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Abstract

This paper compares two state-of-the-art but very distinct methods used in macroeconomics: rational-expectations DSGE and bounded rationality behavioural models. Both models are extended to include financial frictions on the supply side. The result in both frameworks is that production, supply of credit and the front payment to capital producers depend heavily on stock market cycles. During phases of optimism, credit is abundant, access to production capital is easy, the cash-in-advance constraint is lax, risks are undervalued, and production booms. But with a reversal in market sentiment, the contraction in all these parameters is deep and sometimes asymmetric. This is all the more evident in the behavioural model, where economic agents’ cognitive limitations exacerbate the contraction. While both models capture the empirical regularities very well, the validation exercise is even more favourable to the behavioural model.

Keywords: supply side, beliefs, financial frictions, model validations.

Resumen

En este trabajo se comparan dos metodologías macroeconómicas punteras pero a la vez muy distintas: el modelo DSGE con expectativas racionales y el modelo de comportamiento con racionalidad limitada. Procedemos a ampliar los dos para incluir fricciones financieras por el lado de la oferta agregada. En ambos casos, los resultados apuntan a que la producción, la oferta de crédito y el pago anticipado a los productores de capital dependen en gran medida de los ciclos bursátiles. Durante las fases de optimismo, el crédito es abundante, hay facilidad de acceso al capital, existen pocas restricciones al pago por adelanto, los riesgos se infravaloran y se produce un boom en la producción. Sin embargo, cuando el clima de los mercados cambia, la contracción de todas estas variables se vuelve más profunda y asimétrica. Esto es aún más evidente en el modelo de comportamiento, en el que las limitaciones cognitivas de los agentes económicos generan una contracción aún mayor. Aunque ambos modelos captan correctamente las regularidades empíricas, los ejercicios de validación son incluso más favorables para el modelo de comportamiento.

Palabras clave: oferta agregada, creencias, fricciones financieras, validación de modelos.

1 Motivation

It has long been recognized that the (aggregate) supply-side and financial markets can be powerful generators and propagators of shocks. An increase (a drop) in the price of capital, commodities or cost of labour can cause a significant contraction (expansion) in output via the production or investment channel. Equally, a rise in the cost of credit or a shortage of (financial market) liquidity will in many cases cause a fall in output via a multitude of channels (consumption, investment, production, trade) depending on the segment of the market that is hit by the shock. Yet less efforts have so far been invested in understanding the interaction between financial markets and the aggregate supply side. In particular, it is less well understood in what ways and under what conditions the supply side can work as a propagator of shocks generated in the financial sector, or more generally of financial shocks. Nonetheless, a number of recent empirical studies suggest that this interaction is at the core of the contraction in GDP during the Great Recession. Despite the fact that the original negative shocks were generated in the financial sector, a sharp drop in aggregate supply is observed.

One of those studies is Broadbent (2012, 2013), who finds that the main reason for the most recent contraction in the UK economy has been a fall in underlying productivity growth.\(^1\) The observed contraction in output coupled with a relatively high level of employment and inflation in the Great Recession period up to 2013 is better explained by an independent hit to supply rather than a weak demand. He argues that a combination of uneven demand across sectors combined with an impaired financial system that is not capable of reallocating capital resources sufficiently quickly to respond to shocks has lead to a reduction in aggregate output per employee. Such a process would equally result in a rise in volatility in relative prices and the widening sectoral dispersion of profitability that is observed in the UK data. Moreover, while investment has risen in some sectors, on aggregate it has fallen meaning that the fall in some sectors has more-than-outpassed the rise in others. Likewise, Barnett et al (2014a) employ a highly stylized model of the economy to show that the increased price dispersion is a consequence of frictions to efficient capital allocation. Along similar lines, Barnett et al (2014b) believe that inefficient allocation of credit is the most plausible factor behind low productivity growth in the UK since the crisis.

\(^1\) The predicted productivity growth is equally expected to be below-par.
Chadha and Warren (2012) estimate a business cycle accounting (BCA) model to find that the main cause of the output variation during the Great Recession to be the variation in the efficiency wedge of production, rather than any of the other factors, such as the labour supply, investment or total expenditure wedge. Moreover, they run a BCA decomposition on a version of the BGG model that includes a dominant asset price shock and find that the shock does neither appear as a consumption nor investment wedge in the BCA analysis. Taken together, this implies that the asset price shock may show up in other wedges in the BCA framework, and that the shocks transmitted via the supply-side may be generated elsewhere. Hence, the role of asset prices for the wider economy must be considered more broadly, as their impacts in general equilibrium may be to shift labour supply, or even to shift the ratio of outputs to inputs.

At the same time, Manasse (2013) argues that the cause for the most recent recession in Italy is a weak and anemic supply side. A lack of reform in the product, labor and credit markets has resulted in weak (if not zero) innovation, competitiveness and productivity performance for more than a decade.\(^2\)

On the contrary, many financial friction models have concentrated on the demand-side effects from financial cycles.\(^3\) Many have investigated the impact of asset prices and/or risks on the demand for credit, investment, demand for mortgages, consumption, labour supply, demand for capital, etc. In (most of) these models, financial prices alter the value of collateral, perception of risks, probability of default, or future propensity to save, which alters the aggregate demand allocations.

However, in the current paper we wish to investigate the impact of financial swings on the supply side of the economy.\(^4\)

In particular, we are interested in examining how imperfect credit and stock markets affect the allocations on the production side of the economy, and specifically the effects it has on the supply of capital and credit, demand for labour and technology. In addition, we wish to understand whether this mechanism is more accurately cap-

\(^2\) This empirical tendency applies to several other Eurozone countries, such as Spain, Portugal, Greece, and to some extent France.


\(^4\) A related literature to this one is Justiniano et al (2010, 2011) who investigate the business cycle role of shocks to marginal efficiency of investments, or Altig et al (2011) who explore the importance of firm-specific capital for driving the business cycle. Lastly Smets and Wouters (2005, 2007) Gerali et al (2010) and Christiano et al (2010, 2013) also include a more elaborate supply side in their models, but do not specifically focus on the role of the supply-side as a propagator of financial shocks by altering the input-output ratio, labour productivity, or aggregate output per employee.
tured in a rational expectations framework, or whether a relaxing of this assumption is necessary. The alternative model we use to evaluate the importance of rational expectations hypothesis in terms of empirical validity of the mechanism is a behavioural model. In this framework, all the dynamics and transmission mechanisms are identical to the DSGE model, except for the premise on agent behaviour. Here, instead, agents have cognitive limitations and base their inferences on standardized heuristics. We perform model validations using impulse response analyses, statistical comparisons, moment matching and business cycle comparisons using more than 60 years of filtered macroeconomic and financial data.

We find that allowing financial shocks to be propagated via the supply side in an otherwise standard financial accelerator model intensifies the transmission of shocks by between 15 and 25%. Compared to a model where only the stock market mechanism is incorporated, the impulse responses to a financial shock, for instance, are on average 25% lower. Variance decomposition further affirms the importance of aggregate supply-financial market interaction since approximately 75% of the model variation can be explained by the financial and TFP shocks jointly. In addition, the model is capable of replicating many of the statistical moments of the US data, in particular for labor, investment, marginal costs, capital and inflation. On a deeper level, we contrast the quantitative results from a fully rational DSGE model to a bounded rationality behavioural one. The comparative analysis confirms that both models perform well in matching the data moments, as well as generating powerful propagation of shocks. The empirical fit is much better compared to competing models where those mechanisms are excluded. Nonetheless, to additionally relax the rational expectations hypothesis improves even further the empirical fit to data, and the asymmetric nature of many macroeconomic and financial variables. The trade-off, however, is that the supply side becomes (in relative terms to the DSGE model) a weaker propagator of financial (and monetary) shocks. To conclude, while the model construction in the behavioural version is very intuitive and agents’ behaviour micro-founded, the tractability of the model solution is to some extent compromised in comparison to the rational DSGE counterpart.
2 Model set-up

To incorporate a supply side with an asset price bubble and financing constraints in a general equilibrium framework, we apply the following (and equal) modifications in both models. The first modification is an extension of the financial accelerator mechanism onto input market. We allow a firm’s purchasing position on the input markets to directly depend on their financial state. A higher value of net worth means that the collateral constraint the producing firm faces is lower. As a result it can borrow more, which will press the marginal costs down, and therefore they will be able to buy capital inputs at a lower price.

The second modification is a pay-in-advance constraint on the input market. We impose the condition that (a share of) the cost of capital must be paid in advance of purchase in order to insure capital good producers that they will sell what they produce. It is a kind of depository insurance. Firms will finance it with a share of the (liquid) external financing that they get. Since this in turn depends on the cash position that they will hold in the next period, the expected (stock) market price will de facto reflect the price they have to pay in advance for the capital. A higher expected value of the (stock) market price improves the borrowing conditions of the firm today, meaning that she can already in the current period commit to pay more for the inputs. This will increase the quantity of outputs produced in the next period. Once they reach the next period, the (stock) market price will be realised, pushing their net worth up and therefore they will be able to repay their debt in full. We make the down payment time varying over the business cycle in order to capture the asymmetries in financial (or liquidity) positions over the cycle.

The third modification we introduce is a rate of utilization of capital. Producing firms, apart from choosing the amount of capital to purchase and use in the production, also choose the rate at which capital will be used in the production. The higher the rate, the more effective use is made of capital in the production function and the more (intermediate) products can be produced for the same amount of capital. However, increasing the capital utilization cost is also costly because it causes a faster rate of capital depreciation. Hence in this modified version of the model, entrepreneurs do not only choose the quantity of capital to be purchased from the capital good producers, but also the rate at which they will use this capital in production.

Keeping these modifications constant in both models, the difference between the two frameworks will however lie in agents’ information set and expectations formation. While agents use perfect information (or equally distributed imperfect
information, i.e. an ignorance factor which is equal in size throughout the entire population) to form rational expectations in the DSGE model, in the behavioural framework agents have limited cognitive abilities which forces them to form expectations using an incomplete information set (i.e. bounded rationality). As a result, they need to make intertemporal decisions using (imperfect) forecasts. Forecasts, in turn, are chosen using a historical performance record of alternative forecast errors. The model producing the smallest forecast error is chosen. There is therefore a strong history dependance in the forecasts, resulting in strong market-sentiment swings. We will outline the mechanism in further detail under section 3.

We proceed by incorporating these mechanisms in a DSGE model, followed by the behavioural. For the sake of tractability, in the second part we will only focus on the supply-side extensions that differ from the DSGE version.

2.1 DSGE model construction

In what follows, we will disentangle capital production from capital utilization rate, and introduce variable capital usage in an otherwise standard financial accelerator model (augmented with stock market cycles) as in Gerba (2014). The production side of the economy consists of three types of nonfinancial firms: capital good producers, entrepreneurs, and retailers. Let us describe their optimization problems.

2.1.1 Capital Good Producers

Following Gerali et al (2010), perfectly competitive capital good producers (CGP) produce a homogeneous good called 'capital services' using input of the final output from entrepreneurs $(1 - \delta)k_{t-1}$ and retailers $(i_t)$ and the production is subject to investment adjustment costs. They sell new capital to entrepreneurs at price $Q_t$. Given that households own the capital producers, the objective of a CGP is to choose $K_t$ and $I_t$ to solve:

$$\max_{K_t,I_t} E_0 \sum_{t=0}^{\infty} \lambda_{0,t} [Q_t [K_t - (1 - \delta)K_{t-1}] - I_t]$$

subject to:

$$K_t = (1 - \delta)K_{t-1} + \left[1 - \frac{\kappa}{2} \frac{I_t}{I_{t-1}} - 1\right]^2 I_t$$

For the remaining model set-up, we refer to aforementioned paper.
where \(1 - \frac{\kappa}{2}\left(\frac{\kappa}{\nu} - 1\right)^2\) is the adjustment cost function. \(\kappa\) denotes the cost for adjusting investment. Including adjustment costs of investment in the production of capital solves the so-called 'investment puzzle' and produces the hump-shaped investment in response to a monetary policy shock (Christiano et al, 2011).

2.1.2 Entrepreneurs

Perfectly competitive entrepreneurs produce intermediate goods using the constant returns to scale technology:

\[
Y_t = A_t[\psi(u_t)K_t]^{\alpha}L^{1-\alpha}
\]  \hspace{1cm} (3)

with \(A_t\) being stochastic total factor productivity, \(u_t\) the capacity utilization rate, and \(K_t\) and \(L_t\) capital and labor inputs. Capital is homogeneous in this model.\(^6\) We assume a fixed survival rate of entrepreneurs in each period \(\gamma_t\) in order to ensure a constant amount of exit and entry of firms in the model. This assumption also assures that firms will always depend on external finances for their capital purchases, and so will never become financially self-sufficient.

Just as in the canonical financial accelerator model (Bernanke, Gertler and Gilchrist, 1999) as well as in the extension (Gerba, 2014), we will continue to work under the framework that all earnings (after paying the input costs) from production are re-invested into the company such that a constant share is paid out to shareholders.\(^7\) This is why entrepreneurs will maximize their value function rather than their production function.\(^8\)

Entrepreneurs also choose the level of capacity utilization, \(\psi(u_t)\) (Kydland and Prescott (1988), Bills and Cho (1994)). As is standard in the capital utilization literature, the model assumes that using capital more intensively raises the rate at which it depreciates.\(^9\) The increasing, convex function \(\psi(u_t)k_t\) denotes the (relative) cost in units of investment good of setting the utilization rate to \(u_t\). This is chosen before the realization of the production shock (see Auernheimer and Trupkin (2014) for similar assumption). This timing assumption is important because it separates the

---

\(^6\) We could have made capital firm-specific, but the set-up would have to be much more complex without altering qualitatively the results. Using homogeneous capital assumption is standard in these type of models, see for instance Bernanke et al (1999), Gerali et al (2010), Gertler et al (2012).

\(^7\) In our exercises, we will set this share to 0, just as in Bernanke et al (1999).

\(^8\) And so \(y_t\) is not a direct argument of the function.

\(^9\) We could equally assume a fixed rate of capital depreciation and impose a cost in terms of output of using capital more intensively, as in Christiano et al (2005) or Gerali et al (2010).
choice of the stock of productive factor $K_t$, taken before the revelation of the states of nature, from the choice of the flow of factor $u_t K_t$, taken during the production process.

The choice of the rate of capital utilization involves the following trade-off. On the one hand, a higher $u_t$ implies a higher output. On the other hand, there is a cost from a higher depreciation of the capital stock. Therefore this rate can be understood as an index that shows how much of the stock of capital is operated relative to the steady state, per unit of time, given a capital-labor services ratio.

Moreover we specify the following functional form for $\psi(u_t)$:

$$
\psi(u_t) = \xi_0 + \xi_1(u_t - 1) + \frac{\xi_2}{2}(u_t - 1)^2
$$

(4)

in line with Schmitt-Grohe and Uribe (2006), Gerali et al (2010), and Auernheimer and Truphin (2014). As a result, an entrepreneur will now maximize its value (profit) function according to:

$$
V = \max E_0 \sum_{k=0}^{\infty} [(1 - \mu) \int_0^{\infty} \omega dF_\omega U_{t+1}^{rk} E_t(R_{t+1}^{ks}) S_t \psi(u_t) K_{t+1} - R_{t+1}[S_t K_{t+1} - N_{t+1}]]
$$

(5)

with $\mu$ representing the proportion of the realized gross payoff to entrepreneurs’ capital going to monitoring, $\omega$ is an idiosyncratic disturbance to entrepreneurs’ return (and $\omega$ is hence the threshold value of the shock), $E_t R_{t+1}^{ks}$ is the expected stochastic return to stocks, and $U_{t+1}^{rk}$ is the ratio of the realized returns to stocks to the expected return ($\equiv R_{t+1}^{ks} / E_t(R_{t+1}^{ks})$).

To understand how a firm’s financial position influences its’ purchasing power in the capital input market, we need to understand the costs it faces. A firm minimizes the following cost function:

$$
S(Y) = \min_{k,l} [R_{t+1}^s K_t + w_l L_t]
$$

(6)

The real marginal cost is therefore $s(Y) = \frac{\partial S(Y)}{\partial (Y)}$, which is:

$$
s(Y) = \frac{1}{1 - \alpha} \frac{1}{\alpha} \left( r_{t+1}^s \right)^{\alpha} (w_t)^{1-\alpha}
$$

(7)
The return on capital is defined as \( R_{t+1}^* = \frac{E_0(S_{t+1}^*) - S_t}{S_t} \).\(^{10}\) Keeping the wage rate constant, an increase in the expected (stock) market value of capital reduces the (relative) cost of capital service inputs, purchased at today’s capital price.\(^{11}\) This is easier to see in the entrepreneur’s budget constraint.\(^{12}\)

\[
\vartheta E_t[S_{t+1}]K_{t+1} + \omega_t L_t + \psi(u_t)k_{t-1} + R_tB_{t-1} + (1 - \vartheta)S_tK_t = \frac{Y_t}{X_t} + B_t + S_t(1 - \delta)K_{t-1} \Rightarrow
\]

\[
\vartheta E_t[S_{t+1}]K_{t+1} + \omega_t L_t + \psi(u_t)K_t + R_t[S_tK_t - N_t] + (1 - \vartheta)S_tK_t = 
\frac{Y_t}{X_t} + [E_t[S_{t+1}]K_{t+1} - N_{t+1}] + E_t[S_t](1 - \delta)K_{t-1} \quad (8)
\]

with \( \delta \) being the depreciation rate of capital, \( \psi(u_t)K_{t-1} \) the cost of setting a level \( u_t \) of the utilization rate, \( \vartheta \) is the front payment share to CGP, and \( \frac{P_t}{R_t} = \frac{1}{X_t} \) is the relative competitive price of the wholesale good in relation to the retail good.\(^{13}\) An increase in the expected market price (right-hand side) has two effects. First, it reduces the relative cost of capital purchases today since firms can borrow more and pay a higher pre-payment share \( \vartheta \) of capital. Second, a higher market price means that the probability of default of an entrepreneur reduces (since the value of the firm is higher) and so CGP will expect entrepreneurs to be solvent in the next period and will therefore require a smaller front payment (i.e. \( \vartheta \) on the left-hand side will fall). Let us explain the second mechanism in further detail.

As a form of depository insurance, CGP will (in some periods) require entrepreneurs to pay in period ‘t’ a share of the total capital produced and delivered to entrepreneurs in period ‘t+1’. In particular, when CGP suspect that entrepreneurs

\(^{10}\)Following on from Gerba (2014) and disentangling Tobin’s Q, we define the (stock) market value of capital \( S_t \) as the total value of the firm, including intangibles, meanwhile the book value \( Q_t \) is the accounting value of the firm that includes tangibles only. The difference between the two is the residual earnings \( RE_t \), which varies positively with (expected) firm performance and economic prospects. Market value of capital determines the level of firm (physical) investments. As a result, periods of high price-to-book ratios and positive economic outlook will drive investment up by significantly more than in standard DSGE models. In return, when corporate and economic outlook worsen, investor confidence on the stock markets will fall, driving down the market value of capital, and therefore also (physical) investment. For a more detailed background on capital prices and a discussion of macroeconomic implications of this stock market mechanism, refer to Gerba (2014).

\(^{11}\)In line with the costs that intermediate firms face in the model of Christiano et al (2005).

\(^{12}\)We assume that entrepreneurs borrow up to a maximum permitted by the borrowing constraint.

\(^{13}\)Note that \( \vartheta E_t[S_{t+1}]K_{t+1} < E_t[S_{t+1}]K_{t+1} \).
will face liquidity problems in the next period, a lower production, or a lower collateral value in the next period, they expect the firm to be less solvent (in relative terms). Because the default probability of entrepreneurs rises, CGP become suspicious of the entrepreneur’s ability to pay for the entire capital purchased. Therefore, as an insurance mechanism, CGP will ask the entrepreneur to pay in advance a share of its capital production.\footnote{We could equivalently assume that legal conditions/constraints stipulate that entrepreneurs need to pay in advance for their inputs as in Champ and Freedman (1990, 1994). Our approach is analogue to the one taken in Fuerst (1995) or Christiano and Eichenbaum (1992) for labor input costs.}

If the entrepreneur’s value is expected to increase in the next period, the financing constraint it faces will loosen, and thus it can borrow more. Since it can borrow more, it has more money to purchase the inputs (i.e. the marginal cost of a unit of capital decreases, \textit{ceteris paribus}) and therefore produce more outputs. This will push the price of capital in the future up. The CGP anticipating this, will require a smaller share of capital production to be pre-paid. On the other end, if the value of the firm is expected to fall, on the other hand, then the cost of financing will increase and the firm will be able to borrow less. Because it can borrow less, it has less money to purchase inputs, and this will push the price of capital down in the future. In anticipation of this, CGP will require a higher front payment. Hence, we expect the share $\vartheta$ to vary over the business cycle. Formally, the pay-in-advance constraint that entrepreneurs face in the input market is:

$$E_t[S_{t+1}]K_{t+1} \leq \vartheta_tB_t \equiv \vartheta_t \left[ \frac{E_t[S_{t+1}]K_{t+1}}{N_t} \right]$$

\hspace{1cm} (9)

So the down payment share of capital purchases will depend on the entrepreneur’s financial position $B_t$. We can equivalently express it in terms of the additional external funds that the entrepreneur needs for its capital purchases (right-hand side in the above expression) using the fact that an entrepreneur will borrow up to a maximum and use it to purchase capital.\footnote{See Bernanke et al (1999) and Gerba (2014) for a more profound discussion of the entrepreneur’s capital demand behaviour.} We allow $\vartheta$ to vary over time in order to capture the variations in CGP’s pre-cautionary motive over the business cycle. A value of 1 means that the entrepreneur will need to use all of his external finances (loan) to pay for the capital purchases since CGP expects its financial (cash) position to worsen in the next period. Equivalently, a value of 0 means that no pre-payment is required as CGP expects the entrepreneur to be able to pay in full for its purchases in the next period. As a result, the constraint will not be binding.
Both the individual and aggregate capital stock evolves according to:

\[ K_t = (1 - \delta \psi(u_t))K_{t-1} + \Psi\left(\frac{I_t}{K_t}\right)K_{t-1} \]  

(10)

where \( \Psi\left(\frac{I_t}{K_t}\right)K_{t-1} \) are the capital adjustment costs in the usage of capital. \( \Psi(.) \) is increasing and convex, and \( \Psi(0) = 0 \). The term \( \delta \psi(u_t) \) follows Burnside and Eichenbaum (1996) and represents the endogenous capital depreciation rate, which is important for the propagation of productivity shocks (see Greenwood et al (2000), or Albonico et al (2014)).16

The remaining equations for entrepreneurs are as in Gerba (2014).

