MARKOV-SWITCHING THREE-PASS REGRESSION FILTER

2017

Pierre Guérin, Danilo Leiva-Leon
and Massimiliano Marcellino

Documentos de Trabajo
N.º 1748

BANCO DE ESPAÑA
Eurosistema

MARKOV-SWITCHING THREE-PASS REGRESSION FILTER

MARKOV-SWITCHING THREE-PASS REGRESSION FILTER (*)

Pierre Guérin (**)

OECD

Danilo Leiva-Leon (***)

BANCO DE ESPAÑA

Massimiliano Marcellino (****)

BOCCONI UNIVERSITY, IGIER and CEPR

(*) We would like to thank the editor Todd Clark, three anonymous referees, Dalibor Stevanovic, James MacKinnon, seminar participants at Queen's University, Banco de España and the 2016 conference of the Canadian Econometric Study Group held at Western University for helpful comments on a previous version of this paper. We also thank Agustín Díaz for excellent research assistance. The views expressed in this paper are those of the authors. No responsibility for them should be attributed to the Banco de España, the Eurosystem, the OECD or its member countries.
(**) pierre.guerin@oecd.org
(***) danilo.leiva@bde.es
(****) massimiliano.marcellino@unibocconi.it

Documentos de Trabajo. N.º 1748

2017

ISSN: 1579-8666 (on line)

Abstract

We introduce a new approach for the estimation of high-dimensional factor models with regime-switching factor loadings by extending the linear three-pass regression filter to settings where parameters can vary according to Markov processes. The new method, denoted as Markov-switching three-pass regression filter (MS-3PRF), is suitable for data sets with large cross-sectional dimensions, since estimation and inference are straightforward, as opposed to existing regime-switching factor models where computational complexity limits applicability to few variables. In a Monte Carlo experiment, we study the finite sample properties of the MS-3PRF and find that it performs favourably compared with alternative modelling approaches whenever there is structural instability in factor loadings. For empirical applications, we consider forecasting economic activity and bilateral exchange rates, finding that the MS-3PRF approach is competitive in both cases.

Keywords: factor model, Markov-switching, forecasting.

JEL classification: C22, C23, C53.

Resumen

En este artículo se presenta un nuevo enfoque para la estimación de modelos de factores de gran dimensión cuyas cargas de factores están sujetas a cambios markovianos de régimen. Dicho enfoque consiste en una extensión del filtro de regresión de tres pasos lineal a casos en los cuales los parámetros del modelo puedan cambiar en función de procesos markovianos. El método propuesto en este artículo, denominado «filtro de regresión de tres pasos con cambios markovianos» (MS-3PRF), es adecuado para tratar bases de datos que contengan un gran número de variables, ya que la estimación y la inferencia son directas, a diferencia de métodos alternativos, en donde la compleja estimación limita su uso a aplicaciones que envuelven pocas variables. Las propiedades en muestra finita del método propuesto se estudian con experimentos de Monte Carlo. Los resultados indican que, cuando los factores de carga están sujetos a inestabilidades, el método propuesto posee una mayor habilidad predictiva que métodos alternativos existentes en la literatura. Esta superioridad en términos predictivos también se observa en dos aplicaciones empíricas enfocadas a pronosticar la actividad económica y los tipos de cambio bilaterales.

Palabras clave: modelos de factores, cambios markovianos de régimen, predicción.

Códigos JEL: C22, C23, C53.

1 Introduction

This paper introduces a new approach for the estimation of high-dimensional factor models with regime-switching factor loadings. The literature on factor models has mostly concentrated on situations where the comovement among variables is assumed to be constant over time. However, there is now a large body of literature that has challenged the assumption of constant parameters to model the macroeconomic environment (see, e.g., Sims (1993) or Canova (1993)), as well as the relevance of modelling time variation for macroeconomic forecasting (see, e.g., D'Agostino et al. (2013) and Aastveit et al. (2017)). The importance of incorporating time instabilities in large-scale factor models has gained traction in the literature in recent years (see, e.g., Eickmeier et al. (2015) and Mikkelsen et al. (2015)), but the number of works on this front remains relatively small. Moreover, this literature has so far been restricted to the estimation of models with time-varying factor loadings where time-variation is modelled using random-walk or autoregressive behaviours, which typically restrict the dynamics of time-variation to gradual changes in the factor loadings that may not be appropriate to all situations (e.g., if time variation is governed by regime-switching dynamics). The literature has also considered the estimation of factor models with temporal instability (structural breaks) in both factor loadings and the number of factors (see, e.g., Cheng et al. (2016)). Moreover, Nakajima and West (2013) introduce a framework where the factor loadings are time-varying, but shrink to zero when they fall below a threshold. In contrast, in this paper we consider factor loadings that vary according to regime-switching processes so as to model recurrent abrupt changes in factor loadings that are potentially highly relevant features in macroeconomic and financial variables (see, e.g., Ang and Timmermann (2012)).

Our modelling approach builds on Kelly and Pruitt (2015), who developed a new estimator for factor models—the three-pass regression filter (3PRF)—that relies on a series of ordinary least squares (OLS) regressions. As emphasized in Kelly and Pruitt (2015), the key difference between principal component analysis (PCA) and the 3PRF approach is that PCA summarizes the cross-sectional information based on the covariance within the predictors, whereas 3PRF condenses cross-sectional information based on the correlation of the predictors with the target variable of the forecasting exercise, thereby extending partial least squares. In this paper, we extend the 3PRF approach by introducing regime-switching parameters in the linear 3PRF filter. This new framework is denoted as Markov-switching three-pass regression filter (MS-3PRF). A key advantage of this approach is that it is well suited to handle high-dimensional factor models, as opposed to the existing regime-switching factor models that can handle only models with limited dimensions due to computational complexity (see, e.g., Camacho et al. (2012)).[1] Our approach is attractive in that our estimation strategy only requires estimating a series of univariate Markov-switching regressions. As such, it is computationally straightforward to implement and offers a great deal of flexibility in modelling time variation since we do not restrict the regime changes in the cross-sectional dimension to be governed by a single or a limited number of Markov chains.[2]

Empirically, we use the MS-3PRF approach for forecasting selected variables based on a large set of predictors. Since the seminal work of Stock and Watson (2002b), a large literature has developed to improve on the forecasting performance of the principal-component approach for macroeconomic forecasting (see, e.g., Forni et al. (2005) and De Mol et al. (2008), among many others). In a paper related to our work, Bai and Ng (2008) find improvements to the principal-component approach by using fewer but informative predictors. They also suggest that additional forecasting gains can be obtained when modelling non-linearities. The MS-3PRF approach is related to this strand of the literature given that factors are extracted by modelling the correlation of the predictors with the forecast target so that the estimation of the factors takes into account how informative the predictors are for the target variable. Moreover, the MS-3PRF approach captures non-linearities by modelling parameters that vary according to unobservable Markov chains.

This paper contributes to the literature along two main dimensions. First, we provide a new framework for the estimation of high-dimensional factor models with regime-switching parameters under classical inference. In a simulation experiment, we study the finite sample accuracy of the MS-3PRF forecasts compared with a number of alternatives. We find that the MS-3PRF performs well when there are instabilities in the data-generating process (DGP) modelled via regime-switching parameters. Second, we provide empirical evidence that the MS-3PRF performs well when forecasting major U.S. macroeconomic variables based on the McCracken and Ng (2015) data set. Moreover, when forecasting major currencies based on a panel of exchange rates, we also find predictive gains when using the MS-3PRF approach. This provides additional evidence of gains one can draw from the use of factor analysis to forecast exchange rates (see, e.g., Engel et al. (2015)) as well as the importance of modelling non-linearities in this context.

[1]Groen and Kapetanios (2016) show that partial least squares (and Bayesian methods) perform better than principal components when forecasting based on a large data set with a weak factor structure. As partial least squares is obtained as a special case of the 3PRF (see Kelly and Pruitt (2015) for details), our method can also be adopted to introduce Markov switching in partial least squares regressions.

[2]For example, extracting one factor from the MS-3PRF approach using a panel of 130 macroeconomic and financial variables with gross domestic product (GDP) growth as a target proxy takes about 350 seconds using a laptop with a 2.7 GHz processor and 16 GB RAM.

This paper is organized as follows: Section 2 introduces the MS-3PRF approach and discusses its main features. Section 3 presents a Monte Carlo experiment to study the finite sample accuracy of the MS-3PRF. Section 4 gathers empirical applications devoted to macroeconomic and exchange rate forecasting. Section 5 concludes. The online appendix contains supplementary material.

2 Markov-Switching Three-Pass Regression Filter

2.1 The algorithm

There is by now a growing literature on dynamic factor models with time-varying parameters. For example, in a Bayesian setting, Del Negro and Otrok (2008) first introduced a dynamic factor model with time-varying factor loadings. In a classical context, see Mikkelsen et al. (2015) and Eickmeier et al. (2015). However, the literature on regime-switching dynamic factor models is limited and, more importantly, restricted to small-scale models (see, e.g., Chauvet (1998), Camacho et al. (2012) or Barnett et al. (2016), who use fewer than 10 variables and focus only on switches in the parameters governing the factor dynamics and not the factor loadings).[3] The same is true for vector autoregression (VAR) models: while there is now a large (both methodological and empirical) literature on time-varying parameter VAR models, the literature using regime-switching VAR models is a lot narrower, although there are a few noticeable exceptions (see, e.g., Sims and Zha (2006) and Hubrich and Tetlow (2015)).

One key reason for the absence of a significant literature on large-scale Markov-switching factor models relates to the computational challenges associated with the estimation of such models. We present here the Markov-switching three-pass regression filter, which circumvents these difficulties and offers a flexible approach in that it imposes very few restrictions on the Markov processes driving the changes in the parameters of the model.

The type of setting we have in mind can be described informally as follows: There is a relatively large number N of predictors $\mathbf{x}$ from which we want to extract factors so as

[3] In a Bayesian context, Guérin and Leiva-Leon (2016) develop an algorithm to estimate a high-dimensional factor-augmented VAR model with regime-switching parameters in the factor loadings to study the interactions between monetary policy, the stock market and the connectedness of industry-level stock returns. See also Von Ganske (2016), who introduces regime-switching parameters in partial least squares regressions from a Bayesian perspective so as to forecast industry stock returns. Using a Bayesian framework, Hamilton and Owyang (2012) develop a framework for modelling common Markov-switching components in panel data sets with large cross-sectional and time-series dimensions to estimate turning points in U.S. state-level employment data.

to forecast a target variable y. While $\mathbf{x}$ depends on two sets of common factors, say f and g (plus idiosyncratic components), y depends only on f, so we would like to extract only f from $\mathbf{x}$. In addition, there exist proxi variables, $\mathbf{z}$, whose common components are also driven only by f. This setting is the same as that in Kelly and Pruitt (2015), who introduced the linear 3PRF for estimation of f and forecasting of y, but the key novelty is that we include time variation in the model parameters via Markov processes.

More formally, let us consider the following model:

$$y_{t+1} = \beta_0(S_t) + \boldsymbol{\beta}'(S_t)\mathbf{F}_t + \eta_{t+1}(S_t), \tag{1}$$
$$\mathbf{z}_t = \lambda_0(\mathbf{S}_t) + \boldsymbol{\Lambda}(\mathbf{S_t})\mathbf{F}_t + \omega_t(\mathbf{S}_t), \tag{2}$$
$$\mathbf{x}_t = \phi_0(\mathbf{S}_t) + \boldsymbol{\Phi}(\mathbf{S_t})\mathbf{F}_t + \varepsilon_t(\mathbf{S}_t), \tag{3}$$

where y is the target variable of interest; $\mathbf{F}_t = (\mathbf{f}_t', \mathbf{g}_t')'$ are the $K = K_f + K_g$ common driving forces of all variables, the unobservable factors; S_t denotes a standard Markov chain driving the parameters of the forecasting equation, while $\mathbf{S}_t = (S_{1,t}, S_{2,t}, ..., S_{N,t})'$ is a vector containing *variable-specific* Markov chains with M regimes driving the parameters of the factor equations; each Markov chain is governed by its own $M \times M$ transition probability matrix,

$$P_i = \begin{pmatrix} p_{i,11} & p_{i,21} & \cdots & p_{i,M1} \\ p_{i,12} & p_{i,22} & \cdots & p_{i,M2} \\ \vdots & \vdots & \ddots & \vdots \\ p_{i,1M} & p_{i,2M} & \cdots & p_{i,MM} \end{pmatrix}, \tag{4}$$

for $i = 1, 2, ..., N$; $\boldsymbol{\beta}(S_t) = (\boldsymbol{\beta}_f'(S_t), \mathbf{0}')'$, so that y depends only on $\mathbf{f}$; $\mathbf{z}$ is a small set of L proxies that are driven by the same underlying forces as y, so that $\boldsymbol{\Lambda}(\mathbf{S_t}) = (\boldsymbol{\Lambda}_f(\mathbf{S}_t), \mathbf{0})$; $\mathbf{x}_t$ is a large set of N variables, driven by both $\mathbf{f}_t$ and $\mathbf{g}_t$; and $t = 1, ..., T$.

To achieve identification, when N and T diverge, the covariance of the loadings is assumed to be the identity matrix in each state, and the factors are orthogonal to one another.[4] For the sake of space, we refer to Kelly and Pruitt (2015) for precise conditions on the factors, allowed temporal and cross-sectional dependence of the residuals, and the existence of proper central limit theorems.

Given the model in equations 1 to 3, our algorithm for the MS-3PRF model consists of the following three steps:

[4] More precisely, defining $\mathbf{J}_T = \mathbf{I}_T - \frac{1}{T}\iota_T\iota_T'$, where $\mathbf{I}_T$ is a T-dimensional identity matrix and ι_T is a T-vector of ones (and similarly $\mathbf{J}_N$), and assuming that $N^{-1}\boldsymbol{\Phi}'(S_t)\mathbf{J}_N\boldsymbol{\Phi}(S_t) \underset{N\longrightarrow\infty}{\longrightarrow} \mathbf{P}(S_t)$, $N^{-1}\boldsymbol{\Phi}'(S_t)\mathbf{J}_N\phi_0(S_t) \underset{N\longrightarrow\infty}{\longrightarrow} \mathbf{P}_1(S_t)$, $T^{-1}\mathbf{F}'\mathbf{J}_T\mathbf{F} \underset{T\longrightarrow\infty}{\longrightarrow} \Delta_F$, for identification we require, as did Kelly and Pruitt (2015), that $\mathbf{P}(\mathbf{S_t}) = \mathbf{I}$, $\mathbf{P}_1(S_t) = \mathbf{0}$, and Δ_F is diagonal and positive definite, and each diagonal element is unique.

- *Step 1:* Time-series regressions of each element of $\mathbf{x}$, x_i, on $\mathbf{z}$; that is, run N Markov-switching regressions

$$x_{i,t} = \phi_{0,i}(S_{i,t}) + \mathbf{z}_t' \boldsymbol{\phi}_i(S_{i,t}) + \epsilon_{i,t}(S_{i,t}), \tag{5}$$

  where $i = \{1, ..., N\}$, $\epsilon_{i,t}|S_{i,t} \sim NID(0, \sigma^2(S_{i,t}))$, and keep the maximum likelihood estimate of $\boldsymbol{\phi}_i(S_{i,t})$, denoted by $\widehat{\boldsymbol{\phi}}_i(S_{i,t})$. All regime-switching models are estimated via (pseudo) maximum likelihood, and we make a normality assumption about the disturbances to write down the log-likelihood function, which is not required when estimating the linear version of the 3PRF. As mentioned previously, $S_{i,t}$ is a standard Markov chain with M regimes and dynamics driven by constant transition probabilities. It is important to stress that the estimated latent processes $S_{i,t}$ differ across all cross-section units i. The pattern of the regime changes in the factor loadings is therefore left unrestricted as opposed to assuming that the changes in the parameters $\phi_{0,i}$ and $\boldsymbol{\phi}_i$ are governed by a single (or a limited number of) Markov chain(s) across all cross-section units. Moreover, a different number of regimes could be used across the N first-pass regressions. As such, the MS-3PRF approach offers a great deal of flexibility in modelling regime changes.

- *Step 2:* Cross-section regressions of $x_{i,t}$ on $\hat{\boldsymbol{\phi}}_{i,t}$; that is, run T linear regressions

$$x_{i,t} = \alpha_{0,t} + \hat{\boldsymbol{\phi}}_{i,t}' \mathbf{F}_t + \epsilon_{i,t}, \tag{6}$$

  where $t = \{1, ..., T\}$, and keep (for each t) the OLS estimates $\mathbf{F}_t$. In this step, the time-varying factor loadings $\hat{\boldsymbol{\phi}}_{i,t}$ can be obtained from the first step of the algorithm by following two alternatives. First, as a weighted average of the regime-specific factor loadings:

$$\hat{\boldsymbol{\phi}}_{i,t} = \sum_{j=1}^{M} \hat{\boldsymbol{\phi}}_i(S_{i,t} = j) P(S_{i,t} = j|\Omega_T), \tag{7}$$

  where $P(S_{i,t} = j|\Omega_T)$ is the smoothed probability of being in regime j given the full sample information Ω_T. Second, as a selected loading:

$$\hat{\boldsymbol{\phi}}_{i,t} = \sum_{j=1}^{M} \hat{\boldsymbol{\phi}}_i(S_{i,t} = j) I(P(S_{i,t} = j|\Omega_T)), \tag{8}$$

  where $I(\cdot)$ is an indicator function that selects the regime with the highest smoothed probability, $P(S_{i,t} = j|\Omega_T)$, at time t.[5]

[5] In the recursive forecasting exercise, the smoothed probabilities are replaced by the filtered probabilities.

- *Step 3:* Time-series regression of y_t on $\hat{\mathbf{F}}_{t-h}$; that is, run one Markov-switching regression for each forecast horizon of interest, h:

$$y_t = \beta_0(S_t) + \hat{\mathbf{F}}'_{t-h}\boldsymbol{\beta}(S_t) + \eta_t(S_t), \tag{9}$$

keep the maximum likelihood estimates $\beta_0(S_t)$ and $\boldsymbol{\beta}(S_t)$, and calculate the forecast $\hat{y}_{T+h|T}$ as:

$$\hat{y}_{T+h|T} = \sum_{j=1}^{M} \left(P(S_{T+h} = j|\Omega_T)\hat{\beta}_0(S_{T+h} = j) + P(S_{T+h} = j|\Omega_T)\hat{\mathbf{F}}'_t\hat{\boldsymbol{\beta}}(S_{T+h} = j) \right), \tag{10}$$

where $P(S_{T+h} = j|\Omega_T)$ is the predicted probability of being in regime j h-step-ahead given the information available up to time T, Ω_T.

In the third pass of the algorithm, the Markov chain S_t allows us to model time variation in the intercept of the forecasting regression, which is a common source of forecast failure. Changes in the slope parameters $\boldsymbol{\beta}$ are relevant in that they allow us to model time variation in the predictive power of the estimated factors $\hat{\mathbf{F}}_t$ for the target variable y_{t+1}. Note that one can estimate a linear model in the third step. We denote this approach as "MS-3PRF (first pass)," while "MS-3PRF (first and third passes)" refers to considering regime changes in both the first and the third pass.

Kelly and Pruitt (2015) develop asymptotic theory for the linear 3PRF approach, showing that the 3PRF-based forecast converges in probability to the infeasible best forecast as cross-section N and sample size T become large. We need additional special conditions to be able to claim that their consistency results could be extended to the non-linear case. Specifically, we need consistency of the parameter estimators for the Markov-switching models in steps 1 and 3. Douc et al. (2004) establish results concerning the consistency and asymptotic normality of the maximum likelihood estimator in Markov-switching models. For the general case of hidden Markov models, Leroux (1992) proved the consistency of the maximum likelihood estimator under mild regularity conditions. Hence, based on this consistency result of the Markov-switching parameters in the first and third steps, the MS-3PRF should conserve the consistency properties of the linear 3PRF. While we do not provide a formal proof for this statement, it seems supported by a comparison of the performance of the 3PRF and MS-3PRF in the simulation experiments reported in section 3 and in the online appendix.

2.2 Specification choices

The algorithm outlined in the previous subsection rests on the choice of the number of factors and proxi variables to be used in the first step of the algorithm, as well as the number of regimes to consider for the MS-3PRF.

There are several ways to assess the number of regimes in a Markov-switching regression under a classical framework. Just to mention a few, Cho and White (2007) and Carter and Steigewald (2012) suggest the use of a quasi-likelihood ratio test; however, they ignore the Markov property of the variable S_t. Other alternatives consist in calculating goodness-of-fit measures that trade off the fit of the likelihood against the number of parameters (e.g., Smith et al. (2006)). For ease of illustration, throughout the simulation exercises and empirical applications, we leave aside this complication and assume that predictor variables experience either $M = 1$ or $M = 2$ regimes. However, the framework can be generalized accordingly.

