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Abstract

We propose a fundamentals-based econometric model for the weekly changes in the euro-dollar rate with the distinctive feature of mixing economic variables quoted at different frequencies. The model obtains good in-sample fit and, more importantly, encouraging out-of-sample forecasting results at horizons ranging from one week to one month. Specifically, we obtain statistically significant improvements upon the hard-to-beat random walk model using traditional statistical measures of forecasting error at all horizons. Moreover, our model improves greatly when we use the direction-of-change metric, which has more economic relevance than other loss measures. With this measure, our model performs much better at all forecasting horizons than a naive model that predicts the exchange rate has an equal chance to go up or down, with statistically significant improvements.

Keywords: euro-dollar rate, exchange rate forecasting, State-space model, mixed frequencies.

JEL classification: F31, F37, C01, C22.
Resumen

Se propone un modelo econométrico basado en fundamentales para el cambio semanal del tipo de cambio del euro frente al dólar con la característica diferencial de mezclar variables con diferentes frecuencias. El modelo obtiene buenas predicciones en la muestra, y, lo que es más importante, mejoras en predicción fuera de la muestra a horizontes entre una semana y un mes. Específicamente, obtenemos mejoras estadísticamente significativas frente al difícil de batir paseo aleatorio, usando las medidas tradicionales basadas en error de predicción cuadrático medio a todos los horizontes. Las ganancias son mayores cuando utilizamos la métrica basada en la dirección del cambio, que tiene más sentido económico en un modelo como este. Con esta medida, nuestro modelo predice los movimientos alcistas y bajistas del tipo de cambio mucho mejor que un modelo basado en igual probabilidad de bajadas y subidas.

Palabras claves: Tipo de cambio euro-dólar, predicción de tipos de cambio, modelo de ecuación de estados y frecuencias mixtas.

Códigos JEL: F31, F37, C01, C22.
1 Introduction

The importance of forecasting the euro-dollar exchange rate is evident. Currently it is the most important currency pair in the foreign exchange market (Brzeszczynski and Melvin, 2006), and fluctuations in the euro-dollar exchange rate are crucial not only for the economic transactions between the two major economic blocks but also for the rest of the countries as both currencies act as numeraire and medium of exchange for international transactions, and as an international store of value. However, understanding and forecasting euro-dollar fluctuations is not an easy task. The euro was introduced as a currency on January 1, 1999, but its use as a legal tender by consumers in retail transactions started on January 1, 2002. That implies that only twelve years have passed since the first date (and nine since the second. Therefore, both, the European Central Bank and the economic agents have been involved in a learning process about the mechanisms of transmission of the monetary policy and its effects on economic activity, the effects of the modification in the exchange rate regime on the economy of member countries, and the role of the euro as an international currency (see, for instance, the contrary views of Chinn and Frankel, 2008, and Posen, 2008). Moreover, the short length of the euro-dollar series and of many of the economic variables of the European Monetary Union, which are also of varying length, poses some additional challenges to traditional econometric methods such as cointegration techniques that usually need several years of data to uncover stable relationships between variables. In addition, if the aim is to explain or forecast weekly or daily exchange rates traditional econometric methods do not allow doing it using fundamental economic variables which are usually available at monthly or quarterly frequencies only.

To deal with these shortcomings we propose a fundamentals-based econometric model for the weekly changes in the euro-dollar exchange rate with the distinctive feature of combining economic variables quoted at different frequencies. This mixture of frequencies allows us to assess the influence of macroeconomic variables quoted at monthly frequency and not available at weekly frequency over weekly movements in the foreign exchange (FX) rate. In addition, our methodology allows us to employ series of differing lengths.

We do this by relying on recent contributions to time-series econometrics by Mariano and Murasawa (2003), and Camacho and Perez-Quiros (2010) who use maximum likelihood factor analysis of time series with mixed frequencies, treating quarterly series as monthly series with missing observations. In our model, we express the dynamic relationship between the exchange rate and its economic fundamentals in a state-space representation with series at weekly and monthly frequencies, and we use the Kalman Filter to sequentially update a linear projection of the system.
The variables we propose as driving the exchange rate are a conventional set of macroeconomic fundamentals derived from the monetary model of exchange rates and fundamentals derived from international parity arbitrage conditions.

At frequencies of one month or higher, the literature has found that it is very difficult to explain the foreign exchange rate changes, and even harder to forecast them. This is reflected by the fact that researchers using structural exchange rate models usually cannot beat a simple random walk model for the exchange rate movements that predicts the exchange rate to remain unchanged. In other words, the current spot rate appears to be the best predictor of the spot rate in the next period, so other economic variables do not help in forecasting the exchange rate. Of course, the literature on FX forecasting is huge and there are several published works that claim success in forecasting exchange rates for certain currencies and data periods. However, these positive results are mainly for low frequency movements of the exchange rate and do not appear to be robust. In fact, since the seminal work of Meese and Rogoff (1983), “beating the random walk” has become the measure by which an exchange rate model is often judged in international macroeconomics.

Against this background, the results we obtain here are encouraging. Our fundamentals-based econometric model obtains a forward exchange rate in-sample fit and, more importantly, satisfactory out-of-sample results. Specifically, our model explains about 80% of the total in-sample variation of the euro-dollar exchange rate. In addition, when we evaluate the out-of-sample forecasting performance of our model at horizons ranging from one to four weeks with the standard recursive-regression procedure, we obtain improvements upon the hard-to-beat random walk model using traditional statistical measures of forecasting accuracy such as the mean squared error. More importantly, we obtain better results when we consider the direction-of-change metric that considers a forecast successful if it can predict the sign of the future variation in the exchange rate regardless of its magnitude, which has great economic importance since it is related to market timing in financial markets and can be more profitable on economic grounds. With this measure, our model performs much better in all forecasting horizons than a naïve model that predicts that the exchange rate has an equal chance to go up or down, with these improvements being statistically significant.

These results are promising given the short forecasting horizons evaluated, where much of the literature considers that noise dominates economic fundamentals in explaining exchange rate fluctuations. Our success may be due to the novel aspect of our econometric model that explains and forecasts exchange rate with economic fundamentals by combining data at different frequencies. Hopefully, our results may contribute to change the perception that economists have about the usefulness of fundamentals in explaining and forecasting exchange rates in the short-run.
The paper is structured as follows. Section 2 develops a brief literature review. Section 3 states the fundamental statements of exchange rates. Section 4 describes the econometric model proposed in the paper. Section 5 presents the empirical results. Section 6 concludes.

2 Brief literature review

Forecasting nominal exchange rates has been an extremely elusive theme in international finance, despite the huge amount of resources devoted to the task, both in the academic and the non-academic (financial markets) professions.

On the academic side, the challenge is posited in the seminal work of Meese and Rogoff (1983) who highlight the poor out-of-sample forecasting performance of a variety of structural exchange rate models such as the monetary model or the portfolio balance model. Specifically, they show for the post-Bretton Woods floating period that structural post-sample forecasts of foreign exchange rates among major countries are bettered, especially in the short-run, by a simple driftless random walk model that does not use any information on “fundamentals” and forecasts the exchange rate to remain unchanged. This occurs even though these authors base the forecasts of the structural models on the realized values of the fundamentals for the forecasting period, giving the structural models an important informational advantage over the random-walk model.\(^1\) An extensive subsequent literature shows the robustness of these results for the post-Bretton Woods floating period by using non-linear econometric techniques, different currencies, data periodicity and samples (e.g., Cheung, et al, 2005). Then, the difficult task to tackle is to model exchange rates using fundamental economic variables and to obtain forward exchange rate fit both in-sample and out-of-sample, to overcome the pessimistic feeling instilled in the profession by Meese and Rogoff (1983) that exchange rates and fundamentals are separated (Frankel and Rose, 1995, p. 1704). That is, solving the “exchange-rate disconnect puzzle” of Obstfeld and Rogoff (2000) has become a challenging purpose in the related literature.\(^2\)

In the mid-90s, some authors reported empirical evidence that monetary fundamentals may contain predictive power for exchange rate movements in the long-run (MacDonald and Taylor, 1994; Mark, 1995; Chinn and Meese, 1995; Kim and Mo, 1995). These works apply a long-horizon

\(^1\)Faust, et al (2003) question that this artificial advantage has really existed finding better predictive power of exchange rate models using real-time data than using ex-post revised data. In any case, any predictability found using realised values of fundamentals is not useful to policymakers and market participants who must forecast exchange rates in real time.

