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Abstract

We compare semi-nonparametric expansions of the Gamma distribution with alternative Laguerre expansions, showing that they substantially widen the range of feasible moments of positive random variables. Then we combine those expansions with a component version of the Multiplicative Error Model to capture the mean reversion typical in positive but stationary financial time series. Finally, we carry out an empirical application in which we compare various asset allocation strategies for Exchange Traded Notes tracking VIX futures indices, which are increasingly popular but risky financial instruments. We show the superior performance of the strategies based on our econometric model.

Keywords: density expansions, exchange traded notes, multiplicative error model, volatility index futures.

JEL classification: G13, C16.

Resumen

En este trabajo comparamos expansiones seminoparamétricas de la distribución Gamma con expansiones de Laguerre alternativas, demostrando que amplían sustancialmente el rango de momentos factibles de variables aleatorias positivas. Posteriormente, combinamos dichas expansiones con una versión con componentes de un Modelo de Error Multiplicativo, con el fin de capturar la reversión a la media característica de series temporales positivas y estacionarias. Finalmente, llevamos a cabo una aplicación empírica en la que comparamos distintas estrategias de selección de cartera para *Exchange Traded Notes,* que son activos financieros cada vez más populares a pesar de sus riesgos que replican índices sobre futuros del VIX. Los resultados demuestran que las estrategias basadas en nuestro modelo econométrico producen rendimientos superiores.

Palabras clave: expansiones de densidades, *exchange traded notes,* modelo de error multiplicativo, futuros sobre índices de volatilidad.

Códigos JEL: G13, C16.

1 Introduction

Asset prices are generally non-stationarity, which explains why the majority of empirical studies work with financial returns. Given that those returns can be both positive and negative, researchers have mostly considered distributions with support on the entire real line. However, there are other important financial applications where the original data is always positive but stationary in levels. Interest rates and volatility measures are perhaps the two most prominent examples. Still, an important feature of those financial time series is a slow reversion to their long run mean. Many discrete and continuous time models have been proposed to capture this strong persistence. An increasingly popular example is the discrete-time Multiplicative Error Model (MEM) proposed by Engle (2002), which has been applied not just to volatility modelling but also to trading volumes and durations (see Brownlees, Cipollini, and Gallo, 2012, for a recent review). In this model, a positive random variable is treated as the product of a time varying, recursive mean times a positive random error with unit conditional mean. The MEM literature has generally neglected the distribution of this multiplicative random error because its main goal has been prediction. In this regard, Engle and Gallo (2006) show on the basis of earlier results by Gourieroux, Monfort, and Trognon (1984) that the mean parameters can be consistently estimated assuming a Gamma distribution for the error term even when the true distribution is not Gamma, as long as the conditional mean is correctly specified. Unfortunately, this pseudo-likelihood approach is insufficient when the interest goes beyond the first conditional moment.

In this paper, we study in detail one relevant example for which the entire conditional distribution matters. The introduction of the new VIX index by the Chicago Board Options Exchange (CBOE) in 2003 meant that volatility became widely regarded as an asset class on its own. As is well known, VIX captures the volatility of the Standard & Poor's 500 (S&P500) over the next month implicit in stock index option prices, and for that reason it has become a widely accepted measure of stock volatility and a market fear gauge. In addition, since March 26, 2004 it is possible to directly invest in volatility through futures contracts on the VIX negotiated at the CBOE Futures Exchange (CFE). More recently, several volatility related Exchange Traded Notes (ETNs) have provided investors with equity-like long and short exposure to constant maturity futures on the

VIX, and even dynamic combinations of long-short exposures to different maturities (see Rhoads, 2011). By 2013, there were already about 30 ETNs with a market cap of around \$3 billion and a trading volume on some of them of close to \$5 billion per day (see Alexander and Korovilas, 2013, for further details).

This surge in interest on volatility futures ETNs might seem surprising on the basis of the evolution of the iPath S&P 500 VIX short term futures ETN (VXX), which, introduced on January 29, 2009, was the first VIX related equity-like ETN. The VXX, which is a 1-month constant-maturity VIX futures tracker, yielded an 8.6% profit during its first month of existence, but from then on until January 2013 it experienced losses of close to 100% due to the fall in volatility over this period. Its poor performance led some commentators to question the potential benefits of VIX futures ETNs (see e.g. Dizard, 2012). However, a short position on a 1-month constant maturity VIX futures has been available since December 2010 through the XIV ETN. Not surprisingly, by January 2013 this inverse ETN had yielded 95% accumulated profits, which confirms that ETNs might give rise to significant but risky returns. The main problem, though, is how to choose the most appropriate investment strategy using only the information available at each point in time.

In assessing trading strategies involving those financial instruments, risk averse investors must take into account not only the expected value of the resulting payoffs, which can be obtained from the mean forecasts generated by the MEM, but also some suitable measures of the risks involved, which necessarily depend on other features of the conditional distribution. In this sense, we develop a comprehensive dynamic asset allocation framework to invest in VIX futures ETNs, which may avoid the losses associated to existing ETNs. Specifically, we first model the mean-reverting features of the VIX with a component MEM specification analogous to the Garch model proposed by Engle and Lee (1999). As we will see, our slowly mean reverting, discrete time dynamic specification captures the main features of the VIX observed by Mencía and Sentana (2013).

Then, we augment this conditional mean model with a flexible functional form for the conditional distribution of the VIX given its past history in order to adequately capture the risks involved. In particular, we make use of a semi-nonparametric expansion of the Gamma density (Gamma SNP or GSNP for short). SNP expansions were introduced by

Gallant and Nychka (1987) for nonparametric estimation purposes as a way to ensure by construction the positivity of the resulting density (see also Fenton and Gallant, 1996; Gallant and Tauchen, 1999). In our case, though, we follow León, Mencía, and Sentana (2009) in treating the SNP distribution parametrically as if it reflected the actual data generating process instead of an approximating kernel. Interestingly, we can show that the GSNP distribution not only adds flexibility to the Gamma distribution, but it also retains its analytical tractability. In particular, we obtain closed-form expressions for its moments and analyse its flexibility by studying the range of coefficients of variation, skewness and kurtosis that it can generate. We also compare the GSNP expansion with a direct Laguerre expansion, which only ensures the positivity of the resulting density with complex parametric restrictions (see e.g. Amengual, Fiorentini, and Sentana, 2013).

Next, we employ derivative valuation methods to transform our time series model for the VIX into a tractable structural model for the excess returns of the VIX futures ETNs. In this regard, it is important to remember that since the VIX index is a risk neutral volatility forecast, not a directly traded asset, there is no cost of carry relationship between the price of the futures and the VIX (see Grünbichler and Longstaff, 1996, for more details). There is no convenience yield either, as in the case of futures on commodities. Therefore, absent any other market information, VIX derivatives must be priced according to some model for the risk neutral evolution of the VIX. This situation is similar, but not identical, to term structure models. For that reason, we specify a stochastic discount factor (SDF) with which we derive an equivalent risk-neutral measure that allows us to obtain closed-form expressions for the prices of VIX futures ETNs.

We use our theoretical framework in an empirical application that compares feasible dynamic investment strategies involving short and mid-term VIX futures indices. In particular, we develop an asset allocation strategy that maximises the conditional Sharpe ratio of a portfolio of those two futures indices. We compare our strategy with buy and hold positions on existing ETNs, some of which are already dynamic combinations of the VIX futures indices, as well as other strategies that have been previously proposed in the literature. Finally, we conduct robustness checks to assess the sensitivity of our results to the evaluation criterion, and compare our model with two alternative approaches: (i) a reduced form model and (ii) the autoregressive Gamma process proposed by Gourieroux and Jasiak (2006).

The rest of the paper is organised as follows. In the next section, we study the statistical properties of the GSNP density and compare them with those of Laguerre expansions. In Section 3, we describe our pricing framework, relate the real and risk-neutral measures, and obtain futures prices. Section 4 presents the empirical application. Finally, we conclude in Section 5. Proofs and auxiliary results can be found in the appendices.

2 Gamma density expansions

2.1 Density definition

Consider the Gamma distribution, whose probability density function (pdf) can be expressed as

$$f_G(x,\nu,\psi) = \frac{1}{\Gamma(\nu)\psi^{\nu}} x^{\nu-1} \exp(-x/\psi), \tag{1}$$

where $\Gamma(\cdot)$ denotes the Gamma function, ν are the degrees of freedom and ψ the scale parameter. For the sake of brevity, we will denote this density as $G(\nu,\psi)$. Following Gallant and Nychka (1987), we consider SNP expansions of this density (GSNP for short):

$$f_{GSNP}(x,\nu,\psi,\boldsymbol{\delta}) = f_G(x,\nu,\psi)\left[\sum_{j=0}^{m}\delta_j\left(\frac{x}{\psi}\right)^j\right]^2\frac{1}{d}, \tag{2}$$

where $\boldsymbol{\delta} = (\delta_0,\delta_1,\cdots,\delta_m)'$, and d is a constant that ensures that the density integrates to 1.

In order to interpret (2), it is convenient to expand the squared term. This yields the following result.

Proposition 1 *Let x be a $GSNP_m(\nu,\psi,\boldsymbol{\delta})$ variable with density $f_{GSNP}(x,\nu,\psi,\boldsymbol{\delta})$ given by (2). Then*

$$f_{GSNP}(x,\nu,\psi,\boldsymbol{\delta}) = f_G(x,\nu,\psi)\frac{1}{d}\sum_{j=0}^{2m}\gamma_j(\boldsymbol{\delta})\left(\frac{x}{\psi}\right)^j, \tag{3}$$

$$= \frac{1}{d}\sum_{j=0}^{2m}\gamma_j(\boldsymbol{\delta})\frac{\Gamma(\nu+j)}{\Gamma(\nu)}f_G(x,\nu+j,\psi), \tag{4}$$

where

$$\gamma_j(\boldsymbol{\delta}) = \sum_{k=\max\{j-m,0\}}^{\min\{j,m\}}\delta_j\delta_{j-k}.$$

Using Proposition 1, it is straightforward to show that the constant of integration can be expressed as

$$d = \sum_{j=0}^{2m} \gamma_j(\boldsymbol{\delta}) \frac{\Gamma(\nu + j)}{\Gamma(\nu)}.$$

But since (2) is homogeneous of degree zero in $\boldsymbol{\delta}$, there is a scale indeterminacy that we must solve by imposing a single normalising restriction on these parameters, such as $\delta_0 = 1$, or preferably $\boldsymbol{\delta}'\boldsymbol{\delta} = 1$, which we can ensure by working with hyperspherical coordinates.[1]

2.2 Moments

From Proposition 1, we can interpret the GSNP distribution as a mixture of $2m + 1$ Gamma distributions.[2] We can exploit the mixture interpretation together with the results in Appendix A to write the moment generating function of a GSNP variable x as

$$E\left[\exp(nx)\right] = \frac{1}{d} \sum_{j=0}^{2m} \gamma_j(\boldsymbol{\delta}) \frac{\Gamma(\nu + j)}{\Gamma(\nu)} (1 - \psi n)^{-(\nu+j)}.$$

Similarly, its characteristic function can be expressed as

$$\psi_{GSNP}(i\tau) = \frac{1}{d} \sum_{j=0}^{2m} \gamma_j(\boldsymbol{\delta}) \frac{\Gamma(\nu + j)}{\Gamma(\nu)} (1 - i\psi n)^{-(\nu+j)},$$

where i is the usual imaginary unit. As a result, we can write the moments of x as

$$E(x^n) = \frac{\psi^n}{d} \sum_{j=0}^{2m} \gamma_j(\boldsymbol{\delta}) \frac{\Gamma(\nu + j + n)}{\Gamma(\nu)}.$$

Hence, it is straightforward to show that the condition

$$\psi = d \left[\sum_{j=0}^{2m} \gamma_j(\boldsymbol{\delta}) \frac{\Gamma(\nu + j + 1)}{\Gamma(\nu)} \right]^{-1} \tag{5}$$

ensures that $E(x) = 1$. Since we plan to use the GSNP distribution to model the residual in MEM models, we assume in what follows that (5) holds to fix its scale.

