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Danilo Leiva-Leon

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(**) danilo.leiva@bde.es

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Abstract

This paper proposes a Markov-switching framework to endogenously identify periods where economies are more likely to (i) synchronously enter recessionary and expansionary phases, and (ii) follow independent business cycles. The reliability of the framework is validated with simulated data in Monte Carlo experiments. The framework is applied to assess the time-varying intra-country synchronization in US. The main results report substantial changes over time in the cyclical affiliation patterns of US states, and show that the more similar the economic structures of states, the higher the correlation between their business cycles. A synchronization-based network analysis discloses a change in the propagation pattern of aggregate contractionary shocks across states, suggesting that the US has become more internally synchronized since the early 1990s.

Keywords: business cycles, Markov-Switching, network analysis.

JEL classification: E32, C32, C45.

Resumen

Este artículo propone un modelo econométrico de regímenes markovianos para identificar endógenamente períodos en los que las economías tienden a experimentar fases del ciclo económico de manera sincronizada e independiente. La fiabilidad del modelo econométrico se ha validado con datos simulados utilizando experimentos de Monte-carlo. El modelo se aplica para identificar cambios en la sincronización regional de Estados Unidos (EEUU). Los resultados indican la presencia de cambios significativos en los patrones cíclicos de afiliación de los estados de EEUU, y muestran que, cuanto más similares son las estructuras económicas de los estados, mayor es la correlación entre sus ciclos económicos. Adicionalmente, un análisis de redes, basado en las medidas de sincronización estimadas, revela un cambio en la propagación de choques contractivos a través de estados, lo que sugiere que EEUU se ha sincronizado internamente con mayor intensidad desde principios de los años noventa.

Palabras clave: ciclos económicos, cambios markovianos, análisis de redes.

Códigos JEL: E32, C32, C45.

1 Introduction

Since Hamilton (1989), Markov-switching (MS) models have become a useful tool for policy makers and investors to construct inferences about the state of the economy (expansionary or recessionary regimes), financial markets (high or low volatile regimes), monetary policy (active or passive policy regimes), etc. Also, multivariate extensions of MS models have been used to provide helpful insights about issues such as business cycles synchronization (Camacho and Perez-Quiros (2006)), business cycles and stock market volatility interdependence (Hamilton and Lin (1996)), real activity and inflation cycles synchronization (Leiva-Leon (2014)), monetary and fiscal policy interaction (Davig and Leeper (2006)), among other types of relationships. In these studies, a key component of the analysis is the dependency relationship between the underlying Markovian latent variables governing the model's dynamics.

The modelling approaches of multivariate MS specifications can be sorted into two categories. The first category includes studies where the relationship between the latent variables is *a priori* defined. Hence, it is based on the researcher's judgment, relying on four different settings (Hamilton and Lin (1996) and Anas et al. (2007)). The first refers to the case where all series in the model are subject to a single latent variable (Krolzig (1997) and Sims and Zha (2006)). The second uses different latent variables which are modelled as totally independent Markov chains (Smith and Summers (2005) and Chauvet and Senyuz (2008)). In the third, the dynamics of one latent variable precedes those of other latent variables (Hamilton and Perez-Quiros (1996) and Cakmakli et al. (2011)), allowing for a possibly different number of lags.[1] Fourth, there is also the case of a general Markovian specification that involves the full transition probability matrix (Kim, Piger and Startz (2007)). However, it raises computational difficulties and is less straightforward to interpret as the number of series, states or lags, increase. Accordingly, the obtained regime inferences and final interpretations of the model's output may vary substantially depending on the approach chosen.

The second category focuses on making *a posteriori* assessments of the synchronization between MS processes, providing "average" dependency relationship estimates. Works in this line are Guha and Banerji (1998) and Artis et al. (2004), which focus on business cycles synchronization. The authors first estimate different MS univariate models and then compute cross-correlations between the probabilities of being in recession as measures of synchronization.[2] Phillips (1991) points out the two extreme cases presented in the literature: the case of complete independence (two independent Markov processes are hidden in the bivariate specification) and the case of perfect synchronization (only one Markov process for both variables). Camacho and Perez-Quiros (2006) and Bengoechea et al. (2006) focus on assessing whether the latent variables in multivariate models are either unsynchronized or perfectly synchronized by modelling the data-generating process as a linear combination between the two cases. Leiva-Leon (2014) extends this approach to state-space representations, where the state vector is driven by latent variables following dynamics that are modelled as linear combination between the two polar cases. However, Leiva-Leon (2014) and previous related studies, assume that the weights assigned to each polar case, which are used to measure the synchronization between the latent variables, are assumed to be constant over time.

[1] Another type of relationship, under a univariate framework, is presented in Bai and Wang (2011), where the state variable governing the mean of the process is conditional to the one governing the variance of that process.

[2] However, as shown in Camacho and Perez-Quiros (2006), these approaches may lead to misleading results, since they are biased toward showing relatively low values of synchronization precisely for countries that exhibit synchronized cycles. This suggests that a bivariate framework would provide a better characterization of pairwise synchronization than two univariate models.

Despite the usefulness of the approaches used in the literature stream to deal with multivariate MS models, they assume, or estimate, constant over time dependency relationships between the underlying latent variables governing the model's dynamics. This assumption makes unfeasible assessments of endogenous changes in the structural relationship between the latent variables. For example, in the case of business cycles synchronization, two economies may become more synchronized due to trade agreements, economic unions, etc. Therefore, the analysis of changes in the structural relationship between the business cycles of theses economies (identified with the underlying latent variables) becomes crucial for the evaluation of specific policies.

Moreover, the study of business cycle synchronization is useful to assess the degree of exposure that a given economy has to its external environment. Previous works have used multivariate MS models to study the synchronization of national economies (Smith and Summers (2005) and Camacho and Perez-Quiros (2006)), or regional economies (Owyang et al. (2005) and Hamilton and Owyang (2012)), providing synchronization patterns that are constant over time. However, such degree of exposure may experience changes over time, which can be caused by a variety of factors, such as global recessionary shocks, global financial crises, etc. Therefore, changes in synchronization over time by using MS models can only be captured by splitting the sample into sub-periods. The problem with this approach is that its output relies on specific date breaks, which sometimes may be controversial and might increase the risk of pretesting bias (Diebold, 2015). To the best of my knowledge, the time-varying relationship between the latent variables of a MS model is an issue that still has not being studied from an endogenous perspective.

This paper proposes an approach to endogenously infer structural changes in the relationship between the latent variables governing multivariate MS models. For simplicity of the presentation and without loss of generality, in the sequel, I focus on the case of business cycles synchronization, however, the proposed framework can be applied to a wide range of applications of multivariate MS models. The proposed framework endogenously identify regimes where two economies enter recessions and expansions synchronously, from regimes where the economies are unsynchronized and experience independent business cycle phases. In contrast to existing MS models in the literature, the filter of the proposed framework not only provides the inferences associated to each latent variable, but it also provides simultaneous inferences on the dependency relationship between the latent variables for each period of time. The model is estimated by Gibbs sampling and its reliability is assessed with Monte Carlo experiments, suggesting it as a suitable approach to track changes in the synchronization of cycles.

Dynamic Factor Models have been widely used is assessing business cycles synchronization by looking at the variability of an economy's output growth explained by a "global component", see Kose et al. (2012), Kose et al. (2003) for a constant parameter version, and Del Negro and Otrok (2008) for a time-varying parameter version. However, they provide no information on bilateral synchronizations, i.e. economy-specific business cycles pairwise interlinkages, which is fundamental to study the dynamic propagation mechanism of business cycle shocks from a disaggregated perspective. The proposed framework provides time-varying pairwise synchronizations obtained from bivariate MS models that can be easily converted into measures of dissimilarity, or business cycle distances. These distances can be used to assess changes in the interdependence and clustering patterns experienced by a large set of economies by relying on network analysis. In such network, the economies take the interpretation of nodes, and the stochastic links between pairs of nodes is given by the estimated synchronicity, fully characterizing a business cycle network governed by Markovian dynamics.

The proposed framework is applied to investigate potential variations in the business cycles interdependence of U.S. states, and to assess the explanatory factor of the complex interactions at the regional level, obtaining four main findings. First, the results report the existence of "interdependence cycles", which are associated with recessions as identified by the National Bureau of Economic Research (NBER). Such cycles are defined as periods characterized by low cyclical heterogeneity across U.S. states, experienced during the recessionary and recovery phases, followed by longer periods of high cyclical heterogeneity, which occurs during the phases of stable growth. Second, there are substantial variations in the grouping pattern of states over time, going from a scheme characterized by several clusters of states to a core and periphery structure, composed of highly and lowly synchronized states, respectively. Third, the network analysis documents a change in the propagation pattern of contractionary shocks across states. Until the 1990s, recessions were characterized by the spread of shocks mainly across a few big states in terms of their share of GDP. Since that time, recessionary shocks have been more uniformly spread across all states, suggesting that regions of the U.S. economy have become more interdependent over the past two decades. Fourth, the main factor driving US intra-synchronization is the similarity of the economic structure across states, the more similar the structures, the more similar the responsiveness to shocks, and therefore, the higher the correlation between their business cycles. Also, more similar states in terms of household wealth tend to experience higher business cycles synchronization.

The paper is structured as follows. Section 2 presents the proposed time-varying synchronization approach. Section 3 reports the Monte Carlo simulation results. Section 4 analyzes the dynamic synchronization of business cycle phases in U.S. states, relying on bivariate, multivariate and network analyses. Finally, Section 5 concludes.

2 The Model

Let $y_{i,t}$ be the growth rate of an economic activity index of economy i, which can be modelled as a function of a latent or unobserved state variable ($S_{i,t}$) that indicates whether the economy is in a recessionary or expansionary regime, an idiosyncratic component, $\epsilon_{i,t}$, and a set of additional parameters, θ_i. Accordingly, for $i = a, b$,

$$y_{a,t} = f(S_{a,t}, \epsilon_{a,t}, \theta_a) \tag{1}$$

$$y_{b,t} = f(S_{b,t}, \epsilon_{b,t}, \theta_b). \tag{2}$$

The goal of this section is to provide assessments on the synchronization between $S_{a,t}$ and $S_{b,t}$ for each period of time; that is,

$$sync(S_{a,t}, S_{b,t}) = \Pr(S_{a,t} = S_{b,t}), \text{ for } t = 1, ..., T. \tag{3}$$

Following Owyang et al. (2005) and Hamilton and Owyang (2012), who rely on AR(0) MS specification, the following tractable bivariate two-state Markov-switching specification is considered:

$$\begin{bmatrix} y_{a,t} \\ y_{b,t} \end{bmatrix} = \begin{bmatrix} \mu_{a,0} + \mu_{a,1} S_{a,t} \\ \mu_{b,0} + \mu_{b,1} S_{b,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{a,t} \\ \varepsilon_{b,t} \end{bmatrix}, \tag{4}$$

where the innovations $\varepsilon_t = (\varepsilon_{a,t}, \varepsilon_{b,t})'$ are assumed to have a variance-covariance matrix that experiences changes of regimes, that is, $\varepsilon_t \sim N(0, \Sigma_t)$, where

$$\Sigma_t = \Sigma_0(1 - G_t) + \Sigma_1 G_t, \tag{5}$$

and G_t denotes an unobserved state variable that accounts for volatility regimes, and it is assumed to be independent from $S_{a,t}$ and $S_{b,t}$.

It is worth noting that the results derived in this section can be straightforwardly extended to specifications including lags in the dynamics. However, Camacho and Perez-Quiros (2007) show that positive autocorrelation in macroeconomic time series can be better captured by shifts between business cycle states rather than by the standard autoregressive coefficients.[3] This result agrees with Albert and Chib (1993), who show that the posterior distribution of autoregressive parameters tend to be centered at zero when modelling US output growth with regime-switching models.