\(^2\)In spite of the scant evidence favouring fundamentals-based explanations of exchange rate fluctuations, international economists and market analysts put great weight on them when evaluating or predicting these fluctuations in non-technical papers (see Salvatore, 2005, page 460).
regression approach to model the relationship between the exchange rate and fundamentals, and although they do not have short-run predictive power (since they use monthly or quarterly data, their shorter-run is one-month or one-quarter ahead), they do find evidence of long-run exchange rate predictability. While these findings were confirmed later for some authors (e.g., Mark and Sul, 2001) they do not appear to be robust (Cheung, et al., 2005) and are not exempt of critics (Kilian, 1999; Berkowitz and Giorgianni, 2001; Boudoukh, et al., 2008). In any case, the forecasting puzzle remains unsolved at short-run horizons.

Although fundamentals do not appear to help in forecasting short-run exchange rate returns, the existence of links between exchange rates and fundamentals in the short-run is stated in the important work of Andersen, et al (2003). They find, using real time data, that macroeconomic announcement surprises produce quick jumps in the conditional mean of five US dollar exchange rates from January 1992 to December 1998. Andersen et al. (2007) and Faust, et al. (2007) confirm this result for the euro-dollar exchange rate. A problem with the use of high-frequency data in checking the relationship between exchange rates (which are quoted second-by-second, if necessary) and economic fundamentals (money stocks, prices, etc.) is that typically there are no available high-frequency series of fundamentals. Hence, most works employ monthly or quarterly data because traditional econometrics methods do not allow for empirically testing the existence of a relationship between these fundamentals and the exchange rate using weekly or daily data, which are usually the frequencies of interest for foreign exchange market participants and policymakers. Of course, it also implies that it is not possible to empirically test at high frequencies models that propose a stable relationship between those fundamentals and the exchange rate.

These drawbacks are important because in the voluminous FX market, foreign currencies are traded continuously through a network of dealers located in large money centres situated around the world, and new information about many relevant economic variables should certainly influence the exchange rate, regardless of the frequency at which it is quoted. Hence, it is relevant to study the usefulness of economic fundamentals in explaining and predicting exchange rate changes using data with mixed frequencies. This is the main objective of this paper.

3 Fundamental determinants of exchange rates

The existing literature employs an extensive list of economic determinants in its attempt to explain and forecast the exchange rate. In this work, we aim to explain and forecast short-run changes in the euro-dollar exchange rate using financial and macroeconomic fundamentals. With this purpose, we employ the conventional set of fundamentals derived from the monetary model of exchange rate determination, enlarged by a set of forward exchange rates.
We use fundamentals from the monetary model for its importance in international economics, where it is the “standard workhorse” (Frankel and Rose, 1995, p. 1691), and because these fundamentals are the same as those derived from modern micro-founded exchange rate models. Forward rates are employed because basic parity conditions suggest that they should help to forecast the exchange rate. In fact, one of the most important research questions in international finance is whether or not the forward exchange rate helps to predict the future spot rate. While the answer is usually “no”, Clarida and Taylor (1997) show that there is important information in the term structure of forward exchange rates about future movements of the spot rate.

By relying on these two sets of fundamentals, which have been widely used in the literature but with limited success in forecasting short-run exchange rate changes, the positive results found when applying our econometric methodology are reinforced. We now briefly explain the theories behind the variables we use.

3.1 Forward exchange rates

Denote $s_t$ as the logarithm of the spot exchange rate at time $t$ defined as the domestic price of foreign currency (hence raises in $s$ imply domestic currency depreciation), and $f_{k,t}$ as the log of the $k$-period forward exchange contracted at time $t$. Spot and forward exchange rates are connected by two fundamental international parity conditions, the Covered Interest Parity (CIP) and the Uncovered Interest Parity (UIP). To see it, let $i_{k,t}$ and $i^*_{k,t}$ be the date $t$ nominal interest rate on similar domestic and foreign securities with a maturity of $k$ periods, respectively. If both deposits have the same risk characteristics and only differ by the currency of denomination, CIP arbitrage condition states that nominally risk-free returns from both deposits should be equal. Using a logarithmic approximation, the CIP condition is expressed as:

$$i_{k,t} \simeq i^*_{k,t} + f_{k,t} - s_t.$$  

(1)

It implies that, in equilibrium, expected forward speculation is driven to zero because if (1) is violated, a riskless arbitrage profit opportunity is available in a zero-net investment strategy. The empirical evidence, in general, supports the validity of CIP (Taylor, 1989).

Uncovered interest parity (called uncovered because forward markets are not used as a hedge) is based on the proposition that with risk-neutral agents (who care only about the mean value and not the variance of asset returns), expected forward speculation profits should be driven to zero. Since $f_{k,t} - s_{t+k}$ is the profit from taking a position in forward foreign exchange, the $k$-period forward exchange in equilibrium must be equal to the market agents’ expected future spot
exchange rate at time $k$. Hence,

$$f_{k,t} = E_t(s_{t+k})$$  \hfill (2)

Where $E_t(s_{t+k})$ is the mathematical expectation of $s_{t+k}$ conditioned on the date-$t$ available information set $I_t$. Several works have studied whether the forward exchange rate is a forward exchange rate predictor of future spot rate, but the evidence indicates that the current spot rate is a better predictor of the future spot rate than the current forward exchange rate (Meese and Rogoff, 1983).

Substituting (2) into the CIP we get the UIP arbitrage condition:

$$i_{k,t} \simeq i_{k,t}^* + E_t(s_{t+k}) - s_t.$$  \hfill (3)

UIP is used as an approximation to equilibrium in the asset markets and is the cornerstone parity condition for testing FX market efficiency. If (3) is violated, a zero net investment strategy of borrowing in one currency and simultaneously lending uncovered in the other currency has a positive expected pay-off. When it holds, the interest rate differential is an estimate of the future exchange rate change. For instance, a positive interest rate differential for a country should cause a proportional depreciation of the domestic currency.\footnote{This is exactly the opposite effect that the carry trade effect suggests, where a positive interest rate differential should cause a FX rate appreciation. In this sense, the returns obtained by following a “carry trade” strategy come from the violation of UIP.} Under rational expectations and risk neutrality, this estimate should also be unbiased. Moreover, plugging (1) into (3), we get:

$$E_t(s_{t+k} - s_t) \simeq f_{k,t} - s_t.$$  \hfill (4)

Hence, if (4) holds, the forward premium should be an optimum predictor of the future exchange rate depreciation. Note that it requires that UIP holds, and that agents have rational expectations and be risk neutral. Building on this, the most common empirical strategy for testing the risk-neutral efficient markets hypothesis is based on the following equation:

$$s_{t+k} - s_t = \alpha + \beta(f_{k,t} - s_t) + \nu_{t+k},$$  \hfill (5)

where the rate of depreciation $(s_{t+k} - s_t)$ is projected onto the lagged forward premium $f_{p,k,t} = (f_{k,t} - s_t)$. Risk-neutral efficient market hypothesis requires $\alpha = 0$ and $\beta = 1$, but the empirical evidence suggests that in general it does not hold (see Engel, 1996, for a survey). Estimations typically find values for $\beta$ closer to negative unity than to positive unity, and coefficients of determination $R^2$ closer to zero, with no support for the hypothesis. However, since the estimated slope coefficient $\beta$ is often statistically significantly different from zero, Clarida and Taylor (1997)
suggest that there is important information in the forward premium regarding subsequent spot rate movements and develop a model to extract this information.\(^4\)

They depart from three stylized facts of forward and spot exchange rates (Meese and Singleton, 1982; Baillie and Bollerslev, 1989; Hai, et al 1997). First, spot and forward exchange rates are integrated of order one processes. Second, spot and forward exchange rates for the same currency are cointegrated with a cointegrated vector pretty close to (-1, 1). Third, forward premiums are stationary. Building on these results, and only assuming that deviations from the risk-neutral efficient market hypothesis are stationary, Clarida and Taylor (1997) propose that in a system of one spot rate and \(J\) forward exchange rates, there exists \(J\) cointegrating vectors and exactly one common trend, which causes the non-stationarity of the \(J+1\) exchange rates, the vector of the \(J\) forward premiums being a basis for the space of cointegrating relationships.

