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Abstract

This article presents a dynamic growth model with energy as an input in the production function. The available stock of energy resources is ordered by a quality parameter based on energy accounting: the “Energy Return on Energy Invested” (EROI). To our knowledge this is the first paper where EROI fits in a neoclassical growth model (with individual utility maximization and market equilibrium), setting the economic use of “net energy analysis” on firmer theoretical ground. All necessary concepts to link neoclassical economics and EROI are discussed before their use in the model, and a comparative static analysis of the steady states of a simplified version of the model is presented.

Keywords: EROI, net energy analysis, growth, Ramsey-Hotelling, energy depletion.

JEL Classification: Q00, Q43, O13.

Resumen

Este artículo presenta un modelo de crecimiento dinámico con la energía como un input en la función de producción. El stock de recursos energéticos disponibles se ordena por un parámetro de calidad basado en la contabilidad energética: la tasa de retorno energético (TRE). Por lo que sabemos esta es la primera vez que la TRE encaja en modelo de crecimiento neoclásico (con maximización individual de la utilidad y equilibrio de mercado) estableciendo así el uso del “análisis de rendimiento energético" sobre una base teórica más firme. Todos los conceptos necesarios para enlazar la economía neoclásica con la TRE se discuten antes de ser usados en el modelo, y se presenta un análisis comparativo del estado estacionario de una versión simplificada del modelo.

Palabras clave: EROI, análisis de energía neto, crecimiento, Ramsey-Hotelling, agotamiento energético.

Códigos JEL: Q00, Q43, O13.

1 Introduction

The economic impact of energy depletion has been a classical issue in economics at least since “The Coal Question” (Jevons, 1865) was published, introducing the problem of the physical sustainability of a productive system significantly reliant on non-renewable resources. The description of the optimal path of depletion of a non-renewable resource was a problem solved using variational calculus in the earlier years of neoclassical economics (Hotelling,1931), but neoclassical growth theory (Cass,1965; Koopmans,1965) was based on production functions that only included capital and labour as inputs. The reasons for this deviation from the classical economic thought of Smith and Ricardo (where natural resources were the third production factor, named “land”), were mainly empirical: post-war in the United States, the importance of the agricultural sector was decreasing, and raw materials and energy sources were cheap and abundant.

On the other hand, in the two hundred years since the Industrial Revolution, economic growth has been related not only to an increasing level of productivity and capital accumulation, but also to *an equally sustained increase in energy use*. This happened in a feedback process, where technical progress itself (let us take the invention of the steam engine as the canonical example) created an increasing demand for energy (coal) and provided the means to accordingly increase supply (the steam engine was, first of all, used in coal mining). Energy use and economic growth moved (between 1890 and 1973) in a co-integrated fashion (Cleveland et al, 1984), but the statistical relation weakened after the oil crisis.

As a reaction to the neoclassical neglect of the extensive use of natural resources (especially energy) in industrial development, a theoretical body of economic thought emerged (Ecological Economics) stating that economic growth after the Industrial Revolution was based on the depletion of the stock of fossil fuels (Cotterel,1955; Hubbert, 1956; Georgescu-Roegen, 1971; Odum, 1976; Cleveland, 1999a; Mayumi 2001), and asserting that economic scarcity was at least partially derived from thermodynamic constraints.

A main tool used by ecological economists was “net energy analysis”, defined (Cleveland, 1992) as a “*technique for evaluating energy systems […] which compares the quantity of energy delivered to society by an energy system with the direct and indirect energy used in the delivery process”.* The technical development of net energy analysis was conducted by engineers to compute the energy life cycle of some products and installations (Thomas, 1977; Hendrickson, Lave & Matthews, 2006) and by ecological scientists, extending to human civilization the energy flow analysis developed for ecosystems (Odum, 1983). A relevant measure derived from net energy analysis is the EROI (“Energy Return on Energy Investment”), defined as “*the ratio of energy delivered to energy costs*”. These costs are the direct energy costs (fuel and electricity used in the process to obtain the final useful energy) and the indirect energy costs (the energy embedded in the capital goods used by the energy production sector).

The economic relevance of “net energy analysis” and particularly of EROI is still a controversial issue (Cleveland 1991, 2001), and is discussed in section 2 of this article. The first advantage of EROI is that it is a physical measure (instead of a monetary one). The classification of natural resources by “monetary costs” (Hotelling, 1931; Chakravorty, Roumasset & Kinping Tse, 1997) is a reasonable first approach to describe quality heterogeneous natural resources, but it cannot be directly used in a general equilibrium model, because monetary costs should be the result of the market interaction between demand (derived from subjective preferences) and supply, derived from the endowments of resources and production functions, which are the mathematical description of technologically feasible transformations of commodities into other commodities (Mas-Collel, Whinston & Green, 1995). Physical descriptions of resource scarcity (such as EROI) are the natural inputs to describe natural resource in general equilibrium models, while "monetary costs of extraction"-based models are making hidden hypotheses that can lead to significant biases when production conditions change significantly from their present state. For example, energy depletion could impact the replacement cost of capital or the cost of labour (wages), which are significant determinants of energy production costs; in a model where energy resources are classified by (fixed) production costs, these second-round effects of energy depletion in the cost of energy goods are not considered (Pearce, 2008; Kenny, Law & Pearce, 2010).

The second advantage of EROI is that it is an aggregate measure of energy efficiency at the scale of the whole economy. The physical efficiency of energy production processes is a determinant of prices and produced quantities of energy goods, and EROI is a sensible measure of energy efficiency that can be compared across very different energy sources. Further, average EROI of the energy production system of a given economy is an aggregate, and a more detailed description of the physical scarcity of the different energy sources (the ore grade of uranium mines for nuclear fuel, the thickness of seam for coal mines, the size and deepness of oil and gas fields) can be used in a Ramsey-Hotelling model to predict depletion paths of natural resources and the impact of natural resource depletion in consumption[1]. As usual in economics, there is a trade-off between low-level modelling (where detailed descriptions of physical scarcity and technology are used) and a high-level description of the relations between the economy and the energy system. In low-level descriptions, the exactness, precision and greater realism of the model imply more sensitivity to modelling choices and the description of the technology, while high-level models are less sensitive to particular technology and modelling choices (but still depend on high-level assumptions), and their results are more transparent and understandable.

In our opinion EROI is the most natural candidate to introduce efficiency in energy production (affected by natural resource depletion and technological change) in long-run macroeconomic models of growth, and that is precisely what we have done in this paper. While the issue of the macroeconomic impact of energy efficiency has been considered in previous articles (van Zon & Yetkiner, 2003), the energy efficiency parameters included in those models were theoretical, while EROI is an observable quantity, and its use in a long-run macroeconomic model could be a first step to the introduction of energy efficiency in growth accounting (Solow, 1957; Barro 1998).

Since 2000 there have been a few peer-reviewed articles and some books about the methodology and economic applications of EROI (Cleveland, 2001; Hall, 2009; Pimentel 2008). The interest in economic applications of EROI by environmental scientists was reflected in an array of publications in natural science reviews, including publications in Nature (Hall et al, 2003), the American Scientist (Hall & Day 2009), the Annals of the New York

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[1] See Chakravorty, Roumasset & Kinping Tse, 1997 for a similar model, but based on monetary extraction costs.

Academy of Sciences (Murphy & Hall, 2010) and “Ambio” (Mulder & Hugens, 2008) and a special issue of “Sustainability” (Hall, 2011).

This article makes two original contributions: first (see section 4) in the conceptual realm we point out that EROI is a leverage ratio between energy as an input and energy as an output for the energy production sector of an economy, and its economic impact depends on the non-energy costs of running the energy sector. Secondly, the previous observation allows us to naturally include EROI in a neoclassical growth model (section 6).

To summarize the conceptual contribution, the economic impact of a change in the EROI of the energy sector of a society depends on some non-energy cost of the expansion and maintenance of the energy production sector of that society, because if the energy production sector could be expanded for free, any decline (not below one) in the EROI of the available energy sources (which is an efficiency ratio) could be compensated for free of charge by an expansion of the scale of activities of the energy sector. Once EROI is properly interpreted and the “cost of leverage” is modelled, the EROI parameter naturally fits in a neoclassical growth model[2]. We present that model in section 6.