2.1.3 Retailers

To incorporate nominal rigidities, a standard feature of New-Keynesian models, we incorporate a retail sector into this model. Let us look at retailers’ problem. The prices are sticky (Calvo, 1983) and indexed to a combination of past and steady-state inflation, with relative weights parametrized by \( l_p \). If retailers want to change their price beyond what indexation allows, they face a quadratic adjustment cost, governed by \( \kappa_p \). Retailers choose \( P_t(j) \) so as to maximize:

\[
\max_{P_t(j)} \Omega_R = E_0 \sum_{k=0}^{\infty} \theta^k \Lambda_{t,k}[P_t(j)yt(j) - P_t^*yt(j)] - \frac{\kappa_p}{2} \left( \frac{P_t(j)}{P_{t-1}(j)} - \frac{l_p}{1 - l_p}\right)^2 P_t yt 
\]

(11)

with \( \theta^k \) being the probability that a retailer does not change his price in a given period, \( \Lambda_{t,k} \equiv \beta \frac{C_{t+1}}{C_t} \) denoting the household intertemporal marginal rate of substitution (since households are the shareholders of the retail firms), which they take as given (\( X_t \) is the gross markup of retail goods over wholesale goods). They face a demand curve equal to:

\[
y_t(j) = \left[ \frac{P_t(j)}{P_t} \right]^{1-\epsilon^p_y} yt 
\]

(12)

where \( \epsilon^p_y \) is the stochastic demand price elasticity.17 Lastly, profits from retail activity are rebated lump-sum to households.

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16 The log-linearized version of this expression is: \( k_t = (1 - \delta \psi(u_t))k_{t-1} + \delta_i t \), as in Bernanke, Gertler and Gilchrist (1999) or Gerba (2014) and the one used in the simulations. \( \delta_i \) is the steady state version of \( \Psi\left(\frac{I_t}{K_t}\right)K_{t-1} \).

17 It would be dynamically equivalent to have entrepreneurs operate in monopolistically competitive markets without altering our results. However, the derivation of the optimal financial contracts and the aggregation would be more cumbersome since in that case the demand for capital by individual firms is no longer in net worth. Therefore we opt for separating the two.
2.2 DSGE model derivations

2.2.1 Optimizations

We begin by describing the optimization problems of the three agents in the DSGE model economy. The household-, financial-, and the government sectors are equivalent to the model in Gerba (2014). We therefore refer to the paper for a detailed derivation of their optimization problems.

Capital Good Producers

The capital good producer maximizes its expected present value stream of profits expressed in equation I.15. Since the owners of CGP are households, the pricing kernel is \( \beta \lambda_t \), with \( \lambda_t \) denoting the Lagrange multiplier in the representative household’s optimization problem. The FOC for CGP is therefore:

\[
Q_k^t - \beta E_t(\frac{\lambda_{t+1}}{\lambda_t}) (1 - \delta) Q_{t+1}^k = \lambda_{t}^{cpp} - \beta E_t(\frac{\lambda_{t+1}}{\lambda_t})(1 - \delta) Q_{t+1}^{cpp} \tag{13}
\]

The condition has to hold in each period, which allows us to split it into two equations. The real price of capital is just the Lagrange multiplier (see Christiano, Eichenbaum and Evans (2005)). Hence, we get:

\[
Q_k^t = \lambda_t^{cpp} \Rightarrow E_t[Q_{t+1}^k] = E_t[\lambda_{t+1}^{cpp}] \tag{14}
\]

\[
\lambda_t^{cpp}[1 - \frac{\kappa}{2} i_t - 1] - \kappa \frac{i_t - i_{t-1}}{i_t} + \beta E_t(\frac{\lambda_{t+1}}{\lambda_t}) \lambda_{t}^{cpp} \kappa \frac{i_{t+1}}{i_t} - 1)[\frac{i_{t+1}}{i_t}]^2 = 1 \tag{15}
\]

Substituting \( Q_k^t = \lambda_t^{cpp} \) into the second FOC, we get that the real price of capital is determined by:

\[
Q_k^t[1 - \frac{\kappa}{2} \frac{i_t}{i_{t-1}} - 1 - \kappa \frac{i_t - i_{t-1}}{i_t}] + \beta E_t(\frac{\lambda_{t+1}}{\lambda_t}) \lambda_{t}^{cpp} \kappa \frac{i_{t+1}}{i_t} - 1)[\frac{i_{t+1}}{i_t}]^2 = 1 \tag{16}
\]

In the steady state, the price of capital will be equal to 1. Lastly we have the constraint I.16 as our last FOC. In the steady state, investment will be equal to: \( \delta^k K \).
Lastly, the capital production inclusive of investment adjustment costs is\(^\text{18}\):

\[
K_t = (1 - \delta)K_{t-1} + \left[1 - \frac{\kappa_t}{2}\left(\frac{i_t}{\kappa_t} - 1\right)^2\right]I_t
\]

\[(17)\]

**Entrepreneurs**

Entrepreneurs maximize their value in equation I.17 subject to the production technology I.18, the pay-in-advance constraint I.23, and the borrowing constraint:\(^\text{19}\)

\[
B_{t+1} \leq \psi(s_t)N_{t+1}
\]

\[(18)\]

\(\psi(s_t)\) is the cost of borrowing, or the external finance premium. Remember that \(\psi'(\cdot) > 0\) and \(\psi(1) = 1\). From Bernanke et al. (1998) and Gerba (2014) we know that entrepreneurs will borrow up to a maximum so that this constraint will bind. By model construction it can easily be shown that in equilibrium, \(D_t = B_t\), so that household deposits in intermediaries equal total loanable funds supplied to entrepreneurs.

The utilization cost function in equation I.18 is described by 4, and the capital accumulates according to 10. Lastly, the entrepreneur minimizes the cost function specified in 6. Entrepreneurs FOC with respect to labor demand \(L_t\), physical capital \(K_t\), and the degree of capacity utilization \(u_t\) are therefore:\(^\text{20}\)

\[
(1 - \alpha)\frac{y_t}{\ell_t} = x_tw_t
\]

\[(19)\]

\[
\lambda_t q_t^k = E_t[S_{t+1}]K_t - N_{t+1} + \lambda_{t+1}E_t[\alpha a_{t+1}[k_t u_{t+1}]^{\alpha-1} l_{t+1}^{1-\alpha}] u_{t+1} + (1 - \delta) q_{t+1}^k - \psi(u_{t+1})
\]

\[(20)\]

\[
\alpha a_t[k_{t-1} u_t]^{\alpha-1} l_t^{1-\alpha} k_{t-1} - [\xi_1 + \xi_2(u_t - 1)] k_{t-1} = 0 \Rightarrow
\]

\[
\xi_1 + \xi_2(u_t - 1) = \alpha a_t[k_{t-1} u_t]^{\alpha-1} l_t^{1-\alpha} \equiv r_t^k
\]

\[(21)\]

\(^{18}\)Note that around the aggregate steady state, the part of capital that will matter is the one purchased and processed by entrepreneurs in the intermediate sector. This means that, on aggregate, the investment adjustment costs will be parametrized to a very small number and therefore can be neglected in the full system. Only the depreciation and utilization rates of capital will matter on aggregate.

\(^{19}\)The derivation of this constraint is the same as in Bernanke et al (1999). We therefore refer to the paper for more details.

\(^{20}\)The derivation to the FOC for borrowing is slightly more complicated as it includes idiosyncratic and aggregate risk. That is the same as in Appendix A of Bernanke et al (1999) and we therefore refer to the paper appendix for further details.
In our calibrations, we will normalize the $\xi$’s, which implies that $r_t^k = u_t$.

$$\lambda_t = \gamma_t(1 + r_t) + E_t[\lambda_{t+1}](1 + r_t)$$

(22)

In the steady state, this will reduce to $\frac{\gamma}{\lambda}(1 + r) = 1$ which means that we can pin down analytically the stochastic return to be $r = \frac{\lambda}{\gamma} - 1$. In our simulations, the interest rate will be set to the 3-year (pre-crisis) average of 2.5%.

Finally, we have the two additional constraints in this model: the cash-in-advance and the utilization costs. The cash in advance can be re-written from 1.23 as:

$$\theta_t \geq \frac{1}{1 - \frac{N_t}{E_t[S_{t+1}]/K_{t+1}}}$$

(23)

, and the utilization cost function has again the following functional form:

$$\psi(u_t) = \xi_0 + \xi_1(u_t - 1) + \frac{\xi_2}{2} (u_t - 1)^2$$

(24)

**Final Good Producers**

Continuing our analysis with the retailers, they maximize their profits:

$$\max_{P_t(j)} \Omega_R = E_0 \sum_{k=0}^{\infty} \theta^k \Lambda_{t,k}[P_t(j)y_t(j) - P^*_t y_t(j)] - \frac{\kappa_p}{2} \left[ \frac{P_t(j)}{P_{t-1}(j)} - \pi_{t-1}' \pi_{1-t'} \right]^2 P_t y_t]$$

(25)

subject to:

$$y_t(j) = \left[ \frac{P_t(j)}{P_t} \right]^{\ell_t^*} y_t$$

(26)

following Gali (2008) on page 5 and Kwok Ping Tsang (2008) on page 6, we can substitute the constraint in the objective function. The FOC with respect to price is:

$$\theta^k\lambda_t^k[(1 - \epsilon^y_t)(P_t(j)^{-\ell_t^y} P_t^{\ell_t^y}) y_t - (-\epsilon^y_t) P_t^* (P_t(j)^{-\ell_t^y} P_t^{\ell_t^y}) y_t - \kappa_p \frac{P_t(j)}{P_{t-1}(j)} - \pi_{t-1}' \pi_{1-t'}]\frac{1}{P_{t-1}(j)} P_t y_t - \beta^{t+1} E_t\left[ \frac{P_t(j)}{P_{t-1}(j)} - \pi_{t-1}' \pi_{1-t'} \right] - \left[ \frac{P_{t+1}(j)}{P_t^2(j)} P_{t+1} y_{t+1} \right] = 0$$

(27)

In a symmetric equilibrium, we have that all $j$’s are equal, and so we can reduce the FOC to:

$$(1 - \epsilon^y_t) y_t + \epsilon^y_t P_t^* P_t^{\ell_t^y} y_t - \kappa_p \left[ \frac{P_t}{P_{t-1}} - \pi_{t-1}' \pi_{1-t'} \right] \frac{P_t}{P_{t-1}} y_t$$

$$+ \beta^{t+1} E_t\left[ \frac{\lambda_t^{\ell_t^y+1}}{\lambda_t} \kappa_p \left[ \frac{P_t}{P_{t-1}} - \pi_{t-1}' \pi_{1-t'} \right] - \left[ \frac{P_{t+1}^2(j)}{P_t^2(j)} P_{t+1} y_{t+1} \right] = 0$$

(28)
2.2.2 Aggregation

We aggregate amongst capital good producers, entrepreneurs, and retailers. Because capital good producers and entrepreneurs are homogeneous within group, we can easily aggregate in a symmetric equilibrium to a representative capital good producer, and a representative entrepreneur. Apart from the timing of price setting, retailers are also homogeneous, and therefore in a symmetric equilibrium, all 'j's' will be equal. The aggregate price evolution is expressed above, and the total profits of the sector are:

\[ j_t^R = y_t \left[ 1 - \frac{1}{x_t} \right] \]

We could have introduced a cost for differentiating goods in the retailer’s objective function. However, around steady state this cost would disappear from the the optimal price and the profit function, and so would be equivalent to this expression.

2.2.3 Market clearing

The (de-centralised) market clearing conditions in this model are:

The market for capital services clears when the demand for capital by entrepreneurs equals the supply by capital good producers:

\[ s_t k_{t+1} = q_t [k_t - (1 - \delta) k_{t-1}]; \quad \frac{s_t}{q_t} > 0 \]
The market for labor clears if entrepreneurs’ demand for labor equals labor supply at the wage level set by households:

\[ \frac{y_t}{l_t c_t} x_t = \frac{1}{\eta} I_t \]  

(33)

The left-hand side is the marginal product of labor weighted by the marginal utility of consumption. In equilibrium, it varies proportionally with the retail-over-wholesale good markup.

The market for deposits clears:

\[ d_t = b_t \]  

(34)

The market for external financing clears:

\[ b_t \frac{E_t [v_{t+1}^s]}{R_{t+1}} = s \frac{N_{t+1}}{S_{t+1} K_{t+1}} \]  

(35)

The total amount of financing supplied to entrepreneurs is equal to the quantity of demand deposits multiplied by a risk premium on those deposits (reflecting the financial contracting problem involved in the financial accelerator model), and this is equal to the demand for capital by entrepreneurs taking into account the collateral constraint they face.

Aggregating across goods markets in this economy, we get that the final resource constraint is:

\[ y_t = c_t + g_t + s_t [k_t - (1 - \delta) k_{t-1}] + \psi(u_t) k_{t-1} + Adj_t + \mu \int_0^\infty \omega dF \omega R^s_{t-1} S_{t-1} K_t \]  

(36)

where \( Adj_t \) are the total adjustment costs in production, i.e. \( [1 - \frac{\kappa}{2} \frac{k_t}{k_{t-1}} - 1]^2 \) \( i_t \) and \( \psi(u_t) k_{t-1} \) is the usable capital in the production of goods. The last term, \( \mu \int_0^\infty \omega dF \omega R^s_{t-1} S_{t-1} K_t \) reflects the aggregate monitoring costs in the financial contracting problem between entrepreneurs and financial intermediaries.
2.2.4 Log-Linearization and Model Solution

The model is log-linearized around a (non-stochastic) steady state. This means that we cannot capture nonlinearities, such as precautionary savings, buffer-stock behaviours, or state-dependent outcomes. We apply a linear approximation method to our solution which means that our perturbation only works around the steady state. The system of log-linearized equations is provided in the appendix.

2.3 Calibrations and simulations

Table II.1 reports the full list of calibrated parameters. Most of these are calibrated following the values given in BGG (1999), and are standard to the literature. There are only a few minor differences. Our consumption-output ratio in the steady state includes both the private and public consumption, hence why the value is slightly larger in our calibration.\(^\text{21}\) We calibrate the share of capital in production, \(\alpha\) to 0.20. For robustness purposes, we also tried with \(\alpha = 0.30, \alpha = 0.35\), the other common values in the literature, but no noticeable differences were observed. Finally, in order to replicate the stylized facts of the asset price wedge (including the market and book values) of Gerba (2014), we parameterize \(\nu\), the elasticity of EFP to leverage to 0.13. It is slightly higher than the 0.05 in the original BGG model, but follows the estimation results for the US of Caglar (2012), and it represents well the post-2000 period, when the leverage of firms increased drastically, and so the sensitivity of financial lending rates to leverage was high.\(^\text{22}\) In the same wave, we consider an accommodative monetary policy, replicating thus the Fed’s stance during most of the past decade, and use the Taylor rule parameters of 0.2 for the feedback coefficient on expected inflation, \(\zeta\) along with a value of 0.95 for the smoothing parameter.

Borrowing from the insights in the corporate finance literature, and the US estimation results for the residual earnings process of Caglar (2012), we set the value of the autoregressive process of residual earnings equal to 0.67. Lastly, the weight on expected evolution of the economy is 0.18.

The parameters specific to this model are set to standard values in the literature. The share of capital in the production \(\alpha\) is set to 0.30 as in Boissay et al (2013). Following Christiano et al (2005), Smets and Wouters (2003, 2007) and Gerali et al

\(^{21}\) In the canonical BGG (1999) model, the \(C/Y\) ratio is calibrated to 0.568. However, if we also include the public consumption in that ratio, which they calibrate to 0.2, the value is almost the same to our, which we calibrate to 0.806.
\(^{22}\) See Gerba (2015) on the balance sheet changes and the financial exposure that firms underwent during the past decade.
(2010), we set the capital depreciation rate $\delta$ to 0.025. The elasticity of the capital utilization adjustment cost function $\psi(i_t)$ is parametrized to 0.5 as in Smets and Wouters (2007).\\(^{23}\)

To conclude, the parameters of the function determining adjustment costs for capacity utilization $(\xi_0, \xi_1, \xi_2)$ are set to $(0.8, 0.3, 0.25)$ in order to capture the estimation results of Smets and Wouters (2005) who find that the capital utilization adjustment costs are between 0.14 and 0.38 (Euro Area 1983-2002) and 0.21 and 0.42 (US 1983-2002), with a mean of 0.25 (Euro Area) and 0.31 (US). If we normalize $u_t$ to 1 (as in Christiano et al (2005), Miao et al (2013) or Auernheimer and Trupkin (2014)), then the cost for utilizing capital will be 0.20 $(1 - \xi_0)$, which is well within the estimated intervals of Smets and Wouters (2005).

The standard error of all shocks is, for reasons of comparability with the behavioural model, set to 0.5. The autoregressive components of the various shocks are set to standard values in the literature. For the monetary policy shock, it is set to 0.90 and for technology shock to 0.99. For the financial shock, and the shock to utilization costs, we set the AR-component to 0 and only consider a 1-period white noise shock. This is because we do not find convincing evidence in the literature for incorporating a persistence parameter in these shocks.

3 Animal Spirits and Credit Cycles on the Supply Side

The next task is to incorporate the same supply-side mechanisms and financial frictions in the behavioural model. The only difference will be that the rational expectations hypothesis is highly violated in the behavioural model.\\(^{24}\)

---

\(^{23}\)This is equivalent to setting a $\kappa$, equal to the estimated range $(10.18 - 12.81)$ as in Gerali et al (2010).

\(^{24}\)Notice that, just as in De Grauwe and Macchiarelli (2015), the share price is derived from the stable growth Gordon discounted dividend model: $S_t = \frac{E_t[\Lambda_{t+1}]}{\delta}$ where $\Lambda_{t+1}$ are expected future dividends net of the discount rate, $R_t$. Agents in this set-up assume that the 1-period ahead forecast of dividends is a fraction $f$ of the nominal GDP one period ahead, and constant thereafter in $t+1$, $t+2$, etc. Since nominal GDP consists of a real and inflation component, agents make forecast of future output gap and inflation according to the specification in subsection 3.3. This forecast is reevaluated in each period. As a result, in order to get the expected (stock) market price, the expected output gap and inflation needs to be defined. That is the reason why stock markets depend on (imperfect) forecasts and thus lead easily to market sentiments. Hence at a deeper level, we are evaluating a fully rational expectations-consistent stock market to a market which is governed by imperfect information and market sentiment.
cognitive limitation of the agents by the fact that understanding complex economic systems requires agents to acquire a large amount of data and cognitive ability. This is very costly so it dis-incentivizes a full acquisition of these resources at each $t$. Therefore agents optimize using limited information regarding the variables in the model that they don’t directly control (or markets they do not directly engage in). This will enable us to compare the relative importance of agents’ beliefs in generating aggregate (non-linear) dynamics, both qualitatively and quantitatively. In what follows, we will only expose the parts that are different from the DSGE extensions above.\footnote{One of the core questions will of course be whether it is necessary to accede to strict bounded rationality in order to create realistic non-linearities in the transmission of shocks and asymmetric business cycle fluctuations. The empirical fit is the next step in the comparison.}

3.1 Capital

Capital is homogeneous in this model. The stock of capital accumulates according to the following process:

$$K_t = (1 - \delta \psi(u_t))K_{t-1} + \Psi\left(\frac{I_t}{I_{t-1}}\right)I_t$$  \hspace{1cm} (37)

where $\Psi\left(\frac{I_t}{I_{t-1}}\right)I_t = [1 - \frac{\kappa}{2}(\frac{I_t}{I_{t-1}} - 1)^2]i_t$ is the adjustment cost function.\footnote{For a full model description, we refer to the baseline behavioural macro model in Macchiarelli and DeGrauwe (2015) and DeGrauwe (2008, 2012).} $\kappa_t$ denotes the cost for adjusting investment. Including adjustment costs of investment in the production of capital solves the so-called 'investment puzzle' and produces the hump-shaped investment in response to a monetary policy shock (see Smets and Wouters, 2007).

3.2 Aggregate dynamics

Since we have introduced a production economy in the baseline behavioural model, we also need to adapt the aggregate equations. First we need to link the capital accumulation with the real interest rate. Linking the investment demand equation from DeGrauwe and Macchiarelli (2015):

$$i_t = i(\rho)_t = \varepsilon_1 E_{t+1} y_{t+1} + \varepsilon_2 (\rho - E_t \pi_{t+1}); \varepsilon_2 < 0$$  \hspace{1cm} (38)
with the aggregate capital accumulation 37, we find that the relation between capital and the real rate is:

\[
k_t = (1-\delta)k_{t-1} + \Psi \left( \frac{i_t}{\rho} \right) i(\rho) t = (1-\delta)k_{t-1} + \Psi \left( \frac{i_t}{\rho} \right) e_1 E_t y_{t+1} + e_2 (r_t + x_t - E_t \pi_{t+1}); e_2 < 0
\]

Incorporating a supply side into the aggregate equations - by means of equations I.18, 37 and 4 - gives:

\[
y_t = a_1 E_t y_{t+1} + (1-a_1) y_{t-1} + a_2 (r_t - E_t \pi_{t+1}) + (a_2 + a_3) x_t + (a_1 - a_2) \psi(u_t) k_t + Adj_t \epsilon_t; \ (a_1 - a_2) > 0
\]

The aggregate demand now also depends on the usable capital in the production, \(u_t k_t\) but discounted for the cost of financing \((x_t)\). Christiano et al. (2005), Smets and Wouters (2007), and Gerali et al. (2010) arrive at the same resource constraint expression in their models. There is an adjustment cost in investment, which we capture by \(Adj_t\). However, it will be calibrated in such a way to equal \(\delta\), as in the DSGE model.

