, having increasing returns in innovation increases optimal intensity for all n . Interestingly, so long as the degree of increasing returns is not too strong, smaller firms may still optimally choose to invest relatively more in R&D in order to rip the benefits of advertising spillovers, as is the case in the figure. This occurs even though the marginal benefit from innovation is lower than in the other two cases (so that the R_x/n line flattens but it is still decreasing).

G Model Extensions

G.1 Patents and Major Innovations

In our baseline model, smaller firms are more innovative because of the advertising spillover effect, in spite of there being constant returns to scale in R&D. Besides differences in R&D intensity across size, however, the literature has emphasized other motives why smaller firms might be relatively more efficient in conducting innovations. One such literature has focused on patent behavior. Firstly, entrants and smaller firms typically produce relatively more major and radical innovations (when the quality of a patent is based on the number of external citations that it receives).⁵³ Secondly, these firms also tend to patent relatively more on average. In this section, we show that a simple extension of our baseline model with advertising can deliver both of these additional facts even in the absence of decreasing returns to scale in R&D.

We first demonstrate how to obtain the first stylized fact, namely that smaller firms and entrants generate more major technological improvements. Intuitively, since in our specification these firms invest relatively more than larger firms in external innovation because they benefit marginally more from the advertising effect, the result directly obtains if the innovation step size for external innovations is larger than that of internal innovations.

Let us assume that each innovation creates a new patent that potentially cites other patents that exist at the time the new patent is introduced. Following Akcigit and Kerr (2018), each innovation belongs to a technological cluster, and there exist two types of technological advances: follow-up and major advances. A major advance in a production line creates a whole new cluster of innovation, while a follow-up innovation belongs to the same cluster as the patent that it improves upon. Each patent within a technological cluster is assumed to cite all the previous innovations in the same cluster with some positive probability. Consequently, major advances in the model do not cite any other existing patent and potentially receive citations from follow-up innovations within the same cluster. Once a new major innovation creates a new technological cluster, the cluster that it replaces receives no more citations. Likewise, follow-up innovations can also receive citations from subsequent follow-up innovations in the same cluster. However, on average, major advances receive more citations than follow-up innovations.

Formally, we extend the baseline model by allowing external innovations to result in major technological advances with some probability. Whereas internal R&D can only result in a follow-up innovation, with step size $\lambda^I > 0$, external innovation can either be a follow-up within an existing technological cluster, or be a major advance and create a new technological cluster altogether. Let ω denote the probability with which a successful external innovation leads to a major technological advance. In this case, the step size for quality improvement is equal to $\lambda^H > \lambda^I$. With the remaining probability $(1 - \omega)$, the successful external innovation is a follow-up, which leads to a step size $\lambda^L \in (0, \lambda^H]$. In sum, for any product line j and a small interval $\Delta t > 0$, intrinsic quality is given by

$$q_{j,t+\Delta t} = q_{jt} + \begin{cases} \lambda^H q_{jt} & \text{w.prob. } \omega \tau_t \Delta t + o(\Delta t) & \text{[External, major advance]} \\ \lambda^L q_{jt} & \text{w.prob. } (1 - \omega) \tau_t \Delta t + o(\Delta t) & \text{[External, follow-up]} \\ \lambda^I q_{jt} & \text{w.prob. } z_{jt} \Delta t + o(\Delta t) & \text{[Internal]} \\ 0 & \text{w.prob. } 1 - \tau_t \Delta t - z_{jt} \Delta t - o(\Delta t) & \text{[No innovations]} \end{cases}$$

⁵³Indeed, Akcigit and Kerr (2018) show that the fraction of a firm's patent in the top patent quality decile is decreasing with firm size.

Therefore, the average quality improvement from a successful external innovation is equal to $\lambda^E \equiv \omega\lambda^H + (1 - \omega)\lambda^L$, which means that the growth and creative destruction rates of the economy are still given by (20) and (21), respectively. Therefore, our baseline model can be thought of a special case of the extended model with patents, and none of the results derived in the baseline case will change. Additionally, the model now predicts that smaller firms, while exhibiting a higher R&D intensity because of the advertising spillover effect, also produce relatively more major innovations.