The economic impact of the reduction in EROI that our society would face as the result of the depletion of the best-quality fossil fuels has been a permanent concern for ecological economists. There have been many qualitative hints in the Ecological Economics literature along the lines that the impact of the reduction of EROI on final consumption, welfare and population could be significant and even catastrophic: an article by Manno (2011) describes alternative societal scenarios under the hypothesis of a declining EROI, some of them implying societal collapse or generalized authoritarian regimes to manage the transition from a high- to a low-EROI society, while Hall & Day (2009), after reviewing the history of the controversy between mainstream and ecological economists, warn of the possibility of a sudden reversal in economic growth (“peak everything”) as a result of peak oil.

Apart from qualitative assessments of the earlier-discussed impact of EROI decline on consumption, Bassi, Powers & Schoenberg (2009) published the results of a detailed simulation exercise, where average EROI for US energy production is explicitly used to model the economic outcomes of energy depletion. Their results for 2050 are that with a change in the EROI of oil and gas from around 11 (2005) to around 6 (2050), the “discretionary consumption” (consumption after basic needs are met) will reduce its weight in all economic flows from 36% to 15% (implying a severe reduction in total per capita consumption) and discretionary investment (beyond capital replacement) would totally disappear. The weight of energy production in all economic flows would increase from around 10% to 22%. This forecast exercise (not based on standard economic equilibrium tools) implies a severe reduction in personal consumption mainly derived from the impact of depletion in the EROI of the energy sector. On the other hand, in this paper the impact of a reduction in average EROI for the energy sector from 10 to 5 would imply (depending on capital-energy substitutability) a reduction in per capita consumption of less than 10%, a significant but not catastrophic decline, but our impact computation is based on steady-state comparative statics (the cost of transition is not taken into account) and does not include the effect of population growth.

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[2] The use of net energy analysis in neoclassical economics has been unusual only with a few interesting applications, mainly concentrated in international trade: Baumol & Wolff (1981), Hong (2007).

The following sections of this article are organized as follows: in section 2 the main concepts of energy accounting and net energy analysis (with a special emphasis on EROI) are presented. Section 3 sets out a description of mineral resource accounting and some stylized facts on natural resource depletion (with a special focus on energy). Then, some limitations of classical natural resource accounting are discussed and, in order to overcome them, a mathematical description of global energy scarcity based on EROI is proposed: the energy auto-consumption curve. In section 4, the relationship between the stocks of reserves of primary fuels, the capital installed in the energy sector and the flow of useful energy delivered into the economy is explored. Section 5 presents a mixed CES-Cobb-Douglas production function with energy, capital and labour that allows a flexible specification of long-run capital-energy substitution. In section 6 we present the Ramsey-Hotelling-EROI model whose elements were discussed in the previous sections. In Section 7 the steady-state of the model for different levels of energy auto-consumption is computed, and a comparative static analysis of these steady states is performed. Section 8 draws the conclusions of the paper and points to possible avenues for our future research.

2 Energy accountancy: EROI and its economic relevance

The first issue before discussing energy accountancy is to assess to which extent “energy” is really a composite good. The oil, gas, coal or uranium, which form the bulk of primary fuels are different goods that are bought and sold in different markets and whose prices do not always move together. In addition, these primary energy sources are used to produce different forms of secondary energy, being the most important the liquid fuels for internal combustion engines, heat, and electricity.

In addressing these goods jointly it should be considered the degree of substitutability among them. Empirical work has found a low substitutability between energy and capital and limited inter-fuel substitution in the short-term, and a higher one is found in the medium and long-term (Sweeney, 1984; Atkenson & Kehoe, 1999). This is not a surprise, because with a given stock of installed capital there is a very tight relation between economic output and energy inputs. On the other hand, in the long run, capital installation decisions depend on the relative prices of different energy inputs, making the long-run fuel demand far more elastic.

From a more fundamental viewpoint, there are known physical processes that allow the transformation of any form of free energy[3] into other: that is, chemical energy can be easily turned into a temperature differential (combustion), a temperature differential can be turned into mechanical work (in turbines or engines), mechanical work can be transformed in electricity (with an electric generator), that can be turned into a chemical potential (in a battery) or a temperature differential (through a resistance). With a positive (but arbitrarily small) increase in entropy (a reduction of free energy) and the use of physical capital, energy forms can be transformed among themselves. Additionally, the laws of thermodynamics imply that any macroscopic process has to be sustained with the consumption of free energy. Both facts point to the fact that energy is an essential input for all economic processes and that there are physical processes that guarantee substitution between different energy sources.

As a result of the fundamental laws of physics and the stylized facts from energy economics we consider that (at least in the long-run) energy can be aggregated as composite good.

As explained before, a measure of the cost of delivering a unit of useful energy into the economic system is the EROI. The definition of EROI, (*the ratio of energy delivered to energy costs)*, can be expressed mathematically. Supposing that the production of E units of the final deliverable energy needs the use of $E_1,\ldots,E_n$ thermal units of the energy sources 1 to n, the EROI is:

$$EROI = \frac{E}{E_1 + \ldots + E_n}$$

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[3] The energy that can be used to do useful (macroscopic) work. In a closed system, where the energy of the system if fixed by the First Law of Thermodynamics , free energy decreases when the entropy of the system increases.

A well-known problem of net energy analysis is the issue of energy quality. In some net energy analyses, inputs and outputs of different types of energy are aggregated by their thermal equivalents (measured in British thermal units, joules, or equivalent barrels of oil), as in the previous formula. The thermal equivalent approach ignores the important fact that there are relevant economic characteristics beyond heat content: a thermal unit of electricity is able to generate more economic output than a thermal unit of petroleum, and a thermal unit of petroleum generates more economic output than the same thermal unit of coal (Kaufmann, 1994). Most net energy analysts accept the principle of energy quality correction, but no standard methodology is still on place.

All methodologies for quality-corrected energy accounting are based on the "quality correction" of the original definition of EROI. Being $\lambda, \lambda_1, \ldots, \lambda_n$ quality correction weights, the quality corrected EROI is:

$$EROI\_q = \frac{\lambda E}{\lambda_1 E_1 + \ldots + \lambda_n E_n}$$

Plain heat-content EROI is a metric based in the First Law of Thermodynamics (energy conservation law), while quality-corrected EROI is associated with the Second Law of Thermodynamics, which creates a hierarchy among energy forms based in the ability of energy to do useful (macroscopic) work. Some entropy-based methodologies for quality correction in energy analysis have been developed (exergy and emergy analysis, for example). But all thermodynamic measurements of quality are based on ideal energy cycles and not in the actual use energy by the economic system. To build economically relevant energy quality correction weighs, it has been proposed (Cleveland, 1992, 2001) the use of some long-run averages of relative energy prices. Cleveland's methodology is supported by Kauffman's (1994) paper, that concludes that *"rational agents manipulate their use of coal, oil, natural gas and electricity so that the marginal product of these energies adjust to changes in the relative prices of these energies."*

In order to include both direct and indirect energy costs of an economic process there is a methodology named Economic Input-Output Life Cycle Analysis[4] (Hendrickson, Lave & Matthews, 2006). This methodology takes a given economic process, computes the direct inputs used by that process, and for all those inputs, the input-output Leontief matrix in inverted to obtain the energy costs incurred across the added value chain from the primary sources (raw materials) to the considered output. The combination of life-cycle analysis and input-output accounting provides a comprehensive and system-wide computation of the energy costs of a given economy (Murphy, Hall, Dale & Cleveland, 2011; Henshaw, King & Zarnikau, 2011). When applied to an economic process whose output is an energy product (the gasoline production process from the oil rig to the service station, or coal produced electricity from the coal mine to the plug) the result of dividing (quality corrected) energy outputs by quality corrected energy inputs is precisely the quality corrected EROI.