When $S_{k,t} = 0$, the state variable $S_{k,t}$ indicates that y_{kt} is in regime 0 with a mean equal to $\mu_{k,0}$. When $S_{k,t} = 1$, y_{kt} is in regime 1 with a mean equal to $\mu_{k,0} + \mu_{k,1}$, for $k = a, b$. Moreover, $S_{a,t}$ and $S_{b,t}$ evolve according to irreducible two-state Markov chains, whose transition probabilities are given by

$$\Pr(S_{k,t} = j | S_{k,t-1} = i) = p_{k,ij}, \text{ for } i_k, j_k = 0, 1 \text{ and } k = a, b. \tag{6}$$

Analogously, when $G_t = 0$, the state variable G_t indicates that ε_t is in regime 0 with a variance-covariance matrix Σ_0. When $G_t = 1$, ε_t is in regime 1 with variance-covariance matrix Σ_1. The state variable G_t follows a two-state Markov chain with transition probabilities given by

$$\Pr(G_t = j_g | G_t = j_g) = p_{g,ij}, \text{ for } i_g, j_g = 0, 1 \tag{7}$$

To characterize the dynamics of $y_t = [y_{a,t}, y_{b,t}]'$, the information contained in $S_{a,t}$, $S_{b,t}$, and G_t, can be summarized in the state variable, $S_{ab,t}$, which accounts for the possible combinations that the vector $\mu_t = [\mu_{a,0} + \mu_{a,1} S_{a,t},\ \mu_{b,0} + \mu_{b,1} S_{b,t}]'$ and Σ_t could take through the different regimes:

$$S_{ab,t} = \begin{cases} 1, \text{ if } S_{a,t} = 0, S_{b,t} = 0, G_t = 0 \\ 2, \text{ if } S_{a,t} = 0, S_{b,t} = 1, G_t = 0 \\ 3, \text{ if } S_{a,t} = 1, S_{b,t} = 0, G_t = 0 \\ 4, \text{ if } S_{a,t} = 1, S_{b,t} = 1, G_t = 0 \\ 5, \text{ if } S_{a,t} = 0, S_{b,t} = 0, G_t = 1 \\ 6, \text{ if } S_{a,t} = 0, S_{b,t} = 1, G_t = 1 \\ 7, \text{ if } S_{a,t} = 1, S_{b,t} = 0, G_t = 1 \\ 8, \text{ if } S_{a,t} = 1, S_{b,t} = 1, G_t = 1 \end{cases}. \tag{8}$$

Similar to Harding and Pagan (2006), the objective of the proposed model is to differentiate regimes where the phases of $y_{a,t}$ and $y_{b,t}$ are unsynchronized, implying that $S_{a,t}$ and $S_{b,t}$ follow independent dynamics; that is,

$$\Pr(S_{a,t} = j_a, S_{b,t} = j_b, G_t = j_g) = \Pr(S_{a,t} = j_a) \Pr(S_{b,t} = j_b) \Pr(G_t = j_g), \tag{9}$$

from regimes where the phases of $y_{a,t}$ and $y_{b,t}$ are fully synchronized, entering expansions and recessions synchronously, implying that $S_{a,t} = S_{b,t} = S_t$; that is,

$$\Pr(S_{a,t} = j_a, S_{b,t} = j_b, G_t = j_g) = \Pr(S_t = j) \Pr(G_t = j_g). \tag{10}$$

In order to do so, I introduce into the framework another latent variable, V_t, that takes the value of 1 if business cycle phases are in a synchronized regime, and the value of 0 if they are under an unsynchronized regime at time t; that is,

$$V_t = \begin{cases} 0 \text{ if } S_{a,t} \text{ and } S_{b,t} \text{ are unsynchronized} \\ 1 \text{ if } S_{a,t} \text{ and } S_{b,t} \text{ are synchronized.} \end{cases} \tag{11}$$

The latent variable V_t also evolves according to an irreducible two-state Markov chain whose transition probabilities are given by

$$\Pr(V_t = j_v | V_{t-1} = i_v) = p_{v,ij}, \quad \text{for } i_v, j_v = 0, 1. \tag{12}$$

[3] This finding is in line with the results obtained by Kim, Morley and Piger (2005) and Morley and Piger (2006) who focus on assessing the importance of the "third phase" in the business cycle, and find that there is no need for autoregressive coefficients in the growth rates once the nonlinearities are correctly specified.

The advantage of introducing V_t, rather than analyzing the general Markovian specification with the full transition probability matrix, as in Sims et al. (2008), is that all the information about the dependency relationship between the latent variables remains summarized in a single variable, V_t, providing an easy-to-interpret way of assessing synchronization changes. It is also able to provide information about the expected duration of regimes where economies are synchronized or unsynchronized based on their associated transition probabilities. Notice that the analysis in this paper focuses on dependency, not on correlations, since the objective is to determine if two economies are either synchronized or unsynchronized.

Accordingly, there is an enlargement of the set of regimes in Equation (8), which remains fully characterized by the latent variable $S^*_{ab,t}$, that simultaneously collects information regarding joint dynamics, individual dynamics and their dependency relationship over time:

$$S^*_{ab,t} = \begin{cases} 1, \text{ if } S_{a,t}=0, S_{b,t}=0, G_t=0, V_t=0 \\ 2, \text{ if } S_{a,t}=0, S_{b,t}=1, G_t=0, V_t=0 \\ 3, \text{ if } S_{a,t}=1, S_{b,t}=0, G_t=0, V_t=0 \\ 4, \text{ if } S_{a,t}=1, S_{b,t}=1, G_t=0, V_t=0 \\ 5, \text{ if } S_{a,t}=0, S_{b,t}=0, G_t=1, V_t=0 \\ 6, \text{ if } S_{a,t}=0, S_{b,t}=1, G_t=1, V_t=0 \\ 7, \text{ if } S_{a,t}=1, S_{b,t}=0, G_t=1, V_t=0 \\ 8, \text{ if } S_{a,t}=1, S_{b,t}=1, G_t=1, V_t=0 \\ 9, \text{ if } S_{a,t}=0, S_{b,t}=0, G_t=0, V_t=1 \\ 10, \text{ if } S_{a,t}=0, S_{b,t}=1, G_t=0, V_t=1 \\ 11, \text{ if } S_{a,t}=1, S_{b,t}=0, G_t=0, V_t=1 \\ 12, \text{ if } S_{a,t}=1, S_{b,t}=1, G_t=0, V_t=1 \\ 13, \text{ if } S_{a,t}=0, S_{b,t}=0, G_t=1, V_t=1 \\ 14, \text{ if } S_{a,t}=0, S_{b,t}=1, G_t=1, V_t=1 \\ 15, \text{ if } S_{a,t}=1, S_{b,t}=0, G_t=1, V_t=1 \\ 16, \text{ if } S_{a,t}=1, S_{b,t}=1, G_t=1, V_t=1 \end{cases}. \tag{13}$$

Inferences on the latent variable $S^*_{ab,t}$, can be computed by conditioning on V_t[4]:

$$\begin{aligned} \Pr(S^*_{ab,t} &= j^*_{ab}) = \Pr(S_{a,t}=j_a, S_{b,t}=j_b, G_t=j_g, V_t=j_v) \\ &= \Pr(S_{a,t}=j_a, S_{b,t}=j_b, G_t=j_g|V_t=j_v)\Pr(V_t=j_v), \end{aligned} \tag{14}$$

where $\Pr(S_{a,t}=j_a, S_{b,t}=j_b, G_t=j_g|V_t=j_v)$ indicates the inferences on the dynamics of $S_{ab,t}$, conditional on total independence if $V_t=0$, or conditional on full dependence if $V_t=1$. In the former case, the joint probability of $S^*_{ab,t}$ is given by

$$\begin{aligned} \Pr(S_{a,t} &= j_a, S_{b,t}=j_b, G_t=j_g, V_t=0) = \Pr(S_{a,t}=j_a, S_{b,t}=j_b, G_t=j_g|V_t=0)\Pr(V_t=0) \\ &= \Pr(S_{a,t}=j_a)\Pr(S_{b,t}=j_b)\Pr(G_t=j_g)\Pr(V_t=0), \end{aligned} \tag{15}$$

while, in the latter case, it is given by

$$\begin{aligned} \Pr(S_{a,t} &= j_a, S_{b,t}=j_b, G_t=j_g, V_t=1) = \Pr(S_{a,t}=j_a, S_{b,t}=j_b, G_t=j_g|V_t=1)\Pr(V_t=1) \\ &= \Pr(S_t=j)\Pr(G_t=j_g)\Pr(V_t=1). \end{aligned} \tag{16}$$

[4]Notice that states 10, 11, 14 and 15 in Equation (13) are truncated to zero by construction, since the two state variables cannot be in different states if they are perfectly synchronized, i.e., $\Pr(S_{a,t}=j_a, S_{b,t}=j_b|V_t=1)=0$ for any $j_a \neq j_b$.

Therefore, inferences on the state variable $S_{ab,t}$, in Equation (8), after accounting for synchronization, can be easily recovered by integrating $\Pr(S_{a,t} = j_a, S_{b,t} = j_b, G_t = j_g, V_t = j_v)$ through V_t; that is,

$$\begin{aligned} \Pr(S_{a,t} &= j_a, S_{b,t} = j_b, G_t = j_g) = \Pr(V_t = 1) \Pr(S_t = j) \Pr(G_t = j_g) + \\ (1 - \Pr(V_t &= 1)) \Pr(S_{a,t} = j_a) \Pr(S_{b,t} = j_b) \Pr(G_t = j_g), \end{aligned} \tag{17}$$

which implies that the joint dynamics of $S_{a,t}$, $S_{b,t}$, and G_t remain characterized by a weighted average between the extreme dependent and independent cases, where the weights assigned to each of them are endogenously determined by

$$\Pr(V_t = 1) = \delta_t^{ab}. \tag{18}$$

Therefore, from now on, the term δ_t^{ab} will be referred to as the dynamic synchronicity between $S_{a,t}$ and $S_{b,t}$.

2.1 Filtering Algorithm

This section develops an extension of the Hamilton (1994) algorithm to estimate the model described in Equations (4) and (17). The algorithm is composed of two unified steps. In the first one, the goal is the computation of the likelihoods, while in the second, the goal is the prediction and updating of probabilities.

STEP 1: The parameters of the model are assumed to be known for the moment and are collected in the vector

$$\theta = (\mu_{a,0}, \mu_{a,1}, \mu_{b,0}, \mu_{b,1}, \Sigma_0, \Sigma_1, p_{a,00}, p_{a,11}, p_{b,00}, p_{b,11}, p_{00}, p_{11}, p_{v,00}, p_{v,11}, p_{g,00}, p_{g,11})'. \tag{19}$$

The conditional joint density corresponding to the state variable that fully characterizes the model's dynamics, $S^*_{ab,t}$, can be expressed as a function of its components,

$$f(y_t, S^*_{ab,t} = j^*_{ab}|\psi_{t-1}; \theta) = f(y_t, S_{a,t} = j_a, S_{b,t} = j_b, G_t = j_g, V_t = j_v|\psi_{t-1}; \theta), \tag{20}$$

which is the product of the density, conditional on the realization of the set of regimes times the probability of occurrence of such realizations,

$$\begin{aligned} f(y_t, S_{a,t} &= j_a, S_{b,t} = j_b, G_t = j_g, V_t = j_v|\psi_{t-1}; \theta) = f(y_t|S_{a,t} = j_a, S_{b,t} = j_b, G_t = j_g, V_t = j_v, \psi_{t-1}; \theta) \times \\ \Pr(S_{a,t} &= j_a, S_{b,t} = j_b, G_t = j_g, V_t = j_v|\psi_{t-1}; \theta). \end{aligned} \tag{21}$$

The joint probability of $S_{a,t} = j_a$, $S_{b,t} = j_b$, $G_t = j_g$ and $V_t = j_v$ is obtained by using conditional probabilities,

$$\begin{aligned} \Pr(S_{a,t} &= j_a, S_{b,t} = j_b, G_t = j_g, V_t = j_v|\psi_{t-1}; \theta) = \Pr(S_{a,t} = j_a, S_{b,t} = j_b, G_t = j_g|V_t = j_v, \psi_{t-1}; \theta) \times \\ \Pr(V_t &= j_v|\psi_{t-1}; \theta), \end{aligned} \tag{22}$$

where the term $\Pr(S_{a,t} = j_a, S_{b,t} = j_b, G_t = j_g|V_t = j_v, \psi_{t-1}; \theta)$ is fully characterized with the results derived in Equations (15) and (16). Thus, Equation (22) remains a function of only $\Pr(S_{k,t} = j_k|\psi_{t-1}; \theta)$ for $k = a, b$, $\Pr(G_t = j_g|\psi_{t-1}; \theta)$, $\Pr(V_t = j_v|\psi_{t-1}; \theta)$ and $\Pr(S_t = j|\psi_{t-1}; \theta)$. The steady state or ergodic probabilities can be used as starting values to initialize the filter.