Empirically, they show for the sterling, mark and yen, against the dollar that the spot and four forward exchange rates for each currency are well represented by a Vector Error Correction Model (VECM) with one common trend and four cointegrating vectors defined by the vector of forward premiums. They compare the forecasts from the VECM with forecasts from a random walk, from the appropriate forward exchange rate, and from those produced by fitting a lagged equation (5). While at a four-week horizon they find that there is little to choose from between those forecasting methods, at 13-, 26-, and 52-week horizons each of the alternative forecasts is outperformed by the VECM forecasts. Clarida, et al (2003) extend this analysis allowing for nonlinearities in the data-generating process for the term structure of the forward premiums, obtaining even better forecasting performance. Clarida and Taylor (1997, p. 361) conclude that their although results constitute tentative evidence on a stylized fact regarding the high information content of the forward exchange rate, further empirical work might be addressed toward establishing the robustness of these conclusions. Since we include forward exchange rate variations in our attempt to forecast the euro-dollar spot exchange rate, we precisely evaluate here the robustness of their conclusions in the short-run. Our work can be considered as an inquiry if, with an alternative econometric methodology, Clarida and Taylor results hold at the shorter one-week, two-week and four-week horizons.\(^5\)

\(^4\)Using our dataset, we find values for \(\beta\) ranging from -0.1 to 1.1. When \(\beta\) is statistically significant, it assumes values ranging from 0.7 to 1.1 (closer to what is expected from UIP). The \(R^2\) coefficient fluctuates around 0.0 to 0.1, taking the greater values when the coefficients are statistically significant.

\(^5\)Clarida and Taylor (1997) and Clarida, et al (2003) do not report results at a one-week or two-week horizons. Also missing in their out-of-sample results is some economic measure of the accuracy of their forecasts, such as the direction of change metric that we use below.
3.2 The monetary model

Although it is a vintage ad-hoc model, the monetary model of exchange rate determination is still very important in international macroeconomics. It provides a set of underlying long-run fundamentals for the exchange rate, and many of its predictions are qualitatively the same as those of more modern optimizing micro-founded models. The model consists of a pair of stable money demand functions with continuous stock equilibrium in the money market, and it rests on two basic assumptions: Purchasing-power parity holds in the long run, and uncovered interest rate parity characterizes the equilibrium in the international capital market. Although there are different versions of the model, A general specification of the sticky price monetary model is subsumed in the following equation for the determination of the exchange rate:

\[ s_t = \beta_0 + \beta_1 \overline{M}_t + \beta_2 i^*_t + \beta_3 \overline{\Pi}_t + \beta_4 \overline{T}B_t + \beta_5 \overline{I}P_t + u_t. \]  

In this expression, \( \overline{M} \) is the growth of money supply between two successive periods, \( IP \) is the industrial production growth, \( i^* \) is the short-term interest rate, \( i \) is the long-term interest rate, \( \Pi \) is the inflation rate, \( TB \) is the trade balance as a proportion of the GDP, \( u \) is an error term, and the circumflex the intercountry difference (so for any variable \( x, \hat{x} = x - x^* \)). Alternative versions of the monetary model (due to, among others, Dornbusch, 1976; Frankel, 1979, 1982; Hooper and Morton, 1982) impose different restrictions on the beta parameters that we do not discuss here, since we are not testing the monetary model, but just verifying if these monetary fundamentals are useful in explaining and predicting the euro-dollar exchange rate.\(^6\)

As noted above, the variables included in this specification subsumes those predicted by the currency substitution model of Calvo and Rodriguez (1977) and the micro-based general equilibrium model of Lucas (1982), among others, which makes the analysis more general. Another argument for applying this model for the Euro Area in relation with the US is that some researchers find support for the monetary model to explain both in-sample and out-of-sample the euro-dollar exchange rate (e.g., Nautz and Offermanns, 2006; Altavilla, 2008).

We model and forecast the log-returns of the exchange rate as opposed to the log-levels. Hence, we use the first difference of equation (6). While forecasts for the log-levels are perhaps more useful, this task is complicated by the non-stationarity of the model in log-levels. In any case, it is possible to obtain forecasts for the level of the exchange rate based on its initial value and using the predicted values for the returns.

\(^6\)See Frankel and Rose (1995, pp. 1691-7) for an exposition of the monetary model. Fundamentals enter in intercountry differences, so we assume that the beta parameters are identical for local and foreign countries, an assumption that some people found very restrictive (Haynes and Stone, 1981; Boothe and Glassman, 1987).
4 The econometric model

4.1 Mixing frequencies

We use data at two frequencies, weekly and monthly. To mix these two frequencies, we consider all series as being of weekly frequency and treat monthly data as weekly series with missing observations. We use end of period (e.o.p.) data. For weekly data, the e.o.p. is the last Friday of each week. For monthly data we assign the monthly e.o.p. value to the last Friday of each month. In particular, let $Y_t$ be a monthly series which is observable on the last Friday of each month. We have to take into account that some months have four Fridays and others five.

Consider first the case of a four-Friday month. The low frequency series is the monthly aggregate of weekly series, $X_t$, which we assume to be observable in this sub-section. To avoid using a nonlinear state-space model, we follow Mariano and Murasawa (2003) and Camacho and Perez-Quiros (2010) and approximate the arithmetic mean with the geometric mean. Hence, in the four-Friday case we assume that the flow data is four times the geometric mean of the weekly series within the given month

$$Y_t = 4(X_t \cdot X_{t-1} \cdot X_{t-2} \cdot X_{t-3})^{1/4}. \quad (7)$$

Applying logs and taking the four-period differences for all $t$, we obtain

$$\Delta_4 \ln Y_t = \frac{1}{4}(\Delta_4 \ln X_t + \Delta_4 \ln X_{t-1} + \Delta_4 \ln X_{t-2} + \Delta_4 \ln X_{t-3}). \quad (8)$$

Denoting $\Delta_4 \ln Y_t = g_t$, and $\Delta \ln X_t = x_t$, and after a little algebra, we obtain

$$g_t = \frac{1}{4}x_t + \frac{2}{4}x_{t-1} + \frac{3}{4}x_{t-2} + \frac{3}{4}x_{t-3} + \frac{3}{4}x_{t-4} + \frac{2}{4}x_{t-5} + \frac{1}{4}x_{t-6}. \quad (9)$$

So we express the monthly-on-monthly growth rate ($g_t$) as a weighted average of the weekly-on-weekly past growth rates ($x_{t-i}, i = 0, ..., 6$) of the weekly series.

Operating analogously, for the case of five-Friday month, we arrive at

$$g_t = \frac{1}{5}x_t + \frac{2}{5}x_{t-1} + \frac{3}{5}x_{t-2} + \frac{4}{5}x_{t-3} + \frac{4}{5}x_{t-4} + \frac{4}{5}x_{t-5} + \frac{3}{5}x_{t-6} + \frac{2}{5}x_{t-7} + \frac{1}{5}x_{t-8} \quad (10)$$

4.2 State-space representation

In factor modelling literature, it is standard to consider that each indicator used in the models is the sum of two orthogonal components. The component is the common factor, $f_t$, and captures the co-movements among the series that are due to the existence of common shocks. The idiosyncratic component aims to capture the effect on each series’ dynamics of series-specific shocks.
Below, we present the model for the case in which the variables used in the estimation are the weekly euro-dollar exchange rate variation, three euro-dollar forward exchange rate variations (at one-week, two-week and three-week maturities), the intercountry short-term interest rate differential, the intercountry long-term interest rate differential, the inflation growth differential and the intercountry differential in the rate of money growth. We call it the basic model.

