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Abstract

Correlations of equity securities have varied substantially over time and remain a source of continuing policy debate. This paper studies stock market correlations in an equilibrium model with heterogeneous risk aversion. In the model, preference heterogeneity causes countercyclical variations in the volatility of aggregate risk aversion. At times of high volatility of aggregate risk aversion, which is a common factor in returns, we see high correlations. The calibrated model matches average industry return correlations and changes in correlations from business cycle peaks to troughs, and replicates the cyclical dynamics of expected excess returns and standard deviations. A proxy for model-implied aggregate risk aversion jointly explains average industry correlations, expected excess returns, standard deviations and turnover volatility in the data. We find supportive evidence for the model's prediction that industries with low dividend-consumption correlation have low average return correlation but experience disproportionate increases in return correlations in recessions.

Keywords: equity return correlations, heterogeneous risk aversion, volatility of turnover, cyclical dynamics of stock price moments.

JEL classification: G10, G11.

Resumen

Las correlaciones entre los títulos de renta variable han variado sustancialmente con el tiempo y siguen siendo una fuente de constante debate de política económica. Este artículo estudia las correlaciones bursátiles en un modelo de equilibrio con aversión al riesgo heterogénea. En el modelo, la existencia de preferencias heterogéneas causa variaciones contracíclicas en la volatilidad de la aversión al riesgo agregada. En episodios de elevada volatilidad de la aversión al riesgo agregada, que es un hecho común en los rendimientos, se observa una correlación muy alta. El modelo calibrado ajusta las correlaciones promedio de las rentabilidades de la industria y los cambios en las correlaciones entre los picos y valles del ciclo económico, al tiempo que replica la dinámica de los excesos de rentabilidad esperados y las desviaciones estándar a lo largo del ciclo. Una *proxy* para la aversión al riesgo agregada implícita del modelo explica conjuntamente las correlaciones medias de la industria, el exceso de los rendimientos esperados, las desviaciones estándar y la volatilidad de los rendimientos observadas en los datos. Finalmente, encontramos evidencia a favor de la predicción del modelo, según la cual las industrias con baja correlación entre dividendo y consumo tienen una baja correlación en las rentabilidades medias, pero experimentan subidas desproporcionadas en las correlaciones de las rentabilidades durante las recesiones.

Palabras clave: tenencias de liquidez, correlación rentabilidades bursátiles, volatilidad del volumen de facturación, dinámica cíclica de los momentos del precio de las acciones, aversión al riesgo heterogénea.

Códigos JEL: G10, G11.

1 Introduction

Correlations rise even between seemingly unrelated assets during recessions or financial crisis. Economists, regulators or the financial press frequently interpret such events as driven by "contagion" and claim that there is "no place to hide" during such events. In turn, we study correlations in a heterogeneous investor exchange economy and thereby provide a rational macroeconomics or consumption based explanation. In the language of the model, correlations rise in economic downturns and fall over boom periods. Remarkably, expected excess returns, standard deviations, and many other equilibrium quantities show joint cyclicality with correlations. These findings raise the question whether policy makers can cure markets from excess correlations, volatility and trade without shrinking financial markets ability to facilitate consumption risk sharing across investors over the business cycle.

Since correlations determine the extent of diversification benefits it might be particularly bad for investors if, as the data suggest, high correlations coincide with economic downturns. However, such a view is too simplistic as stock return correlations are not the only drivers of performance. Consistent with this concern, Figure 1 shows that the average correlation of ten US industry portfolios comoves with average stock return volatility and average expected excess return. Further, variations in correlations, volatilities and expected returns during NBER contractions appear large and can, therefore, cause significant portfolio rebalancing. Indeed, the dynamics of turnover volatility in Figure 1 seem consistent with brisk portfolio rebalancing during contractions. The contributions of this paper, that are most relevant to investors, are to document and justify the business cycle dynamics of correlations and to tie correlations theoretically and empirically to expected stock returns and volatilities, turnover volatility and the business cycle.

In response to the empirical facts stated above and the questions that emerge from them, we build an economy with many Lucas trees that is populated by consumers with heterogeneous risk aversion and external relative habit formation and show that it accounts for the key empirical features in Figure 1. The main economic mechanism that generates the stylized facts about stock return correlations is heterogeneity in risk aversion. When consumption falls, consumers with low risk aversion find it optimal to sell stocks to more risk averse consumers, and consequently the marginal consumer becomes more risk averse. This, in turn, leads to higher compensation for risk and higher expected excess returns. In equilibrium, the volatility of risk aversion rises in economic downturns. The volatility of aggregate risk aversion is driven by optimal consumption risk sharing between heterogeneous consumers with constant relative risk aversion. In contractions, small shocks to aggregate consumption lead to large fluctuations in the distribution of consumption across consumers.

When aggregate risk aversion becomes more volatile, the discount rate volatility rises, and, hence, we see an increase in stock return volatilities. Intuitively, as the volatility of aggregate risk aversion drives every discount rate, we see higher stock return correlations. Finally, we compute the models implications for turnover volatility as heterogeneity leaves a footprint in portfolio trade. In the model, turnover volatility is countercyclical just as correlations.

The consumption sharing rule is implemented by trade in the stock market and in a risk-free security. Since trade is observable, it provides indirect information about consumer heterogeneity, something that is difficult to measure directly. To understand the heterogeneous risk aversion based origins of turnover volatility, we solve for the volatility, or quadratic variation, of model implied portfolio policies as a measure of trading intensity. What we learn from this exercise is that in the model turnover volatility is high during bad times and that it correlates positively with stock return correlations.[1]

To assess the models ability to quantitatively match the dynamics of stock return correlations, we calibrate our model to ten industry portfolios. Further, to evaluate the heterogeneous risk aversion channel it is import to also aim at matching other key asset pricing moments and their cyclical dynamics. Through the calibration, we learn that the model accounts for the unconditional level of correlations together with the change in correlation over the business cycle. Moreover, the model generates high comovement between stock return correlations, volatilities and expected excess returns as in the data. We also show that a measure of habit backed out from our preferred calibration explains average industry return correlations, volatilities and expected excess returns both inside the model and in the data. Specifically, the slope coefficients and the R-squareds in the model based regressions are close to the data. In an effort to further shed light on the model's ability to replicate correlation and stock return dynamics, we calculate the model implied stock return correlations, volatilities and expected excess returns by feeding the model with realized consumption shocks. From this exercise we observe that model implied values replicate the swings in stock return correlations, volatilities and expected excess returns over the business cycles. In particular, the correlation between model implied and actual correlations, volatilities and expected excess returns are 0.4, 0.73 and 0.34 and appear large as the series implied by the model are constructed using consumption data alone.

In the data, industry portfolios exhibit a significant cross-section of dividend-consumption correlations. When we calibrate the model economy to reflect the cross-section of dividend-

[1]Cochrane (2008) argues for a risk aversion explanation, as evidenced by the amount of deleveraging and forced selling taking place within capital markets, as the cause of the daily volatility of 75 percent of such activity occurring in October 2008. Cochrane (2011), in his presidential address to the American Economic Association, says: "... all of these theories are really about discount rates, risk bearing, risk sharing, and risk premiums" by which he essentially says that risk sharing must be a key factor in all asset price moments.

consumption correlation, we find interesting asymmetries in return correlations. In good states, the dividend stream with the lowest dividend-consumption correlation produces lower average correlations with other industries than the dividend stream with the highest dividend-consumption correlation. In bad states, the difference washes out. The industry with the lowest dividend-consumption correlation has only slightly higher expected excess returns and standard deviations in good states than the industry with highest dividend-consumption correlation. These differences increase, though, considerably in bad states. We test these cross-sectional predictions and find supportive evidence.

We emphasize the role of heterogeneity in risk aversion in explaining the cyclical dynamics of correlations and turnover volatility. For this channel to have a significant quantitative impact, the heterogeneity in risk aversion across consumers has to be quite large, at least ranging from 0.2 to 10. Guiso et al. (2011) provide survey based evidence, validated with actual data on portfolio choices, that is consistent with significant heterogeneity in risk aversion, namely ranging from 1 and 10. Based on their estimates, we find it plausible to use risk aversion outside of this range since the distribution is truncated and the frequencies show large weights on truncated values. Several other studies also report significant heterogeneity in risk aversion: Barsky et al. (1997) provide an estimate for risk aversion of 12.1 with a standard deviation of 16.6; Kimball et al. (2008) report 8.2 for the mean and 6.8 for the standard deviation of the distribution of individual consumers' relative risk aversion. Still, our preferred calibration succeeds in reproducing the quantitative cyclical dynamics of equity price moments at the expense of a high upper bound on risk aversion relative to most estimates in the literature, with the notable exception of Barsky et al. (1997). One potential resolution to this problem would be to introduce an additional source of heterogeneity into the model that correlates with preference heterogeneity, thereby allowing reducing the required heterogeneity in risk aversion. We leave this extension to future research.[2]

On a technical note, we solve the model through Monte Carlo simulations; yet our model is analytical in the sense that all quantities, including return correlations and portfolio volatility, can be expressed as Riemann integrals. The integrands are non-linear and partially solved by Newton's method; however, both Newton's method and Riemann integration allow for any desired level of precision. The Monte Carlo simulations allow going beyond the dimensionality hurdle imposed by multi-dimensional Riemann integration while still maintaining the advantage of using quasi-closed form solutions along simulation paths. The quasi-closed form solutions for return correlations and volatility and turnover volatility involve explicitly

[2] In an earlier version of the paper we allowed for heterogeneity in beliefs and time preferences in addition to risk aversion. If the most risk averse investor is also relatively pessimistic about aggregate dividend growth (of all or at least most trees), then the slope of the sharing rule steepens for a given degree of risk aversion heterogeneity as when risk aversion heterogeneity is heightened.

computable Malliavin derivatives (see Nualart (1995)) which were introduced to economics and finance in the work of Detemple and Zapatero (1991). Malliavin calculus is useful in our analysis as it allows characterizing the long-run effects of Brownian shocks. Finally, observe that our simulation-based approach for computing asset price dynamics and turnover or portfolio volatility allows for economic environments with a large numbers of assets.

Our paper combines four strands of the literature: Cochrane et al. (2008) study an economy in which a representative consumer with log preferences consumes the dividends or fruits of two Lucas trees. The representative fruit grower in Martin (2013) has power preferences and occasionally experiences that disaster hits the fruit crop in his Lucas orchard. In both models, dividend growth rates are i.i.d. over time. The growth rates in the economy of Menzly et al. (2004) instead take complicated forms to generate stationary dividend shares. We use simple processes for dividends to isolate the equilibrium impact of the risk aversion volatility effect but match average industry and aggregate dividends and their relation to aggregate consumption.

The paper relates to works studying the role of heterogeneous risk aversion in frictionless economies. Dumas (1989) studies risk sharing in a production economy with heterogeneous risk aversion, Wang (1996) analyzes the dynamics of the real interest rate yields in a Lucas economy, Bhamra and Uppal (2009) and Weinbaum (2009) examine the volatility of stock returns, Bhamra and Uppal (2011) derive closed form solutions for asset prices in an economy with heterogeneous preferences and beliefs, Cvitanic et al. (2012) examine equilibrium properties of an economy with differences in preferences and beliefs, Longstaff and Wang (2012) look at the role of leverage for asset prices. Chan and Kogan (2002) study an economy with "Catching up with the Joneses" preferences and show that such preferences lead to stationary asset price moments through a stationary wealth distribution, Garleanu and Panageas (2012) solve an overlapping generation's model with heterogeneous recursive preferences that also leads to a stationary wealth distribution.[3] Zapatero and Xiouros (2010) solve for the consumption sharing rule in closed form and compare the performance of the heterogeneous risk aversion model to Campbell and Cochrane (1999). Common for these papers is that they focus on the aggregate stock market, and hence do not model multiple Lucas trees. We extend this literature by studying correlations that require a cross-section of Lucas trees, by emphasizing the role of the volatility of aggregate risk aversion and by focusing on the joint implications of heterogeneous risk aversion on correlations, expected excess returns, standard deviations and turnover volatility.

Our research also relates to the literature that theoretically study stock return corre-

[3]Cvitanic and Malamud (2011) and Yan (2008) study the survival of agents, among other things, with heterogeneous risk aversion when there is no stationary wealth distribution.

lations. Dumas et al. (2003) find, in a representative consumer framework, that the level of correlations can be matched, and so there is no international excess correlation puzzle. Chue (2005) employs the Campbell and Cochrane (1999) model with a representative world consumer to study international equity correlations. Chue (2005) shows that the diversification benefits tend to be higher in times when stock return correlations are high as the representative world consumer values diversification more in bad times. Aydemir (2008) extends the model in Chue (2005) and contrasts correlations with perfect and imperfect risk sharing. Ribeiro and Veronesi (2002) analyze fundamental country processes that are jointly affected by an unobservable global business cycle factor. Time variation in correlations of asset returns arises from the learning activity of the representative consumer. Buraschi et al. (2011) studies correlation risk premium in a model with heterogeneous beliefs and multiple consumption goods. Pavlova and Rigobon (2008) study stock prices, exchange rates and the correlations of stock prices with multiple consumption goods. In their model, spill-over effects arise because of binding portfolio constraints. Chabakauri (2013) shows how portfolio constraints and heterogeneous risk aversion help to generate sizeable correlations in a two tree economy. Kyle and Xiong (2001) study the role of convergence traders on stock return correlations. In addition to proposing an alternative channel for return correlation dynamics, namely heterogeneous risk aversion, our work differs from the above papers as we quantitatively calibrate our model to unconditional asset pricing moments together with the dynamics of correlations over the business cycle. Moreover, our paper highlights the tight connection between stock return correlations and volatilities, expected excess returns and turnover volatility.

Although there is a large body of empirical literature devoted to the evidence on time variation in return correlations,[4] there is less recognition that such phenomena imply joint predictions on other related moments of financial security prices. Exceptions to this shortcoming include Lamoureux and Lastrapes (1990) who argue that trading volume has significant explanatory power for equity standard deviations; Tauchen and Pitts (1983) and Gallant et al. (1992) show that trading volume has a positive relation with volatility;[5] Longin and Solnik (1995), Moskowitz (2003), and Goetzmann et al. (2005) argue that correlations or covariances and standard deviations move together. Our paper sheds new light on the findings in this strand and related strands[6] of the empirical literature by providing one possible

[4]We refer the reader, for example, to Bollerslev et al. (1988), Erb et al. (1994), Ang and Chen (2001), Longin and Solnik (2001), Ledoit et al. (2003), Moskowitz (2003), Barberis et al. (2005), Goetzmann et al. (2005) and Chordia et al. (2011).

[5]It is well-documented in the market microstructure literature that volatility is associated with trading volume.

[6]Fama and French (1989) and Ferson and Harvey (1991) show that expected excess returns increase during economic contractions and peak near business cycle troughs. Harrison and Zhang (1999) and Campbell and

theoretical foundation for the joint cyclicality. Importantly, our empirical results relate correlations, expected excess returns, standard deviations and the volatility of turnover to each other in a way that cannot be read off the extant empirical literature.

2 The Economy

This section introduces a continuous-time exchange economy defined on the time span $[0, T]$, in which N risky securities and one locally risk-free security are traded.[7]

Risky securities pay dividends in the form of a single consumption good. Dividends follow

$$d\delta_i(t) = \delta_i(t)\left(\mu_{\delta_i} dt + \sigma_{\delta_i}^\top dW(t)\right), \quad \text{where} \quad \sigma_{\delta_i} \in R^N, \quad \delta_i(0) > 0, \quad i = 1, ..., N. \tag{1}$$

Let the positive definite $N \times N$ matrix σ_δ contain the diffusion terms of all dividend process. Similarly, the $N \times 1$ vector μ_δ contains the drift terms of each dividend process. The N-dimensional Brownian motion, W, defined on a filtered probability space $(\Omega, \mathcal{F}, P, \{\mathcal{F}_t\})$ completes the description of dividend dynamics.[8]

Define dividend shares, which are state variables, as

$$s_\delta(t) = (s_{\delta_1}(t), s_{\delta_2}(t), ..., s_{\delta_N}(t))^\top = \left(\frac{\delta_1(t)}{C(t)}, \frac{\delta_2(t)}{C(t)}, ..., \frac{\delta_N(t)}{C(t)}\right)^\top, \quad \text{where} \quad C(t) = \sum_{i=1}^N \delta_i(t) \tag{2}$$

By Ito's lemma, dividend share processes evolve according to

$$\begin{aligned} ds_{\delta_i}(t) &= s_{\delta_i}(t)\left(\left(\mu_{s_i}(t) - \sigma_{s_i}(t)^\top \sigma_C(t)\right) dt + \sigma_{s_i}(t)^\top dW(t)\right), \\ \text{where} \quad \mu_{s_i}(t) &= \mu_{\delta_i} - s_\delta(t)^\top \mu_\delta, \quad \sigma_{s_i}(t) = \sigma_{\delta_i} - \sigma_\delta^\top s_\delta(t). \end{aligned} \tag{3}$$

Aggregate consumption dynamics derive from the evolution equation of dividends

$$dC(t) = C(t)\left(\mu_C(t)dt + \sigma_C(t)^\top dW(t)\right), \quad \text{where} \quad \mu_C(t) = s_\delta(t)^\top \mu_\delta, \quad \sigma_C(t) = \sigma_\delta^\top s_\delta(t). \tag{4}$$

Diebold (2009) also conclude that expected excess returns are countercyclical. Schwert (1989) argues that stock market volatility is higher in recessions than in booms. Hamilton and Lin (1996) argue that stock market volatility and the business cycle comove.

[7] Our setup allows for finite and infinite horizon economies (provided that additional conditions are placed on the primitives of the economy).

[8] The probability space is defined over the time horizon $[0, T]$, where Ω is the state space, $\mathcal{F}$ denotes the σ-algebra, P represents the probability measure, and the information structure $\mathcal{F}_{(.)}$ is generated by the natural filtration with $\mathcal{F}_T = \mathcal{F}$. In the remainder, random variables appearing in equalities are in the almost surely sense relative to P.

2.1 Financial Markets

Stock price processes follow

$$dS_i(t) + \delta_i(t)dt = S_i(t)\left(\mu_i(t)dt + \sigma_i(t)^\top dW(t)\right), \quad \text{where} \quad S_i(T) = 0. \tag{5}$$

The locally risk-free asset, B, a zero net supply contract, evolves according to

$$dB(t) = r(t)B(t)dt, \quad \text{where} \quad B(0) = 1. \tag{6}$$

The set of price process coefficients μ_i, σ_i, and r derive from equilibrium conditions.

2.2 Consumers

Consumers derive utility over consumption through external habit preferences, similar to Abel (1990, 1999), Chan and Kogan (2002), and Zapatero and Xiouros (2010),

$$\begin{aligned} U_j\left(C,X\right) &= E_0\left[\int_0^T e^{-\rho t}u_j(C_j(t),X(t))dt\right], \qquad (7) \\ \text{where} \quad u_j\left(C_j(t),X(t)\right) &= \frac{1}{1-\gamma_j}C_j(t)^{1-\gamma_j}X(t)^{\gamma_j-\eta}, \quad \rho>0, \quad \eta \le min\left(\gamma_j\right) = \gamma_L \end{aligned}$$

and where u represents the instantaneous utility function, C_j stands for individual consumption rates, X denotes the external economy-wide living standard, γ measures the local curvature of u, i.e., the relative risk aversion parameter. Consumers either have low, $j = L$, or high risk aversion, $j = H$. The parameter η, which is common to all consumers, is set to ensure that the habit level is perceived as a negative externality by both consumer types.[9]

The economy-wide living standard evolves as in Chan and Kogan (2002) accordingly to

$$x(t) = x(0)e^{-\lambda t} + \lambda\int_0^t e^{-\lambda(t-u)}c(u)du, \quad \text{where} \quad x(t) = \log(X(t)), \quad c(t) = \log(C(t)). \tag{8}$$

In Equation 8, λ governs the dependency of x on past aggregate consumption.

With these assumptions, external relative habit, $\omega = c - x$, serves as another state variable besides the dividend shares. By Ito's lemma,

[9]External habit preferences are "neutral" to growth, i.e. consumers feel equally happy or unhappy when their consumption growth rate is high or low as consumption and habit level are cointegrated. This might be interpreted as a counterintuitive property of preferences. However, such a feature of preferences is consistent with trends of well-being over time, Blanchflower and Oswald (2004), and with the observation that measures of happiness such as "Happy" from the General Social Survey (GSS), http://www3.norc.org/GSS+Website/, appear stationary and do not trend up in lockstep with consumption.

$$d\omega(t) = \lambda\left(\overline{\omega}(t) - \omega(t)\right)dt + \sigma_C(t)^\top dW(t), \quad \text{where} \quad \overline{\omega}(t) = \frac{\mu_C(t) - \frac{1}{2}\sigma_C(t)^\top \sigma_C(t)}{\lambda}. \tag{9}$$

We emphasize that in states of nature with current consumption in excess (short) of recent past consumption, external relative habit is in excess (falls short) of its long-run mean. The procyclicality of ω allows to define booms (recessions) in our calibrations that are analogous, in a probabilistic sense, to booms (recessions) in the data.

2.3 Equilibrium

Conditional on endowments and preferences, equilibrium is a collection of allocations and prices such that individuals' consumption are optimal and markets clear. Complete markets allow to solve for the central planner problem in state by state and time by time form[10]

$$u(C(t), X(t), t) = \max_{C_L(t), C_H(t)} \left\{ \begin{array}{c} a e^{-\rho t} \frac{1}{1-\gamma_L} C_L(t)^{1-\gamma_L} X(t)^{\gamma_L - \eta} \\ +(1-a)\, e^{-\rho t} \frac{1}{1-\gamma_H} C_H(t)^{1-\gamma_H} X(t)^{\gamma_H - \eta} \end{array} \right\} \tag{10}$$

$$\text{s.t.} \quad C_L(t) + C_H(t) = C(t)$$

where a denotes the weight on consumer type L in the objective of the aggregate consumer.

Heterogeneous consumers optimally share consumption risk. The shape of the sharing rule depends on the degree of preference heterogeneity and the weight on consumers in the objective function of the aggregate consumer.

Proposition 1. *Pareto optimal consumption allocations are given by*

$$\begin{aligned} C_L(t) &= f(t)C(t) \quad \textit{and} \quad C_H(t) = (1 - f(t))\, C(t), \\ \textit{where} \quad f(t) &= \left(\frac{a}{1-a}\right)^{\frac{1}{\gamma_L}} e^{\left(\frac{\gamma_H}{\gamma_L} - 1\right)\omega(t)} \left(1 - f(t)\right)^{\frac{\gamma_H}{\gamma_L}}. \end{aligned} \tag{11}$$

It is apparent from Proposition 1 that the consumption share, f, only depends on the state variable ω. Moreover, f converges to zero when ω approaches minus infinity and to one when it approaches infinity. Therefore, the less risk averse consumer, L, dominates in very good states of nature, while the more risk averse consumer, H, dominates in bad states.

Aggregate risk aversion is defined as the consumption share weighted harmonic average of individual consumers' risk aversion, Bhamra and Uppal (2011).

[10] A sufficient condition for the market to be complete is that the stock price diffusion matrix is invertible for almost all states and times. We verify numerically that the market is complete for our choice of parameters. For general results on completeness in continuous time economies see Anderson and Raimondo (2008).

Proposition 2. *The coefficient of relative risk aversion and the relative prudence of the aggregate consumer are given by*

$$\begin{aligned}\mathcal{R}(t) &= \frac{1}{f(t)\frac{1}{\gamma_L} + (1-f(t))\frac{1}{\gamma_H}} \quad \textit{and} \qquad (12)\\ \mathcal{P}(t) &= (1+\gamma_L)\left(\frac{\mathcal{R}(t)}{\gamma_L}\right)^2 f(t) + (1+\gamma_H)\left(\frac{\mathcal{R}(t)}{\gamma_H}\right)^2 (1-f(t)).\end{aligned}$$

The above proposition shows that $\mathcal{R}$ is bounded in between γ_L and γ_H and that it is countercyclical: high in bad states and low in good states. Aggregate relative prudence, however, is not bounded in between the prudence of the two types of consumers inhabiting the economy.[11]

Proposition 3. *The diffusion coefficient of aggregate risk aversion is given by*

$$\mathcal{R}(t)\,(1+\mathcal{R}(t)-\mathcal{P}(t))\,\sigma_C(t). \qquad (13)$$

We stress that $\mathcal{P}$ considerably drives the volatility of aggregate risk aversion. The only way volatility of risk aversion remains constant as the economy evolves is when the aggregate relative prudence equals $1+\mathcal{R}$, i.e., the relative prudence with standard CRRA. In the current model this can only happen in the limit when one of the two consumer types dominates.[12]

Equilibrium quantities depend directly, or indirectly via aggregate risk aversion, on consumptions shares and external relative habit.

Proposition 4. *In equilibrium, the state price density is given by*

$$\xi(t) = e^{-\rho t}\left(\frac{f(t)}{f(0)}\right)^{-\gamma_L}\left(\frac{C(t)}{C(0)}\right)^{-1} e^{(1-\gamma_L)(\omega(t)-\omega(0))} \qquad (14)$$

and the risk-free rate is characterized through

$$r(t) = \rho + \eta\lambda\omega(t) + \mathcal{R}(t)\,(\mu_C(t) - \lambda\omega(t)) - \frac{1}{2}\mathcal{R}(t)\,\mathcal{P}(t)\sigma_C(t)^\top\sigma_C(t). \qquad (15)$$

Market prices of risk are given by

$$\theta(t) = \mathcal{R}(t)\,\sigma_C(t). \qquad (16)$$

From the proposition, we see that market prices of risk are determined by the product

[11] See Wang (1996) for a discussion of the consequences of this result for the risk-free interest rate.

[12] One can show that the diffusion coefficient of aggregate risk aversion is countercyclical but we were unable to find an analytical expression for its maximum.

of $\mathcal{R}$ with σ_C. Aggregate risk aversion depends only on the state variable ω via optimal consumption allocation, which generate excess or endogenous stock return correlations and volatilities. While the amount of risk in the economy, as measured by σ_C, depends on the other state variables, namely, the purely exogenous dividend shares.