The reader will notice that aggregate demand also depends on the external finance (or risk) premium \(x_t\). This is a reduced form expression for investment, since investment is governed directly by this premium, and therefore it is the dependent variable (see DeGrauwe and Macchiarelli (2015) for a derivation of this term).

The aggregate supply (AS) equation is obtained from the price discrimination problem of retailers (monopolistically competitive):

\[
\pi_t = b_1 E_t \pi_{t+1} + (1 - b_1) \pi_{t-1} + b_2 y_t + \nu_t
\]

As explained in DeGrauwe and Macchiarelli (2015), \(b_1 = 1\) corresponds to the New-Keynesian version of AS with Calvo-pricing (Woodford (2003), Branch and McGaugh (2009)). Setting \(0 < b_1 < 1\) we incorporate some price inertia in the vein of Gali and Gertler (1999). Equally, the parameter \(b_2\) varies between 0 and \(\infty\) and reflects the degree of price rigidities in the context of a Calvo pricing assumption (DeGrauwe, 2012). A value of \(b_2 = 0\) corresponds to complete price rigidity and \(b_2 = \infty\) to perfect price flexibility (firms have a probability of 1 of changing prices in period \(t\)).
3.3 Expectations formation and learning

Under rational expectations, the expectational term will equal its realized value in the next period, i.e. $E_t X_{t+1} = X_{t+1}$, denoting generically by $X_t$ any variable in the model. However, as anticipated above, we depart from this assumption in this framework by considering bounded rationality as in DeGrauwe (2011, 2012). Expectations are replaced by a convex combination of heterogeneous expectation operators $E_t(y_{t+1} = \tilde{E}_t y_{t+1}$ and $E_t \pi_{t+1} = \tilde{E}_t \pi_{t+1}$. In particular, agents do not have control over (aggregate) output and inflation and therefore make imperfect forecast of those two variables.\(^{28}\) They forecast using two alternative forecasting rules: fundamentalist rule vs. extrapolative rule. Under the fundamentalist rule, agents are assumed to use the steady-state value of the output gap - $y^*$, here normalized to zero against a naive forecast based on the gap’s latest available observation (extrapolative rule). Equally for inflation, fundamentalist agents are assumed to base their expectations on the central bank’s target - $\pi^*$ against the extrapolists who naively base their forecast on a random walk approach.\(^{29}\) We can formally express the fundamentalists in inflation and output forecasting as:

$$\tilde{E}_t^f \pi_{t+1} = \pi^*$$  \(42\)

$$\tilde{E}_t^f y_{t+1} = y_*$$  \(43\)

and the extrapolists in both cases as:

$$\tilde{E}_t^e \pi_{t+1} = \theta \pi_{t-1}$$  \(44\)

$$\tilde{E}_t^e y_{t+1} = \theta y_{t-1}$$  \(45\)

This particular form of adaptive expectations has previously been modelled by Pesaran (1987), Brock and Hommes (1997, 1998), and Branch and McGough (2009), amongst others, in the literature. Setting $\theta = 1$ captures the ”naive” agents (as they have a strong belief in history dependence), while a $\theta < 1$ or $\theta > 1$ represents an ”adaptive” or an ”extrapolative” agent (Brock and Hommes, 1998). For reasons of tractability, we set $\theta = 1$ in this model.

\(^{28}\)The definition of agents excludes the central bank. Firms ’control’ output insofar that they control their production of final goods, but do not have an oversight over the demand-side.

\(^{29}\)The latest available observation is the best forecast of the future.
Note that for the sake of consistency with the DSGE model, all variables here are expressed in gaps. Focusing on their cyclical component makes the model symmetric with respect to the steady state (see Harvey and Jaeger, 1993). Therefore, as DeGrauwe and Macchiarelli (2015) show, it is not necessary to include a zero lower bound constraint in the model since a negative interest rate should be understood as a negative interest rate gap. In general terms, the equilibrium forecast/target for each variable will be equal to its’ setady state value.

Next, selection of the forecasting rule depends on the (historical) performance of the various rules given by a publically available goodness-of-fit measure, the mean square forecasting error (MSFE). After the time ‘t’ realization is revealed, the two predictors are evaluated ex post using MSFE and new fractions of agent types are determined. These updated fractions are used to determine the next period (aggregate) forecasts of output-and inflation gaps, and so on. Agents’ rationality consists therefore in choosing the best-performing predictor using the updated fitness measure. There is a strong empirical motivation for inserting this type of switching mechanism amongst different forecasting rules (see DeGrauwe and Macchiarelli (2015) for a brief discussion of the empirical literature, Frankel and Froot (1990) for a discussion of fundamentalist behaviour, and Roos and Schmidt (2012), Cogley (2002), Cogley and Sargent (2007) and Cornea, Hommes and Massaro (2013) for evidence of extrapolative behaviour, in particular for inflation forecasts).

The aggregate market forecasts of output gap and inflation is obtained as a weighted average of each rule:

\[ \tilde{E}_t \pi_{t+1} = \alpha_t^f \tilde{E}_t^f \pi_{t+1} + \alpha_t^e \tilde{E}_t^e \pi_{t+1} \]  (46)

\[ \tilde{E}_t y_{t+1} = \alpha_t^f \tilde{E}_t^f y_{t+1} + \alpha_t^e \tilde{E}_t^e y_{t+1} \]  (47)

where \( \alpha_t^f \) is the weighted average of fundamentalists, and \( \alpha_t^e \) that of the extrapolists. These shares are time-varying and based on the dynamic predictor selection. The mechanism allows to switch between the two forecasting rules based on MSFE / utility of the two rules, and increase (decrease) the weight of one rule over the other at each \( t \). Assuming that the utilities of the two alternative rules have a deterministic and a random component (with a log-normal distribution as in Manski and McFadden (1981) or Anderson et al (1992)), the two weights can be defined based on each period utility for each forecast \( U_{i,t}^x, i = (y, \pi), x = (f, e) \) according to:
\[
\alpha_{\pi,t}^f = \frac{\exp(\gamma U_{\pi,t}^f)}{\exp(\gamma U_{\pi,t}^f) + \exp(\gamma U_{\pi,t}^e)} \quad (48)
\]
\[
\alpha_{\pi,t}^e = 1 - \alpha_{\pi,t}^f = \frac{\exp(\gamma U_{\pi,t}^e)}{\exp(\gamma U_{\pi,t}^f) + \exp(\gamma U_{\pi,t}^e)} \quad (50)
\]
\[
\alpha_{y,t}^f = 1 - \alpha_{y,t}^f = \frac{\exp(\gamma U_{y,t}^f)}{\exp(\gamma U_{y,t}^f) + \exp(\gamma U_{y,t}^e)} \quad (51)
\]

where the utilities are defined as:

\[
U_{\pi,t}^f = -\sum_{k=0}^{\infty} w_k [\pi_{t-k-1} - \tilde{E}_{t-k-2}^f \pi_{t-k-1}]^2 \quad (52)
\]
\[
U_{y,t}^f = -\sum_{k=0}^{\infty} w_k [y_{t-k-1} - \tilde{E}_{t-k-2}^f y_{t-k-1}]^2 \quad (53)
\]
\[
U_{\pi,t}^e = -\sum_{k=0}^{\infty} w_k [\pi_{t-k-1} - \tilde{E}_{t-k-2}^e \pi_{t-k-1}]^2 \quad (54)
\]
\[
U_{y,t}^e = -\sum_{k=0}^{\infty} w_k [y_{t-k-1} - \tilde{E}_{t-k-2}^e y_{t-k-1}]^2 \quad (55)
\]

and \(w_k = (\rho^k(1 - \rho))\) (with \(0 < \rho < 1\)) are geometrically declining weights adapted to include the degree of forgetfulness in the model (DeGrauwe, 2012). \(\gamma\) is a parameter measuring the extent to which the deterministic component of utility determines actual choice. A value of 0 implies a perfectly stochastic utility. In that case, agents decide to be one type or the other simply by tossing a coin, implying a probability of each type equalizing to 0.5. On the other hand, \(\gamma = \infty\) implies a fully deterministic utility, and the probability of using the fundamentalist (extrapolative) rule is either 1 or 0. Another way of interpreting \(\gamma\) is in terms of learning from past performance: \(\gamma = 0\) implies zero willingness to learn, while it increases with the size of the parameter, i.e. \(0 < \gamma < \infty\).

As mentioned above, agents will subject the performance of rules to a fit measure and choose the one that performs best. In that sense, agents are ‘boundedly’ rational and learn from their mistakes. More importantly, this discrete choice mechanism allows to endogenize the distribution of heterogeneous agents over time with
the proportion of each agent using a certain rule (parameter \( \alpha \)). The approach is consistent with the empirical studies (Cornea et al, 2012) who show that the distribution of heterogeneous agents varies in reaction to economic volatility (Carroll (2003), Mankiw et al (2004)).

### 3.4 Firm equity

To complete the model, we need to characterize the evolution of net worth. In DeGrauwe and Macchiarelli (2015), it is shown that:

\[
n_t^{f,m} = \frac{1}{\tau}(L_{t-1}^D + i_t)
\]

and

\[
n_t^{f,m} = \bar{n}_t S_t
\]

where \( \bar{n}_t \) represents the number of (time-varying) shares of the firm and \( S_t \) is the current (stock) market price. Combining the two, we get that the number of shares is:

\[
\bar{n}_t = \frac{1}{\tau}(L_{t-1}^D + i_t)
\]

Inserting the investment demand equation \( i(\rho)_t = e_1 \bar{E}_t(y_{t+1}) + e_2(r_t + x_t - \bar{E}_t(\pi_{t+1})) \) from DeGrauwe and Macchiarelli (2015) into the expression above, we get:

\[
S_t \bar{n}_t = \frac{1}{\tau}(L_{t-1}^D + e_1 \bar{E}_t(y_{t+1}) + e_2(r_t + x_t - \bar{E}_t(\pi_{t+1})))
\]

We observe three things. First, the net capital (or loans) the firm has after repaying the cost of borrowing is scaled by the inverse leverage ratio. The more it borrows, the smaller will be its equity in the next period. Second, a higher (expected) production increases its revenues and therefore the capital level (via the capital accumulation function). However, a portion of the production is financed by external funds and thus it will need to pay a cost for those funds, represented by the risky interest rate \( r_t + x_t \). However, the more leveraged the firm is, the higher the downpayment on loans and therefore the more ‘exposed’ the firm will be in recessions. Third, a higher expected inflation implies a reduction in the cost of external financing. For a given level of leverage, this reduces firm’s debt exposure...
today and permits her, ceteris paribus to take on additional loans. Finally, note that the more leveraged the firm is, the higher is the effect from movements in (stock) market prices on the equity (shares) of the firm. This set-up is analogous to the state equation shown in Gerba (2014).

3.5 Model solution in the behavioural model

We solve the model using recursive methods (see DeGrauwe (2012) for further details). This allows for non-linear effects. The model has six endogenous variables, output gap, inflation, financing spread, savings, capital and interest rate. The first five are obtained after solving the following system:

$$
\begin{bmatrix}
1 & -b_2 & 0 & 0 & 0 \\
-a_2 c_1 & 1 - a_2 c_2 & -(a_2 + a_3) & 0 & (a_1 - a_2) \psi(u_t) \\
-\psi^{-1} e_2 c_1 & -\psi^{-1} e_2 c_1 & (1 - \psi^{-1} e_2) & 0 & 0 \\
d_3c_1 & -(1 - d_1 - d_3c_2) & 0 & 1 & 0 \\
0 & 0 & e_2 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\pi_t \\
y_t \\
x_t \\
s_t \\
k_t
\end{bmatrix} =
\begin{bmatrix}
b_1 & 0 & 0 & 0 & -e_2 \\
-a_2 & 1 - a_1 & 0 & 0 & \Psi(\frac{\tilde{u}}{u_{t-1}}) e_1 \\
-\psi^{-1} e_2 & -\psi^{-1} e_2 & 0 & 0 & 0 \\
d_3 & -d_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\tilde{E}_t[\pi_{t+1}] \\
\tilde{E}_t[y_{t+1}] \\
\tilde{E}_t[x_{t+1}] \\
\tilde{E}_t[s_{t+1}] \\
\tilde{E}_t[k_{t+1}]
\end{bmatrix} +
\begin{bmatrix}
1 - b_2 & 0 & 0 & 0 & 0 \\
0 & 1 - a'_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & -(1 - d_1 - d_2) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & (1 - \delta \psi(u_t))
\end{bmatrix}
\begin{bmatrix}
\pi_{t-1} \\
y_{t-1} \\
x_{t-1} \\
s_{t-1} \\
k_{t-1}
\end{bmatrix} +
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & a_2 & 0 & 1 & (a_1 - a_2) \\
0 & \psi^{-1} e_2 & 1 & 0 & 0 \\
0 & -d_3 & 0 & -(1 - d_1) & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\eta_t \\
u_t \\
\psi \\
e_t \\
u_{cf_t}
\end{bmatrix}
Using matrix notation, we can write this as: \( AZ_t = B \tilde{E}_t Z_{t+1} + CZ_{t-1} + DX_{t-1} + Ev_t. \) We can solve for \( Z_t \) by inverting: \( Z_t = A^{-1}(B \tilde{E}_t Z_{t+1} + CZ_{t-1} + DX_{t-1} + Ev_t) \) and assuring \( A \) to be non-singular.

Solution for the interest rate \( r_t \) is obtained by substituting \( y_t \) and \( \pi_t \) into the Taylor rule. Investments, utilization costs, bank equities, loans, labor and deposits are determined by the model solutions for output gap, inflation, financing spread, savings and capital.\(^{30}\)

Expectation terms with a tilde \( \tilde{E}_t \) implies that we do not impose rational expectations. Using the system of equations above, if we substitute the law of motion consistent with heterogeneity of agents (fundamentalists and extrapolators), then we can show that the endogenous variables depend linearly on lagged endogenous variables, their equilibrium forecasts and current exogenous shocks.

Note that for the forecasts of output and inflation gap, the forward looking terms in equations 39, 40 and 41 are substituted by the discrete choice mechanism in 46. For a comparison of solutions in the 'bounded rationality' model and rational expectations framework, see section 3.1 in DeGrauwe and Macchiarelli (2015).

### 3.6 Calibration and simulations

To simplify the discussion, we will only present the calibrations of the parameters that are new to this model. A full parameter list can be found in Appendix.

In line with De Grauwe and Macchiarelli (2015), we calibrate the aggregate demand parameters \((d_1, d_2, e_1)\) to \((0.5, 0.15, 0.1)\) which is consistent with standard macroeconomic simulation results. \( \tau \) (or a firms’ average leverage ratio) is again set to 1.43, following Pesaran and Xu (2013), and \( \kappa \) (or banks’ equity ratio) is, following Gerali et al (2010), set to 0.09.

The parameters specific to this model are set to standard values in the literature. The share of capital in the production \( \alpha \) is set to 0.30 as in Boissay et al (2013). Following Christiano et al (2005), Smets and Wouters (2003, 2007) and Gerali et al (2010), we set the capital depreciation rate \( \delta \) to 0.025. The elasticity of the capital utilization adjustment cost function \( \psi(u_t) \) is parametrized to 0.5 as in Smets and Wouters (2007).\(^{31}\)

---

\(^{30}\)However, capital, savings and the external financing spread do not need to be forecasted as these do not affect the dynamics of the model (i.e. there is no structure of higher order beliefs as LIE does not hold in the behavioural model). See section 3.1 in DeGrauwe and Macchiarelli (2015) for comparison of solutions under rational expectations and bounded rationality (“heuristics”).

\(^{31}\)This is equivalent to setting a \( \kappa_t \) equal to the estimated range \((10.18 – 12.81)\) as in Gerali et al (2010).
The sensitivity of capital (or investment) to changes in the real interest rate $e_2$ is, in line with the empirical evidence, set to $e_2 < 0$. To conclude, the parameters of the function determining adjustment costs for capacity utilization ($\xi_0, \xi_1, \xi_2$) are set to $(0.8, 0.3, 0.25)$ in order to capture the estimation results of Smets and Wouters (2005) who find that the capital utilization adjustment costs are between 0.14 and 0.38 (Euro Area 1983-2002) and 0.21 and 0.42 (US 1983-2002), with a mean of 0.25 (Euro Area) and 0.31 (US). If we normalize $u_t$ to 1 (as in Christiano et al (2005), Miao et al (2013) or Auernheimer and Trupkin (2014)), then the cost for utilizing capital will be $0.20 (1 - \xi_0)$, which is well within the estimated intervals of Smets and Wouters (2005).

All shocks, except to the capital utilization, are parametrized as white noise which means that their autoregressive component is set to 0. Likewise the standard deviations of shocks are set to 0.5 across the entire spectrum.\footnote{The AR-component of the shock to capital utilization cost is set conservatively to 0.1, just enough to generate some persistence in the capital cost structure.}

4 Quantitative results in the DSGE model

Analysis of the quantitative results is split into three parts. First, we will analyze a selected number of impulse responses. This will be followed by a moment-matching exercise. Finally, we will conclude the section with a variance decomposition exercise in order to understand the shocks that are most important in explaining the variations in the model. In particular, we will be interested to see whether the DSGE model is capable of capturing the stylized fact of financial shocks being the most important source of variability in the US (and EU) economies, transmitted via the supply side, described above.

4.1 Forcing variables

Next we will examine the impulse responses to two supply shocks (TFP and utilization rate), two financial shocks (firm financing costs and asset price wedge), and one monetary shock (monetary policy). The five shocks have the following model structure:
• (Positive) technology (TFP) shock, $\epsilon_{zt}$

\[
y_t = z_t \epsilon_{zt} K^\alpha L^{1-\alpha}
\]  

(60)

, where $\epsilon_{zt}$ is a white noise shock to the technology factor in the Cobb-Douglas technology function. We model the autoregressive structure of the TFP shock as:

\[
z_t = \rho_z z_{t-1} + \epsilon^z
\]  

(61)

• (Negative) shock to firm financing costs, $\epsilon^{rk}$, which we introduce in the cost of external financing equation:

\[
r^k_t = (1 - \epsilon)(y_t - k_t - x_t) + \epsilon s_t - s_{t-1} + \epsilon^{rk}
\]  

(62)

Another way of interpreting this shock is to consider it as (an unexpected) improvement in the financial condition of firms, which relaxes their cost of external financing.

• Standard (negative) monetary policy shock ($\epsilon$):

\[
r_t = r_{t-1} + \gamma \pi_t + (1 - \gamma) y_t + \epsilon
\]  

(63)

, a shock to capital utilization rate:

• (Positive) shock to utilization cost, $uc_t$, in the utilization cost function:

\[
\psi(u_t) = \xi_0 + \xi_1 (u_t - 1) + \frac{\xi_2}{2} (u_t - 1)^2 + uc_t
\]  

(64)

, where $uc_t$ has the following AR structure:

\[
uc_t = \rho_{uc} uc_{t-1} + \epsilon_{uc}
\]  

(65)

Finally, to examine a full boom-bust cycle in asset prices, we introduce a second financial shock to our model. More specifically, we introduce an exogenous disturbance to the residual earnings equation:

• (Positive) shock to residual earnings, $\epsilon^{re}$:

\[
r_{re_t} = \rho_{re} r_{e_{t-1}} + (\chi)(E_t[y_{t+1}] + n_t - E_t[r_{t+1}]) + \epsilon^{re}
\]  

(66)
The shock can be viewed as unexpected news (good or bad) regarding future economic performance that arrives, and influences stock market investments in that period. We label it a *wedge shock*.

and $\epsilon_{uc}$ is a white noise shock. In our simulations, we calibrate the AR component $\rho_{uc}$ to 0.1 in order to strictly limit the possibility of the shock driving the model dynamics. However, a simple white noise utilization cost shock is excessively short-lived, and does not allow us to study the endogenous dynamics in full. All the white noise shock parameters ($\epsilon, \epsilon_{rk}, \epsilon_{re}, \epsilon_{z}$ and $\epsilon_{uc}$) are calibrated to 0.5.

Remember that the standard errors of all shocks are calibrated to 0.5. Later on, this will allow us to make a qualitative as well as quantitative comparison between the two model responses.

4.2 Impulse response analysis

To maintain the focus, we will only discuss the *TFP* and *financial* shocks in this section. For a discussion of the other 3 shocks, please refer to the Appendix. Note that the numbers on the x-axis indicate the number of quarters.

We will also include a comparison of the results in the current framework with model versions where the stock market and interaction term are omitted (original BGG, 1999), or a version where only the stock market mechanism is included (Gerba, 2014). For sake of comparability, we have calibrated the aforementioned models in the same way as for the current model, as well as applied the same shock structure and parametrization.

4.2.1 Productivity shock

Figure II.1 depicts the responses to an expansionary TFP shock. An increase in productivity of 0.5% results in an expansion in production and sales, which increases the rentability of the firm. This pushes up its market value by 0.8%, its net worth by 1.75%, marginal costs down by 0.2% and labour demand up by 0.8%. Moreover, because the profitability outlook of the firm is positive, the CGP become less worried about the repayment of their capital sold, which relaxes the down payment share by 2.5% (i.e. less of the capital purchased has to be pre-paid). The book value of the firm also goes up by 0.5%. However, since the profitability of the firm is expected to remain high for multiple periods ahead, and its investment demand is highly positive and stable (initially at 3.5%, and then 0.8% after 16 quarters), market value should be higher than the book value for several periods ahead. That
is exactly what we observe in residual earnings, which increases by up to 0.7% in quarter 4. The effects on inflation are, however, non-standard. While a positive TFP shock reduces inflation, the demand effects from an expansion in investment, stock prices, consumption (0.5%) and external financing are so strong that they offset the initial fall, which results in a final increase of inflation by 0.04%. This triggers a positive (albeit marginal) increase in the policy rate of 0.005%, resulting in a very short-lived inflation. Therefore, the real rate falls (-0.018%). Lastly, the total effect of this supply-side expansion is that output expands by 1%, and remains above its steady-state level for multiple quarters.

