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Abstract

This paper proposes a parsimonious approach to test non-linear dependence on the conditional mean and variance of hedge funds with respect to several market factors. My approach introduces non-linear dependence by means of empirically relevant polynomial functions of the factors. For comparison purposes, I also consider multifactor extensions of tests based on piecewise linear alternatives. I apply these tests to a database of monthly returns on 1,071 hedge funds. I find that non-linear dependence on the mean is highly sensitive to the factors that I consider. However, I obtain a much stronger evidence of non-linear dependence on the conditional variance.

Keywords: Generalised Hyperbolic Distribution, Correlation, Asymmetry, Multifactor Models.

JEL classification: C12, G11, C32, C22.

1 Introduction

Understanding the risk exposures of financial assets is crucial for investors. They need to assess these risks before choosing their portfolio allocations. These exposures have traditionally been measured with linear factor models with respect to a set of market variables. Its popularity has been partly due to the simplicity and intuitiveness of this methodology, but also to the fact that, under joint normality, linear models give a complete characterisation of dependence. In some cases, such models have been shown to work extremely well. For instance, a high proportion of mutual funds risks can be explained using a linear model with just a few risk factors (see Sharpe, 1992). However, this methodology can be far from fully informative if the dependence between variables goes beyond a simple linear relationship.

Hedge funds are probably the assets in which non-linearities can be more important. Their managers try to exploit investment opportunities with highly leveraged dynamic strategies in order to obtain high returns regardless of market conditions. In this sense, Fung and Hsieh (1997) have found that linear models of market factors are only able to explain a minor proportion of returns in this industry. Because of this low correlation with the market, traditional measures, such as the Sharpe ratio, might induce a naive investor to overestimate the diversification benefits of hedge funds. However, this picture can be quite misleading in the presence of non-linear dependence. In this sense, Fung and Hsieh (2001) and Agarwal and Naik (2004) have found evidence of option-like dependence of hedge funds with respect to the market. In addition, Lo (2001) has remarked the asymmetric relationship between hedge funds correlations and economic news, which can make them react almost in unison when there are strong negative shocks in the market.[1]

Two main testing approaches have emerged in the literature to analyse this problem. Glosten and Jagannathan (1994), Mitchell and Pulvino (2001) and Díez de los Ríos and Garcia (2009) introduce non-linearities on the conditional mean with piecewise linear functions of one market factor, while Treynor and Mazuy (1966) and Admati, Bhattacharya, Pfleiderer, and Ross (1986) consider a conditional mean that depends on a

[1]This feature is known as "Phase-locking" behaviour. One of the most famous examples of this phenomenon is the general debacle that followed the Russian debt crisis in August 1998, with LTCM as the most prominent victim.

polynomial function of the market. These studies focus on a univariate context, typically analysing non-linearities with respect to only one market factor at a time. More recently, Patton (2009) has extended the concept of non-linear dependence to higher order moments, but again he only considers dependence with respect to a single factor. However, it may be necessary to test dependence jointly for several factors. For instance, a non-linear exposure with respect to one factor might be explained as linear dependence once some additional variables are included. Alternatively, some funds may have an exposure to a non-linear function of a set of factors, such as their product. This feature cannot be detected unless non-linearities are studied from a multifactor perspective.

In this paper, I develop a multivariate framework to investigate non-linear dependence in both the conditional mean and variance. One of the problems of extending non-linearity tests into a multivariate context is that the number of potential moment conditions grows with 2^{N_m} for each fund, where N_m is the number of factors. Unfortunately, it may be infeasible to reliably test all of them due to the relatively small histories in hedge funds databases. I deal with this problem by trying to focus only on empirically relevant types of non-linear exposures. In particular, I introduce a set of parsimonious polynomial functions under the alternative hypothesis on the conditional mean and variance that only grow with N_m. Importantly, these conditions yield tests that are equivalent to using the Generalised Hyperbolic distribution (GH) as the alternative hypothesis. The GH distribution, which was originally introduced by Barndorff-Nielsen (1977), is a flexible family of multivariate asymmetric and leptokurtic distributions. It nests as particular cases the multivariate Gaussian, Student t, as well as the Hyperbolic, the Normal Inverse Gaussian, the Normal Gamma associated to the Variance Gamma process, the Multivariate Laplace and their asymmetric generalisations. Its empirical relevance has already been widely documented in the literature (see e.g. Madan and Milne, 1991; Chen, Härdle, and Jeong, 2008; Aas, Dimakos, and Haff, 2005; or Cajigas and Urga, 2007). In addition, it has been shown that it can account for asymmetric tail dependence in a high dimensional context (see McNeil, Frey, and Embrechts, 2005; Mencía and Sentana, 2009b). I derive both LM and Wald type tests and compare this approach with a multivariate extension of the piecewise linear dependence tests for more than one factor in a Monte Carlo study. For the two approaches, I consider tests that are robust to the presence of non-normality in the marginal distributions of both the hedge fund returns and market factors, since there is strong evidence of deviations from Gaussianity in Hedge funds returns (see e.g. Lo, 2008).

I consider an empirical application in which I revisit the database considered by Patton (2009), which combines information from the HFR and TASS databases about monthly returns in more than 1,000 funds from December 2002 until August 2003. I analyse the dependence structure between hedge fund returns and the S&P 500 index, as well as the Fama and French (1993) factors. I also compare the results obtained for individual hedge funds with those that I obtain for the hedge fund indices constructed by HFR. This comparison may provide some light on the informativeness of indices about the non-linear dependence on individual funds. Finally, I conduct several robustness checks to analyse the impact of the most important biases in hedge fund data on my results. In particular, I deal with the end-game, backfill and survivorship biases.

The rest of the paper is organised as follows. I derive the tests and conduct a Monte Carlo study in Section 2. I describe the data in Section 3, and compute the non-linear dependence tests in Section 4. Section 5 contains the robustness checks. Finally, I conclude in Section 6. Auxiliary results can be found in the Appendix.

2 Tests of non-linear dependence and statistical features

2.1 Non-linear structures

Assume that we have data on N_h hedge funds, with returns denoted by the vector $\mathbf{y}_{ht}$, for $t = 1, \cdots, T$. I am interested in assessing the exposure of these variables to N_m market indices with returns $\mathbf{y}_{mt}$. I analyse dependence in terms of the conditional mean vector and covariance matrix of $\mathbf{y}_{ht}$ given $\mathbf{y}_{mt}$. In this sense, I use the "mean neutrality" and "variance neutrality" concepts defined by Patton (2009), and extend them into a multivariate context where $N_h \geq 1$ and $N_m \geq 1$. My baseline case is linear dependence, which implies that all the exposure of hedge fund returns to the market factors can be explained by their correlations. More specifically, the exposure of $\mathbf{y}_{ht}$ to the market factors $\mathbf{y}_{mt}$ will be linear if

$$\mathbf{y}_{ht} \overset{H_0}{=} \boldsymbol{\alpha} + \sum_{i=1}^{p} \boldsymbol{\beta}_{hi} \mathbf{y}_{ht-i} + \sum_{j=0}^{q} \boldsymbol{\beta}_{mj} \boldsymbol{\varepsilon}_{mt-j} + \boldsymbol{\varepsilon}_{h|mt}, \quad (1)$$

where $\boldsymbol{\varepsilon}_{mt}$ are the unexpected shocks to the market variables or $\boldsymbol{\varepsilon}_{mt} = \mathbf{y}_{mt} - E(\mathbf{y}_{mt}|I_{t-1})$, and I_{t-1} denotes information known at $t-1$. In addition, $\boldsymbol{\varepsilon}_{h|mt}$ is a vector martingale difference sequence satisfying $E\left(\boldsymbol{\varepsilon}_{h|mt}\middle|\,\mathbf{y}_{mt}, I_{t-1};\boldsymbol{\theta}\right) = \mathbf{0}$ and $V\left(\boldsymbol{\varepsilon}_{h|mt}\middle|\,\mathbf{y}_{mt}, I_{t-1};\boldsymbol{\theta}\right) = \boldsymbol{\Gamma}$, while $\boldsymbol{\alpha}$, $\boldsymbol{\beta}_{hi}, \boldsymbol{\beta}_{mj}$ and $\boldsymbol{\Gamma}$ are, respectively, $N_h \times 1$, $N_h \times N_h$, $N_h \times N_m$ and $N_h \times N_h$ matrices, and $\boldsymbol{\theta}$ is a vector containing all the parameters that define the processes followed by $\mathbf{y}_{mt}$ and $\mathbf{y}_{ht}$ under linear dependence.