The following two propositions introduce the key building blocks to equilibrium correlations and quadratic variation of portfolios in an economy driven by preference heterogeneity.

Proposition 5. *In equilibrium, the stock price diffusion processes are*

$$\sigma_i(t) = \frac{E_t\left[\int_t^T \xi(u)\delta_i(u)\left([\mathcal{R}(u)-\eta]\,\lambda\int_t^u e^{-\lambda(t-v)}\sigma_C(v)dv\right)du\right]}{E_t\left[\int_t^T \xi(u)\delta_i(u)du\right]} + \frac{E_t\left[\int_t^T \xi(u)\delta_i(u)\left(\theta(t)-\theta(u)\right)du\right]}{E_t\left[\int_t^T \xi(u)\delta_i(u)du\right]} + \sigma_{\delta_i}. \quad (17)$$

The diffusion processes depend on innovations in current and future $\mathcal{R}$ embedded in the habit process (first term), innovations between current and future θ (second term), and fundamentals or σ_{δ_i} (third term). The first two terms of Proposition 5 are known as discount factor news, related to habit and market prices of risk, while the last term is cash flow news.

Proposition 6. *Equilibrium wealth allocations, Y, and portfolio policies, π, are*

$$Y_j(t) = \frac{1}{\xi(t)}E_t\left[\int_t^T \xi(u)C_j(u)du\right] \quad \text{and} \quad \pi_j(t) = \left(\sigma(t)^\top\right)^{-1}\left[\theta(t)Y_j(t) + \frac{\psi_j(t)}{\xi(t)}\right], \quad (18)$$

where

$$\psi_j(t) = \left(\frac{1}{\gamma_j}-1\right)E_t\left[\int_t^T \xi(u)C_j(u)\mathcal{R}(u)\left[\sigma_C(u)-\lambda\int_t^u e^{-\lambda(t-v)}\sigma_C(v)dv\right]du\right] + (1-\eta)\,E_t\left[\int_t^T \xi(u)C_j(u)\left[\lambda\int_t^u e^{-\lambda(t-v)}\sigma_C(v)dv\right]du\right].$$

The above proposition illustrates that consumers' hedging terms, ψ_j,[13] depend on realizations of aggregate risk aversion and aggregate consumption volatility in the future. Hedging terms also relate to habit via its dependency on the path of aggregate consumption volatility (the integral between t and u).[14]

[13] See Cox and Huang (1989) and Detemple et al. (2003) and the references therein.

[14] In Detemple et al. (2003) the interest rate and market prices of risk drive the hedging terms. We can rewrite Proposition 6 in terms of these quantities. However, in equilibrium the interest rate and market prices of risk depend on aggregate risk aversion, aggregate prudence and consumption volatility.

2.4 Correlations, Returns, Volatilities, and Quadratic Variations in Portfolio Policies

We now explicitly compute conditional correlations and the quadratic variations of portfolio policies: Instantaneous equity returns, $dR(t)$, evolve according to

$$dR_i(t) = \frac{dS_i(t) + \delta_i(t)dt}{S_i(t)} = \mu_i(t)dt + \sigma_i(t)^\top dW(t). \tag{19}$$

The conditional variance, of security i's instantaneous return, is defined as follows

$$Var_t(dR_i(t)) = \|\sigma_i(t)\|^2 dt, \quad \text{where} \quad \|\sigma_i(t)\| = \sqrt{\sigma_i(t)^\top \sigma_i(t)}. \tag{20}$$

The conditional covariance between security i and k, with $k = 1, .., N$, is given by $Cov_t(dR_i(t), dR_k(t)) = \sigma_i(t)^\top \sigma_k(t)dt$. The conditional correlation[15] between i and k is

$$Corr_t(dR_i(t), dR_k(t)) = \frac{\sigma_i(t)^\top \sigma_k(t)}{\|\sigma_i(t)\| \, \|\sigma_k(t)\|}. \tag{21}$$

Since trading volume or turnover in continuous-time economies is infinite, we employ the quadratic variation or volatility of portfolio policies as a measure of trading intensity.[16] In general form, the "dollar" denominated dynamics of portfolio policies follow

$$d\pi_j(t) = \mu_{\pi_j}(t)dt + \sigma_{\pi_j}(t)^\top dW(t) \tag{22}$$

with drift and diffusion terms specified by Equation 18. The diffusion of the fraction of stock i held by consumer j is given by $\frac{\pi_{i,j}(t)}{S_i(t)}\sigma_{\frac{\pi_{i,j}}{S_i}}(t)$ where

$$\sigma_{\frac{\pi_{i,j}}{S_i}}(t) = \sigma_{\pi_{i,j}}(t) - \sigma_i(t). \tag{23}$$

Market clearing implies that $\frac{\pi_{i,L}(t)}{S_i(t)} + \frac{\pi_{i,H}(t)}{S_i(t)} = 1$. Applying Ito's lemma to both sides of the market clearing requirement leads to

$$\frac{\pi_{i,L}(t)}{S_i(t)}\sigma_{\frac{\pi_{i,L}}{S_i}}(t) + \frac{\pi_{i,H}(t)}{S_i(t)}\sigma_{\frac{\pi_{i,H}}{S_i}}(t) = 0. \tag{24}$$

[15] In the remainder, we use the terms "conditional correlation" and "correlation" interchangeably.

[16] Grossman and Zhou (1996) and Longstaff and Wang (2012), among others, also employ the quadratic variation of portfolio policies to measure trading intensity.

From Equation 24, we see that in an economy with two types of consumers $\frac{\pi_{i,L}(t)}{S_i(t)}\sigma_{\frac{\pi_{i,L}}{S_i}}(t)$ fully captures the degree of trading intensity in stock i. Hence,

$$RQV_t\left(d\left(\frac{\pi_{i,L}(t)}{S_i(t)}\right)\right) = \sqrt{\left(\frac{\pi_{i,L}(t)}{S_i(t)}\right)^2 \sigma_{\frac{\pi_{i,L}}{S_i}}(t)^\top \sigma_{\frac{\pi_{i,L}}{S_i}}(t)} \tag{25}$$

the relative quadratic variation, RQV, measures trade volatility in stock or portfolio i. Appendix B contains additional details regarding Equation 25.

We note that changing the asset structure in the economy will change the level of RQV. As long as assets or trees pay out dividends and the economy stays dynamically complete, the cyclical dynamics of RQV remain unaffected by the asset structure. For our purposes, only the salient cyclicality of RQV matters as we cannot quantitatively compare RQV to turnover volatility. There would be no trade and consequently no RQV, however, if assets pay out the optimal consumption stream of each consumer.

3 Correlations with Two Trees

This section presents numerical examples of the effects of risk aversion and heterogeneity in risk aversion on stock market correlations in a series of economies with two Lucas trees.[17] We solve the economies through Monte Carlo simulations. Details are found in Appendix C.

Equilibrium correlations vary as a function of the dividend share, s_{δ_1}, and the state of the economy. The state of the economy is identified with the consumption share of L consumers, f, where a low value of f indicates low aggregate consumption when consumer do not have habits and through external relative habit, ω, with ratio habit preferences.

Each economy contains two i.i.d. dividend processes with $\mu_\delta = 0.02$ and $\sigma_\delta = 0.05$.[18] We set starting values for dividends to 1, the maturity of the economy to 50 years, the weight on L consumers in the aggregate consumer objective to 0.5,[19] the rate for the time preference of both consumer types to 1%, η to 1, while the habit persistence, λ, is set at 0.1.[20]

[17] The Internet Appendix presents examples for the influence of other fundamentals on correlations.

[18] Annualized consumption expenditure grew by an average rate of 2 percent (continuously compounded) with a 3.5 percent standard deviation between 1890 and 2007 (Source: Robert Shiller's web page). Two i.i.d. dividend streams with $\mu_\delta = 0.02$ and $\sigma_\delta = 0.05$ roughly match these two moments.

[19] Heterogeneous consumer models usually set weights on consumers in the aggregate utility function. Changing the risk aversion of one of the consumers while keeping the weight on the consumers constant implies that endowments in the market portfolio change. Keeping endowments constant slightly alters the shape of correlations, and other equilibrium quantities, by moving the location of the peak. However, the maximum correlation in a particular economy is not affected by changing endowments or weights.

[20] As the goal of this section is to illustrate the economic mechanism, we use slightly different parameters than in the calibration. This is mainly due to the fact that here we use only two trees.

3.1 Homogeneous Preferences

We consider three economies populated by a representative consumer with (i) logarithmic utility, (ii) power utility with $\gamma = 3$, and (iii) power utility with $\gamma = 10$.

The top left plot in Figure 2 shows the correlation as a function of the dividend share, $s_{\delta_1}(0)$, for the economies i) - iii). Although dividend processes are uncorrelated, stock returns show positive correlation ranging from 0 to 0.029 when the aggregate consumer is logarithmic (blue dotted line). Ceteris paribus, correlations are higher the higher is risk aversion. For example, correlations almost double from 0.05 in the economy with a risk aversion coefficient of 3 (green dashed line) to 0.086 in the economy with a risk aversion of 10 (red solid line).[21]

What is the intuition for positive correlations? Consider an economy with uncorrelated dividends. The net effect of a positive shock to dividend 1 on the price and return of stock 1 is positive. The return has two components, a positive dividend effect and a negative discounting effect, i.e., the discount rate increases. The discount rate increases since investors require a higher expected return to compensate for the increase in exposure to stock 1 via its increased dividend. For other stocks, all else equal, the discounting effect is positive as their market prices of risk decrease. Hence, in equilibrium all stocks experience positive returns after one positive dividend shock to stock 1. We see this formally from applying Equation 16 to the economies in Figure 2; the application leads to the following expression for the market prices of risk

$$\theta_1(t) = \gamma s_{\delta_1}(t)\sigma_{\delta_1} \quad \text{and} \quad \theta_2(t) = \gamma \left(1 - s_{\delta_1}(t)\right)\sigma_{\delta_2}. \tag{26}$$

Cochrane et al. (2008) study in detail this market clearing mechanism with a logarithmic representative investor and two Lucas trees. We note that this effect is diminishing if there are many stocks and dividend correlations are low.

The top left plot in Figure 2 shows that the discounting effect for all returns, from one dividend shock, increases in risk aversion. What is the intuition for the level effect of γ? First, the larger the risk aversion, the larger is the negative discounting effect, and the smaller is the net return of the asset which experiences a positive dividend shock, first part of Equation 26. Second, the larger the risk aversion, the larger is the positive discounting effect, and the larger is the return on the second or other assets, second part of Equation 26. Hence, the

[21] In the economy iii) with $\gamma = 10$ the correlation is bimodal. This particular feature of the correlation is not due to the heightened risk aversion. The economies i) and ii) also exhibit this feature, although we cannot see it from the plot as the drop in correlations around $s_{\delta_1}(0) = 0.5$ is tiny. In an economy with higher dividend diffusion terms, σ_δ, the correlation will be visibly bimodal even in the logarithmic economy (Cochrane et al. (2008)). An explanation for the bimodal correlations, which are driven by tensions between current and future market prices of risk, is found in the Internet Appendix.

higher the risk aversion, the higher is the correlation between assets returns. Martin (2013) studies in detail an economy with three trees that are exposed to occasional disaster where the representative investor exhibits power preferences with sensitivity analysis for γ. The risk aversion level effect, though, cannot produce countercyclical correlations.

The top middle and top right plots of Figure 2 show the effect of the habit. For the logarithmic case, equilibrium is independent of the external standard of living.[22] Therefore, the correlation is identical to the correlation with logarithmic consumer without habits. The procyclical variation in the correlations —low correlations when the habit level is low and vice versa— in the economies ii) - iii) is inconsistent with the data. This shows that homogeneous risk aversion economies with external relative habit preferences fail to replicate the dynamics of correlations. Nevertheless, the habit significantly elevates correlations.

3.2 Heterogeneous Preferences

A consumer with low risk aversion is more willing to take on consumption risk than a consumer with high risk aversion. To do so, a consumer with low risk aversion invests his entire after-consumption wealth into the stock market plus borrows from a consumer with high risk aversion to further elevate his stock market allocation. This portfolio strategy shifts consumption from bad states of nature into good states of nature. Therefore, consistently with Proposition 7 in Appendix A, consumers with low risk aversion face higher consumption volatility than investors with high risk aversion. In the time series, when consumption falls, consumers with low risk aversion sell the market portfolio. Over the course of trading, stock prices fall as consumers with high risk aversion require higher expected returns.

To understand the equilibrium impact of buying during good times and selling in bad times by consumers with low risk aversion, consider the following risk aversion pairs: log $-$ 3 and log $-$ 10. The bottom left and middle plots in Figure 2 show the correlation in these heterogeneous preference economies. We see countercyclical stock market correlation as correlations are the larger the lower the realization of aggregate dividends or the consumption share of L consumers. We learn that the maximum correlation is higher in the log $-$ 10 economy. The magnitude of the effect is noteworthy, as correlations in the log $-$ 3 economy range roughly from 0.05 to 0.2 while in the log $-$ 10 economy correlations reach almost 0.8.[23]

Consumers with high risk aversion hold a larger fraction of the economy when aggregate consumption declines. Since aggregate risk aversion is consumption share weighted, it in-

[22] With $u(C,X) = log\left(\frac{C}{X}\right)$, marginal utility $u'(C,X) = \frac{1}{C}$ does not depend on the habit level X.

[23] In untabulated calibrations we find that the model with heterogeneous power preferences without habits qualitatively accounts for the empirical regularities shown in Table 4. In addition, the calibrated log $-$ 10 economy quantitatively accounts for the level and cyclicality of industry return correlations and volatilities.

creases with the share of H consumers, see top middle-left plot in Figure 3. Heightened risk aversion increases correlations due to the level effect of γ. Aggregate risk aversion, however, cannot explain the entire increase in correlations. For instance, correlations in the top left plot of Figure 2 with aggregate risk aversion set at 3 reach 0.05, while correlations in the bottom left plot of Figure 2 with risk aversion pair set at $\log - 3$ reach 0.2.

Why are correlations significantly larger in a $\log - 3$ economy than the correlations obtained in a homogeneous consumer economy with $\gamma = 3$? With heterogeneous consumers, the marginal utility of the aggregate consumer varies due to changes in aggregate consumption and in consumption shares of individual consumers. When small changes in consumption imply large changes in consumption shares, then endogenous variations, for example in correlations, are large. The variations are driven by the slope of the sharing rule. Specifically, the larger the difference in risk aversion, the higher is the maximum slope of the sharing rule. A steeper sharing rule implies higher volatility of aggregate risk aversion, which again implies more volatile market prices of risk. As aggregate risk aversion drives all market prices of risk correlations reach higher values in economies with more heterogeneity in risk aversion. We call this feature of the model the risk aversion volatility effect.[24]

The bottom left plot of Figure 3 shows correlations between stock 1 and stock 2 in an economy with risk aversion pairs $\log - 3$ (blue solid line). The plot also shows correlations between stock 1 and stock 2 in an economy with homogeneous risk aversion in which γ equals aggregate risk aversion from the heterogeneous consumer economy (green dashed line). This shows that the risk aversion level effect is diminishing the more heterogeneous the consumers are. Yet, the risk aversion level effect matters indirectly through aggregate risk aversion.

With ratio habit preferences, correlations range from below 0.4 to approximately 0.6 in the $\log - 3$ economy (not shown) and from 0.5 to almost 1.0 in the $\log - 10$ economy as can be seen from the bottom right plot of Figure 2.

3.3 On the Relation between Stock Return Correlations, Standard Deviations, Expected Returns and Turnover Volatility

We ask the model for testable implications for stock market return correlations that go beyond a mere countercyclicality.

[24] In the model, the mean and the variance of the market prices of risk are also countercyclical. Though, the variance of the market prices of risk is the key driver of the second moments of returns. However, countercyclical market prices of risk are neither sufficient nor necessary for countercyclical correlations. For instance, consider pricing dividend strips with maturity $\overline{T}$ and dynamics as in Equation 1 with an essentially affine stochastic discount factor driven by purely Gaussian state variables, then the second moments will be constant even with countercyclical market prices of risk.

The plots in Figure 3 show the fraction of aggregate consumption allocated to H consumers, the slope of the sharing rule, aggregate risk aversion, the derivative of aggregate risk aversion with respect to aggregate consumption, the correlation between stock 1 and stock 2, the expected excess return on the market portfolio, the standard deviation of the market and relative quadratic variation, RQV, of the most risk averse consumers' portfolio. We see from the top left plot that the consumption share of the consumers with high risk aversion decreases in aggregate consumption. The slope of the consumption sharing rule, middle-left top plot, reflects essentially the intensity of consumption shifts across consumers per unit of aggregate consumption. Note that the risk aversion of the representative consumer (top middle-right plot) and the sensitivity of the representative consumer's risk aversion to aggregate consumption shocks replicate the shape of the sharing rule and the slope of the sharing rule. That is, when the slope of the sharing rule is high, then the volatility of aggregate risk aversion is high. Further, stock return correlations (bottom left plot), expected excess returns of the market portfolio (bottom middle-left plot), return volatilities of the market portfolio (bottom middle-right plot), and relative quadratic variation of portfolio policies (bottom right plot) all peak jointly with the volatility of aggregate risk aversion. This can be understood in the following way; when the slope of the sharing rule is steep, then a small shock to aggregate consumption causes consumption allocations to shift by a large amount. The shift in consumption allocations leads to large changes in the risk aversion of the representative consumer, and consequently to large changes in the market prices of risk. Importantly, the volatility of the market prices of risk is high in times when the slope of the sharing rule is steep. The high volatility of market prices of risk leads to higher volatility of the equilibrium discount rates, and thus to more volatile stock returns. Note that all market prices of risk, as given by Equation 16, depend on aggregate risk aversion, and thus all discount factors become more volatile (high stock return volatility) and more correlated as all move with aggregate risk aversion. Expected excess returns increase due to higher volatility of stock returns (quantity of risk) and higher price of risk, which is driven by higher aggregate risk aversion. Finally, equilibrium trading intensifies in times when the slope of the sharing rule is steep or aggregate risk aversion is volatile. Hence, the model predicts joint cyclical dynamics for correlations, volatility, expected excess returns and trading volatility.

The plots in Figure 4 show the correlation between stock 1 and stock 2, the expected excess return on the market portfolio, the standard deviation of the market portfolio and the relative quadratic variation of stock 1 with ratio habit preferences. These plots show that habit preferences neither alter nor drive our predictions.

4 Calibration and Empirical Analysis

In this section we calibrate the model to 10 industry portfolios. We construct our sample from the CRSP files for the period January 1927 to December 2009. We employ all firms, surviving and non-surviving, that appear on CRSP. Monthly industry portfolios are derived from Kenneth French's industry classification. For each industry portfolio, we calculate aggregate dividends and returns. Dividends are adjusted for inflation using the consumer price index and for population growth using population estimates from the U.S. Census Bureau. Stock returns are adjusted for realized inflation. Aggregate per capita real consumption data are from Robert Shiller's website. The monthly nominal risk-free rate is obtained from Kenneth French's website. The volatility of the realized real interest rate is substantially higher than the volatility of the expected real rate, which is the right object to match. We compute expected inflation as the mean forecast from the Michigan Surveys of Consumers. Since expectations for inflation one year ahead from the Michigan Surveys of Consumers are only available starting from January 1978, we compute the ratio between the volatility of the expected real rate and the volatility of the realized real interest rate for the period January 1978 to December 2009. We use this ratio to adjust the volatility of the realized real interest rate over the period January 1927 to December 2009. The adjusted volatility of the expected real rate is 3.73% while the annualized volatility, from monthly data, of the realized real short rate is 6.48%.[25]

As we are interested in the average return correlation between the ten industry portfolios, it is convenient to set homogeneous parameters for industry dividends.[26] To match the empirical regularity that aggregate dividend growth is more volatile than, and imperfectly correlated with, per capita real consumption growth, we include another dividend stream in addition to the ten industries.[27] We set the dividend share of the eleventh tree to match the average dividend to consumption ratio over the period January 1927 to December 2009.[28] Table 1 contains the parameters for consumption and dividend processes in the data together

[25]Using data from the CRSP Risk Free Rates File instead of the rates from Kenneth French's website leads to almost identical results for the volatility of the expected and realized real rate. Using a Kalman filter assuming an AR(1) process implies that the volatility of the expected real rate is 4.28%.

[26]Formally, we assume $E_t\left[\frac{d\delta_i}{\delta_i}\right] = E_t\left[\frac{d\delta_j}{\delta_j}\right]$, $Var_t\left[\frac{d\delta_i}{\delta_i}\right] = Var_t\left[\frac{d\delta_j}{\delta_j}\right]$, $\frac{Cov_t\left[\frac{d\delta_i}{\delta_i},\frac{d\delta_j}{\delta_j}\right]}{Var_t\left[\frac{d\delta_i}{\delta_i}\right]} = \rho_\delta$, where $i \neq j$ in $Cov_t\left[.\right]$, for $i, j = 1, .., 10$ and $\delta_1(0) = \delta_2(0) = ... = \delta_{10}(0)$. This simplification reduces the number of sample paths required in the Monte Carlo simulations as we can exploit symmetry. A calibration based on industry dividend streams that match the mean and volatility of industry portfolio dividends and their correlations to the moments from the CRSP/Compustat files are available in the Internet Appendix.

[27]Aggregate consumption is then the sum of the eleven Lucas trees. Below we report moments of equilibrium quantities such as correlations based on the ten "industry trees."

[28]Expected growth of the eleventh tree is set at 0.019970 while its standard deviation is set at 0.029397.

with the corresponding values from the calibrations.[29] To illustrate the role played by preference heterogeneity in explaining the business cycle properties of stock return correlations and other asset pricing moments, we consider two different calibrations of the model: One calibration with homogeneous risk aversion and one with heterogeneous risk aversion. Our choices for the preference parameters are reported in Table 1.

We choose the persistence of the habit level, λ, to match the persistence of the price-dividend ratio in the data.[30] The risk aversion pair, log $-$ 30, together with the weight, a, are set to match unconditional asset pricing moments and changes of these moments over the business cycle. In the heterogeneous (homogeneous) consumer economy, η equals γ_L (1). The homogeneous risk aversion economy cannot jointly match expected excess returns and average industry correlations. We match correlations at the cost of too low returns. The subjective discount factor, ρ, is set to match the level of the risk-free rate.

4.1 Unconditional Asset Pricing Moments

Table 2 shows unconditional asset pricing moments of the calibrated models.[31] We stress that the heterogeneous and the homogeneous consumer economies reproduce the correlation and standard deviation of average industry returns. With the parameter values in Table 1, the homogeneous and the heterogeneous model generate a real risk-free rate of 1.1% that is comparable to that in the data. The heterogeneous risk aversion economy replicates the volatility of the risk-free rate, while in the homogeneous consumer model the volatility of the risk-free rate is too high. Further, the heterogeneous consumer economy generates higher excess returns, 6.6%, than the corresponding homogeneous consumer economy, 4.3%, while it still matches the other asset return moments. The autocorrelation of the price-dividend ratio is matched in both economies.

[29]Using five million paths, we find that after 100 years the aggregate dividend to aggregate consumption ratio ranges from 0.0451 for the lowest percentile to 0.1099 for the largest percentile. The maximum (minimum) industry dividend to aggregate consumption ratio ranges from 0.0070 (0.0029) for the lowest percentile to 0.0153 (0.0065) for the largest percentile.

[30]Our choice for the persistence of the habit is almost identical to the continuous time limit of the habit persistence in Campbell and Cochrane (1999). On a related point, the calibrated model implies a reasonable term structure for yield volatilities: 0.0363, 0.0355, 0.0345, 0.0323, 0.0312 (1- to 5-year maturity).

[31]Strictly speaking, we compute moments conditional on dividend shares, which are non-stationary with log-normal dividends. However, simulating the model over a 100 years period, allowing dividend shares to evolve over time, produces comparable results.

4.2 Conditional Correlations, Standard Deviations, Expected Excess Returns and the Short Rate

Our main goal is to study the theoretical and empirical conditional moments of equity correlations and its relations to the cyclical moments of expected returns and volatilities. In this regard, we compute the average industry return correlation, standard deviation, risk-free rate and expected excess return conditional on the NBER business cycle indicator.[32] Based on 211 recession months over the sample period January 1927 to December 2009 with a total of 996 months, the unconditional recession probability is 21.18%. To find a corresponding recession probability in the model, we simulate the distribution of ω from the calibrated model to back out the ω that correspond to the unconditional recession probability. In our calibration, this corresponds to a value of 0.1046 for ω. Table 3 shows that the average recession length in the data is roughly 14 months, while it is about 12.5 months in the model. The first order autocorrelation of the BCI in the data and the model are also similar with values of 0.8980 and 0.9126, respectively. We compute conditional asset pricing moments in recessions (booms) by integrating over all ω's less than (above) 0.1046.

Table 4 shows the results from this exercise. We see that the homogeneous consumer economy fails to replicate the changes in correlations, standard deviations and excess returns over the business cycle. This follows from the fact that in the homogeneous consumer economy the market price of risk is constant and volatility is procyclical. The heterogeneous consumer economy is capable of simultaneously reproducing changes in correlations, standard deviations and expected excess returns over the business cycle. In particular, one lesson we learn from the calibration exercise is that the model matches simultaneously unconditional and business cycle moments. Importantly, in the heterogeneous consumer economy the volatility of the risk-free rate also evolves countercyclically and the data support this prediction.[33] Taken together, these results suggest that the heterogeneous consumer economy outperforms the homogeneous consumer economy by a wide margin and that it replicates the cyclical dynamics of equity return correlations and standard deviations.

To further examine the model's ability to capture the dynamic properties of correlations, standard deviations and expected returns, we simulate 100 paths of 1000 months of prices from the model. On each path, we estimate a multivariate GARCH (DVEC(1,1)) and compute 3-year ahead average returns. Similarly, we estimate a multivariate GARCH using

[32] Expected excess returns at time t are measured as realized average return from t to $t+36$ (months). Results are robust to changes in the horizon (one and five years ahead), although the difference between booms and recessions shrinks at the five year horizon.