For the choice of the proxi variables, when there is just one $\mathbf{f}_t$ factor, Kelly and Pruitt (2015) suggest using directly the target variable y as proxy $\mathbf{z}$. From a predictive point of view, this is a natural choice, since, in this context, one wants to extract a factor that summarizes how related the predictors are to the the predicted variable. They refer to this case as target-proxy 3PRF. In the case of more factors, they propose using either economic theory to select indicators correlated with the target variable y, or an automated procedure that can be implemented with the following steps, indicating a proxy by z_j with $j = 1, ..., L$.

- Pass 1: Set $z_1 = y$; get the 3PRF forecast $\widehat{y}_t^1$, and the associated residuals $e_t^1 = y_t - \widehat{y}_t^1$.
- Pass 2: Set $z_2 = e^1$; get the 3PRF forecast $\widehat{y}_t^2$ using z_1 and z_2 as proxies. Get the associated residuals $e_t^2 = y_t - \widehat{y}_t^2$.
- ...
- Pass L: Set $z_L = e^{L-1}$; get the 3PRF forecast $\widehat{y}_t^L$ using $z_1, z_2, ...z_L$ as proxies.

For the choice of the number of factors, Kelly and Pruitt (2015) use appropriate information criteria with asymptotic optimality properties. However, empirically it can be more informative to assess the performance of different numbers of factors. As in the case of PCA, using more factors than needed reduces forecast efficiency in finite samples but does not introduce a bias, while using fewer factors generates an omitted-variable problem and therefore biases both the estimators of the loadings and the forecasts.

3 Monte Carlo Simulations

In this section, we conduct Monte Carlo simulations to evaluate the finite sample properties of the MS-3PRF, focusing on its predictive performance. We compare the MS-3PRF with competing approaches that have proved to be successful in dealing with large data sets, such as the linear 3PRF and PCA. We also use two additional benchmark models—targeted PCA (TPCA) and a least angle regression approach, PC-LARS—from which factors are extracted by the method of principal component from a smaller set of predictors than the N predictors used by PCA. These two additional methods are described in the online appendix; Appendix A.1 describes the hard-thresholding approach (TPCA), and Appendix A.2 outlines the soft-thresholding approach (PC-LARS). Our simulation exercises compute the out-of-sample mean square forecasting errors (MSFE) to predict the target variable, y_t, that is generated based on a large set of predictors, $\mathbf{x}_t = (x_{1,t}, x_{2,t}, ..., x_{N,t})'$, driven by a factor structure with regime-switching in the loadings.

3.1 Design

The data on $\mathbf{x}_t$ and y_t, for $t = \{1, 2, ..., T\}$, are generated following the factor structure proposed in Bates et al. (2013) and Kelly and Pruitt (2015):

$$\mathbf{x}_t = \Phi_t F_t + \varepsilon_t, \tag{11}$$

$$y_{t+1} = \Lambda F_t + \eta_t, \tag{12}$$

where $F_t = (f_t, \mathbf{g}_t')'$, $\Phi_t = (\Phi_{f,t}, \Phi_{g,t})$ and $\Lambda = (1, \mathbf{0})$. The relevant and irrelevant factors are generated according to the following dynamics, respectively:

$$f_t = \rho_f f_{t-1} + u_{f,t}, \tag{13}$$

$$\mathbf{g}_t = \rho_g \mathbf{g}_{t-1} + \mathbf{u}_{g,t}, \tag{14}$$

where $u_{f,t} \sim N(0,1)$, and $\mathbf{u}_{g,t} \sim N(0, \mathbf{\Sigma}_g)$, with $u_{f,t}$ and $\mathbf{u}_{g,t}$ uncorrelated. We consider $K_g = 4$ irrelevant factors and $K_f = 1$ relevant factor. The parameters in $\mathbf{\Sigma}_g$ are chosen so that irrelevant factors are dominant; that is, their variances are 1.25, 1.75, 2.25 and 2.75 times larger than the relevant factor. The idiosyncratic terms are assumed to follow autoregressive dynamics,

$$\varepsilon_{it} = \alpha \varepsilon_{i,t-1} + v_{i,t}, \tag{15}$$

and to be cross-sectionally correlated; that is, $v_t = (v_{1,t}, v_{1,t}, ..., v_{N,t})'$, and it is independent and identically distributed (i.i.d.) normally distributed with covariance matrix $\Omega = (\beta^{|i-j|})_{ij}$, as in Amengual and Watson (2007). The starting values for the factors and idiosyncratic terms f_0, $\mathbf{g}_0$, ε_{i0} are drawn from their respective stationary distributions. The disturbances, η_t, associated to the target variable equation are i.i.d. normally distributed with a variance, σ_η^2, that is adjusted to ensure that the infeasible best forecast has a R^2 of 50 per cent. The free parameters of our Monte Carlo simulations are ρ_f, ρ_g, α, β, N and T. In line with Stock and Watson (2002a), Bates et al. (2013) and Kelly and Pruitt (2015), we consider $\rho_f = \{0.3, 0.9\}$, $\rho_g = \{0.3, 0.9\}$, $\alpha = \{0.3, 0.9\}$, $\beta = \{0, 0.5\}$, $N = \{100, 200\}$ and $T = \{100, 200\}$.

The factor loadings, collected in Φ_t, experience changes between two regimes over time,

$$\Phi_t = \Phi_1 \mathbf{S}_t + \Phi_2(1 - \mathbf{S}_t), \tag{16}$$

where $\mathbf{S}_t = (S_{1,t}, S_{2,t}, ..., S_{N,t})$ contains N dichotomous state variables, each following distinct dynamics according to a first-order Markov chain. Since the data in $\mathbf{x}_t$ are generated from a factor structure, it is mechanically subject to a certain degree of comovement. Therefore, the non-linear relationship between the data, $\mathbf{x}_t$, and the factors, F_t, measured by the factor loadings, Φ_t, may also experience a certain degree of comovement.

To provide a more realistic DGP that is relevant for economic data where data are generally weakly dependent (as opposed to i.i.d.), we model comovement in the factor loadings, which is translated in modelling comovement in the Markovian state variables contained in $\mathbf{S}_t$. In doing so, we let $\tilde{S}_{i,t}$ be the state vector of the i-th sequence at time t. If the i-th sequence is in state 1 at time t, then we write $\tilde{S}_{i,t} = (1, 0)'$, and if it is in state 2 at time t, then we write $\tilde{S}_{i,t} = (0, 1)'$. First, we generate a “seed” sequence variable, $\tilde{S}_{0,t}$. For time t, we compute $(q, 1 - q)' = P_{00}\tilde{S}_{0,t}$, where

$$P_{00} = \begin{pmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{pmatrix} \tag{17}$$

is the transition probability matrix, and the realization of the sequence at time $t + 1$ is defined as

$$\tilde{S}_{0,t+1} = \begin{cases} (1, 0)' & \text{If } \ q \geq \theta, \\ (0, 1)' & \text{otherwise} \end{cases} \tag{18}$$

where θ is drawn from a $U[0, 1]$. Next, we generate a Markov chain, $\tilde{S}_{i,t}$, conditional on the dynamics of $\tilde{S}_{0,t}$, using the following system:

$$\begin{bmatrix} (q_0, 1 - q_0)' \\ (q_i, 1 - q_i)' \end{bmatrix} = \begin{bmatrix} \lambda_{00}P_{00} & \lambda_{0i}P_{0i} \\ \lambda_{i0}P_{i0} & \lambda_{ii}P_{ii} \end{bmatrix} \begin{bmatrix} \tilde{S}_{0,t} \\ \tilde{S}_{i,t} \end{bmatrix}, \tag{19}$$

where the coefficients λ measure the comovement between both Markov chains, with $\lambda_{jk} \geq 0$, and $\sum_{k=1}^{2} \lambda_{jk} = 1$. The matrix P_{jk} collects the transition probabilities from the states in the k-th sequence to the states in the j-th sequence.[6] Accordingly, q_k represents the state probability distribution of the k-th sequence at time $t+1$, from which the realization $\tilde{S}_{i,t+1}$ can be generated as follows:

$$\tilde{S}_{i,t+1} = \begin{cases} (1,0)' & \text{If } \; q_i \geq \theta \\ (0,1)' & \text{Otherwise} \end{cases} . \tag{20}$$

For simplicity, we assume that $P_{0i} = P_{i0} = P_{ii} = P_{00}$, and that $p_{11} = p_{22}$, with $p_{11} = 0.9$. Also, we set $\lambda_{00} = \lambda_{ii} = 0.2$ to induce a relatively large degree of comovement between the state variables. Given $\tilde{S}_{0,t}$, we repeat the same procedure for $i = \{1, 2, ..., N\}$, to get all the elements in $\mathbf{S}_t$. Finally, the elements in Φ_κ, for $\kappa = \{1, 2\}$ are generated from a $N(\phi_\kappa, \sigma_\phi)$ with $\phi_1 = 0.5$, $\phi_2 = 1.5$, and $\sigma_\phi = 0.1$.

We also assess the performance of the proposed approaches under different variations of the data generating process. First, we study the case where $\mathbf{x}_t$ and y_t are generated by following the same processes described above but with the factor loadings being driven by Markov chains that are totally independent of each other and that do not experience comovement. Second, we study the performance of the models when considering different degrees of instability contained in the data $\mathbf{x}_t$. Third, we also consider a data generating process where the factor loadings follow random walks rather than Markovian switches.

3.2 Models and evaluation criteria

We perform $L = 500$ Monte Carlo replications for each configuration of parameters ρ_f, ρ_g, α and β and sample sizes T and N. Once $\mathbf{x}_t$ and y_t are generated, we apply the MS-3PRF to extract the factor and predict the target variable. In particular, first, we estimate a time-series (Markov-switching) regression, $x_{i,t} = \mathbf{z}_t' \boldsymbol{\phi}_i(S_{i,t}) + \epsilon_{i,t}$, for $i = \{1, 2, ..., N\}$. For simplicity, we take the proxy variable as the target variable, $\mathbf{z}_t' = y_t$. Second, we run a cross-section OLS regression, $x_{i,t} = \hat{\boldsymbol{\phi}}_{i,t}' \mathbf{F}_t + \epsilon_{i,t}$, for $t = 1, 2, ..., T$, using the weighted average of the regime-switching factor loadings obtained in the previous step. Third, we run a time-series OLS regression, $y_t = \beta_0 + \hat{\mathbf{F}}_{t-1}' \boldsymbol{\beta} + \eta_t$, and produce the forecast $\hat{y}_{t+1} = \beta_0 + \hat{\mathbf{F}}_t' \hat{\boldsymbol{\beta}}$, obtained with the MS-3PRF approach introduced in this paper. Also, we produce forecasts with the version of the MS-3PRF when the loadings are selected instead of being averaged (MSS-3PRF); that is, the time-varying loadings are set to the regime-specific loadings of the most likely regime. To ease the computational burden, in our Monte Carlo simulations, we do not model regime switches in the third pass of the algorithm. This is not detrimental to our simulation exercise, since we are interested only in studying situations characterized by

[6] Ching et al. (2002) proposed a multivariate Markov chain approach for modelling multiple categorical data sequences.

instabilities in the factor loadings (and not time instability in the relationship between the predicted variable and the predictors). However, in the empirical applications, we consider the case of regime switches in the third pass.

We compare the predictive performance of the two variants of our proposed method with several benchmark methodologies. First, we compute the forecast obtained with the linear version of the 3PRF proposed in Kelly and Pruitt (2015). Second, Bates et al. (2013) show that PCA methods can be applied to consistently estimate dynamic factor models under certain instabilities in the loadings. Therefore, we compute the forecast obtained with the method of principal components. Third, Bai and Ng (2008) argue that the principal components methodology, as it stands, does not take into account the predictive ability of $\mathbf{x}_t$ for y_{t+h} when the factors are estimated. Therefore, Bai and Ng (2008) propose using only predictors that are informative for y_t in estimating the factors in order to take explicitly into account that the object of interest is ultimately the forecast of y_t. Accordingly, we also compute the forecast obtained with TPCA. Fourth, we consider the elastic net soft-thresholding rules, which are special cases of the LARS algorithm developed in Efron et al. (2004), and compute the forecast using the PC-LARS method. Ultimately, our focus is on comparing the median out-of-sample MSFE over the L replications associated with each of the six methods (two MS-3PRF approaches and four competitors) to evaluate their relative predictive performance. Finally, in our simulation experiments, we assume that the number of relevant factors and the number of regimes are known (i.e., across all procedures, we extract one factor, and the non-linear models consider the case of two regimes).

3.3 Results

Table 1 reports the simulation results associated with the different configurations of parameters, methodologies and degrees of instability (regime switching) in the data. In particular, panel A of Table 1 presents the MSFE for the cases when all factor loadings exhibit regime-switching dynamics. Some features deserve to be commented. First, the MSS-3PRF exhibits the lowest MSFE for most of the cases, indicating that it performs best in terms of predictive performance. In particular, the MSS-3PRF outperforms the MS-3PRF, linear 3PRF, PCA, TPCA and PC-LARS. Notice also that, in general, the MS-3PRF exhibits the second-best forecasting performance, suggesting that the non-linear frameworks, MS-3PRF and especially MSS-3PRF, are able to capture in a better way instabilities in the relationship between the set of predictors and its common factor.

Second, TPCA provides the third-best performance, in particular, it shows the lowest MSFE for some particular cases when the idiosyncratic terms have a low autocorrelation.

Also notice that in most cases the linear 3PRF outperforms PCA and PC-LARS. This implies that in the presence of instabilities in the loadings, the linear 3PRF takes better advantage of both time-series and cross-sectional dimensions to provide a more accurate estimation of the underlying factor than PCA and PC-LARS.

Third, the scenarios associated with low autocorrelation in the irrelevant factors, ρ_g, yield the highest MSFE. The fact that irrelevant factors (i) behave close to a white noise, (ii) are linked to $\mathbf{x}_t$ through regime-switching loadings and (iii) are dominant makes them able to introduce a relatively large amount of noise into the set of predictors, creating more difficulty for all the methods to provide more accurate estimates of the underlying relevant factor and consequently better forecasts for y_t. In particular, when not only the irrelevant factors but also the idiosyncratic terms are closer to behaving as a white noise (that is, $\rho_g = 0.3$ and $\alpha = 0.3$), the forecasting performance of all methods deteriorates due to the reasons just described. These results indicate that in the presence of instabilities in the loadings, a lower autocorrelation in any of the components driving the observed data (predictors and target variables) is detrimental to the predictive performance of all factor-extraction methods studied in this paper.

When dealing with high-dimensional data sets, the substantial heterogeneity in the data may be accompanied by different degrees of instabilities contained in the predictors $\mathbf{x}_t$. Therefore, we repeat these simulation exercises along the lines of Bates et al. (2013) and let only a subset J of variables, randomly selected from a uniform distribution, exhibit regime-switching factor loadings. Panel B of Table 1 reports the case where the share of variables experiencing instabilities in their loadings is 0.75. The results are relatively similar to the ones obtained with all the variables experiencing instabilities in the loadings in that the MSS-3PRF in general obtains the best forecasting performance, followed by the MS-3PRF. Also in this case, the method showing the third-best forecasting performance is TPCA. Panel C of Table 1 shows the case when 50 per cent of the variables experience instabilities in the factor loadings. Notice that in approximately one third of the cases the MSS-3PRF shows the best forecasting performance, while for the rest of scenarios, the linear methods 3PRF and the PC-LARS report the lowest MSFE. This result indicates that as the degree of instability in the data decreases significantly, the predictive ability of the linear forecasting methods increases. This is confirmed in panel D of Table 1, that shows the results when the share of variables exhibiting instabilities in the loadings is only 0.25. In this case, the 3PRF is the method showing the best forecasting performance, followed by PC-LARS.

The results reported in Table 1 were based on a DGP where the factor loadings associated with each predictor variable were driven by their own Markov chain, and these Markov chains were assumed to experience a high degree of interdependence between them to mimic the behaviour that macroeconomic and financial data usually tend to exhibit. However, we are also interested in assessing the performance of the methods when the assumption of interdependent Markov chains is no longer valid—that is, when the loadings depend on Markov chains that are independent of each other. Panel A of Table 2 reports the MSFE for the case where factor loadings are driven by independent Markov chains. The results provide the same consistent message obtained from the previous exercises: the MSS-3PRF shows in general the most accurate predictive performance among the methods under consideration, followed by the MS-3PRF. These results imply that regardless of the relationship between the Markov chains driving the factor loadings, the non-linear methods proposed in this paper consistently outperform the competing linear methods.

In addition, we explore the predictive ability of the competing methods when factor loadings follow independent random walks instead of Markov processes. The results are reported in panel B of Table 2, showing that when the relevant factor exhibits a high autocorrelation, the MSS-3PRF method yields the lowest MSFE. However, when the serial correlation of relevant factors is relatively low, TPCA produces the strongest forecasting performance, followed by the MSS-3PRF. This result indicates that even if the underlying instabilities in the data are not Markov-type, the MSS-3PRF approach manages to fit their dynamics better than linear frameworks when the relevant factors exhibit high persistence. Moreover, we also consider the case of constant factor loadings and show the results in panel C of Table 2. As expected, the linear 3PRF is the method showing the best forecasting performance across all the scenarios under consideration, indicating its superiority over other competing linear models when there is no time instability in the factor loadings from the underlying data generating process.

Overall, conditional on our DGPs, we can conclude that, on average, the MSS-3PRF is the framework best able to capture instabilities in the relationship between the set of predictors and its common factor, followed by the MS-3PRF. Both nonlinear frameworks outperform linear approaches. Regarding the linear frameworks, in general, TPCA outperforms the 3PRF and the PC-LARS, and PCA obtains the weakest forecasting performance. In online appendix A.3, we also report Monte Carlo experiments to assess how well the 3PRF and MS-3PRF estimate the underlying factor, and we find that the MS-3PRF performs similarly to the 3PRF approach.

4 Empirical Applications

The first application is related to exchange rate forecasting. This is highly relevant given that it has long been recognized that non-linearities play an important role in the dynamics of exchange rates (see, e.g., the early contribution in Chinn (1991) and more recently Rossi (2013) and Abbate and Marcellino (2016)). However, it is only recently that the literature on exchange rate forecasting has concentrated on the role and importance of factors for predicting exchange rates (see, e.g., Engel et al. (2015) in a linear context). Putting the MS-3PRF approach to work in the context of exchange rate forecasts is highly relevant: it allows us to combine the non-linear dynamics observed in exchange rate movements with the factor structure driving systematic variations in exchange rates, which has recently gained traction in the exchange rate forecasting literature. Our second empirical application is a standard macroeconomic forecasting application in that we use the McCracken and Ng (2015) data set to forecast economic activity in the United States.

4.1 Forecasting exchange rates

In this first forecasting exercise, we construct factors from a cross-section of nominal bilateral U.S. dollar (USD) exchange rates against a panel of 26 currencies. We extract factors from the MS-3PRF, MSS-3PRF, linear 3PRF, PCA, TPCA and PC-LARS. We then use the resulting factors to forecast selected bilateral exchange rates. (All currency pairs use the USD as numéraire.) The choice of the data set draws from the exercise in Greenaway-McGrevy et al. (2016). The data set is monthly, and the full sample size extends from January 1995 to December 2015. The data are obtained from the International Financial Statistics of the International Monetary Fund, and the monthly data are taken as the monthly average of daily data. The data set consists of the currencies of Australia (AUS),[7] Brazil (BRA), Canada (CAN), Chile (CHI), Columbia (COL), the Czech Republic (CZE), the euro (EUR), Hungary (HUN), Iceland (ICE), India (IND), Israel (ISR), Japan (JPN), Korea (KOR), Mexico (MEX), Norway (NOR), New Zealand (NZE), the Philippines (PHI), Poland (POL), Romania (ROM), Singapore (SIN), South Africa (RSA), Sweden (SWE), Switzerland (SUI), Taiwan (TAI), Turkey (TUR) and the United Kingdom (GBR).[8]

We consider forecast horizons, h, ranging from 1 month to 12 months and report predictive results for selected major currencies: the euro, the British pound, the Japanese yen and the Canadian dollar. The first estimation sample runs from February 1995 to March 2007, and it is recursively expanded until we reach the end of the estimation sample. Hence, the forecast evaluation period extends from August 2006 to December 2015. Note that all models are estimated recursively, hence we never use future information to calculate any of the

[7]The three-letter country codes follow the convention from the International Olympic Committee except for Taiwan, labelled as TAI.

[8]Data for the euro before January 1999 and Taiwan were obtained from the U.S. Federal Reserve G.5 table (monthly average of daily data).

models' parameters. As in the Monte Carlo experiment, we compare the forecasts obtained from the MS-3PRF with forecasts derived from PCA, linear 3PRF, TPCA and PC-LARS. Moreover, we use two different versions of the MS-3PRF, one with regime switching only in the first step (i.e., in the factor loadings), and one with regime-switching in the first and third steps (i.e., in the factor loadings and in the parameters of the forecasting equation). In the first step of the algorithm for the MS-3PRF approach, we model regime changes in all parameters of the model (i.e., intercept, slope and innovation variance), since we obtained stronger fit—as measured by the Schwarz information criterion (SIC)—with such a specification. Note also that we consider a model with two factors across all methods. The choice of the number of factors follows the modelling choices in Engel et al. (2015), Greenaway-McGrevy et al. (2016) and Verdelhan (2015), but, qualitatively, our results are robust to the use of one or three factors in the predictive equation. We also include the MSS-3PRF approaches in the set of models we consider (both versions—that is, with regime switches in the first pass only and with regime switches in the first and third passes).