If all variables are observable at weekly frequency, the state representation of the baseline model is subsumed in the measurement and transition equations. The measurement equation, $Y_t = Hh_t + w_t$, with $w_t \sim i.i.d.N(0, R)$, can be expressed as

$$
\begin{pmatrix}
    s_t \\
    f_{u1,t} \\
    f_{u2,t} \\
    f_{u3,t} \\
    \delta^s_t \\
    \delta^\pi_t \\
    \bar{M}_t \\
    \bar{\pi}_t
\end{pmatrix}
= \begin{pmatrix}
    H1_{6x2} & 0_{6x8} & I_{6x6} & 0_{6x5} & 0_{6x5} \\
    \left[ H2_{2x10} \right] & 0_{2x2} & H2_{1x5} & 0_{1x5} & 0_{1x5} \\
\end{pmatrix}
\begin{pmatrix}
    f_{t+1} \\
    f_t \\
    \vdots \\
    f_{t-8} \\
    u_{s,t} \\
    u_{f1,t} \\
    u_{f2,t} \\
    u_{f3,t} \\
    u_{\delta^s,t} \\
    u_{\delta^\pi,t} \\
    u_{\bar{M},t} \\
    \vdots \\
    u_{\bar{\pi},t} \\
    \vdots \\
    u_{\bar{\pi},t-4}
\end{pmatrix}, \quad (11)
$$

where $i = 4, 5$, $H2 = (1, 0, 0, 0, 0)$,

$$H1 = \begin{pmatrix}
\beta_{s,1} & \beta_{1,1} & \beta_{2,1} & \beta_{3,1} & 0 & 0 \\
\beta_{s,2} & \beta_{1,2} & \beta_{2,2} & \beta_{3,2} & \beta_{4,2} & \beta_{5,2}
\end{pmatrix},$$

$$H4 = \begin{pmatrix}
0 & \frac{\beta_{5,2}}{4} & \frac{\beta_{6,2}}{4} & \frac{\beta_{7,2}}{4} & \frac{\beta_{8,2}}{4} & \frac{\beta_{9,2}}{4} & \frac{\beta_{10,2}}{4} & 0_{2x1} \\
0 & \frac{\beta_{5,2}}{4} & \frac{\beta_{6,2}}{4} & \frac{\beta_{7,2}}{4} & \frac{\beta_{8,2}}{4} & \frac{\beta_{9,2}}{4} & \frac{\beta_{10,2}}{4} & 0_{2x1}
\end{pmatrix},$$

and

$$H5 = \begin{pmatrix}
0 & \frac{2\beta_{5,2}}{5} & \frac{2\beta_{6,2}}{5} & \frac{2\beta_{7,2}}{5} & \frac{2\beta_{8,2}}{5} & \frac{2\beta_{9,2}}{5} & \frac{2\beta_{10,2}}{5} & 0_{2x1} \\
0 & \frac{2\beta_{5,2}}{5} & \frac{2\beta_{6,2}}{5} & \frac{2\beta_{7,2}}{5} & \frac{2\beta_{8,2}}{5} & \frac{2\beta_{9,2}}{5} & \frac{2\beta_{10,2}}{5} & 0_{2x1}
\end{pmatrix}.$$
In our empirical applications, $H^4$ is used for the four-Friday month case, while $H^5$ is for the five-Friday month case. The factor loadings $\beta = (\beta_s, \beta_1', \beta_2', \beta_3', \beta_t', \beta_M', \beta_\pi')$ measure the sensitivity of each series to movements in the latent common factor, and $u_{i,t}$, $i = s, f w 1, f w 2, f w 3, i, M, \pi$, is the idiosyncratic component of each series. When all variables are observable at weekly frequencies with no missing observations, $w_i$ is a $(6 \times 1)$ vector of zeroes.

The transition equation, $h_t = F h_{t-1} + \xi_t$, with $\xi_t \sim i.i.d. N (0, Q)$, can be stated as follows. Let $Q$ be a diagonal matrix in which the entries inside the main diagonal are determined by the vector

$$ q = \left( \begin{array}{cccccccc} \sigma_f^2 & 0_{1,9} & \sigma_f^2 & 0_{1,9} & \sigma_f^2 & 0_{1,9} & \sigma_f^2 & 0_{1,9} \\ \sigma_s^2 & \sigma_s^2 & \sigma_s^2 & \sigma_s^2 & \sigma_s^2 & \sigma_s^2 & \sigma_s^2 & \sigma_s^2 \\ \sigma_{f1}^2 & \sigma_{f1}^2 & \sigma_{f1}^2 & \sigma_{f1}^2 & \sigma_{f1}^2 & \sigma_{f1}^2 & \sigma_{f1}^2 & \sigma_{f1}^2 \\ \sigma_{f2}^2 & \sigma_{f2}^2 & \sigma_{f2}^2 & \sigma_{f2}^2 & \sigma_{f2}^2 & \sigma_{f2}^2 & \sigma_{f2}^2 & \sigma_{f2}^2 \\ \sigma_{f3}^2 & \sigma_{f3}^2 & \sigma_{f3}^2 & \sigma_{f3}^2 & \sigma_{f3}^2 & \sigma_{f3}^2 & \sigma_{f3}^2 & \sigma_{f3}^2 \\ \sigma_{f4}^2 & \sigma_{f4}^2 & \sigma_{f4}^2 & \sigma_{f4}^2 & \sigma_{f4}^2 & \sigma_{f4}^2 & \sigma_{f4}^2 & \sigma_{f4}^2 \\ 1_{8x8} & 0_{8x1} & 0_{8x1} & 0_{8x1} & 0_{8x1} & 0_{8x1} & 0_{8x1} & 0_{8x1} \end{array} \right)' \quad (12), $$

where in the empirical applications we impose the standard identification assumption that $\sigma_f^2 = 1$.

Let us now assume that the idiosyncratic components of the interest rate differentials are $I(1)$ processes.\footnote{The unit roots tests performed for this series (not reported) cannot reject the unit root null hypothesis.} In addition, let us assume that the weakly frequencies of idiosyncratic components of monthly money growth and inflation differentials have an AR(5) representation which implies that the are fifth-order autocorrelated at monthly frequencies. Under these assumptions, the matrix $F$ becomes

$$ F = \begin{pmatrix} [0_{1x26}] & \vdots & \vdots \\ \vdots & \ddots & \vdots \\ \vdots & \vdots & [0_{2x14}] \end{pmatrix} \begin{pmatrix} I_{2x2} & [F1] & 0_{5x1} \\ \vdots & \ddots & \vdots \\ \vdots & \vdots & I_{4x4} \\ 0_{4x26} & \vdots & \vdots \\ \vdots & \ddots & \vdots \\ [F2] & \vdots & \vdots \\ I_{4x4} & \vdots & \vdots \\ 0_{4x1} & \vdots & \vdots \end{pmatrix}, $$

where

$$ F1 = \begin{pmatrix} \phi_{17,17} & \phi_{17,18} & \phi_{17,19} & \phi_{17,20} & \phi_{17,21} \\
\phi_{17,18} & \phi_{17,19} & \phi_{17,20} & \phi_{17,21} & \phi_{17,22} \\
\phi_{17,19} & \phi_{17,20} & \phi_{17,21} & \phi_{17,22} & \phi_{17,23} \\
\phi_{17,20} & \phi_{17,21} & \phi_{17,22} & \phi_{17,23} & \phi_{17,24} \\
\phi_{17,21} & \phi_{17,22} & \phi_{17,23} & \phi_{17,24} & \phi_{17,25} \end{pmatrix}, $$

$$ F2 = \begin{pmatrix} \phi_{22,22} & \phi_{22,23} & \phi_{22,24} & \phi_{22,25} & \phi_{22,26} \\
\phi_{22,23} & \phi_{22,24} & \phi_{22,25} & \phi_{22,26} & \phi_{22,27} \\
\phi_{22,24} & \phi_{22,25} & \phi_{22,26} & \phi_{22,27} & \phi_{22,28} \\
\phi_{22,25} & \phi_{22,26} & \phi_{22,27} & \phi_{22,28} & \phi_{22,29} \\
\phi_{22,26} & \phi_{22,27} & \phi_{22,28} & \phi_{22,29} & \phi_{22,30} \end{pmatrix}. $$
4.3 Estimation

