INFLATION DYNAMICS IN A MODEL WITH FIRM ENTRY AND (SOME) HETEROGENEITY

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Abstract

We analyse the incidence of endogenous entry and firm TFP-heterogeneity on the response of aggregate inflation to exogenous shocks. We build up an otherwise standard DSGE model in which the number of firms is endogenously determined and firms differ in their steady state level of productivity. This splits the industry structure into firms of different sizes. Calibrating the different transition rates, across firm sizes and out of the market we reproduce the main features of the distribution of firms in Spain. We then compare the inflation response to technology, interest rate and entry cost shocks, among others. We find that structures in which large (more productive) firms predominate tend to deliver more muted inflation responses to exogenous shocks.

Keywords: firm dynamics, industrial structure, inflation, business cycles.

Resumen

En este trabajo se analiza el impacto que la entrada endógena de empresas y la heterogeneidad en la productividad empresarial tienen sobre la respuesta de la inflación ante perturbaciones exógenas. Partiendo de un modelo DSGE estándar, se endogeniza el número de empresas y se permite que estas difieran en su nivel de productividad en el estado estacionario y, por lo tanto, en su tamaño. Se calibran las probabilidades de transición de las empresas entre distintos tamaños y de salida del mercado para reproducir las principales características de la distribución de empresas en España. A continuación se compara la respuesta de la inflación ante una perturbación de carácter tecnológico, del tipo de interés y de coste de entrada, entre otras. Se muestra que estructuras industriales en las que predominan empresas grandes (más productivas) generan una menor respuesta de la inflación ante perturbaciones exógenas.

**Palabras clave:** dinámica empresarial, estructura industrial, inflación, ciclo económico.

**Códigos JEL:** E31, E32, L11, L16.
1 Introduction

In this paper we look at the effect that the productivity distribution across firms in an economy exerts upon the response to exogenous shocks of the inflation rate and other related macroeconomic variables. We build upon the work of Bilbiie, Ghironi and Melitz (2012) focusing on the role played by the number of productive firms in a DSGE model and extend their framework in a number of ways, in particular to allow for some limited firm heterogeneity in productivity (and hence size). The response of inflation to exogenous disturbances has been extensively studied both empirically and theoretically using general equilibrium models with nominal frictions. A number of factors, other than wage and price inertia, have been identified as potential drivers of inflation (Álvarez et al., 2006, Fabiani et al, 2006 and Vermeulen et al., 2012), such as the way monetary policy responds to exogenous shocks, the exchange rate regime and the presence of real rigidities in the labor market (Campolmi and Faia, 2011) or in the goods market (Andrés, Ortega and Vallés, 2008). It is somewhat striking that while the productivity distribution of firms has been considered a key determinant of competitiveness, the direct relationship of this feature with the dynamics of inflation in an economy has received such scant attention so far in the DSGE literature that very often assumes a monopolistic competition structure with a fixed number of equal firms.

As discussed by Lewis and Poilly (2011), endogenizing the number of firms in an economy opens up two additional channels through which the short-run dynamics of inflation can be affected. One is the competition effect whereby shocks that favor the entry of new firms in the industry enhance the competition among them, thus leading to a reduction in the markup and hence in the inflation rate. Jaimovich and Floettoto (2008) and Etro and Colciago (2010) look at this effect in a model with flexible prices. Besides, the entry of new firms can be thought of as an expansion in the number of varieties produced in an economy having a direct effect on consumers’ welfare and on the aggregate (welfare-based) price level and inflation. Bilbiie, Ghironi and Meltz (2008, 2012) have studied this variety effect and its implications in a number of papers. From an empirical point of view Correa-López, et al. (2010) find that the response of inflation to productivity shocks is indeed amplified in the OECD economies with easier firms’ entry into the market and that these results can be attributed to short run movements in the mark-up. Álvarez et al. (2010, 2011) and Przybyla et al. (2005) also find that product market competition helps in reducing inflation both at the aggregate and at the sector level.

But beyond endogenous entry other features of the industrial structure might be relevant to explain macroeconomic performance. The size and productivity distribution of firms are different across countries and display significant variation over the cycle. Bartelsman, Scarpetta and Schivard (2005) uncover consistent country patterns in the OECD that dominate sector specific ones regarding the size distribution of firms. In particular, they find significant turnover (entry plus exit) rates around 15-20% per year, although this turnover affects to a lower proportion of labor (10% of total employment) which implies that new entrants and dying firms are smaller than the average. Moreover, the dynamics of the distribution of firms by size and productivity
changes over the cycle. Lee and Mukoyama (2008) find that entry rates differ significantly in booms and recessions: plants entering in recessions are larger and more productive than those entering in booms, while such differences are relatively small for exiting plants. Thus, most of the cyclicality of the aggregate net growth rate reflects the variations within broad size and age class cells, rather than changes in the shares at business cycle frequencies. Since the shares are relatively stable over time, fluctuations in the aggregate must be driven by within firm size and firm age group variation in growth rates to which we now turn.

Table 1. Descriptive statistics of firms by size.

<table>
<thead>
<tr>
<th></th>
<th>Spain</th>
<th>France</th>
<th>Italy</th>
<th>Germany</th>
<th>EMU</th>
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<tbody>
<tr>
<td>Percentage of firms by size</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>1-9</td>
<td>78.5</td>
<td>83.1</td>
<td>82.9</td>
<td>60.5</td>
<td>79.5</td>
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<tr>
<td>10-19</td>
<td>10.6</td>
<td>7.3</td>
<td>10.1</td>
<td>21.0</td>
<td>10.4</td>
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<tr>
<td>20-49</td>
<td>7.6</td>
<td>5.8</td>
<td>4.8</td>
<td>8.3</td>
<td>5.9</td>
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<tr>
<td>+50</td>
<td>3.3</td>
<td>3.9</td>
<td>2.2</td>
<td>10.3</td>
<td>4.1</td>
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<tr>
<td>Percentage of employment by size</td>
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<tr>
<td>1-9</td>
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<td>12.2</td>
<td>25.5</td>
<td>6.7</td>
<td>14.6</td>
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<tr>
<td>10-19</td>
<td>12.1</td>
<td>6.7</td>
<td>15.3</td>
<td>8.1</td>
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<tr>
<td>20-49</td>
<td>19.6</td>
<td>12.3</td>
<td>16.0</td>
<td>7.6</td>
<td>12.4</td>
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<tr>
<td>+50</td>
<td>49.5</td>
<td>68.8</td>
<td>43.3</td>
<td>77.6</td>
<td>63.3</td>
</tr>
<tr>
<td>Apparent labour productivity, relative to EMU overall average</td>
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<tr>
<td># employees</td>
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<td></td>
</tr>
<tr>
<td>1-9</td>
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<td>0.66</td>
<td>0.47</td>
<td>0.56</td>
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<tr>
<td>10-19</td>
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<td>0.68</td>
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<td>20-49</td>
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<td>TFP by size, relative to overall median</td>
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<tr>
<td># of employees</td>
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<td>1-9</td>
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<td>1.57</td>
<td>0.61</td>
<td></td>
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</tr>
<tr>
<td>average</td>
<td>1.03</td>
<td>1.30</td>
<td>0.84</td>
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</tbody>
</table>


Finally, there is a growing evidence suggesting that differences in average firms’ productivity across (large) European countries might be mostly due to differences in the size distribution. As shown in table 1, Spain and Italy have a size distribution greatly biased towards micro-firms (1-9 employees), both in terms of the share of number of firms and of employment, in comparison with Germany or France.\(^1\) Moreover, some recent papers (Fernández and López (2014) and

\(^{1}\)Eurostat’s data on the number of firms by size in France is inconsistent with other data sources, like the OECD (see Bartelsman et al. (2005)), which show a similar puct to Germany. However, in the sake of comparability we have used the same source for all countries.
Antrás et al. (2010) indicate that the productivity of Spanish firms may be greater than the one of firms of comparable size from other members of EMU, despite the fact that the average productivity is lower in Spain. This evidence shows how important it is to consider the whole productivity distribution and not only the mean.

To analyze the interplay between these distributional features and the conditional dynamics of inflation we enlarge the standard entry model with heterogeneity in firm’s productivity or size. The model can accommodate any finite number of classes of firms, although most of our theoretical analysis and simulations are conducted in a two group model, small and large firms for simplicity. In addition, we incorporate the possibility of rich internal dynamics of firms by allowing for upwards and downwards transitions in the productive structure. Therefore, even though all firms enter the economy at the bottom of the productivity distribution, they may latter stay small, grow or die; similarly, large firms can (less likely) be downgraded or exit from the market. Unlike entry, these transitions are considered exogenous and calibrated to mimic the industry structure of Spain and other advanced European economies. Notice that not much is lost for keeping these transition rates exogenous at this stage, since the empirical evidence (Haltiwanger, Jarmin and Miranda, 2010) documents that these structures tend to be quite stable, at least at business cycle frequencies.

Bilbiie et al (2008) and Lewis & Poilly (2012) derive a New Keynesian Phillips curve in an economy with firm entry under the assumption of price stickiness à la Rotemberg. However, to the best of our knowledge, we are the first to derive the Phillips curve in an economy with entry and a size distribution. The closest approach to ours in the literature can be found in Lewis and Poilly (2012), who incorporate delayed entry and price stickiness to the multi-industry economy proposed by Jaimovich and Floetotto (2008). However, they concentrate on the interaction between firm creation and the different degree of substitutability of goods within and between industries and thus they do not consider size or productivity differences across firms.

Another departure from the standard literature on entry is that we model price stickiness à la Calvo (1983) and derive an explicit log linear representation of the Phillips curve that features the dynamics of firms as one of the determinants of current inflation. This augmented Phillips curve helps to gauge the relative importance of the new drivers of inflation (versus current and expected marginal costs) that stem directly from the assumption of firm’s entry and heterogeneity, and can also be thought of as providing a structural interpretation to markup shocks that generate a proper output-inflation trade-off in welfare. The number of productive firms (current, past and expected in the future) as well as the firm distribution exert a direct influence on current and expected inflation.

We study the response of inflation to macroeconomic shocks in a model with price heterogeneity by comparing it with the conditional dynamics of this variable in a model with homogeneous firms. Our results confirm that structures in which large (more productive) firms
predominate tend to deliver more muted responses of inflation to exogenous disturbances. This is the result of two effects. On the one hand, in the face of competition from new entrants large and small firms adjust their prices differently. As we shall discuss later on, the technological advantage allows the more productive firms to set lower optimal relative prices, which in turn reinforces the competition effect in the model thus weakening the link among inflation and the (present discounted value of) marginal cost in response to exogenous shocks. On the other hand, the change in the composition of the industry that occurs as new entrants increase or decrease, also changes the average inflation rate since these firms are less productive than the average and thus feature a higher relative price.

Figure 1. Correlation between inflation volatility and % of large firms

These results have relevant policy implications as far as the dynamics of inflation and the competitiveness of economies is concerned. Economies with easier firm creation and, above all, with structures in which large firms predominate tend to display lower inflation volatility. In fact, as figure 1 shows, there is a strong (negative) correlation between the volatility of prices (measured by the non-energy industrial goods component of the HICP or the Industrial Prices index for the manufacturing sector) and the percentage of large firms in manufacturing both in the eurozone (-50% or -53%) and the EU (-38% or -24%). In an open economy this is of great importance since it protects the competitiveness of firms in the face of shocks that increase the inflation rate. Also, by reducing the volatility of aggregate inflation, the presence of more productive firms may also help in easing the trade-off that monetary authorities face among output and inflation volatility to maximize welfare. We see our results as providing support to the extended idea that the productivity and size structure of firms in an economy have a close connection with the response of inflation, and hence competitiveness, to aggregate shocks. Our model helps in identifying different policy sensitive parameters, beyond entry costs, that may help in removing the barriers to grow that many firms face: productivity differentials, transition and death rates, etc. Policy makers aiming at making the existing industrial structures more competitiveness friendly may seek to implement appropriate changes in these and related parameters.

The rest of the paper is organized as follows, in section 2 we present the model and the calibration, and in section 3 the steady state effects of changes in the most relevant parameters of
the model. The main results regarding the response of inflation and other aggregate variables are summarized in sections 4, entry versus non entry, and 5, homogeneous firms versus heterogeneity in terms of productivity and size. Section 6 concludes.

2 A model with heterogenous firms

2.1 Households

There is a continuum of households in our economy indexed by \( j \in [0, 1] \). Each household maximizes the following lifetime utility function, which is separable in per capita consumption, \( c_{jt} \), and per capita hours worked, \( l_{jt} \) (in terms of proportion of the day spent at work):

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t d_t \left\{ \frac{c_{jt}^{1-\sigma}}{1-\sigma} - \varphi_t \psi \left( \frac{l_{jt}}{1+\vartheta} \right)^{1+\vartheta} \right\}
\]

where \( \beta \) is the discount factor and \( \vartheta \) is the inverse of Frisch labor supply elasticity. \( d_t \) is an intertemporal preference shock and \( \varphi_t \) is a labor supply shock both with an AR(1) law of motion.