[1] In particular, $\nu_0 = \cos\theta_1$; $\nu_i = (\prod_{k=1}^{i} \sin\theta_k)\cos\theta_{i+1}$ for $0 < i \leq m - 1$; and $\nu_m = \prod_{k=1}^{m} \sin\theta_k$, where $\theta_k \in [0, \pi)$, for $1 < k \leq m - 1$, and $\theta_m \in [0, 2\pi)$.

[2] This interpretation is consistent with Bowers (1966), who expands general density functions for positive random variables using sums of Gamma densities. Interestingly, the mixing variable of the equivalent mixture might have some negative weights, as in Steutel (1967) and Bartholomew (1969). However, this causes no inconsistencies because by construction the GSNP density is positive for all values of the parameters.

2.3 Relationship with Laguerre expansions

The Gamma distribution can also be used in place of the normal distribution as the parent distribution in a Gram Charlier expansion. In particular, if we consider a non-negative random variable y, under certain assumptions its density function $h(y)$ can be expressed as the product of a Gamma density times an infinite series of polynomials,

$$h(y) = f_G(y, \nu, \bar{\psi}) \sum_{j=0}^{\infty} c_j P_j(y, \nu, \bar{\psi}), \tag{6}$$

where $P_j(y, \nu, \bar{\psi})$ denotes the polynomial of order j that forms an orthonormal basis with respect to the Gamma distribution (see Johnson, Kotz, and Balakrishnan, 1994).[3] Following Bontemps and Meddahi (2012), we can express those polynomials as $P_0(y, \nu, \bar{\psi}) = 1$,

$$\begin{aligned} P_1(y, \nu, \bar{\psi}) &= \frac{\bar{\psi}^{-1} y - \nu}{\sqrt{\nu}}, \\ P_2(y, \nu, \bar{\psi}) &= \frac{[(\bar{\psi}^{-1} y)^2 - 2(\nu + 1)\bar{\psi}^{-1} y + \nu(\nu + 1)]}{\sqrt{2\nu(\nu + 1)}}, \end{aligned}$$

and in general

$$P_n(y, \nu, \bar{\psi}) = \frac{(\bar{\psi}^{-1} y - \nu - 2n - 2) P_{n-1}(y, \nu, \bar{\psi}) - \sqrt{(n-1)(\nu + n - 2)} P_{n-2}(y, \nu, \bar{\psi})}{\sqrt{n(\nu + n - 1)}}.$$

Given that

$$P_n(y, \nu, \bar{\psi}) = (-1)^n L_n(\bar{\psi}^{-1} y, \nu - 1) \sqrt{\frac{\Gamma(\nu) n!}{\Gamma(\nu + n)}}, \tag{7}$$

where $L_n(\cdot, \cdot)$ is the generalised Laguerre polynomial of order n, we will refer to (6) as the Laguerre expansion of the density of y. The orthonormal properties of these polynomials imply that we can obtain the coefficients of the expansion as

$$c_n = \int_0^{\infty} P_n(y, \nu, \bar{\psi}) h(y) dy. \tag{8}$$

Importantly, we can interpret the GSNP distribution as a finite order Laguerre expansion by reordering the terms in (3) appropriately. We can formally express this relationship as follows.

[3] Consider a random variable $x \sim G(\nu, \kappa)$. Then, $E[P_j(x, \nu, \kappa)] = 0$, $V[P_j(x, \nu, \kappa)] = 1$ and $E[P_k(x, \nu, \kappa) P_j(x, \nu, \kappa)] = 0$, for all $j, k \geq 0$ and $j \neq k$.

Proposition 2 *Let x be a $GSNP_m(\nu, \psi, \boldsymbol{\delta})$ variable with density $f_{GSNP}(x, \nu, \psi, \boldsymbol{\delta})$ given by (2). Then, this density can be expressed as a Laguerre expansion (6) of order $2m$ with coefficients*

$$c_n = \frac{(-1)^n}{d}\sqrt{\frac{\Gamma(\nu)n!}{\Gamma(\nu+n)}}\sum_{i=0}^{n}\sum_{j=0}^{2m}\frac{(-1)^i}{i!}\binom{n+\nu-1}{n-i}\frac{\Gamma(\nu+i+j)\psi^i}{\Gamma(\nu)\bar{\psi}^i}\gamma_j(\boldsymbol{\delta})$$

for $n = 0, \cdots, 2m$.

2.4 Comparison with other distributions

Given that the GSNP distribution is a finite order Laguerre expansion, it is natural to consider a truncated Laguerre expansion by treating the $c_j's$ as free parameters,

$$h(x) = f_G(x, \nu, \nu^{-1})\left[1 + \sum_{j=2}^{k} c_j P_j(x, \nu, \nu^{-1})\right], \tag{9}$$

where we have imposed that $c_1 = 0$ and $\bar{\psi} = 1/\nu$ so that this distribution has unit mean too. Unfortunately, this approach does not ensure the non-negativity of the resulting density function, a property that is satisfied by construction by the GSNP distribution.[4] In this sense, Amengual, Fiorentini, and Sentana (2013) have studied the parametric restrictions that the c_j coefficients must satisfy to ensure positivity in second and third-order Laguerre expansions.

Since both the GSNP distribution and the truncated Laguerre expansion have unit mean, one may ask which of them can generate a wider range of higher order moments. We address this question by comparing the coefficients of variation, skewness and kurtosis of the two distributions, which we will denote as τ, ϕ and λ, respectively. In particular, we compare (9) for $k = 3$ with a GSNP distribution of order $m = 2$ since both have the same number of free parameters. Figures 1a to 1c show the regions generated by both distributions on the $\tau - \phi$, $\tau - \lambda$ and $\phi - \lambda$ spaces. We have computed these regions using numerical methods.[5] We also include as a reference the values generated by the Gamma distribution, which are available in closed form,[6] and the lower bounds

[4] The GSNP satisfies sufficient conditions for positivity. See Meddahi (2001) and León, Mencía, and Sentana (2009) for a discussion of necessary and sufficient conditions.

[5] For the GSNP, we simulate values for $\boldsymbol{\delta}$ in the unit sphere for a dense grid of values for ν, and compute the envelope of the coefficients on the $\tau - \phi$, $\tau - \kappa$ and $\phi - \kappa$ spaces. For the Laguerre expansion we obtain the envelopes by combining a dense grid for ν with another dense grid for the frontier, as parametrised by Amengual, Fiorentini, and Sentana (2013).

[6] We can use the results in Appendix A to show that in the case of the Gamma those coefficients are $\tau_G = \sqrt{1/\nu}$, $\phi_G = \sqrt{4/\nu}$ and $\kappa_G = 3 + 6\nu^{-1}$.

that no properly-defined density can exceed (see Appendix B). As can be observed, both distributions provide similar flexibility for coefficients of variation smaller than 0.5. For larger coefficients of variation, the GSNP turns out to be superior in terms of feasible values of skewness and kurtosis. Interestingly, the flexibility of the Laguerre distribution relative to the Gamma distribution decreases drastically for coefficients of variation larger than around 1.8. In contrast, we do not observe this phenomenon in the GSNP distribution. In terms of skewness and kurtosis, the Laguerre expansion remains less flexible than the GSNP, but the differences are smaller.

Another way of adding flexibility would be to shift the expanded distribution by a constant amount Δ. This shift would affect τ, but not ϕ or λ. We shall revisit this issue in the empirical application.

3 Component MEM applied to the valuation of volatility futures

3.1 Real measure

Consider a non-traded volatility index whose value at time t is $V_t \geqslant 0$. We model this variable using the Multiplicative Error Model (MEM) proposed by Engle (2002). Specifically, we model the volatility index under the real measure $\mathbb{P}$ as

$$V_t = \mu_t(\boldsymbol{\theta})\varepsilon_t, \quad \mu_t(\boldsymbol{\theta}) = E(V_t|I_{t-1}), \tag{10}$$

where I_{t-1} denotes the information observed at $t-1$, $\boldsymbol{\theta}$ is a vector of parameters and ε_t is a unit mean *iid* non-negative variable. Engle and Gallo (2006) show that we can obtain a consistent estimator of $\boldsymbol{\theta}$ using the Gamma distribution even though the true distribution is not Gamma as long as $\mu_t(\boldsymbol{\theta})$ is correctly specified. However, in our case we are also interested in higher order moments because we want to study asset allocation strategies. Therefore, we will assume that ε_t follows a $GSNP_m(\nu, \psi, \boldsymbol{\delta})$ as a natural flexible generalisation of the Gamma distribution. As we mentioned before, we will use the scale restriction (5) to ensure that ε_t has unit mean.

Figure 2a shows that historically the VIX has mean reverted, but experiencing highly persistent swings. Figure 2b shows the more recent evolution of the VIX together with that of the CBOE S&P500 3-month volatility index, or VXV for short. Both series display similar mean reverting features, which is natural given that they measure volatility on the same variable at different horizons, but they do not coincide. For example, the VIX reached a maximum value of 80.86 on November 20, 2008, which was around 10 points higher than the VXV. As highlighted by Schwert (2011), this indicates that during the financial crisis the market did not expect the volatility of the S&P500 to remain at such high levels forever.

In an earlier paper (Mencía and Sentana, 2013), we modelled the VIX index in a continuous time framework, finding that it is crucial to allow for mean reversion to a time-varying long run mean, which in turn mean reverts more slowly. In this paper, though, we prefer to use a discrete time model because it allows us to uncouple the specification of the mean process from the shape of the conditional distribution. Thus, we are able to easily modify the distribution while keeping the autocorrelation structure of the model fixed.