[33] The GARCH(1,1) volatility of the risk-free rate shows the following correlations: with industry stock market correlations (0.42), with expected excess returns (0.18), with standard deviations (0.41), with quadratic variations of industry turnover (0.68), and with calibrated external relative habit (-0.29).

the actual data. We run regressions of average correlations (Av. CORR), average 3-year ahead excess market returns (Av. EXR) and average standard deviations (Av. STDV) on external relative habit, ω, for returns from the data and the simulation. To calculate external relative habit in the data, we employ consumption data from 1889 to 2009. We assume that ω is in its steady state in 1889 to back it out from the data using the Euler discretization of its dynamics. As the frequency of consumption is annual, we interpolate ω to get a monthly series, which is shown in Figure 5. Figure 5 also shows that aggregate risk aversion increases during NBER recessions. Further, model implied aggregate risk aversion stays below 10 over the period January 1927 to December 2009 except at the end of the Great Depression. We note that changes in aggregate risk aversion, $\partial R(t)/\partial exp\left(\omega(t)\right)$, mirror the dynamics of ω.

Table 5 shows the results from the data and model based regressions. All regressions show the expected negative sign with highly significant coefficient estimates. Further, adjusted R-squared are similar in the model and the data with the exception of average correlations that shows somewhat lower R-squared in the data. Slope coefficients in the model exhibit similar magnitudes and are not significantly different from the coefficient in the data.

To shed further light on the ability of the model to replicate changes in correlations and standard deviations over the business cycle, we calculate model implied values using actual consumption data. We feed the model realized habit adjusted consumption over the period 1927 to 2009.[34] The left plot of Figure 5 shows model implied correlation and the data.[35] The model matches the level of the average industry return correlation over the period well. Importantly, the model replicates the high correlation levels during the early thirties and the 2007-2009 recession. The correlation coefficient between model implied average industry correlation and the data equals 0.4, which appears high given that we only employ aggregate consumption data in the calibrated model. The right plot of Figure 5 shows the model implied standard deviation together with the data. The model is successful in capturing the spike in volatility during the early thirties and the increased volatility over 2007-2009. The correlation coefficient between the model implied standard deviation and the data is 0.73.[36]

Further, we investigate the sensitivity of equilibrium to perturbations in preference heterogeneity. Table 6 shows results for four different economies where we set γ_H at 10, 15,

[34]It is convenient to keep dividend shares fixed over the entire period.

[35]As external relative habit, ω, has annual frequency, we calculate the annual correlation as the within year average correlation based on the monthly GARCH estimates. We follow the same procedure for the standard deviation. Alternatively, we use monthly correlations together with interpolated external relative habit. The resulting correlation between monthly model implied correlations and the data is 0.32. The estimation is robust to using exponentially weighted moving average (EWMA) to estimate correlations and standard deviations; the resulting correlation between the data and model using monthly frequency is 0.36.

[36]The correlation coefficient between model implied expected excess return of the market and the three year forward looking excess return of the market is 0.34. The corresponding correlation coefficients for the realized real risk-free rate and the price dividend ratio are 0.31 and 0.23, respectively.

20, and 40 scattered around our preferred economy with $\gamma_H = 30$. To penalize the models ability to match unconditional and conditional moments, we keep all parameters at their benchmark values except γ_L and the weight a on L consumers. The four economies around the benchmark economy are calibrated to replicate a weighted average of the risk aversion and the prudence of the representative consumer in the benchmark economy for values of ω ranging from 0.1 to 0.2.[37] The second panel in Table 6 shows, unsurprisingly, that the higher γ_H, the higher are the equity premium, the standard deviation of the market, and the average correlation. However, even the economy with $\gamma_H = 10$ produces an equity premium of 3.7%. The third panel in Table 6 shows the conditional average correlation, expected excess return, standard deviation, and real risk-free rate in recessions and booms. Again, even when we use $\gamma_H = 10$, the model delivers a substantial increase in all conditional moments relative to the homogeneous consumer economy.

We perform a series of checks to confirm the models ability to replicate the cyclical dynamics of correlations. These include: we carry out extensive principal component analysis to show that ω explains the first principal component of correlations and the other financial series of interest, we replace ω in the regressions with the price-dividend ratio; we regress model implied external relative habit onto average correlations, expected excess returns, and standard deviations of size, book-to-market and momentum sorted portfolios; we perform standard in-sample and out-of-sample predictive regressions using model implied external relative habit as explanatory variable that extend the Av. EXR regressions in Table 5; and we compare the distribution of aggregate risk aversion and prudence from our two consumer setting to the case with a continuous distribution of consumer types with a generalized Beta distribution. All checks corroborate the models' ability to justify the cyclical dynamics of correlations. They can be found in the Internet Appendix accompanying this paper.

4.3 Dividend-Consumption Correlations and the Sensitivity to Risk Aversion

In this subsection we explore cross-sectional heterogeneity in the dividend-consumption correlation. In the data, there is considerable heterogeneity across the correlations of industry dividend growth rates and consumption growth. Specifically, from Table 7 we see that dividend-consumption correlation ranges from -0.01 (Telecom) to 0.53 (Manufacturing). Accordingly, we set the dividend-consumption correlation equally spaced in the range $[0, 0.5]$. All other parameters remain unchanged.

[37]We put a weight of 70% on matching aggregate risk aversion and 30% on matching aggregate prudence.

Figure 6 shows the average correlation with the other nine industry returns, the standard deviation and the expected excess returns of the two dividend streams with 0 and 0.5 correlation with aggregate consumption as a function of ω. Correlations, standard deviations and expected excess returns for the two industries show different degrees of asymmetry. In good states, the dividend stream with low correlation with aggregate consumption has lower average correlations with other industries than the dividend stream with high dividend-consumption correlation. In bad states, the difference washes out and average correlations reach 0.9. For return standard deviations and expected excess returns, the dividend stream with the higher dividend-consumption correlation shows higher volatility and expected excess returns and the difference widens in bad states. The intuition for the asymmetry in correlations is that for the dividend stream with low correlation with aggregate consumption or the discount factor, most of the variation in good times is explained by the dividend volatility. In bad times, the volatility of aggregate risk aversion increases significantly and for all stocks most of the variation is attributable to the common component or aggregate risk aversion. The increase in volatility in the bad states is mainly through the volatility of risk aversion and not through covariance between the dividends and the discount factor for the stock with low dividend-consumption correlation. Though, for the stock with high dividend-consumption correlation, the increase in volatility is due to the discount factor and higher covariance between dividends and the discount factor and, consequently, the increase in bad states is higher than for the dividend stream with low dividend-consumption correlation. The expected excess return is mostly driven by the higher volatility of stock returns.

We run three sets of regressions to verify whether the asymmetries in the model are present in the data: i) average correlation of industry i with the nine other industries on ω, ii) standard deviation of industry i on ω and iii) excess return over the next three years of industry i on ω. From the top panel in Table 7 we see that on average industries with low (high) dividend-consumption correlation exhibit return correlations with higher (lower) sensitivity to ω and standard deviations and expected excess returns that have lower (higher) sensitivity to ω, just as in Figure 6. To compare the data with the model, we simulate 100 paths of 1000 months from the model. The results are reported in the lower panel in Table 7. Overall, the model replicates the cross-section in the data. To test if the difference in dividend-consumption correlation drives the cross-section, we regress the slope coefficients of the ten industries onto the dividend-consumption correlations. The results are presented in the Table 8. From the table, we see that slope coefficients for correlations, standard deviations and expected excess returns are all significant and show the same sign as that predicted by the model. However, the cross-sectional difference within the model is lower than that of the data.

4.4 Volatility of Trading Volume

We now turn to the relation between correlations and trading activity. As we illustrated in Section 3.3, the intensity of trade peaks when correlations peak. Moreover, this also coincides with high volatility and high expected excess returns. As a measure of trading intensity we calculate a GARCH(1,1) of the log changes of turnover of the market portfolio. This measures the volatility of turnover and is our empirical counterpart to the quadratic variation of the portfolio policies in Equation 25. The correlation between the trading intensity and average industry correlation, standard deviations and expected excess returns are 0.52, 0.39 and 0.16. Regressing turnover volatility onto the backed out habit yields -0.6031 for the coefficient estimate with a Newey-West corrected t-statistics of -6.5560 and adjusted R-squared of 0.1946. These estimates are in line with our results in Table 5 and are an important piece of evidence in support of the model.

5 Conclusions

We study stock market correlations in an equilibrium model with heterogeneous risk aversion. We show that stock return correlations are countercyclical and comove with stock return volatilities, expected excess returns, and volatility of turnover. The main intuition for the dynamics of stock return correlations comes from the optimal risk sharing between consumers with different risk aversion. After a negative aggregate consumption shock, less risk averse consumers sell stocks or the stock market to consumers with higher risk aversion, and this creates a downward price pressure on the stock market as the marginal consumer-investor becomes more risk averse. Importantly, in bad times small shocks to aggregate consumption lead to large changes in the risk aversion of the marginal consumer, thus we see a large increase in the volatility of risk aversion. This has important implications for stock return correlations, expected excess returns and volatilities and the volatility of turnover. Specifically, the volatility of aggregate risk aversion is a common factor in any discount rate and, therefore, we see higher stock return correlations and volatilities in times when volatility of risk aversion is high. Expected returns increase both due to higher compensation for risk as the marginal investor becomes more risk averse and due to the increased volatility of stock returns. In addition, we also see a higher intensity of trading during bad times as the required shift in consumption allocations to equate marginal utilities of agents with different risk aversion is higher in bad times.

Empirically, we show that correlations are countercyclical and that they comove with stock return volatilities, expected excess returns and volatility of turnover. We also show that the model matches the dynamics of stock return correlations and the relation between stock return correlations, volatilities and expected excess returns in addition to matching unconditional moments.

The model gives some advice for portfolio choice and risk management problems. Investors, portfolio managers and risk managers might ask, "Are stock return correlations significantly different over the business cycle?" The data and the model seem to suggest that the answer is "yes." In fact, the model in this paper suggests something else, namely that the affirmative answer to the above question is difficult to interpret in isolation. Yes, correlations and volatilities rise in recessions; recessions, fortunately, forecast high returns. Therefore, it is important to understand how correlations vary jointly with expected stock returns, volatilities and other potentially relevant time series.

We believe that there is need to carry on follow up research in this area: For example, in our model it is not possible to separate the elasticity of intertemporal substitution from risk aversion. Therefore, it is conceivable that heterogeneity in elasticity of intertemporal substitution instead of risk aversion drive the results in our paper. While we think that both sources of heterogeneity matter in equilibrium, it appears that risk aversion is more important for correlations and volatilities. Indeed, the results in Garleanu and Panageas (2012) point towards an exclusive role of heterogeneity in risk aversion as determinant of asset price volatilities. In their model, heterogeneity in elasticity of intertemporal substitution, instead, helps to reduce the volatility of the risk-free rate. In the end, our model setting is not suited to help answer the question how heterogeneity in risk aversion might interplay with heterogeneity in elasticity of intertemporal substitution and possibly with external relative habit. We leave this important extension to future research.

A Individual Consumption Dynamics

Proposition 7. *Consumers' consumption dynamics evolve according to*

$$\begin{aligned} dC_j(t) &= C_j(t)\left(\mu_{C_j}(t)dt + \sigma_{C_j}(t)^\top dW(t)\right), \qquad \text{(A.1)} \\ \textit{where} \quad \mu_{C_j}(t) &= \left(\frac{\mathcal{R}(t)}{\gamma_j}\right)\mu_C(t) + \left(1 - \frac{\mathcal{R}(t)}{\gamma_j}\right)\lambda\omega(t) \\ &\quad + \frac{1}{2}\left[(1+\gamma_j)\left(\frac{\mathcal{R}(t)}{\gamma_j}\right) - \mathcal{P}(t)\right]\left(\frac{\mathcal{R}(t)}{\gamma_j}\right)\sigma_C(t)^\top\sigma_C(t), \\ \sigma_{C_j}(t) &= \left(\frac{\mathcal{R}(t)}{\gamma_j}\right)\sigma_C(t). \end{aligned}$$

From Proposition 7, we see that diffusion processes of individual consumption growth depend on the ratio of aggregate relative risk aversion to individual relative risk aversion. As aggregate risk aversion is bounded in between the risk aversion of the two consumer types, consumers with low risk aversion have higher consumption volatility than consumers with high risk aversion.

B Quadratic Variation of Portfolio Policies

We compute the diffusion coefficients of portfolio policies from its dynamics given by

$$d\pi_j(t) = \mu_{\pi_j}(t)dt + \sigma_{\pi_j}(t)dW(t). \qquad \text{(B.1)}$$

Below we ignore dt terms as we are only interested in the local volatility of portfolio policies: the $N \times N$ matrix σ_{π_j}. Recall optimal portfolio policies from Proposition 6:

$$\pi_j(t) = \left(\sigma(t)^\top\right)^{-1}\left[\theta(t)Y_j(t) + \frac{\psi_j(t)}{\xi(t)}\right]. \qquad \text{(B.2)}$$

From Equation B.2, the optimal portfolio policy is a function of stock price diffusion coefficients (σ), market prices of risk (θ), wealth (Y_j), hedging terms (ψ_j) and the state price density (ξ). Thus, if we know the dynamics of these quantities we can calculate $\sigma_{\pi_j}(t)$ by a standard application of Ito's lemma to Equation B.1. The dynamics of ξ can be found in the proof of Proposition 6 in the Internet Appendix. Wealth dynamics are given by

$$dY_j(t) = \left(r(t)Y_j(t) + \pi_j(t)^\top\left(\mu(t) - r(t)I\right) - C_j(t)\right)dt + \pi_j(t)^\top\sigma(t)dW(t). \qquad \text{(B.3)}$$

Hence, we have that $\sigma_{Y_j} = \pi_j(t)^\top \sigma(t)$. The dynamics of $\theta(t)$, $\sigma(t)$ and $\psi_j(t)$ are given by

$$\begin{aligned} d\theta(t) &= \mu_\theta(t)dt + \sigma_\theta(t)dW(t), &\text{(B.4)}\\ d\sigma_i(t) &= \mu_{\sigma_i}(t)dt + \sigma_{\sigma_i}(t)dW(t), &\text{(B.5)}\\ d\psi_j(t) &= \mu_{\psi_j}(t)dt + \sigma_{\psi_j}(t)dW(t), &\text{(B.6)}\end{aligned}$$

where σ_θ, σ_{σ_i} and σ_{ψ_j} are $N \times N$ matrices. Recall the market prices of risks: $\theta(t) = \mathcal{R}(t)\sigma_C(t)$. Thus, we need to compute the dynamics of $\mathcal{R}(t)$ and $\sigma_C(t)$. We obtain

$$\begin{aligned} d\mathcal{R}(t) &= d\left(-\frac{u_{CC}\left(C(t),X(t),t\right)}{u_C\left(C(t),X(t),t\right)}C(t)\right)\\ &= [.]\,dt + \left(\left(\frac{u_{CC}\left(C(t),X(t),t\right)}{u_C\left(C(t),X(t),t\right)}\right)^2 C(t) - \frac{u_{CC}\left(C(t),X(t),t\right)}{u_C\left(C(t),X(t),t\right)} - \frac{u_{CCC}\left(C(t),X(t),t\right)}{u_C\left(C(t),X(t),t\right)}C(t)\right)\\ &\quad \times C(t)\sigma_C^\top dW(t)\\ &= [.]\,dt + \mathcal{R}(t)\left(1+\mathcal{R}(t)-\mathcal{P}(t)\right)\sigma_C^\top dW(t); \qquad \text{(B.7)}\end{aligned}$$

$$\begin{aligned} d\sigma_C(t) &= d\left(\sigma_\delta^\top s_\delta(t)\right)\\ &= \sigma_\delta^\top ds_\delta(t)\\ &= \sigma_\delta^\top I_{s_\delta}(t)\left(\nu_s(t)dt + \sigma_s(t)dW(t)\right)\\ &= \mu_{\sigma_C}(t)dt + \sigma_{\sigma_C}(t)dW(t) \qquad \text{(B.8)}\end{aligned}$$

where

$$\mu_{\sigma_C}(t) = \sigma_\delta^\top I_{s_\delta}(t)\nu_s(t) \quad \text{and} \quad \sigma_{\sigma_C}(t) = \sigma_\delta^\top I_{s_\delta}(t)\sigma_s(t) \qquad \text{(B.9)}$$

and where

$$ds_\delta(t) = I_{s_\delta}(t)\left(\nu_s(t)dt + \sigma_s(t)dW(t)\right) \quad \text{with} \quad \nu_s(t) = \begin{bmatrix} \mu_{s_1}(t) - \sigma_{s_1}(t)^\top \sigma_C(t) \\ . \\ . \\ \mu_{s_N}(t) - \sigma_{s_N}(t)^\top \sigma_C(t) \end{bmatrix}, \qquad \text{(B.10)}$$

which is the compact form of Equation 4. Thus, by Ito's lemma we have that

$$\begin{aligned} d\theta(t) &= d\left(\mathcal{R}(t)\sigma_C(t)\right)\\ &= [.]\,dt + \left[\mathcal{R}(t)\left(1+\mathcal{R}(t)-\mathcal{P}(t)\right)\sigma_C(t)\sigma_C(t)^\top + \mathcal{R}(t)\sigma_{\sigma_C}(t)\right]dW(t). \qquad \text{(B.11)}\end{aligned}$$

From Proposition 5, the stock price diffusion coefficients are

$$\sigma_i(t) = \theta(t) + \frac{Q_i(t)}{\xi(t)S_i(t)} + \sigma_{\delta_i} \quad \text{where} \quad Q_i(t) = E_t\left[\int_t^T \xi(u)\delta_i(u)h(t,u)du\right]. \tag{B.12}$$

Next we rewrite Q_i as

$$\begin{aligned} Q_i(t) &= E_t\left[\int_0^T \xi(u)\delta_i(u)h(t,u)du\right] - \int_0^t \xi(u)\delta_i(u)h(t,u)du \\ &= Q_i^M(t) - \int_0^t \xi(u)\delta_i(u)h(t,u)du \end{aligned} \tag{B.13}$$

where Q_i^M stands for the martingale part of Q_i. Clark-Ocone's theorem renders

$$dQ_i^M(t) = \sigma_{Q_i}(t)dW(t) \quad \text{where} \quad \sigma_{Q_i}(t)^\top = E_t\left[\int_0^T D_t\left(\xi(u)\delta_i(u)h(t,u)^\top\right)du\right], \tag{B.14}$$

in which $\sigma_{Q_i}(t)$ is a $N \times N$ matrix. Calculating the Malliavin derivative, leads to

$$\begin{aligned} D_t\left(\xi(u)\delta_i(u)h(t,u)^\top\right) &= \delta_i(u)h(t,u)^\top D_t\xi(u) + \xi(u)h(t,u)^\top D_t\delta_i(u) + \xi(u)\delta_i(u)D_t h(t,u)^\top \\ &= \xi(u)\delta_i(u)h(t,u)h(t,u)^\top + \xi(u)\delta_i(u)\sigma_{\delta_i}h(t,u)^\top \\ &\quad +\xi(u)\delta_i(u)g(t,u). \end{aligned} \tag{B.15}$$

Using the above expressions and the following relation

$$d\left(\xi(t)G_i(t)\right) = \xi(t)S_i(t)\left(\sigma_i(t) - \theta(t)\right)^\top dW(t) \tag{B.16}$$

leads to

$$\begin{aligned} d\sigma_i(t) &= [.]\,dt + \left(\sigma_\theta(t) + \sigma_{Q_i}(t) + (\theta(t) - \sigma_i(t))\,(\theta(t) - \sigma_i(t))^\top\right)dW(t) \\ &= [.]\,dt + \sigma_{\sigma_i}(t)dW(t). \end{aligned} \tag{B.17}$$

Next we derive the diffusion coefficient of ψ_j. We have that

$$\psi_j(t) = E_t\left[\int_0^T \xi(u)C_j(u)\left(h(t,u) + H_j(t,u)\right)du\right]. \tag{B.18}$$

Factoring out the martingale part of Equation B.18 and using the Clark-Ocone theorem yields

$$\begin{aligned}
\sigma_{\psi_j}(t)^\top &= E_t\left[\int_0^T D_t\left(\xi(u)C_j(u)\left(h(t,u)+H_j(t,u)\right)\right)\right] \\
&= E_t\left[\int_t^T \left\{\xi(u)C_j(u)h(t,u)h(t,u)^\top + \xi(u)C_j(u)g_j(t,u)\right\}du\right] \\
&\quad + E_t\left[\int_t^T \left\{\xi(u)C_j(u)h(t,u)H_j(t,u)^\top + \xi(u)C_j(u)G_j(t,u)\right\}du\right]. \quad \text{(B.19)}
\end{aligned}$$

C Technical Details of Monte Carlo Simulations

Below, we describe the numerical procedures employed to solve for equilibrium quantities.

The model is solved using Monte-Carlo simulations. State variables are simulated forward using an Euler scheme with 10,000 sample paths and 1,000 time steps. We use antithetic sampling to reduce sampling variance. For each time step, we compute optimal allocations of external habit adjusted consumption between the two consumers by solving the problem in Equation 10. Specifically, the sharing rule or consumption share f in Proposition 1 is solved by Newton's method. Derivatives of optimal allocations are computed using finite differences. Time integrals are calculated by using the trapezoid rule with 1,000 steps.

We benchmark our code in the following way: The two trees version of the model with logarithmic utility function is available in closed form, Cochrane et al. (2008). Further, we solve 2, 3 and 4 stock versions of our economy with quadrature methods. First, our three codes (closed form, quadrature, Monte-Carlo) yield identical results in a two-stock economy when preferences are logarithmic and consumers do not exhibit habits. Second, our Monte-Carlo simulations coincide with the quadrature method for 2, 3 and 4 stock economies. Especially, Monte-Carlo simulations and the quadrature methods imply and yield the same results independently of the assumed pair of risk aversion and the choice of the utility functions (standard power preferences versus power preferences with external relative habit).

Monte-Carlo errors are not crucial for our purposes. Nevertheless, we have checked whether model implied quantities such as the expected excess return, volatilities, correlations or the risk-free rate vary substantially across runs. Overall, we find that model implied statistics are accurately estimated by the Monte-Carlo simulations.

Finally, all plots reported in the paper are based on 1,000 sample paths since increasing the size of the Monte-Carlo simulations makes no difference.

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Table 1: **Calibration — Parameters.** This table summarizes real and population adjusted moments of consumption, aggregate industry dividends, and average industry dividends as well as relations between consumption, aggregate dividends, and average industry dividends for the period January 1927 to December 2009. The table also shows corresponding moments from a heterogeneous and a homogeneous consumer model calibration and contains in the bottom panel the preference parameters employed in the heterogeneous and the homogeneous consumer model calibration. The utility weight, a, of 0.02 is equivalent to a wealth share of 0.074 in the steady state.

	Data	Model	
		Heterogeneous	Homogeneous
Consumption and Dividend Moments			
Mean consumption growth	0.020	0.020	0.020
Standard deviation of consumption growth	0.030	0.030	0.030
Mean aggregate dividend growth	0.024	0.024	0.024
Standard deviation of aggregate dividend	0.082	0.082	0.082
Consumption-aggregate dividend correlation	0.539	0.539	0.539
Aggregate dividend-consumption ratio	0.030	0.030	0.030
Average industry mean dividend growth	0.024	0.024	0.024
Average standard deviation of industry dividends	0.133	0.148	0.148
Average industry dividend correlation	0.229	0.229	0.229
Preference Parameters			
Risk aversion, high (γ_H)	-	30	7
Risk aversion, low (γ_L)	-	1	7
Utility weight (a)	-	0.020	-
Subjective discount factor (ρ)	-	0.03	0.010
Habit curvature parameter (η)	-	1	1
Degree of habit history dependence (λ)	-	0.130	0.130

Table 2: **Calibration — Asset Pricing Moments.** This table summarizes real moments of the market portfolio, average industry portfolio, risk-free rate, and price-dividend ratio for the period January 1927 to December 2009 and corresponding moments from a heterogeneous and a homogeneous consumer model calibration. Returns are annualized from monthly frequency. Model risk-free rate is $r(t)$, see Equation 15, while the expected excess return of the market portfolio is $\mu_M(t)-r(t)$, see Equation 5, where M denotes the market portfolio. ACF(1) denotes the first-order autocorrelation coefficient.

	Data	Model	
		Heterogeneous	Homogeneous
Expected excess return of the market portfolio	0.079	0.066	0.043
Standard deviation of the market portfolio	0.190	0.238	0.215
Average industry correlation	0.719	0.744	0.730
Risk-free rate	0.006	0.011	0.011
Standard deviation of risk-free rate	0.038	0.038	0.045
Average price-dividend ratio of the market portfolio	30.0	39.1	69.4
Standard deviation of log price-dividend ratio of the market portfolio	0.412	0.364	0.315
ACF(1) of the log price-dividend ratio of the market portfolio	0.874	0.871	0.877

Table 3: **Calibration — BCI properties.** This table shows the unconditional recession probability, the average length of recessions and booms in months, and the first order autocorrelation of the BCI in the data and the model. The data is based on the NBER business cycle indicator. To find a corresponding recession probability in the model, we simulate the distribution of ω from the calibrated model to back out the ω that correspond to the unconditional NBER recession probability. We then record the average recession time based on the threshold for ω corresponding to the unconditional recession probability.

	Data	Model
Recession probability	0.21	0.21
Average duration of recessions (months)	14.07	12.47
Average duration of booms (months)	52.33	46.11
ACF(1) BCI	0.898	0.913

Table 4: **Calibration — Asset Pricing Moments over the Business Cycle.** This table summarizes unconditional and conditional (booms and recessions) moments of three year ahead excess return and standard deviation of the market portfolio, average industry portfolio correlation, and risk-free rate for the period January 1927 to December 2009 and corresponding moments from a heterogeneous and a homogeneous consumer model calibration. Returns are annualized from monthly frequency. Model risk-free rate is $r(t)$, see Equation 15, while the expected excess return of the market portfolio is $\mu_M(t) - r(t)$, see Equation 5, where M denotes the market portfolio. In the data, recessions are defined by NBER recession dates. In the calibrated model, recessions have the same unconditional probability as in the data.