All exchange rate series are taken as the first difference of their logarithm before performing factor analysis. In the case of the 3PRF approaches, we use the automatic proxy-selection procedure from Kelly and Pruitt (2015); that is, we use the exchange rate we are interested in forecasting as a target proxy when extracting the first factor and then proceed sequentially as outlined in Table 2 from Kelly and Pruitt (2015). For all methods, we standardize the data recursively in the estimation before estimating the factors.

For the prediction step, in the linear cases—that is, for PCA, TPCA, PC-LARS, 3PRF, MS-3PRF (first pass) and MSS-3PRF (first pass)—the h-period-ahead forecasts for a specific currency $R^j_{t+h|t}$ are constructed in level based on the following equation:

$$R^j_{t+h|t} = R^j_t(1 + \hat{\alpha} + \boldsymbol{F_t'}\hat{\boldsymbol{\beta}}), \tag{21}$$

where $\hat{\alpha}$ and $\hat{\boldsymbol{\beta}}$ are obtained from the following regression (for simplicity of the notation, we omit h subscripts from the coefficients α and $\boldsymbol{\beta}$):

$$\Delta r^j_{t,h} = \alpha + \boldsymbol{F}'_{t-h}\boldsymbol{\beta} + \epsilon_t, \tag{22}$$

where $\Delta r^j_{t,h}$ indicates the h-period change in the logarithm of the exchange rate R^j_t (i.e., $\Delta r^j_{t,h} = \ln(R^j_t) - \ln(R^j_{t-h})$), and $\boldsymbol{F}_t$ indicates the factors extracted from either PCA, TPCA, PC-LARS, 3PRF or MS-3PRF approaches. In the case of the MS-3PRF (first and third passes) and MSS-3PRF (first and third passes), equation 22 is modified as follows:

$$\Delta r^j_{t,h} = \alpha(S^j_t) + \boldsymbol{F}'_{t-h}\boldsymbol{\beta}(S^j_t) + \epsilon_t(S^j_t), \tag{23}$$

where S^j_t is a two-regime Markov chain, distinct across all predicted currencies j, with constant transition probabilities. In the case of the MS-3PRF (first and third passes), as is commonly done in forecasting exercises with Markov-switching models, the forecasts are calculated as a weighted average of forecasts conditional on the parameters being in a given regime. The predicted probabilities of being in a given regime k, h periods ahead, are obtained recursively as

$$P(S^j_{t+h|t} = k) = \sum_{i=1}^{2} p^j_{ik} P(S^j_{t+h-1|t} = k), \tag{24}$$

where p^j_{ik} indicates the constant transition probabilities, and 2 is the total number of regimes.

In the case of the MSS-3PRF (first and third passes) approach, the forecasts are calculated conditional on being in a given regime; that is, the predicted probabilities are obtained as

$$P(S^j_{t+h|t} = k) = I(P(S^j_{t|t} = k)), \tag{25}$$

where $I(\cdot)$ is an indicator variable that indicates the regimes with the highest smoothed probability at the origin of the forecast horizon. As such, this corresponds to the approach often used to plot (regime-specific) impulse responses in MS-VAR models (see, e.g., Hubrich and Tetlow (2015)).

As an illustration of the MS-3PRF approach to extract factors from a panel of exchange rates, Figure 1 reports the Markov-switching factor loadings based on the MS-3PRF approach using the Canadian dollar as a target proxy, that is, the factor loadings associated with the first factor. From this figure, one can see that there is substantial time variation in factor loadings for a number of currencies (e.g., the Australian dollar and the New Zealand dollar), whereas for other currencies, there is little time variation in the factor loadings (e.g., the euro and the Swiss franc).

A number of additional comments are in order. First, we do not use observable factors to model the dynamics of exchange rates. Verdelhan (2015) finds that exchange rate variations are driven by a two-factor structure: a U.S.-dollar factor that serves as a proxy for global macroeconomic risk and a carry factor that is interpreted as capturing uncertainty risk. Our analysis differs from this study in that our focus is on out-of-sample predictive ability. Hence, we do not aim to provide a structural interpretation to the factors we extract.

The left-hand side of Table 3 reports point forecasting results for specific currencies: the Canadian dollar (CAD), the euro (EUR), the Japanese yen (JPY) and the British pound (GBP), all relative to the USD. These are G7 currencies, and among the most traded currency pairs according to the Bank for International Settlements Triennal Central Bank Survey.[9] The point forecast results are presented as the MSFE of a specific approach

[9]See `http://www.bis.org/publ/rpfx16fx.pdf`.

relative to the MSFE obtained from the no-change forecast. The no-change forecast is the standard benchmark in the exchange rate forecasting literature (see, e.g., Rossi (2013)). We also report the results of the Diebold and Mariano (1995) test of equal out-of-sample predictive accuracy using the no-change forecast as a benchmark.[10] First, the models' forecasting performance relative to the no-change forecast is typically the strongest for forecast horizon $h = 1$ (except for the JPY/USD). The improvement in forecast accuracy relative to the random walk is also statistically significant according to the Diebold and Mariano test of equal MSFE when forecasting the Canadian dollar at forecast horizon $h = 1$ across most approaches (this is also true to a lesser extent for the British pound). Second, the PC-LARS approach performs best for forecast horizon $h = 1$ when forecasting the British pound. Moreover, the MS-3PRF (first and third pass) approach performs best when forecasting the Canadian dollar for forecast horizons $h > 1$. Third, for the Canadian dollar and the Japanese yen, modelling time variation in the forecasting equation is relevant in that this leads to substantial forecasting improvement over the no-change forecast at distant forecast horizons $h = \{9\}$ for the Japanese yen and $h = \{2, 3, 6, 9, 12\}$ using the MS-3PRF (first and third passes) approach.[11]

Next, the right-hand side of Table 3 shows the directional accuracy forecasting results, which are broadly in line with the point forecast results. Under the null hypothesis of no directional accuracy, one would expect a success ratio of 0.5. We also report the results of the Pesaran and Timmermann (2009) test to evaluate the statistical significance of the directional accuracy results. Across all forecasting approaches, the success ratios tend to be stronger for forecast horizon $h = 1$, except for the JPY/USD. In those cases, the improvements in directional accuracy are often statistically significant according to the Pesaran and Timmermann (2009) test. It is also interesting to note that the success ratios are especially strong at distant forecast horizons for selected currencies, as high as 72.6 per cent for the CAD/USD and 77.0 per cent for the JPY/USD in the case of the MS-3PRF with regime changes in the first and third passes. In the online appendix, we investigate the stability of the directional accuracy results, implementing the Giacomini and Rossi (2010) test of stability in predictive performance when predicting the CAD-USD for forecast horizon $h = 12$. The results, reported in Figure A.5 of the online appendix, show that there is time instability in the forecasting performance in this case, but we do conclude that there were periods when the MS-3PRF approach significantly outperformed the benchmark no-change predictive model.

[10] The Diebold and Mariano (1995) test of equal out-of-sample predictive accuracy is reported to give a sense of statistical significance of the point forecasting results. However, this test is based on the population MSFE (not the actual MSFE), so this test tends to reject the null of equal MSFEs too often.

[11] Admittedly, in the case of the euro, the forecasting performance of the MS-3PRF (first and third passes) and the MSS-3PRF (first and third passes) approaches deteriorates as the forecast horizon lengthens, suggesting that it is not always relevant to model regime shifts in the forecasting equation.

Overall, while the differences in predictive accuracy tend to be small across forecasting approaches in terms of point forecasts, the gains in terms of directional accuracy are strong with the MS-3PRF approach and typically statistically significant according to the Pesaran and Timmermann (2009) test.

4.2 Forecasting economic activity

In this application, we use the McCracken and Ng (2015) data set to forecast eight major quarterly U.S. variables: GDP, consumption, investment, exports, imports, total hours, GDP inflation and personal consumption expenditures (PCE) inflation.[12] We implemented the following outlier corrections to the predictors: observations of the transformed series with absolute median deviations larger than six times the inter-quartile range were replaced with the median value of the preceding five observations. The full sample extends from the third quarter of 1960 to the third quarter of 2015. In the forecasting exercise, the first estimation sample extends from the third quarter of 1960 to the fourth quarter of 1984, and it is expanded recursively until we reach the end of the sample. We consider forecast horizons, h, ranging from one quarter to eight quarters. We use eight competing approaches: PCA from which we extract five factors from the underlying data set, although we use only the first one in the forecasting equation;[13] PCA where hard thresholding has been performed before extracting the first principal component to forecast (TPCA); PCA where soft thresholding has been performed before extracting the first principal component to forecast (PC-LARS); linear 3PRF; MS-3PRF and MSS-3PRF with regime-switching parameters in the first pass only; and MS-3PRF and MSS-3PRF with regime-switching parameters in the first and third passes. For the 3PRF approaches, we use one factor and use the predicted variable as a target proxy in the first step of the 3PRF approach (target-proxy 3PRF).[14]

[12]Data descriptions and details on data transformation are available online at `https://research.stlouisfed.org/econ/mccracken/fred-databases/Appendix_Tables_Update.pdf`. The slight modifications we made to the original data set are reported in the appendix.

[13]Our results are qualitatively robust to the use of a different number of factors in the predictive equation. Detailed results are available on request.

[14]When estimating the number of factors using information criteria, it is common to find a large number of factors summarizing the comovements of U.S. macroeconomic variables (e.g., McCracken and Ng (2015) estimate eight factors in the FRED-MD monthly macroeconomic database). However, in the forecasting exercise, in line with the literature, we use the first factor in the predictive equation. This corresponds to a real economic activity factor that closely follows the U.S. business cycle dynamics (see Figure 2). Using the first two factors in the predictive equation led to little changes in the forecasting performance.

We first report results from an in-sample exercise. Figure 2 shows the estimated factors across all six methods (PCA, TPCA, PC-LARS, 3PRF, MS-3PRF and MSS-3PRF; the latter three methods use GDP growth as a target variable). This shows that the factor estimates are relatively similar across approaches and that they closely follow the U.S. business cycle. As in McCracken and Ng (2015), we calculate diffusion indices ($\hat{F}_t$) based on the partial sums of the factor estimates $\hat{f}_t$; that is, $\hat{F}_t = \sum_{j=1}^{t} \hat{f}_j$. (The reason for doing so is that diffusion indices summarize information contained in the trend as opposed to the "raw" factors that are estimated on stationary data, so the resulting factors are too volatile for turning point analysis.) The factors f_t are extracted with the six aforementioned methods. We then implemented the Bry and Boschan (1971) algorithm to estimate expansions and recessions from these diffusion indices.[15] The resulting classification of U.S. business cycles obtained from the MS-3PRF diffusion index has the strongest correlation with the National Bureau of Economic Research dummy variable of expansions and recessions (0.563) followed by the 3PRF diffusion index (0.543), MSS-3PRF diffusion index (0.506), PC-LARS diffusion index (0.497), PCA diffusion index (0.486) and TPCA diffusion index (0.467). Moreover, only the MS-3PRF and MSS-3PRF approaches obtain a perfect classification of recessions, while PCA, TPCA and 3PRF diffusion indexes have a near perfect classification of recessions (these three methods identify the 1973-1974 recession with a one-quarter lag). This suggests that the MS-3PRF approach has important information related to the state of the business cycle that is not necessarily reflected in competing approaches.

An attractive feature of Markov-switching models is their ability to endogenously estimate regimes. Figure 3 shows a heat map of the smoothed probability of being in the first regime (associated with adverse business cycle conditions) for the in-sample factor loadings obtained from the first step of the MS-3PRF approach. First, it is interesting to note that, across all series, there is substantial time variation in the smoothed probability, suggesting that there is evidence in favour of regime shifts in the factor loadings. Second, the timing of the shifts in the factor loadings coincides with the changes in business cycle phases for a large number of series (e.g., output and income, as well as labour market variables). Additional evidence on regime shifts in the factor loadings is provided in Figure A.2 of the online appendix. This figure shows that there is substantial variation in the factor loadings for the unemployment rate and industrial production related to the state of the business cycle. Selected financial and credit variables (S&P 500 returns and consumer loans) also exhibit substantial time variation, suggesting that the assumption of constant factor loadings often employed with this type of data set is likely to be too restrictive.

[15] The Bry and Boschan (1971) algorithm is a non-parametric method to estimate cycles in time series. We implemented the quarterly version of the Bry and Boschan (1971) algorithm from Harding and Pagan (2002), using the GAUSS code available at `http://www.ncer.edu.au/resources/data-and-code.php`.

A few additional comments related to the out-of-sample forecasting exercise are required. First, note that macroeconomic variables are typically subject to substantial revisions and different publication lags. In this empirical exercise, we abstract from this issue and consider revised data. While this is not a fully realistic approach from a practitioner's perspective, there is no reason to think that one specific approach would benefit more from this simplification. Hence, this remains a useful forecasting exercise to compare the relative merits of each forecasting approach. Second, across all approaches, quarterly factors are extracted from the monthly data set of McCracken and Ng (2015), where quarterly data are taken as quarterly averages of monthly data before performing factor analysis. Obviously, alternative temporal aggregation schemes could be adopted, but we found that the in-sample correlation of the real activity factor was very strong compared with a situation where one would use the last monthly observation of the quarter as a quarterly observation before performing factor analysis (about 0.95 between these two aggregation schemes across the different factor approaches).[16] Our temporal aggregation scheme is standard in the literature (see, e.g., section 6.1 in Stock and Watson (2016)), and we leave the issue of a mixed-frequency setting to future research.[17] Third, the forecasts are constructed as follows:

$$y_{t+h|t} = \hat{\alpha} + \hat{\beta}(L)\hat{f}_t + \hat{\gamma}(L)y_t, \tag{26}$$

where $\beta(L)$ and $\gamma(L)$ are finite-order lag polynomials, whose lag lengths are obtained with the SIC at the beginning of the forecasting exercise, using a maximum lag length of 6 for $\gamma(L)$ and 3 for $\beta(L)$. All predicted variables y_t are taken as the first difference of their logarithm. For the MS-3PRF and MSS-3PRF with switches in the first and third passes, we consider regime-switching parameters in all parameters of equation 26 and in the variance of the error term.

Table 4 shows the out-of-sample forecasting results. All results are reported relative to the forecasts obtained from PCA. Hence, a number below 1 indicates that a given approach outperforms PCA. We also report the results of the Diebold and Mariano (1995) test of equal out-of-sample predictive accuracy using PCA as a benchmark. Overall, across all forecast horizons and predicted variables (40 cases), the MS-3PRF and MSS-3PRF obtain the best forecasting results in 22 cases, PC-LARS in 8 cases, the linear 3PRF in 6 cases and TPCA in 1 case. In the remainder of the cases, PCA performs best. It is interesting

[16] This result still holds when doing PCA at a monthly frequency and then aggregating the factor at a quarterly frequency.

[17] As a side note, the first pass of the 3PRF filter could possibly accommodate mixed-frequency data using the techniques outlined in Foroni et al. (2015); whereas, in the third pass of the filter, unrestricted mixed data sampling (MIDAS) polynomials could be used as in Hepenstrick and Marcellino (2016), and regime-switching parameters in the mixed-frequency predictive equation could be modelled as in Guérin and Marcellino (2013).

to note that the MSS-3PRF (first and third passes) approach performs best for forecasting inflation (both PCE inflation and GDP inflation), and it does so significantly according to the Diebold and Mariano (1995) test at long forecast horizons (i.e., for $h > 4$). We formally investigate the stability of these forecasting results for predicting inflation by implementing the Giacomini and Rossi (2010) test for forecast comparisons in unstable environments. Figure A.3 in the online appendix shows the results of the fluctuation test for forecasting PCE inflation at a 8-quarter horizon with the MSS-3PRF (first and third passes) approach relative to the benchmark PCA model. The local relative MSFE is above the critical value for most of the evaluation sample suggesting that the MSS-3PRF (first and third passes) approach produced better 8-quarter-ahead forecasts than PCA for most of the evaluation sample except in the last years of the evaluation sample. A similar result is obtained in the case of GDP inflation (see Figure A.4 of the evaluation sample). When forecasting aggregate economic activity (GDP), the linear 3PRF approach performs best for $h = \{2\}$ and PC-LARS for $h = \{3\}$ and $h = \{4\}$. The improvements in forecast accuracy are typically statistically significant according to the Diebold and Mariano (1995) test. The MS-3PRF (first pass) approach performs well at forecasting exports and consumption. For predicting investment and hours worked, PC-LARS tends to perform best at short forecast horizons.

5 Conclusion

In this paper, we extended the linear three-pass regression filter to settings where parameters can vary according to Markov processes, introducing the Markov-switching three-pass regression filter. A key advantage of our framework is to circumvent the computational difficulties associated with the estimation of a large-scale dynamic factor model with regime-switching parameters without foregoing flexibility in modelling choices.

In both simulation and empirical examples, our method compares favourably with existing alternatives in terms of forecasting performance. The MS-3PRF approach is also attractive beyond forecasting applications. For example, the MS-3PRF approach would easily allow one to model regime-switching correlations often observed in finance in a high-dimensional setting. This could be relevant in the context of the growing literature aiming at measuring network connectedness among financial firms or asset classes (see, e.g., Billio et al. (2012) or Diebold and Yilmaz (2014)). Likewise, the MS-3PRF framework could be used in the context of structural factor-augmented VAR models that are commonly used in macroeconomics. Overall, thanks to its generality and ease of implementation, the MS-3PRF approach offers a promising framework to model regime changes in high-dimensional settings for a large class of applications in macroeconomics and finance.

References

Aastveit, K., Carreiro, A., Clark, T., and Marcellino, M. (2017). Have standard VARs remained stable since the crisis? *Journal of Applied Econometrics*, 32(5):931–951.

Abbate, A. and Marcellino, M. (2016). Modelling and forecasting exchange rates with time-varying parameter models. *Journal of the Royal Statistical Society, Series A*, forthcoming.

Amengual, D. and Watson, M. W. (2007). Consistent Estimation of the Number of Dynamic Factors in a Large N and T Panel. *Journal of Business & Economic Statistics*, 25:91–96.

Ang, A. and Timmermann, A. (2012). Regime Changes and Financial Markets. *Annual Review of Financial Economics*, 4(1):313–337.

Bai, J. and Ng, S. (2008). Forecasting economic time series using targeted predictors. *Journal of Econometrics*, 146(2):304–317.

Barnett, W. A., Chauvet, M., and Leiva-Leon, D. (2016). Real-time nowcasting of nominal GDP with structural breaks. *Journal of Econometrics*, 191(2):312–324.

Bates, B. J., Plagborg-Moller, M., Stock, J. H., and Watson, M. W. (2013). Consistent factor estimation in dynamic factor models with structural instability. *Journal of Econometrics*, 177(2):289–304.

Billio, M., Getmansky, M., Lo, A. W., and Pelizzon, L. (2012). Econometric measures of connectedness and systemic risk in the finance and insurance sectors. *Journal of Financial Economics*, 104(3):535–559.

Bry, G. and Boschan, C. (1971). Cyclical Analysis of Time Series: Procedures and Computer Programs. *National Bureau of Economic Research.*

Camacho, M., Pérez-Quirós, G., and Poncela, P. (2012). Markov-switching dynamic factor models in real time. CEPR Discussion Papers 8866, C.E.P.R. Discussion Papers.

Canova, F. (1993). Modelling and Forecasting Exchange Rates with a Bayesian Time-Varying Coefficient Model. *Journal of Economic Dynamics and Control*, 17:233–261.

Carter, A. and Steigewald, D. (2012). Testing for Regime Switching: A Comment. *Econometrica*, 80:1809–1812.

Chauvet, M. (1998). An Econometric Characterization of Business Cycle Dynamics with Factor Structure and Regime Switching. *International Economic Review*, 39(4):969–96.

Cheng, X., Liao, Z., and Schorfheide, F. (2016). Shrinkage Estimation of High-Dimensional Factor Models with Structural Instabilities. *Review of Economic Studies*, forthcoming.

Ching, W.-K., Fung, E., and Ng, M. (2002). A multivariate Markov chain model for categorical data sequences and its applications in demand predictions. *IMA Journal of Management Mathematics*, 13:187–199.

Chinn, M. D. (1991). Some linear and nonlinear thoughts on exchange rates. *Journal of International Money and Finance*, 10(2):214–230.

Cho, J. S. and White, H. (2007). Testing for Regime Switching. *Econometrica*, 75:1671–1720.

D'Agostino, A., Gambetti, L., and Giannone, D. (2013). Macroeconomic forecasting and structural change. *Journal of Applied Econometrics*, 28(1):82–101.

De Mol, C., Giannone, D., and Reichlin, L. (2008). Forecasting using a large number of predictors: Is Bayesian shrinkage a valid alternative to principal components? *Journal of Econometrics*, 146(2):318–328.