The economic relevance of net energy analysis is the subject of a hot controversy, between those who consider that the only relevant measure of scarcity is the market price of different fuels, and those that consider energy analysis as important, or even as the most

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[4] An interesting resource to perform Economic-Input Output Life Cycle Analysis is the EIO-LCA model, developed by the Green Design Institute of the Carnegie Mellon University (www.eiolca.net).

important measure of scarcity. In Cleveland (1992) three main arguments are given for the relevance of EROI: net energy analysis assesses the change in the physical scarcity of energy resources, and therefore is immune to the effects of market imperfections that distort monetary data; it is a measure of the potential to do useful work in economic systems; and consequently quality corrected EROI can be used to rank alternative energy supply technologies according to their potential abilities to do useful work in the economy.

The three arguments are relevant, but in our opinion, there is no contradiction between market price and EROI. Competitive market price is (at least under perfect competition and rational expectations) the best available measure of economic scarcity. But economic scarcity is not only the result of the subjective preferences of the agents and the structure of production, but also the result of physical scarcity. Conversely, physical scarcity is an essential determinant (but not the only one) of economic scarcity, which combined in the market process with the production function and the subjective preferences of the agents, generate the best indicators of economic scarcity: the market prices of the different energy sources. In our opinion EROI plays a role in energy economics similar to "ore grade" in mineral resource economics. No mineral resource economist deny the relevance of marginal ore grade to understand the price level of a given mineral (Phillips & Edwards, 1976); on the other hand models of energy prices use ore grade of available resources as a relevant determinant of prices, but of course, not a substitute for them (Shinkuma & Nishiyama, 2000).

Some measures are better than EROI to capture physical scarcity of a given energy source: the ore grade of uranium mines for nuclear fuel, the thickness of seam for coal mines, the size and deepness of oil and gas fields, etc. But all these physical measures are incommensurable among different energy sources. On the other hand, EROI has the advantage of being a mostly physical measurement, and a uniform one across different energy sources. That is, the average quality corrected EROI provides a straightforward comparison between American coal, Saudi oil and Russian gas and provides a one-dimensional and robust (physically based) metrics for energy scarcity, and the time evolution of EROI measures energy resources depletion (Cleveland, 2001). In the following table we provide estimations of the average quality corrected EROI of some energy sources[5].

Quality Corrected EROI for selected energy sources

Energy Source	**Quality corrected EROI**	**Reference**
Oil & Gas, US,1954	18	Cleveland, 2001
Oil & Gas, US,1974	12	Cleveland, 2001
Oil & Gas, US,1997	11	Cleveland, 2001
Refined gasoline, 1997	~7	Cleveland, 2001
Coal, mine mouth, US, 1954	~25	Cleveland, 1992
Coal, mine mouth, US, 1987	~25	Cleveland, 1992
Wind	10-15, site specific	Kubiszewski, Cleveland, Endres, 2009
Solar Photovoltaic	~4	Battisti and Corrado, 2005
Biofuels	<1	Pimentel, 2008

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[5] The case of nuclear energy is involved in such a controversy with estimations that range from less than one to more than ninety that I have decided to no include it in the table, while hydropower is very much site specific, and "average EROI" does not make much sense for it.

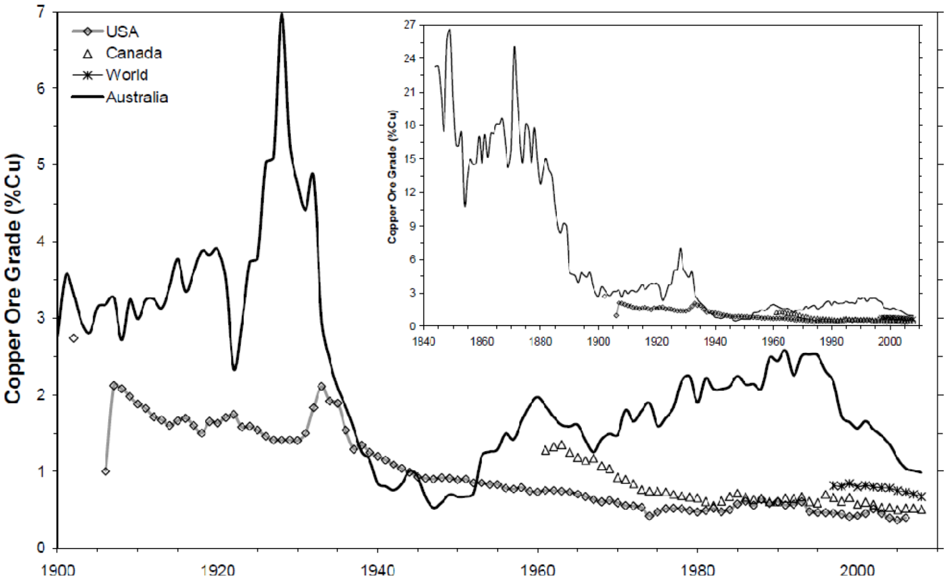
For mathematical convenience, we define “energy auto-consumption” as the inverse of the EROI (the amount of quality corrected energy that is consumed in energy producing processes to deliver a final unit of energy to the non-energy sector of the economy). Auto-consumption moves between 0 and 1 for energy sources (it is bigger than 1 for energy sinks). From now onwards, in this paper both EROI and auto-consumption are quality corrected, unless otherwise indicated.

$$Auto-consumption = \frac{1}{EROI\_q} = \frac{\lambda_1 E_1 + ... + \lambda_n E_n}{\lambda E} = \frac{Quality\text{-}corrected\text{-}energy\text{-}spent}{Quality\text{-}corrected\text{-}energy\text{-}obtained}$$

3 Resource quality, depletion and the energy auto-consumption curve as a comprehensive representation of mineral and energy resources

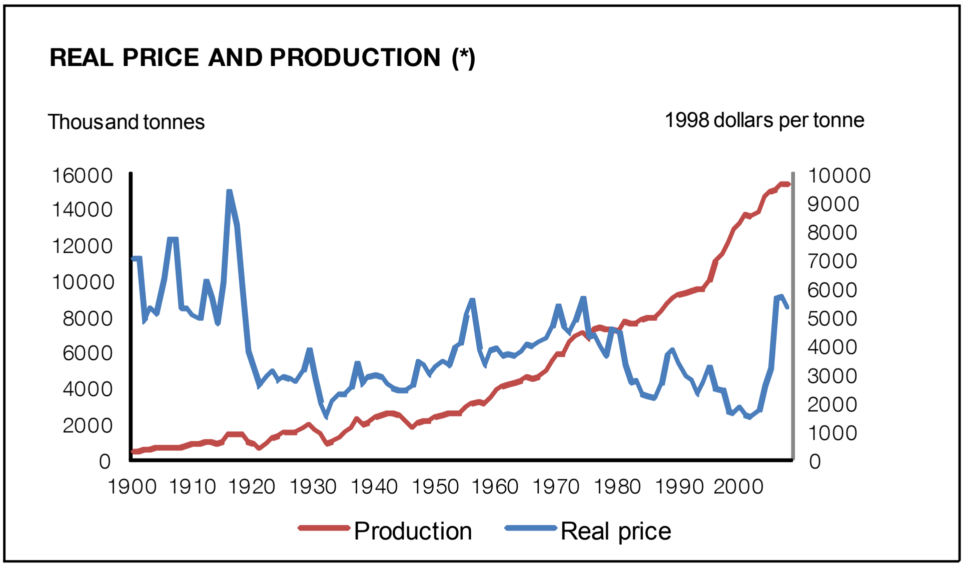
The economic theory of resource depletion is well known at least since Ricardo used the concept of marginal productivity of the land factor as the basis for his theory of land's rent. In almost any country in the world large tracts of uncultivated land exist side by side with a positive income of the land factor: Ricardo managed to explain this apparent paradox pointing out that the rent is not paid by the absolute scarcity of the resource, but by the relative scarcity of high quality land (Ricardo, 1817). Similarly, the total amount of free energy available to the world economy could be virtually infinite, and yet the depletion of the cheapest energy sources would continue to have economic consequences, as it would require the use of larger volumes of labour and capital to obtain the same amount of economically useful energy.