In order to make inferences on the evolution of single-state variables, the marginal densities are obtained as

$$f(y_t, S_{a,t} = j_a|\psi_{t-1};\theta) = \sum_{j_b=0}^{1}\sum_{j_g=0}^{1}\sum_{j_v=0}^{1} f(y_t, S_{a,t}=j_a, S_{b,t}=j_b, G_t=j_g, V_t=j_v|\psi_{t-1};\theta), \quad (23)$$

$$f(y_t, S_{b,t} = j_b|\psi_{t-1};\theta) = \sum_{j_a=0}^{1}\sum_{j_g=0}^{1}\sum_{j_v=0}^{1} f(y_t, S_{a,t}=j_a, S_{b,t}=j_b, G_t=j_g, V_t=j_v|\psi_{t-1};\theta), \quad (24)$$

$$f(y_t, G_t = j_g|\psi_{t-1};\theta) = \sum_{j_a=0}^{1}\sum_{j_b=0}^{1}\sum_{j_v=0}^{1} f(y_t, S_{a,t}=j_a, S_{b,t}=j_b, G_t=j_g, V_t=j_v|\psi_{t-1};\theta), \quad (25)$$

$$f(y_t, V_t = j_v|\psi_{t-1};\theta) = \sum_{j_a=0}^{1}\sum_{j_b=0}^{1}\sum_{j_g=0}^{1} f(y_t, S_{a,t}=j_a, S_{b,t}=j_b, G_t=j_g, V_t=j_v|\psi_{t-1};\theta). \quad (26)$$

The marginal density associated the state variable S_t requires a special treatment. When it is assumed that the model's dynamics are governed by only one state variable, i.e., $S_{a,t} = S_{b,t} = S_t$, the density in Equation (20) collapses to $f^\dagger(y_t, S_t = j|\psi_{t-1};\theta)$, where

$$f^\dagger(y_t, S_t = 0|\psi_{t-1};\theta) = \sum_{j_g=0}^{1} f(y_t, S_{a,t}=0, S_{b,t}=0, G_t=j_g, V_t=1|\psi_{t-1};\theta), \quad (27)$$

$$f^\dagger(y_t, S_t = 1|\psi_{t-1};\theta) = \sum_{j_g=0}^{1} f(y_t, S_{a,t}=1, S_{b,t}=1, G_t=j_g, V_t=1|\psi_{t-1};\theta), \quad (28)$$

Accordingly, the density of y_t, conditional on the past observables, is given by

$$f(y_t|\psi_{t-1};\theta) = \sum_{j_a=0}^{1}\sum_{j_b=0}^{1}\sum_{j_g=0}^{1}\sum_{j_v=0}^{1} f(y_t, S_{a,t}=j_a, S_{b,t}=j_b, G_t=j_g, V_t=j_v|\psi_{t-1};\theta), \quad (29)$$

and under the assumption that $S_{a,t} = S_{b,t} = S_t$, it is given by

$$f^\dagger(y_t|\psi_{t-1};\theta) = \sum_{j=0}^{1} f^\dagger(y_t, S_t = j|\psi_{t-1};\theta). \quad (30)$$

STEP 2: Once y_t is observed at the end of time t, the prediction probabilities $\Pr(S_{k,t} = j_k|\psi_{t-1};\theta)$ for $k = a, b$, $\Pr(G_t = j_g|\psi_{t-1};\theta)$, $\Pr(V_t = j_v|\psi_{t-1};\theta)$ and $\Pr(S_t = j|\psi_{t-1};\theta)$ can be updated:

$$\Pr(S_{a,t} = j_a|\psi_t;\theta) = \frac{f(y_t, S_{a,t} = j_a|\psi_{t-1};\theta)}{f(y_t|\psi_{t-1};\theta)} \quad (31)$$

$$\Pr(S_{b,t} = j_b|\psi_t;\theta) = \frac{f(y_t, S_{b,t} = j_b|\psi_{t-1};\theta)}{f(y_t|\psi_{t-1};\theta)} \quad (32)$$

$$\Pr(G_t = j_g|\psi_t;\theta) = \frac{f(y_t, G_t = j_g|\psi_{t-1};\theta)}{f(y_t|\psi_{t-1};\theta)} \quad (33)$$

$$\Pr(V_t = l|\psi_t;\theta) = \frac{f(y_t, V_t = l|\psi_{t-1};\theta)}{f(y_t|\psi_{t-1};\theta)} \quad (34)$$

$$\Pr(S_t = j|\psi_t;\theta) = \frac{f^\dagger(y_t, S_t = j|\psi_{t-1};\theta)}{f^\dagger(y_t|\psi_{t-1};\theta)} \quad (35)$$

Forecasts of the updated probabilities in Equations (31) to (35) are done by using the corresponding transition probabilities $p_{a,ij}, p_{b,ij}, p_{g,ij}, p_{ij}, p_{v,ij}$, in the vector θ, for $S_{a,t}$, $S_{b,t}$, G_t, S_t and V_t, respectively:

$$\begin{aligned}
\Pr(S_{k,t+1} &= j_k|\psi_t;\theta) = \sum_{i_k=0}^{1} \Pr(S_{k,t+1} = j_k, S_{k,t} = i_k|\psi_t;\theta) \\
&= \sum_{i_k=0}^{1} \Pr(S_{k,t+1} = j_k|S_{k,t} = i_k) \Pr(S_{k,t} = i_k|\psi_t;\theta), \text{ for } k = a, b \qquad (36) \\
\Pr(G_{t+1} &= j_g|\psi_t;\theta) = \sum_{i_g=0}^{1} \Pr(G_{t+1} = j_g, G_t = i_g|\psi_t;\theta) \\
&= \sum_{i_g=0}^{1} \Pr(G_{t+1} = j_g|G_t = i_g) \Pr(G_t = i_g|\psi_t;\theta) \qquad (37) \\
\Pr(V_{t+1} &= j_v|\psi_t;\theta) = \sum_{i=0}^{1} \Pr(V_{t+1} = j_v, V_t = i_v|\psi_t;\theta) \\
&= \sum_{i=0}^{1} \Pr(V_{t+1} = j_v|V_t = i_v) \Pr(V_t = i_v|\psi_t;\theta) \qquad (38)
\end{aligned}$$

$$\begin{aligned}
\Pr(S_{t+1} &= j|\psi_t;\theta) = \sum_{i=0}^{1} \Pr(S_{t+1} = j, S_t = i|\psi_t;\theta) \\
&= \sum_{i=0}^{1} \Pr(S_{t+1} = j|S_t = i) \Pr(S_t = i|\psi_t;\theta) \qquad (39)
\end{aligned}$$

Finally, the above forecasted probabilities are used to predict inferences on the realizations of $S^*_{ab,t+1}$, relying on Equation (22):

$$\begin{aligned}
\Pr(S_{a,t+1} &= j_a, S_{b,t+1} = j_b, G_{t+1} = j_g, V_{t+1} = j_v|\psi_t;\theta) = \\
\Pr(S_{a,t+1} &= j_a, S_{b,t+1} = j_b, G_{t+1} = j_g|V_{t+1} = j_v, \psi_t;\theta) \times \Pr(V_{t+1} = j_v|\psi_t;\theta), \qquad (40)
\end{aligned}$$

where Equation (40) remains a function of $\Pr(S_{k,t+1} = j_k|\psi_t;\theta)$ for $k = a, b$, $\Pr(G_{t+1} = j_g|\psi_t;\theta)$, $\Pr(V_{t+1} = j_v|\psi_t;\theta)$ and $\Pr(S_{t+1} = j|\psi_t;\theta)$.

By iterating these two steps for $t = 1, 2, \ldots, T$, the algorithm provides simultaneous inferences on $S_{a,t}$, $S_{b,t}$, G_t, and their dynamic synchronicity δ_t^{ab} between $S_{a,t}$ and $S_{b,t}$ as defined in Equation (18).

Regarding the estimation of the parameters, notice that, as the number of possible states increases, the likelihood function could be characterized by several local maximums, causing strong convergence problems in performing maximum likelihood estimations, as shown in Boldin (1996). Hence, given the high number of combinations of states through which the likelihood is conditioned in Equation (29), the set of parameters θ along with the inferences on the state variables are estimated by using Bayesian methods. Specifically, a multivariate version of the approach in Kim and Nelson (1999), which applies Gibbs sampling procedures, is used. The estimation method is explained in detail in Appendix A.

3 Simulation Study

In order to validate the reliability of the proposed approach to assess changes in the synchronization of business cycle phases, I rely on the use of Monte Carlo experiments. Each simulation consists of two steps. First, the generation of two stochastic processes subject to regime switching that experience one or more synchronization changes. Second, by letting the econometrician observe only the generated data, but not the data-generating process, the proposed filter in Section 2.1 along with the Gibbs sampler, are applied to obtain estimates of the model's parameters, probabilities of recession for each economy, and, more importantly, the inferences on synchronization changes. I then address how well the parameter estimates and inferences match the real ones.

Given a sample of size T, the data generating process consists of generate a two-state first-order Markovian process, G_t, with transition probability matrix

$$P_g^* = \begin{pmatrix} p_{g,00}^* & 1-p_{g,11}^* \\ 1-p_{g,00}^* & p_{g,11}^* \end{pmatrix}. \tag{41}$$

Given two variance-covariance matrices, Σ_0^* and Σ_1^*, generate the innovations $e_t = [e_{a,t}, e_{b,t}]$ from a $N(0, \Sigma_t^*)$, where

$$\Sigma_t^* = \Sigma_0^*(1-G_t) + \Sigma_1^* G_t. \tag{42}$$

Next, generate a Markovian process, $S_{a,t}$, with a transition probability matrix,

$$P_a^* = \begin{pmatrix} p_{a,00}^* & 1-p_{a,11}^* \\ 1-p_{a,00}^* & p_{a,11}^* \end{pmatrix}. \tag{43}$$

Then, given a vector of means $\mu_a^* = [\mu_{a,0}^*, \mu_{a,1}^*]'$, generate a process $y_{a,t}^I$ as follows:

$$y_{a,t}^I = \mu_{a,0}^* + \mu_{a,1}^* S_{a,t} + e_{a,t}, \tag{44}$$

and given a vector of means $\mu_b^* = [\mu_{b,0}^*, \mu_{b,1}^*]'$, and transition probabilities $p_{b,00}^*$ and $p_{b,11}^*$, the same procedure is repeated to independently generate

$$y_{b,t}^I = \mu_{b,0}^* + \mu_{b,1}^* S_{b,t} + e_{b,t}, \tag{45}$$

where $S_{b,t}$ is a first-order Markovian process. Next, another Markovian process, S_t, is generated by using the transition matrix

$$P_{ab}^* = \begin{pmatrix} p_{00}^* & 1-p_{11}^* \\ 1-p_{00}^* & p_{11}^* \end{pmatrix}. \tag{46}$$

Then, given the two vectors of means μ_a^* and μ_b^*, generate jointly

$$\begin{bmatrix} y_{a,t}^D \\ y_{b,t}^D \end{bmatrix} = \begin{bmatrix} \mu_{a,0}^* + \mu_{a,1}^* S_t \\ \mu_{b,0}^* + \mu_{b,1}^* S_t \end{bmatrix} + \begin{bmatrix} e_{a,t} \\ e_{b,t} \end{bmatrix}. \tag{47}$$

The information generated so far can be collected in two vectors, one in which two stochastic processes are driven by two Markov-switching variables independent from each other, $y_t^I = [y_{a,t}^I, y_{b,t}^I]'$, and the other where two stochastic processes are governed by only one Markov-switching dynamic, $y_t^D = [y_{a,t}^D, y_{b,t}^D]'$.

The premise of this paper is that, during some regimes, the output growth of two economies can follow dynamics similar to those in y_t^D, while during other regimes, things can change in one, or both, of the economies, leading their joint dynamics to behave in the same way as those in y_t^I, following independent patterns. To mimic this situation, I start analyzing the simplest case in which there is just one synchronization change in a sample of size T, occurring at time

τ, with $1 < \tau < T$.[5] Then, I let $y_t = [y_{a,t}, y_{b,t}]'$ be the observed output growth of two economies, which comes from the following unobserved data generating process:

$$y_t = \left\{ \begin{array}{c} y_t^D, \text{ for } t = 1, \ldots, \tau \\ y_t^I, \text{ for } t = \tau+1, \ldots, T \end{array} \right. , \tag{48}$$

which can be alternatively expressed as

$$y_t = y_t^D V_t + (1 - V_t) y_t^I, \tag{49}$$

where V_t is an indicator variable of synchronization, whose dynamics are described by

$$\{V_t\}_1^T = \left[\begin{array}{c} \mathbf{1}_\tau \\ \mathbf{0}_{T-\tau} \end{array} \right], \tag{50}$$

with $\mathbf{1}_\tau$ being a vector, with entries equal to one, of size τ, and $\mathbf{0}_{T-\tau}$ a zero vector of size $T - \tau$. The case of one synchronization change can be easily extended to mimic the case of Z synchronization changes, occurred at $\tau_1, \tau_2, \ldots, \tau_Z$, with $1 < \tau_1 < \tau_2 < \ldots < \tau_Z < T$, just by appropriately modifying the dynamics in $\{V_t\}_1^T$. These experiments are evaluated under $Z = 6$ different scenarios. Each scenario corresponds to z changes in synchronization, for $z = 1, 2, 3, 4, 5$, and the last case considers a random number of synchronization changes, i.e., unlike predefining the dynamics of V_t as in Equation (50), it is modelled as a first-order Markov chain with transition probabilities $p^*_{V,00}$ and $p^*_{V,11}$, i.e. $z = f(V_t)$.