Thus, the conditional mean is linear in the contemporaneous realisation of the market through the factor loadings $\boldsymbol{\beta}_{m0}$, while the conditional variance does not depend on the market. This dependence structure would be the one obtained if $\mathbf{y}_{ht}$ and $\mathbf{y}_{mt}$ were jointly Gaussian. In this sense, I can also denote this structure as Gaussian dependence. However, I do not make any explicit assumption about the marginal distributions of either $\mathbf{y}_{ht}$ or $\mathbf{y}_{mt}$. In addition, I also allow for time series autocorrelation. As Getmansky, Lo, and Makarov (2004) have emphasised, this is an important feature of hedge funds, because of their lack of liquidity. It is also important to take into account that hedge fund returns may respond to market's moves with some lag under the presence of these "stale" prices. Consequently, as Asness, Krail, and Liew (2001) suggest, I follow Scholes and Williams (1977) and consider a feedback effects of lagged market returns on current hedge funds returns in (1).

I introduce the unexpected shocks of the market factors in (1) because the expected component can be explained by the previous history of these variables. My approach does not preclude any stationary model of the dynamics of $\mathbf{y}_{mt}$. For the sake of concreteness, though, I assume the following vector autoregression

$$\mathbf{y}_{mt} = \boldsymbol{\lambda}_0 + \sum_{j=1}^{q'} \boldsymbol{\lambda}_i \mathbf{y}_{mt-j} + \boldsymbol{\varepsilon}_{mt}, \tag{2}$$

such that $\boldsymbol{\varepsilon}_{mt}$ is a vector martingale difference sequence satisfying $E\left(\boldsymbol{\varepsilon}_{mt}\middle|\, I_{t-1}\right) = \mathbf{0}$ and $V\left(\boldsymbol{\varepsilon}_{mt}\middle|\, I_{t-1}\right) = \boldsymbol{\Psi}$. For the sake of simplicity, I am considering a homoskedastic model, but it is also possible to introduce a heteroskedastic parametrisation in which $\boldsymbol{\Gamma}$ and $\boldsymbol{\Psi}$ are time varying functions of I_{t-1}.[2]

[2]This extension is available on request.

As the alternative hypothesis, I consider the following conditional moments

$$E\left(\mathbf{y}_{ht}|\,\mathbf{y}_{mt}, I_{t-1}; \boldsymbol{\phi}\right) = \boldsymbol{\alpha} + \sum_{i=1}^{p} \boldsymbol{\beta}_{hi}\mathbf{y}_{ht-i} + \sum_{j=0}^{q} \boldsymbol{\beta}_{mj}\boldsymbol{\varepsilon}_{mt-j} + \boldsymbol{\Gamma}\boldsymbol{\delta}(\varsigma_{mt} - N_m), \quad (3)$$

$$V\left(\mathbf{y}_{ht}|\,\mathbf{y}_{mt}, I_{t-1}; \boldsymbol{\phi}\right) = \boldsymbol{\Gamma}\left[1 + \boldsymbol{\omega}_1'\boldsymbol{\varepsilon}_{mt} + \omega_2(\varsigma_{mt} - N_m)\right]^2, \quad (4)$$

where $\boldsymbol{\phi} = (\boldsymbol{\theta}', \boldsymbol{\omega}_1', \omega_2, \boldsymbol{\delta}')'$, $\boldsymbol{\delta}$ and $\boldsymbol{\omega}_1$ are, respectively, $N_h \times 1$ and $N_m \times 1$ vectors of parameters, ω_2 is a scalar, and

$$\varsigma_{mt}(\boldsymbol{\theta}) = \left[\mathbf{y}_{mt} - E(\mathbf{y}_{mt}|I_{t-1}; \boldsymbol{\theta})\right]' \boldsymbol{\Psi}^{-1} \left[\mathbf{y}_{mt} - E(\mathbf{y}_{mt}|I_{t-1}; \boldsymbol{\theta})\right]. \quad (5)$$

Hence, the non-linearities on the conditional variance have the form of squared unexpected standardised shocks to the market factors. Thus, testing linearity in this context implies checking whether $\boldsymbol{\delta}$, $\boldsymbol{\omega}_1$ and ω_2 are jointly zero. We can also focus on linearity on the mean by considering a test of $\boldsymbol{\delta} = \mathbf{0}$ under the maintained hypothesis that $\boldsymbol{\omega}_1 = \mathbf{0}$ and $\omega_2 = 0$, or test linearity on the conditional variance with the joint null of $\boldsymbol{\omega}_1 = \mathbf{0}$ and $\omega_2 = 0$ and the maintained hypothesis that $\boldsymbol{\delta} = \mathbf{0}$.

The alternative hypothesis specified in (3) and (4) has several interesting properties. To begin with, it is fully general when $N_m = 1$ for the family of polynomial quadratic expansions of the conditional mean and variance. In this sense, my approach is consistent with the univariate polynomial non-linear models considered by Treynor and Mazuy (1966) and Admati, Bhattacharya, Pfleiderer, and Ross (1986), the Taylor expansions on the conditional mean and variance proposed by Patton (2009), and the asset pricing model with co-skewness considered by Barone-Adesi, Gagliardini, and Urga (2004). For higher dimensions, I introduce up to quadratic terms on the conditional mean and standard deviation in such a way that the number of additional parameters only grows with N_m for each fund. In this sense, I avoid the curse of dimensionality that introducing all the potential moment conditions would cause by focusing on some important deviations from the null. Specifically, it can be shown that testing linearity against (3) and (4) yields exactly the same tests as if I used the rather flexible dependence structure of the GH distribution (see Appendix C). Therefore, I focus on those departures from the null that seem to be relevant from an empirical point of view.

I also compare my approach with the non-linear dependence tests on piecewise linear functions of the market. This strand of the literature was popularised by Glosten and Jagannathan (1994), who considered the following type of non-linear dependence on the mean

$$E(y_{ht}|\,\mathbf{y}_{mt},I_{t-1})=\boldsymbol{\alpha}+\sum_{i=1}^{p}\beta_{hi}y_{ht-i}+\sum_{j=0}^{q}\beta_{mj}\mathbf{y}_{mt-j}+\sum_{k=1}^{N_m}\varphi_k\max(y_{mt,k}-x_k,0), \tag{6}$$

where $y_{mt,k}$ is the $k-th$ element of $\mathbf{y}_{mt}$. One of the advantages of (6) is that this kind of non-linear dependence can be related to option-like features in the performance of hedge funds. However, these models do not study non-linear effects beyond the conditional mean, and it is difficult to derive any specific recommendation for an appropriate multivariate distribution that deals with these characteristics. Furthermore, testing for $\varphi_1=\cdots=\varphi_{N_m}=0$ is a hard non-standard problem, especially for large dimensions, since the cutoff points $x_1,\cdots,x_{N_m}$ are not identified under the null. In this sense, I follow Díez de los Ríos and Garcia (2009) and use the methodology of Hansen (1996) to deal with this issue.