Let us now show how the model can deliver the prediction that smaller firms tend to patent relatively more on average, and that their patents are of higher quality (when quality is measured by the number of external citation that it receives). For this, we can characterize expected patent citations. Let us assume that the probability that the n -th follow-up innovation cites all relevant past patents is κ^n , where $0 < \kappa < 1$. This generates a decline in the relative citation rate as a technological cluster ages. We can then derive the expected number of citations received by major as well as follow-up innovations. For major technological advances, this number is given by:

$$\mathbb{E}[cit^M] \equiv \underbrace{\frac{\tau\omega}{\tau+z} \cdot 0}_{\text{External and major innovation}} + \underbrace{\Lambda \left\{ \kappa + \Lambda \left[\kappa^2 + \Lambda (\kappa^3 + \dots) \right] \right\}}_{\text{Follow-up innovations}} = \sum_{j=1}^{\infty} (\kappa\Lambda)^j \quad (\text{G.1})$$

In the first term, $\frac{\tau\omega}{\tau+z}$ is the probability that a successful innovation in a given product line is external and major, which creates a new cluster and does not add a citation within the existing cluster. In the second term, we have defined

$$\Lambda \equiv \frac{\tau(1 - \omega) + z}{\tau + z}$$

as the probability of a follow-up innovation, coming either from an external or an internal innovation. The probability that such an innovation yields a single citation is κ , and therefore the probability of the n -th follow-up yielding a citation is $(\kappa\Lambda)^n$. The expected number of citations is then the sum of all such probabilities. Since $\kappa\Lambda < 1$, Equation (G.1) can be expressed as:

$$\mathbb{E}[cit^M] = \frac{\kappa\Lambda}{1 - \kappa\Lambda}$$

A similar derivation shows that the expected number of citations for the n -th follow-up innovation in a given technological cluster, denoted $\mathbb{E}[cit_n^F]$, is given by:

$$\mathbb{E}[cit_n^F] = \frac{\kappa^{n+1}\Lambda}{1 - \kappa\Lambda}$$

Therefore, $\mathbb{E}[cit_n^F] < \mathbb{E}[cit^M]$ for any number of follow-ups, $n \in \mathbb{N}$. Since in our baseline model with advertising smaller firms invest relatively more in external R&D, these firms also hold relatively more patents, and these patents are of higher quality on average (as measured by the number of external citations). In sum, our baseline model can generate the observed negative correlation between firm size and the fraction of top quality patents in the firm's patent portfolio, even when there exist non-decreasing returns to scale in the R&D technology.

G.2 Alternative Advertising Functions

The main mechanisms presented in this paper rely on two important aspects of advertising. The first one is that advertising acts as a demand shifter in a way that is comparable to the effect of increased intrinsic quality from innovation. This is a necessary condition for

the substitution between R&D and advertising that we obtain in our calibration exercise. Second, the existence of spillover in advertising implies that R&D intensity can be decreasing in firm size even in the absence of decreasing returns to scale in innovation. In this section, we show different alternative ways to model advertising which also lead to advertising being a demand shifter. This implies that the results that we obtain in this paper would still qualitatively hold under the different alternative models presented below.

G.2.1 Goodwill Accumulation

In our baseline model, we assume that the advertising decision is static. Advertising expenditures affect current demand but have no long-lasting effect on consumer demand. An alternative way of modeling advertising and its effects on demand which is often used in the literature is to assume that advertising expenditures accumulate over time to increase a brand equity (so-called goodwill). Goodwill in turns acts as a demand shifter. In discrete time settings, the evolution of the stock of goodwill for good j at time t ($G_{j,t}$) is usually modeled as follows:

$$G_{j,t} = \delta G_{j,t-1} + d_{j,t} \quad (\text{G.2})$$

where $\delta \in [0, 1]$ controls the rate at which goodwill depreciates.