	Data	Model	
		Heterogeneous	Homogeneous
Expected Excess Return of Market			
Average	0.079	0.066	0.043
Boom	0.071	0.047	0.043
Recession	0.110	0.136	0.042
Recession minus boom	0.040	0.089	-0.001
Standard Deviation of Market			
Average	0.190	0.238	0.215
Boom	0.156	0.211	0.216
Recession	0.280	0.335	0.211
Recession minus boom	0.124	0.124	-0.004
Average Industry Correlation			
Average	0.719	0.744	0.730
Boom	0.655	0.711	0.732
Recession	0.812	0.864	0.723
Recession minus boom	0.157	0.153	-0.009
Risk-Free Rate			
Average	0.006	0.011	0.011
Boom	-0.001	-0.005	-0.007
Recession	0.031	0.065	0.071
Recession minus boom	0.032	0.070	0.077

Table 5: **Calibration — Regressions.** This table summarizes OLS regression results (intercept, coefficient estimate and adjusted R-squared) of model implied external relative habit as explanatory variable for average of industry market correlations (Av. CORR), 3-year ahead expected excess returns (Av. EXR), and standard deviations (Av. STDV). Newey-West corrected t-statistics are in parentheses. Correlations and standard deviations are estimated using a DVEC(1,1) model. Model implied external relative habit is linearly interpolated from the heterogeneous consumer model calibration employing annual consumption data from Robert Shiller's web page. The regressions in the data columns use 996 monthly observations with data ranging from January 1927 to December 2009. The regressions in the model are based on the parameters in Table 1. We simulate 100 paths, each with 1000 monthly observations. For every path we calculate the average correlations, 3-year ahead expected excess returns and standard deviations. The reported results are averages over the 100 sample paths.

	Av. CORR		Av. EXR		Av. STDV	
	Data	Model	Data	Model	Data	Model
Intercept	0.7649	0.7989	0.1866	0.1203	0.3433	0.3099
	(28.8627)	(154.6527)	(6.4946)	(2.8024)	(8.2471)	(30.6899)
Model implied external habit, ω	-0.4574	-0.3052	-0.6740	-0.5775	-0.9200	-0.5186
	(-2.8642)	(-7.9585)	(-4.0504)	(-2.2547)	(-3.8619)	(-8.4881)
Adjusted R-squared	0.1317	0.3155	0.1254	0.0739	0.4292	0.4784

Table 6: **Calibration — Alternative Risk-Aversion Pairs.** This table summarizes unconditional and conditional moments of three year ahead excess return and the standard deviation of the market portfolio, average industry portfolio correlation, and the risk-free rate for the period January 1927 to December 2009 and corresponding moments from five heterogeneous investor model calibrations. Each model uses the parameters from Tables 1 - 2 except for the parameters from panel Preference Parameters. Returns are annualized from monthly frequency. Model risk-free rate is $r(t)$, see Equation 6, while the expected excess log-return of the market portfolio is $\mu_M(t) - r(t)$, see Equation 5, where M denotes the market portfolio. In the data, recessions are defined by NBER recession dates. In the calibrated model, recessions have the same unconditional probability as in the data.

	Data	Model				
Preference Parameters		1	2	3	4	5
Risk aversion, high (γ_H)	-	10	15	20	30	40
Risk aversion, low (γ_L)	-	0.227	0.498	0.734	1.000	1.133
Utility weight (a)	-	0.070	0.043	0.030	0.020	0.024
Habit curvature parameter (η)	-	0.227	0.498	0.734	1.000	1.133
Unconditional Moments						
Expected excess return of market	0.079	0.037	0.039	0.053	0.066	0.072
Standard deviation of market	0.190	0.165	0.173	0.205	0.238	0.252
Average industry correlation	0.719	0.571	0.590	0.677	0.744	0.769
Risk-free rate	0.006	0.023	0.024	0.016	0.011	0.007
Standard deviation of risk-free rate	0.038	0.044	0.037	0.039	0.038	0.035
Average price-dividend ratio of market	30.0	60.2	48.9	43.9	39.1	37.5
STD of log price-dividend ratio of market	0.412	0.366	0.333	0.362	0.364	0.358
ACF(1) of log price-dividend ratio of market	0.874	0.875	0.869	0.871	0.871	0.871
Conditional Moments						
Expected Excess Return of Market						
Average	0.079	0.037	0.039	0.053	0.066	0.072
Boom	0.071	0.030	0.026	0.037	0.047	0.052
Recession	0.110	0.063	0.088	0.113	0.136	0.148
Recession minus boom	0.040	0.033	0.062	0.076	0.089	0.096
Standard Deviation of Market						
Average	0.190	0.165	0.173	0.205	0.238	0.252
Boom	0.156	0.149	0.147	0.179	0.211	0.227
Recession	0.280	0.223	0.269	0.304	0.335	0.348
Recession minus boom	0.124	0.074	0.125	0.121	0.124	0.121
Average Industry Correlation						
Average	0.719	0.571	0.590	0.677	0.744	0.769
Boom	0.655	0.524	0.531	0.632	0.711	0.742
Recession	0.812	0.751	0.808	0.842	0.864	0.872
Recession minus boom	0.157	0.227	0.277	0.210	0.153	0.130
Risk-Free Rate						
Average	0.006	0.023	0.024	0.016	0.011	0.007
Boom	-0.001	0.003	0.009	0.001	-0.005	-0.006
Recession	0.031	0.096	0.079	0.074	0.065	0.059
Recession minus boom	0.032	0.092	0.070	0.073	0.070	0.065

Table 7: **Cross-Section I.** This table summarizes OLS regression results of model implied external relative habit, ω, as explanatory variable for the average correlation of each industry with every other industry, industry standard deviations and expected excess returns measured by the excess return over the next three years. The table reports the slope coefficients (β_{corr}, β_{stdev}, β_{exr}) and Newey-West corrected t-statistics are in parentheses. $\rho_{\delta_i,C}$ is the correlation of the dividend growth of industry i with aggregate consumption over the entire sample period. Correlations and standard deviations are estimated using a DVEC(1,1) model. Model implied external relative habit is linearly interpolated from the heterogeneous consumer model calibration employing annual consumption data from Robert Shiller's web page. The regressions in the data columns use 996 monthly observations with data ranging from January 1927 to December 2009. The regressions in the model are based on the parameters in Table 1. We simulate 100 paths, each with 1000 monthly observations. For every path we calculate the average correlations, 3-year ahead expected excess returns and standard deviations. The reported results are averages over the 100 sample paths.

					Data					
Industry	Telcm	Utils	Hlth	Enrgy	NoDur	HiTec	Other	Shops	Durbl	Manuf
$\rho_{\delta_i,C}$	-0.014	0.169	0.209	0.318	0.355	0.386	0.409	0.419	0.455	0.530
β_{corr}	-0.611	-0.548	-0.356	-0.678	-0.357	-0.675	-0.261	-0.358	-0.400	-0.330
	(-5.154)	(-3.220)	(-2.508)	(-5.651)	(-3.507)	(-6.472)	(-3.270)	(-3.265)	(-3.657)	(-4.299)
β_{stdev}	-0.239	-0.758	-0.520	-0.464	-0.346	-0.702	-0.771	-0.497	-0.976	-0.776
	(-3.102)	(-7.072)	(-3.414)	(-5.968)	(-3.408)	(-4.003)	(-4.048)	(-3.996)	(-5.297)	(-4.705)
β_{exr}	-0.679	-0.383	-0.513	-0.549	-0.458	-1.239	-0.936	-0.843	-1.850	-1.198
	(-7.361)	(-4.273)	(-2.924)	(-4.280)	(-2.862)	(-6.537)	(-4.484)	(-4.551)	(-4.978)	(-5.233)
					Model					
Industry	1	2	3	4	5	6	7	8	9	10
$\rho_{\delta_i,C}$	0.007	0.060	0.120	0.176	0.223	0.287	0.337	0.388	0.443	0.490
β_{corr}	-0.357	-0.348	-0.321	-0.316	-0.314	-0.312	-0.304	-0.302	-0.310	-0.295
	(-8.690)	(-8.559)	(-8.457)	(-8.533)	(-8.388)	(-8.481)	(-8.348)	(-8.465)	(-8.710)	(-8.269)
β_{stdev}	-0.155	-0.156	-0.154	-0.162	-0.156	-0.163	-0.166	-0.164	-0.162	-0.166
	(-7.225)	(-7.403)	(-7.383)	(-7.339)	(-7.275)	(-7.193)	(-7.246)	(-7.262)	(-7.139)	(-7.067)
β_{exr}	-0.423	-0.431	-0.386	-0.455	-0.489	-0.477	-0.488	-0.511	-0.545	-0.556
	(-2.299)	(-2.144)	(-1.922)	(-2.269)	(-2.538)	(-2.340)	(-2.382)	(-2.414)	(-2.524)	(-2.521)

Table 8: **Cross-Section II.** This table summarizes OLS regression results of the slope coefficients β_{corr}, β_{stdev} and β_{exr} from Table 7 on the dividend-consumption correlations, $\rho_{\delta,C}$. The regressions in the data columns use 996 monthly observations with data ranging from January 1927 to December 2009. The regressions in the model are based on the parameters in Table 1. We simulate 100 paths, each with 1000 monthly observations. For every path we calculate the average correlations, 3-year ahead expected excess returns and standard deviations. The reported results are averages over the 100 sample paths.

	Correlations		Stdevs		Exr	
	Data	Model	Data	Model	Data	Model
Intercept	-0.604	-0.345	-0.331	-0.154	-0.318	-0.399
	-12.708	-53.462	-3.152	-141.535	-1.294	-26.505
Slope	0.453	0.108	-0.846	-0.024	-1.689	-0.303
	3.666	5.524	-3.000	-7.603	-2.292	-7.776
R^2	0.220	0.794	0.356	0.721	0.352	0.844

Figure 1: **Mean Industry Correlations, Returns, Standard Deviations, and Quadratic Variation of Turnover.** This figure shows plots of average pairwise industry correlations from a multivariate GARCH model, average annualized 3-year ahead continuously compounded market excess returns, average industry standard deviations, also from the multivariate GARCH model, and average quadratic variations of industry turnover are estimated by a GARCH(1,1) model based on log changes in turnover for the sample period 1927 to 2009 using Kenneth French's industry classification. Gray shaded areas denote NBER recessions. Sample based on the CRSP/Compustat files, using PERMNO.

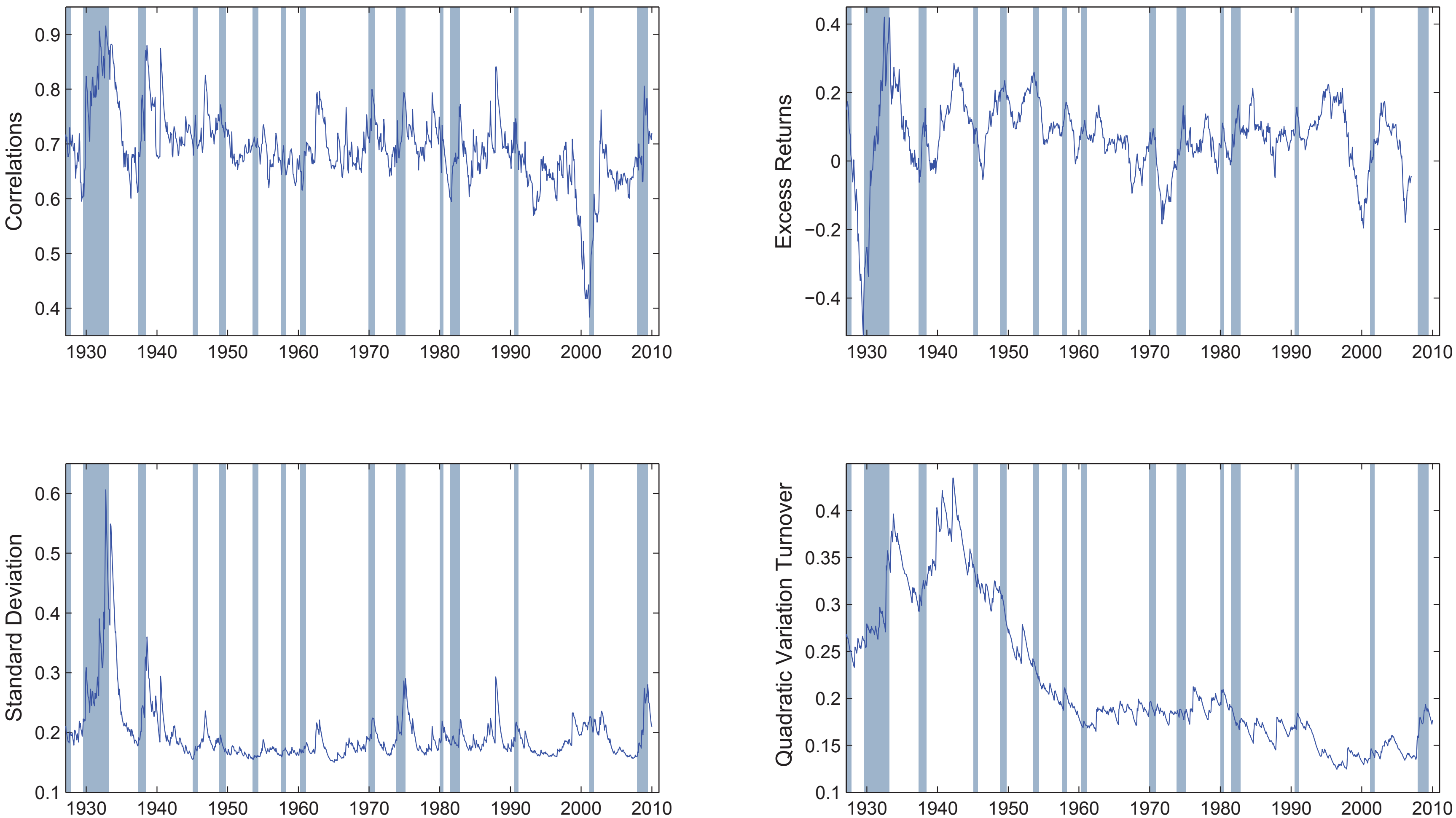


Figure 2: **Correlations with Homogeneous Preferences.** This figure shows the conditional return correlation between stock 1 and stock 2 as a function of dividend share, $s_{\delta_1}(0) = \delta_1(0)/\left(\delta_1(0) + \delta_2(0)\right)$, see Equation 4, or as a function of dividend share and model implied habit, ω, see Equation 9 or as a function of dividend share and consumption share of L consumers, Proposition 1. The figure contains six plots: $log - log$, $3 - 3$, $10 - 10$ (top left plots) with standard power preferences, $3 - 3$ and $10 - 10$ with ratio habit preferences (top middle and right plots), $log - 3$ and $log - 10$ (bottom left and middle plots) with standard power preferences and $log - 10$ with ratio habit preferences (bottom right plot). Investors time preference, ρ, is set at 0.01. Habit persistence, λ, is set at 0.1. Both stocks have identical dividend drift and diffusion coefficients. The drift is set at 0.02 while the diffusion coefficient is 0.05. Dividends are uncorrelated. The horizon of the economy, T, is set at 50 years.

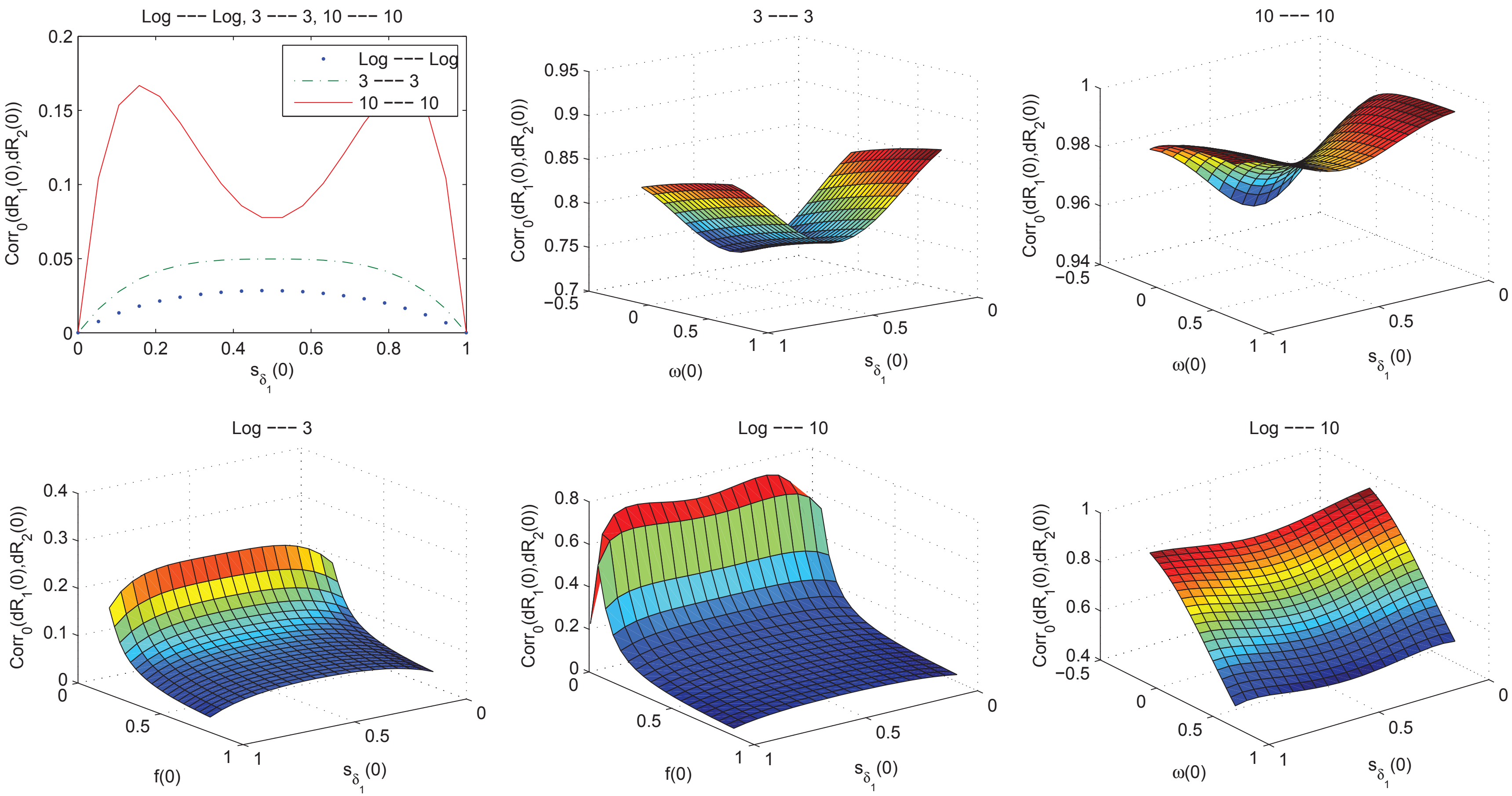


Figure 3: **Risk Aversion Volatility Effect.** This figure shows plots of the consumption share of consumers with high risk aversion, H, slopes of the sharing rules, the risk aversion of the representative agent $(\mathcal{R})$, the negative of the derivative of the risk aversion of the representative agent $(-\frac{\partial \mathcal{R}}{\partial C})$, the conditional return correlations between stock 1 and stock 2 in a heterogeneous consumer economy and an equivalent homogeneous economy $(\gamma_L = \gamma_H = \mathcal{R}(t))$, expected excess return on the market portfolio, conditional standard deviation of the market portfolio, the relative quadratic variation of the portfolio of H consumers as functions of aggregate dividends, $C(0)$. Risk aversion coefficients are $\gamma_L = log$ and $\gamma_H = 3$. All other fundamentals are as in Figure 2.

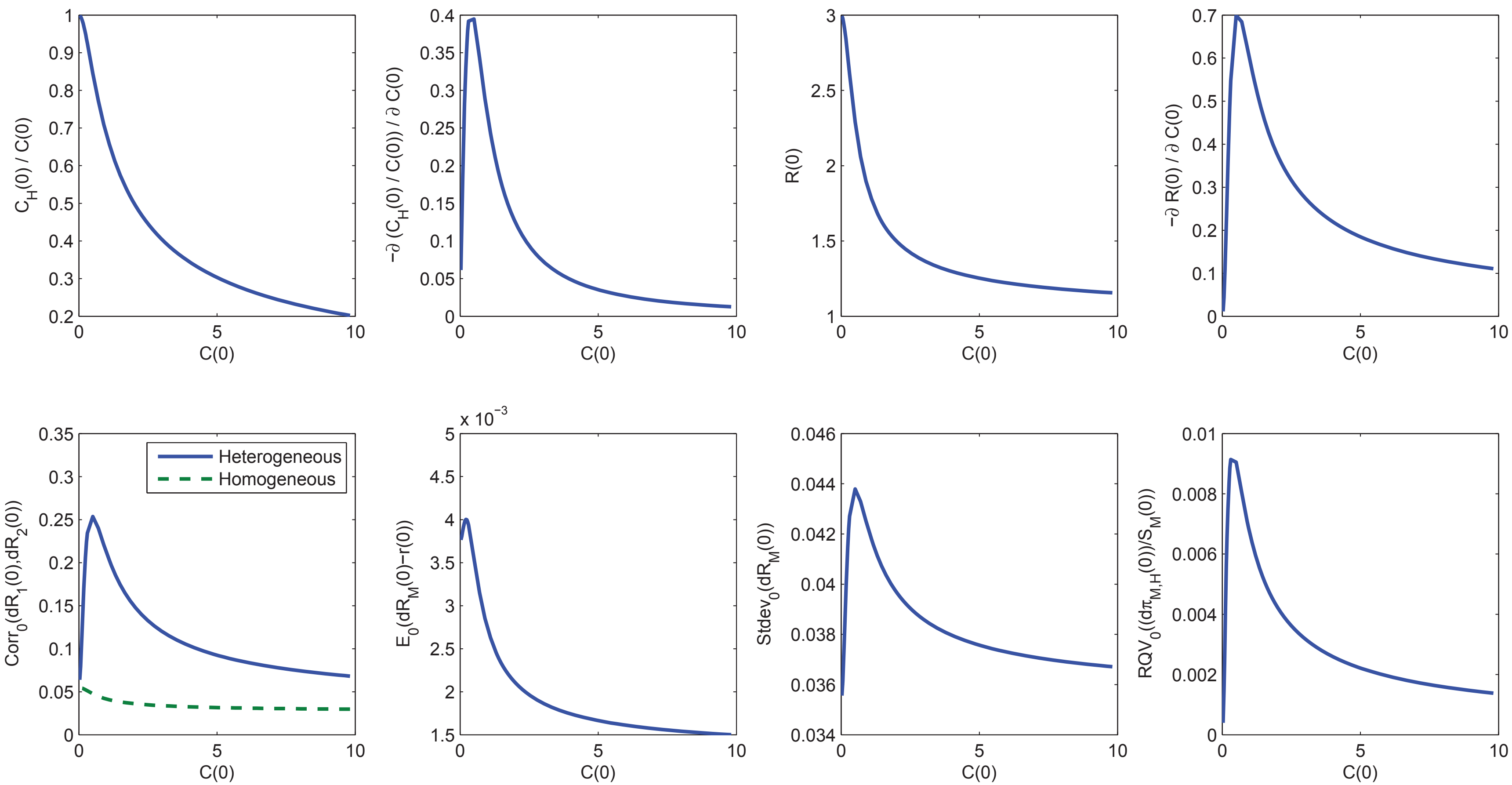


Figure 4: **Empirical Predictions with Ratio Habit Preferences.** The top left figure shows the conditional return correlations between stock 1 and stock 2, the top right figure shows the expected excess return on the market portfolio, the bottom left figure shows the conditional variance of the market portfolio, and the bottom right figure shows the relative quadratic variation of the portfolio of H consumers. Risk aversion coefficients are $\gamma_L = log$ and $\gamma_H = 3$. All other fundamentals are as in Figure 2.

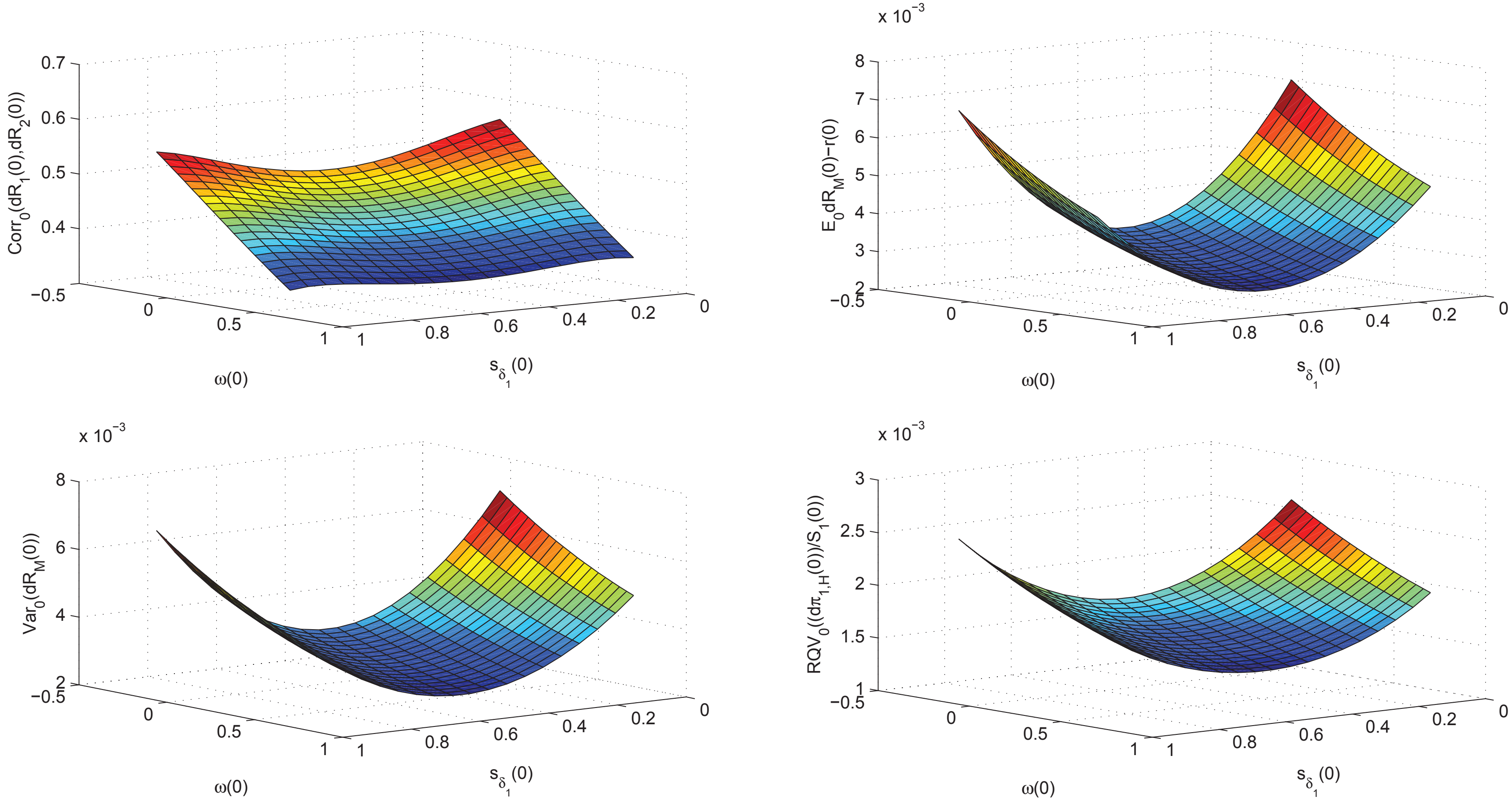


Figure 5: **Model Implied Correlations, Standard Deviations, External Relative Habit and Aggregate Risk Aversion.** The top plots show heterogeneous consumer model implied average pairwise industry correlations and standard deviations and the corresponding data series and over the sample period 1927 to 2009 using Kenneth French's industry classification. Bottom plots show heterogeneous consumer model implied external relative habit, $\omega(t)$, aggregate risk aversion, $R(t)$, and changes in aggregate risk aversion, $\partial R(t)/\partial exp\,(\omega(t))$. Gray shaded areas denote NBER recessions. Sample based on the CRSP/Compustat files, using PERMNO.