2.2 Testing approach

I consider score based tests of polynomial non-linear dependence on the mean and variance. I also study Wald tests for non-linear dependence on the mean, but not for the variance because the estimates of $\boldsymbol{\omega}_1$ and ω_2 under the alternative suffer large distortions for the small sample sizes that I have. The score based or LM tests do not suffer this problem because they only require the estimation of the model under the null hypothesis. In particular, I estimate (1) and (2) by Gaussian Pseudo-Maximum Likelihood (GPML) estimation, which ensures consistency even under absence of Gaussianity as long as the means and variances are correctly specified. Then, I derive a score based test of the null of linear dependence from the GPML of $\mathbf{y}_{ht}$ given $\mathbf{y}_{mt}$. For the sake of brevity, I will focus on the main result. The details of the derivation of the test are available in Appendix A. First, define $\boldsymbol{\varepsilon}_{h|mt}(\tilde{\boldsymbol{\theta}}_T)$ and $\boldsymbol{\varepsilon}_{mt}(\tilde{\boldsymbol{\theta}}_T)$ as the estimates of the error terms in (1) and (2), respectively, under the GPML vector $\tilde{\boldsymbol{\phi}}_T=(\tilde{\boldsymbol{\theta}}_T',\mathbf{0}',0,\mathbf{0}')'$ of restricted parameter estimates. Then, it can be shown that the LM test that assesses that $\boldsymbol{\delta}$, $\boldsymbol{\omega}_1$ and ω_2 are jointly zero can be expressed as

$$\begin{aligned}\tau &= \left[\frac{\sqrt{T}}{T}\sum_t \mathbf{v}_{\eta t}'(\mathbf{y}_{ht}|\,\mathbf{y}_{mt};\tilde{\boldsymbol{\theta}}_T)\right]\\ &\quad\times\mathcal{F}^{-1}\left(\tilde{\boldsymbol{\theta}}_T\right)\left[\frac{\sqrt{T}}{T}\sum_t \mathbf{v}_{\eta t}(\mathbf{y}_{ht}|\,\mathbf{y}_{mt};\tilde{\boldsymbol{\theta}}_T)\right]\end{aligned} \tag{7}$$

where

$$\mathbf{v}_{\eta t}\left(\mathbf{y}_{ht}|\,\mathbf{y}_{mt};\tilde{\boldsymbol{\theta}}_T\right)=\left[\begin{array}{c}\mathbf{v}_{\eta v t}\left(\mathbf{y}_{ht}|\,\mathbf{y}_{mt};\tilde{\boldsymbol{\theta}}_T\right)\\ \mathbf{v}_{\eta\mu t}\left(\mathbf{y}_{ht}|\,\mathbf{y}_{mt};\tilde{\boldsymbol{\theta}}_T\right)\end{array}\right] \tag{8}$$

is the vector of moment conditions, which can be decomposed as

$$\mathbf{v}_{\eta\mu t}\left(\mathbf{y}_{ht}|\,\mathbf{y}_{mt};\tilde{\boldsymbol{\theta}}_T\right)=\left[\varsigma_{mt}\left(\tilde{\boldsymbol{\theta}}_T\right)-k\right]\boldsymbol{\varepsilon}_{h|mt}\left(\tilde{\boldsymbol{\theta}}_T\right), \tag{9}$$

and

$$\mathbf{v}_{\eta v t}\left(\mathbf{y}_{ht}|\,\mathbf{y}_{mt};\tilde{\boldsymbol{\theta}}_T\right)=\left\{\begin{array}{c}\frac{1}{2}\left[\varsigma_{h|mt}\left(\tilde{\boldsymbol{\theta}}_T\right)-N_h\right]\left[\varsigma_{mt}\left(\tilde{\boldsymbol{\theta}}_T\right)-N_m\right]\\ \left[\varsigma_{h|mt}\left(\tilde{\boldsymbol{\theta}}_T\right)-N_h\right]\boldsymbol{\varepsilon}_{mt}\left(\tilde{\boldsymbol{\theta}}_T\right)\end{array}\right\}. \tag{10}$$

In addition, $\varsigma_{h|mt}(\tilde{\boldsymbol{\theta}}_T)=\boldsymbol{\varepsilon}'_{h|mt}(\tilde{\boldsymbol{\theta}}_T)\tilde{\boldsymbol{\Gamma}}_T^{-1}\boldsymbol{\varepsilon}_{h|mt}(\tilde{\boldsymbol{\theta}}_T)$, and the weighting matrix $\mathcal{F}(\tilde{\boldsymbol{\theta}}_T)$, which is defined in Appendix A, is the covariance matrix of $\sum_t \mathbf{v}'_{\eta t}(\mathbf{y}_{ht}|\,\mathbf{y}_{mt};\tilde{\boldsymbol{\theta}}_T)/\sqrt{T}$ under the null of linear dependence. Hence, this test converges asymptotically to a chi-square with N_h+N_m+1 degrees of freedom under the null hypothesis.

The score based test (7) can be interpreted as a moment test of $E\left[\mathbf{v}_{\eta t}\left(\mathbf{y}_{ht}|\,\mathbf{y}_{mt};\boldsymbol{\theta}_0\right)\right]=\mathbf{0}$. It can also be shown that (9) is the set of moment conditions that results from testing $\boldsymbol{\delta}=\mathbf{0}$. Similarly, the first and second components of (10) are due to testing $\boldsymbol{\omega}_1=\mathbf{0}$ and $\omega_2=0$, respectively. Hence, it is not difficult to test each of these hypotheses separately. For instance, we can test the null of linearity on the mean, which corresponds to $\boldsymbol{\delta}=\mathbf{0}$, by computing

$$\tau_\mu=\left[\frac{\sqrt{T}}{T}\sum_t \mathbf{v}'_{\eta\mu t}(\mathbf{y}_{ht}|\,\mathbf{y}_{mt};\tilde{\boldsymbol{\theta}}_T)\right]\mathcal{F}_{\mu\mu}^{-1}\left(\tilde{\boldsymbol{\theta}}_T\right)\left[\frac{\sqrt{T}}{T}\sum_t \mathbf{v}_{\eta\mu t}(\mathbf{y}_{ht}|\,\mathbf{y}_{mt};\tilde{\boldsymbol{\theta}}_T)\right], \tag{11}$$

where $\mathcal{F}_{\mu\mu}(\tilde{\boldsymbol{\theta}}_T)$ is the $N_h\times N_h$ sub-matrix of $\mathcal{F}(\tilde{\boldsymbol{\theta}}_T)$ containing the covariance matrix of (9). In this case, the asymptotic distribution of (11) will be a chi-square with N_h degrees of freedom.