To quantify the importance and the propagating power of the mechanism developed in this paper, we briefly compare the responses in the current model with the benchmark BGG (1999), and the Gerba (2014) extension. Judging from Figure II.2, one can clearly see that by omitting a (stock) market valuation mechanism of firms and an explicit interaction between supply-side and financial markets, the expansionary effects from a TFP shock are considerably smaller. Net worth of firm increases 3 times less (or 0.55%), marginal costs drop 50 times less (or -0.004%), and contrary to above, labour demand falls (since the TFP effect is entirely on employing capital more effectively). As a result, the (book) value of firm increases by less than a half (or 0.2%) and investment demand more than 3 times less (or 1%) compared to the full model. Because of this weaker transmission in the canonical BGG (1999) model, inflation falls instead (by 0.0015%), which pushes the policy rate down and the real rate up, and output increases by 50% less (or just 0.5% above the steady state level).

Even when we include the (stock) market valuation mechanism, but exclude the explicit interaction term between the stock market, the financial market and the supply-side (as in Gerba, 2014), the responses are weaker. More specifically, while the responses in Gerba (2014) and this model are qualitatively the same, the magnitudes of the responses in the current model are, on average, 15% higher. So, for instance, in Gerba (2014) market value of firms rises by 0.6%, net worth by 1.55%, marginal costs down by 0.15%, and labor demand up by 0.5%. In the same vein, investment rises by only 2.9%, inflation increases by 0.03%, the real rate falls by 0.01%, and output rises by 0.9%.33

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33 For the sake of space, we have not reported the impulse responses of the Gerba (2014) model. However, should you wish to see them, please do not hesitate to contact the authors.
4.2.2 Financial shock

The second shock we consider is a 0.5% reduction in the cost of external financing for firms. The impulse responses are depicted in Figure II.8. The immediate impact is that, via equation 62, the return on capital rises by 3% and the real rate falls by nearly 0.1%, which makes borrowing and investment much more attractive for entrepreneurs. They will therefore borrow up to the new maximum, and increase their investment by 10%. Moreover, firms will produce more since their marginal costs have gone down by 0.8% and their demand for labor up by 2.3% so to keep the capital-labor ratio constant. Hence capital increases by 1.1%. Higher production and investment imples a higher net worth in the future, which increases by 6.5%. As a result of the positive outlook on firm finances and its realized cash-flows, both the market and book value of firms increase. The market value rises by 2.5%, meanwhile the book value by 1.8%. Since the expectations of future firm profits and investment returns are high, these are additionally priced in today’s market value, which at the peak (after 4 quarters) pushes the residual earnings up to 1.75% above the steady-state level. The increased activity results in an increase in inflation by 0.18%, which forces the monetary authority to respond by raising the interest rate by 0.03%. However, the final effect on real interest rate is for it to fall by 0.08%. As a consequence, expected consumption falls by 0.1% and increase only when the real rate turns positive. The accumulated rise is, nevertheless, small compared to investment, since consumption does not rise at any point by more than 0.15%.

Note the (positive) supply side effects that a (positive) financial shock has in this framework. Not only does it increase production, reduce the marginal input costs, and increase the market value of firm, but it also relaxes the front payment share by 9%. The financial position and the production possibility of entrepreneurs has improved, which means that CGP are less worried about entrepreneurs repayment-status, and therefore require less of pre-payment. All of this results in an output increase of 2%. That is twice the expansion orginated from a supply shock only.

Once again, let us compare the impulse responses to a canonical BGG (1999) model. These are depicted in Figure II.9. It is clear that the expansion is attenuated in the benchmark version. Return on capital increases by 1% and the real rate falls by 0.015%. The consequence is a much smaller (positive) borrowing gap, and so the increase in investment is 4 times smaller than the one observed in the full model (i.e. 2.5%). In addition the rise in production is also significantly smaller. The fall in marginal costs is 5 times smaller (0.15%) and the rise in the demand for labor
4 times smaller (0.6%). As a consequence, net worth increases only by 2.25% and the (book) value by 0.6% (a third of the values in the full model). The resulting inflation rise is 6 times smaller, the same as for the real interest rate (0.03% and -0.015%). The aggregate effect on output is that it increases by 0.45%, or by less than a fourth to the full model.

Including a (stock) market strengthens the (financial) shock transmission mechanism, even if less significantly than in the full model. The impulse responses are on average 25% lower in the Gerba (2014) extension compared to the model here. On the financial side, the return on capital increases by 2% and the real rate falls by 0.08%. On the demand side, investment increases by 8%, (market) value of capital by 2%, book value by 1.3%, and net worth by 5.8%. On the supply side, marginal production costs fall by 0.6%, labor demand increases by 2% and capital by 1%. The aggregate effect on inflation and output is that they rise by a third and a fifth less than in the full model. These observations show that including the interaction between stock markets, external financing and the supply side does not only amplify the shocks, but that the amplification is stronger when the economy faces a financial shock compared to a real shock only. Moreover, under the current framework financial shocks which are predominantly transmitted via the supply side have stronger macroeconomic effects than those transmitted predominantly via the demand side (see appendix). That is very much in line with the empirical observations made by Broadbent and others outlined earlier in the introduction. The aggregate supply is a powerful propagator of financial shocks in the current model.

4.3 Variance decomposition

Next, we would like to apprehend the most important shocks for explaining the variation in the model. To do so, we decompose the volatilities of all variables using the five shocks discussed above. The percentages are reported in Table II.7.

The first observation is that the financial shock, followed by the monetary policy and technology shocks explain the vast majority of the model volatility. The asset price shock and the shock to the utilization costs are, in contrast, almost irrelevant.

Continuing with output, more than half of its volatility is explained by the financial shock. Approximately a fourth is explained by the monetary policy shock, and just under a fifth by the technology shock. That is not surprising since the financial

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34 Logically, the fall in consumption is also smaller, 0.05%, as the real rate falls by less.
35 See appendix for further analysis of real-financial shocks.
accelerator mechanism plays a significant role in the model construction. What is more surprising and affirmative of the importance of the interaction between the supply-side and finance is that taken together, the TFP and financial shock roughly explain three-fourths of the variation in the majority of the model variables.\footnote{The only exceptions are the real interest rate, the utilization cost function and marginal costs. For marginal costs, 42\% of the variation is explained by the TFP and financial shocks together, which is far from negligible, and only slightly after the monetary policy shock.} On the other hand the wedge shock (which propagates via the demand-side) plays a very minor role. Taking further into account that a monetary policy shock passes through the demand side (via consumption Euler equation and investment demand) as much as the supply side (via cost of capital, intertemporal risk smoothing in capital input market, and more broadly firm marginal production costs), it becomes evident that the supply side in conjunction with financial frictions is the most important motor of the model. Moreover, it suggests that the current model is sufficiently different from the canonical BGG (1999) or Gerba (2014), in which the demand side coupled with financial frictions were the most important motor, in order to be considered as separate.\footnote{For a variance decomposition of the BGG (1999) and Gerba (2014) models using the same calibrations, do not hesitate to contact the authors for details.}

4.4 Statistical moments

The statistical moments are reported in Tables II.4 and II.5. For that, we have calculated the statistical moments for all variables using the longest data sample period available from 1953:I - 2014:IV.\footnote{The most recent data recorded is for 2014:IV using Fed St Louis database on March 2, 2015.} Following Stock and Watson (1998), we choose 1953:I as the starting year of our sample since the (post-war) quarters prior to 1953 include noise and inaccuracies in the data recording. The sample includes 247 quarters (or 62 years) which is the closest approximation available for the long-run (cyclical) moments that is generated by the model. During this period, the US economy experienced 10 cycles (using NBER business cycle dates), and the average GDP increase (quarter-on-quarter) during expansions was 1.05\% while it was -0.036\% during recessions. The data were downloaded from Flow of Funds at the Fed St Louis database. These were de-trended using a standard two-sided HP-filter before their moments were calculated.\footnote{This is in order to allow for a smoother comparison with the model generated (cyclical) moments.}

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4.4.1 Correlations

Let us begin with the correlations reported in Table II.4. The DSGE model is capable of matching quiet a few of the correlations. In particular, it matches relatively well the autocorrelations of output, inflation, and (especially) capital. The matching of demand side correlations is also very good. So, the correlation of investment to output, and consumption to output is almost the same as the value obtained from the US data (0.98 vs 0.90 and 0.30 vs 0.32). A similar accomplishment is also achieved for the stock market variables in the model. The correlation of stock prices to output is 0.97 in the model and 0.83 in the data. Equally, residual earnings to output has a correlation of 0.61 in the model and 0.76 in the data. Lastly, the book value to output has a correlation of 0.59 in the model and 0.90 in the data.

Next, the model manages to capture some of the supply side correlations. In particular, the data correlations of capital-output, marginal costs-output, capital-interest rate, capital-marginal costs, and labor-output are closely matched by the model. Moreover, and confirming the observations in the impulse response analysis, the correlation between the front payment share for capital purchases \( \vartheta_t \) and output is strongly countercyclical (-0.97). Another interesting insight comes from the correlation between the share, \( \vartheta_t \) and capital (-0.31), or residual earnings (-0.67). These numbers are in line with the intuition from the model. A higher (lower) pre-payment share will force entrepreneurs to use a higher (lower) proportion of their liquid funds to fund their capital purchases. However, since these are limited, the total amount of capital they can purchase will be lower (higher) compared to the case without such a constraint. Hence the negative correlation since less (more) capital will be bought and accumulated in total. Along the same lines, a positive (negative) residual earnings means that the economic outlook of the future is positive (negative) since the market value is above (below) the book value. Knowing this, CGP will have less (more) doubt of entrepreneurs repayment status, and therefore will ask for a lower (higher) share.\(^{40}\)

Nevertheless, there is still room for improvement in the data matching. More specifically, the correlation of output to inflation is positive and high in the model (0.83) while it is negative in the data (-0.43).\(^{41}\) Equally, the correlation between in-

\(^{40}\)CGP know that during booms (busts), the probability of default of entrepreneurs will shrink (rise) and they will receive more (less) external funds in the next period, which assures CGP of receiving the full payment for their capital sold.

\(^{41}\)One reason for why it is negative in the data is because the relation is mainly driven by supply-side factors, which increase (decrease) output and decrease (increase) inflation in booms, as noted by Broadbent and others in the introduction.
flation and the interest rate is highly negative in the model (-0.91) while it is positive in the data (0.34). This implies that the correlation between output and the policy rate is positive and large (0.78), but since inflation and output have the ‘wrong’ sign in the model, the relation between inflation and interest rate is also incorrectly captured. There are also a few relations where the sign is correct, but where there is space to improve the magnitudes. Along these lines, the autocorrelation of output, the autocorrelation of inflation, and the correlation between output and book value are higher in the data than in the model. To conclude this section, the DSGE model does a good job in matching a large portion of the US correlations. The supply-side and demand-side relation are correctly matched, and the autocorrelations are much closer to the data than in many other financial friction models. However, there is space for improvement, in particular in capturing the ‘true’ relation between output and inflation, and in bringing the autocorrelations even closer to the data.

### 4.4.2 Second moments

Let us continue with the (relative) standard deviations reported in Table II.5. Note that we follow the standard procedure in the literature by calculating the standard deviation of all variables (except output) with respect to the general business cycle, both in the model and the data. First and foremost, the DSGE model matches most of the volatilities in the data. With the only exception of capital and residual earnings, the model attributes the right type of (relative) standard deviation for all the other variables. In other words, when a variable is less (more) volatile than the business cycle in the data, it is identical in the model. In addition, for variables such as inflation, investment, labor, and net worth of firms, the (relative) standard deviations in the model are very close to the numbers in the data. Taking into account that we did not explicitly follow the procedure of *ex ante* moment matching when calibrating the model, the results are very promising.

Where the model could do better is in replicating the second moments of capital and residual earnings. Whereas capital and residual earnings are more volatile than the business cycle in the data, they are less volatile in the model. This implies that their responsiveness to shocks over the business cycle is higher in the data. Also for some variables, such as the stock market price, consumption or the (policy) interest rate, the standard deviations in the model could be increased in order to approximate the empirical figures, even if their general business cycle characteristic is correct. Taken altogether, however, the DSGE model is effective in replicating...
most of the data volatilities, even better than for correlations, and without having to adopt abstract ad hoc modelling tricks, such as including a large list of shocks, or introducing autoregressive structures on key variables in the model.

5 Quantitative results in the behavioural model

Our analysis of the behavioural model results is also split into three parts. The first part is an analysis of (model consistent) impulse responses to a set of independent white noise shocks. The second is an examination of the (model generated) second-, and higher-order moments to contrast the fit to the US data. The final part consists of depicting and analyzing the nature of the model variables over the business cycle.

5.1 Forcing variables

The four shocks we will examine are:

- (Positive) technology (or TFP) shock, $\epsilon_{zt}$

$$y_t = z_t \epsilon_{zt} K^\alpha L^{1-\alpha}$$

, where $\epsilon_{zt}$ is a white noise shock to the technology factor in the Cobb-Douglas technology function. We explicitly do not wish to model an AR structure for the TFP shock since we want to understand the endogenous transmission power of the model structure without recurring to exogenous (or ad hoc) extensions. See De Grauwe (2011, 2013) for a longer discussion on how autoregressive shocks can and should be omitted in macroeconomic models.

- (Negative) shock to firm financing costs, i.e. a relaxation in the cost of external financing for firms

$$x_t = \rho_x x_{t-1} + \epsilon_x$$

which is introduced in the aggregate demand equation in 40.

- Standard (negative) monetary policy shock ($\epsilon$):

$$r_t = r_{t-1} + \gamma \pi_t + (1 - \gamma) y_t + \epsilon$$

, and lastly,

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42There are actually five shocks in the model, but one, the aggregate demand shock, is not relevant in this model nor does it have a structural interpretation, and therefore we omit it in the general analysis.
• (Positive) shock to utilization cost, $uc_t$, in the utilization cost function:

$$\psi(u_t) = \xi_0 + \xi_1(u_t - 1) + \frac{\xi_2}{2}(u_t - 1)^2 + uc_t$$

(70)

where $uc_t$ has the following AR structure:

$$uc_t = \rho_{uc}uc_{t-1} + \epsilon_{uc}$$

(71)

and $\epsilon_{uc}$ is a white noise shock. In our simulations, we calibrate the AR component $\rho_{uc}$ to 0.1 in order to strictly limit the possibility of the shock driving the model dynamics. However, a simple white noise utilization cost shock is excessively short-lived, and does not allow us to study the endogenous dynamics in full.\textsuperscript{43} All the white noise shock parameters ($\epsilon$, $\epsilon^R$, $\epsilon_z$ and $\epsilon_{uc}$) are calibrated to 0.5.

### 5.2 Impulse response analysis

As for the DSGE model we will only discuss the impulse responses to two shocks, the TFP or technology shock, and the financial shock. For a longer discussion of other shocks, please refer to the Appendix.

Figure II.11 depicts the (median) impulse responses to a TFP shock. Figure II.13 does the same for a (negative) shock to firm external financing conditions. This shock is well representative of the pre-2008 period, where the firm financing conditions were very lax and they were able to borrow at an unprecedentedly low cost. Symmetrically, following the crash on financial markets in 2007-08, the external financing costs spiked, and the transmission to the real economy, via the supply side, can be inversely interpreted from the current impulse responses.\textsuperscript{44} In particular, we would like to test whether the current model is capable of capturing the financial-supply side interactions that were noted by several empirical studies mentioned in the introduction.

\textsuperscript{43}Before we begin with the analysis, bear in mind that the behavioural model does not necessarily have one steady state that is time invariant for the same calibration (as is standard for the DSGE method). Therefore, following a white noise shock, the model may not necessarily return to a previous steady state. If not the same steady state, it can either reach a new steady state, or have a prolonged response to the initial shock. In other words, there is a possibility for the temporary shock to have permanent effects in the model (via the animal spirits channel). However, due to the methodological proximity to the DSGE analogue and because it is a standard evaluation (and comparison) tool in the literature, we will proceed analyzing the impulse responses in the behavioural model.

\textsuperscript{44}Since the transmission is symmetric for a positive or negative shock.
Note that the numbers on the x-axis indicate number of quarters. All shocks are introduced in \( t=100 \) and we observe the responses over a long period of 60 quarters (or 15 years). Observe that in these figures we depict the median impulse response amongst a distribution of impulse responses generated with different intializations (or realization) of shocks. The full distribution of impulse responses are instead depicted in Figures II.14 to II.17. For the sake of clarity in the exposition, we will only concentrate on the median impulse response however, which is a good representation of the overall (non-Gaussian) distribution.\(^{45}\)

### 5.2.1 Technology shock

Let us start with the first of the supply side shocks. An improvement in TFP (or equivalently, increase in productivity) of 0.5\% results in an inflation reduction (1\%) and a more than proportional output expansion (1.15\%). This is a result of both the increased capacity in the final goods market, but also from an increase in investment (0.3\%) following the heavy fall in interest rate (1.3\%) as a response to the falling inflation. Following this general supply-side expansion, deposits and loans to firms also increase (1 and 1.3\% respectively) since the value of firm net worth (i.e. collateral) has increased. As a consequence of the lower marginal cost to investment and higher marginal return on capital, capital accumulation increases significantly in the next period (0.5\%). This results in a general market optimism (animal spirits rise by 0.1\%).

However, as soon as the inflation starts recovering, interest rate react very rapidly to their increase and start rising (0.35\%). Because of this rise in cost of capital, coupled with the fall in external financing for firms, investment and output expansion reverts. However, unlike in the DSGE models, the model has eventually reached a new steady state, where bank loans, deposits and equity are permanently 1.1\%, 0.7\% and 0.1\% above the previous pre-shock level. \(^{46}\) Hence a temporary technology shock in the behavioural model will have long-lasting positive effects on the banking sector and financial efficiency.\(^{47}\)

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\(^{45}\)Keep in mind, when interpreting the results, that the impulse responses in the DSGE model are not conditional on the realization of shocks, since no learning occurs, and thus only a 'representative' unconditional IRF is depicted.

\(^{46}\)In DSGE models, this is only possible to achieve with permanent or continuously inserted shocks.

\(^{47}\)Arising from the additional dynamics generated by learning.
5.2.2 Financial shock

Let us turn to the financial shock. A relaxation in the external financing costs for firms means that they will be able to increase their borrowing by 0.2%, and thus their leverage. The same is true for banks, since they reduce their equity by 0.1% in order to increase their lending to firms. Firms will use this new credit to increase their investments by 0.2%. Production will also increase, which will push firm net worth up in the future. This positive outlook produces an optimism on the market, generating an increase in animal spirits of 0.6%. This acceleration in activity pushes output and inflation up by 0.4% and 0.035%. Monetary authority is rapid in responding to the rise in inflation and raises the policy rate by 0.33%, with the desired consequence of attenuating the initial expansion to bring output and inflation back to their pre-shock level after approximately 3 years (or 12 quarters). Note that, in contrast to the case with supply shocks (see the appendix), the financial market variables (loans, deposits and bank equity) return to their pre-shock level relatively swiftly.\(^{48}\)

We believe the reason lies in the model construction. Since alterations in the cost of corporate financing are transmitted via demand-side channel in this model, the macroeconomic effects are short-term and there is therefore no fundamental reason for why credit should be supplied at a new level. On the other hand, when the economy is faced with supply shocks, the macroeconomic impact is more long-lasting, and the bank can therefore provide more (less) credit at the higher (lower) productivity level. This endogenous mechanism is very much in line with what has been argued in the empirical macroeconomic literature that fundamental changes in the real economy will be reflected in permanent changes in the financial sector activity.\(^{49}\)

Furthermore, notice that this mechanism is very difficult (if not impossible) to capture in the current generation of DSGE models unless permanent shocks are introduced.

\(^{48}\)For a monetary policy shock, the financial market variables reach a new level following the shock, but at a much lower magnitude than any of the supply side shocks.

\(^{49}\)For instance, think about the effects from oil shocks on the subsequent deregulation in, and expansion of the financial sector, or the IT-revolution on the long-term quantity of credit supplied and the balance sheet management policies adopted by banks.
5.3 Distributions and statistical moments in the behavioural model over the business cycle

The second part of the model evaluation consists of analysing and validating the model-generated distribution and statistical moments over the business cycle. These are generated using the entire sample period of 2000 quarters. For our purposes, we will use the data on second and higher moments in Tables II.4 and II.5 to II.6, the evolution of the model variables over the business cycle in Figures II.18 to II.21, as well as histograms of a selection of these variables in Figures II.22 to II.24. For the graphs note that we are plotting the business cycles over a sub-sample period of 100 quarters.

5.3.1 Macroeconomic aggregates

The short-term cycles of output, inflation and the interest rate are asymmetric. While the amplitude of expansions is in general higher for output, the duration of recessions is longer. This is further confirmed by the histogram for output, which is asymmetric and skewed to the right, with a higher probability mass on the left of the mean of the distribution. Moreover, the autocorrelation of output is very high (0.86), as is the volatility (2.17) and it is leptokurtic (kurtosis=10.91).

The opposite applies to inflation. The amplitude of deflationary periods is in general higher, while the duration of inflationary periods is longer. From histogram, the distribution of inflation is slightly skewed to the left. In line with the data, inflation is three times less volatile than output but has a very similar kurtosis to output. Further, inflation is very persistent over time ($\rho = 0.74$) and countercyclical (-0.42), exactly as in the US data (-0.43).

Turning to the (risk-free) interest rate, it is mostly positive and remains above the trend for a longer period over the cycle. It is also highly correlated with the business cycle (0.39) as well as with inflation (0.57), indicating a firm inflation target on the part of the monetary authority. It is almost as volatile as output (0.95), but highly skewed to the left (-4.29) compared to the general business cycle.

5.3.2 Firm and supply-side variables

From Figure II.19, capital stock is mostly positive over the cycle, with a mean-reversion around 1. This is in line with the data on inventories, which shows it is positive mean-reverting. It is highly persistent ($\rho = 0.95$) and positively correlated with output (0.45). It is also highly correlated with animal spirits (0.34).
Distribution-wise, it is less volatile than the business cycle (0.413), but heavily skewed to the right (3.48).

The first thing to observe regarding utilization costs is that while apparently more volatile, it oscillates within a much smaller interval compared to any of the other variables. Hence, the volatility is 4 times smaller compared to output. In addition, it reverts around a mean of approx. 0.5. This is in line with the data, which points towards a largely non-negative cost in utilizing capital over the cycle. It is however weakly countercyclical (-0.1), and symmetric as well as mesokurtic.