In order to incorporate the aforementioned mean-reverting features in a discrete time setting, we use the MEM analogue to the component GARCH model proposed by Engle and Lee (1999). In particular, we model the conditional mean as the sum of two components $\mu_t(\boldsymbol{\theta}) = \varsigma_t(\boldsymbol{\theta}) + s_t(\boldsymbol{\theta})$, where $\varsigma_t(\boldsymbol{\theta})$ captures the slowly moving long run mean, while $s_t(\boldsymbol{\theta})$ captures short-run oscillations around it. We parametrise the long run component as

$$\varsigma_t(\boldsymbol{\theta}) = \omega + \rho\varsigma_{t-1}(\boldsymbol{\theta}) + \varphi(V_{t-1} - \mu_{t-1}(\boldsymbol{\theta})),$$

while

$$s_t(\boldsymbol{\theta}) = (\alpha + \beta)s_{t-1}(\boldsymbol{\theta}) + \alpha(V_{t-1} - \mu_{t-1}(\boldsymbol{\theta})).$$

Hence, the short run term mean reverts to zero, while the long run term mean reverts to ω. The unconditional mean implied by this model is $E[\mu_t(\boldsymbol{\theta})] = \omega/(1-\rho)$. Using the results in Engle and Lee (1999), we can show that the n-period ahead forecast can be easily obtained in closed form as $E(V_{t+n}|I_t) = \varsigma_{t+n|t}(\boldsymbol{\theta}) + s_{t+n|t}(\boldsymbol{\theta})$, where

$$\begin{aligned}
\varsigma_{t+n|t}(\boldsymbol{\theta}) &= \omega\frac{1}{1-\rho} + \rho^{n-1}\left[\varsigma_{t+1}(\boldsymbol{\theta}) - \frac{\omega}{1-\rho}\right], \\
s_{t+n|t}(\boldsymbol{\theta}) &= (\alpha+\beta)^{n-1}s_{t+1}(\boldsymbol{\theta}).
\end{aligned}$$

As expected, if $\rho > \alpha + \beta$ then $\varsigma_{t+n|t}(\boldsymbol{\theta})$ is more persistent than $s_{t+n|t}(\boldsymbol{\theta})$. In addition, notice that the convergence of $E(V_{t+n}|I_t)$ to its long-run value can be non-monotonic.

3.2 Risk-neutral measure

We solve the problem of pricing derivatives on V_t by defining a stochastic discount factor with an exponentially affine form

$$M_{t-1,t} \propto \exp(-\alpha\varepsilon_t). \tag{11}$$

Such a specification corresponds to the Esscher transform used in insurance (see Esscher, 1932). In option pricing applications, this approach was pioneered by Gerber and Shiu (1994), and has also been followed by Buhlman, Delbaen, Embrechts, and Shyraev (1996, 1998), Gourieroux and Monfort (2006a,b) and Bertholon, Monfort, and Pegoraro (2003) among others. On this basis, we can easily characterise the risk-neutral measure as follows.

Proposition 3 *Assume that the volatility index V_t follows the process given by (10) under the real measure $\mathbb{P}$, where the distribution of ε_t is a $GSNP_m(\nu, \psi, \boldsymbol{\delta})$ and (5) holds. Then, if the stochastic discount factor is defined by (11), under the equivalent risk-neutral measure $\mathbb{Q}$ we will have that $V_t = \mu_t(\boldsymbol{\theta})\varepsilon_t$, where $\varepsilon_t \sim^{\mathbb{Q}}$ iid $GSNP_m(\nu, \psi^{\mathbb{Q}}, \boldsymbol{\delta}^{\mathbb{Q}})$, with $\psi^{\mathbb{Q}} = \psi/(1+\alpha\psi)$ and $\delta_i^{\mathbb{Q}} = \delta_i(1+\alpha\psi)^i$ for $i = 0, \cdots, m$.*

Hence, if we model $\mu_t(\boldsymbol{\theta})$ as a Component-MEM process under $\mathbb{P}$, the process under $\mathbb{Q}$ will be another Component-MEM. However, the residual $\varepsilon_t^{\mathbb{Q}}$ will no longer have unit mean since

$$E^{\mathbb{Q}}[\varepsilon_t] = \varkappa = \frac{\psi}{d(1+\alpha\psi)} \sum_{j=0}^{2m} \gamma_j(\boldsymbol{\delta}^{\mathbb{Q}}) \frac{\Gamma(\nu+j+1)}{\Gamma(\nu)} \tag{12}$$

will be generally different from 1. We can exploit this feature to extract from VIX futures prices relevant economic information about the risk premia implicit in the CBOE market.

In order to price futures defined on V_t it is important to keep in mind that since V_t is not a directly traded asset, there is no cost of carry relationship between the price of the futures and V_t (see Grünbichler and Longstaff, 1996, for more details). Therefore, absent any other market information, the price at time t of a futures contract maturing at $t+n$ must be priced according to its risk-neutral expectation, i.e.

$$F_{t,t+n} = E^{\mathbb{Q}}(V_{t+n}|I_t). \tag{13}$$

On this basis, we can obtain the following analytical formula for (13).

Proposition 4 *The price at time t of a future written on the volatility index V_{t+n} under the risk-neutral measure defined in Proposition 3 can be written as*

$$F_{t,t+n} = \varkappa E^{\mathbb{Q}}[\varsigma_{t+n}(\boldsymbol{\theta}) + s_{t+n}(\boldsymbol{\theta})|I_t],$$

where

$$E^{\mathbb{Q}}\left[\begin{array}{c} \varsigma_{t+n}(\boldsymbol{\theta}) \\ s_{t+n}(\boldsymbol{\theta}) \end{array}\middle| I_t\right] = (\mathbf{I}_2 - \mathbf{A}_1)^{-1}\left[\mathbf{I}_2 - \mathbf{A}_1^{n-1}\right]\mathbf{A}_0 + \mathbf{A}_1^{n-1}\left[\begin{array}{c} \varsigma_{t+1}(\boldsymbol{\theta}) \\ s_{t+1}(\boldsymbol{\theta}) \end{array}\right],$$

$\mathbf{I}_2$ *is the identity matrix of order* 2, $\mathbf{A}_0 = (\ \omega \quad 0\ )'$ *and*

$$\mathbf{A}_1 = \left[\begin{array}{cc} \rho + \varphi(\varkappa - 1) & \varphi(\varkappa - 1) \\ \alpha(\varkappa - 1) & \alpha\varkappa + \beta \end{array}\right].$$

Hence, the futures price is an affine function of the short and long term components of the MEM process, whose coefficients depend on the time to maturity. Proposition 4 also shows that the change of measure not only affects the mean of the residual, but also the term structure of the forecasts of V_{t+n} for $n > 1$.

4 Empirical application

4.1 Estimation

As we mentioned in the introduction, nowadays volatility is widely regarded as an asset class on its own. For that reason, we apply our methodology to a relevant and realistic asset allocation context in which we compare static and dynamic strategies that invest in exchange traded notes (ETNs) tracking the S&P500 VIX short and mid term futures indices. The short term index measures the return from a daily rolling long position in the first and second VIX futures contracts that replicates the evolution of a one-month constant-maturity VIX futures. In turn, the mid term index takes long positions in the fourth, fifth, sixth and seventh month VIX futures contracts (see Standard & Poor's, 2012, for more details). Figure 3 shows the evolution of the short and mid-term indices. Both indices experienced large gains from the beginning of their history until the peak of the financial crisis in the Autumn of 2008. From then on, though, they have lost most of their value due to the reversion of the VIX to lower volatility levels. In the same figure we also display the contrarian strategies that would be obtained if it were possible to short the S&P500 VIX futures indices. As expected, those contrarian strategies would yield losses of value in the first half of the sample, and substantial gains after volatility started to decrease. In practice, it is actually possible to obtain direct and inverse exposure to both futures indices since they are a popular reference on which many ETNs are constructed. For instance, the goal of the VXX and VXZ ETNs is to mirror the short and mid term indices, respectively, while the XIV and ZIV ETNs replicate inverse positions on them. Given that a comparison of the original futures indices with those ETNs shows that the counterparty risk implicit in them is negligible, in what follows we will ignore such tracking errors and directly model the S&P500 VIX futures indices.

We will also model the VIX directly, and infer the distribution of the futures index returns conditional on the values of this volatility index. In this way, we can exploit the much larger historical information available on the VIX[7] (see Figure 2a). Specifically, let $\mathbf{y}_t$ denote the two dimensional vector which contains the VIX futures index returns at time t. Using the results from Section 3.2, we assume the following pricing structure,

$$\mathbf{y}_t = E^{\mathbb{Q}}(\mathbf{y}_t | V_t, I_{t-1}) + \boldsymbol{\epsilon}_t, \tag{14}$$

where $E^{\mathbb{Q}}(\mathbf{y}_t | V_t, I_{t-1})$ denotes the expected value of the index returns at time t given V_t (the VIX) and the information available at time $t-1$, and $\boldsymbol{\epsilon}_t$ the corresponding pricing errors, which simply reflect the fact that no model will be able to fit actual market futures prices perfectly. In addition, given that Bates (2000) and Eraker (2004) convincingly argue that if an asset is mispriced at time t, then it is likely to be mispriced at $t+1$, we assume that $\boldsymbol{\epsilon}_t \sim iid\ N(\rho_f \boldsymbol{\epsilon}_{t-1}, \boldsymbol{\Sigma}_f)$.

We obtain the model prices by exploiting the fact that the two futures index returns are portfolios of n_f VIX futures contracts maturing at $T_1, T_2, \cdots T_{n_f}$. Hence, we can express the price of the i^{th} element in $\mathbf{y}_t$ as

$$E^{\mathbb{Q}}(y_{it} | V_t, I_{t-1}) = \sum_{j=1}^{n_f} \zeta_{i,T_j-t} F_{t,T_j}(\boldsymbol{\theta}),$$

where $F_{t,T_j}(\boldsymbol{\theta})$ are the model-based futures prices and the loadings ζ_{i,T_j-t} deterministically depend on the time to maturity $T_j - t$ (see Standard & Poor's, 2012, for further details).

Under this setting, we can decompose the joint log-likelihood as

$$l(\mathbf{y}_t, V_t | I_{t-1}) = l(\mathbf{y}_t | V_t, I_{t-1}) + l(V_t | I_{t-1}), \tag{15}$$

[7] Another advantage is that we could value other indices different from the ones used in the estimation.

where $l(\mathbf{y}_t|V_t, I_{t-1})$ denotes the (pseudo) log-likelihood of the two futures index returns given the contemporaneous value of the VIX and I_{t-1}, while $l(V_t|I_{t-1})$ denotes the marginal likelihood of the VIX given I_{t-1}. We model $l(V_t|I_{t-1})$ by assuming that $V_t - \Delta$ follows a Component-MEM process with a $GSNP_m(\nu, \psi, \boldsymbol{\delta})$ conditional distribution given I_{t-1}. We introduce the constant shift Δ because the VIX cannot take values close to zero as they would imply constant equity prices over one month for all the constituents of the S&P500.[8] Thus, we can obtain large gains in fit by assigning zero probability to those events in which $V_t < \Delta$.