The estimation of the model would be standard if all series were observable at the weekly frequency, as we assumed in the last sub-section. However, in the empirical application we actually use series of different length and we mix weekly data with monthly data, which makes estimation more involved. To deal with these complications, we treat all data as coming from weekly frequency, considering monthly series as weekly series with missing observations. Mariano and Murasawa (2003) develop a framework to easily handle with this issue. Their proposal consists on substituting the missing observations with random draws from a standard normal distribution which must be independent of the parameters of the model. The substitutions are applied not only to monthly series treated as weekly series with missed observations but also to weekly series that are of short length.\(^8\)

Let \(\theta\) be the parameter vector. Let \(Y_{i,t}\) be the \(i - th\) element of the \((n \times 1)\) vector \(Y_t\) and let \(R^+_{i,t}\) be its variance. The \(Y_{i,t}\) element takes the following values:

\[
Y_{i,t}^+ = \begin{cases} Y_{i,t} & \text{if } Y_{i,t} \text{ is observable} \\ z_t & \text{otherwise} \end{cases}, \quad i = 1...n,
\]

(13)

where \(z_t\) is a random draw from a standard normal distribution which is, by construction, independent of \(\theta\). Element \(i - th\) of vector \(w_t\) now becomes

\[
w_{i,t}^+ = \begin{cases} 0 & \text{if } Y_{i,t} \text{ is observable} \\ z_t & \text{otherwise} \end{cases}, \quad i = 1...n.
\]

(14)

The variance of \(Y_{i,t}\) becomes

\[
R^+_{i,t} = \begin{cases} 0 & \text{if } Y_{i,t} \text{ is observable} \\ 1 & \text{otherwise} \end{cases}, \quad i = 1...n.
\]

(15)

Finally, let \(H_i\) be the corresponding row \(i\) of matrix \(H_{nxr}\). This row takes the following values

\[
H_i^+ = \begin{cases} H_i & \text{if } Y_{i,t} \text{ is observable} \\ 0_{1xR} & \text{otherwise} \end{cases}, \quad i = 1...n.
\]

(16)

With these assumptions, we obtain a state-space model with no missing observations. We apply then the Kalman filter to \(Y_t^+, H^+, w_t^+\) and \(R^+\), and maximize the log-likelihood of \(\{Y_t^+\}_{t=1}^{T}\) numerically with respect to the unknown parameters in matrices \(F, H^+, Q\) and \(R^+\).

---

\(^8\)For example, the two-week euro-dollar forward exchange rate series used in the empirical application starts in October 10, 2003.
5 Empirical results

5.1 Preliminary analysis of data

We use only post-1998 euro-dollar exchange rate series, so we do not employ the “synthetic euro” or the ECU. Our weekly data sample for the euro-dollar exchange rate starts on January 8, 1999, the first business Friday of existence of the euro, and ends on August 6, 2010, so our sample includes 605 weekly observations for the euro-dollar exchange rate.

The series of euro-dollar exchange rate is plotted in log-levels in Figure 1. From the graph, the series in level appears as non-covariance stationary. To confirm this prior, Table 1 provides the results of Ng and Perron (2001) unit root tests. The test statistics show that the unit-root null hypothesis is not rejected which is consistent with the common perception that the exchange rate follows as a random walk process (possibly with drift) and that it will be hard to predict.

The weekly change in the log-return series is plotted in Figure 2. According to the graphical intuition, Table 1 shows that the unit-root null hypothesis is rejected at 5% of significance in all the tests suggested by Ng and Perron (2001). As can be seen in the graph, the time series shows considerable randomness and suggest that it will be hard to predict.

To analyze it further, in Table 2 we present some descriptive statistics for the log-returns of the euro-dollar exchange rate. According to the table, there are almost the same quantity of positive changes (300) than negative variations (301), so the average change is almost zero. In fact, the average value of the weekly change in the logarithm of the euro-dollar exchange rate is 0.00025, and its t-statistic is 0.36, failing to reject the hypothesis that the expected exchange rate change could be zero. The euro-dollar exchange rate returns are skewed left, as it is indicated by the negative value for the skewness. Moreover, their kurtosis is close to 5 which indicates that the shape of exchange rate returns is not close to a normal distribution. These results are strongly influenced by post-August 2008 events (especially, the three months after Lehman Brother bankruptcy).

The other weekly variables are euro-dollar forward exchange rates at different horizons (from one-week to one-year), short-term interest rate (three-month Libor) and long-term interest rate (ten-year government bond yields). Not all the weekly series start in January 1999, so there are differences in lengths among series due to data availability. Once we express spot and forward exchange rates in weekly growth rates (i.e., the first difference in the log of the weekly series), the effective sample of weekly variables reduces to 489 observations. The sample of monthly variables is from December 1998 until June 2010, with differences between series due to data availability.

# We use the GLS-detrended method, and select the lag-length using the Modified Akaike Information Criteria.
# Using a sample that ends July 2008, the skewness is about zero and the kurtosis is very close to three.
which gives at most 139 monthly observations that reduces to 138 observations when we take monthly growth rates.  

Data is mainly from Datastream, but some series are from the European Central Bank, the US Federal Reserve Bank and the OECD Main Economic Indicators. A detailed description of all series we employ is in the Appendix.

5.2 In-sample results

We show in this section the results of estimating the model outlined in Section 4. Note that since we substitute the missing values of the weekly series that start later than the euro-dollar exchange rate, we can estimate the model with the whole sample and are not restricted to using a smaller sample with no missing values.

In Figure 3, we show the actual series of euro-dollar exchange rate returns, and the estimated values of euro-dollar changes according to the model (i.e., using line 1 of the observation equation). The figure shows that the estimated series of euro-dollar exchange rate variation using our basic model mimic the erratic behavior of actual euro-dollar weekly returns.

Table 3 displays the estimated values for the factor loadings which reflect the degree to which variations in each observed variable are correlated with the latent factor. As expected, the estimated factor loadings of the spot rate indicate that it responds mainly to the factor loading at time $t$, but its response to the factor at $t + 1$ becomes negligible. By contrast, forward exchange rate variations mainly respond to the factor at $t + 1$, especially the forward exchange rate at 1 week, and their response to the factor at $t$ is much smaller and in general statistically insignificant.

The macroeconomic variables long-run interest rate differentials, M1 growth differential and inflation differential present loading factors that are much smaller than those of the spot rate and forward premiums, although they are statistically significant. The lower loading factors are not surprising because in a system of four exchange rates (one spot and three forwards) that display high comovements, the common factor mainly reflects their common movements. However, they are included in the model not because of their in-sample accuracy but because they contribute to improve the forecasting accuracy of exchange rate, as will be showed in the next two sub-sections.

It is worth noting that, although it is not shown in the table, the loading factor for short-run interest rate differential was statistically non-significant and the variable was drop out from the model. Therefore, we consider alternatives for risk for some different countries. In particular, we tried with 5-year credit default swap information and 10-year interest rate differentials from some risky euro-area countries as Portugal, Spain and Italy versus some riskless countries such as

\footnote{We download the data on August 12th, 2010, so the last observations of macro variables were for June 2010.}
as Germany and US. From the 12 measures of risk analyzed, we found that the better in-sample results were achieved for 5-year credit default swap of Portugal versus Germany which is clearly a valuable measure of risk.

Notably, Table 3 also shows the percentage of the variance of the euro-dollar exchange rate variation that is explained by the model which is as high as 0.90. This value appears to be relatively high when it is compared with works that aim to explain exchange rates with economic fundamentals. For instance, MacDonald and Taylor (1994), using and error-correction monetary model and monthly data, explain only 14% of the total variance of the US dollar-UK Pound rate. Moreover, recent works which employ a microstructure approach to the FX market and use order flows as explanatory variable report as evidence of the success of their models R-squared coefficients with lower values than ours. For instance, Chinn and Moore (2008) find for the euro-dollar exchange rate using monthly data a R-squared coefficient of 0.47.