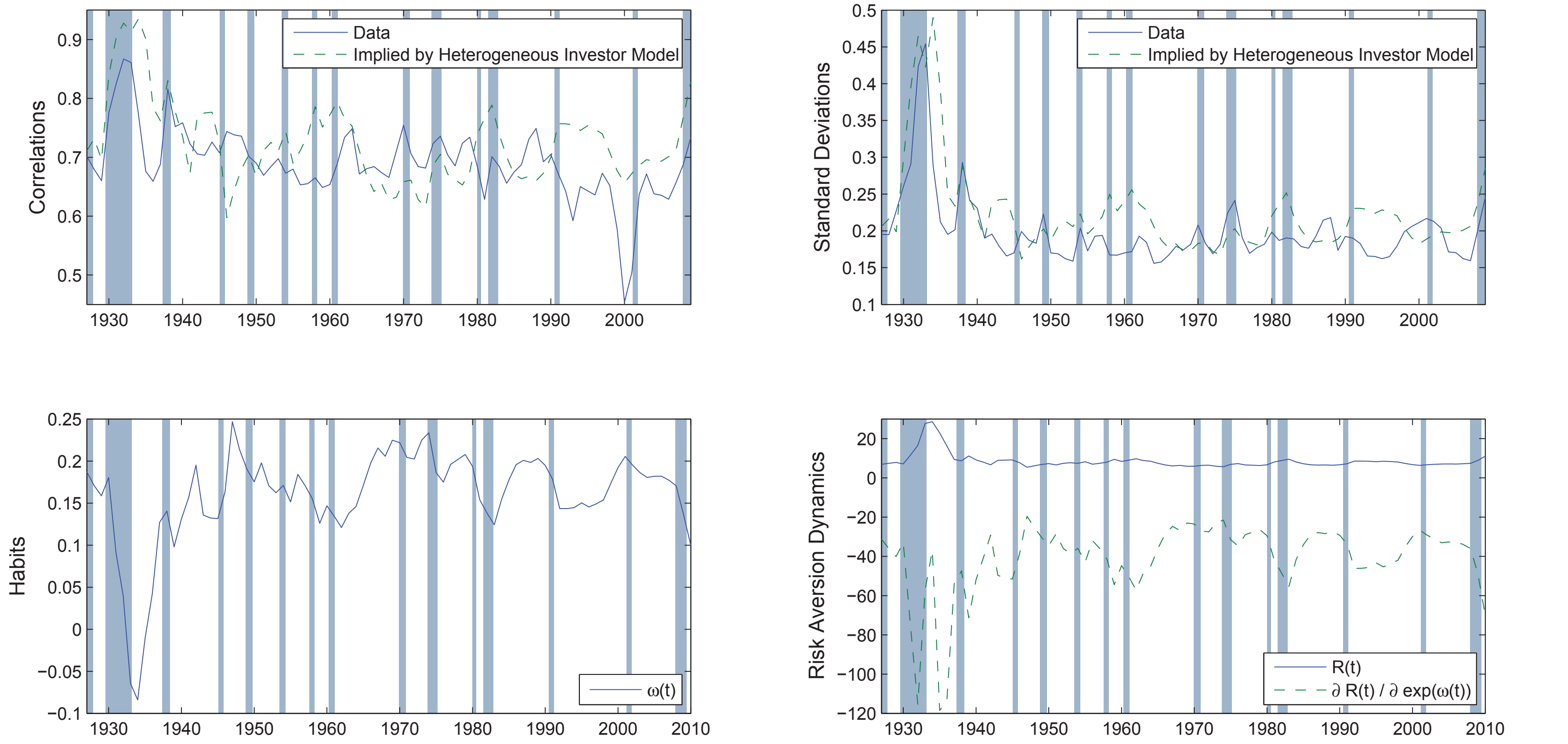
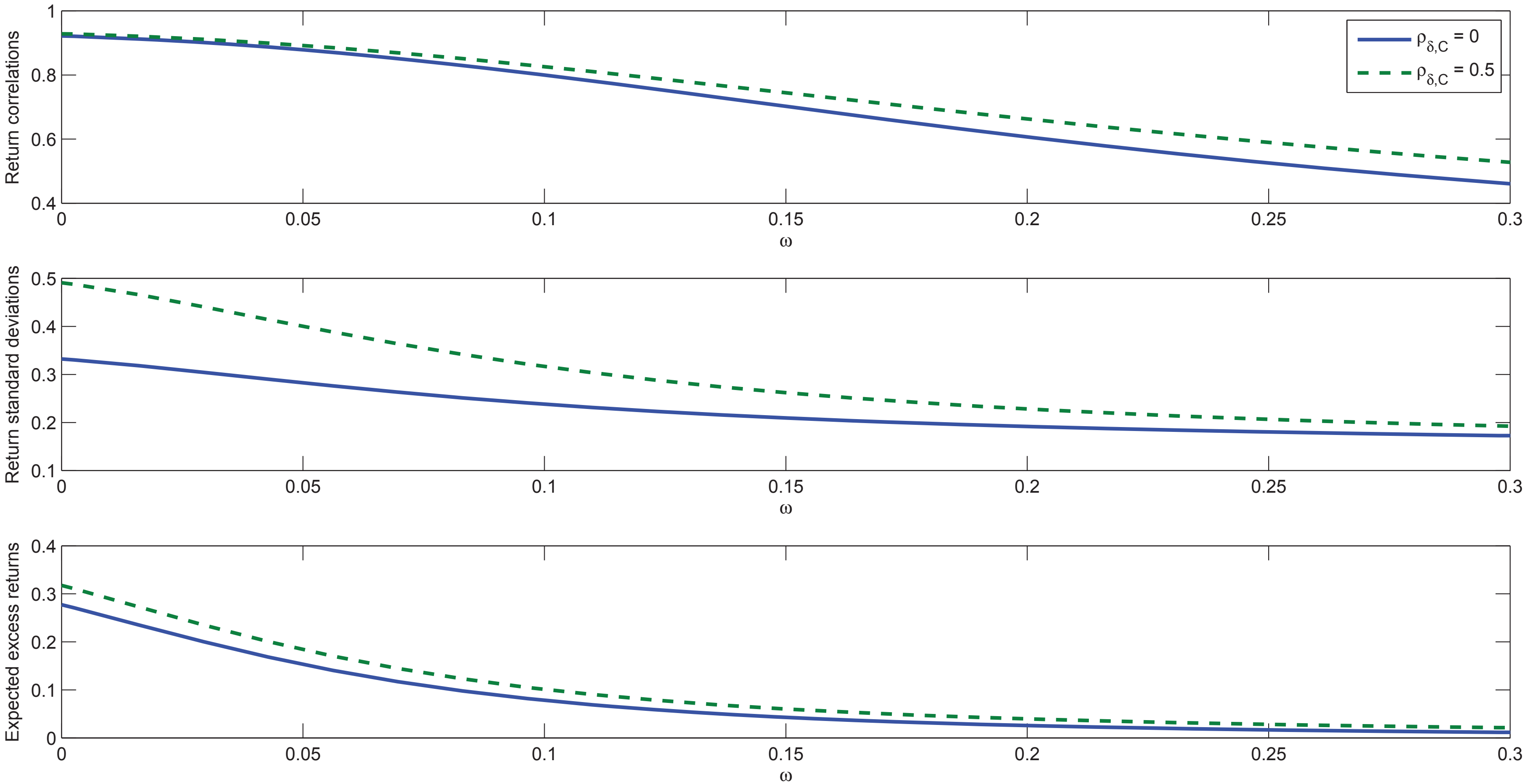


Figure 6: **Sensitivity to ω.** This figure plots the average industry correlation, the standard deviation, and the expected excess return for the industry with lowest, 0, and highest, 0.5, dividend-consumption correlation. Parameters are set as in the benchmark calibration, with the exception that the dividend-consumption correlations are equally spaced in the range $(0, 0.5)$. As in the benchmark calibration, there area 11 Lucas trees in which we interpret the first 10 trees as the dividends from the 10 industries that constitute the market portfolio.



Internet Appendix for "Correlations"

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This Internet Appendix provides additional results that are left out of the main text of the paper. The appendix is organized as follows: Section 1 presents proofs of the propositions in the main text of the paper. Malliavin derivatives are gathered in Section 2. Section 3 extends the two-investor economy in the main text to the case with a continuous distribution of investor types, i.e., a continuous distribution of risk aversion. Section 4 contains a brief explanation for the bimodal shape of stock market correlations in sub-section 3.1 of the main text of the paper. Section 5 presents a sensitivity analysis for the influence of other fundamentals on correlations. Section 6 provides an alternative calibration with dividend streams calibrated to average industry dividend shares, industry dividend growth rates and dividend volatilities. Section 7 performs an extensive principal component analysis and shows that ratio habits explain the first principal component in the time series of correlations and other asset pricing related time series. Section 8 shows the performance of the log price-dividend ratio as explanatory variable instead of ratio habits. Next, Section 9 presents regressions analysis that shows that model implied ratio habit or aggregate risk aversion predicts excess returns in-sample and out-of-sample. Finally, Section 10 presents regression analysis with portfolios sorted on size, book-to-market, and momentum instead of industry sorted portfolios.

1 Proofs of Propositions

Proof of Proposition 1

We solve for equilibrium using the martingale approach (see Cox and Huang (1989) and Karatzas et al. (1987)). Each investor solves the static optimization problem

$$\max_{C_j} E\left[\int_0^T e^{-\rho t}\frac{1}{1-\gamma_j}C_j(t)^{1-\gamma_j}X(t)^{\gamma_j-\eta}dt\right] \tag{1}$$

$$s.t.$$

$$E\left[\int_0^T \xi(t)C_j(t)dt\right] \leq f_{Y,j}(0)E\left[\int_0^T \xi(t)C(t)dt\right], \tag{2}$$

where $f_{Y,j}(0) = \frac{Y_j(0)}{Y_L(0)+Y_H(0)}$ *is the initial wealth fraction of investor type* j. *Necessary and sufficient conditions for optimality are*

$$C_j(t) = \left(y_j e^{\rho t} X(t)^{\eta-\gamma_j}\xi(t)\right)^{-\frac{1}{\gamma_j}}, \tag{3}$$

where $y_j > 0$ *is such that*

$$E\left[\int_0^T \xi(t)\left(y_j e^{\rho t} X(t)^{\eta-\gamma_j}\xi(t)\right)^{-\frac{1}{\gamma_j}} dt\right] = f_{Y,j}(0) E\left[\int_0^T \xi(t) C(t) dt\right], \tag{4}$$

i.e., that the budget condition holds with equality. To solve for equilibrium, it is convenient to introduce an aggregate investor

$$u(C(t), X(t), t) = \max_{C_L(t), C_H(t)} \left\{ \begin{array}{c} a e^{-\rho t} \frac{1}{1-\gamma_L} C_L(t)^{1-\gamma_L} X(t)^{\gamma_j - \eta} \\ +(1-a)\, e^{-\rho t} \frac{1}{1-\gamma_H} C_H(t)^{1-\gamma_H} X(t)^{\gamma_j-\eta} \end{array} \right\} \tag{5}$$

s.t.

$$C_L(t) + C_H(t) = C(t). \tag{6}$$

From the first-order conditions (FOC) of the aggregate investor's problem we have

$$a e^{-\rho t} \left(\frac{C_L(t)}{X(t)}\right)^{-\gamma_L} X(t)^{-\eta} = (1-a)\, e^{-\rho t} \left(\frac{C_H(t)}{X(t)}\right)^{-\gamma_H} X(t)^{-\eta}. \tag{7}$$

Defining the consumption share $f(t) = \frac{C_L(t)}{C(t)}$ *of* L *investors and imposing market clearing, Equation 6, we can rewrite Equation 7 as*

$$f(t) = \left(\frac{a}{1-a}\right)^{\frac{1}{\gamma_L}} e^{\left(\frac{\gamma_H}{\gamma_L}-1\right)\omega(t)} (1 - f(t))^{\frac{\gamma_H}{\gamma_L}}. \tag{8}$$

Proof of Proposition 2

First note that the utility function of the aggregate investor is defined through Equation 5. The coefficient of relative risk aversion is

$$\mathcal{R}(t) = -\frac{u_{CC}(C(t), X(t), t)}{u_C(C(t), X(t), t)} C(t), \tag{9}$$

where u_C *and* u_{CC} *denote the first and second partial derivative with respect to aggregate consumption, respectively. To calculate* $\mathcal{R}$*, we need to compute the partial derivatives of the aggregate investor's utility function. To this end, note that from the FOC of the aggregate investor problem we have that*

$$a\ u_{L,C}(C_L, X(t), t) = (1-a)\ u_{H,C}(C_H, X(t), t). \tag{10}$$

Consequently, we have that

$$\begin{aligned} u_C(C(t),X(t),t) &= a\ u_{L,C}(C_L,X(t),t)\frac{\partial C_L}{\partial C} + (1-a)\ u_{H,C}(C_H,X(t),t)\frac{\partial C_H}{\partial C} \\ &= a\ u_{L,C}(C_L,X(t),t)\left(\frac{\partial C_L}{\partial C} + \frac{\partial C_H}{\partial C}\right) \\ &= a\ u_{L,C}(C_L,X(t),t), \end{aligned} \tag{11}$$

where the second equality follows from Equation 10 and the third equality follows from differentiating both sides of the market clearing condition in Equation 6. Next we calculate the second derivative of the aggregate investor's utility function

$$u_{CC}(C(t),X(t),t) = a\ u_{L,CC}(C_L,X(t),t)\frac{\partial C_L}{\partial C}. \tag{12}$$

Define the absolute risk aversion of investor type j as

$$\mathcal{A}_j(t) = -\frac{u_{j,CC}(C_j(t),X(t),t)}{u_{j,C}(C_j(t),X(t),t)}. \tag{13}$$

We have that

$$\begin{aligned} \mathcal{A}(t) &= -\frac{u_{CC}(C(t),X(t),t)}{u_C(C(t),X(t),t)} \\ &= -\frac{a\ u_{L,CC}(C_L(t),X(t),t)}{a\ u_{L,C}(C_L(t),X(t),t)}\frac{\partial C_L}{\partial C} \\ &= \mathcal{A}_L(t)\frac{\partial C_L}{\partial C}. \end{aligned} \tag{14}$$

Thus, we also have that $\frac{\partial C_L}{\partial C} = \frac{\mathcal{A}(t)}{\mathcal{A}_L(t)}$. Similarly, we get that $\frac{\partial C_H}{\partial C} = \frac{\mathcal{A}(t)}{\mathcal{A}_H(t)}$. Using the fact that $\frac{\partial C_L}{\partial C} + \frac{\partial C_H}{\partial C} = 1$, we obtain

$$\frac{\mathcal{A}(t)}{\mathcal{A}_L(t)} + \frac{\mathcal{A}(t)}{\mathcal{A}_H(t)} = 1, \tag{15}$$

or

$$\mathcal{A}(t) = \left(\frac{1}{\mathcal{A}_L(t)} + \frac{1}{\mathcal{A}_H(t)}\right)^{-1}. \tag{16}$$

Using $\mathcal{R}(t) = \mathcal{A}(t)C(t)$ together with Equation 16, we find

$$\begin{aligned}
\mathcal{R}(t) &= \mathcal{A}(t)C(t) \\
&= \left(\frac{1}{\mathcal{A}_L(t)} + \frac{1}{\mathcal{A}_H(t)}\right)^{-1} C(t) \\
&= \left(\frac{C_L}{C(t)\gamma_L} + \frac{C_H}{C(t)\gamma_H}\right)^{-1} \\
&= \left(\frac{1}{\gamma_L} f(t) + \frac{1}{\gamma_H}(1 - f(t))\right)^{-1}. \qquad (17)
\end{aligned}$$

The absolute prudence of the representative investor, $\mathcal{P}^A(t)$, is

$$\mathcal{P}^A(t) = -\frac{u_{CCC}(C(t), X(t), t)}{u_{CC}(C(t), X(t), t)}. \qquad (18)$$

Similarly, we define the absolute prudence of investor j as

$$\mathcal{P}_j^A(t) = -\frac{u_{j,CCC}(C_j(t), X(t), t)}{u_{j,CC}(C_j(t), X(t), t)}. \qquad (19)$$

To evaluate Equation 18, we need to calculate $u_{CCC}(C(t), X(t), t)$

$$\begin{aligned}
u_{CCC}(C(t), X(t), t) &= \frac{\partial^2 (a\ u_{L,C}(C_L(t), X(t), t))}{\partial C^2} \\
&= \frac{\partial (a\ u_{L,CC}(C_L(t), X(t), t)) \frac{\partial C_L(t)}{\partial C}}{\partial C} \\
&= a\ u_{L,CCC}(C_L(t), X(t), t) \left(\frac{\partial C_L(t)}{\partial C}\right)^2 \\
&\quad + a\ u_{L,CC}(C_L(t), X(t), t) \frac{\partial^2 C_L(t)}{\partial C^2}. \qquad (20)
\end{aligned}$$

Similarly, we calculate

$$\begin{aligned}
u_{CCC}(C(t), X(t), t) &= (1 - a)\ u_{H,CCC}(C_H(t), X(t), t) \left(\frac{\partial C_H(t)}{\partial C}\right)^2 \\
&\quad + a\ u_{H,CC}(C_H(t), X(t), t) \frac{\partial^2 C_H(t)}{\partial C^2}. \qquad (21)
\end{aligned}$$

Using Equation 20 and Equation 21, allow to compute

$$\frac{\partial C_L(t)}{\partial C} \mathcal{P}^A(t) = -\frac{\partial^2 C_L(t)}{\partial C^2} + \mathcal{P}_L^A(t) \left(\frac{\partial C_L(t)}{\partial C}\right)^2 \qquad (22)$$

and

$$\frac{\partial C_H(t)}{\partial C}\mathcal{P}^A(t) = -\frac{\partial^2 C_H(t)}{\partial C^2} + \mathcal{P}_H^A(t)\left(\frac{\partial C_H(t)}{\partial C}\right)^2. \tag{23}$$

Adding up Equations 22 and 23 and noting that $\frac{\partial^2 C_L(t)}{\partial C^2} + \frac{\partial^2 C_H(t)}{\partial C^2} = 0$*, we get*

$$\mathcal{P}^A(t) = \mathcal{P}_L^A(t)\left(\frac{\mathcal{A}(t)}{\mathcal{A}_L(t)}\right)^2 + \mathcal{P}_H^A(t)\left(\frac{\mathcal{A}(t)}{\mathcal{A}_H(t)}\right)^2. \tag{24}$$

The relative prudence of the representative investor is

$$\begin{aligned}\mathcal{P}(t) &= \mathcal{P}^A(t)C(t) \\ &= (1+\gamma_L)\left(\frac{\mathcal{R}(t)}{\gamma_L}\right)^2 f(t) + (1+\gamma_H)\left(\frac{\mathcal{R}(t)}{\gamma_H}\right)^2 (1-f(t)). \end{aligned} \tag{25}$$

Proof of Proposition 3

Applying Ito's lemma to $\mathcal{R}(t) = \mathcal{A}(t)C(t)$ yields

$$\begin{aligned} d\mathcal{R}(t) &= ...dt + \left(C(t)\frac{\partial \mathcal{A}(t)}{\partial \omega}\sigma_C(t) + \mathcal{A}(t)C(t)\sigma_C(t)\right)^{\top} dW(t) \\ &= ...dt + \mathcal{R}(t)\left(1+\mathcal{R}(t)-\mathcal{P}(t)\right)\sigma_C(t)^{\top}dW(t) = ...dt + \sigma_{\mathcal{R}}(t)^{\top}dW(t). \end{aligned} \tag{26}$$

Proof of Proposition 4

First, note that the individual consumption is only a function of aggregate consumption C and the habit level X.[1] *Then, by Ito's lemma we have*

$$dC_j(t) = \frac{\partial C_j(t)}{\partial C}dC(t) + \frac{\partial C_j(t)}{\partial X}dX(t) + \frac{1}{2}\frac{\partial^2 C_j(t)}{\partial C^2}\left(dC(t)\right)^2. \tag{27}$$

To evaluate Equation 27, we need the partial derivatives $\frac{\partial C_j(t)}{\partial C}$, $\frac{\partial C_j(t)}{\partial X}$ *and* $\frac{\partial^2 C_j(t)}{\partial C^2}$*. From the proof of Proposition 2 we have that*

$$\frac{\partial C_j(t)}{\partial C} = \frac{\mathcal{A}(t)}{\mathcal{A}_j(t)}, \tag{28}$$

and

$$\frac{\partial^2 C_j(t)}{\partial C^2} = \mathcal{P}_j^A(t)\left(\frac{\mathcal{A}(t)}{\mathcal{A}_j(t)}\right)^2 - \mathcal{P}^A(t)\left(\frac{\mathcal{A}(t)}{\mathcal{A}_j(t)}\right). \tag{29}$$

[1] The individual consumption is determined by the consumption share f and aggregate consumption. By Equation (71), the consumption share, f, is an implicit function of $e^{\omega = \frac{C(t)}{X(t)}}$. Thus it is only a function of the habit level and aggregate consumption.

Next, we compute

$$\begin{aligned}\frac{\partial C_L(t)}{\partial X} &= \frac{\partial f(t)C(t)}{\partial X}\\ &= C(t)\frac{\partial f(t)}{\partial X}+f(t)\frac{\partial C(t)}{\partial X}\\ &= C(t)\frac{\partial f(t)}{\partial \omega}\frac{\partial \omega}{\partial X}\\ &= -C_L(t)\left(\frac{\mathcal{R}(t)}{\gamma_L}-1\right)\frac{1}{X(t)}, \end{aligned} \tag{30}$$

where in the above we have used the fact that $\frac{\partial\omega}{\partial X} = -\frac{1}{X(t)}$ *and* $\frac{\partial f(t)}{\partial\omega} = f(t)\left(\frac{\mathcal{R}(t)}{\gamma_L}-1\right)$. *Similarly, we get that*

$$\frac{\partial C_H(t)}{\partial X} = -C_H(t)\left(\frac{\mathcal{R}(t)}{\gamma_H}-1\right)\frac{1}{X(t)}. \tag{31}$$

Inserting the partial derivatives together with the dynamics of C and X into Equation (27) yields the proposition.