We use daily VIX index data from December 11, 1990, until February 28, 2014. Our data on the S&P 500 VIX short and mid-term futures indices goes from December 20, 2005 until the same final date as the VIX data. Table 1 compares the estimates that we obtain with the Gamma distribution and a symmetrically normalised GSNP(2) density in which we fix the scale of $\boldsymbol{\delta}$ using hyperspherical coordinates. The parameters of the conditional mean are similar for both distributions. This is reasonable given that the Gamma distribution yields consistent estimates of the conditional mean under misspecification (once again, see Engle and Gallo, 2006). However, a likelihood ratio test shows that the additional shape parameters of the GSNP density provide hugely significant gains. For that reason, in what follows we will focus on the GSNP density.

Table 1 shows that we obtain a negative and significant risk premium parameter with the GSNP density. To analyse its implications, we use the results from Proposition 4 to plot in Figure 4 the coefficients of the affine prediction formulas of the VIX at different horizons under both the real and risk-neutral measures. We can observe that the loadings on the short term factor decrease very quickly, whereas the long run component has a strong effect even at very long horizons. In other words, the VIX mean-reverts more slowly towards a higher mean under $\mathbb{Q}$ than under $\mathbb{P}$. Thus, we can conclude that it incorporates investors' risk-aversion by introducing more harmful prospects for the evolution of the VIX. Our results are consistent with the parameter estimates of the continuous time model in Mencía and Sentana (2013), and therefore confirm earlier findings by Andersen and Bondarenko (2007), among others, who show that the VIX almost uniformly exceeds realised volatility because investors are on average willing to pay a sizeable premium to acquire a positive exposure to future equity-index volatility.

[8] The minimum historical end-of-day value of the VIX has been 9.31 on December 22, 1993.

4.2 Asset allocation

In this section we study asset allocation strategies for investors seeking exposure to the two VIX futures indices. Consider an investor whose wealth at $t-1$ is A_{t-1}, and denote by $\mathbf{w}_t$ the 2×1 vector of portfolio weights chosen with information known at $t-1$. Then, the investor's wealth at t will be $A_t = A_{t-1}(1 + \mathbf{w}_t'\mathbf{y}_t)$, where $\mathbf{w}_t'\mathbf{y}_t$ is the return of the portfolio. We set $\sum_{j=1}^{n_f} |w_{it}| = 1$ to fix the leverage of the portfolio, which implies that the investor allocates all her initial wealth in the two assets. Importantly, we consider the sum of the absolute value of the weights instead of the sum of the signed values because a short position is in practice a long position on the inverse ETN. Subject to this scaling restriction, we consider an investor who chooses $\mathbf{w}_{t-1}$ to maximise the conditional Sharpe Ratio (SR):

$$SR = \frac{E(\mathbf{w}_t'\mathbf{y}_t|I_{t-1})}{\sqrt{V(\mathbf{w}_t'\mathbf{y}_t|I_{t-1})}}. \tag{16}$$

Unfortunately, the conditional distribution of $\mathbf{y}_t$ given I_{t-1} alone that appears in (16) is not directly available in our setting. In contrast, we know the distribution of $\mathbf{y}_t$ conditional on V_t and I_{t-1}. For that reason, we compute the moments of any given function $g(\cdot)$ of $\mathbf{w}_t'\mathbf{y}_t$ via the law of iterated expectations as follows

$$E[g(\mathbf{w}_t'\mathbf{y}_t)|I_{t-1}] = \int_{\Delta}^{\infty} E[g(\mathbf{w}_t'\mathbf{y}_t)|V_t, I_{t-1}]f(V_t|I_{t-1})dV_t, \tag{17}$$

where we exploit that

$$\mathbf{w}_t'\mathbf{y}_t \sim N[\rho_f \mathbf{w}_t'\boldsymbol{\epsilon}_{t-1} + \mathbf{w}_t' E^{\mathbb{Q}}(\mathbf{y}_t|V_t, I_{t-1}), \mathbf{w}_t'\boldsymbol{\Sigma}_f\mathbf{w}_t]$$

conditional on V_t and I_{t-1} to obtain the expectation in the integrand.[9] Importantly, (17) confirms that the SR depends on the entire conditional distribution of the VIX given its past history even though it only involves the first two moments of $\mathbf{y}_t$.

The parameters reported in Table 1 have been obtained using the whole sample. To avoid any look-ahead bias, we consider a feasible allocation procedure which re-estimates the parameters of the Component MEM - GSNP(2) distribution at each day in the sample using prior historical data only. Thus, we rebalance our investment strategies each day using feasible parameter estimates. In order to have sufficient data at the beginning of the sample, we only consider trading days from January 2, 2008, until the end of the sample. Nevertheless, our sample includes the bulk of the financial crisis.

[9]In practice, we compute the required integrals with numerical quadrature procedures.

Figure 5a shows the accumulated value of the SR maximising strategy (GSNP-SR for short) assuming that the initial wealth on January 2, 2008, was \$100. The gains from this strategy are vastly superior to those obtained from just investing in either the direct or inverse indices. As we mentioned before, the original short and mid indices performed better until December 2008, mainly because the VIX consistently grew during 2008. However, as the VIX started to reverse to lower levels in 2009, the short and mid-term indices rapidly lost value. In contrast, our dynamic strategy automatically rebalances the portfolio to deal with mean reversion.

In Figure 5b we consider the strategies of two different ETNs that combine long and short positions on the indices: XVIX and XVZ. The XVIX, launched by UBS, follows a long-short static strategy that allocates -0.5 to the short term VIX futures index and 1 to the mid term index. Barclays XVZ follows a more sophisticated dynamic strategy that rebalances the investment weights on the short and mid-term indices depending on whether the S&P500 volatility term structure is in contango or backwardation (see Standard & Poor's, 2011; UBS, 2012, for further details).[10] In addition, we consider the CVIX and CVZ strategies, which are two artificial indices proposed by Alexander and Korovilas (2013). The CVIX allocates 75% of capital to the XVIX and 25% of capital to the XVZ. Alexander and Korovilas (2013) choose these weights arguing that 75% (25%) is the proportion of days that the S&P500 volatility term structure is in contango (backwardation). The CVZ index follows a dynamic strategy which holds the XVIX when the S&P500 volatility term structure is in contango, and the XVZ when it is in backwardation. Figure 5b shows that these long-short strategies perform better than the pure long strategies, at least until April 2012. Moreover, the accumulated gains from the CVZ index were slightly superior to those of the GSNP-SR strategy until the summer of 2010. However, at this point the VIX, which had been growing steadily in response to the European sovereign crisis, started a downward trend that lasted until the spring of 2012, when it stabilised. Interestingly, this change of trend deteriorated the performance of the CVZ index without affecting the GSNP-SR strategy. As a result, the accumulated gains at the end of the sample are more than twice as big for the GSNP-SR strategy than for the CVZ index.

[10] On a given day there is contango if the VIX (or one-month volatility) is below the VXV index. Backwardation occurs when the VXV is higher than the VIX.

Figure 5 is useful to compare investments beginning on the first day of the sample. However, it does not reliably rank investments initiated at other points in the sample because accumulated gains are sensitive to the starting point. For that reason, we also compare the realised daily returns, which do not suffer from this problem. Table 2 shows descriptive statistics of the different strategies over the whole sample. The first column shows that in terms of annualised ex-post SR, the GSNP-SR strategy yields the highest values, followed by the CVZ, which is another dynamic strategy. In turn, the second column shows the low proportion of days with positive returns that would result from directly investing in the futures indices. Finally, the last columns of Table 2 show some quantiles of realised returns. The numbers indicate that the main benefit offered by the GSNP-SR strategy is that it substantially reduces the left tail. Specifically, we can see that the left-tail quantiles of the SR maximising strategy are higher than in the competing models. Not surprisingly, though, this result is achieved at the cost of giving away part of the benefits offered by some of the other strategies in the right tail.

Figure 6 and 7 show the sample SR and the proportion of positive returns over one-year rolling moving windows. Those figures confirm that the aggregate results observed in Table 2 for the whole sample are relatively stable across different subperiods. For example, Figure 6 shows that the GSNP-SR strategy is consistently among the strategies with highest SR's. The specific values, though, experience substantial swings over the sample, which partly reflect the difficulties in precisely estimating Sharpe ratios with such short sample spans. The rolling SR from the GSNP-SR strategy reached peak levels during the second halves of 2010 and 2013. In contrast, Figure 6a shows that although going short on the original indices was a good strategy during the last year of the sample, such a strategy performed very poorly in 2010 and 2011. Similarly, CVZ yields high SR's in 2010, but negative values afterwards (Figure 6b). Finally, Figure 7a once again shows that long positions on the indices yield too many negative returns, with only a high proportion of days with positive returns at the very beginning of the sample, when the VIX was still at its highest historical values. The long-short static and dynamic strategies shown in Figure 7b perform better, but they still suffer very large swings over the sample.

4.3 Robustness checks

In this subsection, we consider three alternative modifications of our asset allocation procedure. In the first one, we maintain the GSNP distributional assumption, but change the investor's preferences for an alternative profitability measure known as the Upside Potential Ratio (UPR). For a given return threshold r, the GSNP-UPR approach involves choosing the portfolio weights that maximise the conditional UPR, defined as

$$UPR(r) = \frac{E[\max(0, \mathbf{w}_t'\mathbf{y}_t - r)|I_{t-1}]}{\sqrt{E[\min(0, \mathbf{w}_t'\mathbf{y}_t - r)^2|I_{t-1}]}}. \tag{18}$$

Intuitively, the preferences implied by (18) penalise more heavily than the SR the uncertainty coming from the left tail.

The second robustness check that we consider consists of maximising the conditional SR, but based on a reduced form model that disregards the risk neutral valuation approach developed in Section 3.2. In particular, we directly estimate a bivariate Gaussian ARMA(2,1)-GARCH(1,1) with constant conditional correlation on the short and mid VIX futures return indices.

Lastly, we consider an alternative maximisation of the SR using another model not based on the MEM structure. In particular, we model $V_t - \Delta$ using a first order Autoregressive Gamma process (ARG). This discrete time process, which was originally proposed by Gourieroux and Jasiak (2006), can be interpreted as the discrete time counterpart to the popular square root process (see Cox, Ingersoll, and Ross, 1985). Specifically, in this model the conditional distribution of the VIX is a non-central chi-square. We show in Appendix C that we can easily price futures on the VIX in this setting using another Esscher transform.