The 'cash-in-advance constraint’ \( \vartheta_t \) is strictly non-negative and acyclical (0.02). It is also independent from the cycles of capital- \((-0.01)\), and financing spread (0.01). In addition, the distribution of \( \vartheta_t \) is highly volatile, skewed to the right and leptokurtic. Effectively, with 95% probability (or higher) \( \vartheta_t \) is significantly above zero.

On the other hand, the financing spread for firms is highly countercyclical (-0.41), as well as negatively correlated with animal spirits (-0.12). This is consistent with the model set-up and data, which show that during expansions both the real risk (via a higher collateral value) and the perceived risk (via the optimistic sentiment) of loan default falls, which points towards a largely non-negative cost in utilizing capital over the cycle. The opposite holds for recessions. That is why the spread is both negatively correlated with the business cycle (collateral value), and with the market sentiment (agents’ risk perception). Statistically, the spread is as volatile as the general business cycle, but highly skewed to the left, meaning that for most of the time the spreads will be close to zero (or negative). This is further confirmed by the graph in Figure II.20. However, with some non-negligible probability, the spread can spike, causing a severe contraction in liquidity and the banking market.\(^{50}\) These results are also in line with the (model-generated) statistical moments on loan supply, which is procyclical (0.11), positively correlated with animal spirits (0.12) and capital-net worth (0.28), but negatively correlated with the financing spread (-0.1)

### 5.3.3 Market sentiment

An important driver of the business cycle is the market sentiment (or animal spirits). It is highly procyclical (0.84) throughout the entire sample period (see Figure II.26). Moreover, we observe a higher persistence during the pessimistic interval compared

\(^{50}\) However, the spread is not persistent \((\rho = 0.01)\) implying an RBC type of frictionless financial sector, and non-staggered price setting. That is not a surprise for the current model since the financial market is modeled in reduced form. However, future work should try to extend the model by modeling a more complex and empirically consistent financial price setting mechanism.
to the optimistic. This is in line with our previous observation on the general business cycle (or output) showing that recessions have a longer duration compared to expansions. Moreover, market sentiment has fat tails on the left and right of the mean, but is smoother than the general business cycle.

5.4 Moment matching

The next step in model validation consists of matching the (model generated) moments to the US data. A full list of variables and other details can be found in Table II.1.

5.4.1 Correlations

The behavioural model matches precisely the correlations of many supply-side and financial variables. This includes credit to firms, deposits, the (risk-free) interest rate, inflation, and firm financing spread. It is also very successful in reproducing the autocorrelations of output, capital, and inflation, as well as the correlations between capital and credit to firms, and inflation and the (risk-free) interest rate. However, there is room for improvement in matching stock variables, such as firm and bank net worths, some macroeconomic aggregates (investment mainly) as well as the autocorrelation of firm financing spread. While they are all acyclical and not persistent in the model, they are highly procyclical and highly persistent in the data.

5.4.2 Second and higher moments

Turning to (relative) second-, third-, and fourth moments, the model is highly successful in reproducing the moments of inflation, the (risk-free) interest rate, credit to firms, deposits, and net worth of banks. It is also successful in making net worth of firms more skewed and more leptokurtic than output. However, the moments of the latter are higher in the model compared to US data. On the other hand, capital and investment are smoother in the model.

Another strength of the model lies in reproducing irregular business cycles. In contrast to standard first-, second-, or even third order approximated DSGE models, the behavioural model generates substantial asymmetries between expansions and recessions as well as produces non-Gaussian probability distribution functions for most variables. That is much more in line with the observed pattern in the US cyclical data. Nonetheless, for some variables (net worth, consumption, savings,
(risk free) interest rate, and credit to firms) the model generates excessive skewness and/or kurtosis.

To sum up, the model matches most of the US data. This includes supply-side and financial variables such as the (risk-free) interest rate, inflation, credit to firms, deposits, firm financing spread and net worth of banks. It is also successful in matching several supply relations (capital-firm credit, inflation-interest rate) as well as their autocorrelations (output, capital and inflation) There is, however, some scope for improvement in matching demand-side variables (such as consumption, savings, investment) as well as stocks (net worth of firms).

5.5 The nature of business cycles

Next, we wish to understand to what extent the model is capable of generating inertias in the business cycles.

As discussed in Milani (2012) and DeGrauwe and Macchiarelli (2015), business cycle movements in a rational expectations environment arise as a result of exogenous shocks (including the autoregressive structure of shocks), leads and lags in the endogenous transmission of shocks (such as lagged or expected output), habit formation, interest rate smoothing, or nominal rigidities (price and wage stickiness). One could therefore call this ‘exogenously created’ business cycle fluctuations. The behavioural model, on the other hand, generates inertia and business cycle fluctuations even in the absence of endogenous frictions, lags in endogenous transmissions, and autocorrelated shock structures, as shown in DeGrauwe (2012). In the current case, however, we have introduced supply-side and financial market frictions, as well as leads and lags in the output, inflation and capital transmissions. This is in order to set the behavioural model at par with a standard DSGE model, so to facilitate the comparison between the two frameworks.

The evolution of the different model variables over the business cycle are reported in figures II.18 to II.26. The time period covered is 100 quarters, which is enough to cover multiple cycles. The first thing to note is that with this ‘snapshot’ of the business cycle, we have managed to capture one long cycle (with a high amplitude)
followed by several shorter cycles. Not only is the business cycle peak the highest during those 25 years \( t = 295 \), but the amplitude is also the widest (between \( t = [280 : 300] \) counting from trough to trough). Moreover, the subsequent bust is the sharpest, since it takes the economy more than 40 quarters to return to a level above the long-run trend (or above the zero line). In addition, the subsequent expansions are significantly weaker, somewhat implying that some fundamental (or structural) changes occurred in the economy following the preceding boom and bust.\(^{54}\) Compare that to the boom preceding the Great Recession and the subsequent bust in the US.

Closely related to above observations, we find that the other variables experience similar cycles (inflation, interest rate, capital and the financing spread). Because the main propagation mechanism is on the supply side, inflation falls when output rises (and vice versa).\(^{55}\) So during the sharpest boom, inflation experienced its sharpest decline. However, in contrast to output, inflation oscillates relatively evenly around zero (i.e. we don’t observe any temporal shifts in the trend).

As expected, the interest rate responds elastically to the evolution of inflation (see Figure II.18). Nevertheless, it is smoother than inflation since we have included an interest rate lag in the Taylor rule (see DeGrauwe and Macchiarelli, 2014), which smoothens the reaction of the interest rate to inflation. We also observe a lag in the response of inflation to monetary policy over the cycle, in line with observations from the data.

Capital, on the other hand, is positively skewed and is mostly above the zero line during the entire period. Since it is a stock variable, that is to be expected and in line with the US data (see table II.6. In addition, capital accumulates the most during the long expansionary period discussed above, and contracts under the proceeding episode. Just as the general business cycle, the subsequent capital accumulations are weaker, and the stock of capital is still below its pre-crisis level 40 quarters (or 10 years) after the bust. Contrast that to the Great Recession episode.

In the same vein, utilization costs are also positively skewed (see Figure II.24), but more volatile than output. This is to be expected since utilization cost function is of second order (see equation 4) and depends directly on the production capacity. Therefore the volatility of production will be squared, which increases the fluctuations in the cost. Also, as Figure II.19 shows, the more capital is accumulated and

\(^{54}\) However, to confirm this fact one would need to perform a structural breaks analysis on the full data, which includes the trend.

\(^{55}\) See Figure II.18 for the correlation between output and inflation during the entire period.
used in production, the higher utilization costs the producer will face (due to the inherent trade-offs explained in subsection 2.1.2). The correlation between the two is positive throughout the entire period.

Just as in the DeGrauwe (2011,12) and DeGrauwe and Macchiarelli (2015) models, output is highly correlated with animal spirits throughout the entire period. Its correlation with animal spirits is 0.83 (see figure II.26 and table II.4). We can interpret the role of animal spirits in the model as follows. When the animal spirits index clusters in the middle of the distribution we have tranquil periods. There is no particular optimism or pessimism, and agents use a fundamentalist rule to forecast the output gap. At irregular intervals, however, the economy is gripped by either a wave of optimism or of pessimism. The nature of these waves is that beliefs get correlated. Optimism breeds optimism; pessimism breeds pessimism. This can lead to situations where everybody has become either optimist or pessimist. The index then becomes 1 respectively 0. These periods are characterized by extreme positive or negative movements in the output gap (booms and busts).

Let us continue by examining one of the novelties of this model, the share of loan down payment. It is clear from Figure II.20 that when the economy expands and the stock market booms, the share of loans required by CGP for pre-payment is very low, and often zero. This is due to the stock market boom implying a low probability of default for entrepreneurs (since its collateral value is high, or loan-to-value ratio low). Because of this low probability of default, entrepreneurs will be able to borrow more, increasing their (expected) cash positions and so CGP will not require a pre-payment. In contrast during an exceptionally sharp contraction (as in $t = [295, 300]$) CGP become wary of entrepreneur’s ability to pay for their capital purchases in the next period, and therefore require a high share to be pre-paid. The higher the contraction, the higher the share required to be pre-paid (see lower graph in Figure II.20). The model is capable of generating these asymmetries over the cycle.

To conclude, we see a strong co-movement between asset prices on one hand, and net worth and the financing spread on the other. During stock market booms, net worth rises which increases firm’s collateral value and reduces its probability of default, and so it reduces the external financing spread (as it is less risky for banks to lend to firms).
6 DSGE versus behavioural: Two worlds, one vision?

Having completed the separate analysis of the two models, we are in position to compare and discuss the relative performance of the two frameworks. To achieve that, we will first and foremost compare the statistical matching of the two models in order to evaluate which model does overall a better job, as well as what features are more accurately modelled in each of the frameworks. Second, we will compare the impulse responses and examine where the strongest transmission mechanism exists. Lastly, we will discuss the importance of capturing business cycle asymmetries by looking at the empirical regularities in the US data and compare it to the quantitative results in the behavioural model. To facilitate such comparisons, we have kept the endogenous mechanisms and shocks (to the extent possible) the same in both models. As a rough verifier of this synchronicity, notice in Table II.5 that the persistence as well as the amplitude of the business cycles are very similar in both models (0.85 vs 0.59 and 2.47 vs 3.08).

6.1 Second moment matching: One vision?

Starting with the matching of correlations in Table II.4, both models do a good job in capturing many (if not most) of the correlations. However, the behavioural model does even better and matches 13 correlations better than the DSGE, while the opposite number is 4. In addition the behavioural model manages to exactly match 5 of the correlations and the DSGE model 4. Roughly speaking, the correlations that the behavioural model is better in capturing are the autocorrelations (output, capital, inflation), the stock market cycle (stock price-output, animal spirits/residual earnings-output), prices (inflation-output, real rate-output, inflation-interest rate), and many of the supply-side relations (loan supply-output, marginal costs-output, capital-interest rate, capital-loan supply, and labor demand-output). On the other hand, the DSGE model is better in matching the capital series (capital-output, capital-marginal costs and the autocorrelation of capital) and some demand-side variables (investment-output, consumption-output).

56 The only 5 correlations that neither of the models manage to replicate are the savings-output, loan supply-marginal costs, autocorrelation of marginal costs, net worth of banks-output, and net worth of firms-output.
We find a similar pattern for the second moments in Table II.5. Since in both models the amplitude of the business cycles is very similar, we can be safe in directly comparing the relative standard deviations. In 8 cases, the behavioural model matches more precisely the second moments, while the number of cases is 5 for the DSGE. So the behavioural model has a comparative advantage in prices (inflation, interest rate, stock market price), and some of the financial accelerator variables (loan supply, net worth of banks, net worth of firms). The DSGE model, on the other hand, matches more of the supply side variables (marginal costs, labor demand) as well as some demand-side ones (investment, consumption, book value). To sum up the empirical fit, both models do a good job in capturing the standard statistical (second) moments in the US data. While the strength in the behavioural framework lies in replicating the autocorrelations, the statistical moments of prices (including stock prices), some stock variables, and some supply-side relations, the DSGE has a comparative advantage with respect to capital and the demand-side relations. Nonetheless, the behavioural framework outperforms the DSGE in the total number of replicated moments.

### 6.2 Impulse responses: One vision?

Continuing with the impulse responses, we generally observe a starker transmission and higher responses to supply side shocks in the behavioural model but to financial/monetary shocks in the DSGE model. Let us begin with the TFP shock. Comparing the two Figures II.11 and II.1, it seems that the TFP shock is, in relative terms, transmitted more heavily via the demand side onto output in the DSGE model. This inference is based on the fact that while investment responds by significantly more in the DSGE model (3.5% vs 0.3%), inflation rises (while it falls in the behavioural), the interest rate marginally rises (while it falls heavily in the behavioural) and output rises by less (1% vs 1.15%). Furthermore, the financial market variables in the behavioural model converge towards a significantly higher level compared to the pre-shock state. Remembering moreover that the autoregressive parameter in the DSGE model is set to 0.99 while none is included in the behavioural, it implies that the supply-side transmission is much more powerful in the behavioural model compared to the DSGE. A similar pattern is observed for a

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57 The only 3 standard deviations that neither of the models were capable of matching is capital, animal spirits/residual earnings, and savings.
shock in utilization costs. For the financial shock, on the other hand, the impulse responses in the DSGE model are between 5 to 10 times higher. In the DSGE (behavioural) model, output rises by 2% (0.4%), inflation by 0.2% (0.035%), investment by 10% (0.2%), residual earnings by 1.8% (0.6%) and capital by 1% (0.1%). For this shock, no autoregressive parameter has been included in either of the models. Therefore in this case, the cognitive limitation of agents plays a smaller role in the propagation of financial shocks, while the supply channel as a financial shock propagator plays a more important role.\(^{59}\)

### 6.3 Business cycle asymmetries: How important?

The last point of comparison is the relative importance of including model asymmetries over the business cycle. We have already seen that many of the variables in the US data do not have the same amplitude during expansions and recessions. However, the fundamental question is how important these are for the general business cycle modelling and for understanding the core propagation mechanisms in an economy? Is a symmetric approach a good approximation? We will attempt to answer this question under the current framework. Since we make use of a method (linear approximation) that produces symmetric distributions (DSGE), and at the same time a highly non-linear that produces asymmetric distributions (behavioural), we are capable of evaluating the relative fit of linear approximations to data, as well as try to provide an answer to the question of whether highly non-linear (and complex) modelling tools are necessary in order to understand the latent underlying structure of an economy.

If we look at the statistical values of the US business cycle in Table II.6, the series seems to be weakly skewed (skewness factor=-.042), but highly platykurtic (kurtosis=0.22). Thus the business cycle is roughly symmetric and has almost no tails, meaning that the economy has not experienced exceptional expansions or recessions since 1953. This is easy to replicate in a linearly approximated model by removing autoregressive components to shocks. In the behavioural model, on the

\(^{59}\) A similar difference is observed following a monetary policy shock. The impulse responses in the DSGE model are between 3 and 5 times higher. The exception is investment, where the response is 20 times higher in the case of the DSGE model, pointing towards a stronger investment channel of monetary policy in that framework.
other hand, this is more tricky since the cycles are excessively skewed (skewness=-1.66) and the tails excessively fat (kurtosis=15.94). Equally easy to replicate in a linearly approximated model is the distributions of capital, consumption and investment since they are all roughly symmetric and platykurtic. The skewness factors are 0.40, -0.05 and 0.76, and their respective kurtosis are -1.44, 0.36 and 1. In the behavioural model, on the other hand, the series are excessively skewed (-1.669 and -6.27) and highly leptokurtic (16.30 and 24.7). Hence for these three variables, a standard DSGE linear approximation is preferred.

For the other variables, the conclusion is very different. Most of them are skewed and leptokurtic. This is much easier to capture in the behavioural model. So, for instance, the distribution of marginal costs is almost perfectly replicated in the behavioural model. Also the distributions of financial variables (such as loan supply, deposits, interest rate, net worth of firms, net worth of banks) and of prices (such as inflation, stock market prices and animal spirits/residual earnings) are closely characterized in the model. If anything, the asymmetry or kurtosis of these variables is, in general, (much) higher in the model than in the data, even if the general pattern is well captured. In these instances, therefore, it is not appropriate to apply a linear approximation method since these statistical anomalies would not at all be captured. Even non-linear perturbation methods recently applied in the DSGE literature would struggle to accomplish these distributions without including many frictions and shocks. Therefore which model to use depends very much on what you are interested in examining. If the focus is on the general business cycle and/or the aggregate demand, then the linearly approximated DSGE model is a good option since it is tractable and easy to solve without compromising on the complexity. If, on the other hand, the focus is to understand financial frictions, the financial cycle, or the impact of supply-financial interactions on business cycle anomalies, then the behavioural model is the obvious option.

As a final remark on the methodological differentiation, bear in mind that the asymmetries in the behavioural model are endogenously generated from the learning mechanism of the agents. The interaction between market frictions and the learning set-up in the model leads to powerful propagation of shocks. In the DSGE model,

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60 Remember that the third and fourth moments are all reported in relative terms with respect to the general business cycle. Hence, to get the original moments, you have to add the relative one to the values reported for output.

61 Capital is the exception since it is almost symmetric and platykurtic.
on the other hand, this propagation is achieved via the interaction between market frictions and highly persistent shocks.\footnote{Except for the shock to financial costs and utilization costs, where no AR parameter is included, so the forceful propagation is purely generated from the endogenous model dynamics.}

To sum up the comparative section, both models perform well in matching the standard statistical (second) moments in the data, as well as generating powerful propagation of shocks. Including the stock market and the interaction between aggregate supply, financial accelerator and stock markets improves significantly the empirical fit of the financial accelerator model, and it rightly reproduces the recent observations made by Broadbent, Massani and others regarding the most recent business cycle. In it, not only does the supply side amplify the business cycles, but it acts as a powerful propagator of financial shocks. Having said that, to additionally relax the rational expectations hypothesis improves even further the empirical fit, and the asymmetric nature of many macroeconomic and financial variables. Moreover, the model construction is relatively straight-forward, intuitive, and the behaviour of the agents micro-founded. The cost, however, is that the supply side becomes a weaker propagator of financial (and monetary) shocks, as the impulse response comparison showed. Lastly, the tractability of the model solution is to some extent compromised in comparison to the DSGE model.

7 Discussion and concluding remarks

Including credit frictions on the supply side is a novel way of thinking about financial frictions in the macroeconomics literature. Sharp rises in stock prices do not only allow firms to increase their credit and capital demand, but can equally reduce the input costs for firms, or their input-output ratio. Conversely, a sharp drop in asset prices can restrict the supply of credit to firms, increase their production costs, reduce the supply of capital, and (over time) reduce their production capacity (or productivity).

In the current paper, we have examined the role that the aggregate supply plays in propagating shocks generated elsewhere, and quantified the importance of this channel. In addition, we have performed a (theoretical) comparative analysis of this mechanism by including the former in a fully rational DSGE framework and contrast its performance to a bounded rationality behavioural model.
We find that including the above mechanisms in an otherwise standard financial accelerator model intensifies the transmission of shocks by between 15 and 25%. Compared to a model where only the stock market mechanism is incorporated, the impulse responses to a financial shock, for instance, are on average 25% lower. Variance decomposition further affirms the importance of aggregate supply-financial market interaction since approximately 75% of the model variation can be explained by the financial and TFP shocks jointly. On a deeper level, the comparative analysis between the rational expectations DSGE and behavioural models shows that both perform well in matching the data moments, as well as generating powerful propagation of shocks. The empirical fit is much better compared to competing models where those mechanisms are excluded. Nonetheless, to additionally relax the rational expectations hypothesis improves even further the empirical fit to data, and the asymmetric nature of many macroeconomic and financial variables. The trade-off, however, is that the supply side becomes (in relative terms to the DSGE model) a weaker propagator of financial (and monetary) shocks.
References


Appendices

I Model derivations in the DSGE model

I.1 The Ohlson (1995) model

The model uses two standard characteristics of the accounting models, and one behavioral assumption in order to characterize the wedge between market and book values within the neoclassical framework. Therefore, as Rubinstein (1976) shows, the value of an asset can be expressed as the present value of expected dividends (PVED):

\[ P_t = \sum_{i=1}^{\infty} \frac{E_t [d_{t+i}]}{(1+r)^i} \]  

where \( P_t \) is the market value of capital, \( D_t \) are dividends, and \( r \) is the risk-free rate. A two-step procedure derives a particularly parsimonious expression for residual earnings, or goodwill, which collects the difference between the market and book value of assets. First, the clean surplus relation:

\[ Q_t - Q_{t-1} = e_t - d_t \]  

implies the restriction that dividends reduce current book value, but not the current earnings (but negatively the future), i.e.:

\[ \frac{\partial Q_t}{\partial d_t} = -1 \]  
\[ \frac{\partial e_t}{\partial d_t} = 0 \]  
\[ \frac{\partial E_t [e_{t+1}]}{\partial d_t} = -(r - 1) \]

Peasnell (1982) shows that this condition is sufficient to express market valued in terms of future expected earnings and book value (instead of the sequence of expected dividends). To do so, let us first define residual earnings as:

\[ re_t \equiv \frac{e_t}{(r-1)q_{t-1}} \]
Combined with the clean surplus condition above (expression I.2, we can express dividends in terms of:

$$d_t = re_t - Q_t + RQ_{t-1}$$  \hspace{1cm} (I.7)

Iterating the last expression forward for \(d_{t+1}, d_{t+2}, \text{etc.}\) and re-inserting it into PVED, we get:

$$P_t = Q_t \sum_{i=1}^{\infty} \frac{E_t[re_{t+i}]}{r^i}$$  \hspace{1cm} (I.8)

provided that \(\frac{E_t[re_{t+i}]}{r^i} \to 1\) as \(i \to \infty\). Residual earnings is motivated by the concept that ‘normal earnings’ are return on the capital invested at the beginning of the period, which are equal to the (replacement) cost of using the capital, i.e. \(r*Q_{t-1}\) (book value at time \(t-1\) multiplied by the (risk-free) interest rate).\(^6\) Hence, during profitable periods, earnings are above the cost of using the capital, or the same as saying positive ‘residual earnings’. One can link this idea back to the Bank of England 2012 report by conceptualizing the profitable periods as periods of optimism. During periods of high market confidence, the capital is expected to generate a present value of future earnings above the required earnings demanded by investors, or the same as saying, positive residual earnings. In other words, the future profitability of capital, as measured by the present value of future (anticipated) residual earnings sequence reconciles the difference between the market and the book value of capital.