Proof of Proposition 5

The expression for the state price density follows from the standard result that the state price density is proportional to the marginal utility of the representative investor

$$\xi(t) = \frac{u_C(C(t),X(t),t)}{u_C(C(0),X(0),0)}. \tag{32}$$

The dynamics of the state price density follow, Duffie (2001),

$$\frac{d\xi(t)}{\xi(t)} - \left(r(t)dt+\theta(t)^\top dW(t)\right). \tag{33}$$

Next, applying Ito's lemma to $u_C(C(t),X(t),t)$ we obtain

$$\begin{aligned} du_C(C(t),X(t),t) &= u_{Ct}(C(t),X(t),t)dt + u_{CC}(C(t),X(t),t)dC(t) + u_{CX}(C(t),X(t),t)dX(t)\\ &\quad +\frac{1}{2}u_{CCC}(C(t),X(t),t)\left(dC(t)\right)^2\\ &= \left(u_{CC}(C(t),X(t),t)C(t)\mu_C(t)+u_{CX}(C(t),X(t),t)X(t)\lambda\omega(t)\right)dt\\ &\quad +\left(\frac{1}{2}u_{CCC}(C(t),X(t),t)C(t)^2\sigma_C(t)^\top\sigma_C(t)+u_{Ct}(C(t),X(t),t)\right)dt\\ &\quad +u_{CC}(C(t),X(t),t)C(t)\sigma_C^\top dW(t). \end{aligned} \tag{34}$$

To evaluate Equation 34, we need in addition to $u_{CC}(C(t),X(t),t)$ also expressions for

$u_{Ct}(C(t), X(t), t)$ and $u_{CX}(C(t), X(t), t)$. First note that

$$u_{Ct}(C(t), X(t), t) = -\rho u_C(C(t), X(t), t). \tag{35}$$

Next, we calculate $u_{CX}(C(t), X(t), t)$ as follows

$$\begin{aligned}
u_{CX}(C(t), X(t), t) &= \frac{\partial a \ u_{L,C}(C_L(t), X(t), t)}{\partial X} \\
&= (\gamma_L - \eta)\, a \ u_{L,C}(C_L(t), X(t), t) X(t)^{-1} \\
&\quad + \gamma_L \frac{C(t)}{C_L(t)} f'(t) a \ u_{L,C}(C_L(t), X(t), t) X(t)^{-1} \\
&= \left(\gamma_L - \eta + \gamma_L \frac{C(t)}{C_L(t)} \left[\frac{\mathcal{A}(t)}{\mathcal{A}_L(t)} - f(t)\right]\right) a \ u_{L,C}(C_L(t), X(t), t) X(t)^{-1} \\
&= (\mathcal{R}(t) - \eta)\, u_C(C(t), X(t), t) X(t)^{-1}.
\end{aligned} \tag{36}$$

Since $f(t) = f(\omega(t))$, its derivative is given by $f'(t) = \frac{df(\omega(t))}{d\omega}$. Next, we use the fact that $\frac{\partial C_L(t)}{\partial C(t)} = \frac{\partial f(t)C(t)}{\partial C(t)} = f(t) + C(t) f'(t) \frac{\partial \omega(t)}{\partial C(t)} = f(t) + f'(t)$ together with $\frac{\partial C_L(t)}{\partial C(t)} = \frac{\mathcal{A}(t)}{\mathcal{A}_L(t)}$. Now, note that we have that

$$u_{CCC}(C(t), X(t), t) = u_C(C(t), X(t), t) \mathcal{R}(t) \mathcal{P}(t) \frac{1}{C(t)^2}. \tag{37}$$

Inserting Equations 12, 35, 36 and 37 together with the corresponding dynamics of $C(t)$ and $X(t)$ into Equation 34 we get

$$\begin{aligned}
\frac{du_C(C(t), X(t), t)}{u_C(C(t), X(t), t)} &= -\left(\rho + \eta\lambda\omega(t) + \mathcal{R}(t)\left(\mu_C(t) - \lambda\omega(t)\right) - \frac{1}{2}\mathcal{R}(t)\, \mathcal{P}(t) \sigma_C(t)^\top \sigma_C(t)\right) dt \\
&\quad - \mathcal{R}(t) \sigma_C(t)^\top dW(t).
\end{aligned} \tag{38}$$

Finally, matching the drift and diffusion coefficients in Equation 33 with Equation 38 we obtain

$$r(t) = \rho + \eta\lambda\omega(t) + \mathcal{R}(t)\left(\mu_C(t) - \lambda\omega(t)\right) - \frac{1}{2}\mathcal{R}(t)\, \mathcal{P}(t) \sigma_C(t)^\top \sigma_C(t) \tag{39}$$

$$\theta(t) = \mathcal{R}(t) \sigma_C(t). \tag{40}$$

Proof of Proposition 6

The price of stock $i = 1, ..., N$ is

$$\begin{aligned} S_i(t) &= \frac{1}{\xi(t)} E_t \left[\int_t^T \xi(u)\delta_i(u)du \right] \\ &= \frac{1}{\xi(t)} \left(M_{S_i}(t) - \int_0^t \xi(u)\delta_i(u)du \right), \end{aligned} \tag{41}$$

where $M_{S_i}(t)$ is the martingale defined by

$$M_{S_i}(t) = E_t \left[\int_0^T \xi(u)\delta_i(u)du \right]. \tag{42}$$

By applying Clark-Ocone theorem to $M_{S_i}(t)$, we have that

$$M_{S_i}(t) = M_{S_i}(0) + \int_0^t \phi_{S_i}(u)^\top dW(u), \tag{43}$$

where

$$\phi_{S_i}(t) = E_t \left[\int_t^T D_t \left(\xi(u)\delta_i(u) \right) du \right]. \tag{44}$$

Next, applying Ito's lemma to Equation 41 and matching diffusion coefficients with the dynamics in Equation 8 in the main text, we get

$$\sigma_i = \theta(u) + \frac{E_t \left[\int_t^T D_t \left(\xi(u)\delta_i(u) \right) du \right]}{E_t \left[\int_t^T \xi(u)\delta_i(u)du \right]}. \tag{45}$$

Using the product rule from Malliavin calculus together with the Malliavin derivatives in Equation 50 and 57, yields the result.

Proof of Proposition 7

The wealth of investor types is

$$Y_j = \frac{1}{\xi(t)} E_t \left[\int_t^T \xi(u)C_j(u)du \right]. \tag{46}$$

Moreover, from Cox and Huang (1989) we have that the optimal portfolio is

$$\pi_j(t) = \left(\sigma(t)^\top \right)^{-1} \left[\theta(t)Y_j(t) + \frac{\psi_j(t)}{\xi(t)} \right], \tag{47}$$

where $\psi_j(t)$ is the integrand in the martingale representation of $E_t\left[\int_0^T \xi(u)C_j(u)du\right]$. By Clark-Ocone theorem, we can identify $\psi_j(t)$ as

$$\psi_j(t) \quad = \quad E_t\left[\int_0^T D_t\left(\xi(u)C_j(u)\right)du\right]. \tag{48}$$

Using the product rule from Malliavin calculus together with the Malliavin derivatives in Equation 59 and 57, yields the result.

2 Malliavin Derivatives

Malliavin calculus is a generalization of the calculus of variations (see Nualart (1995)). One useful result from Malliavin calculus concerns the Clark-Ocone theorem (see Detemple et al. (2003), and the references therein), which allows for the explicit identification of the Ito integral in the martingale representation theorem (see Cox and Huang (1989)).

All the Malliavin derivatives used in the paper are standard. Malliavin derivatives are defined for $u > t$ with $u, t \in [0, T]$. We will denote $D_tF(u) = (D_{1,t}F(u), ..., D_{N,t}F(u))^\top$, where $D_{k,t}F(u)$ is the Malliavin derivative with respect to Brownian motion $k = 1, .., N$.

The Malliavin derivatives of interest are $D_t\delta_i(u)$, $D_tC(u)$, $D_tX(u)$, $D_t\omega(u)$, $D_ts_{\delta_i}(u)$, $D_t\sigma_C(u)$,$D_t\left(C(u)\sigma_C(u)^\top\right)$, $D_t\xi(u)$, $D_tC_j(u)$, $D_t\mathcal{A}(u)$, $D_t\mathcal{R}(u)$, $D_t\theta(u)$, $D_t\left(D_t\log\left(\xi(u)\right)\right)^\top$ and $D_t\left(D_tlog\left(C_j(u)\right)\right)^\top$.

The Malliavin derivative of dividend stream i is

$$D_{k,t}\delta_i(u) = \delta_i(u)\sigma_{\delta_{i,k}}. \tag{49}$$

Or in vector notation

$$D_t\delta_i(u) = \delta_i(u)\sigma_{\delta_i}. \tag{50}$$

Next, we calculate the Malliavin derivative of aggregate consumption

$$\begin{aligned} D_t C(u) &= D_t \sum_{i=1}^{N} \delta_i(u) \\ &= \sum_{i=1}^{N} D_t \delta_i(u) \\ &= \sum_{i=1}^{N} \delta_i(u) \sigma_{\delta_i} \\ &= C(u) \sum_{i=1}^{N} s_{\delta_i}(u) \sigma_{\delta_i} \\ &= C(u) \sigma_C(u). \end{aligned} \tag{51}$$

The Malliavin derivative of the habit level $X(u)$ is

$$\begin{aligned} D_t X(u) &= X(u) D_t \log X(u) \\ &= X(u) \lambda \int_t^u e^{-\lambda(t-v)} D_t c(v) dv \\ &= X(u) \lambda \int_t^u e^{-\lambda(t-v)} \sigma_C(v) dv. \end{aligned} \tag{52}$$

Using Equations 51 and 52, we get

$$\begin{aligned} D_t \omega(u) &= D_t \left(logC(u) - logX(u) \right) \\ &= \sigma_C(u) - \lambda \int_t^u e^{-\lambda(t-v)} \sigma_C(v) dv. \end{aligned} \tag{53}$$

The Malliavin derivatives of the dividend shares are given by

$$\begin{aligned} D_t s_{\delta_i}(u) &= D_t \left(\frac{\delta_i(u)}{C(u)} \right) \\ &= \frac{D_t \delta_i(u)}{C(u)} - \frac{\delta_i(u) D_t C(u)}{C(u)^2} \\ &= \frac{\delta_i(u)}{C(u)} \sigma_{\delta_i} - \frac{\delta_i(u) C(u) \sigma_C(u)}{C(u)^2} \\ &= \frac{\delta_i(u)}{C(u)} \left(\sigma_{\delta_i} - \sigma_C(u) \right) \\ &= s_{\delta_i}(u) \sigma_{s_i}(u), \end{aligned} \tag{54}$$

$$\begin{aligned}
D_t\sigma_C(u)^\top &= D_t\left(\left(\sigma_\delta^\top s_\delta(u)\right)^\top\right) \\
&= D_t\left(s_\delta(u)^\top\sigma_\delta\right) \\
&= D_t\left(\sum_{i=1}^{N} s_{\delta_i}(t)\sigma_{\delta_i}^\top\right) \\
&= \sum_{i=1}^{N}\left(D_t s_{\delta_i}(t)\right)\sigma_{\delta_i}^\top \\
&= \sum_{i=1}^{N} s_{\delta_i}(u)\sigma_{s_i}(u)\sigma_{\delta_i}^\top. \qquad (55)
\end{aligned}$$

Next, we find the Malliavin derivative for aggregate consumption risk

$$\begin{aligned}
D_t\left(C(u)\sigma_C(u)^\top\right) &= D_t\sum_{i=1}^{N}\left(\delta_i(u)\sigma_{\delta_i}^\top\right) \\
&= \sum_{i=1}^{N}\left(D_t\delta_i(u)\right)\sigma_{\delta_i}^\top \\
&= \sum_{i=1}^{N}\delta_i(u)\sigma_{\delta_i}\sigma_{\delta_i}^\top \\
&= C(u)\sum_{i=1}^{N} s_{\delta_i}(u)\sigma_{\delta_i}\sigma_{\delta_i}^\top. \qquad (56)
\end{aligned}$$

The Malliavin derivative of the state price density is calculated as follows

$$\begin{aligned}
D_t\xi(u) &= D_t\left(u_C\left(C(u), X(u), u\right)\right) \\
&= u_{CC}\left(C(u), X(u), u\right) D_t C(u) + u_{CX}\left(C(u), X(u), u\right) D_t X(u) \\
&= u_{CC}\left(C(u), X(u), u\right) C(u)\sigma_C(u) + u_{CX}\left(C(u), X(u), u\right) X(u)\lambda\int_t^u e^{-\lambda(t-v)}\sigma_C(v)dv \\
&= u_C\left(C(u), X(u), u\right)\left(-\mathcal{R}(u)\sigma_C(u) + \left(\mathcal{R}(u)-\eta\right)\lambda\int_t^u e^{-\lambda(t-v)}\sigma_C(v)dv\right) \\
&= \xi(u)h(t,u) \qquad (57)
\end{aligned}$$

where

$$h(t,u) = -\theta(u) + \left(\mathcal{R}(u)-\eta\right)\lambda\int_t^u e^{-\lambda(t-v)}\sigma_C(v)dv. \qquad (58)$$

Note that $h(t,u) = D_t \log(\xi(u))$. Next, we calculate the Malliavin derivative of individual consumption

$$\begin{aligned} D_t C_j(u) &= \frac{\partial C_j(u)}{\partial C} D_t C(u) + \frac{\partial C_j(u)}{\partial X} D_t X(u) \\ &= C_j H_j(t,u), \end{aligned} \tag{59}$$

where

$$H_j(t,u) = \left(\frac{\mathcal{R}(u)}{\gamma_j}\right)\sigma_C(u) - \left[\left(\frac{\mathcal{R}(u)}{\gamma_j}\right) - 1\right]\lambda \int_t^u e^{-\lambda(t-v)}\sigma_C(v)dv. \tag{60}$$

Note that $H_j(t,u) = D_t log(C_j(u))$. Next we calculate the Malliavin derivative of absolute aggregate risk aversion

$$\begin{aligned} D_t\mathcal{A}(u) &= \frac{\partial \mathcal{A}(u)}{\partial C} D_t C(u) + \frac{\partial \mathcal{A}(u)}{\partial X} D_t X(u) \\ &= \left(\left(\frac{u_{CC}(C(u),X(u),u)}{u_C(C(u),X(u),u)}\right)^2 - \frac{u_{CCC}(C(u),X(u),u)}{u_C(C(u),X(u),u)}\right) D_t C(u) \\ &\quad + \left(\frac{u_{CC}(C(u),X(u),u)}{u_C(C(u),X(u),u)^2} u_{CX}(C(u),X(u),u) - \frac{u_{CCX}(C(u),X(u),u)}{u_C(C(u),X(u),u)}\right) D_t X(u) \\ &= \mathcal{A}(u) \times \\ &\quad \left[(\mathcal{R}(u) - \mathcal{P}(u))\left(\sigma_C(u) - \lambda\int_t^u e^{-\lambda(t-v)}\sigma_C(v)dv\right) - \lambda \int_t^u e^{-\lambda(t-v)}\sigma_C(v)dv\right] \end{aligned} \tag{61}$$

The Malliavin derivative of the coefficient of relative aggregate risk aversion is

$$\begin{aligned} D_t\mathcal{R}(u) &= D_t(\mathcal{A}(u)C(u)) \\ &= C(u)D_t\mathcal{A}(u) + \mathcal{A}(u)D_tC(u) \\ &= \mathcal{R}(u)\left[(1 + \mathcal{R}(u) - \mathcal{P}(u))\left(\sigma_C(u) - \lambda\int_t^u e^{-\lambda(t-v)}\sigma_C(v)dv\right)\right]. \end{aligned} \tag{62}$$

Next we calculate the Malliavin derivative of the market prices of risk

$$\begin{aligned} D_t\theta(u)^\top &= D_t\left(\mathcal{R}(u)\sigma_C(u)^\top\right) \\ &= \mathcal{R}(u)\left[(1 + \mathcal{R}(u) - \mathcal{P}(u))\left(\sigma_C(u) - \lambda\int_t^u e^{-\lambda(t-v)}\sigma_C(v)dv\right)\right]\sigma_C(u)^\top \\ &\quad + \mathcal{R}(u)\sum_{i=1}^N s_{\delta_i}(u)\sigma_{s_i}(u)\sigma_{\delta_i}^\top. \end{aligned} \tag{63}$$

The Malliavin derivative of $h(t,u)$ can be calculate as follows

$$\begin{aligned}
D_t h(t,u)^\top &= D_t \left(-\theta(u) + (\mathcal{R}(u) - \eta)\,\lambda \int_t^u e^{-\lambda(t-v)} \sigma_C(v) dv \right) \\
&= -\mathcal{R}(u)\,(1 + \mathcal{R}(u) - \mathcal{P}(u)) \\
&\quad \times \left(\sigma_C(u) - \lambda \int_t^u e^{-\lambda(t-v)} \sigma_C(v) dv \right) \left(\sigma_C(u) - \lambda \int_t^u e^{-\lambda(t-v)} \sigma_C(v) dv \right)^\top \\
&\quad -\mathcal{R}(u) \sum_{i=1}^{N} \left(s_{\delta_i}(u) \sigma_{s_i}(u) \sigma_{\delta_i}^\top + \left(\frac{\eta}{\mathcal{R}(u)} - 1 \right) \lambda \int_t^u e^{-\lambda(t-v)} s_{\delta_i}(v) \sigma_{s_i}(v) \sigma_{\delta_i}^\top dv \right) \\
&= g(t,u). \qquad (64)
\end{aligned}$$

Finally, we calculate $D_t H_j(t,u)^\top$

$$\begin{aligned}
D_t H_j(t,u)^\top &= D_t \left(\left(\frac{\mathcal{R}(u)}{\gamma_j} \right) \sigma_C(u) - \left[\left(\frac{\mathcal{R}(u)}{\gamma_j} \right) - 1 \right] \lambda \int_t^u e^{-\lambda(t-v)} \sigma_C(v) dv \right)^\top \\
&= \frac{\mathcal{R}(u)}{\gamma_j} (1 + \mathcal{R}(u) - \mathcal{P}(u)) \\
&\quad \times \left(\sigma_C(u) - \lambda \int_t^u e^{-\lambda(t-v)} \sigma_C(v) dv \right) \left(\sigma_C(u) - \lambda \int_t^u e^{-\lambda(t-v)} \sigma_C(v) dv \right)^\top \\
&\quad + \frac{\mathcal{R}(u)}{\gamma_j} \left(\sum_{i=1}^{N} \left(s_{\delta_i}(u) \sigma_{s_i}(u) \sigma_{\delta_i}^\top + \left(\frac{\gamma_j}{\mathcal{R}(u)} - 1 \right) \lambda \int_t^u e^{-\lambda(t-v)} s_{\delta_i}(v) \sigma_{s_i}(v) \sigma_{\delta_i}^\top dv \right) \right) \\
&= G_j(t,u). \qquad (65)
\end{aligned}$$

3 Model with General Distribution for Risk Aversion

In this section we generalize the model in the paper to a continuum economy, interpreted as the distribution of risk aversion of a continuum of investors. As in the main text of the paper, preferences are described by

$$U(c, X; \gamma) = E_0 \left[\int_0^T e^{-\rho t} u(c(t), X(t); \gamma) dt \right]. \qquad (66)$$

The instantaneous utility function is given by

$$u(c(t), X(t); \gamma) = \frac{1}{1-\gamma} c(t)^{1-\gamma} X(t)^{\gamma-\eta}, \qquad (67)$$

where γ measures the local curvature of the utility function. Complete markets allow to construct the aggregate investor problem

$$\sup_{c(C(t),X(t),t;\gamma)} \int_1^\infty a(\gamma) \frac{c(C(t),X(t),t;\gamma)^{1-\gamma}}{1-\gamma} X(t)^{\gamma-\eta} d\gamma \tag{68}$$

$$s.t.$$

$$\int_1^\infty c(C(t),X(t),t;\gamma) \leq C(t) \tag{69}$$

time by time and state by state. Above, $a(\gamma)$ denotes the "social" weight attached to investor of type γ.

Proposition IAC 1

Optimal consumption allocations are

$$c(t;\gamma) = f(t;\gamma)C(t) \tag{70}$$

where f is given by

$$f(t;\gamma) = a(\gamma)^{\frac{1}{\gamma}} e^{-\omega(t)} y(\omega(t))^{-\frac{1}{\gamma}}, \tag{71}$$

and where $y(\omega(t))$ is defined through

$$\int_1^\infty a(\gamma)^{\frac{1}{\gamma}} e^{-\omega(t)} y(\omega(t))^{-\frac{1}{\gamma}} d\gamma = 1. \tag{72}$$

Following a similar approach as in the two investor setting in the main text of the paper one obtains the following proposition.

Proposition IAC 2

The coefficient of relative risk aversion for the aggregate investor is given by

$$\mathcal{R}(t) = \left(\int_1^\infty \frac{1}{\gamma} f(t;\gamma) d\gamma \right)^{-1}. \tag{73}$$

The relative prudence of the aggregate investor is:

$$\mathcal{P}(t) = \int_1^\infty (1+\gamma) \left(\frac{\mathcal{R}(t)}{\gamma} \right)^2 f(t;\gamma) d\gamma. \tag{74}$$

The risk-free rate and the market prices of risk are give by the next proposition.

Proposition IAC 3

In equilibrium, the risk-free rate is

$$r(t) = \rho + \mathcal{R}(t)\,\mu_C(t) + (1 - \mathcal{R}(t))\,\lambda\omega(t) - \frac{1}{2}\mathcal{R}(t)\,\mathcal{P}(t)\sigma_C(t)^\top\sigma_C(t). \quad (75)$$

Market prices of risks are as follows:

$$\theta(t) = \mathcal{R}(t)\,\sigma_C(t). \quad (76)$$

Note that the risk-free rate and the market price of risk take the same form as in the two-investor setting presented in the main text of the paper.

Next we solve the sharing rule when $a(\gamma)$ is given by a transformed Beta distribution, see Figure 1, with parameters: $a = 1$ and $b = 10.0038$. Draws from the Beta distribution yield random variables between 0 and 1. We adjust these variables (rv) in the following way: $rv_{adj} = 1 + rv * (\gamma_{max} - 1)$ where $\gamma_{max} = 183.9705$. Thus, the maximum γ is 183.9705 and the minimum γ is 1. We fix $a = 1$ and search for b and γ_{Max} that minimize the squared difference between the aggregate relative risk aversion coefficient in the $30 - log$ economy presented in the main text of the paper and the aggregate relative risk aversion coefficient in the Beta distribution economy. We perform this minimization problem over the range of -0.05 to 0.35 for ω with a 0.01 step size.

Figure 2 shows the distribution of ω, the risk aversion of the representative agent, and the relative prudence as a function of the state of the economy for the $30 - log$ economy and the Beta distribution economy. We see from Figure 2 that the relative risk aversion and the relative prudence, which we did not include in the minimization problem, of the representative investor is similar in the two economies. As the dynamics of the risk aversion of the representative investor is the key driver for the results in the main text of the paper, therefore the model with Beta distributed investors shares the properties of the $30 - log$ economy.

4 Bimodal Correlations

To understand the bimodal shape of stock market correlations, it is useful to consider economies in which investors derive utility from terminal wealth. In the terminal wealth case, correlations peak at $s_{\delta_1} = s_{\delta_2} = 0.5$. To understand this, we inspect the standard deviation of the two dividend shares. At $s_{\delta_1} = s_{\delta_2} = 0.5$, the volatility of dividend shares simultaneously reach their maximum. Because risk aversion and dividend diffusion terms are constant, it is evident from Equation 40 in the main text of the paper that the volatility of dividend shares directly translate into volatility of market prices of risk. Hence, changes in the discount rate for stock 1 and stock 2 reach their maximum at $s_{\delta_1} = s_{\delta_2} = 0.5$. Because discount rates move in opposite directions, the correlation reaches its maximum when dividend shares reach their maximum volatility. In short, the drop in correlations in economies with intermediate consumption, which is not crucial for our analysis below, around $s_{\delta_1} = s_{\delta_2} = 0.5$ is due to the interplay between current market prices of risk and future market prices of risk. More formally, the market prices of risk shift the peaks of the stock return diffusion terms away from $s_{\delta_1} = s_{\delta_2} = 0.5$. For stock 1, the diffusion term exposed to own shocks peaks after $s_{\delta_1} = s_{\delta_2} = 0.5$ while the diffusion term exposed to the shock to the second dividend stream peaks before $s_{\delta_1} = s_{\delta_2} = 0.5$. For stock 2, the peaks of the diffusion terms are reversed. These patterns lead to the drop in covariances and correlations at $s_{\delta_1} = s_{\delta_2} = 0.5$ in economies with intermediate consumption in which dividends evolve as geometric Brownian motion.

5 Correlations and Fundamentals

In this subsection, we report how other fundamentals affect correlations. First, correlations increase in dividend growth rates. Second, studying the influence of the dividend volatility on correlations, we find that correlations also increase in dividend volatility. Third, we introduce correlation at the dividend level and find that return correlations increase. Yet, heterogeneity in risk aversion continues to play a prominent role since return correlations are always above dividend correlations except when dividend correlations approach one. Fourth, we analyze economies with different time horizons, 20 years and 100 years, and learn that correlations increase in the horizon. Figure 3 summarizes these four findings. Fifth, correlations increase in habit persistence (see Figure 4), λ. Increasing (decreasing) λ from 0.1 to 0.2 (0.05) increases (decreases) correlations in a $3 - \log$ economy in the range of 7 to 10.5 (12 to 18) percentage points for reasonable values of relative consumption, ω.

6 Alternative Calibration

In this section we present an alternative calibration: we build industry dividends and add in share repurchases using Kenneth French's industry classification. The industries are Consumer Non-Durables (NoDur), Consumer Durables (Durbl), Manufacturing (Manuf), Energy (Enrgy), Business Equipment (HiTec), Telephone and Television Transmission (Telcm), Shops, Health (Hlth), Utilities (Utils), and Other. The sample consists of all firms appearing jointly in Compustat and CRSP for the sample period, 1950 to 2008. We adjust the series for inflation and population growth. Table 1 shows the mean, the standard deviation and pair-wise correlations of the ten industry processes in this alternative calibration.

Table 2 shows the parameters of this calibration together with the corresponding values in the data (based on the data from the calibration in the main text of the paper, i.e., January 1927 to December 2009). Table 3 shows conditional asset pricing moments. Details of the calibration procedure are found in Section 4 in the main text of the paper.

7 Empirics – Principal Component Analysis

We calculate the first principal component of the forty-five correlation series (PCA CORR), the ten series of 3-year ahead excess returns (PCA EXR), the ten series of standard deviations (PCA STD) and the ten series of quadratic variations of turnover (PCA QV) separately. To calculate the first principal component of all the series, we compute the average of the four sets of series to reduce the impact of the forty-five correlation series and obtain from the averages the first principal component (PCA TOTAL). First, we regress the first principal component of these four series onto model implied external relative habit. Second, we regress the first principal component from all the series, PCA TOTAL, onto model implied external relative habit.[2]

Table 4 shows the results from these principal component regressions. All regression coefficients show negative sign consistent with a heterogeneous consumer version of the model. Further, all coefficient estimates for external relative habit show highly significant Newey-West corrected t-statistics. The adjusted R-squared range from 14.06% to 40.95%. Our results regarding excess returns are essentially unchanged if we correct the nominal short rate with expected inflation instead of realized inflation.[3] Overall, we conclude that signs of the coefficients as well as the explanatory power of the regressions support our theory.

8 Empirics – Log Price-Dividend Ratio Regressions

In our model the log price-dividend ratio is increasing in ω, i.e., the relation represents a one-to-one mapping. Indeed, the log price-dividend ratio leads to comparable results for the

[2]The first principal component explains 54%, 86%, 87% and 71% of the variation in the 45 correlation coefficients, 10 standard deviations, 10 quadratic variations of turnover and 10 three years ahead excess returns. The first principal component explains 51% of the variation of the averages of the four series.

[3]Model implied external relative habit and the real short rate show statistically significant negative relation.

principal component analysis, Table 5, as well as for regressions that explain the averages of the series we study, Table 6.