In addition, I study the performance of the proposed approach under a scenario where the assumption that $S_{a,t}$ and $S_{b,t}$ are either perfectly correlated or totally independent is relaxed. Accordingly, it is assume that the state variables $S_{a,t}$ and $S_{b,t}$ are imperfectly correlated. In particular, I generate a four-state Markovian process, $\bar{S}_{ab,t} = \{1, 2, 3, 4\}$, with its corresponding full 4×4 transition probability matrix, Q. Based on the realizations of $\bar{S}_{ab,t}$, I generate the vector $\mathbf{S}_t$ according to:

$$\mathbf{S}_t = \left\{ \begin{array}{l} (0,0), \text{ if } \bar{S}_{ab,t} = 1 \\ (0,1), \text{ if } \bar{S}_{ab,t} = 2 \\ (1,0), \text{ if } \bar{S}_{ab,t} = 3 \\ (1,1), \text{ if } \bar{S}_{ab,t} = 4 \end{array} \right. \tag{51}$$

Then, given the matrix of means $\mu = [\mu_a^*, \mu_b^*]$, generate observed data, y_t^{IC}, from state variables experiencing imperfect correlation, that is,

$$y_t^{IC} = \mu \odot \mathbf{S}_t + e_t, \tag{52}$$

where $\odot$ represents the Hadamard product. The entries of the matrix Q are set to produce a given level of correlation, δ, between $S_{a,t}$ and $S_{b,t}$. Specifically, three levels of correlation are evaluated, high, medium, and low, with $\delta = 0.7,\ 0.5,\ 0.2$, respectively.[6] Therefore, the data observed by the econometrician is produced following the data generating process:

$$y_t = \left\{ \begin{array}{c} y_t^D, \text{ for } t = 1, \ldots, T/3 \\ y_t^{IC}, \text{ for } t = T/3+1, \ldots, T(2/3) \\ y_t^I,, \text{ for } t = T(2/3)+1, \ldots, T \end{array} \right.$$

[5] The selection of τ is based on a random draw u, generated from a uniform distribution $U[0,1]$, i.e., $\hat{\tau} = uT$, then $\hat{\tau}$ is rounded to the nearest integer number to obtain τ. Also, the use of draws of τ equal to the boundaries, i.e., 1 or T, is avoided.

[6] For each level of correlation, δ, the matrix Q is calibrated such that $\bar{\rho} \simeq \delta$, where $\bar{\rho}$ is the average correlation between $\bar{S}_{a,t}$ and $\bar{S}_{b,t}$ over $10,000$ simulations of $\bar{S}_{ab,t}$, and the state variables are defined as
$\bar{S}_{a,t} = \left\{ \begin{array}{c} 0, \text{ if } \bar{S}_{ab,t} = 3 \text{ or } \bar{S}_{ab,t} = 4 \\ 1, \text{ Otherwise} \end{array} \right. , \quad \bar{S}_{b,t} = \left\{ \begin{array}{c} 0, \text{ if } \bar{S}_{ab,t} = 2 \text{ or } \bar{S}_{ab,t} = 4 \\ 1, \text{ Otherwise} \end{array} \right.$

Since the data-generating process and parameters are unknown by the econometrician, the Gibbs sampler is used to estimate the model's parameters, the probabilities of recession for each economy, the probability of highly volatile output, and, more importantly, inferences on the dynamics of V_t, by relying on the filtering algorithm proposed in Section 2.1. The criterion used to assess the performance of the regime inferences and the synchronization is the Quadratic Probability Score (QPS), defined as

$$QPS(\Xi) = \frac{1}{T}\sum_{t=1}^{T}(\Xi - \Pr(\Xi = 1|\psi_T))^2, \text{ for } \Xi = S_{a,t}, S_{b,t}, G_t, V_t. \tag{53}$$

To illustrate the filtering and estimation strategy's performance, Figure 1 plots one simulation for the cases in which there is one, two and three synchronization changes in a sample of 400 periods, i.e., for $z = 1, 2, 3$, with $T = 400$.[7] For each case, the top charts plot the two observed time series, $y_{a,t}$ and $y_{b,t}$, generated with the parameter values in Table 1 and by using Equation (49), along with the unobserved dynamics of V_t. Both time series show strong coherence in phases when $V_t = 1$, and the opposite occurs when $V_t = 0$. The second row of charts plot the computed inferences on the synchronization changes, i.e., $\Pr(V_t = 1)$, along with the true dynamics of V_t, showing their close relation in all three cases and providing insight into the satisfactory performance of the proposed framework for assessing synchronization changes. The third row of charts plot the two observed series along with the unobserved dynamics of the state variable G_t. Notice that $y_{a,t}$ and $y_{b,t}$ experience more volatile fluctuations during periods where $G_t = 1$, and less volatile dynamics when $G_t = 0$. Finally, the fourth row of charts plot the computed inferences on regimes of high volatility, i.e., $\Pr(G_t = 1)$, along with the true dynamics of G_t, showing that the model is also able to perform accurate inferences of the volatility regimes.

The parameters used in the simulations exercises are specified in Table 1 and the experiments associated to each scenario are replicated $M = 1,000$ times. The results of the Monte Carlo simulations are reported in Table 2, showing the average over the M replications of each estimated parameter

$$\theta_z^* = \frac{1}{M}\sum_{m=1}^{M}\theta_z^{*(m)}, \tag{54}$$

where $\theta_z^{*(m)}$ corresponds to the vector of parameters, as defined in Equation (19), associated to the m-th replica and the z-th case. All parameter estimates appear to be unbiased for the different values of z and δ. Notice that the stochastic process with the highest difference of the within-regime means, in this case $y_{b,t}$, shows more accurate estimates, meaning that higher differences provide a better identification of the phases of the business cycles. Regarding the performance of the regime inferences, Table 3 reports the averages over the M replications with the QPS associated with the state variables $S_{a,t}$, $S_{b,t}$ and V_t, which can be interpreted as the average over the M replications of the squared deviation from the generated business cycles:

$$QPS(\Xi)_\zeta = \frac{1}{M}\sum_{m=1}^{M}QPS(\Xi)_\zeta^{(m)}, \text{ for } \Xi = S_{a,t}, S_{b,t}, G_t, V_t, \text{ and for } \zeta = z, \delta, \tag{55}$$

where $QPS(\Xi)_\zeta^{(m)}$, as defined in Equation (53), corresponds to the m-th replica and the ζ-th scenario, that corresponds to a specific value of z or δ. The results indicate that, although inferences on the state variables in general present high precision, the ones associated with the time series with the highest difference of the within-regime means, $y_{b,t}$, are, in general, the most accurate.

[7] We choose this sample size since it is close to the one used in the empirical application of Section 4.

The precision of the inferences on synchronization changes decreases as the number of changes, k, increases. This feature can also be observed by looking at the histograms of the M replications plotted in Figure 1 of Appendix B, in particular, the last column of charts, where the distribution of $QPS(V_t)_\zeta^{(m)}$ is shown. However, it is natural to think of synchronization changes as events that do not occur as often as the business cycle phases of an economy. They may require longer periods of time to take place, since they originate from changes in the structural relationships among economies. This suggests that the proposed model is suitable for accurately inferring synchronization changes of business cycle phases. Moreover, the model is able to appropriately characterize the underlying level of imperfect correlation between $S_{a,t}$ and $S_{b,t}$, as can be seen in the last row of Table 3, for lowly, moderately and highly correlated state variables, respectively.

4 Monitoring U.S. States Business Cycles Synchronization

The most recent global financial crisis has stimulated interest in the study of the sources and propagation of contractionary episodes, calling for a more careful look at the disaggregation of business cycles in order to assess the mechanisms underlying economic fluctuations.

On the one hand, recent work by Acemoglu et al. (2012), which relies on network analysis, finds that sectoral interconnections capture the possibility of "cascade effects," whereby productivity shocks to a sector propagate not only to its immediate downstream customers, but also to the rest of the economy. On the other hand, two recent papers have shown interesting features of economic activity synchronization when the business cycle is disaggregated at the regional level. In the first, Owyang et al. (2005) investigate the evolution of the individual business cycle phases of U.S. states. By following a univariate approach, the authors find that U.S. states differ significantly in the timing of switches between expansions and recessions, and also differ in the extent to which phases in state business cycles are synchronous with those of the national economy. In the second paper, Hamilton and Owyang (2012) use a unified framework to go through the propagation of regional recessions in the United States, using a multivariate approach that focuses on clustering the states that share similar business cycle characteristics. They find that differences across states appear to be a matter of timing and that they can be grouped into three clusters, with some entering recession or recovering before others. Although these previous studies provide useful insights about the overall synchronization pattern in a given sample period, they are not able to detect changes in patterns occurring in these time spans.

This study intends to unify both concepts: first, the dynamic synchronization of pairwise cycles, by using the framework proposed in Section 2; and second, the dynamic interdependence among all U.S. states, by relying on network analysis, in order to assess the presence and the nature of potential changes in the regional propagation of contractionary shocks. For this purpose, I use data on U.S. states coincident indexes, proposed in Crone and Matthews(2005) and provided by the Federal Reserve Bank of Philadelphia, as monthly indicators of the overall economic activity at the state level. The sample spans from August 1979, when the data for all the states started to be reported, until February 2016 (Alaska and Hawaii are excluded as in Hamilton and Owyang (2012)). The Chicago Fed National Activity Index (CFNAI) is used as a monthly measure of the U.S. national business cycle. All these indexes of real economic activity, for each state and for the entire United States, have been constructed based on the principle of co-movement among industrial production, employment, sales and income measures.

4.1 Bivariate Analysis

The analysis for 48 states plus the United States as a whole requires the modelling of the $\mathbf{C}_2^{49} = 1,176$ pairwise comparisons. To assess the performance of the proposed Markov-switching synchronization model, two selected examples are analyzed in detail.[8]

The first example focuses on the case of two states that have a high share of national GDP: New York (7.68%) and Texas (7.95%). Table 4 reports the Bayesian estimates for the *New York vs. Texas* model, showing almost zero growth rates when $S_t = 0$ and positive growth when $S_t = 1$, for both states. It is worth highlighting the estimates of the transition probabilities associated with the state variable that measures synchronization, V_t. The probability of remaining in a low synchronization regime, 0.97, is slightly higher than the probability of remaining in a low synchronization regime, 0.94. This result is corroborated in the first three rows of Chart A of Figure 2, which plots (i) the probabilities of recession for New York and (ii) for Texas along with (iii) the corresponding time-varying synchronization, $\delta_t^{NY,TX}$, as defined in Equation (18). As can be seen, from the 1980s to the mid-1990s, these states experienced recessions at different times. This is reflected in the low values of the synchronicity. However, since the mid-1990s, both economies have been experiencing the same recession chronology, which is consistent with the increase in the synchronicity observed after the mid-1990s. Also, the model controls for potential changes in the variance-covariance matrix of innovations by inferring the probability of high volatile real activity for the two states, shown in the fourth row of Chart A of Figure 2. These probabilities indicate a high volatility regime during the pre-Great Moderation period and also during the Great Recession.

The second example analyzes the case of two states with different shares of GDP: the state with the highest, California (13.34%); and the state with the lowest, Vermont (0.18%). Table 5 presents the Bayesian parameter estimates of the model. Unlike the previous example, the probability *California vs. Vermont* remain highly synchronized, 0.99, is higher than the probability of remaining unsynchronized, 0.87. This is also illustrated in Chart B of Figure 2, which shows that, in general, both states have experienced the same business cycle chronology, entering recessions and expansions synchronously, with the exception of one period. Specifically, in 1989, Vermont entered a recessionary phase, while California was still growing until mid-1990, when it also started to experience a recession. However, at the beginning of 1992, Vermont started an expansionary phase, while California remained in recession until 1994. These desynchronicities are reflected in the downturn of the dynamic synchronization, $\delta_t^{CA,VT}$, during that period. Also, Chart B of Figure 2 plots the probability of high volatility regime, showing high values during the 1990 recession and during the Great Recession.

Considerable heterogeneity was found in the dynamics of the estimated time-varying synchronizations, finding cases involving significant changes, and cases where the synchronization was almost constant, at low or high levels. Although the proposed framework can provide information on the synchronization between any pair of states for any given period of time, other ways to summarize the information are needed, since policy-makers are interested in the "big picture" of the overall regional synchronization path.

4.2 Multivariate Analysis

As suggested by Tim (2002) and Camacho et al. (2006), the multi-dimensional scaling (MDS) method is a helpful tool for identifying cyclical affiliations between economies, since it seeks to find a low-dimensional coordinate system to represent n-dimensional objects and create a map of lower dimension (k). Traditionally, studies use as input for this method a symmetric matrix, Γ, that summarizes the cyclical distances between economies for a given time span.