It can be useful to consider the case of $N_h=N_m=1$, in which (8) becomes

$$\left[\begin{array}{c}\tilde{\gamma}_T\text{cov}_{t-1}(\varepsilon^*_{h|mt},\varepsilon^{*2}_{mt})\\ .5\text{cov}_{t-1}(\varepsilon^{*2}_{h|mt},\varepsilon^{*2}_{mt})\\ \tilde{\psi}_T\text{cov}_{t-1}(\varepsilon^{*2}_{h|mt},\varepsilon^{*}_{mt})\end{array}\right]=\left[\begin{array}{c}\tilde{\gamma}_T\sqrt{2}H_1(\varepsilon^*_{h|mt})H_2\left(\varepsilon^*_{mt}\right)\\ H_2(\varepsilon^*_{h|mt})H_2\left(\varepsilon^*_{mt}\right)\\ \tilde{\psi}_T\sqrt{2}H_1\left(\varepsilon^*_{mt}\right)H_2(\varepsilon^*_{h|mt})\end{array}\right], \tag{12}$$

where $H_k(\cdot)$ denotes the standardised Hermite polynomial of order k (see Stuart and Ord, 1977), and $\varepsilon^*_{h|mt}=\varepsilon_{h|mt}/\tilde{\gamma}_T$, while $\varepsilon^*_{mt}=\varepsilon_{mt}/\tilde{\psi}_T$.[3] Hence, these moment conditions are the co-skewness and co-kurtosis terms between the standardised residuals $\varepsilon^*_{h|mt}$ and ε^*_{mt}.

[3]For ease of notation, we will not specify the dependence on $\boldsymbol{\theta}$ in this discussion. I use lower case Greek letters to denote the values of $\tilde{\boldsymbol{\Gamma}}_T$ and $\tilde{\boldsymbol{\Psi}}_T$ in the univariate case.

I can compare (12) with the moment conditions of the test of Mencía and Sentana (2009a) for Gaussian vs. GH innovations. In particular, the moment conditions for the joint normality of $\varepsilon^*_{h|mt}$ and ε_{mt} would become

$$\tilde{\gamma}_T \left[\sqrt{6}H_3\left(\varepsilon^*_{h|mt}\right) + \sqrt{2}H_1\left(\varepsilon^*_{h|mt}\right)H_2\left(\varepsilon^*_{mt}\right)\right], \tag{13}$$

$$\tilde{\psi}_T \left[\sqrt{6}H_3\left(\varepsilon^*_{mt}\right) + \sqrt{2}H_1\left(\varepsilon^*_{mt}\right)H_2\left(\varepsilon^*_{h|mt}\right)\right], \tag{14}$$

and

$$\sqrt{3/2}\left[H_4\left(\varepsilon^*_{h|mt}\right) + H_4\left(\varepsilon^*_{mt}\right)\right] + H_2\left(\varepsilon^*_{h|mt}\right)H_2\left(\varepsilon^*_{mt}\right). \tag{15}$$

The kurtosis component of this test is captured by (15), while (13) and (14) are the two elements of the skewness component. If we compare (12) with these equations, we can see that our test only retains from the normality test those components that are related to co-skewness and co-kurtosis, but not those that deal with the marginal skewness and kurtosis of each variable ($H_3(\cdot)$ and $H_4(\cdot)$, respectively). Hence, the test does not impose Gaussianity on the marginal distributions. In general, for N_h or N_m greater than one, I obtain a generalised multivariate measure of co-kurtosis and co-skewness.

2.3 Finite sample properties

In this subsection, I assess the finite sample size and power properties of the testing procedures discussed above by means of several extensive Monte Carlo exercises. I consider linear dependence tests between $N_h = 1$ fund and either $N_m = 1$ or $N_m = 3$ market factors. In particular, I compare the score based and Wald tests of non-linear dependence on the mean that use polynomial alternatives with their analogs for piecewise linear alternatives. In addition, I consider the joint test of non-linear dependence on the mean and variance for the case of polynomial alternatives.

I allow for first order autocorrelation and one-period feedback effects of the market factors on the fund returns. Hence, I set $p = q = q' = 1$ in (1) and (2). The data generating process of the fund's returns is Gaussian with $\alpha = 2.5$, $\beta_{h1} = 0.1, \boldsymbol{\beta}_{m1} = 0$ and $\sqrt{\gamma} = 8$ in all the designs. These parameter values resemble the annualised moments of an average market neutral hedge fund. I also consider Gaussian factors with $\boldsymbol{\lambda}_0 = (5, 4, 3)'$, $\boldsymbol{\lambda}_1 = \mathbf{0}$, and set $\boldsymbol{\Psi}$ to a diagonal matrix whose elements on the diagonal are 14^2, 11^2 and 12^2. Again, these parameter values yield returns whose characteristics are similar to those of the factors that I use in the empirical application. I only consider the first of these factors in the single factor tests, in which $N_m = 1$.

The results, based on 5,000 Monte Carlo replications of samples of size $T = 100$, are summarised by means of Davidson and MacKinnon's (1998) p-value plots. These plots graphically compare actual and nominal test sizes for every possible nominal size. I obtain the nominal sizes from the asymptotic p-values in all tests. These values are available in closed form for the tests against polynomial alternatives, while I need to use the simulation algorithm devised by Hansen (1996) in the case of piecewise linear alternatives.[4]

Single factor tests. Figure 1 shows the results for the tests with respect to one factor. In Figure 1a I consider the size of the tests when I simulate the model from the null of linear dependence. The plot shows the discrepancy between the actual and nominal sizes. The two Wald tests of linear dependence on the mean yield the highest distortions, since they tend to over-reject the null hypothesis. For instance, they reject in around 14% of cases for a nominal size of 10%. In contrast, the smallest distortions are obtained for the LM test of polynomial non-linear dependence on the mean.

Figure 1b shows the power of the tests when I consider polynomial non-linear dependence on the mean. In particular, the fund's returns are generated from (3) and (4), with $\delta = 0.025$, $\omega_1 = 0$, and $\omega_2 = 0$. Since $\delta > 0$ under this parameter configuration, the fund would tend to yield higher returns when the market experiences large shocks, regardless of their sign. The tests that only consider non-linear dependence on the mean yield a slightly higher power than the one that also studies the dependence on the variance. The latter test loses some power by testing that (10) has a zero mean, because the conditional variance does not depend on the market factor in this design. It is interesting to observe that the LM test for piecewise alternatives on the mean is less powerful than the Wald test based on the same alternative hypothesis. However, this may in part be due to the tendency of the Wald test to over-reject under the null. I also observe this feature in Figure 1c, where I consider a data generating process with piecewise linear dependence. Specifically, I generate the returns of y_{ht} with the conditional mean (6), such that $\varphi_1 = 0.5$ and $x_1 = 6$. Hence, the fund would yield higher than usual returns

[4] I obtain these p-values from 1,000 simulations for $N_m = 1$, but I can only use 200 simulations when $N_m = 3$ due to the computational burden of this methodology.

in bull markets in this case. Again, the relative performance of the tests is similar to the one observed in Figure 1b. Hence, regardless of the actual data generating process, both types of tests provide similar power when I consider dependence with respect to one factor. Finally, I consider polynomial non-linear dependence on the variance in Figure 1d, by setting $\delta = 0$, $\omega_1 = 0.02$, and $\omega_2 = 0.02$ in (3) and (4). As expected, now only the test that considers non-linear dependence on the conditional variance is able to yield non-trivial power. All the remaining tests follow closely the 45 degree line, which indicates that their actual and nominal sizes almost coincide.