5.3 Out-of-sample results

We present here the results of a simulated out-of-sample analysis to evaluate the predictive power of our econometric model for the euro-dollar exchange rate variations one-week, two-week and four-week ahead. We carry out this analysis using the conventional recursive forecasts approach that proceeds in the following way: we estimate the model using a reduced data sample and generate a one-week, two-week and four-week out-of-sample forecasts using the model suggested in Section 4. We then add an observation to the effective sample, estimate the model, and generate the forecasts again. We proceed in this fashion, adding one observation, reestimating the model and generating the forecasts, until the end of the sample when all the out-of-sample observations are exhausted. Specifically, we estimate our model with data from January 15, 1999 until December 28, 2007 (468 weekly observations, approximately 77.5% of the sample) and reserve the last 136 observations for the out-of-sample evaluation. Hence, we obtain the last one-week forecast with the model estimated using data up to August 6, 2010, the last two-week forecast from the model estimated using data up to July 30, 2010, and the last four-week forecast with the model estimated using data up to July 16, 2010. We denote $t = 1$ the first observation for which we calculate the out-of-sample, and we carry out the exercise until $t = T$, the last in-sample observation of FX variation.

Once we have the forecasted values for the euro-dollar exchange rate returns, denoted by $\{\hat{y}_{t,t}\}_{t=1}^{T}$, we obtain the forecast errors of our model, $\{e_{t,t}\}_{t=1}^{T}$, as the difference of the actual and predicted value of euro-dollar exchange rate variation. Our benchmark is the Random Walk (RW) model, whose forecasted values, denoted by $\{\hat{y}_{rw,t}\}_{t=1}^{T}$, are for this case a series of zeros
of dimension $Tx1$. This is because, for the RW model, the predictor of the next-period spot rate is the current spot rate, so for the spot rate variation, the prediction is the no change forecast. We then obtain the forecast errors of the RW model, $\{e_{rw,t}\}_{t=1}^{T}$, and compute for both series of forecasting errors the Mean Absolute Error (MAE), the Mean Squared Prediction Error (MSPE), and the Median Absolute Deviation (MAD). The MSPE and the MAE are the usual measures of prediction error performance but are sensitive to the existence of outliers, while the MAD is free of them. For these three important measures of forecast accuracy, the lower the output, the better the forecasting accuracy of the model. However, among two competing models in forecasting a determinate series, lower MSPE or MAE, for instance, does not necessarily imply superior forecasting specification, since the difference between the MAEs or MSPEs must also be statistically significantly different from zero. Hence, it is important to test whether reductions in the MSPE are statistically significant.

To evaluate if the differences between the measures of both models are statistically significant, we carry out several tests of forecast accuracy. Specifically, we use three tests of forecast accuracy from Diebold and Mariano (DM, 1995), a non-parametric test of predictive performance developed by Pesaran and Timmermann (1992), and a test of forecast encompassing proposed by Harvey, et al (1998).

First, we consider the asymptotic DM test. If the $MSPE_i$ is the loss function associated with a forecast, then $d_i = e^2_{1,t} - e^2_{rw,t}$ is the loss differential between the forecasts. Under certain conditions, the large-sample statistic for testing the null hypothesis of equal forecast accuracy, denoted by $S$, is asymptotically normally distributed. In addition, we also compute the Harvey, et al (1997) modification of the $S$ test-statistic, denoted by $S^*$, which distributes as a $t$-Student with $(T - 1)$ degrees of freedom.

Second, we use the Sign Test. It is based on a variable $s_i$ for $i = 1, ..., T$ which equals 1 when the SPE of our model is greater than the SPE of the random walk model, and zero otherwise. Based on the sum of $s_i$, the Sign test-statistic, $S_g$, to test the null hypothesis of equal forecast accuracy is asymptotically distributed as a normal random variable. Values of $S_g$ above critical values of standard normal distribution would then indicate significant differences between the MSPEs.

The third test we use is based on the Direction of Change (DCH) measure, which evaluates out-of-sample forecasts by comparing the sign of the forecasts with the sign of the true observation. This alternative evaluation metric for the relative forecast performance of two models, also called the success ratio, is computed as the number of correct predictions of the direction of change over the total number of predictions. Hence, the DCH metric is just the fraction of the $T$ forecasts that have the same sign as the realization of the exchange rate variation. A value of DCH above...
0.5 indicates a better forecasting performance than a naïve model that predicts that the exchange rate has an equal chance to go up or down. As Leitch and Tanner (1991) argue, the direction of change measure is a relevant alternative metric for establishing forecasting accuracy in financial markets since models which can accurately forecast the sign of future returns are found to be more profitable on economic grounds. They find that the direction of change criterion is the best proxy among several (including mean squared error and mean absolute error) for choosing forecasts of interest rates on their ability to maximize expected trading profits. Hence, this criterion may be more relevant for profitability and other economic concerns, while the criteria used above are based only on statistical motivations. This is particularly relevant for the exchange rate returns that we aim to forecast, since investors may be more interested in accurate forecasts of the direction in which the euro-dollar exchange rate is moving than in the exact magnitude of the change.

Diebold and Mariano (1995) describe a test-statistic, denoted by $DCH_{st}$, to evaluate if model 1 has a $DCH$ significantly better than a naïve “coin-toss” model that predicts that the exchange rate has an equal chance to go up or down. Since the RW model does not provide a sign prediction, we compare our model to the coin-toss model, which is analogous to the RW to this scenario. It can be shown that $DCH_{st}$ is asymptotically normally distributed, so positive values of $DCH_{st}$ above conventional critical values for standard normal distribution will indicate a significant improvement in the correct forecasting of the sign of FX variation.

The fourth test we employ is the Pesaran and Timmermann (1992) nonparametric test of predictive performance which also evaluates the correct prediction of the direction of change. Their test statistic, denoted by $PT$, is also asymptotically distributed as a normal random variable.

Lastly, we use the forecast encompassing described in Harvey, et al (1998). This refers to whether or not the forecasts from a competing model, in our case the random walk model, contain information missing from the forecasts from the original model. If they do not, then the forecasts from the alternative model are said to be encompassed by the forecasts from the original model. The test-statistic, denoted $HLN$, is also asymptotically normally distributed.

Table 4 displays the results of our out-of-sample exercise using the basic model described in Section 4 and compare them with those of the RW model. The table shows that at all horizons there are improvements in forecasting using our model over the random walk when comparing the respective MAE, MSPE and MAD measures. With all these statistical measures, and at all forecasting horizons considered, the ratios between the forecasting error measure of our model and the corresponding measure for the random walk model is below one, indicating that our model consistently drop lower forecasting errors. More importantly, results from the Asymptotic DM test, and the DM Sign test imply that these improvements are statistically significant at least at the 10%
level of significance. It is remarkable that at the very short one-week to four-week horizons, where much of the literature considers that noise dominates economic fundamentals in explaining FX fluctuations, our model does at least slightly better than the random walk in common statistical measures.

When we turn to the DCH measure, which as we stated above has greater economic content for FX forecasting, results show great improvements over a naïve model that predicts that the exchange rate has an equal chance to go up or down, since the fraction of times our model correctly predicts the sign of the FX variation is above 0.58 at all horizons. Moreover, this fraction is increasing in the forecasting horizon, reaching at a four-week horizon a ratio of 0.62 correctly predicted the sign of exchange rate variation. To evaluate if these improvements are statistically significant we use the DCH test and the PT test. Results of the DCH test show that these improvements are statistically significant at conventional levels of significance at all horizons. Using the PT test we also find that at all horizons the improvements in the DCH measure are statistically significant, but at the higher 10% level of significance.

Lastly, when we evaluate the results of the HLN test of forecast encompassing, we cannot reject, at two- and four-week horizons, the null hypothesis that forecasts from the random walk model are encompassed in those of our model. At two-week and four-week horizons, the statistics are negative and their p-values are above 0.15. However, at one-week horizon, the HLN results are less convincing.

Overall, our model forecasts the euro-dollar exchange rate variations better than the random walk using conventional error measures, these being statistically significant improvements, and considerably better than a naïve model that predicts that the exchange rate has an equal chance to go up or down using the Direction of change measure, which is also statistically significant at all horizons. The positive forecasting results of our model are also reflected in the fact that, using the forecast encompassing test, we do not reject the null hypothesis that the forecasts of random walk model are included in our forecasts. Our results are then in contrast to the assertion of Evans and Lyons (2002, p. 170) that macroeconomic models of exchange rates perform poorly at frequencies higher than one year.