9 Predicting Excess Returns In-Sample and Out-of-Sample

Model implied relative consumption forecasts excess returns in-sample and out-of-sample. Because relative consumption does not include the level of market prices, it is unlikely to produce spurious results. In the sense that relative consumption forecasts excess returns in the model, it is a natural predictor of stock market returns.[4]

The relation between the excess return on the market portfolio and relative consumption is

$$Corr_t(E_t(dR_M(t) - r(t)), \omega(t)) < 0, \tag{77}$$

which is negative for most of the distribution of ω. Hence, on average, the model implies a negative relation between expected excess returns and relative consumption. A discrete time formulation implies the following slope coefficient in a predictive regression

$$\beta_t = \frac{Cov_t(E_t(R_{M,t+1} - r_t), \omega_t)}{Var_t(\omega_t)}, \tag{78}$$

which is negative whenever Equation 77 is negative. Therefore, the model predicts on average negative relations between relative consumption and expected excess return of the market portfolio as well as other portfolios.

The first three rows of Table 7 show the coefficient estimate, Newey-West corrected t-statistics and the adjusted R-squared for in-sample regressions using relative consumption from the heterogeneous investor economy as the predictive variable. The table contains 3 sets of regressions: 1 year excesses returns, 3 year excess returns and 5 year excess returns with each set containing regressions for 10 industries using Kenneth French's industry classification and the market excess return. Regressions include a constant with data ranging from 1927 to 2009. The predictive impact of relative consumption is statistically significant at least at the 10 percent level except for the following sectors: Non-Durables (five year horizon), Energy (one and five year horizons), and Health and Utils for all horizons. Importantly, all coefficients appear with negative sign favoring the model with heterogeneity in risk aversion. The adjusted R-squared statistics indicate that regressions with significant coefficients explain at least 2.8 percent of the variation in excess industry and market returns. Overall, adjusted R-squared statistics first increase when the prediction horizon increases, 3

[4]Cooper and Priestley (2009) argue that it is important to link predictability to economic fundamentals.

year versus 1 year, but decrease when the prediction horizon is 5 years.

Next, we ask whether predictive regressions perform also out-of-sample by making nested forecast comparisons. The comparisons are between a model which includes only the constant and a model which includes a constant and relative consumption as a predictor. Theil's U, in the fourth row for each prediction horizon in Table 7, is the ratio of the root-mean-squared forecast errors for the unrestricted and restricted models. A number larger than one for Theil's U indicates that the restricted model (with only a constant) has a lower root-mean-squared error than the model with relative consumption as an explanatory variable. The root-mean-squared error of the regressions which include relative consumption are always lower than the regressions with a constant, except for Non-Durables (three and five year horizons) and Health and Utilities for all horizons. Another standard out-of-sample test is the MSE-F statistic which tests the null hypothesis that the MSE for the unrestricted model forecasts is less than the MSE for the restricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 3.467, 1.636 and 0.819, respectively. The fifth row for each prediction horizon in Table 7 shows the test statistics from our data. The ENC-NEW statistic, in the 6th row for each prediction horizon, tests the null hypothesis that restricted model forecasts encompass the unrestricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 2.566, 1.334 and 0.842, respectively. At the 10% level, we obtain a picture very similar to the previous results.

10 Empirics – Alternative Portfolio Sorts

In the main body of the paper we calibrate the model to ten industry portfolios. According to our model, stock market correlations, standard deviations and expected returns as well as quadratic variation of portfolio policies have negative relation with ω. This negative relation, however, is independent of portfolio sorts. Therefore, we repeat the regression from the main text of the paper and the predictive regression in Section 9, but use alternative portfolio sorts. In particular, we consider ten portfolios sorted on size, book-to-market and momentum, respectively. The Tables 8 - 13 confirm the prediction of our model and, thus, show that our empirical results in the main text of the paper are not an artifact of industry sorted portfolios.

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Table 1: **Calibration — Didividend Parameters.** This table summarizes descriptive statistics (mean, standard deviation (STD), and pair-wise correlations) of industry dividends and equity share repurchases using Kenneth French's industry classification. Industries are Consumer Non-Durables (NoDur), Consumer Durables (Durbl), Manufacturing (Manuf), Energy (Enrgy), Business Equipment (HiTec), Telephone and Television Transmission (Telcm), Shops, Health (Hlth), Utilities (Utils), and Other. The sample consists of all firms appearing jointly in Compustat and CRSP for the sample period, 1950 to 2008.

	Dividends and Repurchases (Inflation and Population Growth Adjusted)									
	NoDur	Durbl	Manuf	Enrgy	HiTec	Telcm	Shops	Hlth	Utils	Other
Mean	0.04920	0.01950	0.03491	0.05487	0.11195	0.05227	0.06098	0.08514	0.02886	0.08625
STD	0.17797	0.22857	0.15574	0.15930	0.26700	0.18397	0.20313	0.16393	0.09411	0.15619
	Pair-Wise Correlations of Dividends and Repurchases (Inflation and Population Growth Adjusted)									
NoDur	1.00000									
Durbl	0.43813	1.00000								
Manuf	0.32118	0.41952	1.00000							
Enrgy	0.24487	0.36414	0.40745	1.00000						
HiTec	0.02949	0.19177	0.47249	0.19020	1.00000					
Telcm	-0.10497	0.20925	0.07696	0.20349	0.03350	1.00000				
Shops	0.25547	0.27865	0.38651	-0.01269	0.18353	0.07082	1.00000			
Hlth	0.16818	0.08646	0.06828	-0.12243	-0.26574	-0.04993	0.16271	1.00000		
Utils	0.13811	0.11247	0.04789	-0.12235	0.10224	0.28798	0.10467	-0.04603	1.00000	
Other	0.28184	0.34657	0.38163	0.30779	0.32310	0.19839	0.36734	0.04747	0.29393	1.00000
Average	0.17512									

Table 2: **Calibration — Parameters.** This table summarizes moments of consumption, aggregate industry dividends, as well as relations between consumption, and aggregate dividends for the period January 1927 to December 2009 and corresponding moments from a heterogeneous investor model calibration for the period January 1950 to December 2008 (based on the dividend and repurchase data in Table 1). The table also shows the preference parameters employed in the alternative heterogeneous investor model calibration.

	Data	Model
Consumption and Dividend Moments		
Mean consumption growth	0.020	0.054
Standard deviation of consumption growth	0.030	0.094
Mean aggregate dividend growth	0.024	0.054
Standard deviation of aggregate dividend	0.082	0.094
Consumption-aggregate dividend correlation	0.539	1
Aggregate dividend-consumption ratio	0.030	1
Preference Parameters		
Risk aversion, high (γ_H)	-	20
Risk aversion, low (γ_L)	-	1
Utility weight (a)	-	0.250
Subjective discount factor (ρ)	-	0.010
Degree of habit history dependence (λ)	-	0.110

Table 3: **Calibration — Asset Pricing Moments over the Business Cycle.** This table summarizes unconditional and conditional (booms and recessions) moments of three year ahead excess return and standard deviation of the market portfolio, average industry portfolio correlation, and risk-free rate one average for the period January 1927 to December 2009 and corresponding moments from the heterogeneous investor model calibration in Table 2 for the period January 1950 to December 2008 (based on the dividend and repurchase data in Table 1). Returns are annualized from monthly frequency. Model risk-free rate is $r(t)$, while the log-return of the market portfolio is $\mu_M(t) - \frac{1}{2}\sigma_M^2(t)$, where M denotes the market portfolio. In the data, recessions are defined by NBER recession dates. In the calibrated model, recessions have the same unconditional probability as in the data.

	Data	Model
Expected Excess Return of Market		
Average	0.079	0.032
Boom	0.071	0.027
Recession	0.110	0.050
Recession minus boom	0.040	0.023
Standard Deviation of Market		
Average	0.190	0.211
Boom	0.156	0.198
Recession	0.280	0.260
Recession minus boom	0.124	0.062
Average Industry Correlation		
Average	0.719	0.660
Boom	0.655	0.635
Recession	0.812	0.754
Recession minus boom	0.157	0.119
Risk-Free Rate		
Average	0.006	0.017
Boom	-0.001	0.015
Recession	0.031	0.024
Recession minus boom	0.032	0.009

Table 4: **Empirics — Principal Component Analysis.** This table summarizes OLS regression results (intercept, coefficient estimate and adjusted R-squared) of model implied external relative habit as explanatory variable for the first principal component of industry market correlations (PCA CORR), 3-year ahead expected excess returns (PCA EXR), standard deviations (PCA STDV), quadratic variations of industry turnover (PCA QV), and the first principal component of the four means of the respective series (PCA TOTAL). Newey-West corrected t-statistics are in parentheses. Industry market correlations and standard deviations are calculated using a DVEC(1,1) model. Quadratic variations of industry turnover are estimated by a GARCH(1,1) model based on log changes in turnover. Model implied external relative habit is linearly interpolated from the heterogeneous consumer model calibration employing annual consumption data from Robert Shiller's web page. The regressions use 996 monthly observations with data ranging from January 1927 to December 2009.

	PCA CORR	PCA EXR	PCA STDV	PCA QV	PCA TOTAL
Intercept	0.5387	0.3617	0.3232	0.4423	0.1687
	(2.9547)	(3.6869)	(3.0444)	(6.7586)	(4.7019)
Model implied external habit, ω	-3.3680	-2.2556	-2.0205	-2.7656	-1.0524
	(-3.0384)	(-4.0211)	(-3.3327)	(-7.9762)	(-4.8871)
Adjusted R-squared	0.1406	0.1501	0.4095	0.2392	0.2719

Table 5: **Empirics with the Log Price-Dividend Ratio — Principal Component Analysis.** This table summarizes OLS regression results (intercept, coefficient estimate and adjusted R-squared) of model implied log Price-Dividend ratio (pd) as explanatory variable of the first principal component of industry market correlations (PCA CORR), 3-year ahead expected excess returns (PCA EXR), standard deviations (PCA STDV), quadratic variations of industry turnover (PCA QV), and the first principal component of the four means of the respective series (PCA TOTAL). Newey-West corrected t-statistics are in parentheses. Industry market correlations and standard deviations are calculated using a DVEC(1,1) model. Quadratic variations of industry turnover are estimated by a GARCH(1,1) model based on log changes in turnover. The regressions use 996 monthly observations with data ranging from January 1927 to December 2009.

	PCA CORR	PCA EXR	PCA STDV	PCA QV	PCA TOTAL
Intercept	2.3299	0.8976	0.3122	1.5327	0.5088
	(8.1551)	(5.3126)	(2.3408)	(13.0112)	(8.7718)
pd-ratio	-0.70161	-0.27175	-0.094015	-0.46155	-0.15404
	(-8.1361)	(-5.3423)	(-2.5066)	(-14.1999)	(-8.7895)
Adjusted R-squared	0.41733	0.14183	0.059489	0.45431	0.37974

Table 6: **Empirics with the Log Price-Dividend Ratio — Averages.** This table summarizes OLS regression results (intercept, coefficient estimate and adjusted R-squared) of model implied log Price-Dividend ratio (pd) as explanatory variable of the average of industry market correlations (Av. CORR), 3-year ahead expected excess returns (Av. EXR), standard deviations (Av. STDV), and quadratic variations of industry turnover (Av. QV). Newey-West corrected t-statistics are in parentheses. Industry market correlations and standard deviations are calculated using a DVEC(1,1) model. Quadratic variations of industry turnover is estimated by a GARCH(1,1) model based on log changes in turnover. The regressions use 996 monthly observations with data ranging from January 1927 to December 2009.

	Av. CORR	Av. EXR	Av. STDV	Av. QV
Intercept	1.007	0.3739	0.29082	0.59651
	(25.3129)	(6.6404)	(7.1048)	(18.2762)
pd-ratio	-0.094924	-0.089417	-0.028203	-0.11399
	(-7.9479)	(-5.1959)	(-2.4477)	(-12.6464)
Adjusted R-squared	0.38805	0.14389	0.057661	0.47446

Table 7: **Empirics — Predictive Regressions.** This table reports in-sample coefficient estimates from OLS regressions of 1 year, 3 year and 5 year excess returns on 10 industry returns, and the market return on a constant and relative consumption, ω, implied by the $30 - log$ economy. The constant is not tabulated. Newey-West corrected t-statistics appear in parentheses below the coefficient estimate. The regressions use 83 annual observations with data ranging from 1927 to 2009. The table also reports one period ahead (for 1 year, 3 year and 5 year) nested forecast comparisons of excess returns on 10 industry returns and the market return. The comparisons are between a model which includes a constant and a model which includes a constant and relative consumption. Theil's U is the ratio of the root-mean-squared forecast errors for the unrestricted and restricted models. A number less than one indicates that the restricted model (with only a constant) has lower root-mean-squared error than the model with relative consumption as explanatory variable. The MSE-F statistic tests the null hypothesis that the MSE for the unrestricted model forecasts is less than the MSE for the restricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 3.467, 1.636 and 0.819, respectively. The ENC-NEW statistic tests the null hypothesis that restricted model forecasts encompass the unrestricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 2.566, 1.334 and 0.842, respectively. The out-of-sample regressions use 41, 40 and 39 annual observations with 83 total observation ranging from 1927 to 2009.

		NoDur	Durbl	Manuf	Enrgy	HiTec	Telcm	Shops	Hlth	Utils	Other	Mkt
1 year	Coefficient	-0.7645	-2.2214	-1.3620	-0.5428	-1.0977	-0.8883	-1.1093	-0.3020	-0.1509	-0.8508	-0.8969
	t-statistics	-2.7532	-3.7688	-3.2671	-1.4148	-2.2542	-2.0395	-3.7377	-0.9408	-0.2421	-2.3849	-2.8392
	Adj. R-squared	0.0375	0.1101	0.0828	0.0065	0.0348	0.0527	0.0526	-0.0061	-0.0108	0.0280	0.0457
	Theil's U	0.9731	0.8976	0.9225	0.9970	0.9603	0.9711	0.9550	0.9966	1.0073	0.9648	0.9536
	MSE-F	2.2993	9.8919	7.1780	0.2493	3.4613	2.4753	3.9562	0.2815	-0.5932	3.0437	4.0887
	ENC-NEW	1.6596	6.5249	5.2799	0.3866	1.9220	1.5971	2.5484	0.1757	-0.2791	1.8737	2.5655
3 year	Coefficient	-1.2595	-4.3025	-3.0497	-1.3942	-2.9738	-2.1803	-1.9299	-0.8658	-0.6338	-1.8276	-2.0394
	t-statistics	-2.0500	-3.3514	-3.6276	-1.8990	-4.1732	-4.6367	-3.3049	-1.4796	-0.8203	-3.2314	-3.8722
	Adj. R-squared	0.0386	0.2071	0.1817	0.0331	0.1000	0.1154	0.0710	0.0048	-0.0024	0.0561	0.1008
	Theil's U	0.9729	0.8257	0.8085	0.9896	0.9025	0.9491	0.9464	0.9859	0.9845	0.9344	0.8994
	MSE-F	2.2630	18.6636	21.1884	0.8427	9.1083	4.4055	4.6619	1.1552	1.2670	5.8098	9.4460
	ENC-NEW	1.9320	13.6465	17.5287	1.3477	5.3968	3.2575	3.4095	0.6535	0.7912	3.7521	6.4276
5 year	Coefficient	-1.2952	-4.9389	-3.4826	-1.0383	-3.5356	-2.5932	-2.2884	-1.3108	-0.8938	-1.9686	-2.2946
	t-statistics	-1.5359	-3.5891	-3.1063	-0.9750	-3.3304	-4.7007	-3.1951	-1.3886	-1.3221	-2.0412	-2.8981
	Adj. R-squared	0.0258	0.2259	0.1845	0.0053	0.1043	0.1087	0.0845	0.0118	0.0014	0.0443	0.0939
	Theil's U	0.9726	0.7830	0.7734	0.9784	0.9079	0.9453	0.9294	0.9863	0.9772	0.9420	0.8856
	MSE-F	2.2283	24.6203	26.2042	1.7443	8.3154	4.6483	6.1528	1.0913	1.8392	4.9469	10.7276
	ENC-NEW	1.5420	16.1885	18.2481	1.2092	4.8803	3.4253	4.2000	0.64116	1.2122	3.0579	6.7181

Table 8: **Size Sorted Portfolios — Averages.** This table summarizes OLS regression results (intercept, coefficient estimate and adjusted R-squared) of relative consumption as explanatory variable of the average of ten size sorted portfolio correlations (Av. CORR), 3-year ahead expected excess returns (Av. EXR), and standard deviations (Av. STDV). Newey-West corrected t-statistics are in parentheses. Size sorted correlations and standard deviations are calculated using a DVEC(1,1) model. Relative consumption is linearly interpolated from the heterogeneous investor model calibration employing annual consumption data from Robert Shiller's web page. The regressions use 996 monthly observations with data ranging from January 1927 to December 2009.

	Av. CORR	Av. EXR	Av. STDV
Intercept	0.92608	0.17465	0.3775
	(134.7149)	(7.4878)	(9.2808)
Relative consumption, ω	-0.083886	-0.6404	-0.92581
	(-2.057)	(-4.7258)	(-3.957)
Adjusted R-squared	0.038919	0.10923	0.44339

Table 9: **BM Sorted Portfolios — Averages.** This table summarizes OLS regression results (intercept, coefficient estimate and adjusted R-squared) of relative consumption as explanatory variable of the average of ten book-to-market sorted portfolio correlations (Av. CORR), 3-year ahead expected excess returns (Av. EXR), and standard deviations (Av. STDV). Newey-West corrected t-statistics are in parentheses. Book-to-market sorted correlations and standard deviations are calculated using a DVEC(1,1) model. Relative consumption is linearly interpolated from the heterogeneous investor model calibration employing annual consumption data from Robert Shiller's web page. The regressions use 996 monthly observations with data ranging from January 1927 to December 2009.

	Av. CORR	Av. EXR	Av. STDV
Intercept	0.89778	0.28147	0.37096
	(82.2972)	(4.2825)	(9.1625)
Relative consumption, ω	-0.19937	-0.91342	-0.96231
	(-3.1788)	(-2.5318)	(-4.1551)
Adjusted R-squared	0.10979	0.096109	0.49111

Table 10: **Momentum Sorted Portfolios — Averages.** This table summarizes OLS regression results (intercept, coefficient estimate and adjusted R-squared) of relative consumption as explanatory variable of the average of ten momentum sorted portfolio correlations (Av. CORR), 3-year ahead expected excess returns (Av. EXR), and standard deviations (Av. STDV). Newey-West corrected t-statistics are in parentheses. Momentum sorted correlations and standard deviations are calculated using a DVEC(1,1) model. Relative consumption is linearly interpolated from the heterogeneous investor model calibration employing annual consumption data from Robert Shiller's web page. The regressions use 996 monthly observations with data ranging from January 1927 to December 2009.

	Av. CORR	Av. EXR	Av. STDV
Intercept	0.8573	0.21641	0.3113
	(60.3286)	(4.8501)	(8.0865)
Relative consumption, ω	-0.18107	-0.40609	-0.63045
	(-2.135)	(-1.6217)	(-2.9175)
Adjusted R-squared	0.052038	0.029092	0.28267

Table 11: **Size Sorted Portfolios — Predictive Regressions.** This table reports in-sample coefficient estimates from OLS regressions of 1 year, 3 year and 5 year excess returns on 10 size sorted portfolios on a constant and relative consumption, ω, implied by the $30-log$ economy. The constant is not tabulated. Newey-West corrected t-statistics appear in parentheses below the coefficient estimate. The regressions use 83 annual (monthly are available) observations with data ranging from 1927 to 2009. The table also reports one period ahead (for 1 year, 3 year and 5 year) nested forecast comparisons of excess returns on 10 size sorted portfolio returns and the market return. The comparisons are between a model which includes a constant and a model which includes a constant and relative consumption. Theil's U is the ratio of the root-mean-squared forecast errors for the unrestricted and restricted models. A number less than one indicates that the restricted model (with only a constant) has lower root-mean-squared error than the model with relative consumption as explanatory variable. The MSE-F statistic tests the null hypothesis that the MSE for the unrestricted model forecasts is less than the MSE for the restricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 3.467, 1.636 and 0.819, respectively. The ENC-NEW statistic tests the null hypothesis that restricted model forecasts encompass the unrestricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 2.566, 1.334 and 0.842, respectively. The out-of-sample regressions use 41, 40 and 39 annual observations with 83 total observation ranging from 1927 to 2009.

		Low 10	Dec 2	Dec 3	Dec 4	Dec 5	Dec 6	Dec 7	Dec 8	Dec 9	High 10
1 year	Coefficient	-2.5331	-2.3642	-2.0631	-1.766	-1.5606	-1.522	-1.3315	-1.2363	-1.0641	-0.76187
	t-statistics	-3.7912	-3.9303	-3.4419	-3.678	-3.542	-3.5886	-3.4128	-3.3881	-3.0504	-2.4094
	Adj. R-squared	0.10335	0.12343	0.11171	0.093707	0.084875	0.086283	0.069929	0.072247	0.057857	0.034455
	Theil's U	0.90734	0.90559	0.90648	0.91603	0.93167	0.91258	0.94562	0.9418	0.94773	0.96313
	MSE-F	8.587	8.7749	8.6791	7.6691	6.0827	8.0307	4.7331	5.0969	4.5336	3.1212
	ENC-NEW	6.6204	7.1416	7.0036	5.7212	4.5479	5.6783	3.5434	3.7448	3.1748	1.8997
3 year	Coefficient	-5.4825	-5.0902	-4.4132	-3.915	-3.2542	-3.4706	-2.627	-2.4402	-2.3836	-1.9029
	t-statistics	-2.9939	-3.211	-3.0312	-3.7668	-3.4398	-4.2756	-3.0354	-2.9019	-3.4775	-4.1285
	Adj. R-squared	0.13887	0.17569	0.1748	0.17274	0.14305	0.17962	0.1134	0.12062	0.11901	0.089665
	Theil's U	0.88625	0.85769	0.86816	0.8739	0.89822	0.86268	0.91162	0.88741	0.88705	0.91651
	MSE-F	10.6536	14.0158	12.7447	12.0672	9.3394	13.4044	7.9286	10.5242	10.5642	7.4286
	ENC-NEW	9.312	12.8505	12.1593	10.5713	8.3344	11.5743	6.8144	8.5001	8.1435	4.7055
5 year	Coefficient	-6.2674	-5.5616	-4.7686	-4.5781	-3.6692	-4.0739	-2.7641	-2.5305	-2.668	-2.2446
	t-statistics	-2.6992	-2.7431	-2.7775	-3.4134	-3.1552	-3.8067	-2.4722	-2.4207	-2.7209	-2.9326
	Adj. R-squared	0.12387	0.15503	0.157	0.17529	0.14107	0.19233	0.10297	0.10742	0.11654	0.08405
	Theil's U	0.9001	0.86455	0.87416	0.87178	0.89524	0.85526	0.88538	0.86682	0.85659	0.90839
	MSE-F	8.9036	12.8402	11.7284	12.0001	9.4139	13.9507	10.4762	12.5739	13.7892	8.0513
	ENC-NEW	7.0617	10.0728	9.5515	9.7094	7.7083	11.1572	7.4234	8.497	9.217	4.7559

Table 12: **BM Sorted Portfolios — Predictive Regressions.** This table reports in-sample coefficient estimates from OLS regressions of 1 year, 3 year and 5 year excess returns on 10 book-to-market sorted portfolios on a constant and relative consumption, ω, implied by the $30 - log$ economy. The constant is not tabulated. Newey-West corrected t-statistics appear in parentheses below the coefficient estimate. The regressions use 83 annual (monthly are available) observations with data ranging from 1927 to 2009. The table also reports one period ahead (for 1 year, 3 year and 5 year) nested forecast comparisons of excess returns on 10 book-to-market sorted portfolio returns and the market return. The comparisons are between a model which includes a constant and a model which includes a constant and relative consumption. Theil's U is the ratio of the root-mean-squared forecast errors for the unrestricted and restricted models. A number less than one indicates that the restricted model (with only a constant) has lower root-mean-squared error than the model with relative consumption as explanatory variable. The MSE-F statistic tests the null hypothesis that the MSE for the unrestricted model forecasts is less than the MSE for the restricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 3.467, 1.636 and 0.819, respectively. The ENC-NEW statistic tests the null hypothesis that restricted model forecasts encompass the unrestricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 2.566, 1.334 and 0.842, respectively. The out-of-sample regressions use 41, 40 and 39 annual observations with 83 total observation ranging from 1927 to 2009.