The number of operative firms is endogenous and we assume that firms can have different steady state levels of productivity, which naturally leads to different size classes. All prospective entrants are of the smallest size. After entry, firms may change size, but we only consider transitions to the nearest group size. Therefore, every period a fraction \( \zeta_t \) of firms of size \( s \) become larger and a proportion \( \zeta_t \) diminish in size. In addition, a fraction \( \delta_t \) of firms of each size (including entrant firms) stop producing. The dynamics of firms by class is,

\[
N_{t+1}^s = \zeta_t L_t - \delta_t N_t^s - \zeta_t S_t + \delta_t N_t^s s_{t+1} \quad (1)
\]

and the number of operating and dividend yielding firms (\( N_H \)) evolves as:

\[
N_{t+1}^H = \sum_{s=1}^{N} N_{t+1}^s = (1 - \delta_t^F) N_t^E + \sum_{s=1}^{N} (1 - \delta_t^F) N_t^s \quad (2)
\]

where we assume that these transition and death rates are exogenous, while entry is endogenous.

There is a mutual fund of firms, which pays a dividend each period equal to the total nominal profit of all firms producing in that period, \( p_t d_t F N_t^H \), where \( d_t F = \frac{1}{N_t F} \int_0^{N_t F} d_{i,t} \) is the average real profit and \( p_t \) the aggregate price level. During period \( t \), each household buys \( s h_{j,t} \) shares in the fund of all the firms in the economy, since they do not know what firms will stop producing this period and will not pay dividends at \( t + 1 \). The date \( t \) price of a claim to the future profit stream of the mutual fund of \( N_t \) firms is equal to the average nominal price of claims of future profits of firms, \( p_t v_t N_t^H \), where \( v_t = \frac{1}{N_t F} \int_0^{N_t F} v_{i,t} d_{i} \) is the average real value of producing firms.
Therefore, households hold three types of assets: government bonds $b_{jt}$, which pay a nominal gross interest rate of $R_t$, physical capital $k_{jt}$, that earns a real rate of return of $r_t$, and shares in the mutual fund of firms $sh_{jt}^s$, where the superscript $s$ stands for the class of firms the mutual fund invests on. Then, the $j$th household’s per capita budget constraint is given by
\[
c_{jt} + i_{jt} + \frac{b_{jt}}{p_t} + \sum_{s=0}^{N} v_t^s N_t^s s h_{jt}^s = T_t + w_t l_{jt} + r_t k_{j-1} + R_{t-1} \frac{b_{j-1}}{p_t}
\]
\[
+ \sum_{s=0}^{N} \left( \left( d_t^{F_s+1} + v_t^{s+1} \right) \zeta^{ls} + \left( d_t^{F_s-1} + v_t^{s-1} \right) \zeta^{ss} \right) (1 - \delta^{Fs}) N_t^{s} s h_{jt-1}^s
\]
while investment $i_{jt}$ induces a law of motion for capital of the household:
\[
\mathbb{E}_t k_{jt} = (1 - \delta) k_{j-1} + \mu_t \left( 1 - S \left[ \frac{i_{jt}}{i_{j-1}} \right] \right) i_{jt}
\]
where $S \left[ \cdot \right]$ is an adjustment cost function such that $S \left[ 1 \right] = 0$, $S' \left[ 1 \right] = 0$, and $S'' \left[ \cdot \right] > 0$ and $\mu_t$ is an AR(1) investment-specific technological shock. The value of $\mu_t$ is also the inverse of the relative price of new capital in consumption terms.

Households maximize over $c_{jt}$, $b_{jt}$, $k_{jt}$, $sh_{jt}^s$, $i_{jt}$ and $l_{jt}^s$. Since we assume complete markets and separable utility in labor, we consider a symmetric equilibrium where $c_{jt} = c_t$, $k_{jt} = k_t$, $i_{jt} = i_t$, $\lambda_{jt} = \lambda_t$ and $q_{jt} = q_t$. The aggregate first order conditions associated to the consumer’s problems are:
\[
d_t c_t^{-\sigma} = \beta \mathbb{E}_t \left[ d_{t+1} c_{t+1}^{-\sigma} \frac{R_t}{\Pi_{t+1}} \right] \quad (4)
\]
\[
\lambda_t v_t^s = \beta \mathbb{E}_t \left\{ \lambda_{t+1} \left[ \left( d_t^{F_1} + v_t^{s+1} \right) \zeta^{ls} + \left( d_t^{F_s-1} + v_t^{s-1} \right) \zeta^{ss} \right] (1 - \delta^{Fs}) \right\} \text{ for } s = 0, ..., N \quad (5)
\]
\[
d_t c_t^{-\sigma} v_t^s = \beta (1 - \delta^{FS}) \mathbb{E}_t \left\{ d_{t+1} c_{t+1}^{-\sigma} \left[ \left( d_t^{F_1} + v_t^{L} \right) \zeta^{L} + \left( d_t^{FS} + v_t^{S} \right) (1 - \zeta^{L}) \right] \right\} \quad (6)
\]
\[
q_t = \beta \mathbb{E}_t \frac{d_{t+1} c_{t+1}^{-\sigma}}{d_t c_t^{-\sigma}} \left\{ q_{t+1} (1 - \delta) + r_{t+1} \right\} \quad (7)
\]
\[
1 = q_t \mu_t \left( 1 - S \left[ \frac{i_t}{i_{t-1}} \right] - S' \left[ \frac{i_t}{i_{t-1}} \right] \right) + \beta \mathbb{E}_t q_{t+1} \frac{d_{t+1} c_{t+1}^{-\sigma}}{d_t c_t^{-\sigma}} \mu_{t+1} S' \left[ \frac{i_{t+1}}{i_t} \right] \left[ \frac{i_{t+1}}{i_t} \right]^2 \quad (8)
\]

The first-order conditions with respect to labor and wages are more involved (see Erceg et. al., 2000). The labor used by intermediate good producers is supplied by a representative

\footnote{We define the (marginal) Tobin’s Q as $q_{jt} = \frac{Q_t}{s_{jt}}$, (the ratio of the two Lagrangian multipliers, or more loosely the value of installed capital in terms of its replacement cost), where $\lambda_{jt}$ and $q_{jt}$ are the Lagrangian multipliers associated with the budget constraint and capital accumulation equations,}
competitive firm that hires the labor supplied by each household \( j \), \( l^*_j t \) and aggregates them with the following production function:

\[
l^* t = \left( \int_0^1 \left( l^*_j t \right)^{\frac{n-1}{n}} dj \right)^{-\frac{n}{n-1}}
\]

where \( 0 \leq \eta < \infty \) is the elasticity of substitution among different types of labor, \( l^* t \) is the total labor demand. Thus, the labour demand function and aggregate wage are:

\[
l^* j t = \left( \frac{w_{jt}}{w_t} \right)^{-\eta} l^* t \quad \forall j
\]

\[
w_t = \left( \int_0^1 w_{jt}^{1-\eta} dj \right)^{\frac{1}{1-\eta}}
\]

Households set their wages following a Calvo’s setting. In each period, a fraction \( 1 - \theta w \) of households can change their wages. All other households can only index their wages to either past inflation or steady state inflation, which is controlled by the parameter \( \chi w \in [0, 1] \), where \( \chi w = 0 \) is only indexation to steady state inflation and \( \chi w = 1 \) is total indexation to past inflation. Thus, the relevant part of the Lagrangian for the household is:

\[
\max_{w_{jt}} \sum_{\tau=0}^{\infty} \theta_{jt+\tau} \left\{ -d_{t+\tau} \varphi_{t+\tau} \psi \left( \frac{l^*_t t+\tau}{1+\varphi} \right)^{1+\varphi} + \lambda_{jt+\tau} \prod_{s=1}^{\tau} \frac{\Pi^{1-\chi w}_{t+s-1} w_{jt}}{\Pi_{t+1}} l^*_t t+\tau \right\}
\]

s.t. \( l^*_j t+\tau = \left( \prod_{s=1}^{\tau} \frac{\Pi^{1-\chi w}_{t+s-1} w_{jt}}{\Pi_{t+1}} \right)^{-\eta} l^* t+\tau \)

All households that can optimize their wages in this period set the same wage \( (w^*_t = w_{jt} \forall j) \) that optimizes) because complete markets allow them to hedge the risk of the timing of wage change, hence, we can drop the \( jth \) from the choice of wages and \( \lambda_{jt} \). The first-order condition of this problem is expressed recursively through the use of the auxiliary variable \( f_t \) and substituting for \( \lambda_{jt} = d_t c_{jt}^{\sigma} \), to have:

\[
f_t = \frac{\eta-1}{\eta} (w^*_t)^{1-\eta} c_{jt}^{\sigma} w_{jt}^{\eta} + \beta w_{jt} \mathbb{E}_t \left( \frac{\Pi^{1-\chi w}_{t+1}}{\Pi_{t+1}} \right)^{1-\eta} \left( \frac{w_{t+1}}{w_t} \right)^{\eta-1} f_{t+1}
\]

and

\[
f_t = d_t \varphi_t \psi \left( \frac{w^*_t}{w_t} \right)^{-\eta(1+\varphi)} \left( l^* t \right)^{1+\varphi} + \beta w_{jt} \mathbb{E}_t \left( \frac{\Pi^{1-\chi w}_{t+1}}{\Pi_{t+1}} \right)^{-\eta(1+\varphi)} \left( \frac{w_{t+1}}{w_t} \right)^{\eta(1+\varphi)} f_{t+1}
\]
In a symmetric equilibrium and in every period, a fraction \( 1 - \theta_w \) of households set \( w^*_t \) as their wage, while the remaining fraction \( \theta_w \) partially index their price to past inflation, consequently, the real wage index is a geometric average of past real wage and the new optimal wage:

\[
w_t^{1-\eta} = \theta_w \left( \frac{\prod_{i=0}^{N_t} y_{it}^s} {\prod_t} \right) ^{1-\eta} w_{t-1}^{1-\eta} + (1 - \theta_w) w^*_t^{1-\eta}.
\]

### 2.2 Final Good Producers

There are two final goods, one \( (y_{dE}^t) \) destined to create new firms, produced using intermediate goods from the new entrants and another \( (y_{dH}^t) \) for the rest of destinations, produced using intermediate goods from the established intermediate producing firms (of all sizes \( y_{it}^s \)) with the following technology,

\[
y_{dH}^t = \left( N_t^H \right) ^{-\frac{\xi_v}{\varepsilon-1}} \sum_{i=1}^{N_t^H} \left( y_{it}^s \right) ^{\frac{\varepsilon-1}{\varepsilon}}
\]

where \( \varepsilon \) is the elasticity of substitution, and the term \( \left( N_t^H \right) ^{-\frac{\xi_v}{\varepsilon-1}} \) is included to control for the variety effect, if \( \xi_v = 0 \) there is full variety effect, if \( \xi_v = 1 \) there is no variety effect. When the variety effect is operative the entry of new firms has a direct effect on the welfare-based price index over and above the effect of the new firms on the intensity of competition.

Final good producers are perfectly competitive and maximize profits subject to their production function, taking as given all intermediate goods prices and the final good price. Note that we are assuming that the intermediate products from firms of all sizes are similar, the only difference is that some produce them more efficiently. That is, the elasticity of substitution between goods is identical across firm sizes, since all producers are competing in the same market with similar differentiated goods. This is in contrast with Jaimovich and Floetotto (2008) and Lewis and Poilly (2012) who assume different elasticities across industries than within each industry. The reason being that goods across different industries are less substitutive than varieties within an industry. Thus, they solve the following maximization problem:

\[
\max p_t y_{dH}^t - \sum_{i=0}^{N_t^H} p_{it}^s y_{it}^s \quad \text{for } s = 1, ..., N
\]

s.t.: \( y_{dH}^t = \left( N_t^H \right) ^{-\frac{\xi_v}{\varepsilon-1}} \sum_{i=1}^{N_t^H} \left( y_{it}^s \right) ^{\frac{\varepsilon-1}{\varepsilon}} \)

\[
to get the input demand functions and price indices associated with this problem,
\]

\[
y_{it}^s = \left( \frac{p_{it}^s}{p_t} \right) ^{-\varepsilon} \left( \frac{y_{dH}^t}{N_t^H} \right) ^{\xi_v} \quad \text{for } s = 1, ..., N
\]

\[
p_t = \left( N_t^H \right) ^{\frac{\xi_v}{\varepsilon-1}} \left[ \sum_{s=1}^{N_t^H} \left( \sum_{i=1}^{N_t^H} (p_{it}^s)^{-\frac{\varepsilon-1}{\varepsilon}} \right) \right] ^{\frac{1}{\varepsilon-1}}
\]
Notice here the role of the variety effect. Assume that \( p^*_t = p_{it}, \sqrt{i} \) then \( p_t = (N^H_t)^{1-\xi_v} p_{it}, \) and when the variety effect is operative \( (\xi_v = 0) \) the welfare based aggregate consumer price index is decreasing in the number of operative firms.