[8] The results for the other 1,174 cases are available from the author upon request.

Each entry γ^{ij} of the matrix assigns a value characterizing the distance between economies i and j. The output of the MDS consists of one map showing the general picture for all the cyclical affiliations.

The dynamic synchronization measures obtained in the bivariate analysis, $0 \leq \delta_t^{ij} \leq 1$, can be easily converted into desynchronization measures, $\gamma_t^{ij} = 1 - \delta_t^{ij}$. Accordingly, γ_t^{ij} can be interpreted as cyclical distances, allowing the construction of the dissimilarity matrix Γ, for each time period:

$$\Gamma_t = \begin{pmatrix} 1 & \gamma_t^{12} & \gamma_t^{13} & \dots & \gamma_t^{1n} \\ \gamma_t^{21} & 1 & \gamma_t^{23} & \dots & \gamma_t^{2n} \\ \gamma_t^{31} & \gamma_t^{32} & 1 & \dots & \gamma_t^{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \gamma_t^{n1} & \gamma_t^{n2} & \gamma_t^{n3} & \dots & 1 \end{pmatrix}, \tag{56}$$

which provides the possibility of assessing changes in the general picture of all cyclical affiliations of U.S. states.

In a recent work on MDS, Xu et al. (2012) propose a way to deal with MDS in a dynamic fashion, where the dimensional coordinates of the projection of any two objects, i and j, are computed by minimizing the stress function,

$$\min_{\tilde{\gamma}_t^{ij}} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}(\gamma_t^{ij} - \tilde{\gamma}_t^{ij})^2}{\sum_{i,i}(\gamma_t^{ij})^2} + \beta \sum_{i=1}^{n} \tilde{\gamma}_{t|t-1}^{i}, \tag{57}$$

where

$$\tilde{\gamma}_t^{ij} = (||z_{i,t} - z_{j,t}||^2)^{1/2} \tag{58}$$
$$\tilde{\gamma}_{t|t-1}^{i} = (||z_{i,t} - z_{i,t-1}||^2)^{1/2}, \tag{59}$$

$z_{i,t}$ and $z_{j,t}$ are the k-dimensional projection of the objects i and j, and β is a temporal regularization parameter that serves to zoom in or zoom out changes between frames at t and at $t+1$, always keeping the same dynamics independent of its value. In principle, β can be simply set up to 1; however, since the data in Γ_t belong to the unit interval, for a more adequate visual perception of the transitions between frames it is set up to 0.05. The output of the minimization in Equation (57) provides a two-dimensional representation of Γ_t.

The synchronization maps of U.S. states for the first month of the last four recessions are plotted in the charts of Figure 3. Each point in the charts represents a state, and the middle point refers to the United States as a whole. The closeness between two points in the plane refers to their degree of synchronicity, i.e., the closer the points are, the greater their synchronization. The figure corroborates the premise in the introduction of this paper about the existence of significant changes in the grouping pattern among regional economies over time.

Specifically, Chart A plots the scenario for the 1981 recession, showing a few clusters of states experiencing similar business cycles phases. Notice that only a few states, such as South Carolina and Washington, located inside the first concentric circle of the chart, were highly synchronized with the national business cycle. Instead, most of the states were located in the second concentric circle, experiencing moderated synchronization. The remaining states, located in the third concentric circle, such as Florida, Colorado, Texas, North Dakota, West Virginia, among others, were lowly synchronized between each other and with respect to the national cycle, following mostly independent patterns. Chart B presents the situation for the 1990 recession, showing a slightly different clustering pattern between states, but keeping three groups, highly, moderately and lowly synchronized states with the national cycle. Charts C and D present the scenarios for the 2001 and 2007 recessions, in the left and right corner, respectively. Both charts indicate a stronger synchronization pattern between states and with respect to the national business cycle, characterized by a core (composed of states highly in sync) and periphery (composed of independent states) structure. In both periods, the first concentric circle contains a large number of states, such as Pennsylvania, North Carolina, Georgia, among others, while the third concentric circle contains only a few states, such as the oil producing states, Texas, Oklahoma, and North Dakota. The full animated representation can be found at the author's webpage.[9]

An additional advantage of the proposed framework is the possibility of recovering the stationary measures of synchronization, by using the ergodic probabilities associated with the latent variable V_t. Chart A of Figure 2 of Appendix B plots the stationary grouping pattern, which can be interpreted as the *average* pattern from August 1979 to March 2013. It shows three groups of states, corresponding to the three concentric circles: one is close to the U.S. cycle, the second is less but still close to the U.S. cycle, while the third is characterized by the states following independent dynamics. To assess whether this result reconciles with the one in Hamilton and Owyang (2012), Chart B of Figure 2 of Appendix B plots the clusters obtained by those authors. The results show that clusters found in Hamilton and Owyang (2012) are consistent with the grouping pattern of states found in this paper. Moreover, this result is not only robust to the methodology employed, but also to the data used, since Hamilton and Owyang (2012) use annualized quarter-to-quarter growth rates of payroll employment, while I use monthly growth rates of state coincident indexes of economic activity. These facts show one of the main contributions of the proposed framework, which is to provide synchronization measures that may change over time, and that can be collapsed into ergodic measures that yield results consistent with those in previous work.

Regarding the cyclical relationship between states and the national business cycle, Table 6 reports the corresponding ergodic synchronizations, showing the range from the highest ones, which are Illinois and Pennsylvania with 0.86 and 0.85, respectively, to the lowest ones, Louisiana and Oklahoma with 0.18 and 0.17, respectively. To provide a visual perspective, Chart A of Figure 3 of Appendix B plots a U.S. map with the estimates obtained in this paper, and Chart B plots the concordance pattern obtained in Owyang et al. (2005) by calculating the percentage of the time two economies were in the same regime, based on univariate MS models for each state. Although both results report high values in most of the states located in the east region and moderated values in a few states located in the west, the stationary synchronization measure presents higher dispersion than the concordance, as can be seen in the associated histograms. This comparison helps to differentiate in a more precise way the strenghtness of cyclical relationships between the business cycles of states and the nation.

4.3 Network Analysis

Recent works by Carvalho (2008), Gabaix (2011), Acemoglu et al. (2012), among others, rely on network analysis to show how idiosyncratic shocks, at the firm or sectoral level, may originate macroeconomic fluctuations, given their interlinkages. Although, such analysis primarily relies on the economy's sectoral disaggregation, it may be interesting to assess if another type of disaggregation, e.g., regional, may also have significant implications on aggregate fluctuations.

The intuition behind the synchronization measure in Equation (18) relies on the fact that if δ_t^{ij} is close to 1, it is likely that at time t, economies i and j are sharing the same business cycle phases, creating a link of interdependence between them. On the other hand, if δ_t^{ij} is close to 0, it means that the economies are following independent phases and thus are not linked.[10]

[9] `https://sites.google.com/site/daniloleivaleon/media`

[10] Notice that the proposed synchronization modelling approach distinguishes between the state in which two economies are in recession but their cycles are independent and just coincided, from the state where the two economies are in recession because they are under a regime of dependence, i.e., states 1 and 5 of $S^*_{ab,t}$ in Equation (13), respectively.

Therefore, by letting $H = \{h_i\}_1^n$ be the set of n economies taking the interpretation of nodes, h_i for $i = 1, \ldots, n$, and defining δ_t^{ij} as the probability that nodes h_i and h_j are linked at time t, the matrix $\Delta_t = \mathbf{1}_n - \Gamma_t$, can be interpreted as a weighted network of synchronization with Markovian dynamics.[11] Consequently, the cyclical interdependence of a large set of economies can be dynamically assessed under a unified framework by relying on network analysis. It is worth noting that although the construction of Δ_t requires the computation of several bivariate models of the type in Equation (4), it may be less restrictive and involve less parameter and regime uncertainty than the computation of a framework with a similar non-linear nature but involving all n economies simultaneously. However, further research in this respect is needed.

To provide a glimpse of the shape that the Markov-switching synchronization network (MSYN) has taken during contractionary episodes, the charts of Figure 4 plot the corresponding network graph for the first month of the last four recessions. Given that the MSYN is a weighted network, in order to make the graphical representation possible, a link between nodes i and j is plotted if $\delta_t^{ij} > 0.5$; otherwise, no link is plotted between them. The figure corroborates the grouping pattern shown in the MDS analysis, which is consistent with a relatively disperse network structure during the 1981 and 1990 recessions, while showing a core and periphery structure in the 2001 and 2007 recessions.[12]

The main advantage of providing a network analysis for the present framework is that all the information on synchronicities in the current analysis can be summarized in just one measure, the closeness centrality. There are several measures regarding the centrality of a network, but given that desynchronization measures are interpreted as distances, the most appropriate one for this context is the closeness centrality.

For robustness purposes, two variations of the closeness centrality are analyzed in this section. For each of them, it is necessary to first compute the centrality of each node,

$$C_t(i) = \frac{1}{\sum_{j \neq i|t} d_t(i,j)}, \text{ for } i = 1, 2, ..., n, \tag{60}$$

where $d(i,j)$ is the length of the shortest path between nodes i and j, which can be computed by the Dijkstra (1959) algorithm.[13] Thus, the more central a node is, the lower the total distance from it to all other nodes. Closeness can be regarded as a measure of how fast it will take to spread information, e.g., risk, economic shocks, etc., from node i to all other nodes sequentially. For an overview of definitions in network analysis, see Goyal (2007).

Once the dynamic centrality of each node has been computed, the information about the whole network's centrality can be typically assessed as follows:

$$C_t^N = \sum_{i=1|t}^{k} [C_t(i^*) - C_t(i)], \tag{61}$$

where i^* is the node that attains the highest closeness centrality across all nodes at time t. The second measure, consists on the average across all nodes' centralities, $C_t(i)$, defined by

$$C_t^A = \sum_{i=1|t}^{k} C_t(i). \tag{62}$$

[11] The term $\mathbf{1}_n$ represents a squared matrix of size n with all entries equal to 1.

[12] Notice that, although the U.S. business cycle is not included in the network analysis, only those of the states, each chart in the figure shows a close relation with the corresponding one in Figure 3.

[13] For example, in a set $H' = \{a, b, c\}$ where the distances $\gamma = 1 - \delta$ are given by $\gamma^{ab} = 0.5$, $\gamma^{ac} = 0.9$ and $\gamma^{bc} = 0.2$, the shortest path between a and c will be 0.7, since $\gamma^{ab} + \gamma^{bc} < \gamma^{ac}$. Thus, notice that $d(a,c)$ does not necessarily have to be equal to γ^{ac}.

These two measures, which provide information on the changes in the degree of aggregate synchronization among the economies in the set H, for the present case between the states of the United States, can be used to investigate the relationship between regional business cycle interdependence and aggregate fluctuations.[14]

One of the main findings in Hamilton and Owyang (2012) is the substantial heterogeneity across regional recessions in the United States at the state level. How such heterogeneity could change over time, however, is an issue that has remained uninvestigated. The proposed framework is used to dynamically quantify the substantial regional heterogeneity under the unified setting MSYN. The intuition behind the state's centrality in Equation (60) is the following: if, at time t, state i is highly synchronized with respect to the rest of U.S. states, its total distance from them, $\sum_{j\neq i|t} d_t(i,j)$, would tend to be low and its centrality, $C_t(i)$, to be high. If a similar behaviour occurs with the remaining $n-1$ states, the MSYN's centrality would also tend to take high values. This means that high global interdependence, or, equivalently, high homogeneity of regional recessions, is associated with high values of the MSYN's centrality C_t^{Υ}, for $\Upsilon = N, A$.

Chart A of Figure 5 plots the network centrality, C_t^N, and the average centrality, C_t^A. Both measures show similar dynamics, experiencing substantial changes over time that have a close relation with the national recessions dated by the NBER, and showing some interesting features. First, the centrality shows a markedly high tendency to increase some months before national recessions take place, implying that sudden increases in the degree of interdependence among states may be useful to signal upcoming national recessions. Second, once national recessions have ended, the centrality measures also increase. This is because the whole economy is recovering from the recession and most of the states are synchronized, although, this time, in a expansionary regime. Third, after this phase of recovery has ended and the U.S. economy starts its moderated expansionary path, the centrality decreases until it reaches a certain stable level, which prevails until another recession takes place and the cycle repeats. Notice that the periods with higher heterogeneity across regional business cycles do not occur during turning points, but during periods of stable economic expansion. These three observations reveal that regional economies in the United States at the state level are subject to cycles of interdependence that are highly associated with the national business cycle, in particular, to the periods around the turning points.