In mineral accounting (Nordhaus, 2006) "reserves" are defined as the quantity of known recoverable resources with the available technology in the neighbourhood of the current market prices. The stock of proven reserves of a mineral is determined by the result of three accountancy flows: *production, discoveries* and *reclassifications*. Production reduces the amount of proven reserves by the produced quantity, where discoveries increase it by the discovered amount. Reclassifications link the accountancy flows with economic forces: when prices increase or technology improves, some old resources not economically recoverable can turn profitable, and consequently they are "reclassified" as reserves. Two opposite forces dominate the history of metallic mining in the last two centuries: on one hand, there has been a sustained reduction in ore grades, both in average value and variance (Mudd, 2009; Philips and Edwards, 1976). In line with the economic theory, in the beginning of geological exploration there are swings in average ore grades, related to the discovery of new mining provinces, but when geological knowledge improves, the rule of resource extraction by order of cost drives the market (Reynolds, 1999). This means (see next graph, from Mudd 2009) that the reduction in the quality of the mineral resources as a result of depletion is a well-documented reality.



On the other hand, real prices of metals look stationary and production[6] growths exponentially, without a reduction in reserves (see next graph, for the case of copper, which is typical). This means that during the Industrial Revolution, productivity in the discovery and extraction branches of the mining business have always been able to overcome the economic effects of depletion. If we make the hypothesis that productivity growth can be indefinitely sustained, and the exhaustible resource is substitutable in the production function the exhaustible resources can be used in a sustainable fashion if any consumption of natural capital is compensated with an increase of the stock of produced capital (Hartwick 1977, Dasgupta 2001, Ridley 2010; World Bank, 2010; Gelb, Kaiser, Viuela, 2011). On the other hand, for models with capital depreciation, no productivity growth and an essential exhaustible resource (like the one proposed in this article), no positive level of consumption is sustainable (Dasgupta & Heal, 1974).

COPPER, WORLD, 1900-2010



(*) Only primary production, recycling excluded Source: US Geological Survey

In the case of fossil fuel mining we find the same stylized facts that we have discussed in the case of metallic minerals (see next graph). Reserves increase in the 1970-2010 period: that is, discoveries and reclassifications have been able to more than replace production. Real prices do not show any discernible trend: their high levels in the first decade of the XXI century are analogous to those reached in the seventies of the XX century. In any case, physical limitations look more binding in the case of oil than in the case of natural gas: oil prices are more volatile and its reserves to production ratio is higher (46 years for oil, 58 for gas). Coal, on the other hand, show both very low price volatility and an extremely high reserve to production ratio (118 years).The estimated total amount of available oil (the main energy source) is involved in much controversy (Hirsch, Bezdeck & Wendling, 2005; Cleveland & Kauffman,1991), and indeed the date on which its production will peak.

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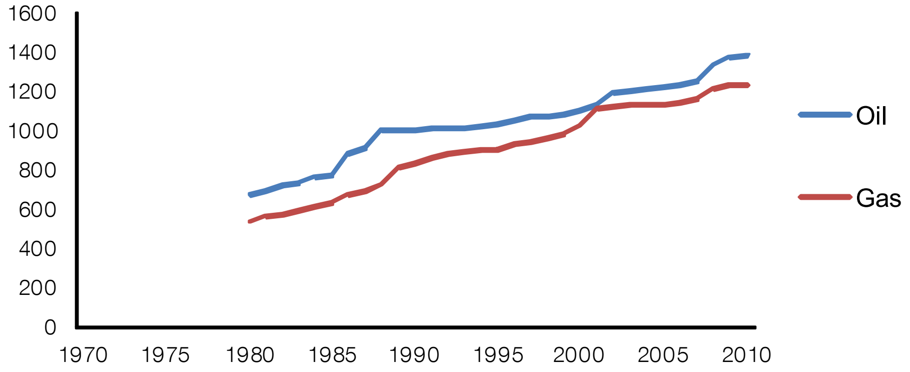
[6] Production from mines, excluded recycling.

FOSSIL FUELS, WORLD, 1970-2010

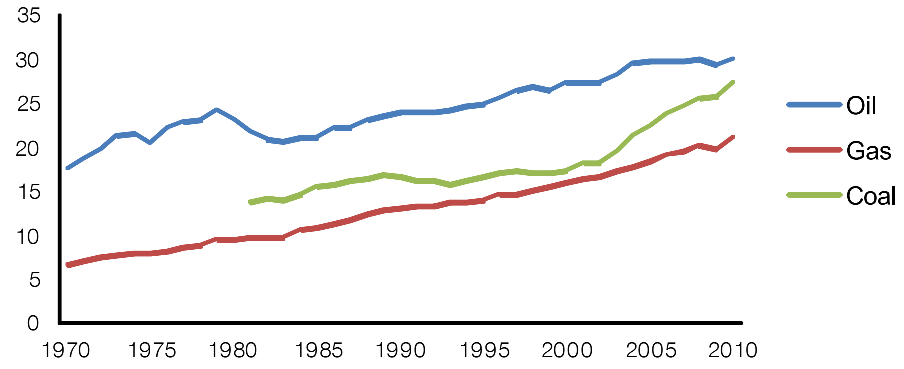
PROVEN RESERVES

Thousand million thermal equivalent oil barrels



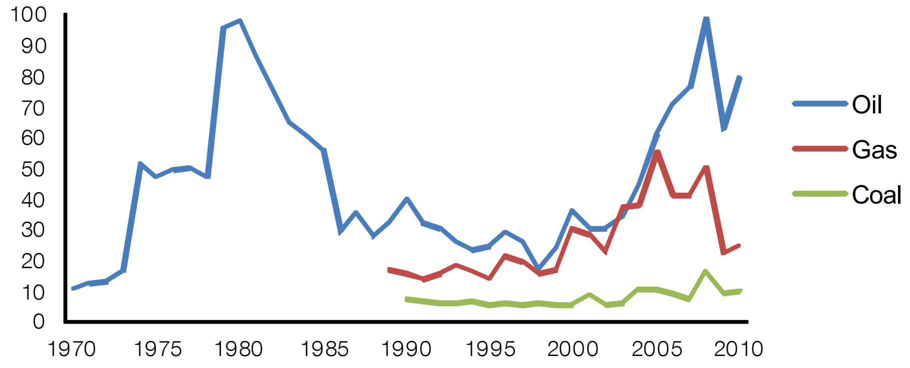
PRODUCTION

Thousand million thermal equivalent oil barrels



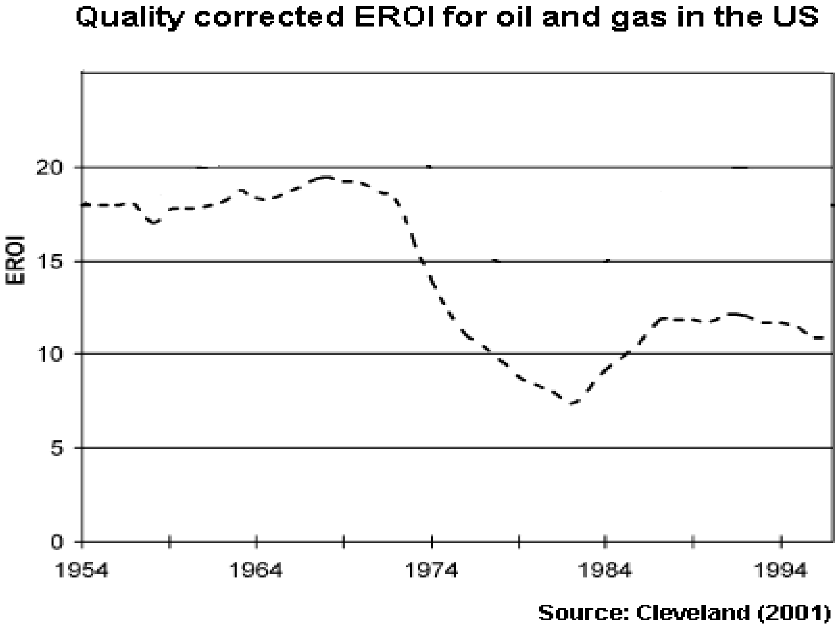
REAL PRICE

2010-real dollars per thermal equivalent oil barrels (year average)



Source: BP Statistical Review of World Energy

Using EROI as the quality measurement for the fossil fuel resources (see graph from Cleveland 2001) the same depletion pattern of the rest of the mining industry is apparent: the quality of resources declines over time (EROI decreases) for the oil and gas industry in the US, and EROI plays the same role as a quality indicator in the fossil fuel mining that ore grade plays in metallic minerals mining.