Del Negro, M. and Otrok, C. (2008). Dynamic factor models with time-varying parameters: measuring changes in international business cycles. Staff Reports 326, Federal Reserve Bank of New York.

Diebold, F. X. and Mariano, R. S. (1995). Comparing Predictive Accuracy. *Journal of Business & Economic Statistics*, 13(3):253–63.

Diebold, F. X. and Yilmaz, K. (2014). On the network topology of variance decompositions: Measuring the connectedness of financial firms. *Journal of Econometrics*, 182(1):119–134.

Douc, R., Moulines, E., and Rydén, T. (2004). Asymptotic properties of the maximum likelihood estimor in autoregressive models with Markov regime. *Annals of Statistics*, 32(5):2254–2304.

Efron, B., Hastie, T., Johnstone, I., and Tibshirani, R. (2004). Least Angle Regression. *Annals of Statistics*, 32(2):407–499.

Eickmeier, S., Lemke, W., and Marcellino, M. (2015). Classical time varying factor-augmented vector auto-regressive modelsestimation, forecasting and structural analysis. *Journal of the Royal Statistical Society Series A*, 178(3):493–533.

Engel, C., Mark, N. C., and West, K. D. (2015). Factor Model Forecasts of Exchange Rates. *Econometric Reviews*, 34(1-2):32–55.

Forni, M., Hallin, M., Lippi, M., and Reichlin, L. (2005). The Generalized Dynamic Factor Model: One-Sided Estimation and Forecasting. *Journal of the American Statistical Association*, 100:830–840.

Foroni, C., Guérin, P., and Marcellino, M. (2015). Using low frequency information for predicting high frequency variables. *Norges Bank Working Paper*, 2015/13.

Giacomini, R. and Rossi, B. (2010). Forecast comparisons in unstable environments. *Journal of Applied Econometrics*, 25(4):595–620.

Greenaway-McGrevy, R., Mark, N., Sul, D., and Wu, J.-L. (2016). Identifying Exchange Rate Common Factors. *Mimeo Notre Dame*.

Groen, J. and Kapetanios, G. (2016). Revisiting useful approaches to data-rich macroeconomic forecasting. *Computational Statistics & Data Analysis*, 100:221–239.

Guérin, P. and Leiva-Leon, D. (2016). Monetary Policy, Stock Market and Sectoral Comovement. *Mimeo*.

Guérin, P. and Marcellino, M. (2013). Markov-Switching MIDAS Models. *Journal of Business & Economic Statistics*, 31(1):45–56.

Hamilton, J. D. and Owyang, M. T. (2012). The Propagation of Regional Recessions. *The Review of Economics and Statistics*, 94(4):935–947.

Harding, D. and Pagan, A. (2002). Dissecting the cycle: a methodological investigation. *Journal of Monetary Economics*, 49(2):365–381.

Hepenstrick, C. and Marcellino, M. (2016). Forecasting with Large Unbalanced Datasets: The Mixed-Frequency Three-Pass Regression Filter. *SNB Working Papers*, 2016-04.

Hubrich, K. and Tetlow, R. J. (2015). Financial stress and economic dynamics: The transmission of crises. *Journal of Monetary Economics*, 70(C):100–115.

Kelly, B. and Pruitt, S. (2015). The three-pass regression filter: A new approach to forecasting using many predictors. *Journal of Econometrics*, 186(2):294–316.

Leroux, B. (1992). Maximum-likelihood estimation for hidden Markov models. *Stochastic Processes and their Applications*, 40:127–143.

McCracken, M. W. and Ng, S. (2015). FRED-MD: A Monthly Database for Macroeconomic Research. *Journal of Business & Economic Statistics*, forthcoming.

Mikkelsen, J. G., Hillebrand, E., and Urga, G. (2015). Maximum Likelihood Estimation of Time-Varying Loadings in High-Dimensional Factor Models. CREATES Research Papers 2015-61, School of Economics and Management, University of Aarhus.

Nakajima, J. and West, M. (2013). Bayesian Analysis of of Latent Threshold Dynamics Models. *Journal of Business Economics and Statistics*, 31(2):151–164.

Pesaran, M. H. and Timmermann, A. (2009). Testing Dependence Among Serially Correlated Multicategory Variables. *Journal of the American Statistical Association*, 104(485):325–337.

Rossi, B. (2013). Exchange Rate Predictability. *Journal of Economic Literature*, 51(4):1063–1119.

Sims, C. (1993). A 9 Variable Probabilistic Macroeconomic Forecasting Model. *Business Cycles, Indicators and Forecasting, NBER studies in business cycles*, 28:179–214.

Sims, C. A. and Zha, T. (2006). Were There Regime Switches in U.S. Monetary Policy? *American Economic Review*, 96(1):54–81.

Smith, A., Prasad, N., and Tsai, C.-L. (2006). Markov-switching Model Selection using Kullback-Leibler Divergence. *Journal of Econometrics*, 134:553–577.

Stock, J. and Watson, M. (2002a). Forecasting Using Principal Components From a Large Number of Predictors. *Journal of the American Statistical Association*, 97:1167–1179.

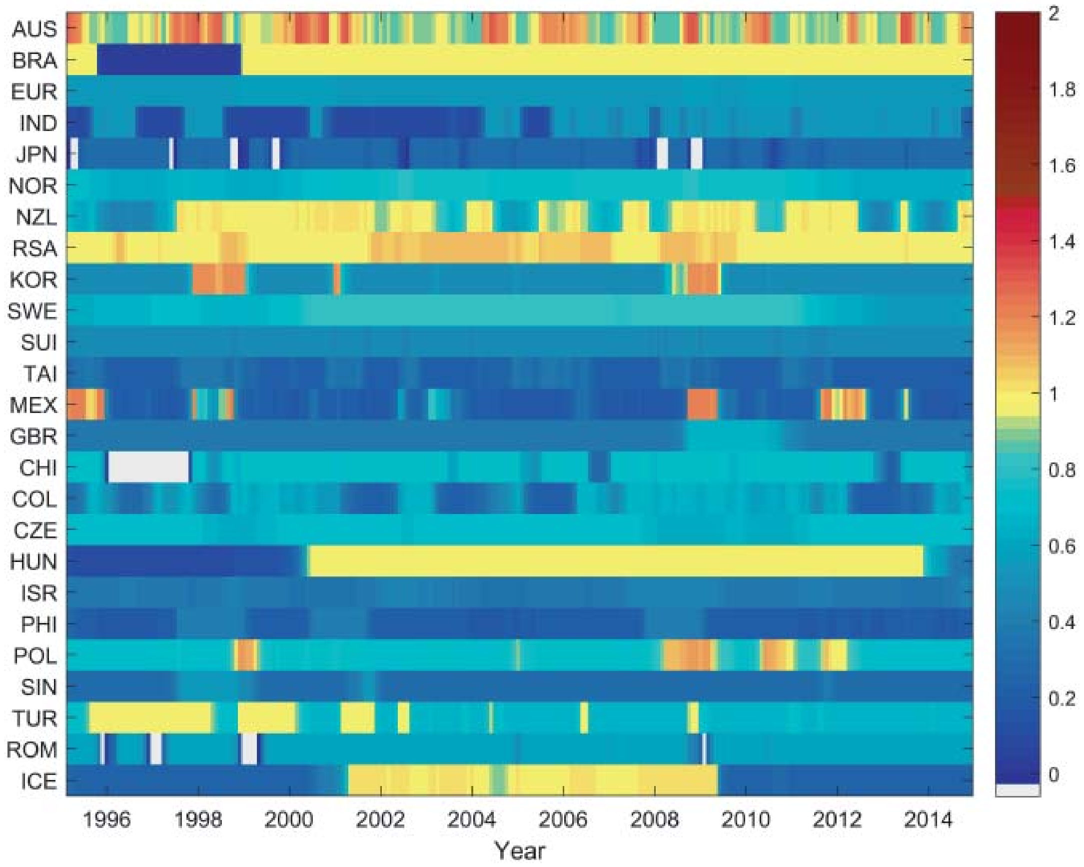
Stock, J. H. and Watson, M. W. (2002b). Macroeconomic Forecasting Using Diffusion Indexes. *Journal of Business & Economic Statistics*, 20(2):147–62.

Stock, J. H. and Watson, M. W. (2016). Factor Models and Structural Vector Autoregressions in Macroeconomics. *Mimeo.*

Verdelhan, A. (2015). The Share of Systematic Variation in Bilateral Exchange Rates. *Journal of Finance*, forthcoming.

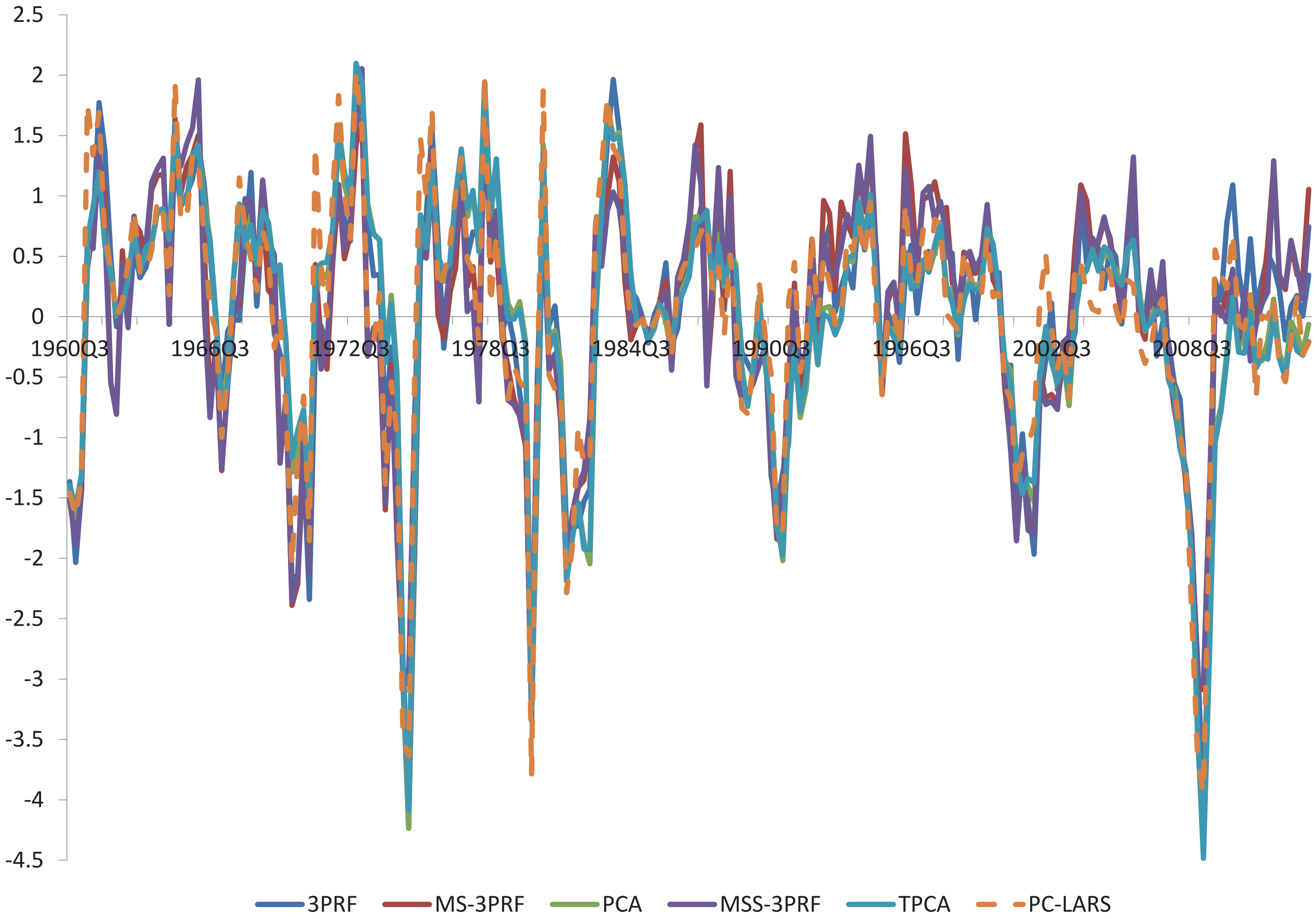
Von Ganske, J. (2016). A Regime Switching Partial Least Squares Approach to Forecasting Industry Stock Returns. *Mimeo Edhec-Risk Institute.*

Figure 1: Markov-switching factor loadings—Canadian dollar as a target proxy



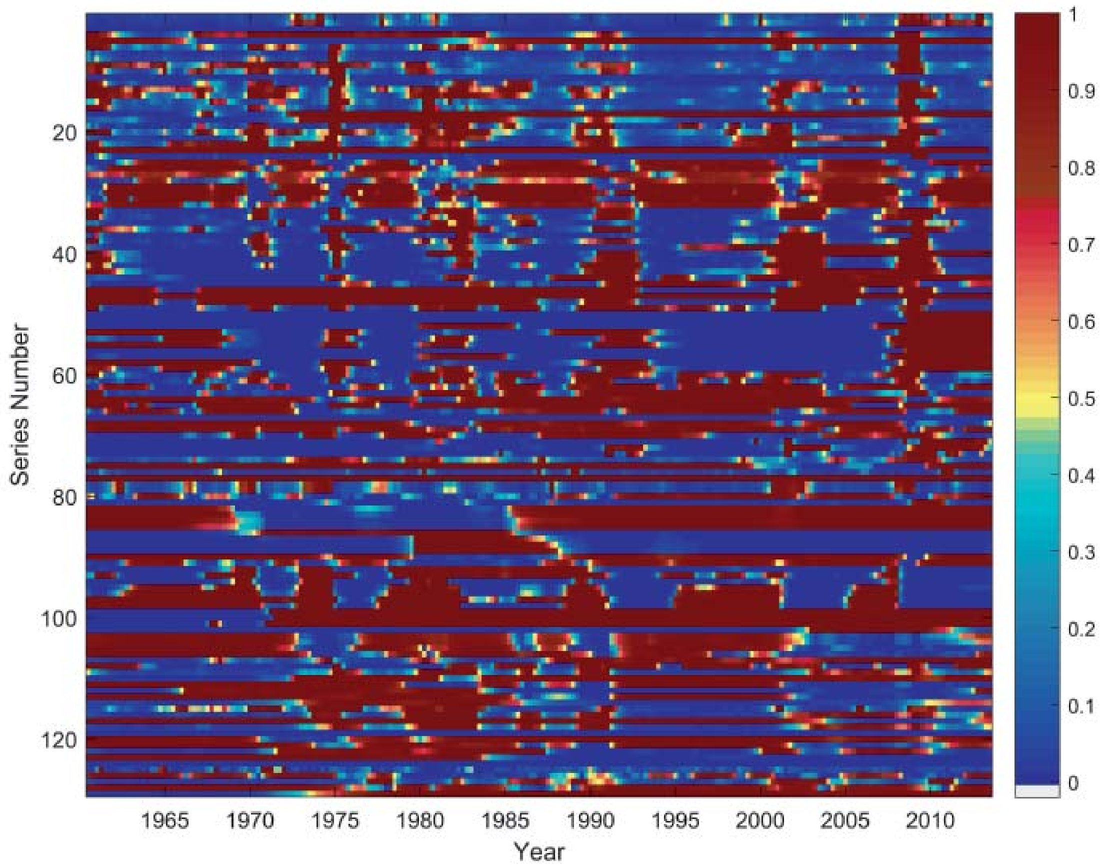
Note: Dark red indicates higher values for the factor loadings obtained with the MS-3PRF approach with the Canadian dollar as a target proxy.

Figure 2: Factor estimates across different approaches



Note: Factor estimates across different approaches: linear 3PRF, Markov-switching 3PRF (both MS-3PRF and MSS-3PRF), PCA, TPCA and PC-LARS. GDP growth is used as a target proxy for the 3PRF approaches.

Figure 3: Probability of being in the first regime for the factor loadings



Note: Dark red indicates higher value for the probability of being in the first regime, which is normalized to correspond to the lowest intercept of the two regimes. Factor loadings are obtained from the MS-3PRF approach with GDP growth as a target proxy.

Table 1: Simulation results with different degrees of instabilities in the loadings

Panel A. Instability in 100 per cent of the loadings

ρ_f	ρ_g	α	β	T=100 , N=100						T=200 , N=200					
				MS-3PRF	MSS-3PRF	3PRF	PCA	TPCA	PC-LARS	MS-3PRF	MSS-3PRF	3PRF	PCA	TPCA	PC-LARS
0.3	0.9	0.3	0	2.09	2.06	2.09	2.12	**2.04**	2.12	2.09	2.08	2.08	2.10	**2.04**	2.09
0.3	0.9	0.3	0.5	2.14	**2.11**	2.15	2.18	2.11	2.18	2.09	2.08	2.08	2.10	**2.04**	2.09
0.3	0.9	0.9	0	2.03	**1.99**	2.07	2.12	2.00	2.11	2.01	**1.97**	2.05	2.12	2.00	2.09
0.3	0.9	0.9	0.5	2.04	2.00	2.06	2.13	**1.98**	2.13	1.97	**1.93**	2.04	2.09	2.00	2.08
0.9	0.3	0.3	0	8.95	**8.77**	9.05	9.17	9.06	9.17	9.51	9.42	9.39	9.48	**9.36**	9.45
0.9	0.3	0.3	0.5	9.03	**8.87**	9.13	9.28	9.08	9.27	9.31	**9.16**	9.31	9.40	9.33	9.40
0.9	0.3	0.9	0	8.51	**8.17**	9.04	9.23	8.88	9.29	9.30	**8.82**	9.60	9.71	9.65	9.65
0.9	0.3	0.9	0.5	8.43	**8.04**	9.14	9.41	9.06	9.35	9.03	**8.75**	9.35	9.56	9.35	9.50

Panel B. Instability in 75 per cent of the loadings

ρ_f	ρ_g	α	β	T=100 , N=100						T=200 , N=200					
				MS-3PRF	MSS-3PRF	3PRF	PCA	TPCA	PC-LARS	MS-3PRF	MSS-3PRF	3PRF	PCA	TPCA	PC-LARS
0.3	0.9	0.3	0	1.93	1.87	1.95	2.16	**1.85**	2.14	1.89	1.89	1.90	2.10	**1.80**	2.09
0.3	0.9	0.3	0.5	1.95	1.94	1.93	2.10	**1.82**	2.09	1.89	1.88	1.88	2.08	**1.73**	2.06
0.3	0.9	0.9	0	1.95	1.92	1.95	2.11	**1.81**	2.11	1.93	**1.88**	1.92	2.11	1.95	2.09
0.3	0.9	0.9	0.5	2.01	**1.93**	1.99	2.14	1.94	2.13	1.94	**1.88**	1.91	2.10	1.93	2.07
0.9	0.3	0.3	0	7.23	**7.15**	7.41	8.82	7.78	8.27	7.91	7.66	8.08	9.50	9.21	**7.56**
0.9	0.3	0.3	0.5	7.42	**7.25**	7.67	9.15	8.00	8.65	7.86	**7.67**	8.15	9.48	9.28	7.80
0.9	0.3	0.9	0	6.78	**6.27**	8.07	9.53	8.63	8.97	7.04	**6.44**	8.11	9.07	8.83	7.79
0.9	0.3	0.9	0.5	6.37	**5.96**	7.33	8.59	7.63	8.14	7.70	**7.02**	8.71	9.75	9.40	8.07

Panel C. Instability in 50 per cent of the loadings

ρ_f	ρ_g	α	β	T=100 , N=100						T=200 , N=200					
				MS-3PRF	MSS-3PRF	3PRF	PCA	TPCA	PC-LARS	MS-3PRF	MSS-3PRF	3PRF	PCA	TPCA	PC-LARS
0.3	0.9	0.3	0	1.84	1.82	**1.75**	2.15	1.75	2.11	1.74	1.71	**1.70**	2.12	1.62	2.01
0.3	0.9	0.3	0.5	1.87	1.84	1.78	2.14	**1.72**	2.13	1.74	1.73	1.68	2.11	**1.65**	2.02
0.3	0.9	0.9	0	1.94	1.89	**1.86**	2.14	1.89	2.14	1.86	1.81	**1.76**	2.08	1.89	2.08
0.3	0.9	0.9	0.5	1.92	**1.86**	1.89	2.16	1.93	2.16	1.88	1.80	**1.78**	2.09	1.92	2.08
0.9	0.3	0.3	0	6.32	**6.16**	6.27	8.88	7.30	6.97	6.66	6.38	6.51	9.25	8.60	**5.31**
0.9	0.3	0.3	0.5	5.99	**5.77**	5.99	8.55	6.64	6.53	6.40	6.07	6.27	9.27	8.58	**5.12**
0.9	0.3	0.9	0	6.12	**5.70**	6.46	8.84	7.52	7.56	6.61	6.06	6.98	9.47	8.83	**5.72**
0.9	0.3	0.9	0.5	6.19	**5.76**	6.52	8.56	7.49	7.53	6.22	5.74	6.71	8.79	8.28	**5.53**

Panel D. Instability in 25 per cent of the loadings

ρ_f	ρ_g	α	β	T=100 , N=100						T=200 , N=200					
				MS-3PRF	MSS-3PRF	3PRF	PCA	TPCA	PC-LARS	MS-3PRF	MSS-3PRF	3PRF	PCA	TPCA	PC-LARS
0.3	0.9	0.3	0	1.68	1.69	**1.53**	2.12	1.61	2.04	1.58	1.55	**1.36**	2.08	1.49	1.86
0.3	0.9	0.3	0.5	1.75	1.73	**1.59**	2.13	1.63	2.05	1.56	1.56	**1.39**	2.08	1.48	1.82
0.3	0.9	0.9	0	1.92	1.86	**1.71**	2.15	1.91	2.12	1.84	1.78	**1.57**	2.12	1.88	2.09
0.3	0.9	0.9	0.5	1.90	1.85	**1.71**	2.12	1.88	2.10	1.86	1.80	**1.62**	2.12	1.85	2.07
0.9	0.3	0.3	0	5.29	5.05	**4.91**	6.99	5.76	5.13	5.53	5.38	4.96	8.09	6.80	**4.75**
0.9	0.3	0.3	0.5	5.46	5.35	**5.17**	7.14	5.88	5.49	5.50	5.37	5.01	7.88	6.69	**4.83**
0.9	0.3	0.9	0	5.66	5.31	**5.09**	7.38	6.14	6.87	5.94	5.51	5.43	8.25	7.11	**5.26**
0.9	0.3	0.9	0.5	5.58	5.26	**5.16**	7.77	6.62	7.17	5.96	5.54	5.41	8.30	7.41	**5.28**

Note: The table reports the median MSFE based on 500 replications. Serial correlation in the factors is governed by ρ_f and ρ_g, while α and β govern serial and cross sectional correlation in the predictors' residuals, respectively. Entries in bold represent the lowest median MSFE for each specification. See text for additional details.