Second, to complete the model the time-series behavior of residual earnings need to be specified. Ohlson (1995) assumes an autoregressive process

$$re_{t+1} = \alpha re_t + v_t$$  \hspace{1cm} (I.9)

where \(\alpha\) is restricted to be positive, and \(v_t\) is a scalar variable that represents information regarding future expected (residual) earnings other than the accounting data and dividends. Ohlson (1995) motivates it by the idea that some value-relevant events may affect future expected earnings as opposed to current earnings which means that accounting measures incorporate these value-relevant events only after some time. The scalar information variable is independent of past residual earnings since the value relevant events have yet to have an impact on the financial

\(^6\)The model assumes risk neutrality and homogeneous beliefs, even though Ohlson (1995) has extended the model to include other risk preferences.
earnings since the value relevant events have yet to have an impact on the financial statements. This is the same as saying:

\[
\frac{\partial v_t}{\partial re_{t-1}} = 0 \quad (I.10)
\]

since it captures all non-accounting information used in the prediction of future residual earnings. On the other hand, the variable may depend on past realizations of the same scalar (even if that is not necessary), since they can feed expectations about future earnings via past beliefs.

Given the assumption of the stochastic process of residual earnings, one can evaluate \( \sum_{i=1}^{\infty} r_i \), and reduce expression I.8 to:

\[
P_t = Q_t \alpha re_t v_t \quad (I.11)
\]

Market value can now be reduced to a composite of book value, residual earnings measuring current profitability and other information that modifies the prediction of future profitability. Rearranging this expression and using the definition of residual earnings in I.6, one can also express next period’s expected (total) earnings as:

\[
E_t [e_{t+1}] = (r - 1)Q_t \alpha re_t v_t \quad (I.12)
\]

Note that future earnings only partially depend on the current book value.

Since next period (expected) earnings are formed using information set available up to period \( t \) for all the three components (book value, residual earnings, and information), the expression poses no problem. However, for earning forecasts two periods ahead, the model yields no prediction since information from period \( t + 1 \) is necessary in order to forecast this variable.

To conclude, though the process \( [P_t - Q_t] \) allows for serial correlations over sufficiently long periods of time, the average realization approximates zero. This means that in the very long-run, book value will become the unbiased estimator of market value.
I.2 The financial accelerator model and the optimization problems

I.2.1 Households

The representative risk-averse household maximizes its lifetime utility, which depends on consumption $C_{t+k}$, real money balances $M/P_{t+k}$ and labor hours (fraction of hours dedicated to work = $H_{t+k}$):

$$\max_{c_t,m_t} E_t \sum_{k=0}^{\infty} \beta^k \left[ \ln(C_{t+k}) + \varsigma \ln \left( \frac{M_{t+k}}{P_{t+k}} \right) + \theta \ln(1 - H_{t+k}) \right]$$

constrained by lump-sum taxes he pays in each period $T_t$, wage income $W_t$, and dividends he earns in each period from owning the representative capital good and retail firms $\prod_t$, and real savings he deposits in the intermediary, $D_t$ according to the budget constraint:

$$C_t = W_t H_t - T_t + \prod_t + R_t D_t - D_{t+1} + \left[ \frac{M_{t-1} - M_t}{P_t} \right]$$

The household takes $W_t$, $T_t$, and $R_t$ as given and chooses $C_t$, $D_{t+1}$, $H_t$ and $M/P_t$ to maximize its utility function subject to the budget constraint.

I.2.2 Capital Good Producer

The first key agent in this model is the raw material producer that sells capital to entrepreneurs (or intermediary good producers). They operate in perfectly competitive markets and are owned by households. Capital Good producer (CGP) chooses capital $K_t$ and investment good $I_t$ to produce capital services $K_{CGP}$, which it sells to entrepreneurs at price $Q_t$. CGP maximizes its profit function

$$\max_{K_t,I_t} E_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \left[ Q_t [K_t - (1 - \delta)K_{t-1}] - I_t \right]$$

subject to the investment adjustment cost function:

$$K_t = (1 - \delta)K_{t-1} + [1 - \kappa \left( \frac{i_t}{i_{t-1}} - 1 \right)]^2 I_t$$

where $[1 - \frac{\kappa}{2} (\frac{i_t}{i_{t-1}} - 1)]^2 I_t$ is the adjustment cost function. $\kappa$ denotes the cost for adjusting investment.
I.2.3 Entrepreneur

The other key agent in this model, the representative entrepreneur, chooses capital $K_t$, labor input $L_t$ and level of borrowings $B_{t+1}$, which he determines at the beginning of each period and before the stock market return has been determined, and pays it back at the end of each period as stated by $R_{t+1}[Q_tK_{t+1} - N_{t+1}]$ (where $R_{t+1}$ is the risk-free real rate that borrowers promise lenders to pay back on their loans, $R_{t+1}^k$ is the return on market value of assets, $K_{t+1}$ is the quantity of capital purchased at 't+1' and $N_{t+1}$ is entrepreneurial net wealth/internal funds at 't+1') to maximize his profits according to:

$$V = \max E_0 \sum_{k=0}^{\infty} [(1-\mu) \int_0^\omega \omega dF \omega U_{t+1}^r E_t(R_{t+1}^k) S_t \psi(u_t) K_{t+1} - R_{t+1}[S_tK_{t+1} - N_{t+1}]]$$

(I.17)

with $\mu$ representing the proportion of the realized gross payoff to entrepreneurs' capital going to monitoring, $\omega$ is an idiosyncratic disturbance to entrepreneurs' return (and $\omega$ is hence the threshold value of the shock), $E_tR_{t+1}^k$ is the expected stochastic return to stocks, and $U_{t+1}^r$ is the ratio of the realized returns to stocks to the expected return ($\equiv R_{t+1}^k/E_tR_{t+1}^k$). The entrepreneur uses household labor and purchased capital at the beginning of each period to produce output on the intermediate market according to the standard Cobb-Douglas production function:

$$Y_t = A_t[\psi(u_t)K_t]^\alpha L^{1-\alpha}$$

(I.18)

where $Y_t$ is the output produced in period 't', $A_t$ is an exogenous technology parameter, $K_t^\alpha$ is the share of capital used in the production of output, $\psi(u_t)$ are the utilization costs, and $L^{1-\alpha}$ is the labor share. Utilization costs are defined as:

$$\psi(u_t) = \xi_0 + \xi_1(u_t - 1) + \frac{\xi_2}{2}(u_t - 1)^2$$

(I.19)

The physical capital accumulates according to the law of motion:

$$K_{t+1} = \Phi \frac{I_t}{K_t} K_t + (1-\delta) K_t$$

(I.20)

with $\Phi \frac{I_t}{K_t} K_t$ denoting the gross output of new capital goods obtained from investment $I_t$, under the assumption of increasing marginal adjustment costs, which we capture by the increasing and concave function $\Phi$. $\delta$ is the depreciation rate of capital.
The entrepreneur borrows funds from the financial intermediary, as a complement to its internal funds, in order to finance its purchase of new capital, which is described by the following collateral constraint:

$$B_{t+1} \leq \psi(s_t)N_{t+1} \quad (I.21)$$

and the cost of external funding, which is represented by the external finance premium (EFP) condition:

$$E_t\left(\frac{R^{ks}_{t+1}}{R_{t+1}}\right) = s \left[ \frac{N_{t+1}}{S_tK_{t+1}} \right] \quad (I.22)$$

By assuming a fixed survival rate of entrepreneurs in each period, the model assures that entrepreneurs will always depend on external finances for their capital purchases, and are further assumed to borrow the maximum amount, subject to the value of their collateral (which means that the collateral constraint will bind with equality).

The entrepreneur also faces the cash-in-advance constraint in its purchases of capital in the input market. More specifically, it must pre-pay a share of its total capital purchases, which depends on its net borrowing position according to:

$$E_t[S_{t+1}]K_{t+1} \leq \vartheta_tB_t = \vartheta_t[E_t[S_{t+1}]K_{t+1} - N_t] \quad (I.23)$$

To complete the model, let us look at the maximization problem of the remaining agents in the model: financial intermediaries, retailers and government.

### I.2.4 Financial intermediary

The role of the financial intermediary in this model is to collect the deposits of savers, and to lend these funds out to borrowers through 1-period lending contracts against a risk-free return $R_t$ that households demand on their deposits. Because of information asymmetries between lenders and borrowers, the intermediary needs to invest some costly monitoring of the borrowers in order to assure that borrowers survive to the next period and pay back the return on deposits. Therefore the wedge between the rate they charge entrepreneurs for their borrowings, $R^{ks}_t$, and the one that they pay out to households for their deposits $R_t$ reflects this monitoring cost. Most importantly, the intermediary can not lend out more than the deposits they have (incentive constraint), and it operates in a perfectly competitive market. This
means that in each period, intermediaries choose a level of borrowings $B_t$ in order to maximize:

$$\max_{B_t} F = E_t \sum_{t=0}^{\infty} (R_{t-1}^{ks} B_t - B_{t+1}) - (R_{t-1} D_t - D_{t+1}) - \mu(\omega R_{t+1} Q_t K_{t+1}) = \pi_t$$  \hspace{1cm} (I.24)

where $\mu \omega R_{t+1} Q_t K_{t+1}$ is the monitoring cost of borrowers. The amount of lending is constrained by the incentive constraint:

$$B_{t+1} \leq D_{t+1}$$  \hspace{1cm} (I.25)

and the intermediary makes zero profits in each period:

$$\prod_{t=0}^{\infty} \pi_t = 0$$  \hspace{1cm} (I.26)

### I.2.5 Retailer

To incorporate the nominal rigidities, a standard feature of New-Keynesian models, we incorporate a retail sector into this model. Let us look at retailers’ problem. They set their price of the final good according to the standard Calvo process (1983), where $P_t^*$ is the price set by retailers who are able to change prices in period ‘t’, and let $Y_t^*(z)$ denote the demand given this price. Then, retailer ‘z’ chooses $P_t^*$ in order to maximize:

$$\max_{P_t(j)} \Omega_t = E_0 \sum_{k=0}^{\infty} \theta^k \Lambda_{t,k}[P_t(j)y_t(j) - P_t^*y_t(j) - \frac{\kappa_p}{2} \frac{P_t(j)}{P_{t-1}(j)} - \pi_{t-1}^{l_p} \pi_{t-1}^{-1-l_p}]^2 P_t y_t]$$  \hspace{1cm} (I.27)

with $\theta^k$ being the probability that a retailer does not change his price in a given period, $\Lambda_{t,k} \equiv \beta C_t/C_{t+1}$ denoting the household intertemporal marginal rate of substitution (since households are the shareholders of the retail firms), which they take as given, and $P_t^* \equiv P_t/X_t$ denoting the nominal price of goods produced by a retailer ($X_t$ is the gross markup of retail goods over wholesale goods). They face a demand curve equal to:

$$y_t(j) = [\frac{P_t(j)}{P_t}]^{-\epsilon} y_t$$  \hspace{1cm} (I.28)

where $P_t(z)/P_t^{-\epsilon}$ is the nominal price ratio of wholesale goods for retailer ‘z’, $\epsilon > 1$ is a parameter on retail goods, $Y_t^f$ is the total final output in the economy which is composed by a continuum of individual retail goods, and $P_t$ is the composite nominal price index of a continuum of individual prices set by retailers.
I.2.6 Government

Finally, a government plans spending, and finances it by either lump-sum taxes, or money creation (Central Bank division). In each period, it chooses spending $G_t$, and a combination of taxes $T_t$ and money creation $M_t$ so to fulfil the balanced budget condition:

$$G_t = \frac{M_t - M_{t-1}}{P_t} + T_t$$  \hspace{1cm} (I.29)

It chooses money creation for budget financing according to a standard Taylor rule:

$$(1 + R^n_t) = (1 + R^n_{t-1})^\rho + \left[ \frac{E_t[\pi_{t+1}]}{\pi} \right]^{\xi}$$  \hspace{1cm} (I.30)

where $R^n_t$ is the policy rate in period ‘t’, $\rho$ is the coefficient of interest rate growth, and $\xi$ is the coefficient on expected inflation in the Taylor rule.

I.3 Solutions

Solving the households optimization problem yields standard first order conditions for consumption,

$$\frac{1}{C_t} = E_t \left( \beta \frac{1}{C_{t+1}} \right) R_{t+1}$$  \hspace{1cm} (I.31)

labor supply,

$$W_t \frac{1}{C_t} = \theta \frac{1}{1 - H_t}$$  \hspace{1cm} (I.32)

and real money holdings,

$$\frac{M_t}{P_t} = \varsigma C_t \left[ \frac{R^n_{t+1} - 1}{R^n_{t+1}} \right]^{-1}$$  \hspace{1cm} (I.33)

The last equation implies that real money balances are positively related to consumption, and negatively related to the nominal/policy interest rate.

In the input market, the solution to CGP’s problem is:

$$Q^k_t - \beta E_t[\frac{\lambda_{t+1}}{\lambda_t}(1 - \delta)Q^{k}_{t+1}] = \lambda^{cpp}_t - \beta E_t[\frac{\lambda_{t+1}}{\lambda_t}(1 - \delta)Q^{cpp}_{t+1}]$$  \hspace{1cm} (I.34)

and

$$Q^k_t = \lambda^{cpp}_t \Rightarrow E_t[Q^k_{t+1}] = E_t[\lambda^{cpp}_{t+1}]$$  \hspace{1cm} (I.35)
Turning to the entrepreneur, his choice of labor demand, capital, and utilization costs yields the following optimization conditions:

\[
(1 - \alpha) \frac{y_t}{l_t} = x_t w_t \tag{I.36}
\]

\[
\lambda_t q^k_t = E_t[S_{t+1}] K_t - N_{t+1} + \lambda_{t+1} \alpha a_t u_t \left[ \alpha^{-1} - \lambda_{t+1} \right] u_{t+1} + (1 - \delta) q^k_{t+1} - \psi(u_{t+1}) \tag{I.37}
\]

\[
\alpha a_k [k_{t-1} u_t]^{\alpha - 1} l_t^{1 - \alpha} k_{t-1} - [\xi_1 + \xi_2 (u_t - 1)] k_{t-1} = 0 \Rightarrow \\
\xi_1 + \xi_2 (u_t - 1) = \alpha a_k [k_{t-1} u_t]^{\alpha - 1} l_t^{1 - \alpha} \equiv r^k_t \tag{I.38}
\]

As is common in the literature, marginal product of labor equals their wage. Labor input in the production of wholesale goods can either come from household or entrepreneurs own labor supply (i.e. they can devote a small fraction of their own time to the production activity). Therefore entrepreneurs receive income from supplying labor based on the wage rate above. Since that income stream is assumed to be marginal in this model however, we can assume that the proportion of entrepreneurial labor used for production of wholesale goods is so low that it can be ignored, so that all of labor supply is provided by the household sector.

Continuing our analysis with the retailers, and differentiating their objective functions with respect to \( P_t^* \) gives us the following optimal price rule:

\[
1 - \epsilon_t^y + \frac{\epsilon_t^y}{x_t} - \kappa_p (\pi_t - \pi_{t-1}^l - \pi_t^p) \pi_t + \beta_p^{t+1} E_t \left[ \frac{\lambda^k_{t+1}}{\lambda_t^k} \kappa_p (\pi_{t+1} - \pi_{t-1}^l - l_{t+1}) (\pi_{t+1})^2 \frac{y_{t+1}}{y_t} \right] = 0 \tag{I.39}
\]

We can aggregate across retailers, and express their profits (with Calvo pricing) as:

\[
j_t^R = y_t \left[ 1 - \frac{1}{x_t} - \frac{\kappa_p}{2} (\pi_t - \pi_{t-1}^l \pi_1 - l_{t+1})^2 \right] \tag{I.40}
\]

To conclude the optimizations, we turn to the representative intermediary. He will set the maximum level of lending such that the incentive constraint is satisfied, and subject to the competitive market condition. Differentiating his value function
with respect to $B_{t+1}$ will mean that the level of credit given to the entrepreneurial sector will be:

$$B_{t+1} = D_{t+1}$$  \hspace{1cm} \text{(I.41)}$$

This condition will hold in each period, which means that the intermediary’s balance sheet expansion is limited to its deposit holdings in each period.

### I.4 General equilibrium

The competitive equilibrium is characterized by a set of prices and quantities, such that:

- Given $W_t, R_t, \xi, \theta, D_t$ the household optimizes $C_t, H_t, D_{t+1}$
- Given $K_t, I_t$ the capital good producer optimizes $K^C_{t-GP}$
- Given $W_t, R_t, R^{ks}_t, \gamma, \delta, z_t, \mu, N_t, K_t, \psi(u_t)$ the entrepreneur optimizes $I_t, H_t, K_{t+1}, B_{t+1}, N_{t+1}, Y_t$
- Given $R_t, R^{ks}_t, D_{t+1}, B_t$ the financial intermediary optimizes $B_{t+1}$
- Given $A_{t,k}, \theta, Y_t(z), P_t^o$ the retailer ’z’ optimizes $P_t^*$

Labor, capital and financial markets clear: $H^*_t = H^d_t, K^*_t = K^d_t, D_t = B_t$

In the final goods market, the production is governed by the following resource constraint: $y_t = c_t + q^k_t[k_t - (1 - \delta)k_{t-1}] + k_{t-1}\psi(u_t) + Adj_t$ with $Adj_t = \frac{\pi^p_t}{2}(\pi^p_t - \pi^p_{t-1})^{1/2} y_t$

and

Aggregate demand holds: $Y_t = C_t + I_t + C^e_t + G_t$

The complete log-linearized model is presented below by the Equations I.42 through I.61. In all the equations, lower case letters denote percentage deviations from steady state, and capital letters denote steady state values:
I.5 Log-linearized model

Aggregate Demand:

Resource constraint

$$y_t = \frac{C}{Y} c_t + \frac{I}{Y} i_t + \frac{C_e}{Y} c^e_t$$  (I.42)

Consumption Euler equation

$$c_t = -r_t + E_t(c_{t+1})$$  (I.43)

Entrepreneurial consumption

$$c^e_t = n_t$$  (I.44)

Financial accelerator

$$r^k_{t+1} - r_t = -\nu(n_t - (q_t + k_t))$$  (I.45)

External Finance Premium

$$efp_t = r^k_t - r_t$$  (I.46)

Return on capital

$$r^k_t = (1 - \epsilon)(y_t - k_t - x_t) + \epsilon s_t - s_{t-1}$$  (I.47)

Investment accelerator

$$s_t = \psi(i_t - k_t)$$  (I.48)

(Stock) Market value of capital

$$s_t = q_t + re_t$$  (I.49)

Residual earnings and formation of stock market expectations

$$re_t = \rho_{re} re_{t-1} + (\chi)(E_t[y_{t+1}] + n_t - E_t[r_{t+1}]) + e_i$$  (I.50)

Aggregate Supply:

Cobb-Douglas production function

$$y_t = a + \alpha k_t + \alpha \psi(u_t) + (1 - \alpha) \omega h_t$$  (I.51)
Capital (service) production

\[ i_t = \frac{1}{1 + \beta} i_{t-1} + \frac{\beta}{1 + \beta} E_t[i_{t+1}] + \frac{1}{\kappa(1 + \beta)} q_t \] (I.52)

Utilization costs

\[ \psi(u_t) = \xi_1(u_t - 1) + \xi_2(u_t - 1) \] (I.53)

Cash-in-advance constraint

\[ 1 + n_t - (E_t[s_{t+1}] + k_{t+1}) = \vartheta_t \] (I.54)

Marginal cost function

\[ y_t - h_t - x_t - c_t = \frac{1}{\eta} h_t \] (I.55)

Approximated Philips curve

\[ \pi_t = \kappa(-x_t) + \beta\pi_{t+1} \] (I.56)

Evolution of State Variables:

Capital accumulation

\[ k_t = \delta i_t + (1 - \delta \psi(u_t)) k_{t-1} \] (I.57)

Net worth accumulation

\[ n_t = \gamma R \frac{K}{N} (r_t^k - r_t) + r_{t-1} + n_{t-1} \] (I.58)

Monetary Policy Rule and Shock Processes

Monetary policy

\[ r_t^n = \rho r_{t-1}^n + \zeta E_t[\pi_{t+1}] \] (I.59)

Technology shock

\[ a_t = \rho_a a_{t-1} + e_a \] (I.60)

Real interest rate (Fisher relation)

\[ r_t^n = r_t - E_t(\pi_{t+1}) - e_r^n \] (I.61)
I.6 Residual earnings

Let us characterize the process governing residual earnings. Our main purpose is to establish a bridge between the residual earnings and the general state of the economy. We assume that $X^{re}$ follows an AR (1) process and make it in addition contingent on economic fundamentals in the next period, $F_{t+1}|I_{t+1}$ according to:

$$X^{re}_{t+1} = \rho (X^{re}_t) + F_{t+1}|I_{t+1}$$  \hspace{1cm} (I.62)

where $\rho$ is restricted to be positive. Following a vast number of empirical studies (outlined in section 2.3) who find a strong link between stock prices and economic fundamentals, this definition gives an important role to the evolution of the economy in determining residual earnings. Since residuals earnings are related to a firm’s growth perspectives and their future earnings, the economic fundamentals that are relevant in this case are entrepreneurial output or industrial production $Y_{t+1}$ (from I.18 in the firm’s optimization problem), firm equity $N_{t+1}$ (from I.17 and I.21 in the firm’s optimization), and the nominal interest rate, $R_{t+1}$ (from the minimization of the borrowing cost in I.22). Thus, we have given the monetary authority an additional channel through which it can influence the stock markets, by altering the prospects for residual earnings of firms. The full residual earnings process can therefore be expressed as (in log-linearized format):

$$re_{t+1} = \rho_{re} re_t + (\chi) E_t[Y_{t+1} + N_t - R_{t+1}]$$  \hspace{1cm} (I.63)