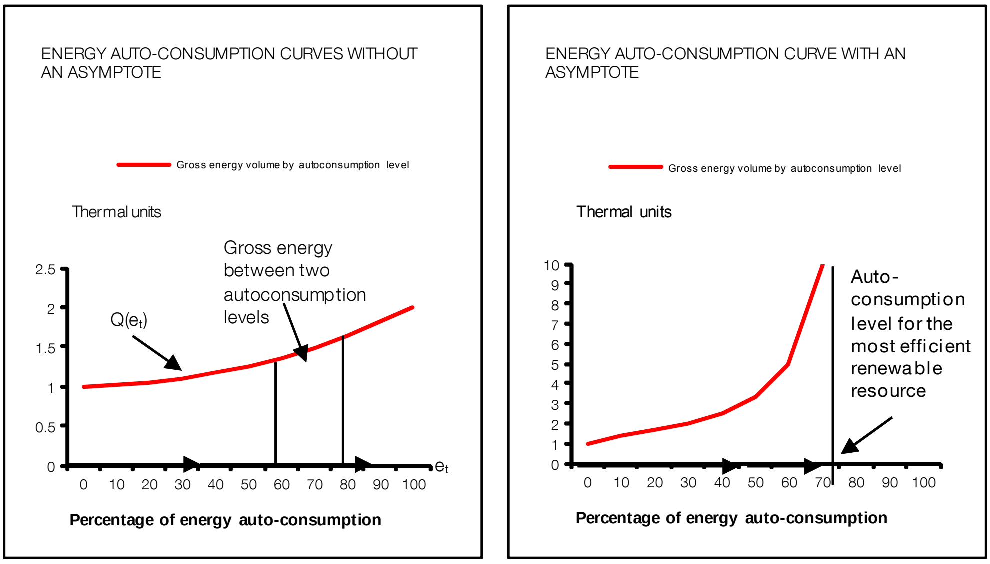
Source: Cleveland (2001)

In any case, the mere display of the stock of reserves of the main primary fuels (oil, gas, coal and uranium) is incomplete because infra-marginal resources that are not presently profitable to recover are not included in the proven reserves, but some of them will become profitable in the course of the depletion process. From an economic standpoint, a more comprehensive economic description of the available resources of a given mineral commodity would be a curve classifying its stock by extraction cost. Although no institution is currently publishing these curves for the world reserves of the main primary fuels, a similar analysis was done for the US coal resources (Zimmerman, 1977).

With energy auto-consumption as the measure of relative depletion, we can classify all energy resources (measured in quality corrected thermal units) by their auto-consumption levels. The auto-consumption curve would take for every level of auto-consumption the total amount of energy available at that energy auto-consumption level (we will call that curve Q(.) in the rest of this paper).

In the chart below, the sketch of an auto-consumption curve is shown, giving for each level of auto-consumption (percentage) the volume of energy (corrected for quality) in thermal units. In the left panel energy available to the world economy is finite, while on the right one there is a scalable renewable energy source (represented as an asymptote).

ENERGY AUTO-CONSUMPTION CURVES



Given the Q(.) curve the total energy available for the World economy would be:

$$E_{gross} = \int_0^1 Q(s)ds$$

But, a part of that energy would have to be used to produce energy for the energy sector itself, so net deliverable energy would be:

$$E_{net} = \int_0^1 (1-s)Q(s)ds$$

The energy auto-consumption curve will be the one-dimensional representation of physical energy scarcity in our model.

4 Installed capital in the energy sector: the link between the stock of reserves and current production

There is not controversy about the fact that EROI (or energy auto-consumption) is a good measure of absolute depletion. The economic system (as all macroscopic systems in general) should be able to obtain free energy from its environment, and it cannot work based on energy sources that deliver less useful energy than they consume. Energy sources with an EROI smaller that one (more than 100% auto-consumption) are "energy sinks", instead of sources.

But for energy sources with an EROI higher than 1 (auto-consumption smaller than 100%), the relevance of EROI is not so direct. Let's suppose that an economy runs its energy sector with an EROI of 10 (10% auto-consumption) and the energy sector produces 100 units of energy for the rest of the economy (it produces, then, 100/(1-0.1)=111 energy units, consuming itself 11.1 energy units). When the EROI decreases to 5 (20% auto-consumption), in order to produce 100 units of energy for the economy, the energy sector would need to produce (100/(1-0.2)=125) total units. Following this example it is obvious that for every EROI higher than 1 (auto-consumption smaller than 100%), it is always possible to expand the activities of the energy sector to attain any desired level of energy production.

As a consequence, the problem with a decrease in the EROI of the energy sources in use is not that it is necessary to use more energy to produce energy, but that it is necessary a bigger amount of labour (both current and/or stored in form of capital) to deliver the same energy to the economy as a whole. Then, any model trying to assess the economic effects of a decrease in the EROI of the energy sector should explicitly model some non-energy cost that is supposed to be linked to the expansion of the energy sector (if that expansion is free in terms of all non-energy inputs, the EROI is not a good measure of relative depletion). In our model, the non-energy cost linked to the expansion of the energy sector when EROI decreases will be the cost of the installed capital in the energy sector.

The most important issue in the short and medium term controversy on energy shortages and their impact is referred to the difficulties in converting reserves into production when the world economy is forced to use more marginal energy deposits (Robelius, 2007). Turning reserves into production is not a process that can be made at will: even though on many occasions (notably 1973) oil prices have increased very significantly, the ability of the oil industry to meet these price increases with a fast supply expansion has proven to be limited. The reason for this rigidity is that turning reserves into production requires the installation of an additional stock of capital[7]. The capital stock necessary to produce a unit of energy is higher the more marginal (higher energy auto-consumption) is the energy resource to be extracted.

In the model that will be presented in the next section it is assumed that the amount of energy that can be obtained with a given stock of extractive capital is proportional to the level of energy auto-consumption of the energy source we are considering. For example, if a given amount of extractive capital is capable of extracting 100 (quality corrected) thermal units of energy in a 10% energy auto-consumption site, for a 20% auto-consumption site, the same amount of capital can extract only 50 (quality corrected) thermal units of energy: this means that we suppose that the energy sector has a capital-energy fixed proportions production function.

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[7] Apart from the lags related to the geological exploration process that will not be considered in this paper.

5 Energy-capital substitution: the CES-Cobb-Douglas production function

In the late seventies, after the oil crisis, there was a great deal of controversy between those who considered that resources put a cap to growth, and those who considered that increasing productivity and resource substitution could always allow to the world economy to grow even when some finite resource is essential for production (Stiglitz, 1974; Georgescu-Roegen, 1975). After that, ecological economists and mainstream energy economics have continued the controversy about the energy-capital substitution, with ecological economics having a more pessimistic view of substitution limits, and mainstream energy economists being more optimistic on the long-run value of this parameter.

There is a significant empirical evidence of long-run substitutability between energy and capital. In Sweeney (1984) the following stylized facts on inter-fuel and capital-energy substitution are stated:

"*Demand responses to higher energy prices typically involve substitution of other factor for energy, one energy carrier for another or both mechanisms".*

"Most energy is used in conjunction with long lived capital equipment which once in place has fairly fixed energy requirements per unit of equipment use"

"Long-run energy price adjustments tend to be substantially greater than adjustments occurring after several years or even a decade. Thus conservation and inter-fuel substitution motivated by price raises can continue to increase for many years after prices stop rising".

In order to account for the discrepancies between long and short-run substitution, a putty-putty capital model of energy demand can be used (Atkenson & Kehoe, 1999). Our interest in the model developed in this article are the long run trends in consumption and welfare under energy depletion, so the substitution parameter in the production functions should be interpreted as the long-run one.