Table 3 compares the performance of the realised returns of these three alternative approaches with those of the GSNP-SR strategy. We can observe that the GSNP-UPR strategy is able to yield a higher realised SR and UPR, and a very similar proportion of days with positive returns. In contrast, the strategy based on the bivariate ARMA-GARCH model yields much smaller values for the SR and UPR, although the proportion of days with positive returns is slightly higher in this case. Finally, the ARG process, estimated with the pricing error structure in (14), yields a slightly higher SR and UPR than the ARMA-GARCH model, but they are still noticeably smaller than those obtained with the GSNP framework.

Figure 8a shows that investing $100 on January 2, 2008, would have yielded similar gains at the end of the sample under both the GSNP-SR and GSNP-UPR strategies. However, the ARMA-GARCH bivariate model and the ARG process would have yielded much smaller gains. In the ARMA-GARCH case, it is mainly due to its bad performance in 2008. In the ARG case, the restrictive AR(1) time series structure does not seem to adapt well to the decreasing futures prices over the last year of the sample. Figure 8b and 8c show the evolution of the realised SR and UPR, respectively, computed over one-year moving windows. We can observe that the GSNP-SR and GSNP-UPR strategies are very similar in terms of the SR, while the GSNP-UPR strategy is slightly superior in terms of the UPR. Once again, the strategy based on the bivariate ARMA-GARCH model clearly underperforms in 2008, while the ARG framework performs poorly in 2013. The ARMA-GARCH model works better over the following years, but it systematically yields lower performance statistics than the strategies based on the GSNP distribution.

5 Conclusions

In this paper we develop a flexible distributional framework to model positive but stationary discrete time processes. We begin by proposing SNP expansions of the Gamma density to obtain a flexible family of GSNP distributions. We also compare our proposed distributions, which are positive by construction, to Laguerre expansions, for which it is difficult to ensure positivity. For the same number of parameters, our distribution turns out to be much more flexible in terms of the range of feasible coefficients of variation, skewness and kurtosis that it can achieve.

Since positive but stationary financial time series are typically highly persistent and mean-reverting, we consider the Multiplicative Error Model (MEM) of Engle (2002) which we combine with a unit-mean GSNP residual. In particular, we specify a component version of the MEM to describe the conditional mean of the VIX index as the sum of a short run component that mean-reverts to zero and a long run component, which mean-reverts more slowly towards a long run mean. In addition, we define an exponentially affine stochastic discount factor that allows us to price futures on the VIX index in closed form.

We use this framework to study asset allocation strategies in ETNs tracking the VIX futures short and mid-term indices. ETNs on VIX futures have attracted a lot of attention over the last few years, although the poor performance of some of them during decreasing volatility periods have raised some concerns about their risks. We show that the GSNP expansion yields significant likelihood gains with respect to the original Gamma distribution. For that reason, we consider an investment strategy that each day maximises the conditional Sharpe Ratio (SR), which depends on the GSNP expansion through a convolution formula.

We compare this strategy with the original ETNs, short positions on them, as well as long-short static and dynamic strategies. Our results show that having a flexible distribution is very relevant in practice because the GSNP strategy yields realised returns with the highest ex-post SRs over the whole sample. In effect, our strategy manages to increase the left tail quantiles of the return distribution, at the cost of having a somewhat thinner right tail than other strategies. We also observe that we generally obtain a superior performance with our GSNP strategy when we assess performance over rolling one-year sample sub-periods.

Finally, we investigate the extent to which our results are related to our choice of performance measure and modelling approach. To do so, we consider the Upside Potential Ratio (UPR) as an alternative performance measure, maintaining the GSNP distributional assumption. In addition, we check the impact of the GSNP distribution by keeping the SR preferences but considering either a bivariate ARMA-GARCH model that we directly estimate on the VIX futures index returns, or an Autoregressive Gamma process. We find that the alternative preferences yield minor improvements in performance, but the elimination of our flexible distributional assumption clearly leads to underperformance relative to GSNP-based strategies.

A fruitful avenue for future research would be to consider multivariate expansions, which could be used to invest simultaneously in ETNs on different volatility indices. It would also be interesting to explore time varying specifications of the shape parameters.

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A Properties of the Gamma distribution

Assume that x is a Gamma random variable whose pdf is given by (1). We summarise here the main properties of this distribution, as described in Johnson, Kotz, and Balakrishnan (1994). Its moment generating function is

$$E\left[\exp(nx)\right] = (1 - \psi n)^{-\nu},$$

for $n < \psi^{-1}$, while its characteristic function is $\psi_G(i\tau) = (1 - i\psi n)^{-\nu}$. Similarly, we can express the moments of x as

$$E(x^n) = \psi^n \frac{\Gamma(\nu + n)}{\Gamma(\nu)}. \tag{A1}$$

B Feasible moments of distributions

Stuart and Ord (1977) explain that regardless of the shape of the distribution, the skewness-kurtosis relationship

$$\kappa \geq 1 + \phi^2 \tag{B2}$$

must hold. In a similar spirit, we can apply the Cauchy-Schwarz inequality to show that for a positive random variable x:

$$[E(x^{3/2}x^{1/2})]^2 \leq E(x^3)E(x),$$

so that $\mu_2'^2 \leq \mu_1'\mu_3'$. If we introduce in this expression the relationships between the central and non-central moments, $\mu_2' = \mu_2 + \mu_1'^2$ and $\mu_3' = \mu_3 + 3\mu_1'\mu_2 + \mu_1'^3$, we can show that

$$\phi \geq \tau - \tau^{-1}. \tag{B3}$$

Finally, if we combine (B3) with (B2), we can show that $\kappa \geq 1 + [\max\{\tau - \tau^{-1}, 0\}]^2$.

C Futures pricing based on the ARG process

Let V_t follow an Autoregressive Gamma process of order 1 under the real measure, or $ARG(1)$ for short. Then, it can be shown that the distribution of $2V_t/c$ conditional on I_{t-1} is a non-central chi-square with noncentrality parameter $2\beta V_{t-1}$ and degrees of freedom 2δ. If we consider the exponentially affine stochastic discount factor

$$M_{t-1,t} = \exp(-\alpha V_t),$$

then it can be easily shown that $2(1+2\alpha)V_t/c$ will be, under the risk-neutral measure, a non-central chi-square with degrees of freedom 2δ and non-centrality parameter $2\beta V_{t-1}/(1+2\alpha)$. In practice, this process can be reinterpreted as an ARG(1) process with parameters $\delta_{\mathbb{Q}}=\delta$,

$$c_{\mathbb{Q}}=\frac{c}{1+2\alpha},\quad \beta_{\mathbb{Q}}=\frac{\beta}{1+2\alpha}.$$

Hence, the futures price can be written as

$$F_{t,t+n}=E^{\mathbb{Q}}[V_{t+n}|I_t]=c_{\mathbb{Q},n}\delta+c_{\mathbb{Q},n}\beta_{\mathbb{Q},n}V_t,$$

where

$$c_{\mathbb{Q},n}=\frac{1-c_{\mathbb{Q}}^n\beta_{\mathbb{Q}}^n}{1-c_{\mathbb{Q}}\beta_{\mathbb{Q}}}c_{\mathbb{Q}},\quad \beta_{\mathbb{Q},n}=\frac{c_{\mathbb{Q}}^{n-1}\beta_{\mathbb{Q}}^n(1-c_{\mathbb{Q}}\beta_{\mathbb{Q}})}{1-c_{\mathbb{Q}}^n\beta_{\mathbb{Q}}^n}.$$

D Proofs of propositions

D.1 Proposition 1

We can show through tedious but straightforward algebra that

$$\left[\sum_{j=0}^{m}\delta_j\left(\frac{x}{\psi}\right)^j\right]^2=\sum_{j=0}^{2m}\gamma_j(\boldsymbol{\delta})\left(\frac{x}{\psi}\right)^j.$$

Then, we can use (1) to show that

$$\begin{aligned}\left(\frac{x}{\psi}\right)^j f_G(x,\nu,\psi) &= \frac{1}{\Gamma(\nu)\psi^{\nu+j}}x^{\nu+j-1}\exp(-x/\psi)\\ &= \frac{\Gamma(\nu+j)}{\Gamma(\nu)}f_G(x,\nu+j,\psi).\end{aligned}$$

D.2 Proposition 2

Introducing (4) in (8), we can express the coefficients of the Laguerre expansion as

$$c_n=\frac{1}{d}\sum_{j=0}^{2m}\gamma_j(\boldsymbol{\delta})\frac{\Gamma(\nu+j)}{\Gamma(\nu)}\int_0^\infty P_n(y,\nu,\bar{\psi})f_G(y,\nu+j,\psi)dy. \tag{D4}$$

If we write $P_n(y,\nu,\bar{\psi})$ in terms of the n-order Laguerre polynomial, as in (7), we obtain

$$c_n=(-1)^n\sqrt{\frac{\Gamma(\nu)n!}{\Gamma(\nu+n)}}\frac{1}{d}\sum_{j=0}^{2m}\gamma_j(\boldsymbol{\delta})\frac{\Gamma(\nu+j)}{\Gamma(\nu)}\int_0^\infty L_n(\bar{\psi}^{-1}y,\nu-1)f_G(y,\nu+j,\psi)dy.$$

Then, if we use the following property

$$L_n(\bar{\psi}^{-1}y,\nu-1)=\sum_{i=0}^{n}\frac{(-1)^i}{i!}\binom{n+\nu-1}{n-i}(\bar{\psi}^{-1}y)^i.$$

from Abramowitz and Stegun (1965) (page 775), then we obtain

$$\int_0^{\infty} L_n(\bar{\psi}^{-1}y, \nu-1) f_G(y, \nu+j, \psi) dy = \sum_{i=0}^{n} \frac{(-1)^i}{i!} \binom{n+\nu-1}{n-i} \bar{\psi}^{-i} \int_0^{\infty} y^i f_G(y, \nu+j, \psi) dy, \tag{D5}$$

where

$$\int_0^{\infty} y^i f_G(y, \nu+j, \psi) dy = \psi^i \frac{\Gamma(\nu+i+j)}{\Gamma(\nu+j)} \tag{D6}$$

from (A1). Introducing (D5) and (D6) in (D4), we obtain the final result.

D.3 Proposition 3

The risk-neutral density of ε_t will be proportional to

$$\begin{aligned} & f_{GSNP}(\varepsilon_t, \nu, \psi, \boldsymbol{\delta}) \exp(-\alpha\varepsilon_t), \\ = \quad & f_G(\varepsilon_t, \nu, \psi) \exp(-\alpha\varepsilon_t) \left[\sum_{j=0}^{m} \delta_j \left(\frac{\varepsilon_t}{\psi} \right)^j \right]^2 \end{aligned}$$

It can be easily shown that $f_G(\varepsilon_t, \nu, \psi) \exp(-\alpha\varepsilon) \propto \varepsilon^{\nu-1} \exp\left(-\varepsilon/\psi^{\mathbb{Q}}\right)$, where $\psi^{\mathbb{Q}} = \psi/(1+\alpha\psi)$. Similarly, we can write

$$\sum_{j=0}^{m} \delta_j \left(\frac{\varepsilon_t}{\psi} \right)^j = \sum_{j=0}^{m} \delta_j (1+\alpha\psi)^j \left(\frac{\varepsilon_t}{\psi^{\mathbb{Q}}} \right)^j .$$

Hence, we can always define $\delta_j^{\mathbb{Q}} = \delta_j (1+\alpha\psi)^j$. This proves that the resulting density is a $GSNP_m(\nu, \psi^{\mathbb{Q}}, \boldsymbol{\delta}^{\mathbb{Q}})$.