The parameters $\rho_{re}$ and $\chi$ determine the importance of each factor (the autoregressive process and the (expected) economic fundamentals in the next period) in determining the value of residual earnings.
II Tables and Figures

II.1 Tables

Table II.1: Parameters and descriptions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<tr>
<td>$C/Y$</td>
<td>Share of consumption in resource constraint</td>
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<tr>
<td>$I/Y$</td>
<td>Share of investment in resource constraint</td>
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<td>$C_e/Y$</td>
<td>Share of entrepreneurial consumption in resource constraint</td>
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<td>$X$</td>
<td>Gross markup over wholesale goods</td>
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<td>Share of capital in production</td>
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<td>$\Omega$</td>
<td>Share of household labour in production</td>
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<td>$\eta$</td>
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<td>$R$</td>
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<td>Parameter on the expected state of the economy in the residual earnings equation</td>
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<td>0.3</td>
</tr>
<tr>
<td>$\xi_2$</td>
<td>Parameter 3 in the utilization cost function</td>
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</tr>
<tr>
<td>$\varepsilon_n$</td>
<td>Std. deviation of technology shock</td>
<td>0.5</td>
</tr>
<tr>
<td>$\varepsilon_m$</td>
<td>Std. deviation of nom. interest rate shock</td>
<td>0.5</td>
</tr>
<tr>
<td>$\varepsilon_k$</td>
<td>Std. deviation of financial shock</td>
<td>0.5</td>
</tr>
<tr>
<td>$\varepsilon_u$</td>
<td>Std. deviation of shock in the utilization cost function</td>
<td>0.5</td>
</tr>
<tr>
<td>$\rho$</td>
<td>AR parameter in monetary policy rule</td>
<td>0.95</td>
</tr>
<tr>
<td>$\zeta_f$</td>
<td>MP response to expected inflation</td>
<td>0.20</td>
</tr>
<tr>
<td>$\rho_n$</td>
<td>AR parameter of productivity shock</td>
<td>0.99</td>
</tr>
<tr>
<td>$\rho_{rk}$</td>
<td>AR parameter in financial shock</td>
<td>0</td>
</tr>
<tr>
<td>$\rho_{us}$</td>
<td>AR parameter in shock to utilization costs</td>
<td>0</td>
</tr>
</tbody>
</table>

Notes: The calibrated values are standard in the literature. Following Caglar (2012), the new AR parameter in the extended model, $\rho_{re}$ is calibrated to 0.67, in line with the corporate finance literature. Elasticity of external finance premium to leverage, we calibrate to 0.13.
### Table II.1: Parameters and descriptions

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibrated Calibration</td>
<td></td>
</tr>
<tr>
<td>C/Y Share of consumption in resource constraint</td>
<td>0.806</td>
</tr>
<tr>
<td>I/Y Share of investment in resource constraint</td>
<td>0.184</td>
</tr>
<tr>
<td>C_e/Y Share of entrepreneurial consumption</td>
<td>0.01</td>
</tr>
<tr>
<td>ϵ Marginal product in investment demand</td>
<td>0.99</td>
</tr>
<tr>
<td>X Gross markup over wholesale goods</td>
<td>1.10</td>
</tr>
<tr>
<td>α Share of capital in production</td>
<td>0.20</td>
</tr>
<tr>
<td>Ω Share of household labour in production</td>
<td>0.99</td>
</tr>
<tr>
<td>η Labour supply elasticity</td>
<td>5.00</td>
</tr>
<tr>
<td>κ Share of marginal cost in Phillips Curve</td>
<td>0.086</td>
</tr>
<tr>
<td>θ Calvo pricing</td>
<td>0.75</td>
</tr>
<tr>
<td>β Quarterly discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>δ Depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>γ Survival rate of entrepreneurs</td>
<td>0.973</td>
</tr>
<tr>
<td>R Steady state quarterly riskless rate</td>
<td>1.010</td>
</tr>
<tr>
<td>K/N Steady state leverage</td>
<td>2.082</td>
</tr>
<tr>
<td>ν Elast. of EFP to leverage</td>
<td>0.092</td>
</tr>
<tr>
<td>φ Elast of inv. demand to asset prices</td>
<td>0.25</td>
</tr>
<tr>
<td>ρ re AR parameter on residual earnings</td>
<td>0.67</td>
</tr>
<tr>
<td>χ Parameter on the expected state of the economy in the residual earnings equation</td>
<td>0.18</td>
</tr>
<tr>
<td>Ψ Adjustment cost function in investment</td>
<td>0.05</td>
</tr>
<tr>
<td>Κ Adjustments cost in investment parameter</td>
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</tr>
<tr>
<td>ζ Technological development parameter</td>
<td>0.5</td>
</tr>
<tr>
<td>ξ Parameter 1 in the utilization cost function</td>
<td>0.8</td>
</tr>
<tr>
<td>ξ1 Parameter 2 in the utilization cost function</td>
<td>3.3</td>
</tr>
<tr>
<td>ξ2 Parameter 3 in the utilization cost function</td>
<td>25</td>
</tr>
<tr>
<td>εa Std. deviation of technology shock</td>
<td>0.5</td>
</tr>
<tr>
<td>εrn Std. deviation of nominal interest rate shock</td>
<td>0.5</td>
</tr>
<tr>
<td>εrk Std. deviation of financial shock</td>
<td>0.5</td>
</tr>
<tr>
<td>εuc Std. deviation of shock in the utilization cost function</td>
<td>0.5</td>
</tr>
<tr>
<td>ρ AR parameter in monetary policy rule</td>
<td>0.95</td>
</tr>
<tr>
<td>ζf MP response to expected inflation</td>
<td>0.20</td>
</tr>
<tr>
<td>ρa AR parameter of productivity shock</td>
<td>0.99</td>
</tr>
<tr>
<td>ρrk AR parameter in financial shock</td>
<td>0</td>
</tr>
<tr>
<td>ρuc AR parameter in shock to utilization costs</td>
<td>0</td>
</tr>
</tbody>
</table>

**Notes:**
The calibrated values are standard in the literature. Following Caglar (2012), the new AR parameter in the extended model, $\rho_{re}$, is calibrated to 0.67, in line with the corporate finance literature. Elasticity of external finance premium to leverage, we calibrate to 0.13.

### Table II.2: Model variables and descriptions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$y$</td>
<td>Output</td>
</tr>
<tr>
<td>$c$</td>
<td>Household consumption</td>
</tr>
<tr>
<td>$c_e$</td>
<td>Entrepreneurial consumption</td>
</tr>
<tr>
<td>$i$</td>
<td>Investment</td>
</tr>
<tr>
<td>$g$</td>
<td>Government spending</td>
</tr>
<tr>
<td>$r^*$</td>
<td>Nominal interest rate</td>
</tr>
<tr>
<td>$r^b$</td>
<td>Real interest rate (also the (net) deposit rate of households)</td>
</tr>
<tr>
<td>$r^h$</td>
<td>Rate of return on capital</td>
</tr>
<tr>
<td>$q$</td>
<td>Book value of capital</td>
</tr>
<tr>
<td>$s$</td>
<td>Market value of capital</td>
</tr>
<tr>
<td>$re$</td>
<td>Residual/Abnormal earnings</td>
</tr>
<tr>
<td>$efp$</td>
<td>External finance premium</td>
</tr>
<tr>
<td>$k$</td>
<td>Capital stock</td>
</tr>
<tr>
<td>$n$</td>
<td>Entrepreneurial net worth</td>
</tr>
<tr>
<td>$h$</td>
<td>Mark-up of final good producers</td>
</tr>
<tr>
<td>$b$</td>
<td>Hours of labour input in production</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Inflation</td>
</tr>
<tr>
<td>$u$</td>
<td>Utilization rate</td>
</tr>
<tr>
<td>$\vartheta$</td>
<td>Cash-in-advance to CGP</td>
</tr>
<tr>
<td>$a$</td>
<td>Technological progress</td>
</tr>
<tr>
<td>$e_i$</td>
<td>Information shock</td>
</tr>
<tr>
<td>$e_u$</td>
<td>Shock to the utilization rate</td>
</tr>
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</table>
Table II.3: Parameters of the behavioural model and descriptions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^* )</td>
<td>The central bank’s inflation target</td>
<td>0</td>
</tr>
<tr>
<td>( d_1 )</td>
<td>Marginal propensity of consumption out of income</td>
<td>0.5</td>
</tr>
<tr>
<td>( e_1 )</td>
<td>Coefficient on expected output in investment eq.</td>
<td>0.1</td>
</tr>
<tr>
<td>( d_2 )</td>
<td>Coefficient on expected output in consumption eq. to match ( a_1 = 0.5 )</td>
<td>( 0.5 \times (1 - d_1) - e_2 )</td>
</tr>
<tr>
<td>( d_3 )</td>
<td>Coefficient on real rate in consumption eq.</td>
<td>(-0.01)</td>
</tr>
<tr>
<td>( e_2 )</td>
<td>Coefficient on real rate in investment eq. to match ( a_2 = -0.5 )</td>
<td>((-0.5) \times (1 - d_1) - d_3)</td>
</tr>
<tr>
<td>( a_1 )</td>
<td>Coefficient of expected output in output eq.</td>
<td>((e_1 + d_2)/(1 - d_1))</td>
</tr>
<tr>
<td>( a_1' )</td>
<td>Coefficient of lagged output in output eq.</td>
<td>((d_1 + e_3)/(1 - d_1))</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>Interest rate elasticity of output demand</td>
<td>(-d_3/(1 - d_1))</td>
</tr>
<tr>
<td>( a_3 )</td>
<td>Coefficient on spread term in output eq.</td>
<td>(-d_3/(1 - d_1))</td>
</tr>
<tr>
<td>( b_1 )</td>
<td>Coefficient of expected inflation in inflation eq.</td>
<td>0.5</td>
</tr>
<tr>
<td>( b_2 )</td>
<td>Coefficient of output in inflation eq.</td>
<td>0.05</td>
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<tr>
<td>( c_1 )</td>
<td>Coefficient of inflation in Taylor rule eq.</td>
<td>1.5</td>
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<tr>
<td>( \psi )</td>
<td>Parameter of firm equity</td>
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<tr>
<td>( \tau )</td>
<td>Firms’ leverage</td>
<td>1.43</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>Banks’ inverse leverage ratio</td>
<td>0.09</td>
</tr>
<tr>
<td>( e )</td>
<td>Equity premium</td>
<td>0.05</td>
</tr>
<tr>
<td>( \alpha^d )</td>
<td>Fraction of nominal GDP forecast in expected future dividends</td>
<td>0.2</td>
</tr>
<tr>
<td>( \bar{n} )</td>
<td>Number of shares in banks’ balance sheets</td>
<td>40</td>
</tr>
<tr>
<td>( \tilde{n} )</td>
<td>Initial value for number of firms’ shares</td>
<td>60</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Bubble convergence parameter</td>
<td>0.98</td>
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<tr>
<td>( c_2 )</td>
<td>Coefficient of output in Taylor equation</td>
<td>0.5</td>
</tr>
<tr>
<td>( c_3 )</td>
<td>Interest smoothing parameter in Taylor equation</td>
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<tr>
<td>( \delta )</td>
<td>Depreciation rate of capital</td>
<td>0.025</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Share of capital in production</td>
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</tr>
<tr>
<td>( \Psi )</td>
<td>Adjustment cost function in investment</td>
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</tr>
<tr>
<td>( \kappa )</td>
<td>Adjustment cost in investment parameter</td>
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<tr>
<td>( \gamma )</td>
<td>Switching parameter in Brock-Hommes (or intensity of choice parameter)</td>
<td>1</td>
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<tr>
<td>( \rho )</td>
<td>Speed of declining weights in memory (mean square errors)</td>
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<tr>
<td>( z )</td>
<td>Technological development parameter</td>
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<td>( \xi_1 )</td>
<td>Parameter 1 in the utilization cost function</td>
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<tr>
<td>( \xi_2 )</td>
<td>Parameter 2 in the utilization cost function</td>
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<tr>
<td>( \xi_3 )</td>
<td>Parameter 3 in the utilization cost function</td>
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<tr>
<td>( \epsilon )</td>
<td>Std. deviation of technology shock</td>
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</tr>
<tr>
<td>( \epsilon_x )</td>
<td>Std. deviation of nom. Interest rate shock</td>
<td>0.5</td>
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<tr>
<td>( \epsilon_{uc} )</td>
<td>Std. deviation of shock in the utilization cost function</td>
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<tr>
<td>( \rho_k )</td>
<td>AR process of shock to utilization cost function</td>
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Table II.4: Model correlations - comparisons

<table>
<thead>
<tr>
<th>Correlations</th>
<th>Value - behavioural model</th>
<th>Value - DSGE model</th>
<th>Value - US data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho(y_t, y_{t-1})$</td>
<td>0.85</td>
<td>0.59</td>
<td>0.85</td>
</tr>
<tr>
<td>$\rho(y_t, k_t)$</td>
<td>0.67</td>
<td>0.26</td>
<td>0.15</td>
</tr>
<tr>
<td>$\rho(y_t, \pi_t)$</td>
<td>-0.46</td>
<td>0.83</td>
<td>-0.43</td>
</tr>
<tr>
<td>$\rho(y_t, S_t)$</td>
<td>0.80</td>
<td>0.97</td>
<td>0.83</td>
</tr>
<tr>
<td>$\rho(y_t, as_t)$</td>
<td>0.80</td>
<td>0.61</td>
<td>0.76</td>
</tr>
<tr>
<td>$\rho(y_t, AD_t)$</td>
<td>0.15</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\rho(y_t, AS_t)$</td>
<td>-0.12</td>
<td>-</td>
<td>-</td>
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<tr>
<td>$\rho(y_t, \psi(u_t))$</td>
<td>-0.1</td>
<td>$-1 \times 10^{-3}$</td>
<td>-</td>
</tr>
<tr>
<td>$\rho(y_t, \vartheta_t)$</td>
<td>0.38</td>
<td>-</td>
<td>0.32</td>
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<tr>
<td>$\rho(y_t, l_t^g)$</td>
<td>0.25</td>
<td>-</td>
<td>0.18</td>
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<tr>
<td>$\rho(y_t, r_t)$</td>
<td>0.37</td>
<td>0.78</td>
<td>0.45</td>
</tr>
<tr>
<td>$\rho(y_t, i_t)$</td>
<td>0.24</td>
<td>0.98</td>
<td>0.90</td>
</tr>
<tr>
<td>$\rho(y_t, c_t)$</td>
<td>0.21</td>
<td>0.30</td>
<td>0.32</td>
</tr>
<tr>
<td>$\rho(y_t, s_t)$</td>
<td>0.28</td>
<td>0.46</td>
<td>-0.28</td>
</tr>
<tr>
<td>$\rho(y_t, x_t)$</td>
<td>-0.39</td>
<td>-0.84</td>
<td>-0.49</td>
</tr>
<tr>
<td>$\rho(y_t, \vartheta_t)$</td>
<td>-0.02</td>
<td>-0.97</td>
<td>-</td>
</tr>
<tr>
<td>$\rho(k_t, k_{t-1})$</td>
<td>0.94</td>
<td>0.95</td>
<td>0.88</td>
</tr>
<tr>
<td>$\rho(k_t, as_t)$</td>
<td>0.51</td>
<td>0.81</td>
<td>-</td>
</tr>
<tr>
<td>$\rho(k_t, \vartheta_t)$</td>
<td>$2 \times 10^{-3}$</td>
<td>-0.31</td>
<td>-</td>
</tr>
<tr>
<td>$\rho(k_t, r_{t^n}^n)$</td>
<td>0.15</td>
<td>0.49</td>
<td>0.31</td>
</tr>
<tr>
<td>$\rho(l_t^n, k_t)$</td>
<td>0.27</td>
<td>-</td>
<td>0.38</td>
</tr>
<tr>
<td>$\rho(l_t^n, x_t)$</td>
<td>-0.1</td>
<td>-</td>
<td>0.26</td>
</tr>
<tr>
<td>$\rho(\pi_t, \pi_{t-1})$</td>
<td>0.79</td>
<td>0.41</td>
<td>0.93</td>
</tr>
<tr>
<td>$\rho(\pi_t, as_t)$</td>
<td>-0.44</td>
<td>0.13</td>
<td>-</td>
</tr>
<tr>
<td>$\rho(\pi_t, r_{t^n}^n)$</td>
<td>0.56</td>
<td>-0.91</td>
<td>0.34</td>
</tr>
<tr>
<td>$\rho(\pi_t, r_{t-1}^n)$</td>
<td>0.51</td>
<td>-</td>
<td>0.34</td>
</tr>
<tr>
<td>$\rho(x_t, x_{t-1})$</td>
<td>-0.01</td>
<td>-0.1</td>
<td>0.68</td>
</tr>
<tr>
<td>$\rho(x_t, as_t)$</td>
<td>-0.12</td>
<td>-0.23</td>
<td>-</td>
</tr>
<tr>
<td>$\rho(x_t, k_t)$</td>
<td>-0.27</td>
<td>0.07</td>
<td>0.09</td>
</tr>
<tr>
<td>$\rho(x_t, \vartheta_t)$</td>
<td>0.01</td>
<td>0.8</td>
<td>-</td>
</tr>
<tr>
<td>$\rho(\vartheta_t, as_t)$</td>
<td>0.07</td>
<td>-0.67</td>
<td>-</td>
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<tr>
<td>$\rho(y_t, n_{t^n}^b)$</td>
<td>0.03</td>
<td>-</td>
<td>0.45</td>
</tr>
<tr>
<td>$\rho(y_t, n_{t^n}^d)$</td>
<td>0.02</td>
<td>0.97</td>
<td>0.22</td>
</tr>
<tr>
<td>$\rho(y_t, h_t)$</td>
<td>0.84</td>
<td>0.97</td>
<td>0.88</td>
</tr>
<tr>
<td>$\rho(y_t, q_t)$</td>
<td>-</td>
<td>0.59</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Note: GDP deflator was used as the inflation indicator, 3-month T-bill for the risk-free interest rate, the deposit rate as the savings indicator and the Corporate lending risk spread (Moody’s 30-year BAA-AAA corporate bond rate) as the counterpart for the firm borrowing spread in the models. The variables that are left blank do not have a direct counterpart in the data sample. These are also called ‘deep variables’. The only way is to estimate a structural model (using for instance Bayesian techniques) and to derive a value based on a (theoretical) structure. Alternatively, one could also approximate values using micro data. However, this is outside the scope of this paper.
### Table II.5: Second moments - comparison

<table>
<thead>
<tr>
<th>Variable</th>
<th>Behavioural model</th>
<th>DSGE model</th>
<th>US data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t$</td>
<td>2.47</td>
<td>3.08</td>
<td>0.016</td>
</tr>
<tr>
<td>$\pi_t$</td>
<td>0.33</td>
<td>0.27</td>
<td>0.50</td>
</tr>
<tr>
<td>$k_t$</td>
<td>0.38</td>
<td>0.39</td>
<td>1.50</td>
</tr>
<tr>
<td>$x_t$</td>
<td>1.07</td>
<td>0.50</td>
<td>0.18</td>
</tr>
<tr>
<td>$\alpha_{it}$</td>
<td>0.13</td>
<td>0.98</td>
<td>5.68</td>
</tr>
<tr>
<td>$d_t$</td>
<td>1.60</td>
<td>-</td>
<td>1.36</td>
</tr>
<tr>
<td>$l_t^u$</td>
<td>2.61</td>
<td>-</td>
<td>3.55</td>
</tr>
<tr>
<td>$s_t^u$</td>
<td>0.91</td>
<td>0.02</td>
<td>0.76</td>
</tr>
<tr>
<td>$i_t$</td>
<td>0.26</td>
<td>5.03</td>
<td>3.08</td>
</tr>
<tr>
<td>$\psi(u_t)$</td>
<td>0.21</td>
<td>0.16</td>
<td>-</td>
</tr>
<tr>
<td>$AD_t$</td>
<td>0.2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$AS_t$</td>
<td>0.21</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\theta_t$</td>
<td>79.4</td>
<td>4.38</td>
<td>-</td>
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<tr>
<td>$c_t$</td>
<td>0.25</td>
<td>0.27</td>
<td>0.81</td>
</tr>
<tr>
<td>$s_t$</td>
<td>0.25</td>
<td>-</td>
<td>8</td>
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<tr>
<td>$n_f^l$</td>
<td>4.77</td>
<td>-</td>
<td>1.32</td>
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<tr>
<td>$n_f$</td>
<td>79.4</td>
<td>3.16</td>
<td>2.21</td>
</tr>
<tr>
<td>$S_t$</td>
<td>1.33</td>
<td>1.23</td>
<td>10.33</td>
</tr>
<tr>
<td>$q_t$</td>
<td>-</td>
<td>1.02</td>
<td>2.01</td>
</tr>
<tr>
<td>$h_t$</td>
<td>1.15</td>
<td>1.18</td>
<td></td>
</tr>
</tbody>
</table>

Note: The moments are calculated taking output as the denominator. Following a standard approach in the DSGE literature, this is in order to examine the moments with respect to the general business cycle.