		Low 10	Dec 2	Dec 3	Dec 4	Dec 5	Dec 6	Dec 7	Dec 8	Dec 9	High 10
1 year	Coefficient	-0.82169	-0.8233	-0.76847	-1.1144	-0.90286	-1.2255	-0.97298	-1.1637	-1.206	-1.5153
	t-statistics	-2.3926	-2.8631	-2.898	-2.8092	-2.32	-2.3933	-2.4825	-2.7033	-2.4605	-2.5837
	Adj. R-squared	0.026426	0.041171	0.034161	0.064196	0.036934	0.07056	0.036174	0.046697	0.044599	0.050486
	Theil's U	0.97336	0.95999	0.97043	0.96016	0.9571	0.93103	0.96008	0.93666	0.92978	0.9342
	MSE-F	2.2191	3.4037	2.4746	3.3882	3.6665	6.1463	3.3957	5.5929	6.2696	5.8331
	ENC-NEW	1.3681	2.1445	1.7087	2.7441	2.4577	4.3464	2.3702	3.7847	4.2616	4.0595
3 year	Coefficient	-1.9894	-1.8852	-1.4569	-2.436	-1.8727	-3.5065	-2.1905	-2.4929	-2.5124	-2.3348
	t-statistics	-3.1101	-3.8753	-2.9266	-3.003	-2.6954	-4.4321	-3.7471	-3.3561	-3.0991	-1.8417
	Adj. R-squared	0.07155	0.10998	0.059748	0.13227	0.066723	0.19868	0.09042	0.098653	0.084035	0.043883
	Theil's U	0.9377	0.90651	0.95494	0.93532	0.90063	0.84483	0.91744	0.85973	0.88695	0.90971
	MSE-F	5.3543	8.4587	3.7672	5.5802	9.081	15.6425	7.3346	13.7645	10.5757	8.126
	ENC-NEW	3.4442	5.7063	2.9289	5.9387	5.9431	14.7487	5.9752	10.2236	8.0908	5.7096
5 year	Coefficient	-2.4954	-2.3527	-1.5835	-2.4631	-1.7428	-4.1236	-2.4591	-2.4624	-2.6212	-1.3322
	t-statistics	-2.911	-3.1794	-2.3926	-2.3457	-1.6343	-3.9262	-2.6903	-2.2095	-2.3383	-0.75529
	Adj. R-squared	0.083094	0.12797	0.054045	0.10096	0.036314	0.19746	0.088374	0.067935	0.073144	0.0020927
	Theil's U	0.92249	0.88364	0.93753	0.91574	0.93415	0.80051	0.89156	0.8689	0.88681	0.96878
	MSE-F	6.6544	10.6666	5.2324	7.3151	5.5462	21.2986	9.8059	12.3314	10.3195	2.4883
	ENC-NEW	3.9994	6.7739	3.3904	5.7852	3.2813	17.3771	6.8504	7.7807	7.1821	1.5034

Table 13: **Momentum Sorted Portfolios — Predictive Regressions.** This table reports in-sample coefficient estimates from OLS regressions of 1 year, 3 year and 5 year excess returns on 10 momentum sorted portfolios on a constant and relative consumption, ω, implied by the $30 - log$ economy. The constant is not tabulated. Newey-West corrected t-statistics appear in parentheses below the coefficient estimate. The regressions use 83 annual (monthly are available) observations with data ranging from 1927 to 2009. The table also reports one period ahead (for 1 year, 3 year and 5 year) nested forecast comparisons of excess returns on 10 momentum sorted portfolio returns and the market return. The comparisons are between a model which includes a constant and a model which includes a constant and relative consumption. Theil's U is the ratio of the root-mean-squared forecast errors for the unrestricted and restricted models. A number less than one indicates that the restricted model (with only a constant) has lower root-mean-squared error than the model with relative consumption as explanatory variable. The MSE-F statistic tests the null hypothesis that the MSE for the unrestricted model forecasts is less than the MSE for the restricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 3.467, 1.636 and 0.819, respectively. The ENC-NEW statistic tests the null hypothesis that restricted model forecasts encompass the unrestricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 2.566, 1.334 and 0.842, respectively. The out-of-sample regressions use 41, 40 and 39 annual observations with 83 total observation ranging from 1927 to 2009.

		Low 10	Dec 2	Dec 3	Dec 4	Dec 5	Dec 6	Dec 7	Dec 8	Dec 9	High 10
1 year	Coefficient	-1.4134	-1.3645	-0.64953	-1.1966	-0.81324	-0.98901	-0.91562	-0.90842	-1.0725	-0.74576
	t-statistics	-3.2985	-3.1765	-1.8873	-3.3075	-2.4874	-3.2436	-2.5844	-2.4358	-2.7167	-1.5307
	Adj. R-squared	0.042124	0.063928	0.010663	0.068581	0.037633	0.053421	0.047874	0.04295	0.053237	0.0091011
	Theil's U	0.95863	0.95785	0.98744	0.94953	0.95497	0.94866	0.93964	0.95353	0.95291	0.98042
	MSE-F	3.527	3.5974	1.0238	4.3657	3.8612	4.4468	5.3037	3.9942	4.0511	1.6139
	ENC-NEW	2.2202	2.6941	0.67696	3.3608	2.3963	2.9122	3.3162	2.6541	2.8495	0.97364
3 year	Coefficient	-2.4182	-3.2404	-1.2261	-2.4368	-1.7952	-2.072	-2.3744	-1.9882	-2.3738	-1.9526
	t-statistics	-3.1389	-4.0428	-2.8487	-3.2629	-4.1023	-3.7515	-5.4936	-2.7736	-2.912	-2.4118
	Adj. R-squared	0.060771	0.16318	0.019003	0.11417	0.090235	0.098443	0.1265	0.081617	0.1037	0.043638
	Theil's U	0.94223	0.91922	0.98599	0.90756	0.89059	0.89092	0.85984	0.91074	0.91304	0.95019
	MSE-F	4.9292	7.1561	1.1161	8.3493	10.1706	10.1344	13.7511	8.0195	7.7828	4.1957
	ENC-NEW	3.3809	7.9184	1.0365	7.205	6.6044	6.8477	9.4183	5.2956	5.9867	2.7626
5 year	Coefficient	-3.0006	-4.2482	-1.6043	-2.776	-2.1352	-2.1092	-3.0671	-2.0398	-2.1098	-1.8455
	t-statistics	-2.5646	-4.3167	-2.664	-2.8204	-2.9244	-2.6365	-4.5601	-1.9102	-1.8279	-1.8869
	Adj. R-squared	0.071098	0.20035	0.026245	0.11867	0.08461	0.071213	0.1501	0.058496	0.058741	0.028411
	Theil's U	0.92636	0.85332	0.96323	0.85252	0.87129	0.89095	0.81768	0.93075	0.93319	0.95987
	MSE-F	6.2819	14.1862	2.9563	14.2846	12.0557	9.8713	18.8358	5.8653	5.636	3.2442
	ENC-NEW	4.1534	13.0081	2.1192	10.0006	7.2018	5.9492	12.3747	3.5552	3.6923	1.9515

Figure 1: **Distribution of Risk Aversion.** This figure shows the distribution of risk aversion of a non-atomic continuum of investors based on a transformed Beta distribution with parameters: $a = 1$ and $b = 10.0038$. We adjust the Beta distribution from 0 to 1 in the following way: $rv_{adj} = 1 + rv * (\gamma_{max} - 1)$ where $\gamma_{max} = 183.9705$. Thus, the maximum γ is 183.9705 and the minimum γ is 1. This distribution is obtained from a minimization of the difference between aggregate risk aversion in the heterogeneous investor model of Section 4 in the main text of the paper and aggregate risk aversion based on the transformed Beta distribution.

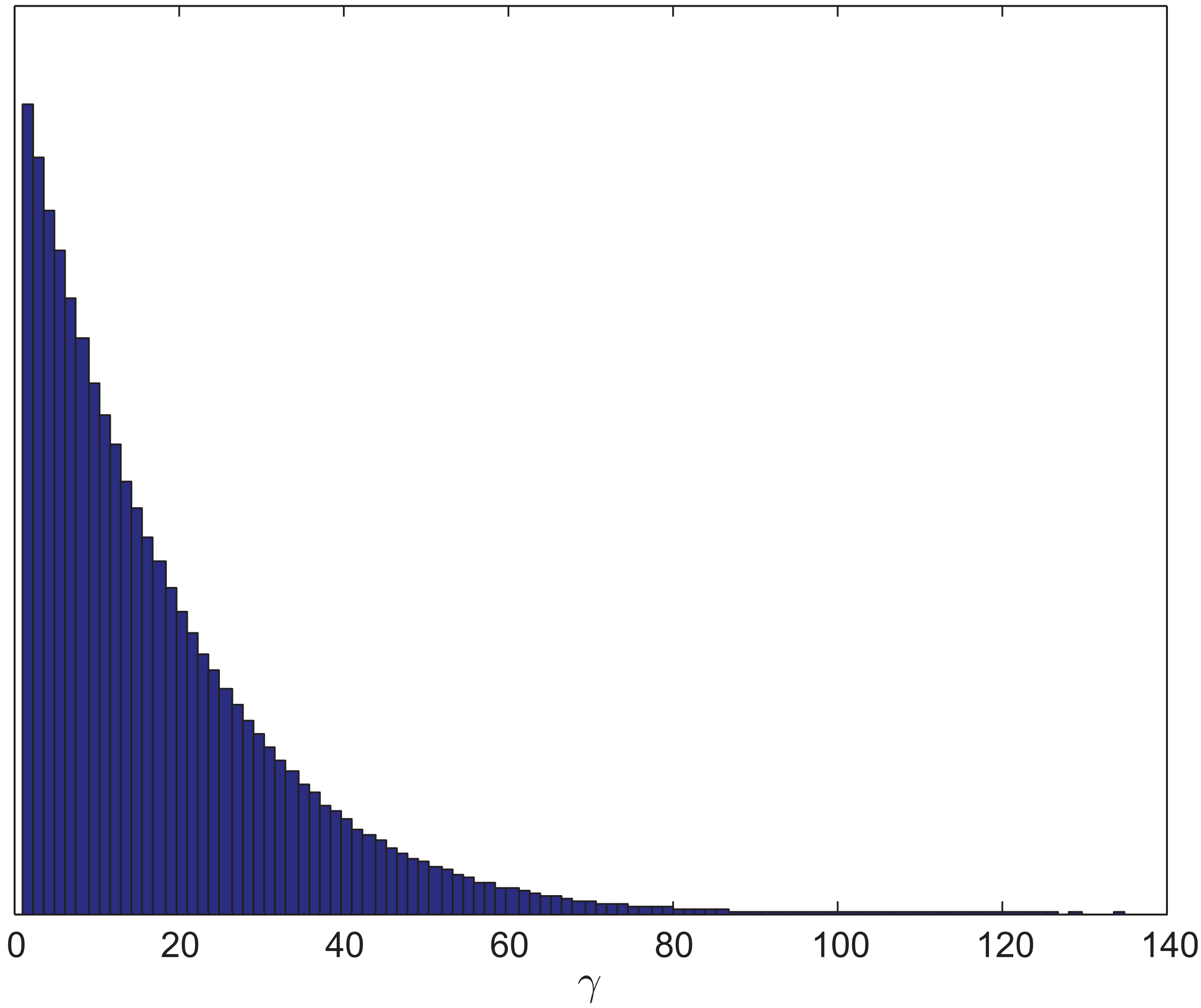


Figure 2: **Beta Distribution versus 30-Log.** This figure shows plots of the distribution of ω implied by the heterogeneous investor model (left plot), aggregate risk aversion of the heterogeneous investor models, $30 - log$ and a transformed Beta distribution, as a function of ω, and aggregate prudence of the heterogeneous investor models. For details of the $30 - log$ heterogeneous investor model see Section 4 in the main text of the paper.

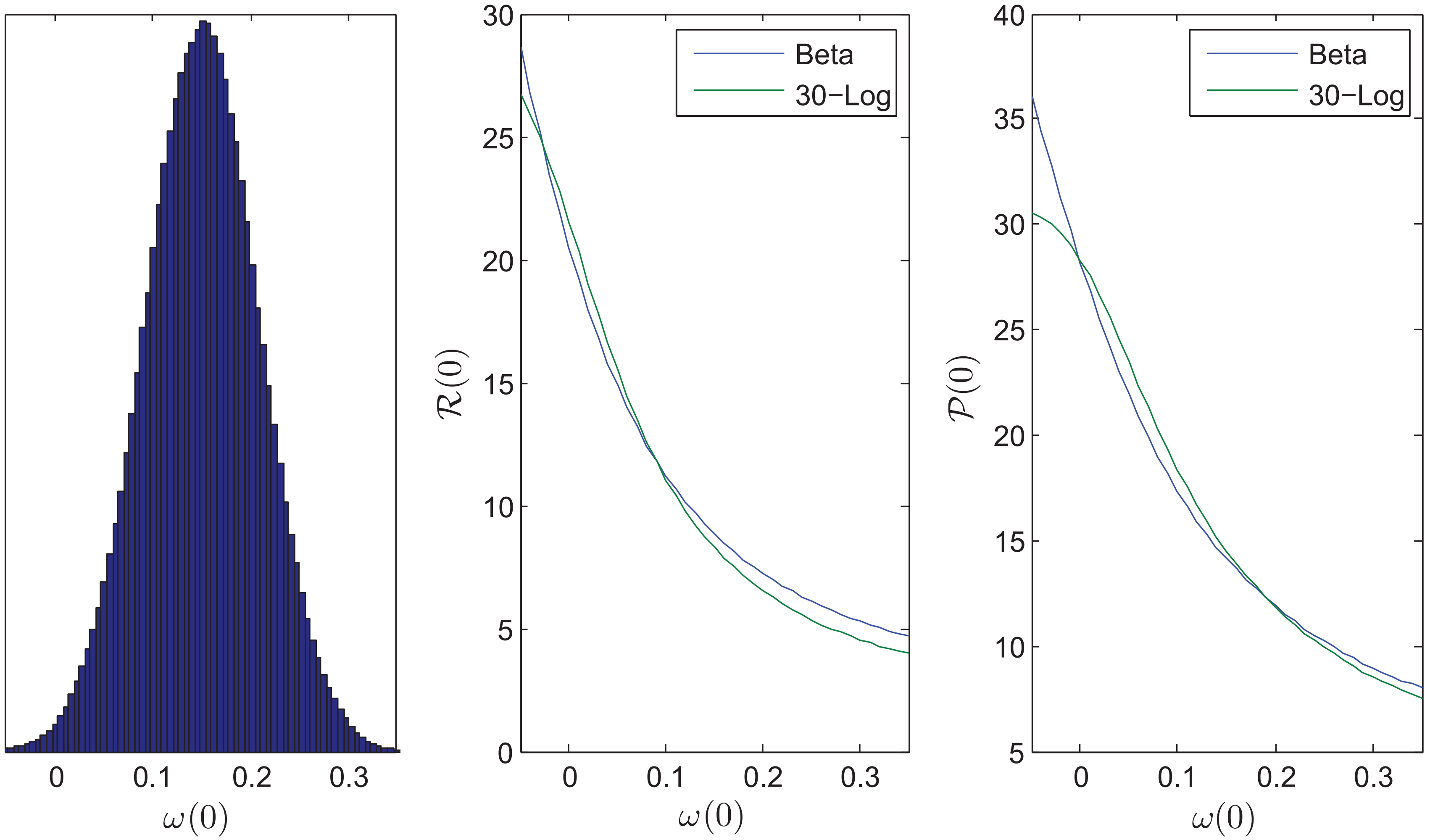


Figure 3: **Sensitivity of Equilibrium Correlation without Ratio Habits.** This figure shows plots with a sensitivity analysis of the conditional return correlations between stock 1 and stock 2 in a heterogeneous investor economy as functions of aggregate dividends, $C(0)$. Risk aversion coefficients are $\gamma_L = log$ and $\gamma_H = 3$. Fundamentals, other than the ones in the legend of the figures, are set as in Figure 4 in the main text of the paper.

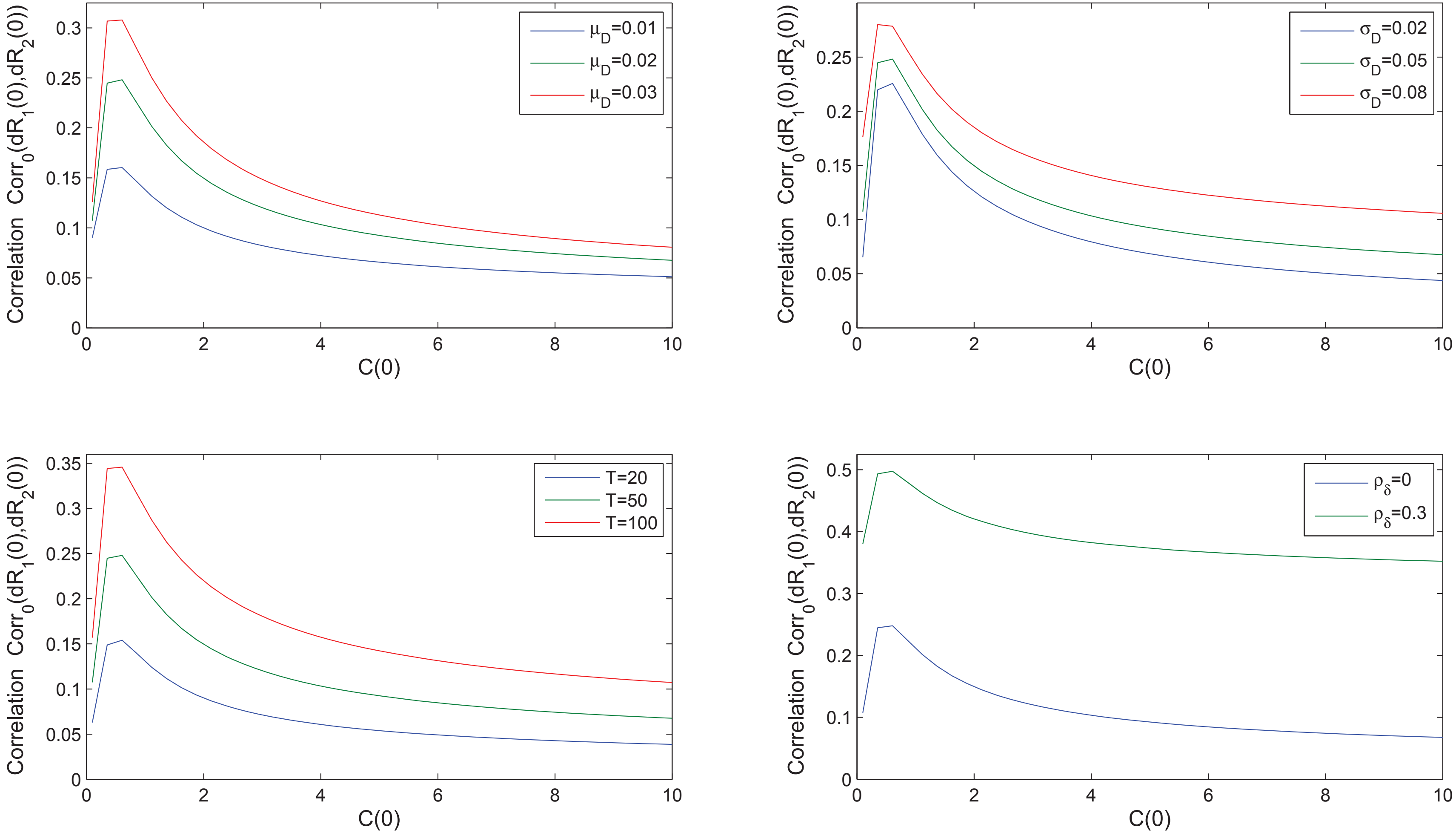


Figure 4: **Sensitivity of Equilibrium Correlation with Ratio Habits.** This figure shows plots with a sensitivity analysis of the conditional return correlations between stock 1 and stock 2 in a heterogeneous investor economy with ratio habits as functions of model implied habit, $\omega(0)$. Risk aversion coefficients are $\gamma_L = log$ and $\gamma_H = 3$. Fundamentals, other than the ones in the legend of the figures, are set as in Figure 4 in the main text of the paper.

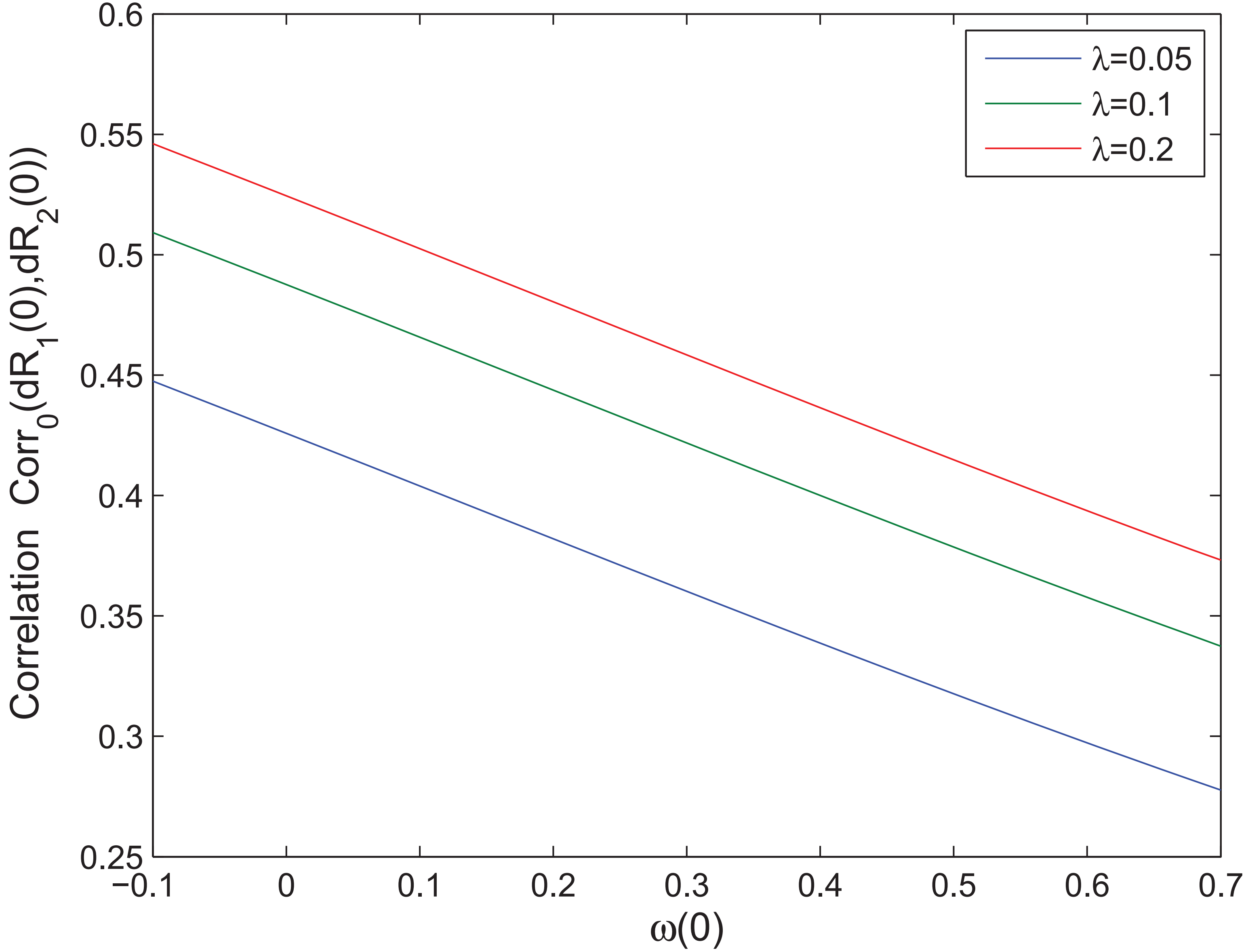
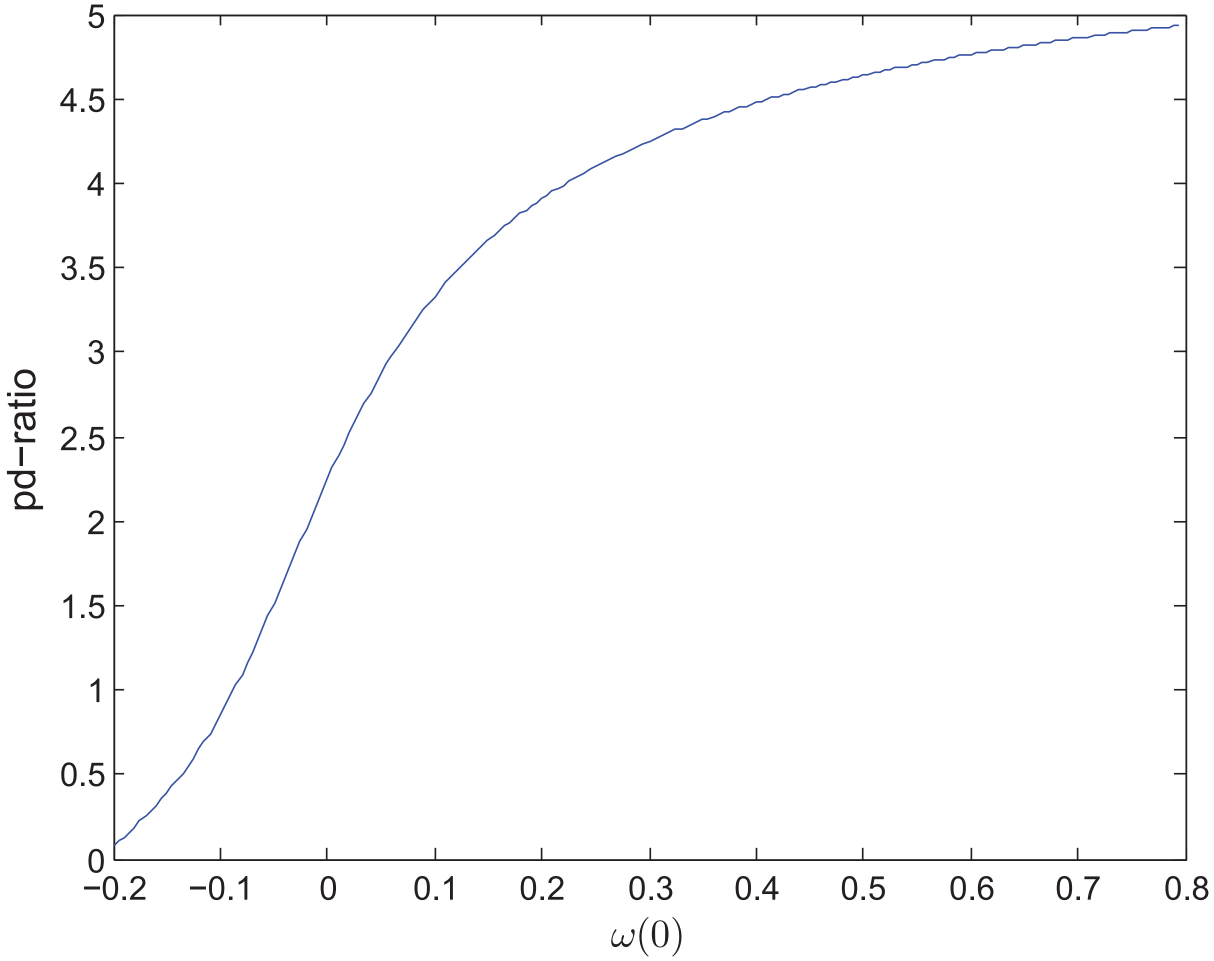


Figure 5: **Log Price-Dividend Ratio in Calibrated Economy.** This figure shows the log price-dividend ratio in the heterogeneous investor model. For details see Section 4 in the main text of the paper.



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