D.4 Proposition 4

If we use (12), we can show that

$$F_{t,t+n} = E^{\mathbb{Q}}[V_{t+n}|I(t)] = \varkappa E^{\mathbb{Q}}[\varsigma_{t+n}(\boldsymbol{\theta}) + s_{t+n}(\boldsymbol{\theta})|I_t]$$

and

$$E^{\mathbb{Q}}[\varsigma_{t+n}(\boldsymbol{\theta})|I_{t+n-2}] = \omega + [\rho + \varphi(\varkappa - 1)]\varsigma_{t+n-1}(\boldsymbol{\theta}) + \varphi(\varkappa - 1) s_{t+n-1}(\boldsymbol{\theta}).$$

Similarly, we can obtain

$$E^{\mathbb{Q}}[s_{t+n}(\boldsymbol{\theta})|I_{t+n-2}] = \alpha(\varkappa - 1)\varsigma_{t+n-1}(\boldsymbol{\theta}) + [\alpha\varkappa + \beta] s_{t+n-1}(\boldsymbol{\theta}).$$

Hence, we have

$$E^{\mathbb{Q}} \left[\begin{array}{c} \varsigma_{t+n}(\boldsymbol{\theta}) \\ s_{t+n}(\boldsymbol{\theta}) \end{array} \middle| I_{t+n-2} \right] = \mathbf{A}_0 + \mathbf{A}_1 \left[\begin{array}{c} \varsigma_{t+n-1}(\boldsymbol{\theta}) \\ s_{t+n-1}(\boldsymbol{\theta}) \end{array} \right].$$

By applying the law of iterated expectations recursively to condition on $I_{t+n-3}, I_{t+n-4}, \cdots, I_t$, we can obtain the final result after some straightforward algebraic manipulations.

Table 1

Maximum likelihood estimates of Component-MEM models

	Gamma	s.e.	GSNP(2)	s.e.
α	0.662	0.010	0.666	0.010
β	0.286	0.011	0.282	0.011
ω	0.025	0.002	0.025	0.002
ρ	0.998	0.000	0.998	0.000
ϕ	0.221	0.009	0.224	0.009
Δ	5.179	0.134	5.386	0.160
ν	139.903	4.113	115.187	4.556
θ_1			0.019	0.001
θ_1			3.137	0.000
Risk premium	-0.387	0.159	-0.248	0.113
σ_{short}	0.019	0.000	0.019	0.000
σ_{mid}	0.012	0.000	0.012	0.000
$\rho_{short,mid}$	0.689	0.009	0.689	0.009
ρ_f	0.990	0.002	0.990	0.002
Likelihood	3183.954		3465.495	
LR test	563.083 ($p - value = 0.000$)			

Notes: The estimation uses VIX data from December 11, 1990, until February 28, 2014, as well as data on the S&P 500 VIX short and mid-term futures indices from December 20, 2005 until the same final date. "Gamma" denotes a Component-MEM model whose conditional distribution given the information known at $t-1$ is Gamma, while in "GSNP(2)" the conditional distribution is a SNP expansion of order 2 of the Gamma distribution. Standard errors have been computed from the outer product of the analytical score.

Table 2

Descriptive statistics of the realised returns of different asset allocation strategies.

	SR	Ret>0(%)	Mean	Std. Dev.	Skewness	Kurtosis	5% Perc.	25% Perc.	Median	75% Perc.	95% Perc.
Short	-0.594	42.6	-0.146	3.974	0.844	6.663	-5.943	-2.277	-0.581	1.541	6.935
Mid	-0.281	45.1	-0.035	1.995	0.632	6.619	-3.042	-1.052	-0.186	0.875	3.307
-1×Short	0.594	57.8	0.146	3.974	-0.844	6.663	-6.935	-1.541	0.581	2.277	5.943
-1×Mid	0.281	55.1	0.035	1.995	-0.632	6.619	-3.307	-0.875	0.186	1.052	3.042
XVIX	0.586	53.1	0.032	0.880	-0.139	5.888	-1.346	-0.443	0.058	0.518	1.378
XVZ	0.398	49.1	0.034	1.384	0.885	11.260	-1.796	-0.558	-0.012	0.507	2.127
CVIX	0.611	52.1	0.032	0.860	0.210	6.461	-1.312	-0.433	0.047	0.480	1.357
CVZ	0.911	53.3	0.074	1.324	0.764	11.641	-1.655	-0.505	0.073	0.568	1.964
GSNP-SR	1.868	57.7	0.147	1.274	-0.249	14.944	-1.613	-0.339	0.128	0.631	2.036

Notes: The sample used is 1-Jan-2008 to 27-Feb-2014. SR denotes the Sharpe Ratio, expressed in annualised terms. The column labelled "Ret> 0 (%) indicates the proportion of days with positive returns. "Short" and "Mid" denote the S&P 500 VIX short and mid futures indices. "-1×" denote short sales on those indices. XVIX is a UBS ETN following a long-short static strategy on the VIX futures indices, while XVZ is a Barclays ETN following a dynamic strategy. CVIX and CVZ are investment strategies proposed by Alexander and Korovilas (2013). GSNP-SR denotes the returns obtained by maximising the conditional Sharpe Ratio, based on the parameters obtained from a Component MEM for the VIX with a GSNP(2) expansion of the Gamma distribution. The parameters are estimated each day using the information available at that point.

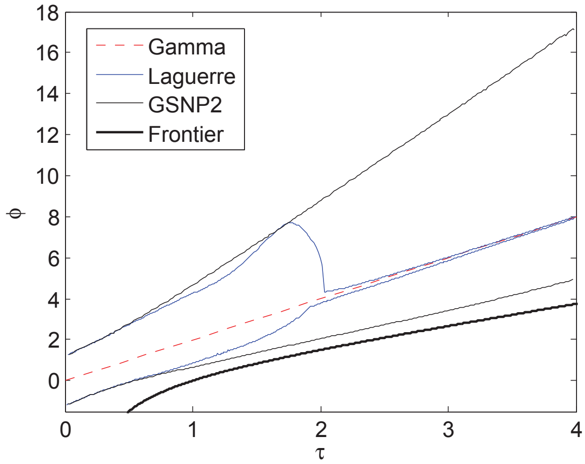
Table 3
Profitability measures of the realised returns of alternative dynamic asset allocation strategies.

	SR	Ret>0(%)	UPR
GSNP-SR	1.868	57.7	9.027
GSNP-UPR	1.917	57.2	9.582
ARMA-GARCH	0.781	58.5	7.288
ARG-SR	1.063	56.0	8.235

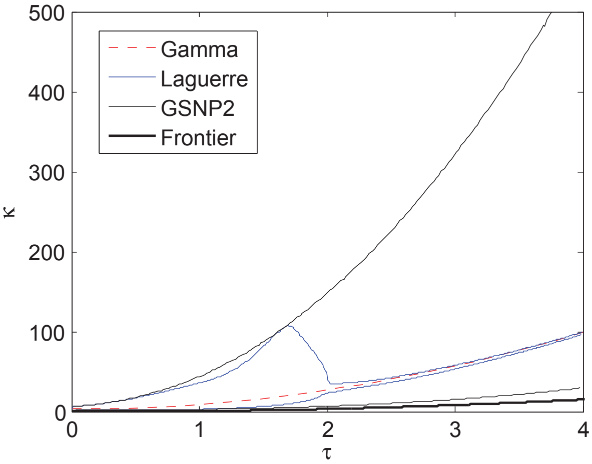
Notes: The sample used is 1-Jan-2008 to 27-Feb-2014. SR denotes the Sharpe Ratio, while UPR denotes the Upside Potential Ratio with zero as the return threshold. Both the SR and UPR are expressed in annualised terms. The column labelled "Ret> 0 (%) indicates the proportion of days with positive returns. GSNP-SR (GSNP-UPR) denotes the returns obtained by maximising the conditional SR (UPR), based on the parameters obtained from a Component MEM for the VIX with a GSNP(2) expansion of the Gamma distribution. ARMA-GARCH denotes the returns obtained by maximising the conditional SR, based on the parameters obtained from a bivariate ARMA(2,1)-GARCH(1,1) with constant conditional correlation, estimated on the short and mid VIX future index returns. ARG-SR denotes the returns obtained by maximising the conditional SR, based on the parameters obtained from a first order Autoregressive Gamma process. The parameters are estimated each day using the information available at that point.

Figure 1: Regions of the coefficients of variation, skewness and kurtosis credit institutions

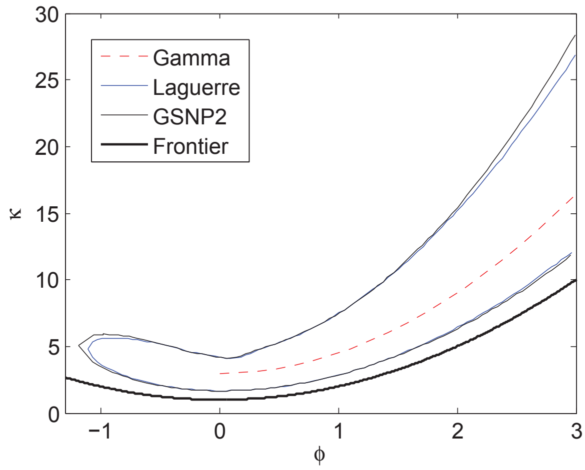
(a) Variation vs. Skewness



(b) Variation vs. Kurtosis



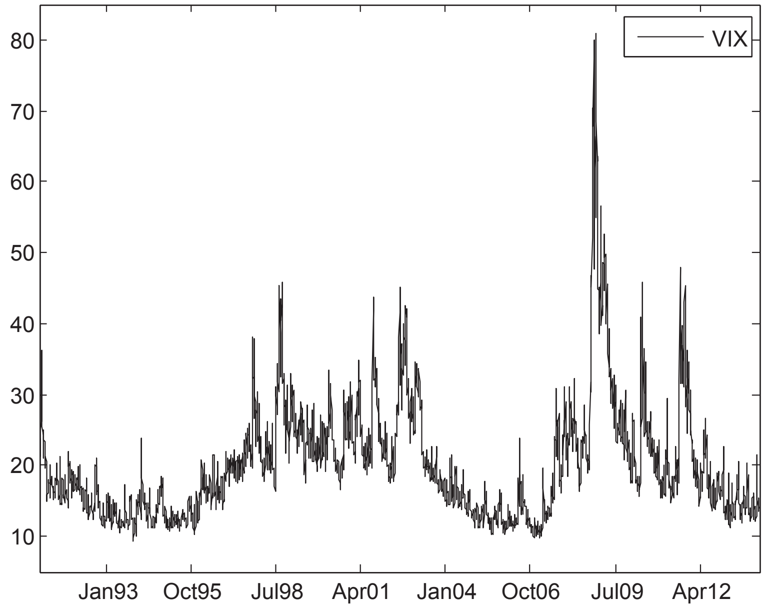
(c) Skewness vs. Kurtosis



Notes: τ, ϕ and κ denote the coefficients of variation, skewness and kurtosis, respectively. The lines labelled “Frontier” denote the limits that no density can surpass. “Laguerre” denotes a truncated third order Laguerre expansion of the Gamma distribution, while “GSNP2” denotes a second order SNP expansion of the Gamma distribution.