Then, we propose a production function where labour and "capital services" are combined in a Cobb-Douglas function that accounts for the most important long-run stylized facts of growth (Stresing, Lindenberger & Kümmel, 2008). On the other hand, capital services are the result of a CES production function combining capital and energy.

$$F(K,E,L) = \left[\left(a\,[K]^{-\beta} + (1-a)\,[E]^{-\beta} \right)^{-\frac{1}{\beta}} \right]^{\alpha} L^{1-\alpha}$$

Where, K is capital, L is labour, and E is energy. The α parameter is the marginal productivity of capital services ([1-α] is labour marginal productivity), and β is the long-run capital-energy substitutability parameter.

The β parameter in the CES-Cobb Douglas production function is related to the capital-energy substitutability, with the elasticity of substitution between capital and energy being:

$$\sigma = \frac{1}{1+\beta}, \beta \in (-1, \infty)$$

For $\beta \to -1,$ the elasticity of substitution between capital and energy tends to be perfect, meaning that it is easy to compensate energy depletion with an increase in the capital stock, while for $\beta \to \infty,$ the CES function tends to be closer to a fixed proportion function, where the production level is determined by the less abundant input. For β=0, the CES function is a mere Cobb-Douglas. For CES functions, σ is exactly the Morishima elasticity (Anderson & Moroney, 1993), a parameter whose long-run value is estimated to move $\sigma \in (0.70, 1.73)$ with a best estimate of 1.21 (Koetse, De Groot & Florax, 2006), implying $\beta \in (-0.42, 0.42)$ with a best estimate of -0.17.

6 The Ramsey-Hotelling-EROI model

We present a Ramsey-Hotelling (Ramsey, 1928; Hotelling, 1931; Chakravorty, Roumasset & Kinping Tse, 1997; Chakravorty & Moreaux, 2008) type model, which describes the evolution of an economy consisting of a representative risk-averse household whose level of consumption appears discounted at a rate ρ.

$$\max_{c,l,m}\int_0^T e^{-\rho t}u\left(c_t\right)dt$$

The representative agent is endowed with a fixed amount of labour (a unit in every moment per person) and the production function is the CES-Cobb Douglas presented in Section 5. In the CES-Cobb-Douglas function, the raw energy extracted (v) is corrected by the energy auto-consumption (e) at time t. The rest is a classical equation of accumulation-depreciation of capital.

$$\dot{k}_t=\left[\left(ak_t^{-\beta}+(1-a)\left[(1-e_t)v_t\right]^{-\beta}\right)^{-\frac{1}{\beta}}\right]^{\alpha} l_t^{1-\alpha}-c_t-\delta k_t$$

The energy sector is defined as the one that turns natural inputs into usable energy: to fix ideas, a car engine is not part of the energy sector, but the station serving fuel is. The energy sector is modelled separately from the rest of the economy to avoid double counting, because the direct energy costs and energy content of the capital in the energy sector are already discounted in the definition of EROI (or energy auto-consumption). For example, a piece of machinery (a hammer) used in the overall economy requires energy consumption to be produced, which appears in the CES-Cobb-Douglas function above. If the same hammer is used in the energy sector, the energy invested in its production is detracted as energy auto-consumption $(1-e_t)$ and must not appear on the production function of that unit of capital.

That is, k^E represents the capital necessary to produce m that is the flow of investment that accumulates into the stock of extractive capital M . Then the following equation is the same capital accumulation-depreciation equation as the accumulation of capital, but without the energy input:

$$\dot{k}_t^E=(k_t^E)^{\alpha}(1-l_t)^{1-\alpha}-m_t-\delta k_t^E$$

And machinery for the energy sector (m) accumulates and depreciates building up a stock of energy extraction and processing machinery (M).

$$\dot{M}_t=m_t-\delta M_t$$

So M represents the energy-producing capital (nuclear power plants, oil rigs, coal mines and power plants that convert primary into useful energy), while k^{E} represents the capital used to produce that capital (including the factories producing the energy equipment, the foundries that produce the steel used for that equipment, and the construction equipment used to build any facility in the energy sector, but without any energy cost included, since these are directly detracted in the energy auto-consumption curve). The installation of capital in the energy sector can increase energy production, but as the economy moves towards greater levels of auto-consumption (because of depletion), a given amount of capital can extract a decreasing volume of raw energy. Energy production is directly proportional to the capital installed in the energy sector and inversely proportional to current energy auto-consumption (see section 4):

$$v_t = \frac{M_t}{e_t}$$

Given Q(.), the energy auto-consumption curve (see section 3), depletion is represented as the movement of the contemporary auto-consumption level (e_t) towards a higher level, and the movement of that depletion variable is directly proportional to the extraction of raw energy, and inversely proportional to the amount of resources at each level of auto-consumption.

$$\dot{e}_t = \frac{v_t}{Q(e_t)}$$

No negative levels of labour ($l_t, 1-l_t$), consumption (c_t), or investment are allowed. Then:

$$\begin{aligned}
&0 \le l_t \le 1 \\
&0 \le m_t \le (k_t^E)^{\alpha}(1-l_t)^{1-\alpha} \\
&0 \le c_t \le \left[\left(ak_t^{-\beta} + (1-a)\left[(1-e_t)v_t\right]^{-\beta}\right)^{-\frac{1}{\beta}}\right]^{\alpha} l_t^{1-\alpha}
\end{aligned}$$

So finally the following optimal control model, with {c, l^E, m} as controls and $\{k, k^E, M, e\}$ as state variables, should be solved to assess the evolution of consumption, welfare, and the amount of labour devoted to the energy sector:

$$\max_{c,l,m} \int_0^T e^{-\rho t} u\left(c_t\right) dt$$

$$\dot{k}_t = \left[\left(a k_t^{-\beta} + (1-a)\left[(1-e_t) v_t\right]^{-\beta} \right)^{-\frac{1}{\beta}} \right]^{\alpha} l_t^{1-\alpha} - c_t - \delta k_t$$

$$\dot{k}_t^E = (k_t^E)^{\alpha} (1-l_t)^{1-\alpha} - m_t - \delta k_t^E$$

$$\dot{M}_t = m_t - \delta M_t$$

$$v_t = \frac{M_t}{e_t}$$

$$\dot{e}_t = \frac{v_t}{Q(e_t)}$$

$$0 \leq l_t \leq 1$$

$$0 \leq m_t \leq (k_t^E)^{\alpha} (1-l_t)^{1-\alpha}$$

$$0 \leq c_t \leq \left[\left(a k_t^{-\beta} + (1-a)\left[(1-e_t) v_t\right]^{-\beta} \right)^{-\frac{1}{\beta}} \right]^{\alpha} l_t^{1-\alpha}$$

7 Steady-state comparative statics

If the energy resources available to the world economy are finite, there is no steady-state for the Ramsey-Hotelling-EROI model: given that no productivity growth term has been included in the model, the depletion of resources gives place to a consumption path that tends to zero (see Dasgupta and Heal, 1974 for general results on exhaustible resources with no productivity growth).