Here, we show that allowing for goodwill accumulation would not change the results derived in our baseline model. We can also notice that the marketing literature shows that the depreciation rate is relatively high so that the effect of advertising on sales has almost entirely vanished after one year. In addition, the introduction of goodwill in the model only affects the decision of firms of different ages. Since the focus of our paper is on firm behavior across firm size, not modeling the evolution of goodwill over firm age is not a major concern.⁵⁴

We now present a discrete-time version of our model with goodwill and show that it does not significantly change the problem of intermediate good producers. We use Equation (G.2) as the evolution of goodwill over time.

The final good is now defined as:

$$Y_t = \frac{1}{1 - \beta} \int_0^1 q_{j,t}^\beta (1 + G_{j,t})^\beta y_{j,t}^{1-\beta} dj$$

The inverse demand function for good j is given by:

$$p_{j,t} = q_{j,t}^\beta (1 + G_{j,t})^\beta y_{j,t}^{-\beta}$$

Advertising goodwill ($G_{j,t}$) is thus a demand shifter. Intermediate good producers maximize their profit subject to the inverse demand function and the dynamics of goodwill. Operational profit (profit before R&D and advertising expenditures) can be written as:

$$\pi_{j,t} = (1 - \beta)^{\frac{1-\beta}{\beta}} \beta \left(\frac{\bar{Q}}{w_t} \right)^{\frac{1-\beta}{\beta}} q_j (1 + \delta G_{j,t-1} + d_{j,t})$$

which is similar to what was obtained in Section 4.4.

G.2.2 Advertising in Utility

In this section, we show that a slight modification of the baseline model which allows advertising to feature directly into the consumers' utility function delivers a very similar allocation, and identical qualitative predictions, as the model in the main text.

⁵⁴We can further notice that firm age does not significantly affect advertising intensity and the relative use of R&D and advertising in Table 1.

There is no final good sector and the household consumes goods $j \in [0, 1]$ directly. The representative household's preferences are now represented by:

$$U = \int_0^{+\infty} e^{-\rho t} \ln(C_t) dt$$

where C_t is a consumption aggregator over a mass-one continuum of quality-weighted good quantities, indexed by $j \in [0, 1]$, which takes the form:

$$C_t = \frac{1}{1-\beta} \int_0^1 \tilde{q}_{jt}^\beta y_{jt}^{1-\beta} dj$$

with $\beta \in (0, 1)$. The flow budget constraint is, therefore:

$$\dot{A}_t = r_t A_t + w_t - \int_0^1 p_{jt} y_{jt} dj$$

where $A_0 \geq 0$ is given, and p_{jt} is the price of good j . Each good variety j is produced with technology:

$$y_{jt} = \bar{Q}_t l_{jt}$$

where good $j = 0$ is the numeraire (so $p_{0,t} = 1, \forall t$).

Taking initial wealth A_0 as given, the representative consumer chooses a path for consumption to maximize utility subject to the flow budget constraint and the no-Ponzi condition $\lim_{t \rightarrow +\infty} e^{-\int_0^t r_s ds} A_t \geq 0$. The optimality condition for good j yields:

$$\omega_t p_{jt} = \frac{1}{C_t} \left(\frac{y_{jt}}{\tilde{q}_{jt}} \right)^{-\beta} \quad (\text{G.3})$$

where $\omega_t \geq 0$ is the Lagrange multiplier, solving

$$-\frac{\dot{\omega}_t}{\omega_t} = r_t - \rho$$

i.e., $\omega_t = \omega_0 e^{-\int_0^t (r_s - \rho) ds}$. Recalling that $\tilde{q}_{jt} = q_{jt} + \phi_{jt}$, Equation (G.3) says that the inverse demand function for goods from households firms is iso-elastic, with β being the price-elasticity. Solving the incumbent firm's problem, one can show easily that $\omega_t = \frac{1}{Y_t}$, where $Y_t \equiv \int_0^1 y_{jt} dj$ denotes aggregate output in the economy. Hence, the Euler equation reads $g_t = r_t - \rho$ and the demand function becomes identical to that of the baseline model (Equation (7)). The two models are therefore qualitatively equivalent.