Our results are also in contrast to those obtained by Dueker and Neely (2007). These authors use a Markov-switching model on daily data on several currencies against the US dollar between 1974 and 2006 (including the euro after 1998). They obtain that at one-, two-, and four-week forecasting horizons the MAEs and MSPEs of their model are almost identical to those of the random walk model (the lowest ratio of MSPEs was 0.999, the highest ratio of MSPEs was 1.002), and DCH measures slightly better than the naïve model, being the greatest DCH of 0.533 for the mark/euro-dollar exchange rate at one-week horizon.
5.4 Are macroeconomic variables useful in forecasting the euro-dollar exchange rate?

So far, we have shown that our basic model is useful in forecasting the weekly variations in the euro-dollar exchange rate. Specifically, we obtain smaller values of usual statistical measures of forecasting accuracy with respect to the RW model and bigger gains in the direction of change measure. In this sub-section, we aim to evaluate if these results are due to the incorporation of the macroeconomic variables (interest rate differentials, M1 growth differentials, and inflation) in the model. With this purpose, we estimate the model using only the spot and forward exchange rate variation, and perform the out-of-sample analysis for this reduced model.

The results of this counterfactual out-of-sample analysis are shown in Table 5. Overall, this table shows that the results of the model using only forward exchange rates cannot improve the results from the no-change random walk model. All the statistical measures of forecasting accuracy of the model that uses only forward exchange rates but the MAD at two- and four-week horizons, are greater than those of the random walk model. Accordingly, all the corresponding ratios between the forecasting error measure of our model and that of the random walk model are above one. In addition, although in any case the reduced model can improve the random walk, all the equal accuracy tests considered in the table indicate that the differences in forecasting accuracy are not statistically significant.

Table 5 also displays the results of the comparisons of the reduced model (i.e., only with forward exchange rates and excluding the macroeconomic fundamentals) with the basic model (i.e., the model that includes the economic fundamentals). According to the table, the basic model obtains smaller values for MAE and MSPE, and greater values for the DCH metric. In addition, the table shows that the results from forecast accuracy tests tend to find that these differences are statistically significant, perhaps with the exception of one-week forecasting horizon.

6 Conclusions

In this work, we propose an econometric model for the euro-dollar exchange rate that has the distinctive feature of utilizing economic variables quoted at different frequencies in explaining and forecasting weekly exchange rate variations. At this high frequency, the literature finds that it is very difficult to explain the foreign exchange rate movements, and even harder to forecast them. This is reflected by the fact that researchers usually cannot beat a simple random walk model
for the exchange rate movements that predicts the exchange rate to remain unchanged. In other words, the current spot rate appears to be the best predictor of the spot in the next period.

Against this background, in this paper we show that our dynamic factor model fits the in-sample weekly fluctuations of the euro-dollar exchange rate quite well, with an in-sample goodness of fit of about 80%. However, the extensive literature on exchange rate forecasting shows that good in-sample results do not always ensure good out-of-sample results. In our case, when we analyze the out-of-sample forecasting performance of the dynamic factor model using the standard recursive-regression procedures, we obtain that the model is able to improve upon the hard-to-beat random walk model in terms of traditional error measures such as the MAE or the MSPE. Moreover, several forecast accuracy tests show that these improvements appear to be statistically significant. It is an important result since at the very short one-, two-, and four-week horizons, the literature usually considers that noise dominates economic fundamentals in explaining exchange rate fluctuations.

The main positive result of our out-of-sample exercise is found when we turn attention to the direction of change metric, which has more economic content than the statistical measures mentioned above. With this metric, forecasting successful appears when the model can predict the sign of future exchange rate returns, regardless of the magnitude of the movement. In this case, our dynamic factor model performs much better than a naïve model that predicts that the exchange rate has an equal chance to go up or down at all forecasting horizons, these being statistically significant improvements at all forecasting horizons.

These results are promising and might hopefully give place to further research. In particular, we highlight two possible extensions of this paper. The first extension is based on recent research on exchange rates that has relied on the development of high-frequency models of the exchange rate based on microeconomic variables related to the structure of the market (Lyons, 2001). Among other things, the failure of macroeconomic fundamentals in explaining and forecasting exchange rates has turned the attention of academics to the study of the microstructure of FX markets. The objective is to identify which specific characteristics of this market bring insights that help to understand the exchange rate. A main result from this literature is the importance of order flows—a measure of net currency buying pressure defined as the net of buyer and seller initiated FX transaction—in understanding and forecasting the exchange rate.\footnote{An early statement of the use of flow data by FX market participants is done by Goodhart (1988, p. 456), but the literature relating order flows to exchange rates only explodes after the seminal study of Lyons (1995). Some papers that explore this theme further are Gehrig and Meunhoff (2004), and Evans and Lyons (2002, 2005). See Lyons (2001) for an excellent introduction to this literature and Osler (2006) for an evaluation of the lessons extracted from this currency microstructure literature about short-run FX rate dynamics.} Given that, the inclusion...
of microeconomic variables taken from these models, such as order flow data might improve the forecasting power of our model. The second extension is to use real-time macroeconomic data in the estimations and forecasts to evaluate the usefulness of our model in predicting with the same information which market participants have at each moment.
Appendix 1: Description of the data

From DATASTREAM

Nominal exchange rate: US$ to €, weekly e.o.p series from 8-1-1999 to 5-23-2008. Datastream Code: USECBSP.

Forward exchange rate 1 week: US$ to €, weekly from 1-1-1999 to 5-23-2008. Code: USEUR1W.


Credit-default swaps (5-year) for sovereign nations (Portugal vs Germany). Weekly from 30-1-2004 to 5-23-2008. Code: CDSPG.


From European Central Bank


From US Federal Reserve Bank


From OECD Main Economic Indicators

References


Table 1. Unit root tests for Euro-Dollar exchange rate

<table>
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<th>Panel A. Null hypothesis: Euro-Dollar exchange rate has a unit root</th>
<th>MZa</th>
<th>MZt</th>
<th>MSB</th>
<th>MPT</th>
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<td></td>
</tr>
<tr>
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<td>2.927**</td>
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</table>

Notes. These tests are proposed by Ng and Perron (2001). *, **, and *** denote rejection of the unit root null at 10%, 5% and 1%, respectively.
Table 2. Descriptive statistics of the euro-dollar exchange rate in log-returns

<table>
<thead>
<tr>
<th></th>
<th>Sample statistics</th>
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<tbody>
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<td>Sample mean</td>
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<td></td>
</tr>
<tr>
<td>Maximum value</td>
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<td>Minimum value</td>
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<td>Sample standard deviation</td>
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<td></td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.23573</td>
<td></td>
</tr>
<tr>
<td>Kurtosis</td>
<td>5.09699</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Value</th>
<th>Count</th>
<th>Percent</th>
<th>Cumulative count</th>
<th>Cumulative percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>[-0.1,-0.05]</td>
<td>3</td>
<td>0.50</td>
<td>3</td>
<td>0.50</td>
</tr>
<tr>
<td>[-0.05,0]</td>
<td>297</td>
<td>49.17</td>
<td>300</td>
<td>49.67</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>0.005</td>
<td>303</td>
<td>50.16</td>
</tr>
<tr>
<td>(0,0.05)</td>
<td>298</td>
<td>49.34</td>
<td>601</td>
<td>99.50</td>
</tr>
<tr>
<td>[0.05,0.1]</td>
<td>3</td>
<td>0.50</td>
<td>604</td>
<td>100</td>
</tr>
</tbody>
</table>