The centrality measures have experienced higher levels during the 2001 and 2007 recessions that during the previous recessions, corroborating the core-periphery structure observed in the MDS analysis for the corresponding periods and plotted in the bottom charts of Figure 4. This result discloses a change in the propagation pattern of aggregate recessionary shocks. During the pre-2000 recessions, those business cycles shocks were spread mainly toward a few but relatively large states, in terms of share of GDP, while during the post-2000 recessions, such shocks were more uniformly and synchronously distributed across states, in particular, to the ones in the core, as can be seen in the charts of Figure 3.

To address changes in the clustering pattern in a statistical rather than visual manner, I compute the clustering coefficient of the MSYN for every time period by following Strogatz and Watts (1998), which allows the measurement of the level of cohesiveness between the business cycle phases of U.S. states. The dynamic clustering coefficient is plotted in Figure 6, showing that in the mid-1990s there was a significant change in the regional cohesiveness. Before that time, the clustering coefficient followed short cycles, but after the mid-1990s, it remained almost stable at higher values, corroborating the change in the propagation of contractionary shocks that occurred since the 2001 recession and providing evidence that the U.S. economy's regions have become more interdependent since the early 1990s.

[14] A third measure was also computed by extracting the common component among the nodes' centralities using principal component analysis. However, the results were similar to those of obtained with the average centrality. Therefore, they are not shown.

4.4 Explanatory Factors

In order to provide assessments about the origins of the complex interactions between the business cycles of US states, I investigate whether changes in US business cycles intra-synchronization may be explained by certain macroeconomic and financial factors. In particular, the interest is place on explaining desynchronization measures, γ_t^{ij}, with a set of variables that represent dissimilarities about certain features of the states, such as, sectoral composition, income, financial activity, and fiscal policy.

If two states possess similar economic structures, both states may experience similar responsiveness to business cycles shocks. Therefore, I follow the line of Imbs (2004) and use the shares in aggregate employment associated to sector l, of state i, at time t, denoted by $\eta_{l,t}^i$, to compute a time-varying measure of industry specialization:

$$SEC_t^{ij} = \sum_{l=1}^{L} |\eta_{l,t}^i - \eta_{l,t}^j|. \tag{63}$$

The variable SEC_t^{ij} measures the differences in the economic structure of states i and j over time, and represents the one of the potential explanatory factors of changes in synchronization to be assessed. Another potential factor is related to the wealth of states, since states with similar levels of household wealth may experienced similar economic fluctuations. Accordingly, I use the real median household income of state i at time t, denoted by, INC_t^i, to construct the variable:

$$INC_t^{ij} = |\ln(INC_t^i) - \ln(INC_t^j)|, \tag{64}$$

where INC_t^{ij} measures the differences in the household wealth of states. The financial structure of states may also play an important role in explaining their business cycles synchronization patterns. I follow Francis et al. (2012) and use the total banking deposits of state i and time t, denoted by DEP_t^{ij}, to measure differences in financial structures, INC_t^{ij}, analogously to Equation (64). Finally, I consider the fiscal sector as a potential explanatory factor of business cycles comovement and use government expenditures of state i at time t, GOV_t^i, to measure differences in fiscal policy of states, denoted by, GOV_t^{ij}, and computed following Equation (64).

The data used in this analysis spans from 1992 until 2013, the longest available sample at the present time, and was taken from different sources. Data on employment, at the monthly frequency, and data on household income, at the yearly frequency, were retrieved from the Federal Reserve Economic Data. Data on bank deposits and government expenditures, at the yearly frequency, was taken from the Federal Deposit Insurance Corporation and the Census Bureau, respectively. The objective of this section is assessing the relationship between those factors measuring economic differences at the state level and the measures of business cycles dissimilarities by estimating the following panel regression:

$$\gamma_t^{ij} = \alpha + \beta_1 SEC_t^{ij} + \beta_2 INC_t^{ij} + \beta_3 DEP_t^{ij} + \beta_4 GOV_t^{ij} + \upsilon_t^{ij}, \tag{65}$$

for $ij = 1, 2, ..., \mathbf{C}_2^{49}$, hence, the cross-sectional unit in the panel model is pairs of US states.

The estimated coefficients of Equation (65) are reported in Table 7, showing that only difference in sectoral composition and in household income are significant factors explaining differences in business cycles synchronization.[15] Notice that in both cases the estimated coefficient is positive, implying that increases in the similarity of the US states economic structures are associated to increases in their business cycles synchronization. Analogously, the more similar are the states household incomes the more synchronized their business cycles tend to be.

[15] I use robust standard errors in all the estimations.

The effect of total deposits and government expenditures on business cycles dissimilarities is also positive, however, it is not statistically significant. It is worth mentioning that because of potential simultaneity bias and reverse causality, I cannot claim any causal relationship between factors and dissimilarities, and only correlation statements can be claimed.

In previous sections, this paper documents an overall increase in the synchronization of US states since the early 1990s, as can be seen in Figure 3, implying potential instabilities in the relationship between the synchronization and its drivers. Therefore, I investigate potential changes over time in those relationships, measured by the coefficients β_ι, for $\iota = 1, 2, 3, 4$, of Equation (65). In particular, I estimate Equation (65) by using only the information contained in a given year, τ, and save all the coefficients associated to each year, that is, $\beta_{\iota,\tau}$, for $\iota = 1, 2, 3, 4$ and $\tau = 1992, 1993, ..., 2013$. Figure 7 plots the time-varying betas associated to each explanatory factor, showing that the importance of sectoral composition in explaining synchronization patterns has significantly increased during the 1990s, reaching to a stable level thereafter. The explanatory power of household income remained positive and significant until the Great Recession, indicating that wealth differences across states did not play an important role in explaining the last simultaneous downturn of regional economies. Finally, the effect of total deposits and government expenditures on synchronization patterns is not statistically significant for most of the years, which is consistent with the full sample estimates reported in Table 7. These results indicate that the main factor driving US intra-synchronization is the similarity of the economic structure across states. The more similar the structures, the more similar the responsiveness to shocks, and therefore, the higher the correlation between their business cycles.

5 Conclusions

Most of the studies on business cycle synchronization provide a general pattern of cyclical affiliations between economies for a given time span. However, little has been done to assess potential pattern changes that may occur during such a time span. This paper proposed an extended Markov-switching framework to assess changes in the synchronization of cycles by inferring the time-varying dependency relationship between the latent variables governing Markov-switching models. The reliability of the approach to track synchronization changes is confirmed by Monte Carlo experiments.

The proposed framework is applied to investigate potential variations in the cyclical interdependence between the states of the United States. There are four main findings. First, the results report the existence of interdependence cycles that are associated with NBER recessions. Such cycles are defined as periods characterized by low cyclical heterogeneity across states, experienced during the recessionary and recovery phases, followed by longer periods of high cyclical heterogeneity that occur during the phases of stable growth. Second, there are substantial variations in the grouping pattern of states over time that can be monitored on a monthly basis, ranging from a scheme characterized by several clusters of states to a core and periphery structure, composed of highly and lowly synchronized states, respectively. Third, there is evidence of a change in the propagation pattern of recessionary shocks across states. Up to the 1991 recession, recessionary shocks were spread mainly toward a few large states, in terms of share of GDP. But after that, contractionary shocks were more synchronously and uniformly spread toward most of the U.S. states, implying that U.S. regions have become more interdependent since the early 1990s. Fourth, the main factor explaining the business cycles synchronization patterns of US states is the similarity of their economic structure, followed by how similar is the wealth, measured by household income, across states. The more similar the structures and the wealth of states, the higher their business cycles synchronization.

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Table 1: Parameter values for generating processes

State 0		State 1	
Parameter	Value	Parameter	Value
$\mu^*_{a,0}$	-1	$\mu^*_{a,1}$	2
$\mu^*_{b,0}$	-2	$\mu^*_{b,1}$	4
$\sigma^*_{a,0}$	0.20	$\sigma^*_{a,1}$	1
$\sigma^*_{b,0}$	0.60	$\sigma^*_{b,1}$	3
$p^*_{a,00}$	0.80	$p^*_{a,11}$	0.90
$p_{b,00}$	0.80	$p^*_{b,11}$	0.90
p^*_{00}	0.80	p^*_{11}	0.90
$p^*_{G,00}$	0.98	$p^*_{G,11}$	0.98
$p^*_{V,00}$	0.98	$p^*_{V,11}$	0.98

Note: The table shows the parameter values used to generate the stochastic processes y_t in Equation (49) for the simulation study in Section 3.

Table 2: Performance of parameter estimations

	$z=1$	$z=2$	$z=3$	$z=4$	$z=5$	$z=f(V_t)$	$\delta=0.2$	$\delta=0.5$	$\delta=0.7$
$\mu^*_{a,0}$	-0.99	-0.98	-0.98	-0.98	-0.99	-0.98	-0.95	-0.97	-0.98
$\mu^*_{a,1}$	1.97	1.97	1.97	1.97	1.97	1.97	1.92	1.95	1.96
$p^*_{a,11}$	0.89	0.89	0.89	0.89	0.89	0.89	0.81	0.80	0.79
$p^*_{a,00}$	0.79	0.79	0.79	0.78	0.79	0.78	0.71	0.69	0.68
$\mu^*_{b,0}$	-1.95	-1.95	-1.95	-1.95	-1.95	-1.95	-1.91	-1.90	-1.94
$\mu^*_{b,1}$	3.93	3.93	3.93	3.93	3.93	3.93	3.86	3.90	3.91
$p^*_{b,11}$	0.89	0.89	0.89	0.89	0.89	0.89	0.80	0.80	0.79
$p_{b,00}$	0.79	0.79	0.79	0.78	0.78	0.78	0.70	0.69	0.68
p^*_{11}	0.89	0.88	0.89	0.88	0.89	0.88	0.80	0.79	0.78
p^*_{00}	0.77	0.76	0.76	0.76	0.76	0.76	0.69	0.67	0.66
$\sigma^*_{a,0}$	0.26	0.26	0.26	0.26	0.26	0.26	0.34	0.30	0.27
$\sigma^*_{a,1}$	0.75	0.76	0.75	0.77	0.76	0.77	0.94	0.88	0.80
$\sigma^*_{b,0}$	1.01	1.01	1.01	1.01	1.02	1.02	1.08	1.04	1.02
$\sigma^*_{b,1}$	3.03	3.05	3.07	3.05	3.06	3.04	3.26	3.16	3.16
$p^*_{G,11}$	0.96	0.92	0.95	0.93	0.95	0.93	0.95	0.95	0.96
$p^*_{G,00}$	0.94	0.96	0.95	0.96	0.94	0.94	0.96	0.96	0.96
$p^*_{V,11}$	-	-	-	-	-	0.96	-	-	-
$p^*_{V,00}$	-	-	-	-	-	0.96	-	-	-

Note: The entries in the table report the average of the estimated parameter values through the 1,000 replications for different numbers of synchronization changes, z.

Table 3: Performance of regime inferences

	$z=1$	$z=2$	$z=3$	$z=4$	$z=5$	$z=f(V_t)$	$\delta=0.2$	$\delta=0.5$	$\delta=0.7$
$QPS(S_{a,t})$	0.02	0.02	0.02	0.02	0.02	0.02	0.13	0.12	0.13
$QPS(S_{b,t})$	0.01	0.02	0.01	0.02	0.01	0.02	0.13	0.12	0.13
$QPS(G_t)$	0.03	0.03	0.03	0.03	0.03	0.03	0.04	0.04	0.03
$QPS(V_t)$	0.02	0.04	0.04	0.05	0.06	0.08	0.06	0.08	0.07
$\bar{\delta}$	-	-	-	-	-	-	0.27	0.55	0.75

Note: The entries in the table report the average of the Quadratic Probability Score associated with the state variables through the 1,000 replications for different numbers of synchronization changes, z, and levels of imperfect synchronization δ. The term $\bar{\delta}$ makes reference to the average estimated synchronization over a regime of imperfect synchronization.

Table 4: Dynamic synchronization estimates between New York and Texas

	Mean	**Median**	**Std. Dev.**
$\mu_{ny,0}$	-0.10	-0.10	0.04
$\mu_{ny,1}$	0.38	0.38	0.03
$\sigma^2_{ny,0}$	0.02	0.02	0.02
$\sigma^2_{ny,1}$	0.09	0.08	0.02
$p_{ny,11}$	0.98	0.98	0.00
$p_{ny,00}$	0.94	0.94	0.02
$\mu_{tx,0}$	-0.05	-0.05	0.02
$\mu_{tx,1}$	0.43	0.43	0.02
$\sigma^2_{tx,0}$	0.01	0.01	0.00
$\sigma^2_{tx,1}$	0.08	0.08	0.01
$p_{tx,11}$	0.98	0.98	0.00
$p_{tx,00}$	0.93	0.94	0.02
$\sigma_{ny,tx,0}$	0.00	0.00	0.00
$\sigma_{ny,tx,1}$	0.03	0.03	0.02
p_{11}	0.98	0.98	0.00
p_{00}	0.93	0.93	0.02
$p_{G,11}$	0.95	0.97	0.03
$p_{G,00}$	0.95	0.97	0.04
$p_{V,11}$	0.94	0.94	0.02
$p_{V,00}$	0.97	0.98	0.00

Note: The selected example presents the case of two states with high and similar shares of U.S. GDP, New York with 7.68% and Texas with 7.95%.