2.3 Intermediate Good Producers

There are \( N^H_t \) intermediate good producers that result from a process of endogenous entry and exogenous destruction. Each period there is an infinite number of potential entrants in the market that face two frictions associated with entry. First they have to pay an entry cost that is proportional to their marginal cost, \( f^E_t mc^E_t, \) where \( f^E_t \) is exogenous; besides new entrants at \( t (N^E_t) \) remain idle for one period before to start producing at \( t + 1. \) In the meantime though new entrants suffer the same structural shock as the incumbent ones and drop out from the market at a rate \( \delta^F. \) We assume that all entrants are small and larger firms only come from the group of smaller established ones. To determine the optimal entry decision we need to calculate the real value of an entrant and a producing firm. Apart from the sunk cost, entry is free so firms will enter the market until the expected gain from producing \( (v^E_{it}) \) equals the cost

\[
v^E_{it} = f^E_t mc^E_t
\]

2.3.1 Production and input demands

Intermediate goods producers solve a two-stage problem. In the first stage, firms rent \( l^d_{it} \) and \( k_{it} \) in perfectly competitive factor markets in order to minimize their real cost, taking input prices \( w_t \) and \( r_t \) as given. In the second stage, they choose the price that maximizes discounted real profits, taking the input prices and factor demands as given.\(^3\)

In the fist stage, a continuum of intermediate producers in each sector demand inputs and produce output according to the following production function,

\[
y^d_{it} = A^s_t \left(k^s_{it-1}\right)^{\alpha} \left(l^d_{it}\right)^{1-\alpha} \text{ for } s = 0, ..., N
\]

where \( k^s_{it-1} \) and \( l^d_{it} \) are the capital and labor rented by the firm of type \( s, \) while the technological level \( A^s_t \) follows the following process

\[
\log A^s_t = \rho_A \log A^s_{t-1} + \sigma_A \varepsilon_{A,t} \text{ where } \varepsilon_{A,t} \sim N(0, 1),
\]

That is, all firms face the same production function and only differ in the productivity shifter \( A^s_t. \) Thus firms solve a similar minimization problem and since there is perfect mobility of

\(^3\)In the calibrated model we also include some adjustment costs to smooth out the entry process.
labor inputs, the wage, the rental rate of capital and the capital-labor ratio are the same for all firms:

$$w_t = mc^s_t (1 - \alpha) A^s_t \left( \frac{k_{t-1}}{l^d_t} \right)^{\alpha}$$

$$r_t = mc^\alpha_t \left( \frac{k_{t-1}}{l^d_t} \right)^{\alpha-1}$$

$$\frac{k^s_{t-1}}{l^d_{t-1}} = \frac{k_{t-1}}{l^d_t} = \frac{\alpha}{1 - \alpha} \frac{w_t}{r_t}$$

and the real marginal cost, $mc^s_{it}$ is equal to $^4$

$$mc^s_{it} = \left( \frac{1}{\alpha} \right) \left( \frac{1}{1 - \alpha} \right)^{1-\alpha} \frac{1 - \theta_s}{\theta_s} \frac{1 - \delta_F^{s+1}}{1 - \delta_F^{s+1}}$$

for $s = 0, \ldots, N$.

### 2.3.2 Price setting

In the second stage, each producing firm $i$ chooses the optimal price that maximizes its discounted real profits, according to the Calvo’s mechanism. That is, in each period, a fraction $(1 - \theta^s_p)$ of the firms that remain in the same group $(1 - \zeta^s)$ and do not die $(1 - \delta F^s)$ can change their prices. The remaining firms in the group can only index their prices to either past inflation or steady state inflation, which is controlled by the parameter $\chi_s$. We assume that firms exactly replicate after entry or size change the existing distribution of prices in the group they enter. That is, firms changing group are given randomly an existing price of the group they join, so that the price distribution of that group after joining is equal to the one before. Therefore, firm demographics do not affect the optimal price decision.5

Thus, firms solve the following maximization problem:

$$\max_{p_{it}} \mathbb{E}_t \sum_{\tau = 0}^{\infty} \left[ \beta (1 - \delta F^s) \theta^s_p \right]^\tau \frac{\lambda_{t+\tau}}{\lambda_t} \left\{ \left( \prod_{j=1}^{\tau} \frac{\Pi^{1-\chi} \Pi^X_{t+j-1} p_{it}}{\Pi^X_{t+j} p_t} - mc^H_{it+\tau} \right) y^H_{it+\tau} \right\}$$

s.t.: $y^H_{it+\tau} = \left( \prod_{j=1}^{\tau} \frac{\Pi^{1-\chi} \Pi^X_{t+j-1} p_{it}}{\Pi^X_{t+j} p_t} \right)^{-\epsilon} \frac{y^H_{it+\tau}}{N^H_{it+\tau} \xi^s}; p_t = \left( N^H_{it} \right)^{\frac{\epsilon}{1-\epsilon}} \left[ \sum_{i=1}^{N^H_{it}} p_{it}^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}$

---

^4 Note that the marginal cost, or Lagrangian multiplier, depends on firm size, because of the different technology, but not the capital labour ratio, since factor prices are the same.

^5 Alternatively, assuming that firms changing group are able to reoptimize does not change the results qualitatively, since only a small proportion of the total change, but it makes the model less parsimonious.
where the discount factor takes into account the marginal value of a dollar to the household, which is treated as exogenous by the firm. The optimal (relative) price that solves this problem 

\[ \Pi_t^{\ast} = \frac{\Pi_t^H}{p_t} \]

is equal to

\[
\Pi_t^{\ast} = \frac{\eta_{pit}}{1 + \eta_{pit}} \frac{\mathbb{E}_t \sum_{\tau = 0}^{\infty} \left[ \beta (1 - \delta^{\tau}) \theta_p^\tau \frac{\lambda_{\tau+r}}{\lambda_r} \left( \prod_{\tau = 1}^{\tau} \frac{\Pi_1^{1-\chi_t} \Pi_0^{\chi_t} - 1}{\Pi_0^{\chi_t}} \right)^{-\varepsilon} \right]}{\mathbb{E}_t \sum_{\tau = 0}^{\infty} \left[ \beta (1 - \delta^{\tau}) \theta_p^\tau \frac{\lambda_{\tau+r}}{\lambda_r} \left( \prod_{\tau = 1}^{\tau} \frac{\Pi_1^{1-\chi_t} \Pi_0^{\chi_t} - 1}{\Pi_0^{\chi_t}} \right)^{1-\varepsilon} \right] \frac{y_{it+r}^H}{(N_{it+r}^H)^{\xi}}}
\]

(17)

where \( \eta_{pit} \) is the price elasticity of demand, which due to firm entry and size growth, is no longer constant. Instead, it depends positively on the number of producing firms and the optimal price.

\[
\eta_{pit} = \frac{\partial y_{it+r}^H}{\partial p_t} \frac{p_t^H}{y_{it+r}^H} = -\varepsilon \left( 1 - \xi_c \frac{\partial p_t}{\partial p_{it}} \frac{p_t}{p_t} \right) = -\varepsilon \left( 1 - \xi_c \left( N_t^H \right)^{-\varepsilon} \frac{p_t}{p_t} \right)^{-(\varepsilon-1)}
\]

The competition effect (\( \xi_c \)) reflects whether (\( \xi_c > 0 \)) or not (\( \xi_c = 0 \)) each individual firm takes into account the incidence of its own pricing decisions on the aggregate price (\( \eta_{pit} = \frac{\partial p_t}{\partial p_{it}} \frac{p_t}{p_t} \geq 0 \)). This expression is key to understand the mechanism proposed in this paper and the results we obtain regarding the response of inflation to different shocks. When there is entry, individual pricing decisions change the aggregate price in two ways: directly, because now we are aggregating over a larger number of firms, reflected by the fact that the number of firms enters the aggregate price elasticity; but also indirectly, because now each firm takes into account the impact of its price on the aggregate when setting prices, given by the term \( \left( \frac{p_t}{p_t} \right) \) in this expression.

It is convenient to rewrite equation (17) recursively through the use of the auxiliary variables \( g_t^{s1} \) and \( g_t^{s2} \) in terms of the following three equations:

\[
g_t^{s1} = d_t c_t^{-\sigma} m_t^s \frac{y_{it}^H}{(N_t^H)^{\xi_c}} + \beta (1 - \delta^{\tau}) \left( 1 - \zeta^{S} - \zeta^{L} \right) \theta_{p}^s \mathbb{E}_t \left( \frac{\Pi_1^{1-\chi_t - \chi^{s}}}{\Pi_1 + 1} \right)^{-\varepsilon} g_{t+1}^{s1}
\]

(18)

\[
g_t^{s2} = d_t c_t^{-\sigma} m_t^s \frac{y_{it}^H}{(N_t^H)^{\xi_c}} + \beta (1 - \delta^{\tau}) \left( 1 - \zeta^{S} - \zeta^{L} \right) \theta_{p}^s \mathbb{E}_t \left( \frac{\Pi_1^{1-\chi_t - \chi^{s}}}{\Pi_1 + 1} \right)^{(\varepsilon)} g_{t+1}^{s2}
\]

(19)

\[
\Pi_t^{\ast} = \frac{\varepsilon}{\varepsilon - 1} \left[ \left( \varepsilon - 1 \right) \left( 1 - \xi_c \left( N_t^H \right)^{-\xi_c} \left( \Pi_t^{\ast} \right)^{1-\varepsilon} \right) \right] \left( \frac{\varepsilon}{\varepsilon - 1} \left( 1 - \xi_c \left( N_t^H \right)^{-\xi_c} \left( \Pi_t^{\ast} \right)^{1-\varepsilon} \right) - 1 \right) \frac{g_t^{s1}}{g_t^{s2}} = \mu_t^d \frac{g_t^{s1}}{g_t^{s2}}
\]

(20)

Therefore, firms set their optimal (relative) price (\( \Pi_t^{\ast} \)) as a markup (\( \mu_t^d \)) over the discounted sequence of future marginal costs \( \left( \frac{g_t^{s1}}{g_t^{s2}} \right) \). \( \mu_t^d \) can be named "desired markup", since it is equal to the one prevalent under no price rigidities, and is decreasing on the number of firms producing in the economy. Notice that when there is no competition effect (\( \xi_c = 0 \)) the desired markup
is constant and equal to the case under no entry \((\mu^d = \varepsilon - 1)\), while when there is full variety effect \((\xi_v = 0)\), optimal prices do not depend directly on the number of firms.

Finally, to calculate the dynamics of the aggregate price index we must recall that firms’ entry, group change or exit, does not modify the distribution of prices within each group of the economy. Consequently, the group \(s\) price index evolves as,

\[
\Pi_t^{s*} = \left( N_t^s \right)^{-1/\varepsilon} \left[ \left( \frac{\Pi_t}{N_t^{s-1}} \right)^{1-\varepsilon} - \left( \frac{N_t^s}{N_t^{s-1}} \right)^{1-\varepsilon} \left( \frac{\Pi_t^{1-\chi_s \Pi_t^{s-1}}}{\Pi_t} \right)^{1-\varepsilon} \frac{\theta_{p_s}}{1 - \theta_{p_s} (p_{t-1}^{s})^{1-\varepsilon}} \right]^{1/\varepsilon} \tag{21}
\]

where \(\bar{p}_t = \frac{p_t^{s}}{\bar{p}_t}\), and the aggregate price index must satisfy the condition

\[
1 = \sum_{s=1}^{N} \left( \frac{N_t^s}{N_t^{s-1}} \right)^{1-\varepsilon} \left( \frac{\bar{p}_t^{s}}{\bar{p}_t} \right)^{1-\varepsilon}
\]

Log linearizing the pricing block of the model, equations (18), (19), (20) and (21), and after some algebra, we derive the Phillips curve. In our model, the Phillips curve for each size group \(s\) is a function of the steady state price elasticity of demand at the optimum price, which we call the direct competition effect \(\varepsilon\), and the acceleration in the number of firms with productivity \(s\), representing the indirect competition effect and the variety effect, respectively. In addition, the coefficients in front of expected future inflation and current marginal costs are modified by a term which is a function of the steady state price elasticity of demand at the optimum price, which we call the indirect competition effect.\(^6\)

\[
\left( \hat{H}_{st} - \chi_s \hat{H}_{s,t-1} \right) = \frac{1 + \theta_{p_s} \varepsilon_s \xi_s^s \xi_s^s (\varepsilon - 1)}{1 + \xi_s^s \xi_s^s (\varepsilon - 1)} \beta (1 - \delta^{Fs}) (1 - \zeta^{Ls} - \zeta^{Ss}) E_t \left( \hat{H}_{s,t+1} - \chi_s \hat{H}_{s,t} \right)
\]

\[
= \frac{1}{1 + \xi_s^s \xi_s^s (\varepsilon - 1)} \left( \frac{1 - \theta_{p_s}^s}{1 - \beta (1 - \delta^{Fs}) (1 - \zeta^{Ls} - \zeta^{Ss}) \theta_{p_s}^s \varepsilon_s^s \xi_s^s} \right) \frac{\hat{N}_t^{s}}{N_t^{s-1}}
\]

\[
- \xi_s^s \frac{1 - \theta_{p_s}^s}{\theta_{p_s}^s (1 + \xi_s^s \xi_s^s (\varepsilon - 1))} E_t \hat{\Delta}^2 \hat{N}_t^{s} - \frac{1 - \xi_s}{\theta_{p_s}^s (\varepsilon - 1)} E_t \hat{\Delta}^2 \hat{N}_t^{s} - f \left( \hat{\Delta} \hat{p}_t \right)
\]

where \(\hat{\Delta}\) represents the following quasi-difference operator

\[
E_t \hat{\Delta} \hat{N}_t^s = \hat{\Delta} \hat{x}_t^s - \beta (1 - \delta^{Fs}) (1 - \zeta^{Ls} - \zeta^{Ss}) \theta_{p_s}^s E_t \hat{\Delta} \hat{x}_t^{s+1}
\]

\[
E_t \hat{\Delta} \hat{x}_t^s = \left( E_t \hat{\Delta} \hat{x}_t^s - \theta_{p_s}^s \hat{\Delta} \hat{x}_t^{s+1} \right)
\]

Note that the coefficient \(\xi_N = \frac{(\eta_{p_s}^s + \varepsilon)}{\eta_{p_s}^s (1 + \eta_{p_s}^s)} > 0\), \(\eta_{p_s}^s \in (-\varepsilon, -1)\) and \(\frac{\partial \xi_s}{\partial \theta_{p_s}^s} < 0\), \(\frac{\partial \xi_s}{\partial \theta_{p_s}^s} = \frac{\partial \xi_s}{\partial \theta_{p_s}^s} > 0\).