On the other hand, if a scalable and renewable energy source exists (solar or nuclear breeders, for example), there is a path of the model that optimally depletes non-renewable resources up to the point where the first scalable renewable energy source supplies all the energy demand. In this case, there is a steady state, that can be easily computed fixing $e_t = e$. To compute the steady-state under the stated conditions, the three last inequalities in the Ramsey-Hotelling-EROI model that will hold for sure in a steady-state, should be dropped. Then, the model (with e as a parameter instead of a variable) becomes:

$$\max_{c,l,m} \int_0^T e^{-\rho t} u\left(c_t\right) dt$$

$$\dot{k}_t = \left[\left(a k_t^{-\beta} + (1-a) \left[\left(\frac{1-e}{e} \right) M_t \right]^{-\beta} \right)^{-\frac{1}{\beta}} \right]^{\alpha} l^{1-\alpha} - c_t - \delta k_t$$

$$\dot{k}_t^E = (k_t^E)^{\alpha} (1-l_t)^{1-\alpha} - m_t - \delta k_t^E$$

$$\dot{M}_t = m_t - \delta M_t$$

Using the Pontryaguin necessary conditions for the above system and imposing steady-state conditions, an steady-state for the variables $\{l, k, k^E, m, M, c\}$ can be computed given the parameters $\{\alpha, a, \delta, \rho, \beta, e\}$. To simplify the equations, the following parameters are defined:

$$\chi = \left(\frac{\rho+\delta}{\alpha(1-a)} \right)^{\frac{\alpha+\beta}{\beta(\beta+1)}} \left(\frac{(\delta+\rho)}{\alpha} \right)^{\frac{\alpha}{1-\alpha}\frac{\alpha+\beta}{\beta(\beta+1)}} \left[\left(\frac{a\alpha}{\delta+\rho} \right) \right]^{\frac{1}{(\beta+1)}} \left[\frac{(\delta+\rho-\alpha\delta)}{\alpha\delta} \left(\frac{(\delta+\rho)}{\alpha} \right)^{\frac{1}{\alpha-1}} \right]^{\frac{\alpha+\beta}{\beta}}$$

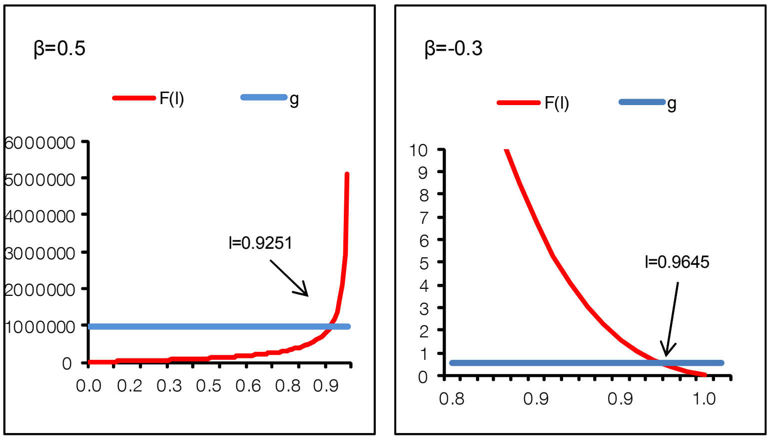
$$\zeta = \chi \left[\left(\frac{1-e}{e} \right) \right]^{\frac{\alpha+\beta}{(\beta+1)}}$$

Then, the following equation allows us to compute labour in the non energy sector (l) in steady state, numerically solving a one-dimensional equation:

$$\left(a\left[\zeta(1-l)^{\frac{(\alpha+\beta)}{\beta}}l^{-\frac{\alpha+\beta\alpha}{\beta(\beta+1)}}\right]^{-\beta}+(1-a)\left[\left(\frac{1-e}{e}\right)\frac{(\rho+\delta-\alpha\delta)}{\alpha\delta}\left[\frac{\alpha}{(\rho+\delta)}\right]^{\frac{1}{1-\alpha}}(1-l)\right]^{-\beta}\right)^{-\frac{1}{\beta}}l^{\frac{1}{\beta}}(1-l)^{-\frac{(\beta+1)}{\beta}}=\left[\left(\frac{\rho+\delta}{\alpha a}\right)\zeta^{(\beta+1)}\right]^{\frac{1}{\alpha+\beta}}$$

The equation has a structure $F_{\alpha,a,\delta,\rho,\beta,e}(l)=g$. The function F(l) has a value of zero for l=0, and grows to infinity if $\beta>0$, and it decreases from infinity to zero if $\beta<0$. As a result, for g positive (that is, 0<e<1) there is always a value of l solving the equation (see chart below).

COMPUTATIONOF THE LABOUR IN THE NON-ENERGY SECTOR IN THE STEADY-STATE



The following equations give the rest of variables in steady state:

$$k=\left[\chi\left[\left(\frac{1-e}{e}\right)\right]^{\frac{\alpha+\beta}{(\beta+1)}}\right](1-l)^{\frac{\alpha+\beta}{\beta}}l^{-\frac{\alpha\beta+\alpha}{\beta(\beta+1)}}$$

$$k^E=\left(\frac{(\delta+\rho)}{\alpha}\right)^{\frac{1}{\alpha-1}}(1-l)$$

$$m=\delta M$$

$$M=\frac{(\delta+\rho-\alpha\delta)}{\alpha\delta}k^E=\frac{(\delta+\rho-\alpha\delta)}{\alpha\delta}\left(\frac{(\delta+\rho)}{\alpha}\right)^{\frac{1}{\alpha-1}}(1-l)$$

$$c=\left[\left(ak^{-\beta}+(1-a)\left[\left(\frac{1-e}{e}\right)M\right]^{-\beta}\right)^{-\frac{1}{\beta}}\right]^{\alpha}l^{1-\alpha}-\delta k$$

With the above equations, steady-state comparative statics will be performed. The full list of parameters determining $\{l, k, k^E, m, M, c\}$ is $\{\alpha, a, \delta, \rho, \beta, e\}$. The parameters $\{\alpha, a, \delta, \rho\}$ will be fixed and a few representative values of β will be chosen, and for those values it will be shown how the percentage of labour used in the non-energy sector (l) and the consumption level (c) evolve for all sensible values of energy auto-consumption (between 2% and 60%). That is, we will graph and briefly comment the curves $l_\beta(e)$ and $c_\beta(e)$ for $e \in (0.02, 0.6)$ and for a few values of β. As discussed in section 5, the empirical range for β is $\beta \in (-0.42, 0.42)$ with a best estimate of -0.17, but the long-run substitutability between capital and energy is the subject of a hot controversy, so in this work we will consider a wider range of values for β $\beta \in \{-0.5,-0.3,-0.1,0.1,0.5,1,2,5\}$.

Depreciation is set at 4% (δ =0.04) as a compromise between very different empirical estimations: Musgrave (1992) estimates depreciation to move between 3% and 3.8% yearly, while Nadiri & Prucha (1996) estimate a 5.9% for tangible assets and R&D in the US manufacturing sector.

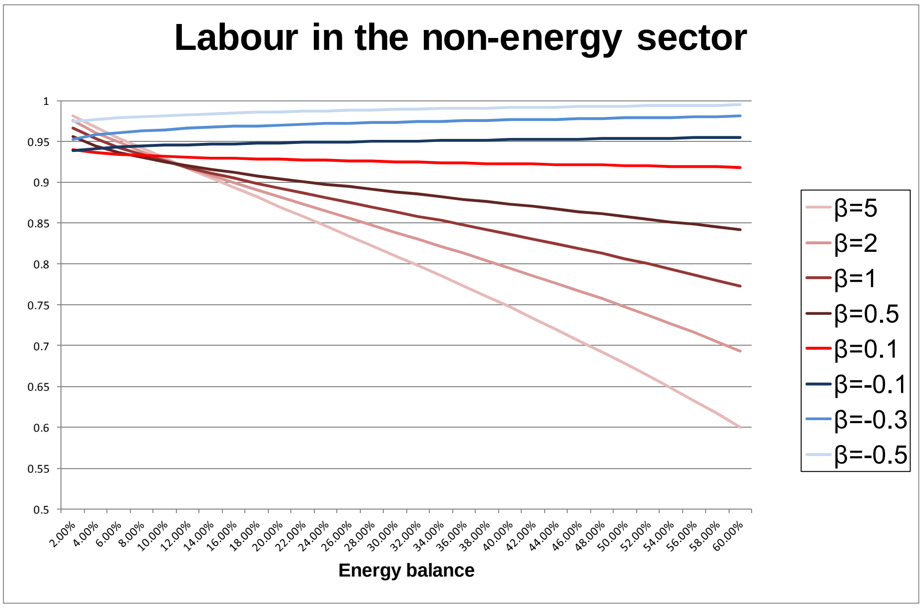
The subjective discount factor is supposed to be a 2% (ρ=0.02), in the middle of Belzil & Hansen (1999) estimated range (1%-5%). The subjective welfare function (u(.), that determines risk aversion) plays no role in the steady-state level of the variables (but it is important to determine the convergence speed), so it will not be chosen in this exercise. For the Cobb-Douglas part of the CES-Cobb-Douglas function, the marginal product of capital services is 40% (α=0.4), while the marginal product of labour is 60% (1-α=0.6), which are their standard levels on the literature for developed countries (see Bentolila, 2003 and Danxia Xie, 2011).