Multifactor tests. In Figure 2, I repeat the same exercise, but now I test dependence with respect to the $N_m = 3$ market factors. Figure 2a shows that the Wald tests may over-reject even more as the number of factors in the test increases. Figure 2b shows the power of the tests when the fund's returns are generated from (3) and (4), with $\delta = 0.0025$, $\omega_1 = 0$, and $\omega_2 = 0$. Notice that, although the Wald tests seem to provide higher power, the distortions in Figure 2a are again responsible for part of this extra power. Figure 2b (polynomial dependence) and Figure 2c (piecewise linear dependence) confirm that polynomial and piecewise linear alternatives are still able to yield non-trivial power even when the alternative hypothesis is incorrectly specified. Again, the joint test of non-linear dependence on the mean and variance is less powerful when the conditional variance does not depend on the market factors. However, as Figure 2d shows, this test is the only one that provides power when non-linear dependence is only present on the conditional variance.

3 Data

I use monthly returns, net of all fees, on the funds in the HFR and TASS databases that belong to the following categories: equity hedge, equity non hedge, market neutral, event driven and fund of funds. According to HFR, equity hedge funds hold long positions on equities that are hedged at all times with short sales of stocks or stock index options. Conservative funds mitigate market risk by seeking an exposure smaller than 100%, while aggressive funds may magnify market risk by exceeding a 100% exposure. Equity non-hedge funds, also known as stock pickers, take predominantly long positions on equities. Although they may hedge with short sales, they do not always have a hedge in place. Market neutral funds try to exploit pricing inefficiencies between related securities, combining long and short positions to neutralise market exposure. Event driven funds obtain their profits by investing in transactional events such as spin-offs, mergers and acquisitions, bankruptcy reorganisations, recapitalisations and share buybacks. Market risk may be hedged in this case with purchases of put options. Finally, funds of funds invest in a diversified portfolio of managers. Since funds are classified based on their self-descriptions, these categories can be less appropriate in some cases. In this sense, non-linear dependence tests can help determine the actual risk exposure of hedge funds.

The database covers the period from December 1992 to August 2003. Both live and dead funds are included, although it is not reported whether those funds with an incomplete history are dead or have just stopped reporting their returns. Hence, the largest sample sizes are only of 129 observations, while the smallest ones can be of just one month of data. Because of this feature, I will only consider those funds with at least 30 observations. Table 1 presents some descriptive statistics of these funds. In particular, I compute the median values by category of the mean and standard deviations of the funds excess returns with respect to the one-month Euro-Dollar rate. In general, hedge funds tend to yield either higher average returns and/or smaller standard deviations than the S&P 500 over the same sample period. I also compute their median correlations by fund category with respect to the S&P 500 and the Small Minus Big (SMB) and High Minus Low (HML) factors of Fama and French (1993). Following Fung and Hsieh (2004) I consider these factors because they are designed to capture the common risks of stock returns, as Fama and French (1993) argue, and this is the type of assets in which the hedge funds that I consider invest. Market neutral funds are clearly the most "neutral" group as far as simple correlations are concerned, since the median correlations are close to zero with respect to the three factors. The other categories have on average a positive exposure to the S&P 500 and the SMB factor, and a negative exposure to HML returns.

However, these are just median correlations. Table 2a reports the proportion of funds in each category with statistically significant correlations with the market factors. Notice that these proportions will not be zero even if the null of no correlation is true.[5] In this sense, asterisks denote the cases in which the proportions are statistically significant.[6] In

[5]For instance, there will be 5% test rejections on average at a 95% confidence level.

[6]If the funds were independent, the critical values of the distribution of test rejections under the null would be binomial. However, since the independence assumption does not seem to be realistic, I use

this case, it turns out that all the values are significant, which implies that there is a non-negligible presence of correlation with the market factors in all categories. Nevertheless, the market neutral group still remains the most "neutral" style, since it has the lowest proportion of test rejections. For instance, only 41.3% of market neutral funds are significantly correlated with the S&P 500, while 84.8% of equity non hedge funds are correlated with this factor. This is not surprising, since equity non hedge funds are generally long on equities. Table 2b shows the proportion of rejections of the normality test of Jarque and Bera (1980). The results show that there is a significant presence of non-normality in this industry. Interestingly, it seems that non-Gaussian kurtosis rather than skewness is the main driver of these rejections, since it is present in about half of the funds in all the categories, and reaches 76.2% in the event driven group. Therefore, it is very important to consider dependence tests that are robust to these deviations form Gaussianity.

There is a lot of discussion in the literature about how representative hedge fund indices are of their respective investment strategy. For the sake of comparison, I will also consider monthly data from January 1990 until September 2009 of the hedge fund indices computed by HFR for market neutral, equity hedge, event driven and fund of funds.[7] Table 3 shows the descriptive statistics for these indices. Again, hedge fund indices display either higher average excess returns, smaller standard deviations or both. Most of them are significantly correlated with the market factors, although the market neutral index only displays a significant albeit small correlation with the S&P 500. Once again, this confirms the higher neutrality of this category as far as linear correlation is concerned. The other indices are positively correlated with the S&P 500 and the SMB factor, and display negative exposure to the HML series. This is consistent with the median results from Table 1. The Jarque and Bera (1980) test is rejected in all cases because of the presence of excess kurtosis, but skewness is also significantly negative in the event driven and the fund of funds indices.

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an alternative approach. In particular, I generate simulated Gaussian data for each fund with the same correlations with the S&P 500 and the Fama-French factors as in the actual data. Then, I compute the proportions of test rejections. By repeating this procedure 1,000 times I can obtain simulations of the empirical distribution of test rejections. Then, the critical value is just the 95% percentile of these simulations.

[7] At the moment of writing, HFR does not compute an index for equity non hedge funds.

4 Non-linear dependence

Individual funds. My goal is to analyse the presence of non-linear exposure to the market in the different hedge fund categories rather than identifying the specific funds displaying non-linear features.[8] One of the most natural statistics for this purpose is probably the proportion of funds with non-linear dependence, since it directly measures on a scale from 0 to 100 the existence of this type of features. In addition, it is directly comparable across different styles. Table 4 reports the proportions of test rejections at the 95% confidence level of the non-linear dependence tests. From a statistical point of view, only those styles with values that are significantly different from zero provide evidence of non-linear dependence. In this sense, I use a bootstrap procedure to assess statistical significance at the 95% level (see Patton, 2009, and Appendix B for more details). In Table 4a, I focus on the dependence with respect to one market factor, proxied by the S&P 500 excess returns. The LM test of linear vs. polynomial dependence on the mean only yields significant test rejections in the event driven and fund of funds categories. In the event driven group, 38.1% seem to display non-linear dependence on the mean, while this figure is 24.8% for funds of funds. The Wald test analog yields similar albeit slightly higher rejection rates, while the tests based on piecewise linear alternatives seem to find a smaller presence of non-linear dependence on the mean. For instance, the piecewise linear LM test finds non-linear dependence in 19% of event driven funds and 11.6% of fund of funds. However, these values are also significant. Hence, non-linear dependence on the mean seems to be limited to two categories. In contrast, non-linear dependence on the conditional variance is much more widespread. All the categories, except for the equity non hedge group, display significant proportions of test rejections. The values range from 14.6% for the equity hedge group to 25% for event driven funds. As a consequence, the joint test of non-linear dependence on the mean and variance obtains significant proportions of test rejections in all but the equity non hedge group.