\(^6\)In addition, this NKPC includes two terms with the relative prices which are not crucial for the behaviour of inflation.
Finally, aggregating across all sizes we get the aggregate NKPC

$$\hat{\Pi}_t = \sum_{s=1}^{N} \frac{N^s (\Pi^{ss})^{1-\varepsilon}}{(NH)^{\xi_v}} \left[ \hat{\Pi}_{st} + \frac{\xi_v}{(1 - \varepsilon)} \Delta \left( \hat{N}_t^s - \hat{N}_t^H \right) \right].$$

Aggregate inflation is a weighted average of the inflation rates across size groups, and the change in the shares of firms of each type, where the weights are inversely proportional to the steady state optimal relative price. These equations are discussed at length at the beginning of the results section.

### 2.3.3 Aggregate Constraints

Finally, to close the model, we derive the economy wide constraint using the aggregate household budget constraint and the equilibrium conditions in the labor and goods markets:

$$\left(c_t + i_t\right) \left(v_t^p\right)^{-\varepsilon} + \frac{N^s_v v_t}{m_c^E} = A_t \left( \frac{k_{t-1}}{l_t^d} \right)^\alpha l_t^d$$  \hspace{1cm} (23)

where aggregate price dispersion is defined as \(\left(v_t^p\right)^{-\varepsilon} = \frac{1}{(NH)^{\xi_v}} \sum_{i=0}^{N^H} \left( \frac{p_t}{P_t} \right)^{-\varepsilon}\), and, using the properties of the index under Calvo’s pricing, evolves as,

$$\left(v_t^p\right)^{-\varepsilon} = (N^H)^{1-\xi_v} (1 - \theta_p) \Pi_t^{-\varepsilon} + \left( \frac{N^H}{N^H_{t-1}} \right)^{1-\xi_v} \theta_p \left( \Pi_t^{1-\chi} \Pi_t^{\chi-1} - \varepsilon \right) \left(v_{t-1}^p\right)^{-\varepsilon}. \hspace{1cm} (24)$$

and the average profit:

$$d_t^F = c_t + i_t \left( 1 - m_c^H \left(v_t^p\right)^{-\varepsilon} \right)$$  \hspace{1cm} (25)

As regards monetary policy we assume that the central bank sets the nominal interest rates according to the following Taylor rule,

$$\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\gamma_R} \left( \frac{\Pi_t}{\Pi} \right)^{\gamma_m} \left( \frac{y_t}{y_t^d} \right)^{\gamma_y} 1^{1-\gamma_R} \exp (m_t)$$  \hspace{1cm} (26)

Where \(\Pi\) represents the target level of inflation (equal to inflation in the steady state), \(R\) the steady state nominal gross return on capital and \(m_t\) is a random shock to monetary policy that follows \(m_t = \sigma_m \varepsilon_{mt}\), where \(\varepsilon_{mt}\) is distributed according to \(\mathcal{N}(0, 1)\). The presence of the previous period interest rate, \(R_{t-1}\), is justified because we want to match the smooth profile of the interest rate over time observed in the data.\(^7\)

\(^7\)Note that \(R\) is beyond the control of the monetary authority, since it is equal to the steady state real gross returns of capital plus the target level of inflation.
2.4 Calibration

The model has 24 calibrated parameters, shown in Table 2. The first two columns display the 14 parameters that determine the steady state solution, the others affecting only the model dynamics. These parameters have been calibrated either using consensus values taken from the literature or were chosen to reproduce some data moments. In particular, we aim at approximating the stylized facts of the industrial structure, as well as the main macroeconomic ratios of the Spanish economy, in bold in the fourth column of Table 3. The first block of these (top block in bold) refers to long run (steady-state) ratios of the whole economy, the second one refers to characteristics of entrant firms, while the last two blocks refer to characteristics of large and small firms.

Table 2. Baseline Calibration

<table>
<thead>
<tr>
<th>steady state parameters</th>
<th>dynamics parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta = 0.99$</td>
<td>$\kappa = 0.1$</td>
</tr>
<tr>
<td>$\vartheta = \sigma = A = 1$</td>
<td>$\gamma_y = 0.125$</td>
</tr>
<tr>
<td>$\varphi = 10$</td>
<td>$\chi = \chi_w = 0.125$</td>
</tr>
<tr>
<td>$\delta = \delta^E = 0.025$</td>
<td>$\theta_p = \theta_w = 0.896$</td>
</tr>
<tr>
<td>$A_L = 1.9$</td>
<td>$\gamma_R = 0.8$</td>
</tr>
<tr>
<td>$A_S = 0.9$</td>
<td>$\gamma_x = 1.7$</td>
</tr>
<tr>
<td>$\varepsilon = \varepsilon_w = 15$</td>
<td>$\rho_A = 0.7$</td>
</tr>
<tr>
<td>$\alpha_H = 0.372$</td>
<td>$\rho_e = 0.9$</td>
</tr>
<tr>
<td>$\alpha_E = 0$</td>
<td>$\rho_{mu} = 0.7$</td>
</tr>
<tr>
<td>$f^E = 0.11$</td>
<td>$\zeta_L = 0.5% (0.2)$</td>
</tr>
<tr>
<td>$\zeta^S = 1.5% (2.6)$</td>
<td>$\rho_{mu} = 0.7$</td>
</tr>
</tbody>
</table>

First, it is worth noting that a model of firm entry with capital poses problems for determinacy and non-explosiveness of the solution, as there might be now increasing returns to an accumulated factor, physical capital. This is exacerbated when a complete variety effect is assumed. Therefore, in this model, a unique, non-explosive solution cannot be guaranteed for a wide range of standard parameter values. This problem can be circumvented in several ways. One may set the parameters determining the steady state so as to reduce the increasing returns to capital, for example, by assuming very fast physical capital depreciation, around 50% per quarter (like in Bilbiie et al., 2008) or Lewis and Poilly, 2009), or through other parameters, mainly $\vartheta$, $\varepsilon$ and $\varphi$. Alternatively, as it is done in this paper, one may assume that new entrants do not need capital to produce (see Bilbiie et al., 2012). This approach allows for a wider parameter space, however, it is still more reduced than when there is no capital accumulation.

We start by setting the parameters that affect only the dynamics of the model (right panel of Table 2). The parameters of the Taylor rule are set to the standard estimation results for the euro area (Clarida, Gali, and Gertler, 2000). On the nominal side, the Calvo and the indexation to inflation parameters for prices and wages are similar to the values generally obtained for the euro area (Smets and Wouters, 2005), while a very small adjustment cost for investment ($\kappa$) is assumed.

---

8 Although the theoretical model and the simulation code admit any finite number of productivity classes, for expository purposes we will carry out the empirical exercises below considering the simplest case of just two types of firms: low productivity or small ($S$) and high productivity or large ($L$).
Then we set the parameters that affect mainly the whole-economy steady state ratios and the characteristics of entrants. The discount factor $\beta$ is set to be consistent with an annualized real interest rate of 2.5 percent and an inflation objective of 2 percent, so that the steady state annual nominal interest rate ($R$) is 4.5 percent. The depreciation rate of capital is consistent with an annual depreciation of 10%. The calibration of utility parameters is quite standard, with log utility of consumption ($\sigma = 1$). The Frisch elasticity of labor supply is $1 \ (1/\vartheta)$ in line with the findings of the recent microeconomics literature (Browning, Hansen, and Heckman, 1999).\footnote{If we assumed higher values, as normally done in macro models (lower $\vartheta$), the steady state level of labour would be too low.}

The labor supply coefficient is set to 10, in line with estimated DSGE models (Fernández-Villaverde, 2009).\footnote{The model cannot be solved for lower values, however these would not help to approximate better the steady state ratios.} The elasticity of substitution between different types of intermediate goods produced ($\varepsilon$) and between different labour types ($\varepsilon_w$) is slightly higher than what is normally used in DSGE models, implying a lower mark-up of around 7 percent.\footnote{Again, the model cannot be solved for lower values, but even if we could it would not help to match the size of entrants. This is another consequence of the somehow smaller parameter space in this model due to the increasing returns to capital.} The labour share of incumbents is set equal to the value in the data for Spain, while the one of entrants is set to 1. Finally, the firms’ death rate $\delta_F$ is such that 10 percent of annual production is destroyed, both as a share of products and as market share (Bilbiie et al., 2012, and Bernard et al., 2010).

The entry cost $J^E$ is one of the main determinants of the characteristics of entrant firms. A low level of this parameter, 0.11, guarantees that entrant firms represent in steady state a small share of firms and of production, similar to the data (2.5%). However entrant firms in this model are much larger in terms of employment (4 times the size of an average incumbent) than in the data (50%). This failure to replicate the data is mainly a consequence of the fact that to increase the parameter space we have assumed that entrants are more labour intensive than incumbents, with a markedly lower labour productivity.

The last set of parameters determine the differences across incumbent firms. First of all, we consider a model without heterogeneity across firms, thus, these parameters are set equal. The probabilities of firms becoming large ($\zeta^L = 52\%$) or small ($\zeta^S = 49\%$) are set so that 50% of firms belong to each group and they represent 50% of employment and production, while firms’ productivity ($A^L, A^S$) is set to the average (1). The result of this calibration is reported in the first column of the right hand side panel of Table 3\footnote{Note that we do not report separately the relevant steady state ratios for the model with and without variety effect, since they are very similar.}. This calibration delivers model steady state ratios for the share of consumption in final demand, the labour share and the capital to GDP ratio which are fairly close to the ones in the data, while entrant firms characteristics are well approximated, except for their size. However, by definition is unable of generating any
differences across incumbent firms.

Table 3. Stylized facts of firms by size model vs data

<table>
<thead>
<tr>
<th>stylized facts</th>
<th>Germany</th>
<th>France</th>
<th>Italy</th>
<th>Spain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Factor Productivity (TFP)</td>
<td>1.29</td>
<td>0.85</td>
<td>0.83</td>
<td>0.77</td>
</tr>
<tr>
<td>TFP small firms (1-19 empl)</td>
<td>2.27</td>
<td>0.86</td>
<td>0.97</td>
<td>0.77</td>
</tr>
<tr>
<td>TFP large firms (20+ empl)</td>
<td>1.84</td>
<td>1.35</td>
<td>1.40</td>
<td>1.37</td>
</tr>
<tr>
<td>Alternative calibrations</td>
<td>flexible wages</td>
<td>sticky wages</td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------------</td>
<td>----------------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>no het</td>
<td>baseline</td>
<td>A=[0.97,1.4]</td>
<td>f0=0.40</td>
<td>A=[0.97,1.4]</td>
</tr>
<tr>
<td>employment by small firm</td>
<td>43.5</td>
<td>37.9</td>
<td>45.1</td>
<td>42.2</td>
</tr>
<tr>
<td>labour share (%)</td>
<td>72.3</td>
<td>73.1</td>
<td>62.4</td>
<td>62.8</td>
</tr>
<tr>
<td>k/4yH</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>w/ldH</td>
<td>0.7</td>
<td>0.7</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>r*kyH</td>
<td>0.84</td>
<td>1.29</td>
<td>0.65</td>
<td>0.86</td>
</tr>
<tr>
<td>size (relative to incumbents)</td>
<td>0.24</td>
<td>0.47</td>
<td>0.49</td>
<td>0.50</td>
</tr>
<tr>
<td>% of all firms</td>
<td>1.3</td>
<td>2.9</td>
<td>1.9</td>
<td>2.6</td>
</tr>
<tr>
<td>% of employment</td>
<td>1.2</td>
<td>5.4</td>
<td>3.7</td>
<td>2.1</td>
</tr>
<tr>
<td>% of small firms</td>
<td>95.3</td>
<td>90.7</td>
<td>95.6</td>
<td>98.3</td>
</tr>
<tr>
<td>% of employment by small firm</td>
<td>43.5</td>
<td>37.9</td>
<td>45.1</td>
<td>42.2</td>
</tr>
</tbody>
</table>

Source: Eurostat, OECD and authors own calculations.