Table 2: Simulation results with different relationships of loadings instabilities

Panel A. Regime changes in the factor loadings are governed by independent Markov chains

ρ_f	ρ_g	α	β	T=100 , N=100						T=200 , N=200					
				MS-3PRF	MSS-3PRF	3PRF	PCA	TPCA	PC-LARS	MS-3PRF	MSS-3PRF	3PRF	PCA	TPCA	PC-LARS
0.3	0.9	0.3	0	2.11	2.08	2.05	2.08	**1.96**	2.07	2.06	2.03	2.05	2.08	**1.99**	2.07
0.3	0.9	0.3	0.5	2.08	2.03	2.08	2.14	**2.01**	2.13	2.05	2.05	2.02	2.07	**1.98**	2.06
0.3	0.9	0.9	0	2.00	**1.94**	2.03	2.12	1.96	2.08	1.98	**1.94**	1.97	2.06	1.94	2.04
0.3	0.9	0.9	0.5	1.97	**1.92**	1.98	2.07	1.92	2.05	1.99	1.95	1.98	2.07	**1.94**	2.05
0.9	0.3	0.3	0	8.62	**8.56**	8.74	8.98	8.60	8.95	9.59	**9.41**	9.47	9.61	9.51	9.56
0.9	0.3	0.3	0.5	9.20	9.21	9.22	9.38	**9.18**	9.35	9.23	9.20	**9.17**	9.32	9.18	9.27
0.9	0.3	0.9	0	8.09	**7.67**	8.73	9.10	8.66	9.10	8.86	**8.50**	9.36	9.54	9.43	9.48
0.9	0.3	0.9	0.5	7.79	**7.45**	8.43	8.73	8.42	8.73	8.78	**8.46**	9.23	9.43	9.27	9.43

Panel B. Random walk factor loadings

ρ_f	ρ_g	α	β	T=100 , N=100						T=200 , N=200					
				MS-3PRF	MSS-3PRF	3PRF	PCA	TPCA	PC-LARS	MS-3PRF	MSS-3PRF	3PRF	PCA	TPCA	PC-LARS
0.3	0.9	0.3	0	2.01	1.96	2.01	2.11	**1.86**	2.10	1.92	1.89	1.91	2.05	**1.82**	2.03
0.3	0.9	0.3	0.5	2.00	1.96	2.00	2.11	**1.84**	2.11	1.95	1.90	1.95	2.09	**1.84**	2.06
0.3	0.9	0.9	0	2.01	1.97	2.02	2.12	**1.82**	2.09	1.91	1.89	1.93	2.07	**1.80**	2.04
0.3	0.9	0.9	0.5	2.03	1.95	1.97	2.10	**1.82**	2.09	1.94	1.91	1.95	2.07	**1.83**	2.03
0.9	0.3	0.3	0	7.44	**7.22**	7.97	9.02	8.21	8.77	8.16	**7.94**	8.60	9.52	8.92	9.07
0.9	0.3	0.3	0.5	7.54	**7.14**	7.91	8.99	8.02	8.66	7.98	**7.66**	8.37	9.34	8.78	8.96
0.9	0.3	0.9	0	7.48	**7.00**	8.12	9.12	8.45	9.11	8.15	**7.61**	8.73	9.54	9.01	9.22
0.9	0.3	0.9	0.5	7.28	**6.77**	8.33	9.14	8.34	9.04	7.87	**7.44**	8.48	9.16	8.74	8.80

Panel C. Time-invariant factor loadings

ρ_f	ρ_g	α	β	T=100 , N=100						T=200 , N=200					
				MS-3PRF	MSS-3PRF	3PRF	PCA	TPCA	PC-LARS	MS-3PRF	MSS-3PRF	3PRF	PCA	TPCA	PC-LARS
0.3	0.9	0.3	0	1.61	1.63	**1.34**	2.09	1.49	2.01	1.40	1.42	**1.16**	1.92	1.27	1.76
0.3	0.9	0.3	0.5	1.61	1.61	**1.38**	2.03	1.51	1.98	1.44	1.43	**1.19**	1.90	1.27	1.73
0.3	0.9	0.9	0	1.91	1.84	**1.64**	2.17	1.88	2.13	1.79	1.72	**1.38**	2.11	1.84	2.06
0.3	0.9	0.9	0.5	1.90	1.84	**1.69**	2.14	1.88	2.12	1.82	1.75	**1.45**	2.13	1.84	2.08
0.9	0.3	0.3	0	5.08	5.01	**4.52**	5.25	4.76	4.97	5.14	5.10	**4.65**	5.02	4.81	4.77
0.9	0.3	0.3	0.5	4.77	4.65	**4.32**	4.83	4.50	4.72	5.21	5.17	**4.73**	4.95	4.79	4.82
0.9	0.3	0.9	0	5.51	5.20	**4.51**	7.16	5.93	6.61	5.79	5.34	**4.72**	6.07	5.37	5.01
0.9	0.3	0.9	0.5	5.69	5.34	**4.85**	7.10	5.83	6.97	5.87	5.48	**4.88**	6.13	5.36	5.14

Note: The table reports the median MSFE based on 500 replications. Serial correlation in the factors is governed by ρ_f and ρ_g, while α and β govern serial and cross sectional correlation in the predictors' residuals, respectively. Entries in bold represent the lowest median MSFE for each specification. See text for additional details.

Table 3: Out-of-sample exchange rate forecasting

Forecast horizon	1	2	3	6	9	12	1	2	3	6	9	12
	CAD/USD (MSPE)						CAD/USD (Success ratios)					
PCA	**0.871***	0.951	0.980	1.001	1.094	1.175	0.637**	0.602**	0.575*	0.522	0.504	0.407
TPCA	0.872*	0.955	0.986	0.996	1.088	1.182	**0.637****	0.620**	0.593**	0.522	0.487	0.407
PC-LARS	0.872	0.949	0.986	0.998	1.095	1.157	0.611**	0.602**	0.513	0.487	0.469	0.407
3PRF	0.963*	1.011	1.033	1.017	1.045	1.047	0.540	0.584**	0.558**	0.566**	0.504	0.451
MS-3PRF (first pass)	0.890	0.962	0.982	0.971	0.993	1.031	0.611**	0.611**	0.566*	0.540	0.522	0.504
MS-3PRF (first and third pass)	0.888**	**0.892**	**0.944**	**0.915**	**0.919**	**0.941**	0.575**	0.575*	**0.620***	**0.620****	**0.726****	**0.717****
MSS-3PRF (first pass)	0.911*	0.938*	0.987	0.994	1.020	1.058	0.620**	**0.620****	0.549	0.496	0.460	0.469
MSS-3PRF (first and third pass)	0.946	1.057	0.973	0.948	1.078	1.026	0.593**	0.478	0.575	0.584	0.620**	0.655**
	EUR/USD (MSPE)						EUR/USD (Success ratios)					
PCA	0.988	1.033	1.000	1.034	1.085	1.138	0.549	0.487	0.540	0.504	0.416	0.372
TPCA	0.995	1.049	1.010	1.061	1.109	1.155	0.558	0.487	0.566	0.487	0.443	0.354
PC-LARS	**0.961**	1.028	1.010	1.072	1.141	1.207	**0.566***	0.531	0.549	0.540	0.460	0.389
MS-3PRF (first pass)	0.994	1.008	0.996	1.002	1.036	1.095	0.549	0.540	0.575	0.566	0.469	0.469
MS-3PRF (first and third pass)	1.253	1.059	1.269	1.210	1.452	1.470	0.522	**0.549**	0.416	0.425	0.319	0.487
MSS-3PRF (first pass)	1.005	**0.985**	1.014	1.016	1.047	1.079	0.531	0.531	**0.593****	**0.558**	**0.531**	0.460
MSS-3PRF (first and third pass)	1.367	1.182	1.329	1.123	1.476	1.845	0.522	0.496	0.487	0.487	0.372	0.372
	JPY/USD (MSPE)						JPY/USD (Success ratios)					
PCA	1.091	1.093	1.108	1.070	1.101	1.145	0.460	0.496	0.469	0.496	0.504	0.425
TPCA	1.081	1.111	1.098	1.066	1.085	1.121	0.460	0.434	0.451	0.504	0.478	0.425
PC-LARS	1.066	1.066	1.078	1.078	1.074	1.117	0.531	0.504	0.496	0.487	0.469	0.425
3PRF	1.017	1.038	1.050	1.039	1.070	1.117	**0.575****	0.513	0.487	0.478	0.451	0.389
MS-3PRF (first pass)	1.031	1.060	1.054	1.066	1.078	1.122	0.487	0.487	0.496	0.451	0.487	0.363
MS-3PRF (first and third pass)	1.136	1.094	1.091	1.009	0.763**	1.051	0.522	0.478	0.451	**0.549**	**0.770****	0.460
MSS-3PRF (first pass)	1.058	1.061	1.055	1.043	1.074	1.078	0.478	0.522	0.531	0.487	0.460	0.443
MSS-3PRF (first and third pass)	1.345	1.240	1.241	1.159	**0.763***	1.068	0.558	**0.540**	**0.531**	0.549	0.735**	**0.513**
	GBP/USD (MSPE)						GBP/USD (Success ratios)					
PCA	0.803*	0.977	1.043	1.045	1.064	1.091	0.593**	0.540	0.513	0.434	0.522	0.575
TPCA	0.796*	0.965	1.023	1.043	1.064	1.091	0.593**	0.540	0.504	0.469	**0.558****	**0.584****
PC-LARS	**0.784***	**0.953**	1.029	1.035	1.073	1.114	**0.611****	**0.593****	0.531	0.531	0.425	0.496
3PRF	0.875	0.953	1.015	1.065	1.057	1.088	0.522	0.584**	**0.575****	0.460	0.549	0.443
MS-3PRF (first pass)	0.908*	0.980	1.010	1.032	1.064	1.094	0.566*	0.531	0.558	0.504	0.478	0.469
MS-3PRF (first and third pass)	0.977	1.083	1.032	1.113	1.128	1.114	0.531	0.549	0.540	0.513	0.416	0.469
MSS-3PRF (first pass)	0.893	0.984	1.037	1.045	1.074	1.119	0.549	0.549	0.566	0.522	0.496	0.460
MSS-3PRF (first and third pass)	1.392	1.523	1.302	1.361	1.397	1.354	0.513	0.478	0.540	**0.531**	0.496	0.496

Note: This table shows the relative mean square forecast error (RMSFE) for selected currency pairs (CAD/USD, EUR/USD, JPY/USD and GBP/USD) using PCA, TPCA, PC-LARS, linear 3PRF, MS-3PRF (first pass), MS-3PRF (first and third passes), MSS-3PRF (first pass) and MSS-3PRF (first and third passes) as forecasting approaches. Entries in bold indicate the best-performing approach for a specific horizon. Statistically significant reductions in the MSFE (or impovements in directional accuracy) relative to the random walk according to the Diebold-Mariano (Pesaran-Timmermann) test are indicated by asterisks (* denotes significance at the 10 per cent level. and ** denotes significance at the 5 per cent level).

Table 4: Out-of-sample macroeconomic forecasting

Forecast horizon	1	2	3	4	8	1	2	3	4	8
			GDP					*Consumption*		
TPCA	**0.966****	0.986**	0.990**	0.996	1.004	0.939**	0.961**	0.981**	0.986*	1.009
PC-LARS	1.024	0.963*	**0.950****	**0.935****	0.950*	1.028	1.011	0.972**	1.016	0.985
Linear 3PRF	0.982	**0.949****	0.957**	0.951**	0.979	**0.890***	0.897**	0.950**	0.922**	0.992
MS-3PRF (first pass)	1.128	0.952**	0.972**	0.956**	0.942**	0.978	0.894**	0.927**	**0.899****	**0.972**
MS-3PRF (first and third pass)	1.176	1.057	1.234	1.104	1.006	1.002	1.158	1.149	1.419	1.112
MSS-3PRF (first pass)	1.108	0.952**	0.977**	0.955**	**0.938****	0.961	**0.883****	**0.920****	0.901**	0.975
MSS-3PRF (first and third pass)	1.190	1.029	1.166	1.129	1.005	1.041	1.108	1.138	1.203	1.273
			Investment					*Exports*		
TPCA	1.010	0.998	0.999	1.001	1.005	1.061	1.029	1.021	1.006	0.998
PC-LARS	0.913	**0.977**	**0.980**	0.991	1.007	1.117	1.027	1.018	1.000	1.003
Linear 3PRF	**0.900**	0.991	0.996	**0.981***	1.013	1.007	1.037	1.015	1.006	0.995
MS-3PRF (first pass)	0.901	0.983*	1.013	1.008	1.007	1.058	**0.968**	**0.985**	**0.951**	**0.992**
MS-3PRF (first and third pass)	0.907*	1.079	1.156	1.150	1.005	0.871**	0.974*	1.018	0.976*	1.030
MSS-3PRF (first pass)	0.903	0.985*	1.013	1.005	1.001	1.055	0.969	0.992	0.955	0.994
MSS-3PRF (first and third pass)	0.938	1.049	1.077	1.065	1.003	**0.861****	1.014	1.056	1.006	1.058
			Imports					*Hours*		
TPCA	1.012	0.995**	0.997	1.004	1.018	1.010	1.001	1.003	1.005	1.008
PC-LARS	1.058	0.991	1.004	1.000	0.972	**0.864****	**0.980***	**0.993**	1.006	1.030
Linear 3PRF	**0.981**	0.995	0.976	**0.992**	0.991	0.934*	1.002	0.999	1.000	1.014
MS-3PRF (first pass)	1.129	0.994	0.956	1.008	0.931	1.003	1.001	1.026	1.023	1.031
MS-3PRF (first and third pass)	1.004	**0.966**	0.989	1.059	0.951	1.005	1.076	1.163	1.447	1.464
MSS-3PRF (first pass)	1.179	0.997	**0.954**	1.006	**0.924***	0.983	0.999	1.027	1.023	1.030
MSS-3PRF (first and third pass)	1.028	0.976	1.028	1.080	0.980	0.988	1.021	1.103	1.160	1.610
			GDP inflation					*PCE inflation*		
TPCA	1.010	1.026	1.037	1.056	1.044	1.031	1.023	1.025	1.023	1.043
PC-LARS	**0.996**	1.031	1.140	1.294	1.085	1.069	1.130	1.169	1.168	1.046
Linear 3PRF	1.083	1.202	1.148	0.991	0.911	1.004	0.976*	0.998	0.980	0.963
MS-3PRF (first pass)	1.061	1.241	1.125	0.898	0.845*	1.009	1.009	1.044	0.995	0.987
MS-3PRF (first and third pass)	1.134	1.599	2.144	1.526	1.455	0.970	1.081	1.363	1.509	1.655
MSS-3PRF (first pass)	1.066	1.243	1.140	0.909	0.861	1.012	1.018	1.055	1.005	0.988
MSS-3PRF (first and third pass)	1.030	**0.910**	**0.869**	**0.658**	**0.421****	**0.947**	**0.868**	**0.893**	**0.827****	**0.679****

Note: This table reports the MSFE of a given approach relative to the MSFE of PCA for forecast horizons ranging from one quarter to eight quarters ahead. Linear 3PRF uses a single target proxy. MS-3PRF (first pass) and MSS-3PRF (first pass) are regime-switching 3PRFs based on a single target proxy and regime-switching parameters in the first pass only; MS-3PRF (first and third passes) and MSS-3PRF (first and third passes) are regime-switching 3PRFs based on a single target proxy and regime-switching parameters in the first and third passes. For these approaches, the target proxy is the variable to forecast. TPCA is PCA where hard thresholding was performed before extracting the first principal component to forecast. PC-LARS is PCA where soft thresholding was performed before extracting the first principal component to forecast. Boldface indicates the best-performing procedure for a specific horizon and variable. The first estimation sample extends from 1960Q3 to 1984Q4, and it is recursively expanded as we progress in the forecasting exercise. The full evaluation sample runs from 1985Q1 to 2015Q3. Statistical reductions in MSFE relative to PCA according to the Diebold and Mariano (1995) test are indicated by asterisks (* denotes significance at the 10 per cent level, and ** denotes significance at the 5 per cent level).

Online appendix for "Markov-Switching Three-Pass Regression Filter"

The online appendix contains the following elements:

- Appendix A1 describes the hard-thresholding forecasting approach (TPCA)
- Appendix A2 describes the soft-thresholding forecasting approach (PC-LARS)
- Appendix A3 reports additional Monte Carlo experiments
- Appendix A4 reports the data treatment for the macroeconomic forecasting application
- Appendix A5 shows the derivations for estimation and filtering in Markov-switching models
- Appendix A6 presents the details for calculating fluctuation tests for the empirical exercises
- Appendix A7 shows additional selected empirical forecasting results

A.1 Description of the hard-thresholding forecasting approach

The hard-thresholding algorithm consists of the following steps (this description stems partly from Bai and Ng (2008)):

1. For each variable $x_{i,t}$, perform a time series regression of the variable to forecast y_t on $x_{i,t}$ and a constant. Let t_i denote the t-statistic associated with $x_{i,t}$.

2. Let k_α^* be the number of series whose $|t_i|$ exceeds a threshold significance level, α. In our application, we use a threshold of 1.65, which corresponds to a one-sided 5 per cent significance level for the t-test.

3. Let $\chi_t(\alpha) = (x_{[1t]}, ..., x_{[k_\alpha^*]})$ be the corresponding set of predictors. Estimate f_t from $\chi_t(\alpha)$ by the method of principal component.

4. Estimate forecasting equation, in the third pass, to calculate the h-period-ahead forecast y_{t+h}.

This approach is denoted as *TPCA*.

A.2 Description of the soft-thresholding forecasting approach

The soft-thresholding approach we adopt follows from the least angle regressions (LARS) method described in Bai and Ng (2008). In detail, we use the set of the first K predictors $x_{i,t}$ resulting from forward stagewise selection regressions to extract principal component(s). In the macroeconomic forecasting application, we use $K = 30$ predictors, since it is the number of predictors retained by Bai and Ng (2008) and Kelly and Pruitt (2015) when forecasting macroeconomic variables with a data set similar to ours. For the exchange rate forecasting application, we retain the first $K = 10$ predictors ordered by LARS to extract principal components, which corresponds to slightly more than a third of the total number of predictors (26). Finally, in the Monte Carlo experiments, we set $K = 30$ across all DGPs.

This approach is denoted as *PC-LARS*.

A.3 Additional Monte-Carlo experiments

As explained in Section 2.1 of the paper, the key difference between the MS-3PRF and 3PRF approaches is the inclusion of Markov-switching dynamics in steps 1 and 3 of the MS-3PRF.[1] We now compare the accuracy of both linear and nonlinear methods in estimating the latent factors for a fixed $N = 100$ and as the sample size T increases. In particular, using the same set-up for the data generating processes as described in Section 3 of the paper, we generate data assuming no time instability in the loadings, estimate the factor using the 3PRF and compute the correlation with the true underlying factor, ρ^{3PRF}. Then, we generate data subject to instability in the loadings, estimate the factor using the MS-3PRF and compute the correlation with the true factor, $\rho^{MS-3PRF}$. Next, we compare ρ^{3PRF} with $\rho^{MS-3PRF}$ and assess how they change as T increases with $T = \{100, 200, 400, 800\}$. Figure A.1 shows the average correlations across Monte Carlo replications for different scenarios. The results indicate that ρ^{3PRF} is systematically larger that $\rho^{MS-3PRF}$ but the differences are small and both correlation coefficients increase at a relatively similar rate as T increases.