I consider multifactor non-linear dependence in Table 4b using the S&P 500 and the SMB and HML Fama-French factors. In this case, the LM test with polynomial alternatives only finds a significant presence of non-linear dependence on the mean in equity hedge funds. However, only 8% of funds are affected. The analogous Wald test

[8] See Romano, Shaikh, and Wolf (2008) for survey of data-snooping techniques to identify funds that reject a test in this kind of context.

yields a much higher proportion of test rejections because this test tends to overreject more in finite samples when there are several factors (see Section 2.3). Nevertheless, the critical values of these proportions correct for these distortions. This is why the results for the Wald test are also insignificant in all categories but the equity hedge group. Hence, the non-linearities on the mean observed in Table 4a for the event driven and fund of fund strategies disappear once we include a broader set of factors. For the piecewise linear alternatives, I find significant non-linear dependence on the mean in the market neutral, equity hedge and equity non hedge groups. Once again, though, the proportions of test rejections are quite small. I obtain a different pattern for the tests of non-linear dependence on the variance. The presence of this kind of non-linear dependence is significant in all categories. The magnitudes, which are higher than 18% in all cases, are also economically relevant. Interestingly, the impact of non-linear exposure on the variance is smaller on the Event driven and fund of funds groups, even though these categories displayed the highest test rejections in Table 4a. In contrast, the remaining categories yield much larger proportions of test rejections. For instance, I obtain the highest proportion of test rejections on the market neutral group (27.2%), which shows that this category is no longer as neutral as it seemed based only on linear correlation or non-linear dependence on the mean with respect to just the S&P 500. Furthermore, this value is much larger than 16.3%, which is the result obtained in Table 4a for this same category. The joint tests of non-linear dependence on the mean and variance reflect these features and find significant values in all cases but the event driven and fund of funds categories.

Hedge fund indices. Table 5 shows the p-values of the non-linear dependence tests for the hedge fund indices. The first panel focuses on non-linear dependence with respect to only the S&P 500. The polynomial LM test obtains significant non-linear dependence on the mean in the equity hedge, event driven and fund of funds strategies, although the result for the equity hedge index is only significant at the 90% level. The Wald tests of polynomial dependence confirm these results, with significant p-values at the 95% level for all cases but the market neutral index. So far, the results are similar to those of Table 4a. However, when I consider piecewise linear alternatives, I can only reject linear dependence in the event driven index. Non-linear dependence on the conditional variance turns out to be highly significant for the event driven and fund of funds, and only significant at the 90% level for the market neutral index.

Therefore, the significance of non-linear dependence seems to be smaller in the indices than in the original funds, although it is still relevant in some cases. In this sense, the construction of indices, which is based on averages, may dilute this type of exposure to market factors. This feature seems to be exacerbated when I consider dependence with respect to the three market factors in Table 5b. In this case, I only find significant non-linear dependence on the mean in the event driven index for piecewise linear alternatives. This index is also the only one that displays significant non-linear dependence on the variance. Thus hedge fund indices, taken individually, do not reflect the actual presence of non-linear dependence on the variance that I observed in Table 4b on many individual funds.

Finally, I compute multivariate tests where I analyse the four indices in a joint setting (i.e. N_h=4). This is a simple way to obtain a global test for the whole industry. In addition, important increases in power can be obtained by exploiting the cross-sectional dimension. The tests, shown in Table 6, show that linear dependence on both the mean and variance can be easily rejected when only the S&P 500 is considered. In the three-factor setting, though, non-linear dependence on the mean is no longer significant, but I still observe highly significant non-linear features on the conditional variance. Hence, the global picture given by the multivariate tests is consistent with what I obtained in Table 4 for the individual funds.

5 Robustness checks

In this section, I carry out several robustness checks to study the impact of the usual biases in hedge fund data on the results of the previous section. In particular, I study end-game behaviour, backfill bias and survivorship bias (see e.g. Lhabitant, 2002). In the three cases, I consider the same checks as Patton (2009). For the sake of brevity, I focus on the LM tests of nonlinear dependence. Table 7 shows the results.

End-game behaviour. Some fund managers may change their behaviour in the last months before they drop out of the database. In these circumstances they might pay less effort to controlling their exposure to market factors. I analyse this issue by recomputing the tests after dropping the last six observations of each fund. The first three columns of panels (a) and (b) in Table 7 show that the results do not change. Non-linear dependence on the mean with respect to the S&P 500 is still significant for the same two funds as before (event driven and fund of funds). In the multifactor tests, again I only find a significant albeit small presence on non-linear dependence on the mean in equity hedge funds. In contrast, non-linear dependence on the conditional variance is significant for all the styles, regardless of whether I consider a single market factor or the three-factor setting.

Backfill bias. When a fund decides to be included in a database, its past performance outside the database is typically added instantaneously. However, since many funds start reporting their returns to draw attention about a recent strong performance, this feature may also introduce a bias in the test results. I have computed the tests after dropping the first 12 observations of each fund, which is the average number of months that are backfilled in hedge fund databases. Columns 4 to 6 in Tables 7a and 7b show that the results are quite similar to the original ones in Table 4. Thus, the backfill bias does not seem to introduce either more or less non-linear dependence.

Survivorship bias. The database used in this paper contains both live and dead funds. In this sense, the results should not be biased towards any of them. Nevertheless, it is interesting to analyse the presence of any different behaviour between live and dead funds. I define as dead funds those funds that stopped reporting before the end of the sample.[9] Then, I compute the proportion of test rejections separately for live and dead funds. The last 6 columns of Tables 7a show the results for the dependence on just the S&P 500. The presence of non-linear dependence on the mean seems to be similar for the two kinds of funds: it is significant for event driven and funds of funds, but insignificant for the remaining styles. However,non-linear dependence on the conditional variance yields some deviations between live and dead funds. For instance, dead market neutral funds yield not only smaller but also insignificant rejection rates. In this sense, it might be the case that some live funds survive by reducing their commitment to market neutrality. Nevertheless, the introduction of the Fama-French factors (last six columns of

[9]In practice, some of the funds may leave the database because of other reasons, such as having closed the fund to new investment. Unfortunately, I have no information about which funds are liquidated and which ones simply stop reporting.

Table 7b) reveals that market neutral dead funds have a significant non-linear exposure with respect to these factors. The opposite effect is obtained on dead event driven and equity non hedge funds, which do not display significant values in the three factor setting. As for non-linear dependence on the mean, I do not find significant test rejections in dead funds once I include the Fama-French factors. In live funds, I obtain significant albeit economically small values on two categories (equity hedge and equity non hedge).

6 Conclusions

This paper analyses the existence of non-linear dependence between hedge fund returns and market conditions. The literature has previously found evidence of non-linear dependence in this industry. Nevertheless, the currently existing tests are designed to study dependence with respect to only one market factor, despite the evidence about the importance of multifactor models for these types of assets. In addition, most of the previous papers typically focus on dependence on the conditional mean, either by introducing piecewise linear or polynomial alternatives. However, the complex investment strategies of hedge fund managers may create a non-linear dependence structure that goes beyond the first conditional moment. In this context, I propose multivariate tests of non-linear dependence on the conditional mean and variance that are based on easily interpretable polynomial alternatives. Importantly, the resulting tests are equivalent to using the dependence structure of the Generalised Hyperbolic (GH) distribution as the alternative hypothesis. The GH is a rather flexible multivariate asymmetric distribution that also nests as particular cases many other well known and empirically realistic examples. I also compare these tests with a multifactor extension of the tests based on piecewise linear alternatives.