Table 5: Dynamic synchronization estimates between California and Vermont

	Mean	**Median**	**Std. Dev.**
$\mu_{ny,0}$	0.03	0.03	0.02
$\mu_{ny,1}$	0.34	0.34	0.01
$\sigma^2_{ny,0}$	0.01	0.01	0.00
$\sigma^2_{ny,1}$	0.23	0.21	0.07
$p_{ny,11}$	0.97	0.98	0.00
$p_{ny,00}$	0.95	0.95	0.01
$\mu_{tx,0}$	0.00	0.00	0.03
$\mu_{tx,1}$	0.37	0.37	0.03
$\sigma^2_{tx,0}$	0.05	0.05	0.00
$\sigma^2_{tx,1}$	0.48	0.45	0.16
$p_{tx,11}$	0.97	0.97	0.00
$p_{tx,00}$	0.94	0.95	0.01
$\sigma_{ny,tx,0}$	0.00	0.00	0.00
$\sigma_{ny,tx,1}$	0.18	0.17	0.09
p_{11}	0.97	0.98	0.00
p_{00}	0.95	0.95	0.01
$p_{G,11}$	0.93	0.94	0.04
$p_{G,00}$	0.96	0.97	0.03
$p_{V,11}$	0.99	0.99	0.00
$p_{V,00}$	0.87	0.88	0.05

Note: The selected example presents the case of the states with the highest and the lowest shares of U.S. GDP, California with 13.34% and Vermont with 0.18%.

Table 6: Stationary synchronization between individual states and the entire United States

State	**Sync**	**State**	**Sync**	**State**	**Sync**
Alabama	0.72	Maine	0.71	Ohio	0.80
Arizona	0.59	Maryland	0.76	Oklahoma	0.17
Arkansas	0.79	Massachusetts	0.69	Oregon	0.78
California	0.74	Michigan	0.69	Pennsylvania	0.85
Colorado	0.75	Minnesota	0.81	Rhode Island	0.59
Connecticut	0.70	Mississippi	0.68	S. Carolina	0.80
Delaware	0.61	Missouri	0.73	S. Dakota	0.46
Florida	0.76	Montana	0.21	Tennessee	0.73
Georgia	0.74	Nebraska	0.50	Texas	0.42
Idaho	0.65	Nevada	0.52	Utah	0.64
Illinois	0.86	N. Hampshire	0.59	Vermont	0.67
Indiana	0.81	New Jersey	0.74	Virginia	0.81
Iowa	0.54	New Mexico	0.52	Washington	0.77
Kansas	0.72	New York	0.80	Wisconsin	0.74
Kentucky	0.75	N. Carolina	0.81	W. Virginia	0.69
Louisiana	0.18	N. Dakota	0.21	Wyoming	0.27

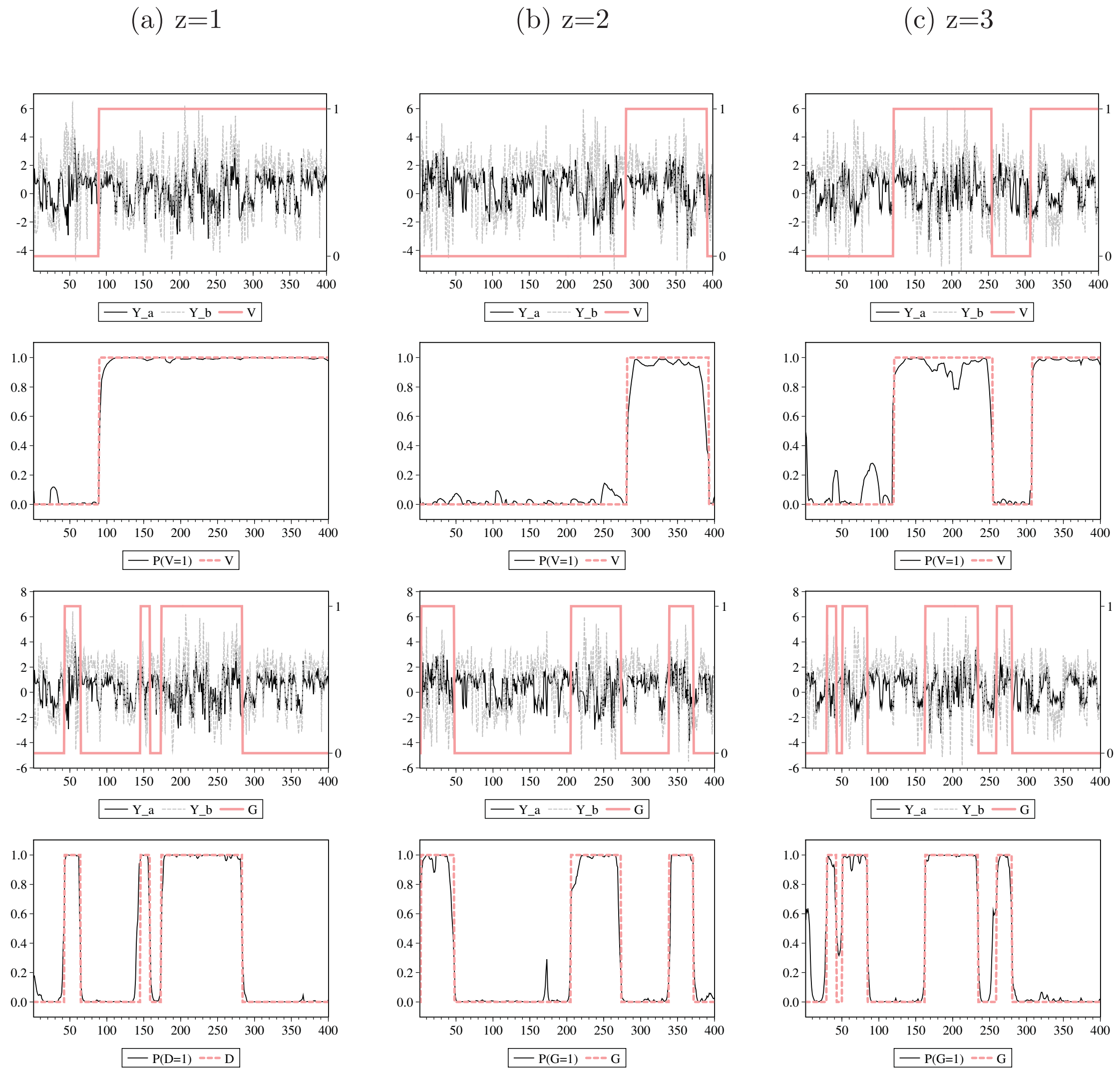
Note: The table reports the stationary synchronization for the period August 1979 to March 2013. These estimates correspond to the ergodic probability that the phases of the state business cycles and U.S. business cycles are the same, i.e., $\Pr(V_t = 1)$. The index used to measure the national business cycle is the Chicago Fed National Activity Index (CFNAI).

Table 7: Panel Regression

	$(1 - \delta_t^{ij})$
Sectorial Composition	1.223^{***}
	(18.97)
Real Median Household Income	0.151^{***}
	(3.94)
Total Deposits	0.00761
	(1.68)
Government Expenditure	0.0136
	(1.82)
Constant	0.363^{***}
	(20.68)
Observations	281640

Note: The table reports the estimates of β_t in Equation (65). t statistics are reported in parentheses. Asterisks are defined as, * for $p < 0.05$, ** for $p < 0.01$, *** for $p < 0.001$.

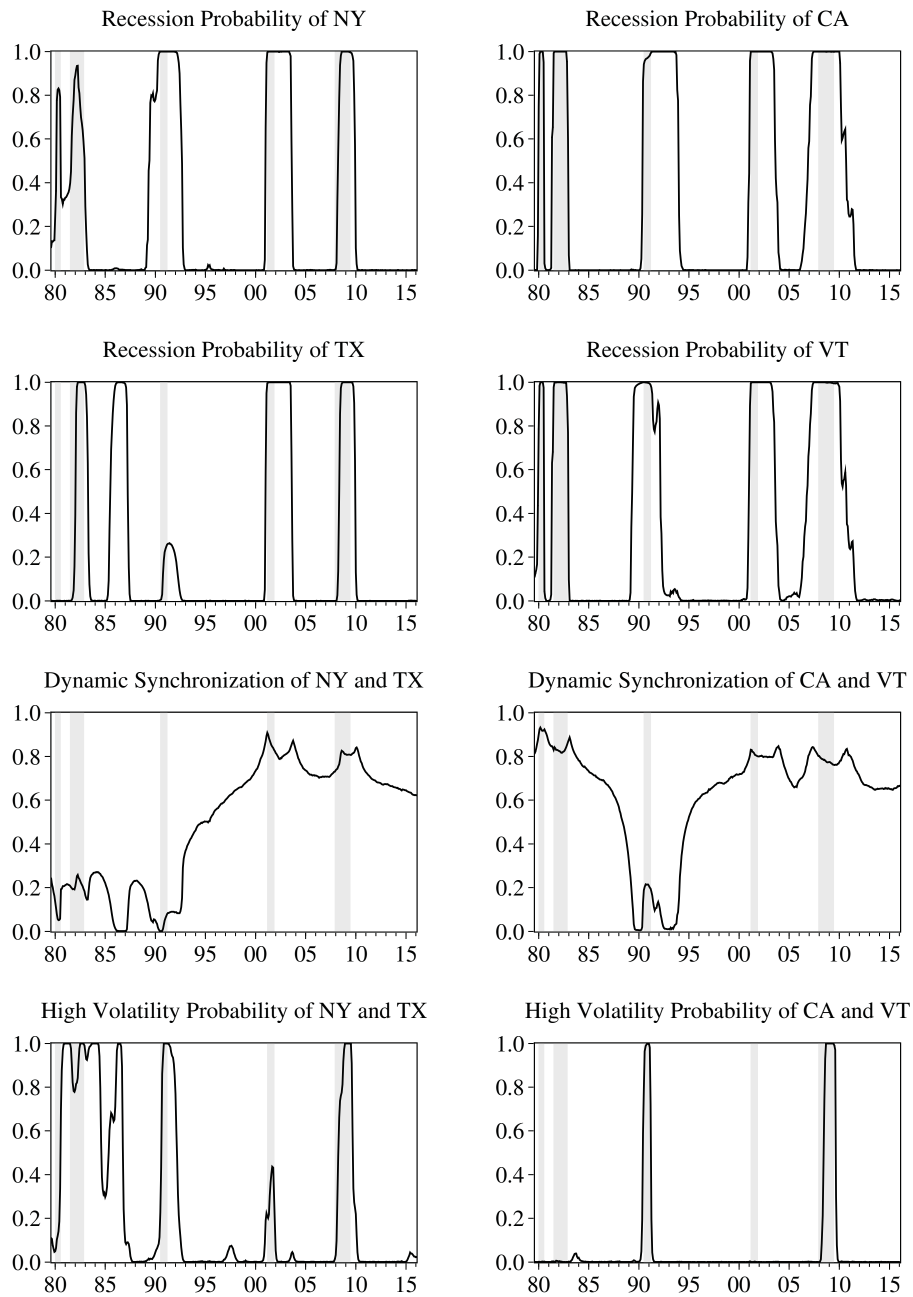
Figure 1: Simulation of changes in synchronization of cycles



Note: The figure plots one simulation for the cases of 1, 2 and 3 changes in the synchronicity of cycles. For each case, the top panels plot the generated pair of time series along with the indicator variable of synchronization changes. The two middle panels plot the probabilities of a low mean regime associated with each time series, along with the indicator variable as reference. The bottom panels plot the estimated dynamics of the indicator variable along with the real one.