In the remaining models we allow for differences across incumbents. In the baseline heterogeneity case we set firms’ productivity equal to the data and set the probabilities of becoming large (small) to match the proportion of large (small) firms in the data (89%). These probabilities are relatively similar to the empirical ones (in brackets in Table 2). As can be seen from column 2 in the RHS panel of Table 3, the productivity gap in the data is too large for this model, so that low productivity firms are far too small in size and represent an almost insignificant proportion of total employment (0.1%) and production (0.03%). This can be circumvented in two ways. First, by increasing (markedly) the cost of entry (until 0.46), see column 4 of the RHS panel of table 3, the size of low productivity firms, relative to high productivity ones, and their share of the whole economy, become identical to the data in terms of both employment
and production. However, in this case entrant firms have to be much larger (22 times the incumbents’ size) to cover for the sunk cost of entry and therefore represent an unrealistic share of total employment and production. Moreover, this calibration also produces unrealistic values for most whole economy steady state ratios, like too high a consumption share of final demand or too low a capital and labour rents’ share of GDP. Secondly, and more promising, one can reduce the productivity gap, while keeping the average constant. In particular, as shown in column 3 of the RHS panel of Table 3, if the productivity gap is reduced from 2.1 to 1.4, the model is able to match fairly well the data on the size of low productivity firms and their share of the whole economy in terms of production, and to a less extent in terms of employment. Moreover, this is achieved while matching slightly better than the baseline case the characteristics of entrants and without worsening the whole economy steady state ratios.

Finally, when we add sticky wages to the model (of a similar magnitude of price stickiness), in the last two columns of table 3, most of the steady state ratios are unchanged, except that now firms’ size is smaller than in the data, due to the fact that this assumption rises steady state wages, since it adds a mark up to them, increasing the costs to setup firms and reducing their size.

3 Steady state analysis

In this section we analyze the steady state effect of changes in some of the more relevant parameters of the model that may be relevant for policy analysis. Table 4 shows the (sign of the) impact of changes in \( \varepsilon, \varepsilon_w, \vartheta, \psi, \delta, \alpha_H, A, A^E, f^E \) and \( \delta^E \) on the most important variables and ratios of the three models considered: a model with entry, with entry and firm heterogeneity (in size) and without entry.

First of all, we look at the parameters not related directly to firms’ entry, included in the first three blocks of the table. The impact of changes in these parameters \( (\vartheta, \psi, \sigma, \varepsilon_w, A) \) is qualitatively identical and quantitatively similar across the models considered. In particular, a decrease in the disutility of labor (either through a greater inverse of the Frisch elasticity \( \vartheta \) or a larger weight of the labour component in the utility function \( \psi \)), a rise in the utility from consumption \( (\sigma) \) or the elasticity of substitution amongst labour types \( (\varepsilon_w) \) all increase the willingness of households to work, which rises firms’ production and total GDP, while a rise in the average TFP level \( (A) \) rises firms’ production and GDP directly. In the models with entry and size, this in turn, improves the value of future entrants, and incentivizes the entry of new firms.

The exception to this is the elasticity of substitution across consumption varieties \( (\varepsilon) \) (see fourth panel of table 4). In a model with no entry, a rise in \( \varepsilon \) implies an increase in competition amongst firms through a fall in the steady state markup, which reduces firms’ profits. Therefore, they react by moving along their demand schedules increasing production and employment, that in turn rises wages and consumption. This is the reason why this parameter is often used as
a proxy for structural reforms in the goods market in a model without entry. However, when one also considers the possibility of firms entry, there are other relevant developments in the economy. In a model with entry, a lower markup initially reduces profits per firm. Thus, a smaller number of firms enter the market until expected profits per producing firm are restored to their original value, when each incumbent produces more, with more capital and labor. In addition, the smaller number of firms reduces competition and partially offsets the fall in markups. Despite this, aggregate output, consumption and employment in the production sector increase by more in the case with entry. On the contrary, in the model with entry and size heterogeneity, the rise in ε reduces the markup of small/low productivity firms (−0.7%) by twice as much as the one of large/more productive firms (−0.3%). This leads to a rise (fall) in employment, capital and production of larger (smaller) firms and of the aggregate, which leads to an increase in total entry, since entrants expect to become large eventually. The reason for this is that large firms, thanks to their technological advantage, face a more inelastic segment of the demand curve, which allows them to lower their optimal relative prices by less than small firms.

Table 4. Impact on the steady state of changes in parameters

<table>
<thead>
<tr>
<th>Parameter Change</th>
<th>Entry</th>
<th>Size</th>
<th>Small</th>
<th>Large</th>
<th>Large/Small</th>
<th>Large/Small</th>
</tr>
</thead>
<tbody>
<tr>
<td>ε increase</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>ε decrease</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Capital</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Investment</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Wages</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Aggregate Output</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

In a model where consumers derive greater utility from a larger number of varieties of goods (variety effect), there is another balancing effect. If consumers value varieties a lot, the reduction in the number of entrants has a negative effect on consumption, which compensates the positive one coming from the lower markup and leaves aggregate production and employment unchanged, although employment in production increases, while capital, investment and wages fall.

The last two columns of table 4 show the impact of a change in the distribution of productivity in the model with heterogeneity of an increase in the productivity of the largest firms (column before last) and of a mean preserving increase in the dispersion of productivity (last
The steady state impacts are qualitatively similar, with a rise in almost all aggregates, except for employment, due to the reduction in small firms’ employment. This is even true in the case of the mean preserving shock, since the positive impact of a rise in the productivity of the largest firms dominates the negative one of reducing the productivity of the small firms.

One of the advantages of modelling firms’ entry is that now we have several parameters that govern the entry process and affect directly the degree of competition in the economy, being therefore much better proxies of structural reforms than the elasticity of substitution between consumption varieties ($\varepsilon$). In particular, the relevant parameters are the cost of entry ($f^E$) and the technological level of entrants ($A^E$), which jointly (together with the real wage) determine the total cost of entry ($f^E mc^E = f^E \frac{w}{A^E}$); and the the firms’ (exogenous) probability of death ($\delta^F$), which determines the exit from the market.

A fall in $f^E$ and an improvement in entrants technology ($A^E$), reduce the total cost of entry ($f^E mc^E = f^E \frac{w}{A^E}$), which increases the number of producing firms. On the other hand, a fall in the producing firms’ death rate ($\delta^F$), also increases the number of surviving firms, while reducing the incentive to enter and therefore the number of entrants. In all cases, this augments the intensity of competition in the economy and lowers the markup and profits of each existing firm. Therefore, the economy has a larger number of firms charging a lower markup, which increases aggregate production and consumption. A side implication of the smaller cost of entry or lower probability of death is that now firms do not need to be as big to afford entry, so in equilibrium they become smaller, both in terms of production and employment.

Table 5. Impact on the mark up of changes in parameters

<table>
<thead>
<tr>
<th></th>
<th>NO ENTRY</th>
<th>ENTRY</th>
<th>VARIETY</th>
<th>SIZE</th>
<th>VARIETY</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon + 1%$</td>
<td>$y^H$</td>
<td>$0.09%$</td>
<td>$0.13%$</td>
<td>$0.00%$</td>
<td>$0.23%$</td>
</tr>
<tr>
<td>(15%-&gt;15.15%)</td>
<td>$c$</td>
<td>$0.07%$</td>
<td>$0.11%$</td>
<td>$-0.25%$</td>
<td>$0.07%$</td>
</tr>
<tr>
<td></td>
<td>$NH, NE$</td>
<td>$-0.86%$</td>
<td>$-0.86%$</td>
<td>$0.003%$</td>
<td>$0.003%$</td>
</tr>
<tr>
<td>$f^E - 7.4%$</td>
<td>$y^H$</td>
<td>$--$</td>
<td>$0.13%$</td>
<td>$0.41%$</td>
<td>$0.93%$</td>
</tr>
<tr>
<td>(0.11-&gt;0.1)</td>
<td>$c$</td>
<td>$--$</td>
<td>$0.11%$</td>
<td>$0.86%$</td>
<td>$0.58%$</td>
</tr>
<tr>
<td></td>
<td>$NH, NE$</td>
<td>$6.63%$</td>
<td>$6.63%$</td>
<td>$3.57%$</td>
<td>$3.57%$</td>
</tr>
<tr>
<td>$AE + 7.4%$</td>
<td>$y^H$</td>
<td>$--$</td>
<td>$0.13%$</td>
<td>$0.41%$</td>
<td>$0.93%$</td>
</tr>
<tr>
<td>Entry (1-&gt;1.07)</td>
<td>$c$</td>
<td>$--$</td>
<td>$0.11%$</td>
<td>$0.86%$</td>
<td>$0.58%$</td>
</tr>
<tr>
<td></td>
<td>$NH, NE$</td>
<td>$6.63%$</td>
<td>$6.63%$</td>
<td>$3.57%$</td>
<td>$3.57%$</td>
</tr>
<tr>
<td>$\delta^F - 11%$</td>
<td>$y^H$</td>
<td>$--$</td>
<td>$0.25%$</td>
<td>$0.53%$</td>
<td>$2.14%$</td>
</tr>
<tr>
<td>(0.025-&gt;0.022)</td>
<td>$c$</td>
<td>$--$</td>
<td>$0.23%$</td>
<td>$0.98%$</td>
<td>$1.60%$</td>
</tr>
<tr>
<td></td>
<td>$NH$</td>
<td>$6.61%$</td>
<td>$6.61%$</td>
<td>$2.69%$</td>
<td>$2.69%$</td>
</tr>
<tr>
<td></td>
<td>$NE$</td>
<td>$-3.60%$</td>
<td>$-3.60%$</td>
<td>$-5.36%$</td>
<td>$-5.36%$</td>
</tr>
</tbody>
</table>

Table 5 compares the quantitative impact on consumption and the number of firms of a change in these parameters that reduces the markup by 0.1 percentage points. Starting with the case of the model with entry but no variety effect (column 2 in the table), firstly, note that the required
percentage change in these parameters to achieve that fall in the markup is around 7-11%, instead of the 1% increase in $\varepsilon$. Secondly, the impact on consumption and output is similar to the baseline for the parameters changing the entry cost, but it doubles for the fall in the death rate. Thirdly, the number of producing (and entrant) firms increases by more than six percent, in contrast with the 1% reduction in the baseline. This is consistent with the different ways of achieving stronger competition through changes in $\varepsilon$ vis a vis changes in $F^E$ or $A^E$; whereas in the former case the reduction in the market power is the cause of a fall in expected profits that discourages entry, in the latter, each firms’ market power falls as more firms are willing to enter due to the lower entry costs. The same logic applies to the reduction (by 3.6%) in the number of entrants when the death rate falls; since the number of producing firms increases, the level of expected profits critical for the entry decision goes down. In the case of the model with variety effect (column 3 in the table), the impact on consumption and output is around eight and four times greater than in the baseline, respectively. This reflects the fact that consumers value the number of varieties in the economy, which reduces the cost of achieving a given level of utility. Finally, when we allow for size heterogeneity we find a much stronger impact on output and consumption (5 and 7 times) and a weaker impact on entry (a half).

4 Results: Firm entry and inflation

In the rest of the paper we investigate the effect of entry and firm heterogeneity on the response of the main macroeconomic variables to different shocks. In particular we first focus on the dynamics of inflation conditional on productivity shocks to describe in detail the mechanism that drives apart the response of this variable with respect to what would occur in a model with a fixed number of productive firms. We then extend our analysis to the responses of other shocks. Key to understanding the differences with the standard non-entry model is expression (22); given the complexity of the mechanisms involved we carry out this analysis in two steps. First we compare the model with homogeneous firms with that of no entry and then in the next section we shall focus on the heterogeneity issue.

The New Keynesian Phillips curve in the model with homogenous firms ($A_t^s = A_t \forall s$) is a particular case of (22) that now becomes:

\[
\left(\hat{\Pi}_t - \chi\hat{\Pi}_{t-1}\right) = \frac{1 + \xi_c\xi_N(\varepsilon - 1)}{1 + \xi_c\xi_N(\varepsilon - 1)} \beta (1 - \delta^E) E_t \left(\hat{\Pi}_{t+1} - \chi\hat{\Pi}_t\right) + \frac{1}{1 + \xi_c\xi_N(\varepsilon - 1)} \left(1 - \theta_p\right) \left(1 - \beta(1 - \delta^E)\theta_p\right) \tilde{m}_{c,t} - \xi_v \frac{\xi_c\xi_N(1 - \theta_p)}{\theta_p(1 + \xi_c\xi_N(\varepsilon - 1))} E_t \tilde{\Delta}\tilde{N}_t^H + \frac{1 - \xi_v}{\theta_p(\varepsilon - 1)} E_t \tilde{\Delta}^2\tilde{N}_t^H
\]

(27)

(indirect) Competition effect (direct) Competition effect Variety effect
where $\tilde{\Delta}$ represents the following quasi-difference operator

$$
\begin{align*}
&\mathbb{E}_t \tilde{\Delta} \tilde{x}_t = \tilde{x}_t - \beta (1 - \delta F) \theta_p \mathbb{E}_t \tilde{x}_{t+1} \\
&\mathbb{E}_t \tilde{\Delta}^2 \tilde{x}_t = (\mathbb{E}_t \tilde{\Delta} \tilde{x}_t - \theta_p \tilde{\Delta} \tilde{x}_{t-1})
\end{align*}
$$

Note that the coefficient $\xi_N = \frac{(\eta^*_p + \varepsilon)}{\eta^*_p (1 + \eta^*_p)} = \frac{\xi_c N^H}{(N^H - 1)(N^H (\varepsilon - 1) - \varepsilon)} > 0$, $\eta^*_p = -\varepsilon \frac{N^H - \xi_c}{N^H} \in (-\varepsilon, -1) < 0$ and $\frac{\partial \xi_N}{\partial \eta^*_p} < 0$, $\frac{\partial \xi_N}{\partial N^H} > 0$.