The capital services are produced combining capital with energy under a CES production function. The CES production function has two parameters: The a parameter gives the weight of factors (capital and energy) in the production of capital services and β is a parameter regulating the elasticity of substitution between energy and capital. When β=0, the production function has a as the parameter of a Cobb Douglas. We set (a=0.1), that for the Cobb-Douglas case (β→0), fixes the marginal product of energy in the 4%, which is in line with the empirical estimations (Atkeson & Kehoe, 1999).

In the following table we present all the parameters in the model, the value set in this steady-state comparative statics exercise and their empirical range in selected references.

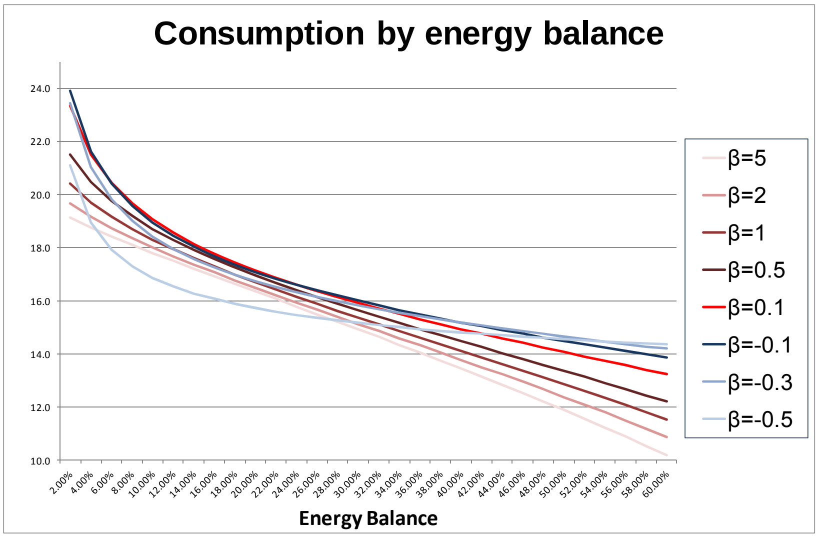
Parameters of the model and their empirical range			
Economic factor	**Name in the model**	**Value in the model**	**Empirical range**
Representative agent preferences			
Subjective Discount	ρ	2%	**1%-5%** (Belzil & Hansen, 1999)
Production function			
Cobb Douglas with capital services and labour as inputs			
Capital services share	α	40%	For OECD Countries: **30%-40%** (Bentolila, 2003). Fon developing countries: 4**0-50%** (Danxia Xie, 2011)
CES for capital services with capital and energy as inputs			
Capital energy substitution	β	{-0.5, -0.3, -0.1, 0.1, 0.5, 1, 2, 5}	95-percentil range: (**-0.428,0.421**); Best Estimate:**-0.173** (Koetse, Groot & Florax, 2006)
Capital energy proportions	a	4%	Atkenson Kehoe, 1999
Capital depreciation	δ	4%	Musgrave, 1992: Range: (**3%-3.8%**); Best Estimate: **3.4%**; Nadiri Prucha, 1996; Best Estimate: **5.9%**
EROI	e	2%-60%	See table in section 2

In steady-state (where capital accumulation stops), for a given e, the only scarce factor is labour, so the best measure of the relative size of the energy and non-energy sectors is precisely the labour use in the non-energy sector (l) and in the energy sector (1-l). In the graph below the curves $l_\beta(e)$ are shown for $\beta \in \{-0.5,-0.3,-0.1,0.1,0.5,1,2,5\}$. The curve corresponding to the Cobb-Douglas case (β→0) should move between the curves with β=-0.1 and β=0.1. In the graph below, $l_\beta(e)$.



For $\beta<0$, when the energy input becomes more costly, it is easily substituted in the production process, and there is an increase in the capital stock of the non-energy sector (for example with investment in energy efficiency) while the amount of resources devoted to energy production is reduced. For $\beta>0$ the economy reacts to depletion with an increase in the size of the energy sector because energy cannot be easily substituted with capital, so to maximize consumption more resources have to be devoted to energy production. For the Cobb-Douglas case ($\beta=0$), it can be proven that the size of the energy sector is constant for every auto-consumption level.

In the following graph the $c_\beta(e)$ curves are shown for the same levels of β as before:



For consumption, the graph always shows a decline with depletion, because a worse energy balance (more scarcity) should always have a negative impact on consumption. It is a more constant decline across auto-consumption levels for higher βs, while for smaller βs the decline is very fast for very low auto-consumption levels (the economy suffers a lot when deprived of its former energy subsidy), but after the first stages of depletion, the easiness of substitution between energy and capital drastically reduces the impact of additional scarcity. In the following table the impact on consumption of the increase in energy auto-consumption (depletion) from 10% to 20% and from 10% to 30% is shown. The impacts would be significant, and in the case of the 10% to 30%, clearly the impact on consumption would be worse for the more inelastic cases.

Per Cent decline in consumption when energy auto-consumption move from			
		10%-20%	**10%-30%**
	β=5	-8,16	-16,17
	β=2	-8,42	-16,01
	β=1	-8,79	-16,04
	β=0.5	-9,37	-16,38
	β=0.1	-10,09	-16,52
	β=-0.1	-9,81	-15,55
	β=-0.3	-9,08	-14,04
	β=-0.5	-6,81	-10,11

8 Conclusions and further research

A Ramsey-Hotelling growth model with energy as an input in the production function has been presented. The quality of the available stock of energy was ordered by its “Energy Return on Energy Invested” (EROI). The energy accountancy issues related to the EROI (energy quality and life-cycle analysis for net energy computation) were discussed, and the energy auto-consumption curve was presented as the best mathematical representation or total energy resources under heterogeneous EROI. A hypothesis for the relationship between EROI and capital use was made: that the flow of energy in the economy is proportional to capital installed in the energy sector and inversely proportional to the energy auto-consumption of currently used energy resources. The mixed CES-Cobb-Douglas was proposed as a production function flexible enough to describe a wide range of opinions on the value of long-run energy-capital substitution. After the full Ramsey-Hotelling-EROI model was presented, its steady-state for different levels of energy auto-consumption was computed, and a comparative statics analysis of those steady-states was conducted.

Two qualitative comparative statics results were obtained. First, when energy scarcity deepens (ie. EROI decreases), the size of the energy sector (as measured by the amount of labour in the energy sector) slightly decreases when long-run capital-energy substitutability is below zero, it is constant if it is zero, and increases if higher than zero. Second, the decline of consumption when auto-consumption worsens is more constant for higher substitutability, while for lower substitutability the decline is very fast for very low auto-consumption levels (the economy suffers a lot when deprived of its former energy subsidy), but after the first stages of depletion, the easiness of substitution between energy and capital drastically reduces the effect of additional scarcity. A numerical experiment on the impact of energy depletion on steady-state consumption was performed leading to the conclusion that an increase in energy auto-consumption from 10% to 20% would have a significant impact for all considered elasticity parameters.

In this article we merely performed a steady-state comparative statics exercise for a simplified model (where the energy auto-consumption curve is substituted by a fixed auto-consumption level). A straightforward extension of this paper would be to provide a full dynamic path of the system under a realistic energy auto-consumption curve. Additional exercises can be performed with the full model: the impact of agents’ myopic decisions on welfare can be readily computed, solving the model for every time under the assumption that an unlimited amount of energy at the present level of auto-consumption is available, obtaining the controls under that hypothesis, and then allowing depletion to occur with the real auto-consumption curve. Apart from answering relevant theoretical questions, the direct introduction of EROI into a growth model can open the door to the introduction of energy efficiency into growth accounting.

Finally, environmental factors can be included in the model,replacing the EROI curve for a surface where the stock of energy resources is classified by auto-consumption and environmental impact.

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