I conduct a Monte Carlo study in which I study the properties of the tests in small samples. I find that the LM tests based on polynomial alternatives yield very small size distortions, while the Wald tests tend to overreject under the null hypothesis. Interestingly, both the tests based on polynomial and piecewise linear alternatives yield non-trivial power even when I consider alternatives for which they were not originally designed. However, only the tests that explicitly consider non-linear dependence on the variance are able to capture this feature.

I study non-linear dependence in a sample of 1,071 hedge fund monthly excess returns obtained from the HFR and TASS databases. I analyse dependence with respect to the S&P 500, as well as a three-factor structure that includes the S&P 500 and the SMB and HML Fama-French factors. Simple correlation statistics seem to indicate that the theoretically most neutral styles, such as the market neutral group, are indeed the categories with less exposure to the market factors. When I study the presence of non-linearities on the conditional mean due to the S&P 500, I still find that the market neutral group remains "neutral", but obtain significant non-linearities in the event driven and fund of funds categories. However, these non-linearities are highly reduced or even disappear once I introduce the Fama-French factors. In contrast, I find a much stronger presence of non-linear dependence on the conditional variance. Specifically, I obtain significant values in four out of the five hedge fund styles in my database when I only study the impact of the S&P 500. This result remains robust when I consider three-factor tests, where I obtain a significant presence of non-linear exposures in all categories. In particular, the market neutral group is the style with more test rejections of the null hypothesis of linear dependence. In this sense, these results show that studying only correlations or linear dependence on the mean may be insufficient to unveil the actual exposures of these funds.

I compute the same tests on individual hedge fund style indices and find that the results of the tests on them are not generally representative of the behaviour of many individual funds from the same styles. Nevertheless, I also compute multivariate joint tests on these indices and obtain overall conclusions similar to those on the individual funds. Specifically, the multivariate tests find non-linear dependence with respect to the S&P 500 on both the conditional mean and variance, but only the non-linearities on the conditional variance remain significant in the three-factor setting.

Finally, I check that neither the presence of end-game behaviour or the backfill bias are causing significant distortions in the results. I also compare the characteristics of live and dead funds. Non-linear dependence on the conditional mean has similar characteristics in both of them, but some categories experience important deviations between live and dead funds in terms of non-linear exposures on the conditional variance.

A fruitful avenue for future research would be to to explore the implications of non-linearities for asset allocation purposes. In this sense, the GH distribution would be a natural candidate given the findings of this paper. It could also be helpful to extend the multi-factor tests of dependence into higher order moments. However, as noted by Patton (2009), such extensions might be highly difficult to implement due to the small sample sizes that are usual in these applications.

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Appendix

A Linear dependence tests

Consider the joint GPML of $\mathbf{y}_{mt}$ and $\mathbf{y}_{ht}$. Due to the conditional independence between $\boldsymbol{\varepsilon}_{mt}$ and $\boldsymbol{\varepsilon}_{h|mt}$, we can express this likelihood as

$$\begin{aligned}
\ell_t(\mathbf{y}_{mt}, \mathbf{y}_{ht}; \boldsymbol{\phi}) &= -\frac{N_m + N_h}{2}\log(2\pi) - \frac{1}{2}\log(|\boldsymbol{\Psi}|\,|\boldsymbol{\Gamma}|) \\
&\quad -\frac{N_h}{2}\log\left[1 + \boldsymbol{\omega}_1'\boldsymbol{\varepsilon}_{mt} + \omega_2(\varsigma_{mt} - N_m)\right]^2 - \frac{1}{2}\varsigma_{mt}(\boldsymbol{\theta}) \\
&\quad -\frac{1}{2}\frac{\varsigma_{h|mt}(\boldsymbol{\theta}) + \left[\varsigma_{mt}(\boldsymbol{\theta}) - N_m\right]^2 \boldsymbol{\delta}'\boldsymbol{\Gamma}\boldsymbol{\delta} + 2\left[\varsigma_{mt}(\boldsymbol{\theta}) - N_m\right]\boldsymbol{\delta}'\boldsymbol{\varepsilon}_{h|mt}(\tilde{\boldsymbol{\theta}}_T)}{\left[1 + \boldsymbol{\omega}_1'\boldsymbol{\varepsilon}_{mt} + \omega_2(\varsigma_{mt} - N_m)\right]^2}. \quad \text{(A1)}
\end{aligned}$$

Hence, the PML estimator under the null of linear dependence can be obtained from

$$\tilde{\boldsymbol{\theta}}_T = \arg\max_{\boldsymbol{\theta}} \frac{1}{T}\sum_{t=1}^{T} \ell_t(\mathbf{y}_{mt}, \mathbf{y}_{ht}; \boldsymbol{\phi}), \text{ s.t. } \begin{cases} \boldsymbol{\delta} = \mathbf{0} \\ \boldsymbol{\omega}_1 = \mathbf{0} \\ \omega_2 = 0 \end{cases}. \quad \text{(A2)}$$

It can be shown through tedious but otherwise straightforward algebra that (8) corresponds the score of (A1) with respect to $(\boldsymbol{\omega}_1', \omega_2', \boldsymbol{\delta})'$ for $\tilde{\boldsymbol{\phi}}_T = (\tilde{\boldsymbol{\theta}}_T', \mathbf{0}', \mathbf{0}', 0)'$. Hence,

$$\mathbf{s}_{\boldsymbol{\phi}t}(\mathbf{y}_{mt}, \mathbf{y}_{ht}; \tilde{\boldsymbol{\phi}}_T) = \left.\frac{\partial \ell_t(\mathbf{y}_{mt}, \mathbf{y}_{ht}; \boldsymbol{\phi})}{\partial \boldsymbol{\phi}}\right|_{\boldsymbol{\phi} = \tilde{\boldsymbol{\phi}}_T} = \begin{bmatrix} \mathbf{s}_{\boldsymbol{\theta}t}(\mathbf{y}_{mt}, \mathbf{y}_{ht}; \tilde{\boldsymbol{\theta}}_T) \\ \mathbf{v}_{\eta t}(\mathbf{y}_{ht}|\,\mathbf{y}_{mt}; \tilde{\boldsymbol{\theta}}_T) \end{bmatrix},$$

where $\mathbf{s}_{\boldsymbol{\phi}t}(\mathbf{y}_{mt}, \mathbf{y}_{ht}; \tilde{\boldsymbol{\phi}}_T)$ and $\mathbf{s}_{\boldsymbol{\theta}t}(\mathbf{y}_{mt}, \mathbf{y}_{ht}; \tilde{\boldsymbol{\theta}}_T)$ denote, respectively, the full score vector and the subvector corresponding to the score with respect to $\boldsymbol{\theta}$. As shown by Bollerslev and Wooldridge (1992)

$$\sqrt{T}[\tilde{\boldsymbol{\theta}}_T - \boldsymbol{\theta}_0] = \mathcal{A}_{\boldsymbol{\theta}\boldsymbol{\theta}}^{-1}(\boldsymbol{\theta}_0)\frac{\sqrt{T}}{T}\sum_t \mathbf{s}_{\boldsymbol{\theta}t}(\mathbf{y}_{mt}, \mathbf{y}_{ht}; \boldsymbol{\theta}_0) + o_p(1), \quad \text{(A3)}$$