Figure 2: Historical evolution of the VIX index

(a) Dec 1990- Jan 2013



(b) Comparison with VXV (Dec 2007- Jan 2013)

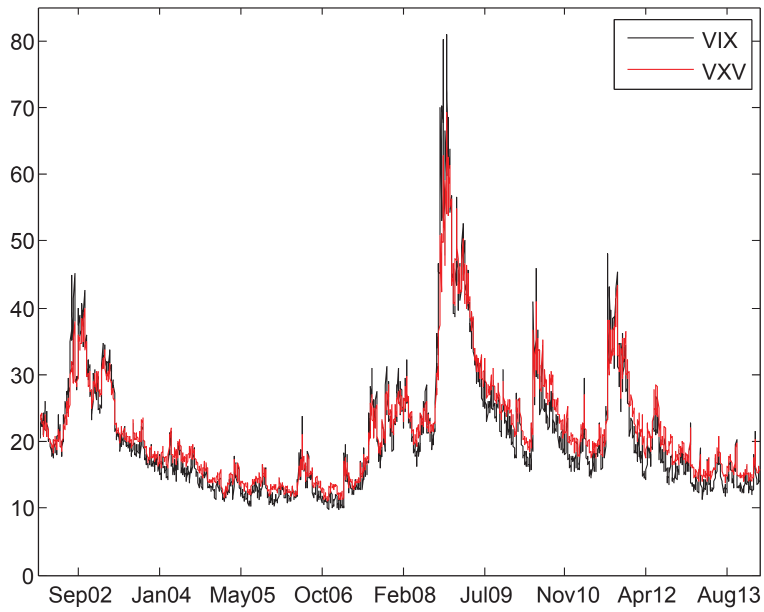
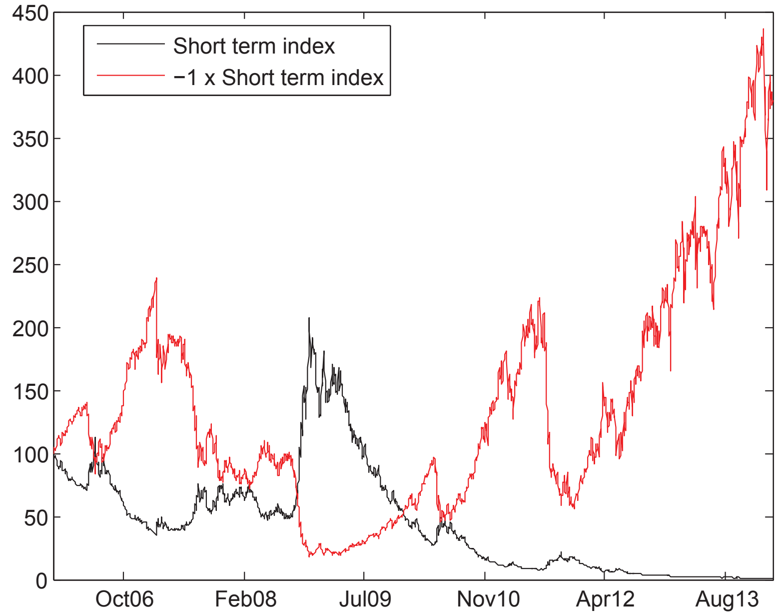
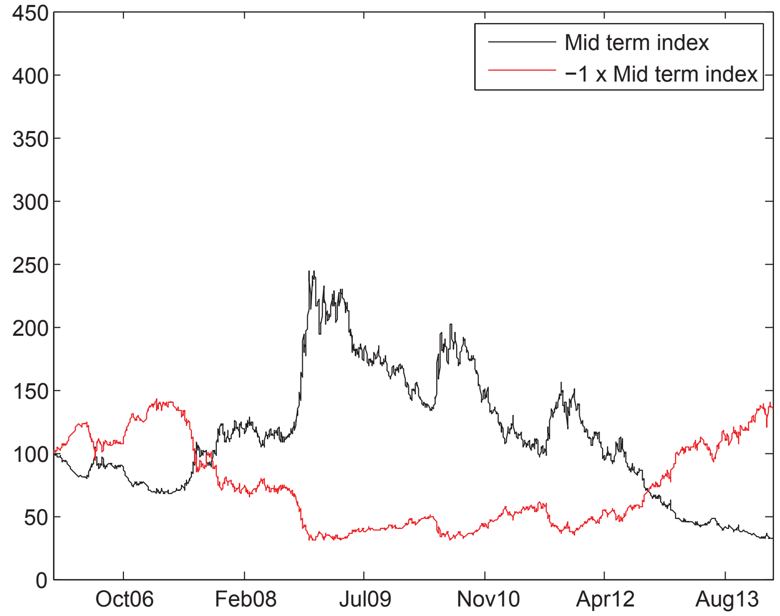


Figure 3: Historical evolution of S&P 500 VIX futures indices

(a) Short term index



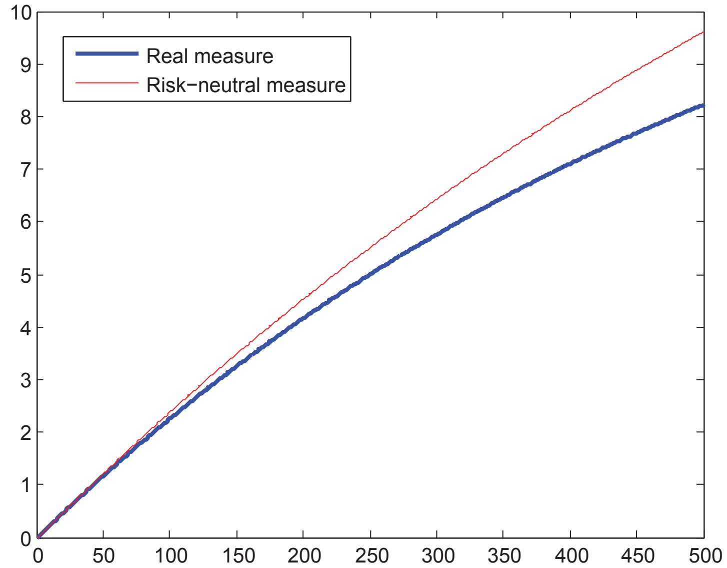
(b) Mid-term index



Note: The black lines show the evolution of the original S&P 500 VIX futures indices, while the red lines show the evolution of indices with exactly the opposite returns from the original ones.

Figure 4: Coefficients of the affine prediction formulas of the VIX at different horizons under the real and risk-neutral densities

(a) Intercept



(b) Coefficients on the short and long-run components

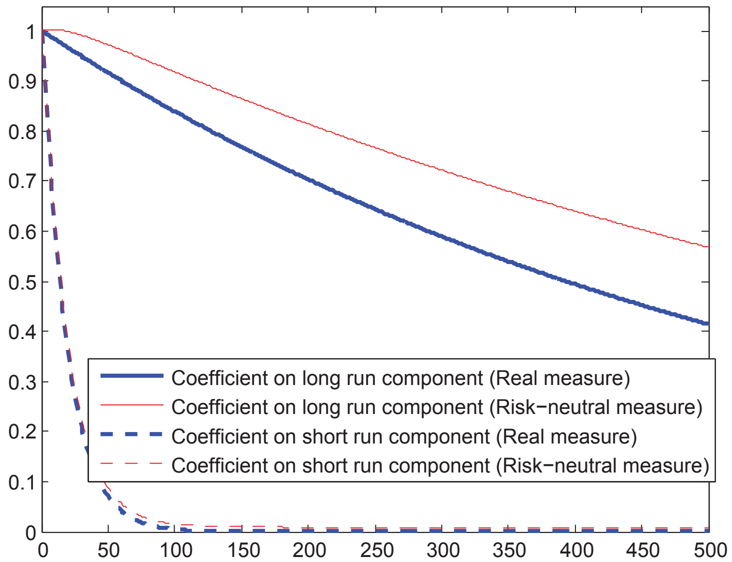
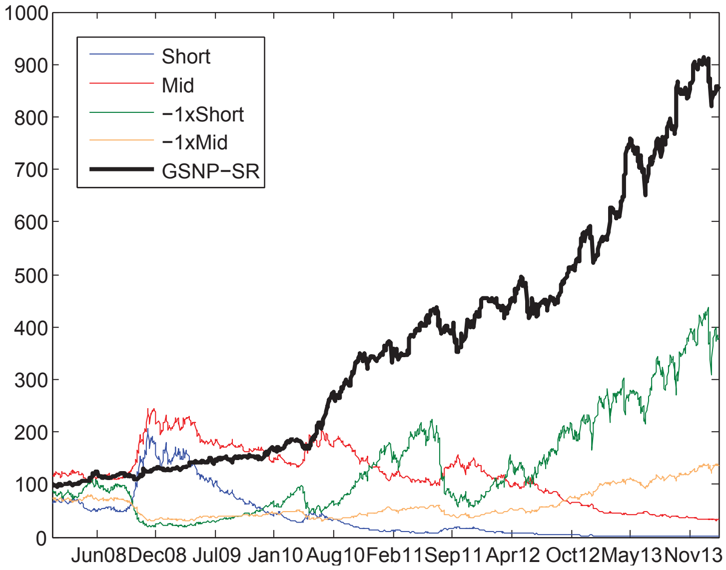
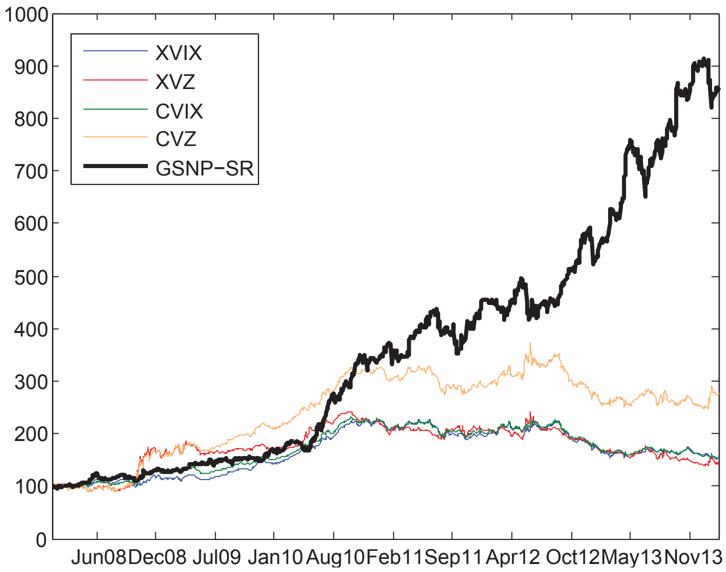