[1] In Appendix A.5 we provide details about the filtering algorithm used to produce time-varying inferences on the regimes in a Markov-switching model.

A.4 Additional details on the macroeconomic forecasting exercise

In the macroeconomic forecasting empirical application, we use the May 2016 vintage of the McCracken and Ng (2015) data set as available online at `https://research.stlouisfed.org/econ/mccracken/fred-databases/`. We use the exact same transformation as suggested by McCracken and Ng (2015). However, due to missing observations, we omit the following five series in our analysis (FRED mnemonics are in parentheses): "New Orders for Consumer Goods" (ACOGNO), "New Orders for Nondefense Capital Goods" (ANDENOx), "Trade-weighted U.S. Dollar Index: Major Currencies" (TWEXMMTH), "Consumer Sentiment Index" (UMCSENTx) and the "VXO" (VXOCLSx).

The eight variables we forecast in the macroeconomic forecasting application are Gross Domestic Product (GDPQ@USNA), Personal Consumption Expenditures (CQ@USNA), Gross Private Domestic Investment (IQ@USNA), Exports of Goods & Services (XQ@USNA), Imports of Goods & Services (MQ@USNA), Business Sector: Hours of All Persons (LXBH@USECON), Gross Domestic Product: Chain Price Index (JGDP@USNA) and Personal Consumption Expenditures: Chain Price Index (JC@USNA). (Haver Analytics mnemonics are in parentheses.)

A.5 Estimation and filtering in Markov-switching models

For ease of exposition, let us consider a simplified version of the regression in equation (3) with one factor, no intercept and no switch in the variance: [2]

$$x_t = \phi(S_t)f_t + \varepsilon_t,$$

where $\varepsilon_t \sim N(0, \sigma^2)$, where

$$\phi(S_t) = \phi(1 - S_t) + \phi S_t,$$

and the state variable may take only two values, $S_t = \{0, 1\}$. If the realizations of S_t were known, the regression above would be nothing more than a dummy variable model, where the log likelihood function would be given by

$$ln(L) = \sum_{t=1}^{T} ln(f(y_t|S_t)),$$

[2] The same procedures explained in this Appendix apply for more complex Markov-switching models.

where

$$f(y_t|S_t) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left(-\frac{(x_t - \phi(S_t)f_t)^2}{2\sigma^2}\right).$$

However, since S_t is unobserved, the marginal density becomes,

$$\begin{aligned} f(y_t|\Omega_{t-1}) = & \frac{1}{\sqrt{2\pi\sigma^2}} exp\left(-\frac{(x_t - \phi_0 f_t)^2}{2\sigma^2}\right) \times P(S_t = 0|\Omega_{t-1}) \\ & + \frac{1}{\sqrt{2\pi\sigma^2}} exp\left(-\frac{(x_t - \phi_1 f_t)^2}{2\sigma^2}\right) \times P(S_t = 1|\Omega_{t-1}), \end{aligned}$$

which can be alternatively expressed as

$$f(y_t|\Omega_{t-1}) = \sum_{j=0}^{1} f(y_t|S_t = j, \Omega_{t-1}) P(S_t = j|\Omega_{t-1})$$

Notice that the marginal density is a weighted average of conditional densities. Therefore, the log likelihood function, when S_t is unobserved, is given by

$$ln(L) = \sum_{t=1}^{T} ln(\sum_{j=0}^{1} f(y_t|S_t = j, \Omega_{t-1}) P(S_t = j|\Omega_{t-1})).$$

Once the corresponding weights, $P(S_t = j|\Omega_{t-1})$ for $j = 0, 1$, are defined as a function of the parameters of the model and the data, the log likelihood function can be maximized with respect to ϕ_0, ϕ_1, σ, and transition probabilities, p_{ij}, to obtain estimates of those parameters.

In order to define the weights, also known as the filtered state probabilities, the next two steps are followed iteratively:

Step 1: Given the updated filtered state probabilities at time $t-1$, $P(S_{t-1} = i|\Omega_{t-1})$,for $i = 0, 1$, the predicted filtered state probabilities are computed as

$$\begin{aligned} P(S_t = j|\Omega_{t-1}) &= \sum_{i=0}^{1} P(S_t = j, S_{t-1} = i|\Omega_{t-1}) \\ &= \sum_{i=0}^{1} P(S_t = j|S_{t-1} = i) P(S_{t-1} = i|\Omega_{t-1}), \end{aligned}$$

where $P(S_t = j|S_{t-1} = i) = p_{ij}$, for $i, j = 0, 1$, are the parameters corresponding to the transition probabilities.

Step 2: Once time t ends and y_t can be observed, the predicted filtered state probabilities can be updated by taking into account that $P(S_t = j|\Omega_t) = P(S_t = j|\Omega_{t-1}, y_t)$, where $\Omega_t = \{\Omega_{t-1}, y_t\}$. Therefore,

$$\begin{aligned} P(S_t = j|\Omega_t) &= \frac{f(S_t = j, y_t|\Omega_{t-1})}{f(y_t|\psi_{t-1})} \\ &= \frac{f(y_t|S_t = j, \Omega_{t-1})P(S_t = j|\Omega_{t-1})}{\sum_{j=0}^{1} f(y_t|S_t = j, \Omega_{t-1})P(S_t = j|\Omega_{t-1})}. \end{aligned}$$

Then $P(S_t = j|\Omega_t)$ is used in the next iteration in *Step 1.* These two steps are followed for $t = 1, 2, ..., T$. To start the iterations at $t = 1$, the ergodic, or stationary, probabilities are used.

Given the filtered probabilities, $P(S_t = j|\Omega_t)$ and the estimated parameters, we compute inferences of S_t using all the available information in the sample at the time that each forecast is performed. In particular, we follow the line of Kim and Nelson (1999) to compute the smoothed probability, denoted by $P(S_t = j|\Omega_T)$ and use it to obtained the time-varying factor loadings in Equation (7).

A.6 Fluctuation tests

In this section, we report the fluctuation tests from Giacomini and Rossi (2010) to evaluate the stability of the out-of-sample forecasting performance of the MS-3PRF approach when forecasting selected variables: GDP inflation, PCE inflation and the CAD–USD exchange rate at specific horizons. We focus on these specific variables, since for them the MS-3PRF approach performed best. In Figures A.3 and A.4 below, we report the standardized local relative 8-quarter-ahead MSPE of a specific MS-3PRF model against the benchmark model (PCA when forecasting macroeconomic variables). In the case of the CAD-USD, in Figure A.5 we report the results of the fluctuation test obtained from the Pesaran and Timmermann (2009) test from the MS-3PRF approach relative to the no-change forecast at a 12-month horizon. In Figures A.3 to A.5, we also report the critical value for testing that the two models have equal out-of-sample performance at each point of time against the alternative that the MS-3PRF approach performs better at least at one point in time. The size of the evaluation window is set to 46 for both macroeconomic and exchange rate forecasting exercises, which corresponds to about 40% of the full evaluation sample. The critical value for a one-sided test at the 10% level is 2.334.

Coming to the results, Figure A.3 shows that the local relative MSPE exceeds the critical value from the early part of the evaluation sample to the Great Recession; hence, we reject the null hypothesis, and conclude that there were periods during which the MSS-3PRF approach produced better 8-quarter-ahead forecasts for PCE inflation than the benchmark PCA model. In the case of predicting GDP inflation 8-quarter ahead, Figure A.4 shows that the local relative MSPE exceeds the critical value for most of the first half of the sample till the Great Recession (with a brief exception between 2001 and 2003), suggesting that there were periods in which the MSS-3PRF approach outperformed the benchmark PCA model. Figure A.5 reporting the results of the fluctuation test for predicting directional changes in the CAD–USD exchange rate 12-month-ahead, indicates that in the latter part of the sample, there is evidence of superior predictive ability from the MS-3PRF approach over the random walk model.

A.7 Recursive MSPE

Figures A.6 to A.11 show the recursive MSPE in selected cases.

References

Bai, J. and Ng, S. (2008). Forecasting economic time series using targeted predictors. *Journal of Econometrics*, 146(2):304–317.

Giacomini, R. and Rossi, B. (2010). Forecast comparisons in unstable environments. *Journal of Applied Econometrics*, 25(4):595–620.

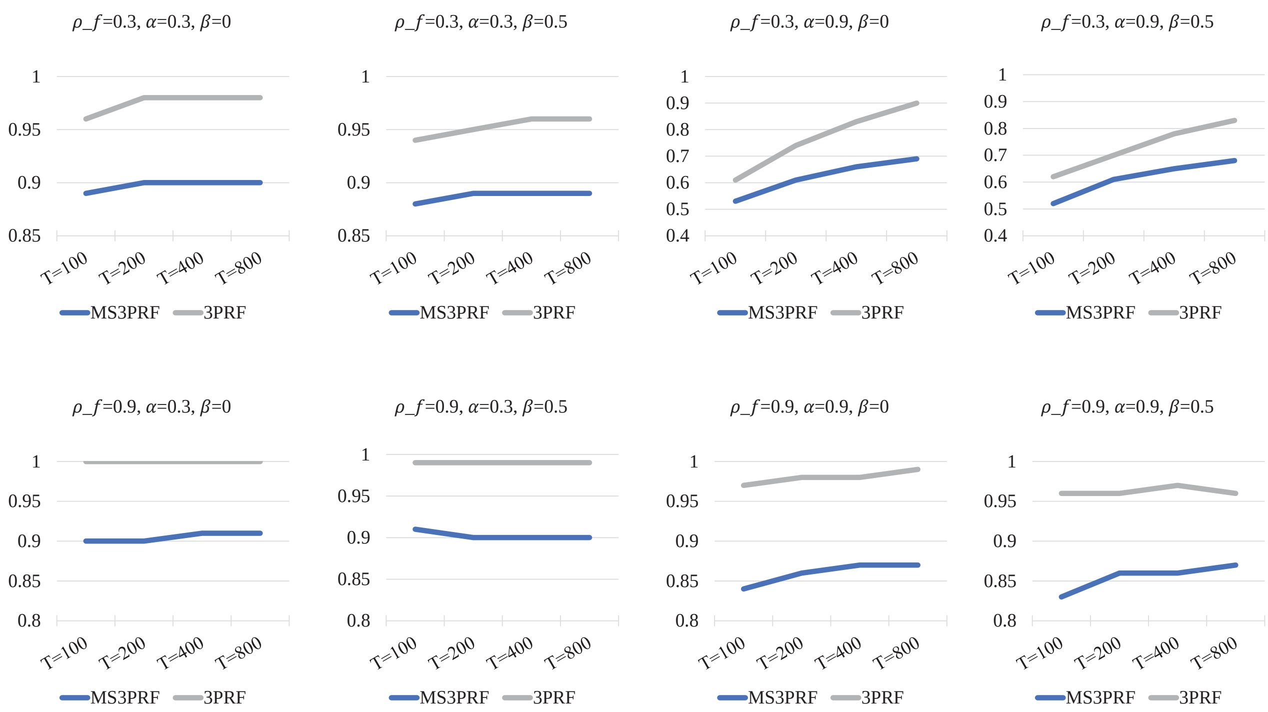
Kelly, B. and Pruitt, S. (2015). The three-pass regression filter: A new approach to forecasting using many predictors. *Journal of Econometrics*, 186(2):294–316.

Kim, C.-J. and Nelson, C. (1999). In *State-Space Model with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications*, MIT press.

McCracken, M. W. and Ng, S. (2015). FRED-MD: A Monthly Database for Macroeconomic Research. *Journal of Business & Economic Statistics*, forthcoming.

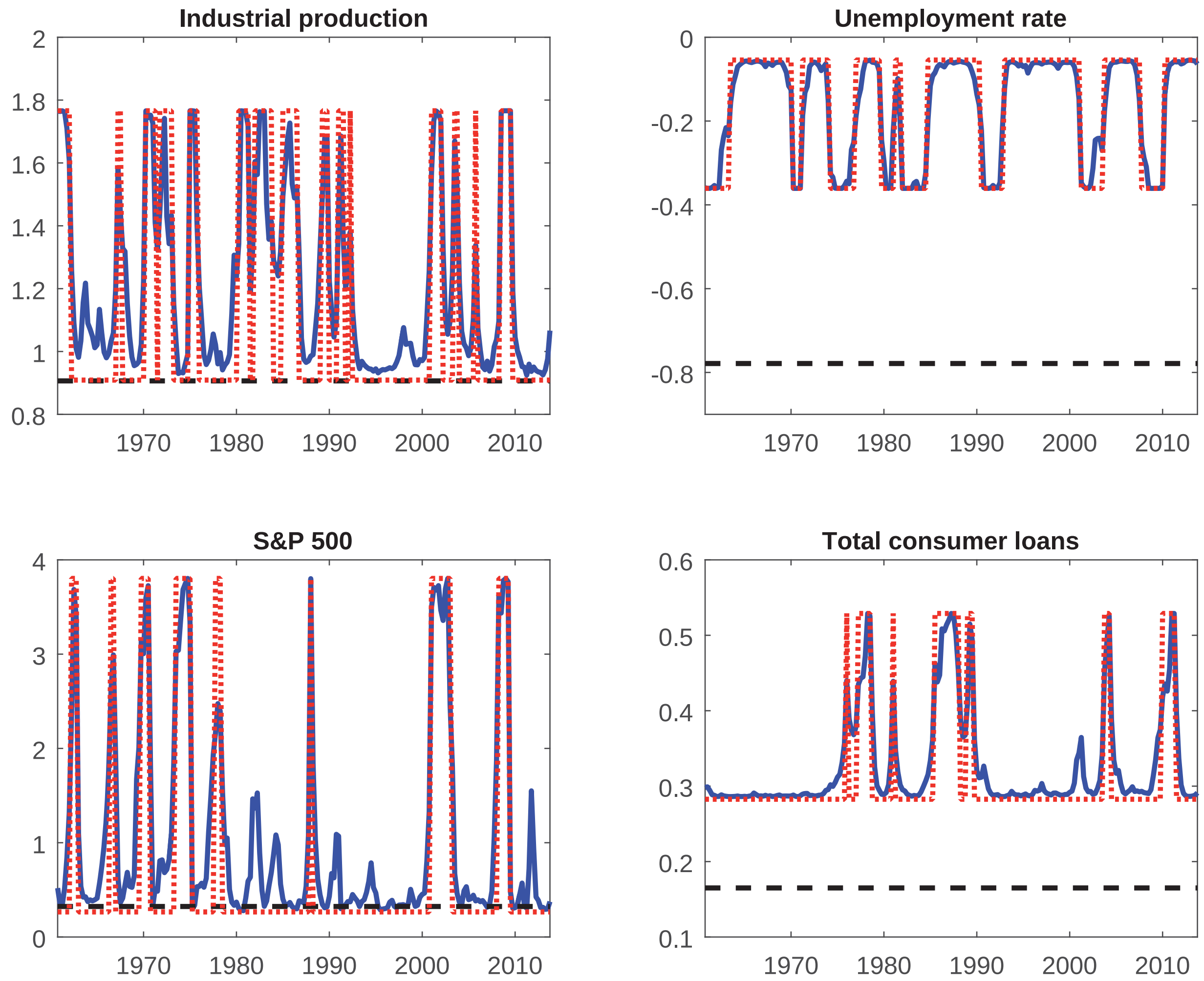
Pesaran, M. H. and Timmermann, A. (2009). Testing Dependence Among Serially Correlated Multicategory Variables. *Journal of the American Statistical Association*, 104(485):325–337.

Figure A.1 Correlation of factor estimates with true factor



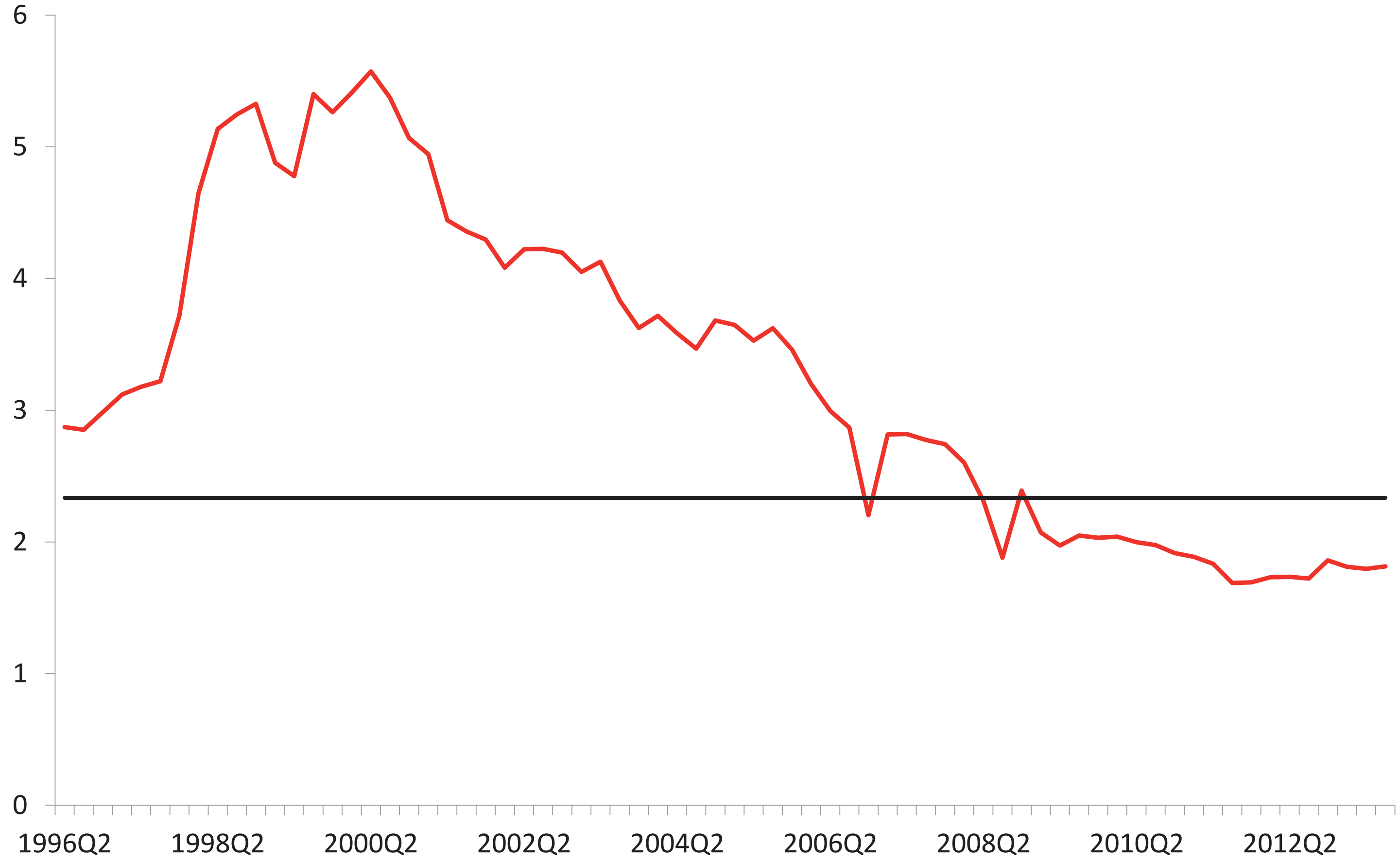
Note: The grey lines in each chart plot the average correlation between the true factor and the estimated factor obtained from a DGP that has no instability in the factor loadings. The blue lines in each chart plot the average correlation between the true factor and the estimated factor obtained from a DGP that has instability in the factor loadings. The average correlation is based on 500 replications and is reported for different sample sizes and configurations of the underlying parameters in the DGP.