That is, endogenous entry reduces the magnitude of the coefficients of the drivers of inflation that appear in the standard NKPC, dampening the response of the current inflation rate to its future expected value and to the current marginal cost, and adds two terms to the standard New Keynesian Phillips Curve: the (quasi) change in the number of producing firms ($\mathbb{E}_t \tilde{\Delta} \tilde{N}^H_t$) and its (quasi) acceleration ($\mathbb{E}_t \tilde{\Delta}^2 \tilde{N}^H_t$). The latter component captures the variety effect and enters the inflation equation by adding up a shifter that operates through the utility based aggregate price index (see equation (13) above), so that an increase in the number of producing firms rises the utility derived by households from consumption and reduces the welfare based aggregate price index and the inflation rate.

The competition effect instead works through the impact of entry on firms price-setting decisions. This occurs through two channels: First, in an economy with entry, reoptimizing firms take into account the impact of their pricing decisions on their competitors’ and the aggregate price, which reduces (in absolute terms) their price elasticity of demand at the optimum. In practice, this means that when allowed to do so firms change their price by a smaller amount than the change in the present discounted value of their marginal costs. This weakening impact on the coefficients results also from competition albeit in a an indirect manner. It does not result from the change in the number of firms in the market, but rather from the very possibility of strategic behavior. We name this the indirect competition effect and it is reflected in the terms pre-multiplying the coefficients of expected future inflation and current marginal costs in equation (22). These terms are decreasing in the price elasticity of demand at the steady state ($\eta^*_p = -\varepsilon \frac{N^H - \xi_c}{N^H}$) and in both cases they are smaller than one. Second, firms also discount the change in the intensity of competition caused by entry and exit of firms. We call this the direct competition effect and it is reflected by the inclusion of an additional term in the NKPC with the change in the number of producing firms. The impact of this channel is slow but very persistent, since the number of firms does not change on impact and converges very slowly back to the steady state. The magnitude of both competition effects will depend crucially on the size of the coefficient $\xi_N$, which is a function of the price-elasticity of demand evaluated at the steady state optimal price. In particular, the more elastic the demand schedule is at the optimum the lower the coefficient.

It must be noticed though that in a model with entry (of homogeneous firms) and full indexation (either to past inflation or to steady state inflation), like the one discussed here, there is no pricing heterogeneity in steady state, all prices equal the optimum, and thus the
The competition effect is likely to be very small. Table 6, reports the values of the coefficients in both versions of the NKPC above for our baseline calibration. When there is entry, the (indirect) competition effect lowers the coefficients of future inflation and current marginal costs around 6%, while the coefficient of the change in the number of firms is very small in magnitude (-.003).

The opposite is true for the variety effect, whose (negative) coefficient is greater than the one of marginal costs. However, since the dynamics of the number of firms is very persistent, its acceleration will not move very much, except just after the initial impact.

Table 6. Parameters of New Keynesian Phillips Curve (equation (27))

<table>
<thead>
<tr>
<th></th>
<th>$\hat{\Pi}_{t+1}$</th>
<th>$\bar{m}c_t$</th>
<th>$E_t\Delta\hat{N}^H_t$</th>
<th>$E_t\Delta^2\hat{N}^H_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>no entry</td>
<td>.990</td>
<td>.086</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>entry</td>
<td>.933</td>
<td>.080</td>
<td>-.003</td>
<td>0</td>
</tr>
<tr>
<td>variety</td>
<td>.933</td>
<td>.080</td>
<td>0</td>
<td>.095</td>
</tr>
</tbody>
</table>

Finally, notice that if we shut down the competition and the variety effects ($\xi_c = 0$, $\xi_v = 1$), this expression boils down to the standard non-entry NKPC with a discount rate augmented by the survival rate of firms ($1 - \delta^F$).

At this stage it might be worth comparing the Phillips curve derived from our model with the one obtained previously by other researchers in models with entry. Differences among (27) and the price equations obtained by other authors in models with entry stem from two sources: first, our assumption of Calvo pricing, which departs from the Rotemberg costly price setting framework in Bilbiie et al (2008) and Lewis & Poilly (2012); and second, other assumptions regarding the industry structure. In terms of our notation the equivalent NKPC under Rotemberg pricing would take the following form:

$$\left(\hat{\Pi}_t - \chi\hat{\Pi}_{t-1}\right) = \beta \left(1 - \delta^F\right) \left(E_t\hat{\Pi}_{t+1} - \chi\hat{\Pi}_t\right) + \frac{\eta^*_p - 1}{\kappa} \bar{m}c_t - \xi_c \frac{\varepsilon}{\eta^*_p \hat{N}^H_t} \hat{N}^H_t$$

where $\kappa$ is the Rotemberg parameter of price rigidities and $\Delta$ is the difference operator. This expression has similar ingredients to the one under Calvo setting, except that in this case the indirect competition effect only changes the slope of the NKPC (marginal cost coefficient), since $\eta^*_p$ is a (negative) function of the number of firms in the economy, but not the forward looking component. In addition, the direct competition effect in this case does not include a

---

14 In deriving this expression we have imposed the same assumptions as in the rest of the paper, namely, a variable variety effect and a standard utility function. Alternatively, if we assumed a Feenstra utility function according to which consumers derive utility from increasing the number of varieties, the coefficient in front of $\hat{N}^H_t$ would be $\left(\eta + \frac{1 - \xi_v}{\varepsilon}\right) \frac{1}{\xi_v}$, where $\eta$ is the elasticity of the desired mark up to the number of varieties in the utility.
forward looking component of the change in the number of firms. Bilbiie et al. (2008) price equation is similar to this expression but assuming no competition effect ($\xi_c = 0$ and $\eta_p^* = -\varepsilon$), and concentrating on the variety effect. On the other hand, Lewis and Poilly (2012) extend this expression to include the supply structure proposed by Jaimovich and Floetotto (2008), where there is a continuum of industries with firm entry. The main difference with the equation derived above is that in their case the steady state price elasticity of demand is a function of the difference between the within ($\varepsilon$) and the between ($\varepsilon_I$) industries’ elasticity of substitution, $\eta_p^* = \varepsilon - (\varepsilon - \varepsilon_I) \frac{1}{N_H}$.

4.1 Productivity shocks

In Figure 2 we depict the impulse responses of the main aggregate variables to a positive shock to total factor productivity. We start by discussing the differential response of the inflation rate in the three models, no entry (NE black continuous line) versus entry (E, red dashed line) and entry cum variety effect (EV, blue dotted line), all of them featuring firm homogeneity. While the NE and the E models display different responses in several macroeconomic variables, there are small differences in the dynamics of inflation: The inflation rate falls sharply on impact by the same amount in both cases but the subsequent return towards equilibrium is slightly more persistent in the entry model, in which the adjustment is not complete after 20 periods while in the non-entry case the adjustment is almost complete after 10 periods. This difference reflects the small but very persistent impact of the number of producing firms (direct competition effect) on inflation. The entry of new firms is triggered by the fall in marginal costs that increases firm profits. The number of hours worked by new entrants rises sharply although new firms become productive with a lag and display an inverted U-shape with great persistence.15

The dynamic response in the variety (V) model is significantly different at $t = 1$. Although the inflation rate falls on impact by a similar magnitude as in the non-entry and entry cases it falls then further as the acceleration in the number of firms affects directly the CPI index.

---

15 This similarity of the dynamics of inflation across the non-entry and entry models depends crucially on the degree of stickiness of marginal costs, that is, of wages, which in our baseline calibration is fairly high. In an economy with entry and flexible wages, marginal costs, and consequently inflation, fall on impact by much less. The reason is that the dynamics of marginal costs are now driven by the fact that the productivity shock triggers an entry of firms. Thus although the demand for labor in incumbent firms falls due to the presence price stickiness, there is a new source of labor demand by entrants. This increase in labor demand comes along with an increase in wages that dampens the reduction in marginal costs.
There are also some differences across models in the response of other macroeconomic variables. While the positive productivity shock triggers a sizable increase of investment in productive capital in the non-entry model, when entry is allowed a significant proportion of new investment takes the form of new firm creation making the response of $\hat{N}_t$ hump-shaped and very persistent. Consumption and production increase by the same amount on impact in the $NE$ and $E$ models, but as the number of operating firms starts to build up, these variables expand by more in the latter. The largest differences however, take place when consumers display a love for variety. When the entrant firms or varieties become operative at $t+1$, the utility from consuming more varieties increases, which pushes up output and factor demands, rising wages, marginal costs and consumption well above the level of the other models.

### 4.2 Other Shocks

The importance of the interaction between firms’ demographics and inflation also extend to the responses to other shocks (Figure 3). In the case of a monetary shock the inflation rate is barely affected by entry, since the movements in the marginal cost are very similar too. In addition, unlike what happened under the productivity shock, now the behavior of consumption...
is very similar across models. The reason is that the productivity shock hits the entry process directly by reducing marginal costs and a substantial amount of resources is devoted to entry and 'distracted' from other uses. Here the opposite happens, with a fall in $N^E$, which diminishes the costs associated to the entry process.

Inflation displays somewhat more different patterns across entry models following other types of shocks. The preference shock and the labor supply shocks, give rise to very different inflation dynamics; in both cases the impact effect under no entry is the largest, but as the number of operating firm diminishes, the inflation increases by more in the variety case. After a positive preference shock that increases consumption and the real wage, the marginal costs rises, reducing firm’s profits and the number of entrants; this in turn reduces the number of active firms which means an additional push on inflation in the $VE$ model long after the effects of the rise in the marginal costs have vanished.

Not surprisingly the largest differences among the $E$ and the $VE$ models occur in response to an entry cost shock. After this contractionary shock output and employment fall whereas inflation rises. The positive response of inflation is most interesting since this occurs, unlike any other type of shock, despite the fact that the marginal cost actually falls due to the sharp drop in the number of entrants that reduces the number of operative units and wages. The rise in $f^E$ reduces the incentive to enter in the market and hence the number of operating firms. In the variety case this acts as a powerful inflationary mechanism that more than compensates the fall in $mc^t$.

The results in this section indicate that the differences in conditional inflation dynamics among models in which the number of firms remain constant along the cycle and those in which there is entry and exit, rely on the strength of the variety effect. In other words, as the theoretical model above suggests strategic pricing behavior and competition among incumbents and potential entrants, that is very often pointed as a key driving force of inflation, plays a minor role. Consequently, competitiveness seems to be disconnected from the barriers of entry in the market. Important as it might be, the variety effect is not highly placed on the policy agenda on this matter, nor is it straightforward to relate this concept with specific policy actions to achieve more stable inflation rates. Admittedly, many of the discussions about policies to foster competition and lower consumer prices include not only the entry/no entry dimension but also look into another relevant feature of the market structure: heterogeneity of firms in terms of productivity and size.
5 Results: Heterogeneous firms and inflation

In order to further explore the importance of competition forces in the market we move on now to study the implications of the presence of heterogeneous firms in the market. We conduct our
analysis by comparing two versions of our general model, the standard entry model we have
discussed so far and a model with two types of firms: large (more productive) and small (less
productive). We then extend our exercise to consider an alternative industry structures in terms
of the dispersion of firms’ size and the number of firm types.