Figure 5: Evolution of investment strategies accumulated gains

(a) GSNP vs. buy and hold strategies



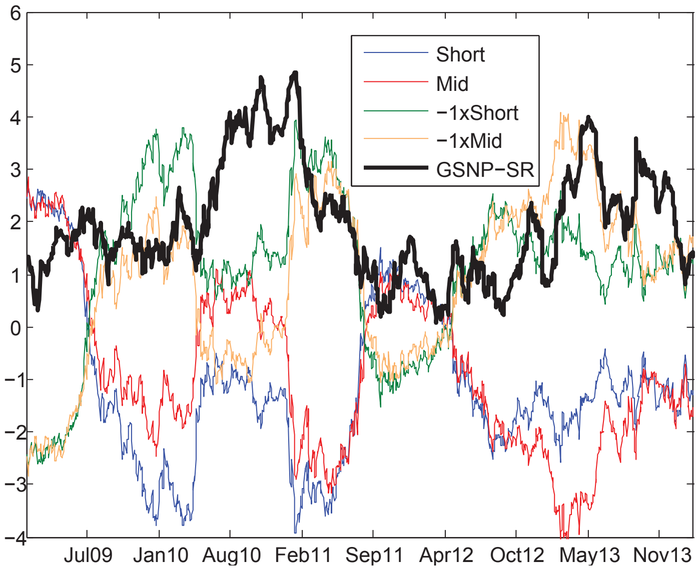
(b) GSNP vs. long-short static and dynamic strategies



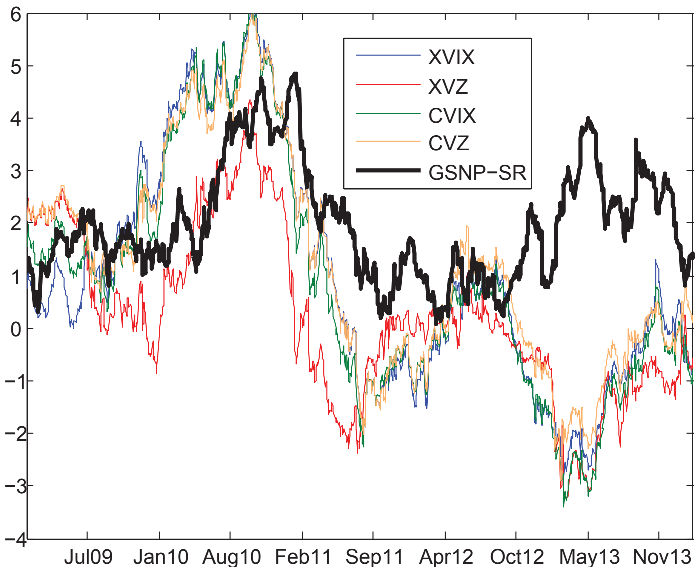
Note: All the strategies start from an initial investment of \$100. “Short” and “Mid” denote the S&P 500 VIX short and mid futures indices. “-1×” denote short sales on those indices. XVIX is a UBS ETN following a long-short static strategy on the VIX futures indices, while XVZ is a Barclays ETN following a dynamic strategy. CVIX and CVZ are investment strategies proposed by Alexander and Korovilas (2013). GSNP-SR denotes the returns obtained by maximising the conditional Sharpe Ratio, based on the parameters obtained from a Component MEM for the VIX with a GSNP(2) expansion of the Gamma distribution. The parameters are estimated each day using the information available at each day.

Figure 6: Sharpe Ratio of realised returns over a one-year moving window

(a) GSNP vs. buy and hold strategies



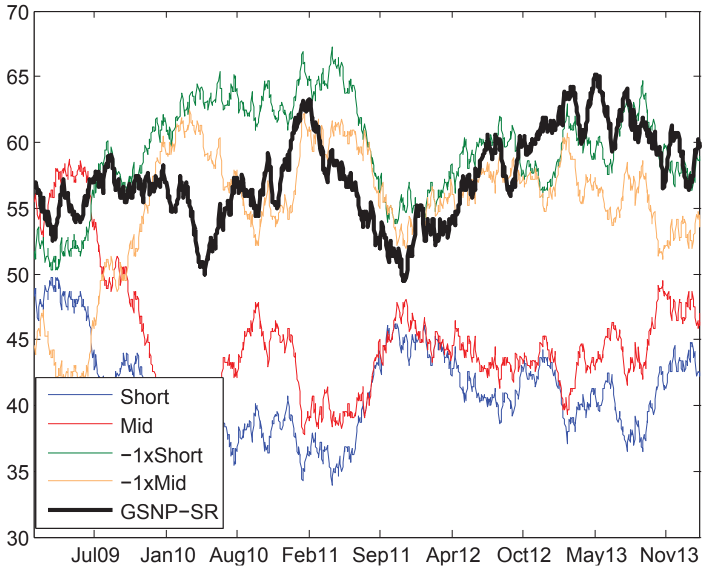
(b) GSNP vs. long-short static and dynamic strategies



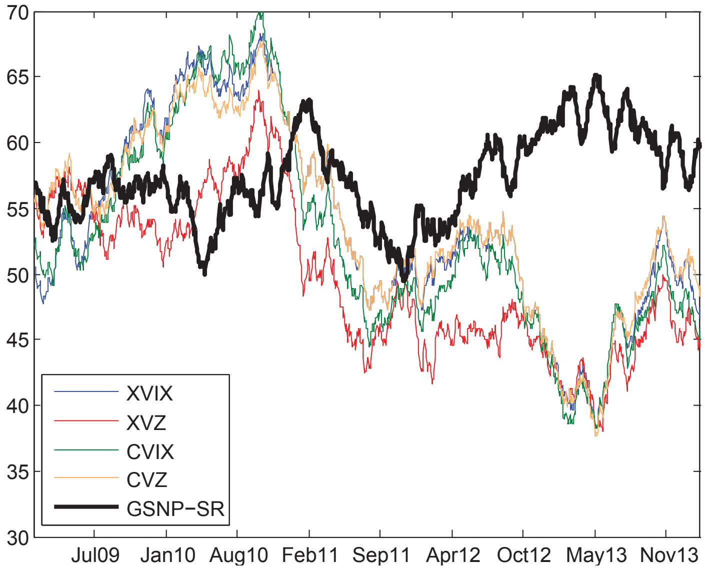
Note: "Short" and "Mid" denote the S&P 500 VIX short and mid futures indices. "-1×" denote short sales on those indices. XVIX is a UBS ETN following a long-short static strategy on the VIX futures indices, while XVZ is a Barclays ETN following a dynamic strategy. CVIX and CVZ are investment strategies proposed by Alexander and Korovilas (2013). GSNP-SR denotes the returns obtained by maximising the conditional Sharpe Ratio, based on the parameters obtained from a Component MEM for the VIX with a GSNP(2) expansion of the Gamma distribution. The parameters are estimated each day using the information available at each day.

Figure 7: Proportion of days with positive realised returns over a one-year moving window (%)

(a) GSNP vs. buy and hold strategies



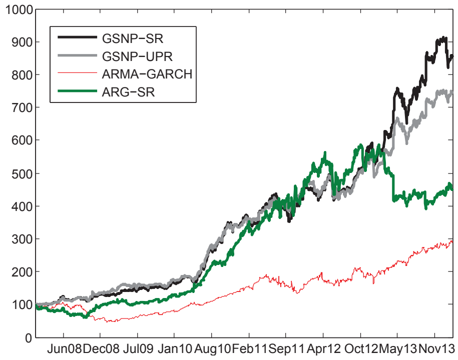
(b) GSNP vs. long-short static and dynamic strategies



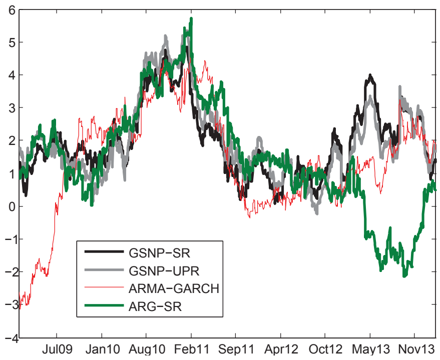
Note: "Short" and "Mid" denote the S&P 500 VIX short and mid futures indices. "-1×" denote short sales on those indices. XVIX is a UBS ETN following a long-short static strategy on the VIX futures indices, while XVZ is a Barclays ETN following a dynamic strategy. CVIX and CVZ are investment strategies proposed by Alexander and Korovilas (2013). GSNP-SR denotes the returns obtained by maximising the conditional Sharpe Ratio, based on the parameters obtained from a Component MEM for the VIX with a GSNP(2) expansion of the Gamma distribution. The parameters are estimated each day using the information available at each day.

Figure 8: Profitability measures of the realised returns of alternative dynamic asset allocation strategies.

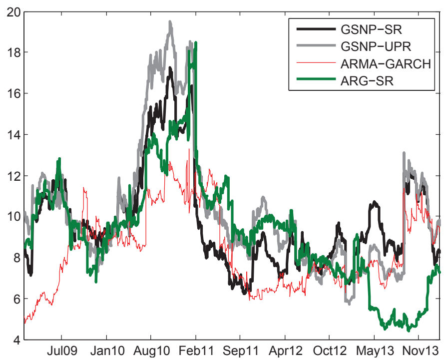
(a) Accumulated gains since Jan-2008



(b) Realised Sharpe Ratio over one-year moving windows



(c) Realised Upside Potential Ratio over one-year moving windows



Note: Both the Sharpe Ratio (SR) and Upside Potential Ratio (UPR) are expressed in annualised terms. GSNP-SR (GSNP-UPR) denotes the returns obtained by maximising the conditional SR (UPR), based on the parameters obtained from a Component MEM for the VIX with a GSNP(2) expansion of the Gamma distribution. ARMA-GARCH denotes the returns obtained by maximising the conditional SR, based on the parameters obtained from a bivariate ARMA(2,1)-GARCH(1,1) with constant conditional correlation, estimated on the short and mid VIX future index returns. ARG-SR denotes the returns obtained by maximising the conditional SR, based on the parameters obtained from a first order Autoregressive Gamma process. The parameters are estimated each day using the information available at each day.

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