| Total       | 604   | 100     | 604              | 100                |

N. sign changes/total possible changes 296/603
Table 3. Estimation results of basic model

<table>
<thead>
<tr>
<th>Series</th>
<th>Parameter(^{(1)})</th>
<th>Estimates</th>
<th>Std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spot rate variations</td>
<td>(\beta_{s,1})</td>
<td>0.091</td>
<td>0.041</td>
</tr>
<tr>
<td></td>
<td>(\beta_{s,2})</td>
<td>0.941</td>
<td>0.027</td>
</tr>
<tr>
<td>Forward one-week var</td>
<td>(\beta_{1,1})</td>
<td>0.047</td>
<td>0.040</td>
</tr>
<tr>
<td></td>
<td>(\beta_{1,2})</td>
<td>0.992</td>
<td>0.025</td>
</tr>
<tr>
<td>Forward two-week var</td>
<td>(\beta_{2,1})</td>
<td>0.039</td>
<td>0.030</td>
</tr>
<tr>
<td></td>
<td>(\beta_{2,2})</td>
<td>0.984</td>
<td>0.025</td>
</tr>
<tr>
<td>Forward two-week var</td>
<td>(\beta_{3,1})</td>
<td>0.037</td>
<td>0.040</td>
</tr>
<tr>
<td></td>
<td>(\beta_{3,2})</td>
<td>0.961</td>
<td>0.028</td>
</tr>
<tr>
<td>CDS diff. Portugal-Germany</td>
<td>(\beta_{hs,2})</td>
<td>-0.015</td>
<td>0.005</td>
</tr>
<tr>
<td>Long-term int. rate diff.</td>
<td>(\beta_{h1,2})</td>
<td>-0.023</td>
<td>0.018</td>
</tr>
<tr>
<td>M1 growth diff.</td>
<td>(\beta_{M,2})</td>
<td>-0.070</td>
<td>0.050</td>
</tr>
<tr>
<td>Inflation diff.</td>
<td>(\beta_{n,2})</td>
<td>-0.066</td>
<td>0.036</td>
</tr>
<tr>
<td>Variance explained by the model(^{(2)})</td>
<td></td>
<td>0.90</td>
<td></td>
</tr>
</tbody>
</table>

Notes. (1) The beta parameters are the loading factors which relate the observed variables with the weakly factor. (2) This is the percentage of the variance of euro-dollar rate variations that are explained by the model.
Table 4. Out-of-sample evaluation with fundamentals

<table>
<thead>
<tr>
<th>Measure</th>
<th>1-week horizon</th>
<th>2-week horizon</th>
<th>4-week horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Basic model</td>
<td>RW</td>
<td>Basic model</td>
</tr>
<tr>
<td>MAE(1)</td>
<td>0.730</td>
<td>0.758</td>
<td>0.748</td>
</tr>
<tr>
<td>MSPE(2)</td>
<td>0.083</td>
<td>0.090</td>
<td>0.086</td>
</tr>
<tr>
<td>MAD(1)</td>
<td>0.582</td>
<td>0.659</td>
<td>0.633</td>
</tr>
<tr>
<td>DCH(3)</td>
<td>0.589</td>
<td>-</td>
<td>0.601</td>
</tr>
</tbody>
</table>

| Ratio MAEs(4) | 0.963 | 0.984 | 0.985 |
| Ratio MAEs(4) | 0.922 | 0.966 | 0.955 |
| Ratio MADs(4) | 0.883 | 0.953 | 0.959 |

Forecast accuracy tests(5)

<table>
<thead>
<tr>
<th>Test statistics</th>
<th>1-week horizon</th>
<th>2-week horizon</th>
<th>4-week horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>-4.58</td>
<td>-2.17</td>
<td>-2.55</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.02)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>S*</td>
<td>-4.47</td>
<td>-2.11</td>
<td>-2.36</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.02)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Sg</td>
<td>-1.34</td>
<td>-1.94</td>
<td>-1.72</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.03)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>DCHst</td>
<td>1.65</td>
<td>1.37</td>
<td>2.01</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.09)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>PT</td>
<td>1.75</td>
<td>1.69</td>
<td>1.88</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.05)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>HLN</td>
<td>2.21</td>
<td>-1.87</td>
<td>-1.43</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.08)</td>
</tr>
</tbody>
</table>

Notes. (MAE) Mean Absolute Error, (MSPE) Mean Squared Prediction Error, (MAD) Median Absolute Deviation, (DCH) Direction of Change. (S) Diebold and Mariano (1995), (S*) Harvey, et al (1997), (Sg) Sign Test (DCHst) Diebold and Mariano (1995), (PT) Pesaran and Timmermann (1992), (HLN) Harvey, et al (1998). (1) Multiplied by 100. (2) Multiplied by 1000. (3) The RW has no sign prediction so DCH is not reported for this model. (4) It is the ratio of the basic model over the RW. Ratios less than one indicate that the basic model forecasts exchange rate better than the RW model. (5) The p-values are in parenthesis.
Table 5 Out-of-sample evaluation without fundamentals

<table>
<thead>
<tr>
<th>Measure</th>
<th>1-week horizon</th>
<th>2-week horizon</th>
<th>4-week horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAE(^{(1)})</td>
<td>0.792</td>
<td>0.782</td>
<td>0.762</td>
</tr>
<tr>
<td>MSPE(^{(2)})</td>
<td>0.099</td>
<td>0.099</td>
<td>0.092</td>
</tr>
<tr>
<td>MAD(^{(3)})</td>
<td>0.687</td>
<td>0.622</td>
<td>0.638</td>
</tr>
<tr>
<td>DCH(^{(3)})</td>
<td>0.534</td>
<td>0.458</td>
<td>0.500</td>
</tr>
<tr>
<td>Ratio MAEs(^{(4)})</td>
<td>1.044</td>
<td>1.030</td>
<td>1.011</td>
</tr>
<tr>
<td>Ratio MAEs(^{(4)})</td>
<td>1.098</td>
<td>1.086</td>
<td>1.021</td>
</tr>
<tr>
<td>Ratio MADS(^{(4)})</td>
<td>1.043</td>
<td>1.037</td>
<td>0.962</td>
</tr>
</tbody>
</table>

Forecast accuracy tests\(^{(5)}\)

<table>
<thead>
<tr>
<th>Test statistics</th>
<th>1-week horizon</th>
<th>2-week horizon</th>
<th>4-week horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Against RW</td>
<td>Against Basic model</td>
<td>Against RW</td>
<td>Against Basic model</td>
</tr>
<tr>
<td>S</td>
<td>0.94</td>
<td>0.81</td>
<td>1.06</td>
</tr>
<tr>
<td>(0.17)</td>
<td>(0.21)</td>
<td>(0.15)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>S'</td>
<td>0.93</td>
<td>0.80</td>
<td>1.03</td>
</tr>
<tr>
<td>(0.18)</td>
<td>(0.22)</td>
<td>(0.16)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>Sg</td>
<td>0.35</td>
<td>2.23</td>
<td>0.94</td>
</tr>
<tr>
<td>(0.37)</td>
<td>(0.01)</td>
<td>(0.18)</td>
<td>(0.18)</td>
</tr>
<tr>
<td>DCHst</td>
<td>0.58</td>
<td>--</td>
<td>-0.70</td>
</tr>
<tr>
<td>(0.28)</td>
<td>--</td>
<td>(0.24)</td>
<td>--</td>
</tr>
<tr>
<td>PT</td>
<td>0.32</td>
<td>--</td>
<td>-1.02</td>
</tr>
<tr>
<td>(0.37)</td>
<td>--</td>
<td>(0.15)</td>
<td>--</td>
</tr>
<tr>
<td>HLN</td>
<td>0.23</td>
<td>0.23</td>
<td>-0.48</td>
</tr>
<tr>
<td>(0.41)</td>
<td>(0.41)</td>
<td>(0.32)</td>
<td>(0.53)</td>
</tr>
</tbody>
</table>

Notes. See notes of Table 5.
1. Previously published Working Papers are listed in the Banco de España publications catalogue.

JAMES COSTAIN AND ANTON NAKOV: Precautionary price stickiness.

ENRIQUE MORAL-BENITO: Model averaging in economics.

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ANTON NAKOV AND GALO NUÑO: A general equilibrium model of the oil market.

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MARCOS DAL BIANCO, MAXIMO CAMACHO AND GABRIEL PEREZ QUIROS: Short-run forecasting of the euro-dollar exchange rate with economic fundamentals.