G.2.3 “Wasteful” Combative Advertising

In our baseline model, advertising not only shifts demand but also has an effect on consumer utility and welfare. In Appendix B, we nevertheless showed that the calibrated version of the model suggests that advertising is welfare decreasing as the level effect (the increase in how consumers value their consumption) is more than offset by the negative effect of advertising on growth through the substitution between advertising and R&D at the firm level. In this section, we show that the same results can be obtained in a model in which advertising does not increase the value of consumption in equilibrium. This can be seen as a model of combative (or predatory) advertising in which the advertising efforts of each firm (partially) cancel out in equilibrium.

Let us define the final good as:

$$Y = \frac{1}{1-\beta} \int_0^1 q_j^\beta (1 + d_j - \iota \Phi^*)^\beta y_j^{1-\beta} dj$$

with $\iota \in [0, 1]$. This implies that the effectiveness of advertising at the good level is a function of the overall level of advertising expenditures in the economy (through Φ^* , i.e. the normalized aggregate extrinsic quality in the economy). Thus, the more other firms invest in advertising, the more one firm has to invest itself in order to obtain a given return to advertising.

We obtain the following demand function:

$$y_j = q_j(1 + d_j - \iota\Phi^*)p_j^{-\frac{1}{\beta}}$$

Intermediate good firms solve their profit maximization problem subject to this demand function and taking the overall level of advertising in the economy as given. From the firm's perspective, advertising acts as a demand shifter in the same way as in our baseline model. Moreover, it is easy to derive Y_t in equilibrium as:

$$Y = \left(\frac{\bar{Q}}{w}\right)^{\frac{1-\beta}{\beta}} (1-\beta)^{\frac{1-2\beta}{\beta}} [1 + (1-\iota)\Phi^*]\bar{Q}$$

If $\iota = 1$, advertising has no direct effect on consumer's utility. It can, nevertheless, have an effect on lifetime utility through its impact on the growth rate of the economy in a way that is similar to our baseline model. If $\iota = 0$, we return to our benchmark model.

G.2.4 Informative Advertising

In our baseline, model advertising is purely persuasive in the sense that it shifts demand toward advertised good though increased marginal utility. Alternatively, one could consider advertising as providing relevant information about the product quality (see for instance Nelson (1974), Butters (1977), Grossman and Shapiro (1984) or Milgrom and Roberts (1986)). In this case, advertising could be socially optimal as it could reduce uncertainty or improve the quality of consumer-firm match.

In this section, we propose a simple model of informative advertising with differentiated products. We look at a static model in which advertising is used to provide information about the quality of the goods. In particular, firms send an imperfect signal about their product quality through advertising. Consumers passively receive the information and update their prior about product quality.

Consumers maximize expected utility. We assume that the utility function is quadratic and given by:

$$U = \int_0^1 q_j y_j \, dj - a \int_0^1 q_j^2 y_j^2 \, dj$$

Before receiving signals through advertising, consumers have a prior about the quality of each good j . This prior is normally distributed with mean μ_j and variance σ_j^2 . Through advertising, firms can send an imperfect signal (s_j) about their product quality, given by:

$$s_j = q_j^* + \omega_j$$

where q_j^* is the actual quality of the good and ω_j is a Gaussian shock with mean 0 and variance σ_ω^2 .