The NKPC for each group is similar to the case of homogeneous firms, except that now all
the parameters are group specific, while the (direct) competition effect refers to the total
number of producing firms,\(^{16}\)

\[
(\hat{\Pi}_{st} - \chi_s \hat{\Pi}_{st-1}) = \left(1 + \frac{\theta_p^s c_s}{\xi_s \xi_N^s (\varepsilon - 1)}\right) \beta (1 - \delta^f_s) (1 - \zeta^{Ls} - \zeta^{Ss}) E_t \left(\hat{\Pi}_{st+1} - \chi_s \hat{\Pi}_{st}\right) \\
\text{(indirect) Competition effect}
\]

\[
+ \frac{1}{\xi_s \xi_N^s (\varepsilon - 1)} \left(1 - \theta_p^s \left(1 - \beta (1 - \delta^f_s) (1 - \zeta^{Ls} - \zeta^{Ss}) \theta_p^s \right) \tilde{N}_t\right)
\text{(direct) Competition effect}
\]

\[
- \xi_s \left(1 - \theta_p^s \right) \left(1 + \xi_s \xi_N^s (\varepsilon - 1)\right) E_t \tilde{\Delta} \hat{N}_t^{H} \text{Variety effect}
\]

where \(\tilde{\Delta}\) represents the following quasi-difference operator

\[
E_t \tilde{\Delta} \hat{x}_t^s = \hat{x}_t^s - \beta (1 - \delta^f_s) (1 - \zeta^{Ls} - \zeta^{Ss}) \theta_p^s E_t \tilde{\Delta} \hat{x}_{t+1}^s
\]

\[
E_t \tilde{\Delta}^2 \hat{x}_t^s = \left(E_t \tilde{\Delta} \hat{x}_t^s - \theta_p^s E_t \tilde{\Delta} \hat{x}_{t-1}^s\right)
\]

Note that the coefficient \(\xi_N = \frac{\eta_p^s + \varepsilon}{\eta_p^s + (1 + \eta_p^s)} > 0, \eta_p^s = -\varepsilon \left(1 - \xi_c \left(N^H\right)^{-\xi_c} \left(N^\varepsilon\right)^{1-\varepsilon}\right), \eta_p^s \in (-\varepsilon, -1) < 0\) and \(\frac{\partial \varepsilon^*_N}{\partial \eta^*_p} < 0, \frac{\partial \varepsilon^*_N}{\partial \eta^*_p} = \frac{\partial \xi^*_N}{\partial \eta^*_p} > 0.\)

This expression resembles the one in the homogeneous firms model, (27). However, in this
case the coefficient \(\xi_N\) may differ across productivity classes since the optimal relative price
is also very different among them due to the technological difference: higher than one in the
case of the smaller/less productive firms and lower than one for the larger/more productive
ones.\(^{17}\) Therefore, the price elasticity of demand is also very different across sizes: greater (in
absolute terms) in the case of small firms, close to the no entry value \((-\varepsilon)\), while it is much
smaller for the more productive firms. This means that when allowed to change their prices more productive firms do so by much less than the present discounted value of marginal costs,
while small firms behave more like the case of homogeneous firms. As a consequence, both the
indirect competition effect (which is a negative function of the optimal relative price in steady
state (\(\Pi^s\))) and the direct competition effect (positive function of \(\Pi^s\)) are much stronger in

\(^{16}\)In addition, this NKPC includes two terms with the relative prices which are not crucial for the behaviour
of inflation.

\(^{17}\)In a model with more than two types, those with productivity above (below) the average will set their optimal
prices in steady state at a level below (above) the average and their relative price will be below (above) one.
the case of the more productive firms. That is, the coefficients in front of future inflation and current marginal costs are smaller in the case of the more productive firms, while the one in front of the change in the number of productive firms is greater.

\[
\hat{\Pi}_t = \sum_{s=1}^{N} \frac{N^s (\Pi^{*s})^{1-\varepsilon}}{\sum_{s=1}^{N} N^s (\Pi^{*s})^{1-\varepsilon}} \left[ \hat{\Pi}_{st} - \frac{\xi_v}{\varepsilon - 1} \Delta \left( \tilde{N}^s_t - \tilde{N}^H_t \right) \right] \\
\hat{\Pi}_t = \sum_{s=1}^{N} \frac{N^s (\Pi^{*s})^{1-\varepsilon}}{\sum_{s=1}^{N} N^s (\Pi^{*s})^{1-\varepsilon}} \left[ \hat{\Pi}_{st} - \frac{\xi_v}{\varepsilon - 1} \frac{1}{N^H_t} \sum_{s=1}^{N} \left( 1 - \frac{(\Pi^{*s})^{1-\varepsilon}}{(\Pi^{*s})^{1-\varepsilon}} \right) \Delta \tilde{N}_t^s \right] 
\]

On the other hand, aggregate inflation is a weighted average of the inflation rate and the change in the share of producing firms of each firm-size, where the weights are the share of firms of each size in steady state times the inverse of their optimal relative price. As shown in (29) this can be re-written in terms of the change in the number of firms of each productivity level, times the difference in optimal relative prices. Given that the optimal relative price of large (small) firms is smaller (greater) than one, this expression puts a much larger weight on the inflation rate of the most productive firms, which will tend to dominate aggregate inflation dynamics. With respect to the change in the number of producing firms of each size, in the case of two sizes, the weights are identical but with different sign (positive for small, negative for large firms). This represents the fact that small firms set a higher price, thus a rise in their share of all firms increases aggregate inflation.

Table 7. Parameters of New Keynesian Phillips Curve equation (29).

<table>
<thead>
<tr>
<th>Entry</th>
<th>$\hat{\Pi}_{t+1}^s$</th>
<th>$\hat{mc}_t^s$</th>
<th>$\hat{\Pi}_{t} \Delta \tilde{N}^H_t$</th>
<th>$\hat{\Pi}_{t} \Delta^2 \tilde{N}_t^s$</th>
<th>$\xi^s_N$</th>
<th>$\eta_{p}^s$</th>
<th>$\Pi^{*s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>no entry</td>
<td>.990</td>
<td>.086</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-15</td>
</tr>
<tr>
<td>entry</td>
<td>.933</td>
<td>.080</td>
<td>-.003</td>
<td>.095</td>
<td>.011</td>
<td>-13</td>
<td>1.0</td>
</tr>
<tr>
<td>size</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>small</td>
<td>.956</td>
<td>.091</td>
<td>-.0005</td>
<td>.095</td>
<td>.002</td>
<td>-14.7</td>
<td>1.1</td>
</tr>
<tr>
<td>large</td>
<td>.767</td>
<td>.022</td>
<td>-.018</td>
<td>.095</td>
<td>.241</td>
<td>-6.5</td>
<td>.87</td>
</tr>
<tr>
<td>aggregate</td>
<td>.226</td>
<td>.774</td>
<td>.048</td>
<td>-.048</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7 shows the coefficients of the size-NKPCs and aggregate NKPC, as well as the values of the steady state optimal relative prices ($\Pi^{*s}$), price elasticity of demand ($\eta_{p}^s$) and the coefficient ($\xi^s_N$) for each firm-size under the baseline calibration. It clearly shows that large firms respond much less (20%) to current marginal costs and a bit less to expected future inflation (80%), while they respond much more (36 times) to changes in the aggregate number of productive firms. In addition, one can see that the price setting decisions of large/more productive firms dominate the dynamics of aggregate inflation, with a weight above 75%. However, the change
in the number of small firms will also have a strong impact on aggregate inflation since they impact with the same magnitude but they represent a larger proportion of all firms.

We depict in Figures 4, 5 and 6 the impulse responses of the model with entry and homogeneous firms (E model, blue dotted lines) and those corresponding to the model with heterogenous firms (S model, red dashed lines). The latter model allows for entry and exit of firms but also for upgrading and downgrading in the level of productivity and size. To highlight the importance of the competition effect in both worlds we abstract from the variety effect throughout the exercise (the results are available from the authors upon request). In addition, in figures 4 and 5 we also display the differential response of variables corresponding to small and large firms.

As was mentioned above, it is worth noting that we are the first to derive the NKPC in an economy with entry and size distribution. Lewis and Poilly (2012) derive the NKPC in an economy with entry into different industries, but their objective is to analyze the impact of the different degree of substitutability of goods within and between industries and thus they do not consider size or productivity differences. In their case the steady state price elasticity of demand is a function of the difference between the within ($\varepsilon$) and the between ($\varepsilon_I$) industries’ elasticity of substitution, $\eta_p = \varepsilon - (\varepsilon - \varepsilon_I) \frac{1}{N_H}$. Note that in their model, when these two elasticities of substitution are identical we are back at the case without competition effect ($\eta_p^* = \varepsilon = \varepsilon_I$), in which the price elasticity of demand is constant and independent of the number of producing firms. This is due to the fact that they assume that firms take into account the impact of a change in their prices on the industry price, but not on the aggregate price.

5.1 Productivity shock

Regarding productivity shocks, the model with large and small firms displays a significantly weaker reaction of the inflation rate than the model with homogeneous firms, despite the fact that marginal costs fall by more and entry of new firms is stronger in the heterogeneity case. The reasons behind this discrepancy have to do with the different behavior of small versus large firms’ inflation as well as with the composition effect behind the aggregate rate. First, notice that the impulse response function of the rate of inflation of small firms basically mimics the response of inflation in the case of homogeneous firms (figure 4, panels A and B). The aggregate behaves differently though due to the combination of two effects. First, large firms adjust their prices significantly less on impact, because, as explained above, the strength of both the direct and indirect competition effects disconnects to some extent the response of prices from the dynamics of the marginal cost.
Figure 4. Neutral Productivity shock

A) Aggregate responses: heterogeneous (red) vs homogeneous (blue)

B) Heterogeneous model: $\Pi^*_t$ and $N^*_t$ by size

C) contributions to aggregate inflation

Second, the entry of firms affects the aggregate rate from $t + 1$ onwards, when entrants become productive. At that point, the industry structure is affected by the massive entrance of small firms with a higher than average price level. Since aggregate inflation is a weighted average of the inflation rate of all different classes of firms it increases when the proportion of less productive firms in the markets increases. Thus, although there are more firms and all of them set a lower price, the aggregate inflation rate does not fall by as much as in the homogeneity case in which all firms reduce prices by the same amount and this composition effect is absent.
This is clear in panel C of figure 4, where most of the dynamics of aggregate inflation is driven by the behavior of the inflation rate of more productive firms (pink bars) and by the change in the number of less productive firms (dark blue bars), while the inflation of less productive firms affects only on impact (magenta bars).

This composition effect is also reflected in the response of consumption, investment and output. The fact that firms discount the probability of being small (low productive) upon entry and remaining so with a given probability reduces their value \( v \) as well as the Tobin’s \( q \), with respect to the model with homogeneous firms. The number of small firms and their employment increases on impact along with the new entrants consuming a great deal of resources, thus dampening more the response of consumption and output in the heterogeneity case. The impact effect of the productivity shock is in this case more moderate although it becomes stronger and more persistent as times goes by and some of the new entrants become large and more productive.

5.2 Other shocks

A similar pattern of response of inflation can be found for other shocks, like a monetary tightening (Figure 5). Again, large firms adjust their prices downwards to a less extent than small/less productive firms do (figure 5, panel B and pink bars in panel C). This is due to the combination of two effects that are much stronger in the case of large firms: On the one hand these firms respond less to changes in the marginal cost and expected inflation (indirect competition effect) and, on the other hand, the fall of new entrants pushes their prices upwards (direct competition effect), partially offsetting the effect of falling marginal costs. There is an additional downwards movement of aggregate inflation once the composition effect becomes operative at \( t + 1 \), since the fall in new (small) entrants increases the share of large (lower price) firms in the industry (dark blue bars in the right hand side panel of figure 5).

For the same reasons, inflation is much less volatile in the model with heterogeneous firms also after other types of shocks (Figure 6). In all cases, large firms dampen the response to the change in expected marginal cost, taking advantage of their better technology. In most of these cases, the response of inflation is decoupled from that of the marginal cost that is more responsive in the model with heterogeneous firms.

One notable exception to this general result is the response of the two models to an increase in the costs of entry in the industry (figure 6, panel D). Again, the entry shock gives rise to an interesting dynamics since not only the inflation rate and the marginal cost move in opposite directions, as in the case of firm homogeneity, but also unlike what happens in response to other shocks in this case the change in inflation is much stronger when different types of firms are allowed. Again, the composition effect (which is absent in the homogeneity case) explains this pattern, but notice first that the impact effect on inflation is the same in the two models. This is due to the fact that the marginal cost of existing small and large firms are equally (un)affected by the entry cost shock, so inflation does not react on impact.
At $t + 1$ however, the entry shock reduces quite dramatically the number of entrants that changes the composition of the population of operating firms in favor of the most productive ones, which are able to set lower prices. Thus a negative supply shock, that makes entry less appealing and affects in the expected way an economy with homogenous firms, ends up heterogeneity: inflation falls and output and consumption increase.\(^{18}\)

\(^{18}\)If the heterogeneity model also featured the variety effect this result would be reversed, since the reduction in the number of varieties would become a strong inflationary shock, lessening the importance of the composition effect.
Figure 6. Other shocks

A) Preference shock
Aggregate irfs: size (red), entry (blue)

B) Labor supply shock
Aggregate irfs: size (red), entry (blue)

C) Investment shock
Aggregate irfs: size (red), entry (blue)

D) Entry cost shock
Aggregate irfs: size (red), entry (blue)

Contributions to aggregate inflation

Contributions to aggregate inflation
The aggregate inflation rate also depends on another dimension of the model: the dispersion in productivity levels among productive firms. The effect of a productivity shock in models with different degrees of price dispersion is depicted in Figure 7, panel A. The inflation rate displays a very similar path in both models although the impact response is more muted in the case with a higher dispersion. This result is a straightforward extension of the one in Figure 4. Higher dispersion means that the productivity of smaller firms is now lower (blue lines) and that of large firms higher than in the benchmark, which widens the gap among low and high productivity firms, and makes it possible for the latter to become less exposed to competition from smaller entrants and to better manage their pricing policy. Thus the counteracting effect of a weaker price response by the latter is now enhanced, so that aggregate inflation responds even less on impact. Also the composition effect is stronger since new entrants being less productive set much higher prices.