### Table II.6: Higher moments - Behavioural vs data

<table>
<thead>
<tr>
<th>Variable</th>
<th>Skewness behavioural</th>
<th>Kurtosis behavioural</th>
<th>Skewness data</th>
<th>Kurtosis data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t$</td>
<td>-1.66</td>
<td>15.94</td>
<td>-0.42</td>
<td>0.22</td>
</tr>
<tr>
<td>$\pi_t$</td>
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</tr>
<tr>
<td>$k_t$</td>
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<td>0.36</td>
<td>0.82</td>
<td>-1.66</td>
</tr>
<tr>
<td>$x_t$</td>
<td>-9.24</td>
<td>20.52</td>
<td>-5.8</td>
<td>58.6</td>
</tr>
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</tr>
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</tr>
<tr>
<td>$l_t^u$</td>
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<td>0.13</td>
<td>-0.61</td>
<td>3.57</td>
</tr>
<tr>
<td>$i_t$</td>
<td>1.27</td>
<td>1.1</td>
<td>-1.27</td>
<td>2.38</td>
</tr>
<tr>
<td>$\psi(u_t)$</td>
<td>-0.001</td>
<td>-0.03</td>
<td>-0.03</td>
<td>-</td>
</tr>
<tr>
<td>$AD_t$</td>
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<td>-0.19</td>
<td>-0.19</td>
<td>-</td>
</tr>
<tr>
<td>$AS_t$</td>
<td>0.01</td>
<td>-0.19</td>
<td>-0.19</td>
<td>-</td>
</tr>
<tr>
<td>$\theta_t$</td>
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<td>48.38</td>
<td>-48.38</td>
<td>-</td>
</tr>
<tr>
<td>$c_t$</td>
<td>-4.61</td>
<td>8.78</td>
<td>0.37</td>
<td>0.14</td>
</tr>
<tr>
<td>$s_t$</td>
<td>4.61</td>
<td>8.8</td>
<td>0.49</td>
<td>8.39</td>
</tr>
<tr>
<td>$n_f^l$</td>
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<td>48.19</td>
<td>-2.34</td>
<td>9.39</td>
</tr>
<tr>
<td>$n_f$</td>
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<td>48.37</td>
<td>-0.34</td>
<td>16.37</td>
</tr>
<tr>
<td>$S_t$</td>
<td>-15.84</td>
<td>48.43</td>
<td>1.57</td>
<td>5.18</td>
</tr>
</tbody>
</table>

Note: The moments are calculated taking output as the denominator. In the US data, moments are calculated taking real GDP as the denominator. These are calculated using the full sample of US data stretching from 1953:I - 2014:IV. During this period, the US economy experienced 10 cycles (using NBER business cycle dates), and the average GDP increase per quarter during expansions was 1.05% while it was -0.036% during recessions. The data were de-trended using a standard two-sided HP filter before the moments were calculated in order to facilitate comparison with the model generated (cyclical) moments. The variables that are left blank do not have a direct counterpart in the data sample. These are also called ‘deep variables’. The only way is to estimate a structural model (using for instance Bayesian techniques) and to derive a value based on a (theoretical) structure. Alternatively, one could also approximate values using micro data. However, this is outside the scope of this paper.
### Table II.5: Second moments - comparison

<table>
<thead>
<tr>
<th>Variable</th>
<th>Behavioural model</th>
<th>DSGE model</th>
<th>US data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t$</td>
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<td>0.39</td>
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</tr>
<tr>
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<tr>
<td>$r_{nt}$</td>
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<td>5.03</td>
<td>3.08</td>
</tr>
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<td>$\psi(u_t)$</td>
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</tr>
<tr>
<td>$AD_t$</td>
<td>-0.03</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$AS_t$</td>
<td>0.01</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\vartheta_t$</td>
<td>15.83</td>
<td>-</td>
<td>48.38</td>
</tr>
<tr>
<td>$c_t$</td>
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<td>0.37</td>
<td>8.78</td>
</tr>
<tr>
<td>$s_l$</td>
<td>4.61</td>
<td>0.49</td>
<td>8.8</td>
</tr>
<tr>
<td>$n^l_t$</td>
<td>-15.78</td>
<td>-2.34</td>
<td>48.19</td>
</tr>
<tr>
<td>$n^f_t$</td>
<td>-15.83</td>
<td>-0.34</td>
<td>48.37</td>
</tr>
<tr>
<td>$S_t$</td>
<td>-15.84</td>
<td>1.57</td>
<td>48.43</td>
</tr>
</tbody>
</table>

Note: The moments are calculated taking output as the denominator. Following a standard approach in the DSGE literature, this is in order to examine the moments with respect to the general business cycle.

### Table II.6: Higher moments - Behavioural vs data

<table>
<thead>
<tr>
<th>Variable</th>
<th>Skewness behavioural</th>
<th>Skewness data</th>
<th>Kurtosis behavioural</th>
<th>Kurtosis data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t$</td>
<td>-1.66</td>
<td>-0.42</td>
<td>15.94</td>
<td>0.22</td>
</tr>
<tr>
<td>$\pi_t$</td>
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<td>-0.66</td>
<td>0.25</td>
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</tr>
<tr>
<td>$k_t$</td>
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<td>0.82</td>
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</tr>
<tr>
<td>$x_t$</td>
<td>-9.24</td>
<td>-5.8</td>
<td>20.52</td>
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</tr>
<tr>
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<td>1.53</td>
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</tr>
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</tr>
<tr>
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<td>-0.61</td>
<td>0.13</td>
<td>3.57</td>
</tr>
<tr>
<td>$r_{nt}$</td>
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<td>1.1</td>
<td>2.38</td>
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<td>0.03</td>
<td>-</td>
</tr>
<tr>
<td>$AD_t$</td>
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<td>-</td>
<td>0.19</td>
<td>-</td>
</tr>
<tr>
<td>$AS_t$</td>
<td>0.01</td>
<td>-</td>
<td>0.19</td>
<td>-</td>
</tr>
<tr>
<td>$\vartheta_t$</td>
<td>15.83</td>
<td>-</td>
<td>48.38</td>
<td>-</td>
</tr>
<tr>
<td>$c_t$</td>
<td>-4.61</td>
<td>0.37</td>
<td>8.78</td>
<td>0.14</td>
</tr>
<tr>
<td>$s_l$</td>
<td>4.61</td>
<td>0.49</td>
<td>8.8</td>
<td>8.39</td>
</tr>
<tr>
<td>$n^l_t$</td>
<td>-15.78</td>
<td>-2.34</td>
<td>48.19</td>
<td>9.39</td>
</tr>
<tr>
<td>$n^f_t$</td>
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<td>-0.34</td>
<td>48.37</td>
<td>16.37</td>
</tr>
<tr>
<td>$S_t$</td>
<td>-15.84</td>
<td>1.57</td>
<td>48.43</td>
<td>5.18</td>
</tr>
</tbody>
</table>

Note: The moments are calculated taking real GDP as the denominator. In the US data, moments are calculated taking real GDP as the denominator. These are calculated using the full sample of US data stretching from 1953:1 - 2014:IV. During this period, the US economy experienced 10 cycles (using NBER business cycle dates), and the average GDP increase per quarter during expansions was 1.05% while it was -0.036% during recessions. The data were de-trended using a standard two-sided HP filter before the moments were calculated in order to facilitate comparison with the model generated (cyclical) moments. The variables that are left blank do not have a direct counterpart in the data sample. These are also called ‘deep variables’. The only way is to estimate a structural model (using for instance Bayesian techniques) and to derive a value based on a (theoretical) structure. Alternatively, one could also approximate values using micro data. However, this is outside the scope of this paper.
Table II.7: Variance decomposition - DSGE model (in percent)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Shock to firm financing costs</th>
<th>Monetary Policy shock</th>
<th>Technology shock</th>
<th>Asset price shock</th>
<th>Utilization cost shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t$</td>
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</tr>
<tr>
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</tr>
<tr>
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<td>21.72</td>
<td>0.40</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$n_t'$</td>
<td>69.85</td>
<td>24.35</td>
<td>5.51</td>
<td>0.28</td>
<td>0</td>
</tr>
<tr>
<td>$r k_t$</td>
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</tr>
<tr>
<td>$r_t$</td>
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<td>0.01</td>
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</tr>
<tr>
<td>$r^n_t$</td>
<td>73.43</td>
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<td>4.72</td>
<td>0.19</td>
<td>0</td>
</tr>
<tr>
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<td>70.29</td>
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<td>8.17</td>
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<td>0</td>
</tr>
<tr>
<td>$\psi(u_t)$</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>$q_t$</td>
<td>69.01</td>
<td>20.36</td>
<td>7.81</td>
<td>2.82</td>
<td>0</td>
</tr>
<tr>
<td>$x_t$</td>
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<td>2.33</td>
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<td>0.88</td>
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<tr>
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<td>6.23</td>
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<td>0</td>
</tr>
<tr>
<td>$h_t$</td>
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<td>0.24</td>
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<tr>
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<td>7.80</td>
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<td>0</td>
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</table>

Note: The moments are calculated taking output as the denominator. Following a standard approach in the DSGE literature, this is in order to examine the moments with respect to the general business cycle.
Table II.8: US variables and sources

<table>
<thead>
<tr>
<th>Variable</th>
<th>US data name</th>
<th>Frequency</th>
<th>Source</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t$</td>
<td>Real GDP</td>
<td>Quarterly</td>
<td>Fed St Louis database</td>
<td>1953:I-2014:IV</td>
</tr>
<tr>
<td>$c_t$</td>
<td>Real Personal Consumption Expenditure</td>
<td>Quarterly</td>
<td>Fed St Louis database</td>
<td>1953:I-2014:IV</td>
</tr>
<tr>
<td>$i_t$</td>
<td>Real Investment</td>
<td>Quarterly</td>
<td>Fed St Louis database</td>
<td>1953:I-2014:IV</td>
</tr>
<tr>
<td>$h_t$</td>
<td>Hours of All Persons: Nonfarm Business Sector</td>
<td>Quarterly</td>
<td>Fed St Louis database</td>
<td>1953:I-2011:IV</td>
</tr>
<tr>
<td>$d_t$</td>
<td>Total Savings and Time Deposits of Households</td>
<td>Quarterly</td>
<td>Fed St Louis database</td>
<td>1953:I-2014:IV</td>
</tr>
<tr>
<td>$s_t$</td>
<td>Net Private Savings Households</td>
<td>Quarterly</td>
<td>Fed St Louis database</td>
<td>1953:I-2014:IV</td>
</tr>
<tr>
<td>$l_t^s$</td>
<td>Credit Market Instruments for Firms</td>
<td>Quarterly</td>
<td>Fed St Louis database</td>
<td>1953:I-2014:IV</td>
</tr>
<tr>
<td>$r_t$</td>
<td>Effective Federal Funds Rate</td>
<td>Monthly</td>
<td>Fed St Louis database</td>
<td>1954:II-2014:IV</td>
</tr>
<tr>
<td>$x_t/efp_t$</td>
<td>Moody’s (30 year) BAA - AAA Corporate Bond Spread</td>
<td>Monthly</td>
<td>Fed St Louis database</td>
<td>1953:I-2014:IV</td>
</tr>
<tr>
<td>$\pi_t$</td>
<td>GDP Deflator</td>
<td>Quarterly</td>
<td>Fed St Louis database</td>
<td>1953:I-2014:IV</td>
</tr>
</tbody>
</table>

Note: All variables were downloaded on March 2, 2015. The latest recorded observation for each variable was 2014:IV (except for capital stock).
II.2 Impulse response analysis in the DSGE model - remaining shocks

II.2.1 Monetary policy shock

Figure II.3 reports the impulse responses to a contractionary monetary policy shock. A 0.5% rise in the interest rate increases the cost of financing for firms as well as the investment rate. As a result, firms will be able to borrow less, which pushes down their available liquidity, and so investment falls by 6%. Hence capital de-accumulates by 0.6% and its return falls by 1.5%. On the production end, input costs become more expensive since the cost of purchasing and financing them are higher. Marginal costs rise by 1.25%, and labor falls by 1.8%. This slowdown in production and investment results in a contraction in the firm balance sheet, whereby net worth contracts by 4%, the (stock) market value of the firm by 1.5%, and the book value by 1%. Observing this contraction in firm activity, CGP turn more averse and demand the entrepreneurs to pay a 6% higher share for their capital purchases, which is signifocant.\(^{64}\) Accordingly, inflation falls by almost 0.2% and output by 1.5%, which forces the monetary authority to decrease the policy rate by 0.018%. However, this fall is smaller than the fall in inflation, which results in a rise of the real rate.\(^{65}\)

Once we remove the interaction term and the stock market, as in BGG (1999), the responses become around 50-60% weaker. Thus, the rise in marginal cost is 0.75%, the fall in labor 1%, and the fall in investment only 2.5%. The de-accumulation of capital is merely 0.2%, which means that capital return is in relative terms higher to the full model (or -0.7%, and it rapidly rises to 0.5% in quarter 2). The net worth falls by 1.5% and the (book) value by 0.5%. Correspondingly, inflation falls by a mere 0.07%, and output by 0.75%.

Even when we include one of the mechanisms, the stock market, the responses are approximately 20% weaker. Investment falls by 5%, labor by 1.5% and marginal costs rise by 0.9%. The balance sheet contraction is therefore also weaker, where net worth contracts by 3.5%, the market value of the firm by 1.2%, and the book value by 0.6%. As a result, output contracts 20% less than in the full model (1.25%), while inflation contracts 17% less (0.125%).

\(^{64}\)Since in the steady state the share is equal to 0, this means that when faced with a small monetary policy shock, CGP ask entrepreneurs to pre-pay a 6% share of their total capital input purchases. This shows that CGP are highly sensitive to firm (and general economic) conditions, and therefore behave highly elastically over the cycle.

\(^{65}\)On the household end, savings rise since the real interest rate goes up, which means that current consumption is sacrificed for future.
II.2.2 Utilization cost shock

A rise in the utilization cost of capital implies that it is now more costly to use the capital in production. Thus we should expect a deacceleration in the production, inflation, and ultimately in output. That is exactly what we observe. A 0.5% increase in the utilization costs (with no AR parameter in the shock), increases the marginal costs of the firm by 0.15% and reduces the labor demand by 0.12%. Less capital is employed in the production and so the labor required in the production falls. Less production and sales means that (future) net worth will fall, which will reduce the borrowing capacity of the firm, and therefore investment. Therefore we observe a fall of 0.003% in net worth, of 0.0015% in residual earnings and of 0.004% in investment. CGP knowing all this turn more doubtious of the repayment status of entrepreneurs and therefore ask for a higher pre-payment share for their capital sold (0.007%). Taking into account the harsher financial condition of firms, the higher marginal costs they face and the higher pre-payment required for their input purchases, capital purchases contract by 0.004% as well as its return by 0.001%. Note, however, that despite the fall in investment, the contraction on the supply-side (marginal costs, labor demand, pre-payment share) is much heavier than on the demand-side. Therefore the fall in inflation is mainly driven by the supply-side contraction since the drop in inflation (-0.012%) is significantly larger than the drop in investment or consumption together (-0.0046%). Lastly, output drops by 0.001%.

We are not able to make a comparison of this shock transmission to previous models since utilization costs are a novel feature to this version. One could possibly try to compare the transmission mechanism here to Christiano and Eichenbaum (2005) or Gerali et al (2010), but since the model constructions differ considerably between them, the comparison would be highly inaccurate and possibly counterproductive.

II.2.3 Asset price wedge shock

In addition to the standard shocks in the literature, we wish to explore the dynamics of the model in relation to updating of beliefs. We consider a positive (0.5 %) shock to residual earnings, and the responses are reported in Figure II.6. We are interested in examining the effects that updating of beliefs has on asset prices, and the wider economy.

---

66For instance, Gertler and Karadi (2011) consider a similar shock in their version of the financial accelerator model with explicit banking.
Overall, the wedge shock generates a strong (boom-bust) cycle in both the asset prices and the general economy. The shock causes optimism on the stock market since capital is expected to generate a much higher return than the fundamental one, causing market value of capital to rise already today. The market value increases by 0.2 %, which is 0.6 % above the book value. There are two effects from this. The immediate effect is that capital is more attractive, and so induces more investment by entrepreneurs. In addition, because the value of net worth increases this eases the borrowing constraints that entrepreneurs face. As a result, they can take out more loans, and use the credit to invest further into capital. The total effect is that net worth and investment increase by 0.4% and 0.8 %. In line with the empirics, there is also a wealth effect on consumption from a higher stock market value. The wealth effect on households is marginal since consumption increases by only 0.011 % but the larger effect comes from entrepreneurial consumption, which rises by 0.8%. The final effect from the demand-side expansion is that output expands by 0.16 %.67

Nonetheless, as soon as expectations about the future return of assets deteriorate (after the first quarter), a negative spiral starts to hit in. The market return on capital falls by 0.28 %, which causes the market value of assets to fall as well as investment, since it is now less attractive to invest because of a lower expected capital returns. Additionally, falling asset prices mean that the value of internal funds starts to deteriorate, which results in higher restrictions to external financing (collateral constraint binds sooner). This will cause a further fall in investment, which will result in lower net worth in the subsequent period, and so on. Hence, 4 quarters after the initial shock, market value of capital drops to below the steady state level, which causes investment to fall below its’ steady state level in the subsequent quarter. The total effect on production is immediate, and output starts to contract (albeit weakly) 4 quarters after the initial shock. Despite the relatively slower fall of market value of capita compared to the book value, the negative economic prospects cause a steady drop in the market value, resulting in output being below its steady state level for almost 15 quarters. Only 5 years (20 quarters) after the initial shock does the economy recover from the contraction, and output turns back to its steady state level. We therefore observe the full cycle in our impulse responses. Our output (and investment) cycle is in line with the empirical literature which finds that output, on

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67Remember that the AR parameter for this shock is set to 0.67. Note therefore how the impulse responses for this shock are smaller than for any other supply-side or financial/monetary shock. This is another proof of how financial shocks transmitted via the supply side have larger macroeconomic effects than when transmitted via the demand side.
average takes longer time to recover after a stock market boom than after any other type of expansion. Moreover, as a result of a stock market boom, the recessionary period is longer than the expansionary.\textsuperscript{68}

II.3 Impulse response analysis in the behavioural model - remaining shocks

II.3.1 Monetary policy shock

The responses to a monetary policy shock are reported in Figure II.10. As is standard, an expansionary monetary policy (0.5% fall) leads to a fall in the external finance premium, which relaxes the credit that firms can access and therefore pushes up investment (0.3%). This pushes up capital accumulation (0.4%). This expansion is perceived by agents as a period of positive outlook, which triggers the optimism (animal spirits up 0.2%). This optimism is translated into an increase in deposits (0.25%) and bank equity (0.3%). The expansion leads to an increase in output (0.20%) and a rise in inflation (0.01%), but with a lag of 1 quarter.\textsuperscript{69}

However, this optimism is very brief as the monetary authority raises the interest rate (0.1%) to combat the rising inflation. By the agents, this is perceived as the end of the expansionary phase, resulting in a reversal of the sentiment to pessimism (animal spirits fall by 0.05%). The consequence is a turn in the response of macroeconomic and financial aggregates, leading to return of these variables to the steady state.

Hence in the behavioural model, we see two waves of responses. The first, standard in the DSGE models, driven directly by a monetary policy expansion. The second, on the other hand, is purely driven by animal spirits. The response of the monetary authority to the initial expansion kills and turns the initial optimism into a pessimism (or negative bubble on the financial market). This results in a reversal in the financial and macroeconomic aggregates, making the initial monetary expansion extremely short-lived. This type of market behaviour are difficult to capture in standard DSGE models (but frequently observed empirically).

\textsuperscript{68}The responses in Gerba (2014) extension are the same as int he full model since the stock market dynamics is modelled in the same way.

\textsuperscript{69}Initially, output falls by 0.25% as well as inflation by 0.05%, but this is reverted after 1 period. This finding is frequent in the literature and denominated as the price puzzle.
II.3.2 Shock to utilization costs

The responses to a shock in utilization costs are reported in Figure II.12. The second of the supply side shocks is a 0.5% decrease in the cost of utilizing capital in production (i.e. a positive supply-side shock). This will therefore increase the marginal benefit or return to capital, which will increase the demand for capital. Hence, capital good producers will produce more, and so investment rises (0.02%). The level of capital will also rise significantly (0.2%) as a result of both capital demand and supply expansion. Therefore, output will expand (0.1%). Because of the higher capital (and thus collateral) and the resulting fall in the financing spread, the quantity of credit to firms will expand (0.7%). Since this is an improvement on the supply side, inflation initially falls (0.03%), and the monetary authority reacts by reducing the interest rate (0.15%). This is reverted as soon as the monetary authority increases the interest rate (0.02%) because of the recovery in the inflation. Following 15 years after the shock, in the new steady state, firm credit and deposits are 0.6% and 0.2% above the pre-shock level. Again a temporary supply-side shock is having permanent effects on financial sector activity.
II.4 Figures in the DSGE model

Figure II.1: Responses to a productivity shock

(continued on next page)
Notes: Impulse responses to an expansionary TFP shock in the DSGE model. The responses of cash-in-advance constraint is \( \vartheta \), utilization costs \( u \), Market asset price \( s \), book value \( q \) and residual earnings \( re \).
Figure II.2: Responses to a productivity shock in BGG(1999)
Notes: Impulse responses to an expansionary TFP shock in the BGG (1999) model.
Figure II.3: Responses to a monetary policy shock

(continued on next page)
Notes: Impulse responses to a contractionary monetary policy shock in the DSGE model. The responses of cash-in-advance constraint is $\vartheta$, utilization costs $u$, market asset price $s$, book value $q$ and residual earnings $re$. 
Figure II.4: Responses to a monetary policy shock in BGG(1999)

(continued on next page)
Notes: Impulse responses to a contractionary monetary policy shock in the BGG (1999) model.
Figure II.5: Responses to a shock in utilization costs

(continued on next page)
Notes: Impulse responses to a positive shock to utilization costs in the DSGE model. The responses of cash-in-advance constraint is $\vartheta$, utilization costs $u$, market asset price $s$, book value $q$ and residual earnings $re$. 
Figure II.6: Responses to an asset price wedge shock

(continued on next page)
Notes: Impulse responses to an expansionary (asset price) wedge shock in the DSGE model. The responses of cash-in-advance constraint is $\vartheta$, utilization costs $u$, market asset price $s$, book value $q$ and residual earnings $re$. 
Figure II.7: Responses to an asset price wedge shock in Gerba (2014)
Notes: Impulse responses to an expansionary (asset price) wedge shock in the Gerba (2014) model.
Figure II.8: Responses to a relaxation in the firm financing costs (external finance premium)

(continued on next page)
Notes: Impulse responses to a relaxation (negative shock) of the firm financing costs in the DSGE model. The responses of cash-in-advance constraint is \( \vartheta \), utilization costs \( u \), market asset price \( s \), book value \( q \) and residual earnings \( re \).
Figure II.9: Responses to a relaxation in the firm financing costs (external finance premium) in BGG(1999)
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II.5  Figures in the behavioural model

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