Figure A.2 Factor loadings for selected variables



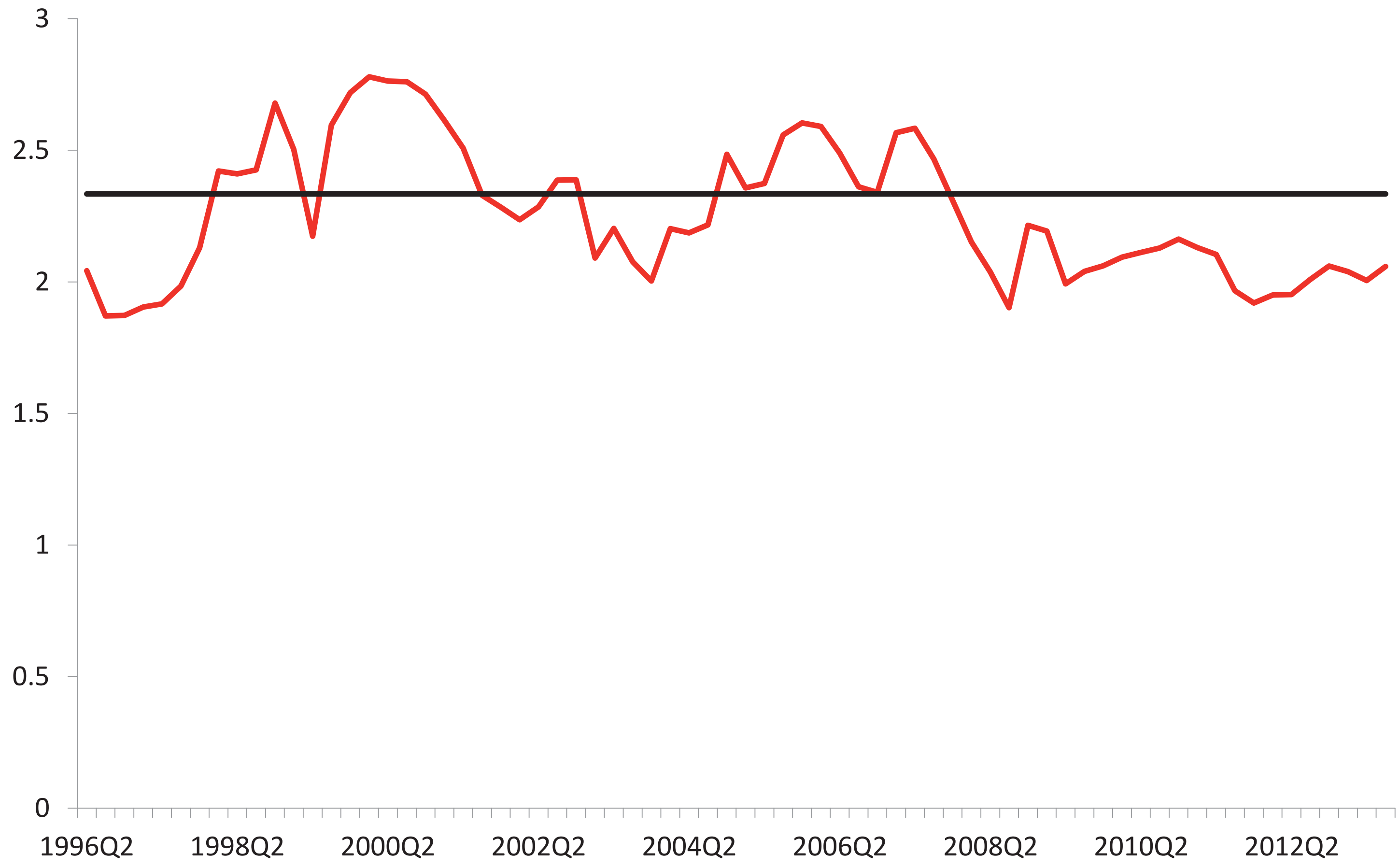
Note: Factor loadings for selected variables obtained from the linear 3PRF (black dashed line), MS-3PRF (blue solid line) and MSS-3PRF (red dotted line). GDP growth is used as a target proxy for the 3PRF approaches.

Figure A.3 Fluctuation test statistic – Forecasting PCE inflation 8-quarter-ahead



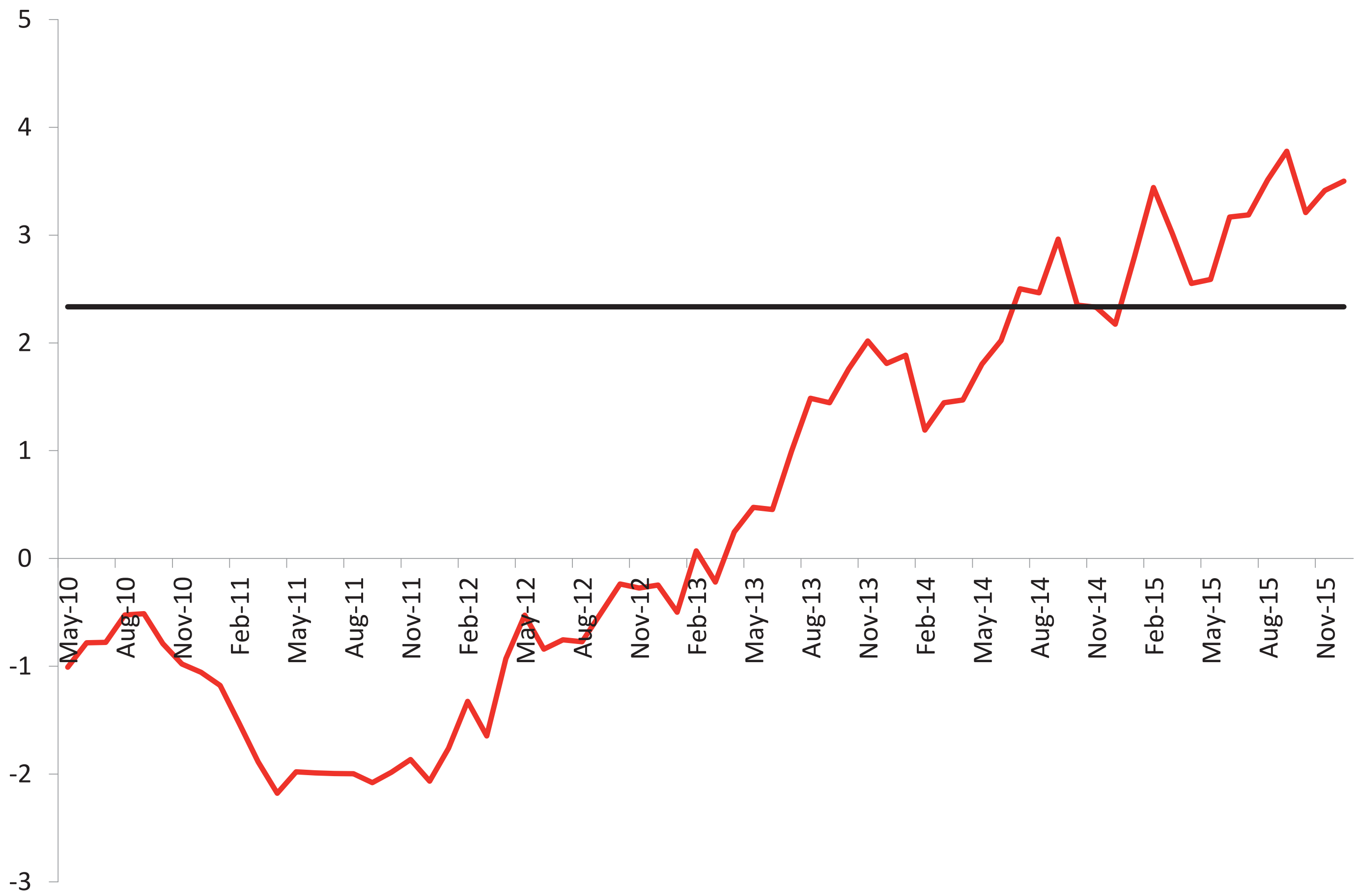
Note: This figure shows the results of the Giacomini and Rossi (2010) fluctuation test statistic, obtained as the standardized difference between the MSPE of PCA and the MSPE of the MSS-3PRF (first and third pass) model. The size of the rolling window is set to 46; that is, about 40 per cent of the full evaluation sample. The critical value for a one-sided test at the 10% level is 2.334. A reading above the critical value indicates that the predictive model statistically outperforms the benchmark model. The horizontal axis denotes the time at which the forecast was made.

Figure A.4 Fluctuation test statistic – Forecasting GDP inflation 8-quarter-ahead



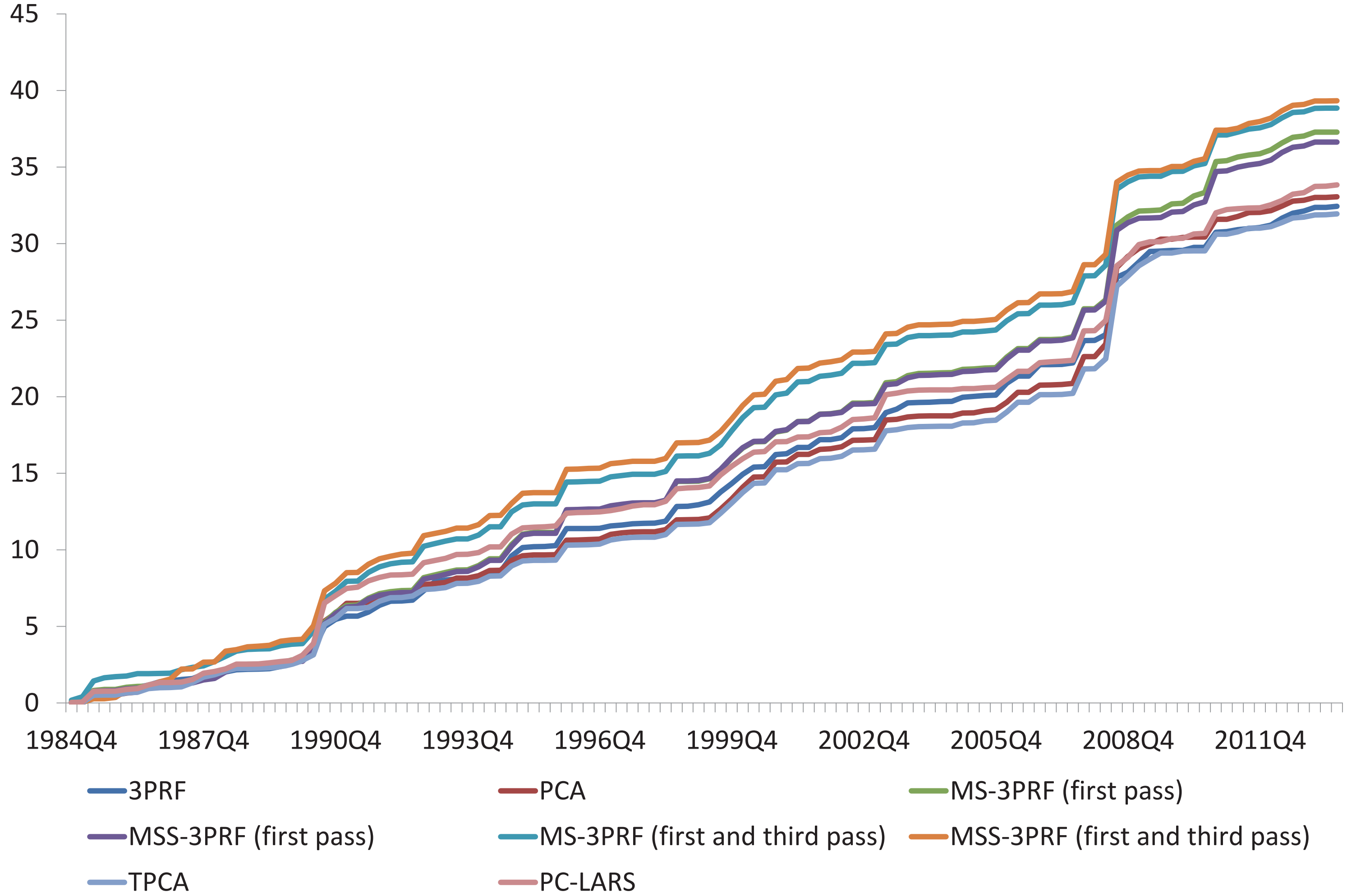
Note: This figure shows the results of the Giacomini and Rossi (2010) fluctuation test statistic, obtained as the standardized difference between the MSPE of PCA and the MSPE of the MSS-3PRF (first and third pass) model. The size of the rolling window is set to 46; that is, about 40 per cent of the full evaluation sample. The critical value for a one-sided test at the 10% level is 2.334. A reading above the critical value indicates that the predictive model statistically outperforms the benchmark model. The horizontal axis denotes the time at which the forecast was made.

Figure A.5 Fluctuation test statistic – Forecasting CAD-USD 12-month-ahead



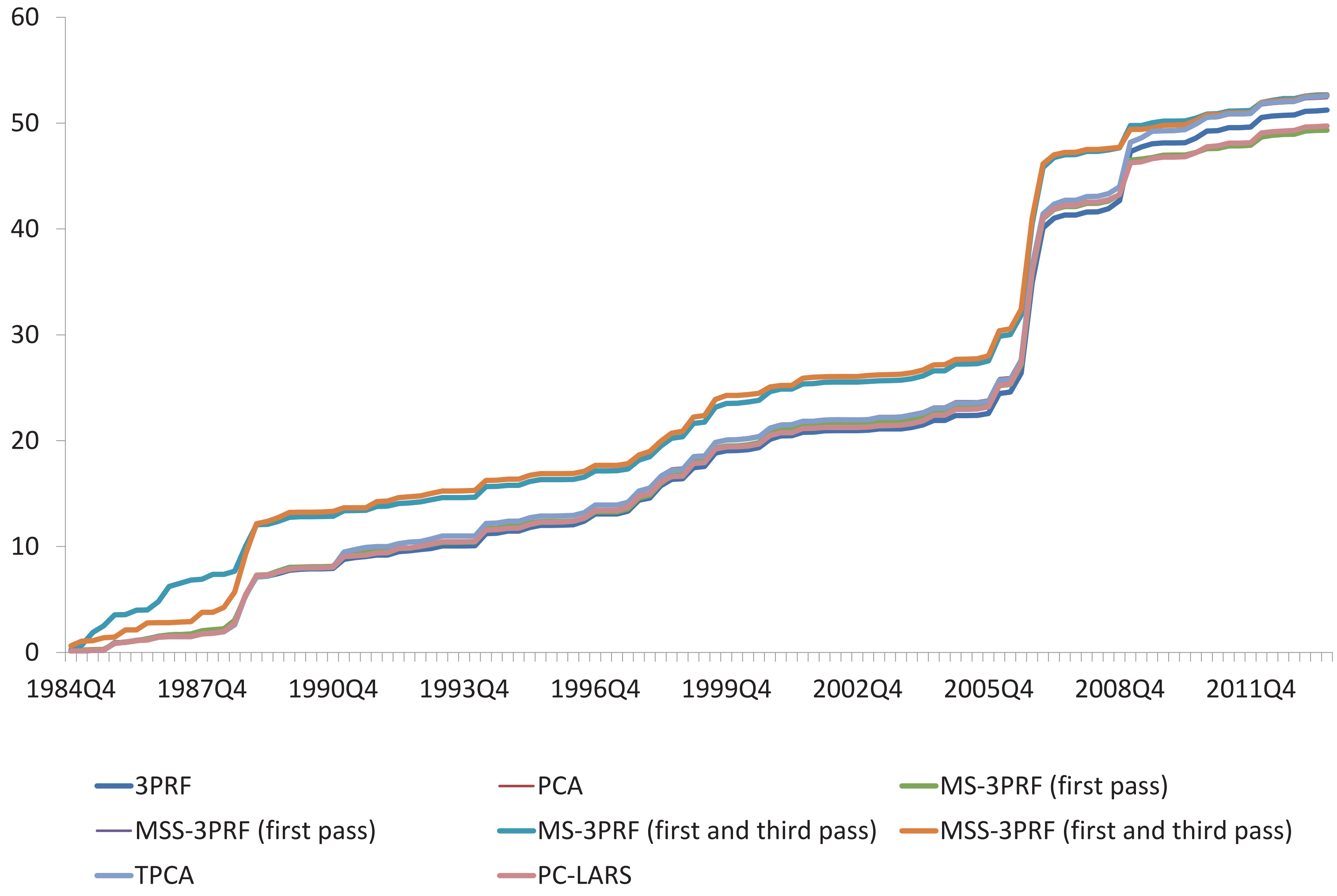
Note: This figure shows the results of the Giacomini and Rossi (2010) fluctuation test statistic, obtained from the Pesaran and Timmermann (2009) test statisic for directional accuracy for the MS-3PRF (first and third pass) model relative to the no-change forecast. The size of the rolling window is set to 46; that is, about 40 per cent of the full evaluation sample. The critical value for a one-sided test at the 10% level is 2.334. A reading above the critical value indicates that the predictive model statistically outperforms the benchmark model. The horizontal axis denotes the time at which the forecast was made.

Figure A.6 Recursive forecasting performance – Forecasting GDP 1-quarter-ahead



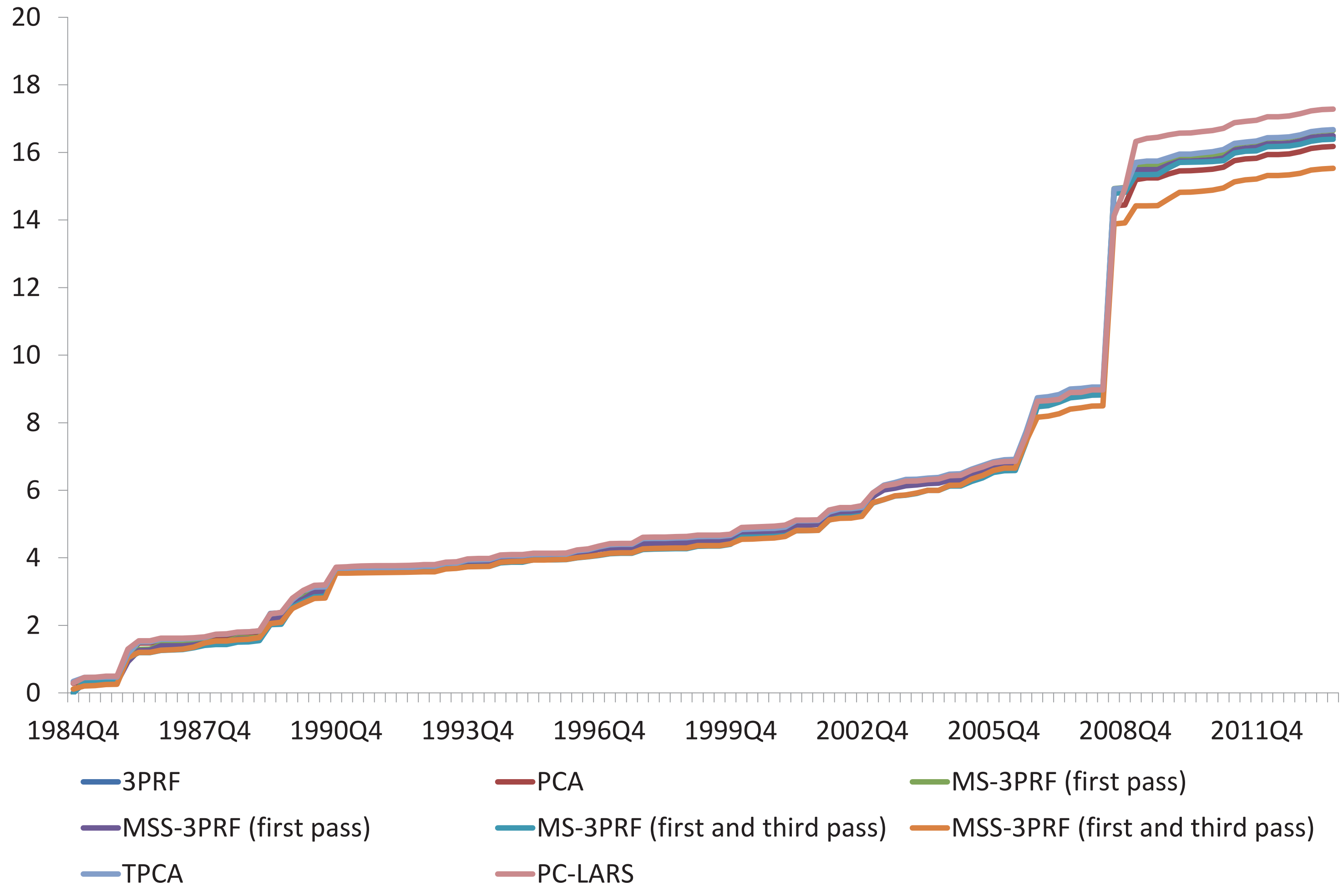
Note: This figure shows the recursive forecasting performance (MSPE) for forecasting GDP inflation 1 quarters ahead.