**Figure 7. Productivity shock**

A) 50% increase in dispersion
Baseline (red) vs higher dispersion (blue)

C) Three types of firms
Baseline (red) vs three types (blue)

Finally, the panel B of figure 7, compares the impact on inflation of a productivity shock in the baseline model with an alternative one in which there are three types of firms, small, medium and large firms, though with the same average productivity and (approximately) similar dispersion. The behaviour of inflation (and the other macro variables) is fairly similar across the two models, confirming that the results in this section are also present when one increases the number of types considered.
6 Conclusions

In this paper we have analyzed the interaction between some key characteristics of the industrial structure of an economy and its response to permanent and transitory shocks. In particular we focus on the role played by the coexistence of firms with different productivity levels and its incidence on the inflation dynamics. We have approached the issue in two steps. First we compare the inflation responses in a DSGE model with endogenous firm entry against those in a model with a fixed number of firms to isolate the impact of entry against the determinants of inflation in a New Keynesian framework: current and expected marginal costs. According to what other authors have found we obtain that given a sufficient degree of wage stickiness there are no significant differences coming through the change in market power that occurs in the entry model. In other words, this pure competition effect tends to be quite small.

Endogenous firm entry though opens up another channel through which inflation may be affected: the variety effect whereby consumers derive utility by having a larger number of imperfectly competitive varieties at their disposal. When this effect is present, the responses of inflation tend to be substantially augmented in the entry model, vis-a-vis the no entry one, in correlation with firms’ entry. This variety effect is not, however, a major theme in policy discussions on competition policy that have focused recently on another feature of the market structure: the heterogeneity of firms in terms of productivity and size and its impact on inflation and competitiveness.

In this paper we augment the entry model with a dynamic structure of heterogenous firms in terms of productivity (size). We allow for a finite number of classes of firms although most of the empirical analysis is conducted in a two group model, small and large firms. Although all firms enter at the bottom of the productivity ladder they may later on move upwards, downwards or out of the market at some exogenous transition rates. These rates are chosen to reproduce alternative industrial structures, with Spain as the benchmark; notice that not much is lost for keeping these transition rates exogenous at this stage, since the empirical evidence documents that these structures tend to be quite stable, at least at business cycle frequencies.

Then we compare the responses of inflation (and other variables) in this model with the ones in the model with endogenous entry but homogenous firms. The general result is that inflation is significantly less responsive, both upwards and downwards, when large firms are present. This result stems from two effects. On the one hand, large and small firms adjust their desired markups differently in such a way that the most productive ones always change their optimal price level (and hence inflation) by less since the competition effects (both direct and indirect) are much stronger, weakening the connection with marginal costs and enhancing the response to the change in the number of firms in the market. Besides, the change in the composition of the industry that occurs as new entrants increase or decrease, also changes the average inflation rate since these firms are less productive and then feature a higher relative price than the average.
One notable exception to this general result is the response to an entry shock in which case not only the inflation rate and the marginal cost move in opposite directions in both models, but also the change in inflation is much stronger when different types of firms are allowed due to a strong composition component.

We see our results as providing support to the extended idea that the productivity and size structure of firms in an economy have a close connection with the response of inflation, and hence competitiveness, to aggregate shocks. Besides, our model identifies a number of channels, beyond changes in entry costs, that might enlighten the policy strategies aimed at reforming the existing industrial structure to achieve a better macroeconomic performance in terms of inflation and competitiveness.
Appendix: The model with endogenous entry and size

A definition of equilibrium in this economy is standard, with 20 aggregate variables \((c_t, R_t, \Pi_t, d_t^F, u_t, l_t^d, l_t^s, l_t^ES, f_t, w_t^s, N_t^H, N_t^E, v_t^p, v_t^w, k_t, q_t, r_t, i_t, \gamma_t, \) \(11\times N\) group \(s\) variables \((v_t^s, d_t^Es, l_t^ds, g_t^s, \tilde{g}_t^s, \Pi_t^s, N_t^s, v_t^{ps}, k_t^s, m_t^s)\), 6 aggregate shocks \((d_t, A_t, \mu_t, \varphi_t, f_t^E, m_t)\) and \(N\) group specific shocks \((A_t^s)\)

Euler eq (bonds): \(d_t c_t^{-\sigma} = \beta E_t \left[ d_{t+1} c_{t+1}^{-\sigma} \frac{R_t}{\Pi_{t+1}} \right] \tag{1} \)

Euler eq (capital): \(q_t = \beta E_t \left[ d_{t+1} c_{t+1}^{-\sigma} \right] \tag{3} \)

Tobin’s q:

\[
1 = q_t \mu_t \left( 1 - S \left[ \frac{i_t}{i_{t+1}} \right] - S' \left[ \frac{i_t}{i_{t+1}} \right] \right) + \beta E_t(q_{t+1}) \left[ d_{t+1} c_{t+1}^{-\sigma} \right] \tag{4} \]

wage setting (intratemporal optimality):

\[
f_t = \frac{\gamma - 1}{\gamma} (w_t^s)^{1-\gamma} d_t c_t^{-\sigma} w_t^p \tag{5} \]

\[
f_t = d_t \varphi_t \psi \left( \frac{w_t^s}{w_t^p} \right)^{1-\gamma} + \beta E_t \left[ \frac{\Pi_{t+1}^{1-\chi_s} \Pi_t^{\chi_s}}{\Pi_{t+1}} \right] \left( \frac{w_t^{s+1}}{w_t^s} \right)^{1-\gamma} f_{t+1} \tag{6} \]

pricing (firms of size \(s\), for \(s = 1, ..., N\),

\[
g_t^{s1} = d_t c_t^{-\sigma} m_t^{s} c_t^{1+i_t \frac{\gamma_t}{(N_t^H)^{\xi_v}}} + \beta (1 - \delta^{Es}) \left( 1 - \zeta^{LS} - \zeta^{SS} \right) \theta_{pE}^{s} \left( \frac{\Pi_{t+1}^{1-\chi_s} \Pi_t^{\chi_s}}{\Pi_{t+1}} \right)^{-\varepsilon} g_t^{s1} \tag{7} \]

\[
g_t^{s2} = d_t c_t^{-\sigma} \Pi_t^{s} c_t^{1+i_t \frac{\gamma_t}{(N_t^H)^{\xi_v}}} + \beta (1 - \delta^{Es}) \left( (1 - \zeta^{LS} - \zeta^{SS}) \right) \theta_{pE}^{s} \left( \frac{\Pi_{t+1}^{1-\chi_s} \Pi_t^{\chi_s}}{\Pi_{t+1}} \right)^{(1-\varepsilon)} g_t^{s2} \tag{8} \]

\[
\Pi_t^{s*} = \frac{\varepsilon}{\varepsilon - 1} \left[ \left( \frac{(N_t^H)^{\xi_v} - \xi_v (\Pi_t^{s*})^{1-\varepsilon}}{(N_t^H)^{\xi_v} - \xi_v (\Pi_t^{s*})^{1-\varepsilon}} \right) \left[ g_t^{s1} \right]^{-\varepsilon} g_t^{s2} \right] \tag{9} \]
price dispersion (firms of size s, for \( s = 1, \ldots, N \)):

\[
(v_t^p)^{-\varepsilon} = \sum_{s=1}^{N} \left( \frac{N_t^s}{N_t^H} \right)^{\xi_v} (v_t^{ps})^{-\varepsilon}
\]

(10)

\[
(v_t^{ps})^{-\varepsilon} = (N_t^s)^{1-\xi_v} (1 - \theta_p^s) (\Pi_t^s)^{-\varepsilon} + \left( \frac{N_t^s}{N_t^{s-1}} \right)^{1-\xi_v} \theta_p^s \left( \frac{\Pi^{1-\chi_s \cdot \Pi_{t-1}}}{\Pi_t} \right)^{-\varepsilon} (v_{t-1}^{ps})^{-\varepsilon}
\]

(11)

variety effect (firms of size s, for \( s = 1, \ldots, N \)):

\[
1 = \sum_{s=1}^{N} \left( \frac{N_t^s}{N_t^H} \right)^{\xi_v} (p_t^s)^{1-\varepsilon}
\]

(12)

\[
\Pi_t^s = (N_t^s)^{1-\xi_v} \left[ \left( \frac{p_t^s}{1 - \theta_p^s} \right)^{1-\xi_v} \left( \frac{\Pi^{1-\chi_s \cdot \Pi_{t-1}}}{1 - \theta_p^s} \right)^{1-\xi_v} \right]^{\frac{1}{\xi_v}}
\]

(13)

aggregate wage

\[
w_t^{1-\eta} = \theta_w \left( \frac{\Pi^{1-\chi_s \cdot \Pi_{t-1}}}{\Pi_t} \right)^{1-\eta} w_{t-1}^{1-\eta} + (1 - \theta_w) w_t^{s1-\eta}
\]

(14)

wage dispersion:

\[
v_t^{w} = (1 - \theta_w) \left( \frac{w_t^{s}}{w_{t-1}^{s}} \right)^{-\eta} + \theta_w \left( \frac{w_{t-1}^{s}}{w_t^{s}} \Pi^{1-\chi_s \cdot \Pi_{t-1}} \right)^{-\eta} v_{t-1}^{w}
\]

(15)

number of firms (firms of size s, for \( s = 1, \ldots, N \)):

\[
N_t^H = \sum_{s=1}^{N} N_t^s
\]

(16)

\[
N_t^s = \left[ \zeta_{Ls-1} (1 - \delta^{Fs-1}) N_{t-1}^{s-1} + \zeta^{Sp+1} (1 - \delta^{Fs+1}) N_{t-1}^{s+1} \right] + (1 - \zeta_{Ls} - \zeta^{Sp}) (1 - \delta^{Fs}) N_{t-1}^{s}
\]

(17)

free entry: \( v_t^E = f_t^E m_{t}^E \)

(18)

profits (firms of size s, for \( s = 1, \ldots, N \)):

\[
d_t^F = \sum_{s=1}^{N} \frac{N_t^s}{N_t^H} d_{t}^{Fs}
\]

(19)

\[
d_{t}^{Fs} = c_t + i_t \left( \frac{N_t^s}{N_t^H} \right)^{\xi_v} \left[ (p_t^s)^{1-\varepsilon} - mc_t^s (v_t^{ps})^{-\varepsilon} \right]
\]

(20)
real marginal costs of producing firms (size $s$):  
\[ mc_s^t = \left( \frac{1}{1 - \alpha_s} \right)^{1-\alpha_s} \left( \frac{1}{\alpha_s} \right)^{\alpha_s} w_t^{1-\alpha_s} r_t^{\alpha_s} A_t^{1-\alpha_s} \]  
for $s = 1, ..., N$  
(21)

real marginal costs of entrants:
\[ mc_E^t = \left( \frac{1}{1 - \alpha_E} \right)^{1-\alpha_E} \left( \frac{1}{\alpha_E} \right)^{\alpha_E} w_t^{1-\alpha_E} r_t^{\alpha_E} A_t^{1-\alpha_E} \]  
(22)

capital-labour ratio:
\[ \frac{k_{t-1}}{l_t} = \frac{\alpha}{1 - \alpha} \frac{w_t}{r_t} \]  
(23)

Capital accumulation:
\[ k_{t-1} = \sum_{s=0}^{N} k_t^s \]  
(24)
\[ k_t - (1 - \delta) k_{t-1} - \mu_t \left( 1 - S \left[ \frac{i_t}{i_{t-1}} \right] \right) i_t = 0. \]  
(25)

Taylor rule:
\[ \frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\gamma_R} \left( \left( \frac{\Pi_t}{\Pi} \right)^{\gamma_m} \left( \frac{y_t^{dH}}{y_t^{dH}} \right)^{\gamma_y} \right)^{1-\gamma_R} \exp(m_t) \]  
(26)

Established firms market:
\[ y_t^H = \sum_{s=1}^{N} A_t^{\alpha_s} \left( \frac{k_t^s}{l_t^s} \right)^{\alpha_s} \]  
(27)
\[ y_t^{dH} = c_t + i_t \]  
(28)
\[ y_t^{dH} = \frac{y_t^H}{(v_t^H)^{\varepsilon}} \]  
(29)

Entrants market:
\[ y_t^E = A_t^{\alpha} \left( \frac{k_{t-1}}{l_t} \right)^{\alpha} y_t^{dE} \]  
(30)
\[ y_t^{dE} = N_t^E v_t^E \]  
(31)
\[ y_t^{dE} = mc_t^E y_t^E \]  
(32)
Labour market:

\[ l_t^s = l_t^d w_t \]  

(33)

\[ l_t^d = \sum_{s=1}^{N} l_t^{ds} + l_t^{dE} \]  

(34)

Aggregate demand & supply:

\[ y_t^s = \frac{y_t^H}{(v_t^p_e)} + mc_t^E y_t^E \]  

(35)

\[ y_t^d = y_t^{dH} + y_t^{dE} \]  

(36)
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