We assume that higher advertising expenditures can decrease the variance of the signal. The posterior distribution of product quality (after receiving the signal) follows a normal distribution with mean and variance:

$$\begin{aligned} \mu_{post} &= \frac{\mu_j/\sigma_j^2 + s_j/\sigma_\omega^2}{\sigma_j^{-2} + \sigma_\omega^{-2}} \\ \sigma_{post}^2 &= \left[\frac{1}{\sigma_j^2} + \frac{1}{\sigma_\omega^2} \right]^{-1} \end{aligned}$$

The representative consumer maximizes expected utility after receiving advertising signals. The demand function for good j can be written as:

$$y_j = \frac{\mu_{post} - p_j}{2a(\mu_{post}^2 + \sigma_{post}^2)}$$

For simplicity, let us assume the limiting case as σ_j goes to infinity (a diffuse prior). In this case, the posterior distribution has mean $\mu_{post} = s_j$ and variance $\sigma_{post}^2 = \sigma_\omega^2$. Therefore, the demand for any good j is a decreasing function of the variance of the signal. Since the precision of the signal is increasing in advertising expenditures, advertising acts as a demand shifter as in our baseline model.

G.2.5 Advertising and the Price-Elasticity of Demand

A strand of the advertising literature has focused on the effect of advertising on the elasticity of demand (see, for instance, Molinari and Turino (2015) and Benhabib and Bisin (2002)). In this section, we present a model in which advertising can change the price-elasticity of demand. We further show that the demand shifting property of advertising is maintained so that the results from our baseline model could be obtained in such a framework as well.

Following Molinari and Turino (2015), we write:

$$Y = \left[\int_0^1 q_j^{\frac{1}{\epsilon}} (y_j + D(d_j))^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}$$

where D is a decreasing function, with $D(0) \geq 0$. The inverse demand function can be written as:

$$p_j = (q_j Y)^{\frac{1}{\epsilon}} [y_j + D(d_j)]^{-\frac{1}{\epsilon}}$$

Setting $D'(d_j) < 0$ as in Molinari and Turino (2015), we obtain that advertising acts as a positive demand shifter. Furthermore, the price-elasticity of demand is equal to:

$$\left| \frac{\partial y_j / y_j}{\partial p_j / p_j} \right| = \epsilon \left(1 + \frac{D(d_j)}{y_j} \right)$$

Thus, advertising decreases the price-elasticity of demand, *ceteris paribus*. Intuitively, by conducting advertising, firms alter the substitutability between goods, and make them more price-inelastic.

H R&D Tax Credits and Additional Tests

H.1 Description of R&D Tax Credit Measures

This section describes the measures of R&D tax credits which are used in our empirical investigation of the substitution between R&D and advertising in Section 6.2. First, we use the statutory credit rate. Second, the tax-adjusted credit rate takes into account the fact that the tax credit is itself subject to corporate taxation in some states. We compute the tax-adjusted credit rate for state s at time t as:

$$Tax \ adj. \ Rate_{st} = Statutory \ Credit \ Rate_{st} \times (1 - \sigma_{st} \times Tax \ Rate_{st})$$

where σ_{st} is the share of the R&D credit which is subject to corporate taxation. When the credit is taxed, the credit rate is now not only influenced by differences in the statutory credit rate over time and across states, but also by changes in corporate taxes.

Third, we use an alternative measure of the marginal effective R&D tax credit, proposed by Wilson (2009) and available until 2006. This measure acknowledges the different definitions of the R&D expenditures which are eligible for tax credit as well as the horizon over which the tax credit is calculated. In some states, all R&D expenditures can lead to a tax credit, while some other states offer a credit only to R&D expenditures above a certain base level. This threshold can in turn be a moving average of past R&D expenditures. For such states, the moving-average base is usually computed as the product of firm R&D-to-sales ratio over the n previous periods times current sales. For a firm with R&D expenditures above the base level, the marginal effective tax credit rate (m_{st}) is computed as:

$$m_{st} = \text{Statutory Credit Rate}_{st} \times (1 - \sigma_{st} \times \tau_{st}^e) \times \left(1 - \frac{1}{n} \sum_{k=1}^n (1 + r_{t+k})^{-k}\right)$$

where r is the real interest rate, n is the number of periods over which the moving-average base is calculated, and τ_{st}^e is the effective marginal tax rate which takes into account the fact that, in some states, taxes paid to the state can be deducted from federal taxable income, and vice-versa.⁵⁵

For states with base definition based on the moving-average of past R&D activity, every dollar spent on R&D today decreases the amount of R&D that qualifies for a tax credit in the future and hence reduces the effective marginal tax credit rate. In some states such as New York and Connecticut, all R&D expenditure qualifies for a tax credit (i.e., there is no moving-average definition of the base level). In this case, the marginal effective credit rate is equal to the (tax-adjusted) statutory credit rate.