Figure A.7 Recursive forecasting performance – Forecasting GDP 8-quarter-ahead



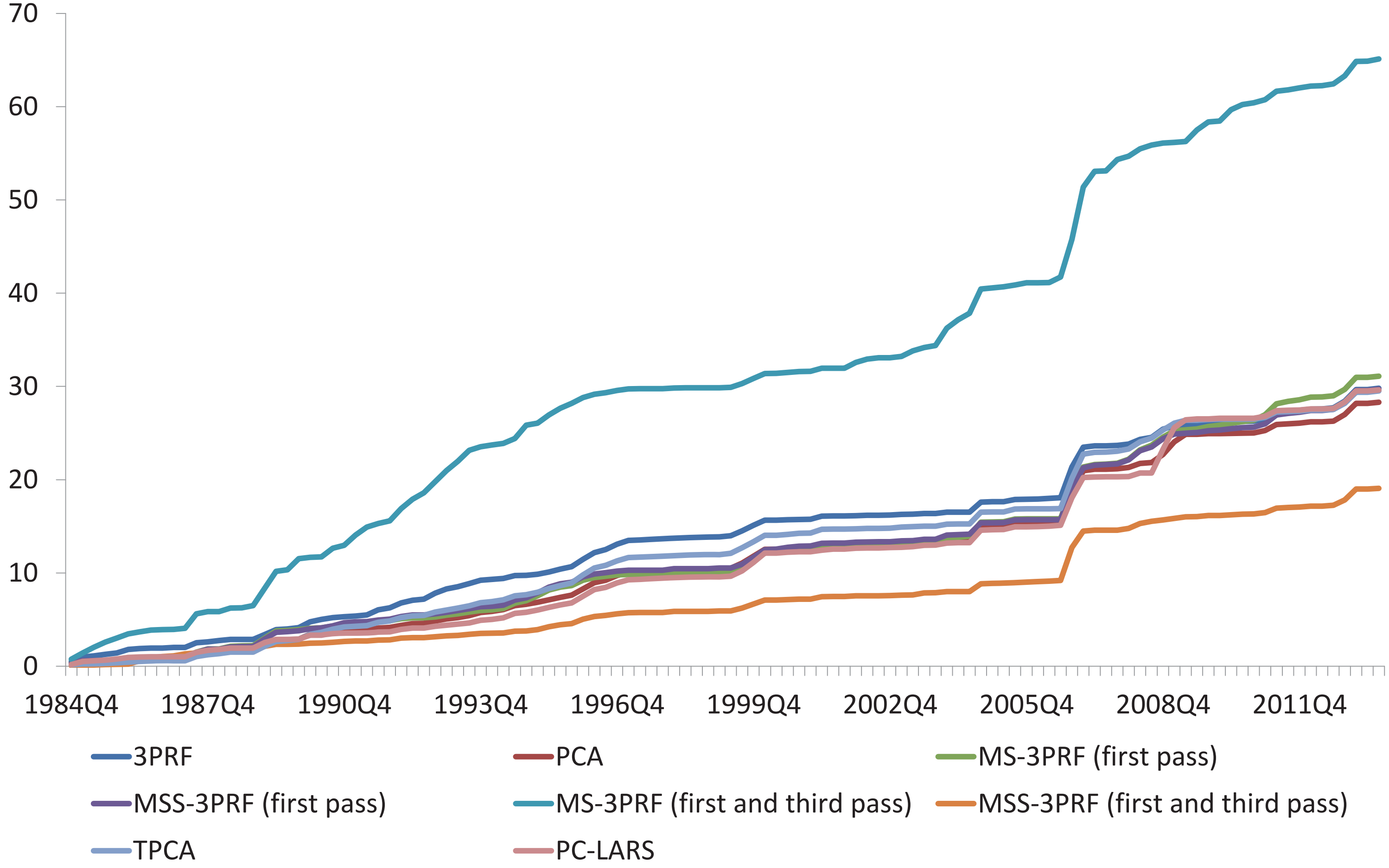
Note: This figure shows the recursive forecasting performance (MSPE) for forecasting GDP inflation 8 quarters ahead.

Figure A.8 Recursive forecasting performance – Forecasting PCE inflation 1-quarter-ahead



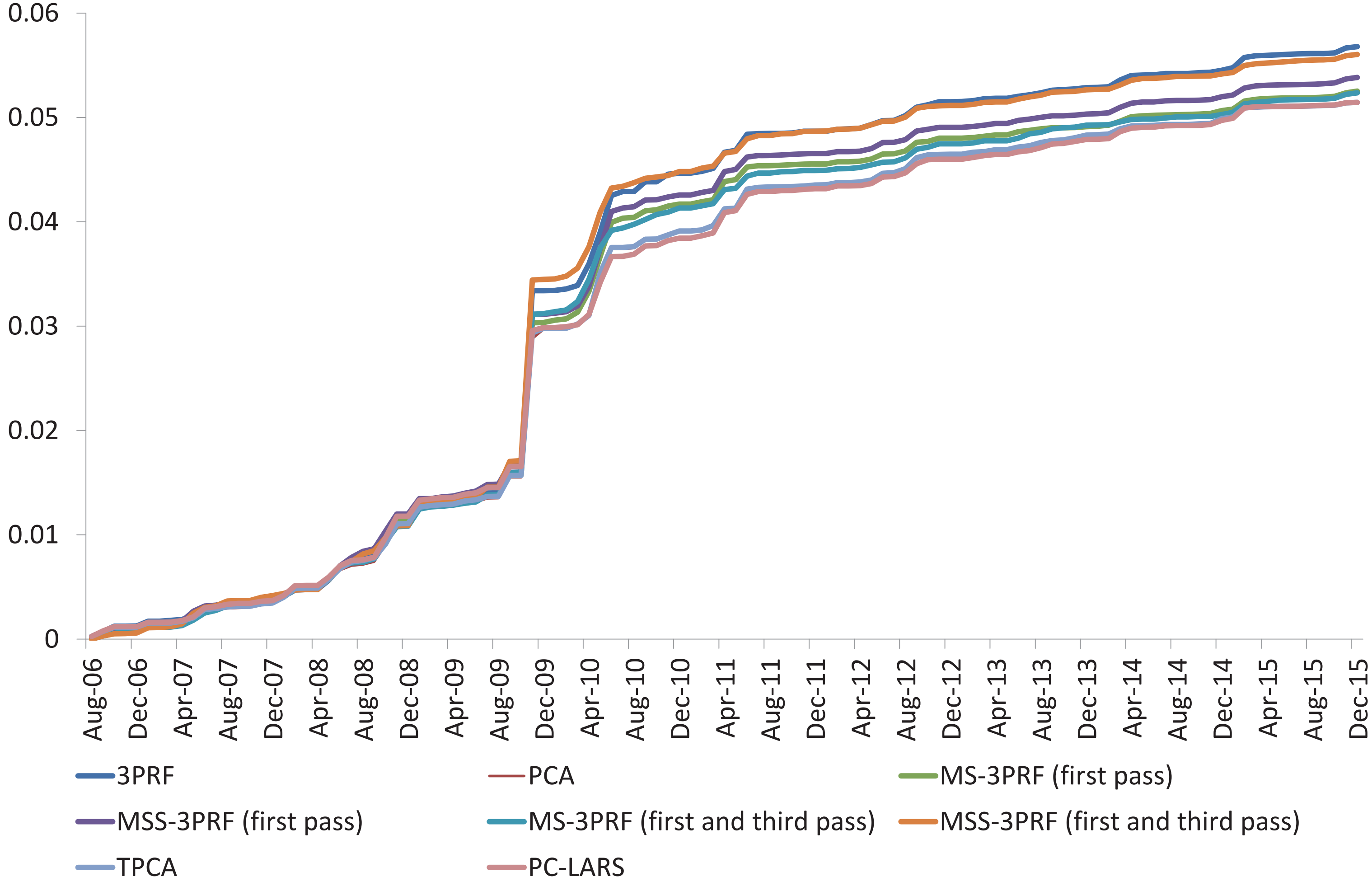
Note: This figure shows the recursive forecasting performance (MSPE) for forecasting GDP inflation 1 quarters ahead.

Figure A.9 Recursive forecasting performance – Forecasting PCE inflation 8-quarter-ahead



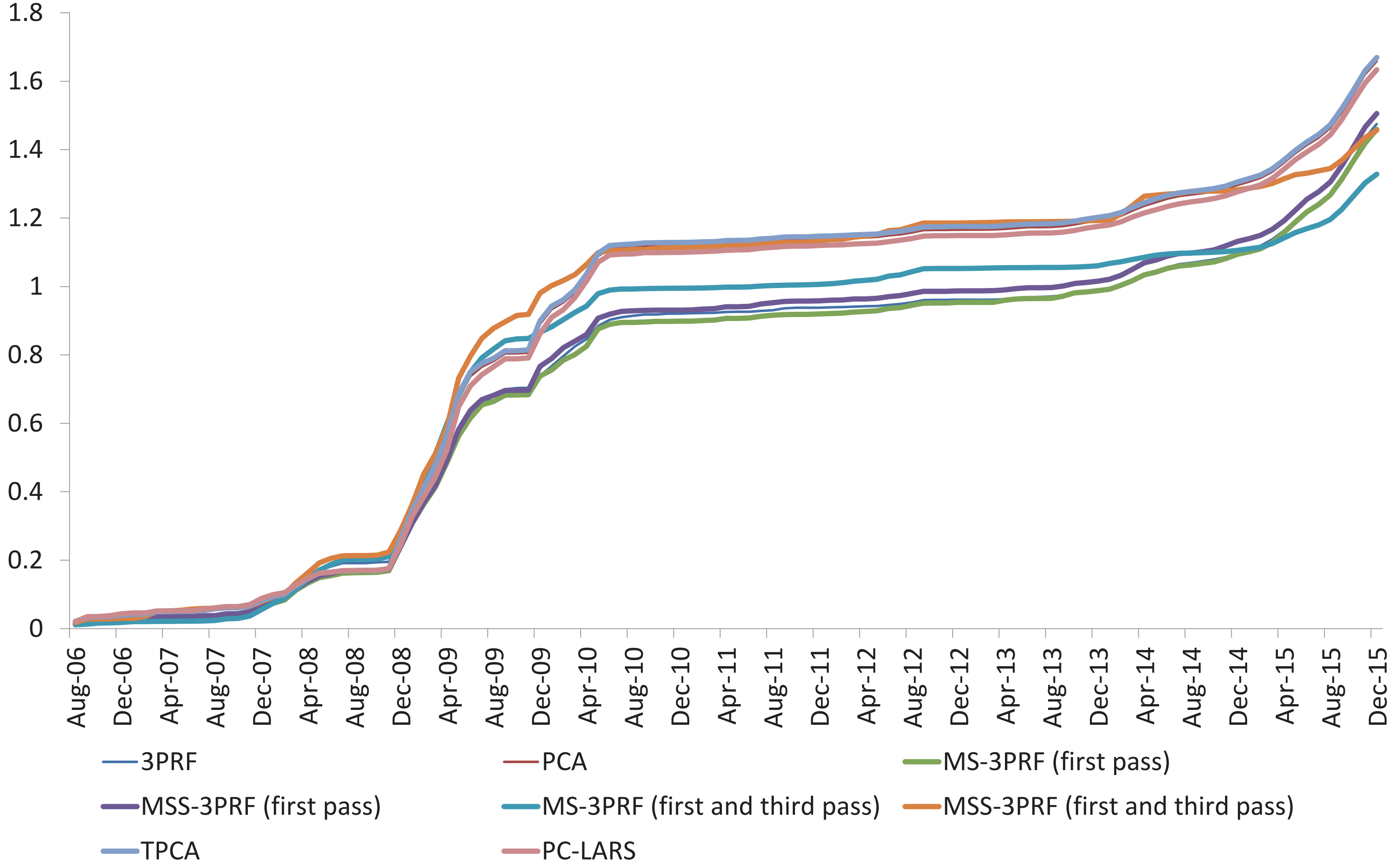
Note: This figure shows the recursive forecasting performance (MSPE) for forecasting GDP inflation 8 quarters ahead.

Figure A.10 Recursive forecasting performance – Forecasting CAD-USD 1-month-ahead



Note: This figure shows the recursive forecasting performance (MSPE) for forecasting the CAD-USD exchange rate 1 month ahead.

Figure A.11 Recursive forecasting performance – Forecasting CAD-USD 12-month-ahead



Note: This figure shows the recursive forecasting performance (MSPE) for forecasting the CAD-USD exchange rate 12 months ahead.

BANCO DE ESPAÑA PUBLICATIONS

WORKING PAPERS

1621 ADRIAN VAN RIXTEL, LUNA ROMO GONZÁLEZ and JING YANG: The determinants of long-term debt issuance by European banks: evidence of two crises.

1622 JAVIER ANDRÉS, ÓSCAR ARCE and CARLOS THOMAS: When fiscal consolidation meets private deleveraging.

1623 CARLOS SANZ: The effect of electoral systems on voter turnout: evidence from a natural experiment.

1624 GALO NUÑO and CARLOS THOMAS: Optimal monetary policy with heterogeneous agents.

1625 MARÍA DOLORES GADEA, ANA GÓMEZ-LOSCOS and ANTONIO MONTAÑÉS: Oil price and economic growth: a long story?

1626 PAUL DE GRAUWE and EDDIE GERBA: Stock market cycles and supply side dynamics: two worlds, one vision?

1627 RICARDO GIMENO and EVA ORTEGA: The evolution of inflation expectations in euro area markets.

1628 SUSANA PÁRRAGA RODRÍGUEZ: The dynamic effect of public expenditure shocks in the United States.

1629 SUSANA PÁRRAGA RODRÍGUEZ: The aggregate effects of government incometransfer shocks - EU evidence.

1630 JUAN S. MORA-SANGUINETTI, MARTA MARTÍNEZ-MATUTE and MIGUEL GARCÍA-POSADA: Credit, crisis and contract enforcement: evidence from the Spanish loan market.

1631 PABLO BURRIEL and ALESSANDRO GALESI: Uncovering the heterogeneous effects of ECB unconventional monetary policies across euro area countries.

1632 MAR DELGADO TÉLLEZ, VÍCTOR D. LLEDÓ and JAVIER J. PÉREZ: On the determinants of fiscal non-compliance: an empirical analysis of Spain's regions.

1633 OMAR RACHEDI: Portfolio rebalancing and asset pricing with heterogeneous inattention.

1634 JUAN DE LUCIO, RAÚL MÍNGUEZ, ASIER MINONDO and FRANCISCO REQUENA: The variation of export prices across and within firms.

1635 JUAN FRANCISCO JIMENO, AITOR LACUESTA, MARTA MARTÍNEZ-MATUTE and ERNESTO VILLANUEVA: Education, labour market experience and cognitive skills: evidence from PIAAC.

1701 JAVIER ANDRÉS, JAVIER J. PÉREZ and JUAN A. ROJAS: Implicit public debt thresholds: an empirical exercise for the case of Spain.

1702 LUIS J. ÁLVAREZ: Business cycle estimation with high-pass and band-pass local polynomial regression.

1703 ENRIQUE MORAL-BENITO, PAUL ALLISON and RICHARD WILLIAMS: Dynamic panel data modelling using maximum likelihood: an alternative to Arellano-Bond.

1704 MIKEL BEDAYO: Creating associations as a substitute for direct bank credit. Evidence from Belgium.

1705 MARÍA DOLORES GADEA-RIVAS, ANA GÓMEZ-LOSCOS and DANILO LEIVA-LEON: The evolution of regional economic interlinkages in Europe.

1706 ESTEBAN GARCÍA-MIRALLES: The crucial role of social welfare criteria for optimal inheritance taxation.

1707 MÓNICA CORREA-LÓPEZ and RAFAEL DOMÉNECH: Service regulations, input prices and export volumes: evidence from a panel of manufacturing firms.

1708 MARÍA DOLORES GADEA, ANA GÓMEZ-LOSCOS and GABRIEL PÉREZ-QUIRÓS: Dissecting US recoveries.

1709 CARLOS SANZ: Direct democracy and government size: evidence from Spain.

1710 HENRIQUE S. BASSO and JAMES COSTAIN: Fiscal delegation in a monetary union: instrument assignment and stabilization properties.

1711 IVÁN KATARYNIUK and JAIME MARTÍNEZ-MARTÍN: TFP growth and commodity prices in emerging economies.

1712 SEBASTIAN GECHERT, CHRISTOPH PAETZ and PALOMA VILLANUEVA: Top-down vs. bottom-up? Reconciling the effects of tax and transfer shocks on output.

1713 KNUT ARE AASTVEIT, FRANCESCO FURLANETTO and FRANCESCA LORIA: Has the Fed responded to house and stock prices? A time-varying analysis.

1714 FÁTIMA HERRANZ GONZÁLEZ and CARMEN MARTÍNEZ-CARRASCAL: The impact of firms' financial position on fixed investment and employment. An analysis for Spain.

1715 SERGIO MAYORDOMO, ANTONIO MORENO, STEVEN ONGENA and MARÍA RODRÍGUEZ-MORENO: "Keeping it personal" or "getting real"? On the drivers and effectiveness of personal versus real loan guarantees.

1716 FRANCESCO FURLANETTO and ØRJAN ROBSTAD: Immigration and the macroeconomy: some new empirical evidence.

1717 ALBERTO FUERTES: Exchange rate regime and external adjustment: an empirical investigation for the U.S.

1718 CRISTINA GUILLAMÓN, ENRIQUE MORAL-BENITO and SERGIO PUENTE: High growth firms in employment and productivity: dynamic interactions and the role of financial constraints.

1719 PAULO SOARES ESTEVES and ELVIRA PRADES: On domestic demand and export performance in the euro area countries: does export concentration matter?

1720 LUIS J. ÁLVAREZ and ANA GÓMEZ-LOSCOS: A menu on output gap estimation methods.

1721 PAULA GIL, FRANCISCO MARTÍ, JAVIER J. PÉREZ, ROBERTO RAMOS and RICHARD MORRIS: The output effects of tax changes: narrative evidence from Spain.

1722 RICARDO GIMENO and ALFREDO IBÁÑEZ: The eurozone (expected) inflation: an option's eyes view.

1723 MIGUEL ANTÓN, SERGIO MAYORDOMO and MARÍA RODRÍGUEZ-MORENO: Dealing with dealers: sovereign CDS comovements.

1724 JOSÉ MANUEL MONTERO: Pricing decisions under financial frictions: evidence from the WDN survey.

1725 MARIO ALLOZA: The impact of taxes on income mobility.

1726 DANILO LEIVA-LEON: Measuring business cycles intra-synchronization in US: a regime-switching interdependence framework.

1727 PIERRE GUÉRIN and DANILO LEIVA-LEON: Model averaging in Markov-Switching models: predicting national recessions with regional data.

1728 MÁXIMO CAMACHO and DANILO LEIVA-LEON: The propagation of industrial business cycles.

1729 JAMES COSTAIN: Costly decisions and sequential bargaining.

1730 MARIO ALLOZA: Is fiscal policy more effective in uncertain times or during recessions?

1731 PIERRE GUÉRIN and DANILO LEIVA-LEON: Monetary policy, stock market and sectoral comovement.

1732 HENRIK JENSEN, IVAN PETRELLA, SØREN HOVE RAVN and EMILIANO SANTORO: Leverage and deepening business cycle skewness.

1733 CÉSAR MARTÍN MACHUCA: External stress early warning indicators.

1734 RODOLFO G. CAMPOS: International migration pressures in the long run.

1735 ANDREA ARIU, ELENA BIEWEN, SVEN BLANK, GUILLAUME GAULIER, MARÍA JESÚS GONZÁLEZ, PHILIPP MEINEN, DANIEL MIRZA, CÉSAR MARTÍN MACHUCA and PATRY TELLO: Firm heterogeneity and aggregate business services exports: micro evidence from Belgium, France, Germany and Spain.

1736 LEONARDO GAMBACORTA, STEFANO SCHIAFFI and ADRIAN VAN RIXTEL: Changing business models in international bank funding.

1737 ENRIQUE MORAL-BENITO and FRANCESCA VIANI: An anatomy of the Spanish current account adjustment: the role of permanent and transitory factors.

1738 MARÍA J. NIETO and LARRY D. WALL: Cross-border banking on the two sides of the Atlantic: does it have an impact on bank crisis management?

1739 JACOPO TIMINI: Currency unions and heterogeneous trade effects: the case of the Latin Monetary Union.

1740 PAULINO FONT, MARIO IZQUIERDO and SERGIO PUENTE: Subsidising mature age employment or throwing coins into a wishing well: a quasi-experimental analysis.

1741 THOMAS FUJIWARA and CARLOS SANZ: Norms in bargaining: evidence from government formation in Spain.

1742 ENRIQUE ALBEROLA, ÁNGEL ESTRADA and FRANCESCA VIANI: Global imbalances from a stock perspective.

1743 ÓSCAR ARCE, RICARDO GIMENO and SERGIO MAYORDOMO: Making room for the needy: the credit-reallocation effects of the ECB's corporate QE.

1744 M. D. GADEA-RIVAS, ANA GÓMEZ-LOSCOS and EDUARDO BANDRÉS: Clustering regional business cycles.

1745 NEZIH GUNER, YULIYA KULIKOVA and JOAN LLULL: Marriage and health: selection, protection, and assortative mating.

1746 SERGIO MAYORDOMO and MARÍA RODRÍGUEZ-MORENO: Did the bank capital relief induced by the supporting factor enhance SME lending?

1747 KATALIN BODNÁR, LUDMILA FADEJEVA, MARCO HOEBERICHTS, MARIO IZQUIERDO PEINADO, CHRISTOPHE JADEAU and ELIANA VIVIANO: ?na`ep odk_go ]j` pda Aqnkla]j H]^kqn i]ngap.

1748 PIERRE GUÉRIN, DANILO LEIVA-LEON and MASSIMILIANO MARCELLINO: Markov-switching three-pass regression filter.

BANCO DE ESPAÑA
Eurosistema

Unidad de Servicios Auxiliares
Alcalá, 48 - 28014 Madrid
E-mail: publicaciones@bde.es
www.bde.es