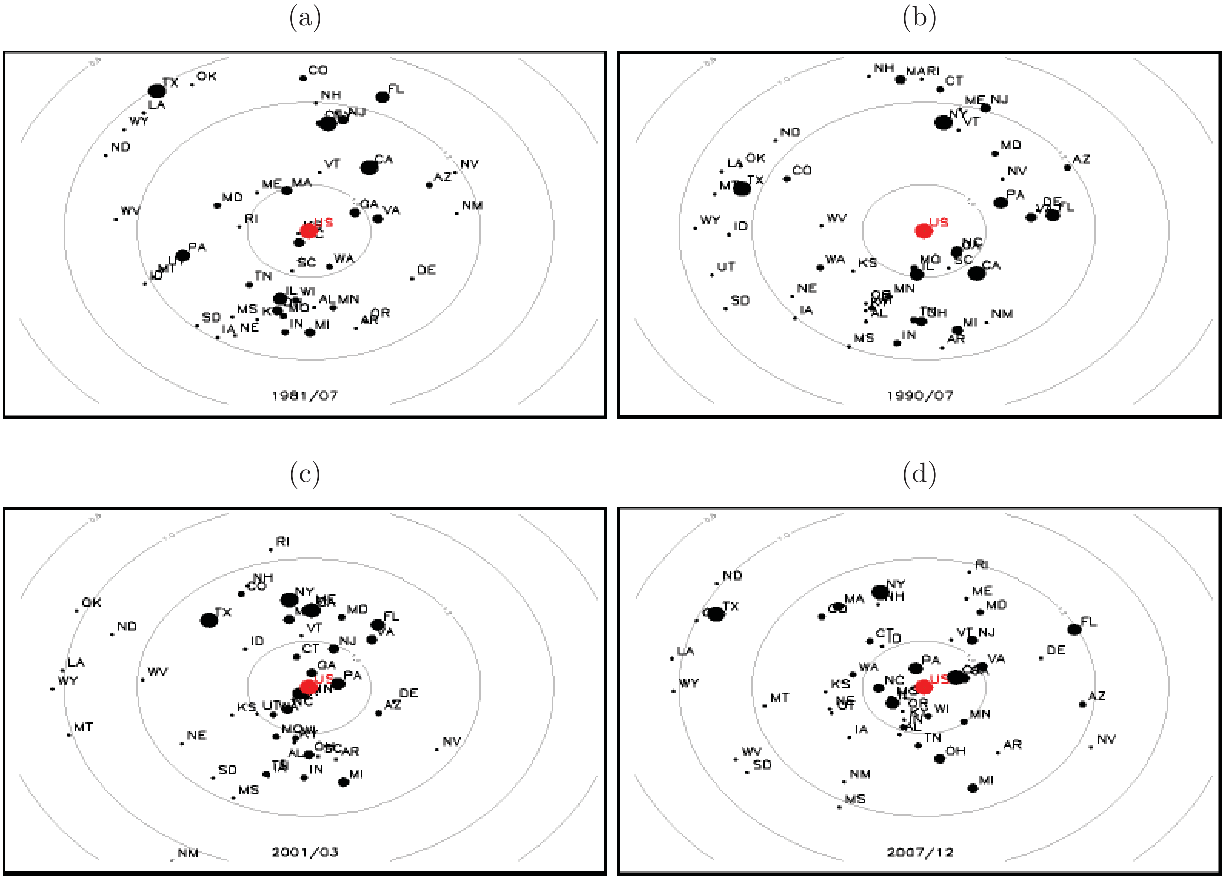
Figure 2: Dynamic synchronization between selected states

(a) New York and Texas (b) California and Vermont



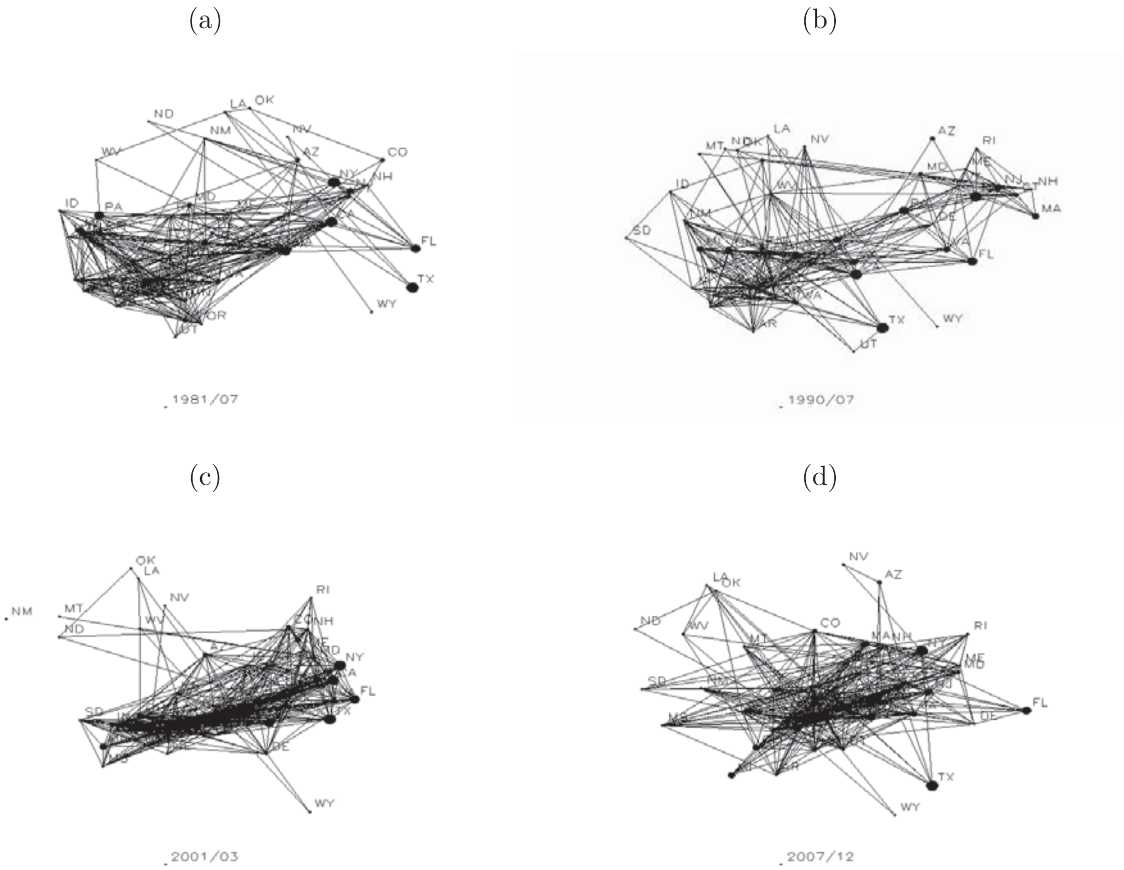
Note: The figure plots the output estimation for two selected pairwise models. Chart A plots the probability of recession for New York and Texas along with their dynamic synchronization. Chart B plots the probability of recession for California and Vermont along with their dynamic synchronization. Shaded areas correspond to NBER recessions.

Figure 3: Dynamic synchronization maps of U.S. states across recessions



Note: Each chart in the figure plots the dynamic multi-dimensional scaling map based on the synchronization distance of the business cycle of U.S. states for different periods. The distances are normalized with respect to the U.S. national economic activity, the grey point in the centre. The size of the points refer to the GDP share of the corresponding state. If two states are placed in the same concentric circle, they are equally in sync with the United States. The full animated version of the synchronization mapping is available at `https://sites.google.com/site/daniloleivaleon/media`.

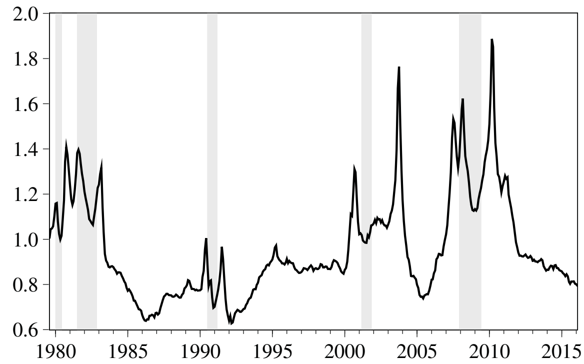
Figure 4: Synchronization network of the U.S. states across recessions



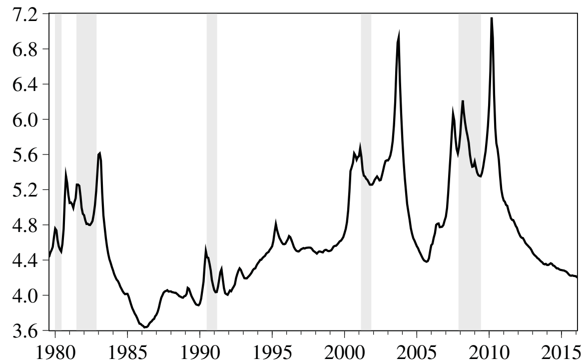
Note: The figure plots the interconnectedness in terms of synchronization between the business cycle phases of U.S. states. Each node represents a state and each line represents the link between two states, which takes place only if $\Pr(Vt = 1) > 0.5$. The full animated version can be found at `https://sites.google.com/site/daniloleivaleon/media`.

Figure 5: Dynamic closeness centrality of the U.S. synchronization network

(a) Closeness centrality

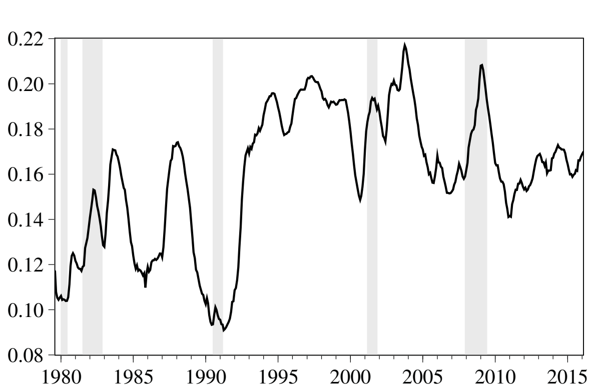


(b) Average closeness centrality



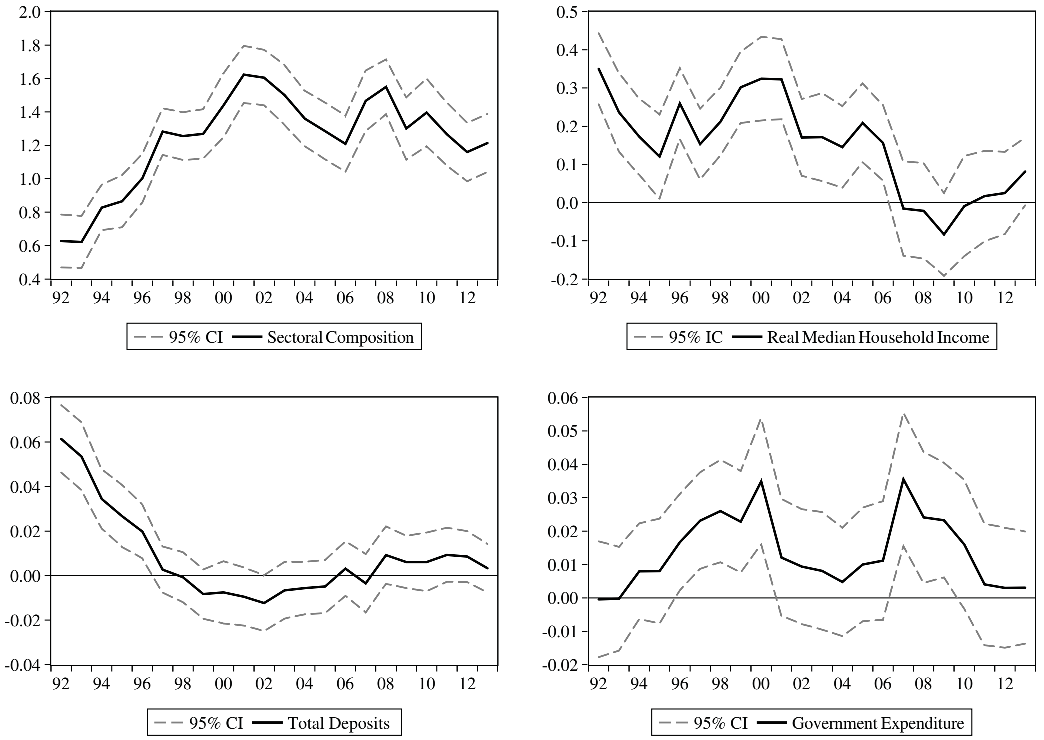
Note: Chart A and Chart B plot the closeness and average closeness centrality measures of the Markov-switching synchronization network, respectively. The solid line plots the network closeness centrality defined in Equation (61) and the dotted line plots the average centrality, as defined in Equation (62). Left axis of Chart B are in percentages units, and left axis of Chart A are in regular units. Shaded bars refer to the NBER recessions.

Figure 6: Dynamic clustering coefficient of the U.S. synchronization network



Note: The figure plots the time-varying clustering coefficient of the Markov-Switching Synchronization Network for U.S. states. Shaded bars refer to the NBER recessions.

Figure 7: Time-varying relationship between explanatory factors and synchronization



Note: The figure plots the estimated coefficients that measure the time-varying relationship between the synchronization and its potential explanatory factors. Dashed lines represents the 95% confidence interval.

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Alcalá, 48 - 28014 Madrid
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