Finally, we use one additional state-level measure of the cost of R&D: the R&D user cost. This measure was extended from Hall and Jorgenson (1967) to R&D investment by Bloom *et al.* (2002), and computed at the U.S. state and federal levels by Wilson (2009). In particular, the user cost of R&D in state j is given by:

$$R\&D\ User\ Cost_{st} = \frac{1 - v(m_{st} + m_{ft}) - z(\tau_{st}^e + \tau_{ft}^e)}{1 - (\tau_{st}^e + \tau_{ft}^e)} (r_t + \delta)$$

where the f subscript stands for *federal*, v is the share of R&D expenditures that qualifies for preferential tax treatment, z is the present discounted value of tax depreciation allowances, and δ is the depreciation rate of R&D capital.⁵⁶

H.2 State-Level Evidence for Advertising-R&D Substitutability

In this Appendix, we provide additional evidence on the substitution between advertising and R&D. We focus on the response of advertising at the state level to changes in R&D tax credits. Our measure of advertising intensity is the ratio of advertising expenditures within the state divided by the overall level of sales in the state. We run the following regressions:

$$\log\left(\frac{Adv_{st}}{Sales_{st}}\right) = \alpha_0 + \beta_1 Tax\ Credit_{st} + \beta_2 \mathbf{X}_{st} + \alpha_s + \alpha_t + u_{st} \quad (\text{H.1})$$

for state s at time t , where α_t and α_s control for time and state fixed effects respectively, $Tax\ Credit_{st}$ is a measure of R&D tax credit, and \mathbf{X}_{st} is a vector of control variables including sales in the state, the average age of firms, the average level of equity financing in the state,

⁵⁵This rate also takes into account the fact that R&D tax credit are subject to corporate taxation in some states.

⁵⁶Wilson (2009) sets $v = 0.5$, $z = 0.525$ and $\delta = 0.15$.

sales in the state, the average age of firms, the average level of equity financing in the state, state corporate tax rates, as well as the average selling, general and administrative expense.

Those results confirm the substitutability between R&D and advertising that is predicted by our model and that was also found at the firm level.

Table A.3: Effect of R&D subsidy on advertising intensity at the state level

	(1)	(2)	(3)	(4)	(5)
	$\log\left(\frac{Adv}{Sales}\right)$	$\log\left(\frac{Adv}{Sales}\right)$	$\log\left(\frac{Adv}{Sales}\right)$	$\log\left(\frac{Adv}{Sales}\right)$	$\log\left(\frac{Adv}{Sales}\right)$
<i>Tax credit dummy</i>	-0.323 (0.182)				
<i>State credit rate</i>		-3.266 (1.382)			
<i>Tax-adjusted state rate</i>			-3.296 (1.389)		
<i>Effective state rate</i>				-3.246 (1.842)	
<i>R&D user cost</i>					2.794 (1.661)
Controls	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓
State FE	✓	✓	✓	✓	✓
Observations	1982	1982	1982	1843	1820
R^2	0.50	0.50	0.50	0.51	0.46

Notes: Data from Compustat from 1950 to 2009, Wilson (2009) and Falato and Sim (2014). Controls include sales (logarithm), age, financial constraint, state tax and selling, general and administrative expense (logarithm). Age is measured as the elapsed time since the first observation in the data. Our measure of financial constraints is sale minus purchases of common and preferred stock, divided by sales. Age and financial constraint are averaged at the state level. Standard errors are clustered at the state level (in parentheses).

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