where

$$\begin{aligned}
\mathcal{A}_{\boldsymbol{\theta}\boldsymbol{\theta}}(\boldsymbol{\theta}_0) &= E\left[\frac{\partial E(\mathbf{y}_{mt}'|I_{t-1}; \boldsymbol{\theta}_0)}{\partial \boldsymbol{\theta}}\boldsymbol{\Psi}^{-1}\frac{\partial E(\mathbf{y}_{mt}|I_{t-1}; \boldsymbol{\theta}_0)}{\partial \boldsymbol{\theta}'}\right] \\
&\quad + \frac{1}{2}\frac{\partial vec'\boldsymbol{\Psi}}{\partial \boldsymbol{\theta}}\left[\boldsymbol{\Psi}^{-1} \otimes \boldsymbol{\Psi}^{-1}\right]\frac{\partial vec\boldsymbol{\Psi}}{\partial \boldsymbol{\theta}'} \\
&\quad + E\left[\frac{\partial E(\mathbf{y}_{ht}'|\,\mathbf{y}_{mt}, I_{t-1}; \boldsymbol{\phi}_0)}{\partial \boldsymbol{\theta}}\boldsymbol{\Gamma}^{-1}\frac{\partial E(\mathbf{y}_{ht}|\,\mathbf{y}_{mt}, I_{t-1}; \boldsymbol{\phi}_0)}{\partial \boldsymbol{\theta}'}\right] \\
&\quad + \frac{1}{2}\frac{\partial vec'\boldsymbol{\Gamma}}{\partial \boldsymbol{\theta}}\left[\boldsymbol{\Gamma}^{-1} \otimes \boldsymbol{\Gamma}^{-1}\right]\frac{\partial vec\boldsymbol{\Gamma}}{\partial \boldsymbol{\theta}'}
\end{aligned}$$

and $\boldsymbol{\phi}_0 = (\boldsymbol{\theta}_0', \mathbf{0}', 0, \mathbf{0}')'$ is the vector of true parameter values. Now consider a Taylor expansion of the moment conditions of the test around the true parameter values

$$\begin{aligned}\frac{\sqrt{T}}{T}\sum_t \mathbf{v}_{\eta t}\left(\mathbf{y}_{ht}|\,\mathbf{y}_{mt};\tilde{\boldsymbol{\theta}}_T\right) &= \frac{\sqrt{T}}{T}\sum_t \mathbf{v}_{\eta t}\left(\mathbf{y}_{ht}|\,\mathbf{y}_{mt};\boldsymbol{\theta}_0\right) \\ &+ \left[\frac{1}{T}\sum_t \frac{\partial \mathbf{v}_{\eta t}\left(\mathbf{y}_{ht}|\,\mathbf{y}_{mt};\boldsymbol{\theta}_0\right)}{\partial \boldsymbol{\theta}'}\right]\sqrt{T}\left[\tilde{\boldsymbol{\theta}}_T - \boldsymbol{\theta}_0\right] + o_p(1) \\ = \frac{\sqrt{T}}{T}\sum_t \mathbf{v}_{\eta t}\left(\mathbf{y}_{ht}|\,\mathbf{y}_{mt};\boldsymbol{\theta}_0\right) &- \mathcal{A}'_{\boldsymbol{\theta}\eta}(\boldsymbol{\theta}_0)\sqrt{T}\left[\tilde{\boldsymbol{\theta}}_T - \boldsymbol{\theta}_0\right] + o_p(1) \qquad \text{(A4)}\end{aligned}$$

where

$$\mathcal{A}'_{\boldsymbol{\theta}\eta}(\boldsymbol{\theta}_0) = p\lim_{T\to\infty}\frac{-1}{T}\sum_t \frac{\partial \mathbf{v}_{\eta t}\left(\mathbf{y}_{ht}|\,\mathbf{y}_{mt};\boldsymbol{\theta}_0\right)}{\partial \boldsymbol{\theta}'}.$$

If we introduce (A3) in (A4), we obtain

$$\begin{aligned}\frac{\sqrt{T}}{T}\sum_t \mathbf{v}_{\eta t}\left(\mathbf{y}_{ht}|\,\mathbf{y}_{mt};\tilde{\boldsymbol{\theta}}_T\right) &= \left[\begin{array}{cc} -\mathcal{A}'_{\boldsymbol{\theta}\eta}(\boldsymbol{\theta}_0)\,\mathcal{A}^{-1}_{\boldsymbol{\theta}\boldsymbol{\theta}}(\boldsymbol{\theta}_0) & \mathbf{I}_{N+k+1}\end{array}\right] \\ &\times \frac{\sqrt{T}}{T}\sum_t \left[\begin{array}{c}\mathbf{s}_{\boldsymbol{\theta}t}\left(\mathbf{y}_{mt},\mathbf{y}_{ht};\boldsymbol{\theta}_0\right) \\ \mathbf{v}_{\eta t}\left(\mathbf{y}_{ht}|\,\mathbf{y}_{mt};\boldsymbol{\theta}_0\right)\end{array}\right] + o_p(1) \qquad \text{(A5)}\end{aligned}$$

In consequence, the asymptotic distribution under the null of (A5) is Gaussian with zero mean and covariance matrix

$$\mathcal{F}(\boldsymbol{\theta}_0) = \mathcal{B}_{\eta\eta}(\boldsymbol{\theta}_0) + \mathcal{A}'_{\boldsymbol{\theta}\eta}(\boldsymbol{\theta}_0)\,\mathcal{A}^{-1}_{\boldsymbol{\theta}\boldsymbol{\theta}}(\boldsymbol{\theta}_0)\,\mathcal{B}_{\boldsymbol{\theta}\boldsymbol{\theta}}(\boldsymbol{\theta}_0)\,\mathcal{A}^{-1}_{\boldsymbol{\theta}\boldsymbol{\theta}}(\boldsymbol{\theta}_0)\,\mathcal{A}_{\boldsymbol{\theta}\eta}(\boldsymbol{\theta}_0), \qquad \text{(A6)}$$

where

$$\begin{aligned}\mathcal{B}(\boldsymbol{\theta}_0) &= \left[\begin{array}{cc}\mathcal{B}_{\boldsymbol{\theta}\boldsymbol{\theta}}(\boldsymbol{\theta}_0) & \mathbf{0} \\ \mathbf{0}' & \mathcal{B}_{\eta\eta}(\boldsymbol{\theta}_0)\end{array}\right] \\ &= \lim_{T\to\infty} V\left\{\frac{\sqrt{T}}{T}\sum_t \left[\begin{array}{c}\mathbf{s}_{\boldsymbol{\theta}t}\left(\mathbf{y}_{mt},\mathbf{y}_{ht};\boldsymbol{\theta}\right) \\ \mathbf{v}_{\eta t}\left(\mathbf{y}_{ht}|\,\mathbf{y}_{mt};\boldsymbol{\theta}\right)\end{array}\right]\right\}, \qquad \text{(A7)}\end{aligned}$$

using the fact that $\mathbf{v}_{\eta t}(\mathbf{y}_{ht}|\,\mathbf{y}_{mt};\boldsymbol{\theta}_0)$ and the score with respect to $\boldsymbol{\theta}$ are orthogonal under the null for known $\boldsymbol{\theta}_0$.

B Distribution of the proportion of funds failing the tests

My simulation method involves two main steps. First, I estimate the distribution of the market return. Secondly, I generate $s = 1 \cdots S$ mutually independent replications of the market index from this distribution. I have generated these simulations from the

Gaussian distribution, but more flexible densities can also be used. For each of these replications, I compute the dependence tests between each hedge fund and the market, obtaining in each simulation an indicator function $\iota_{js} = 1(\tau_{js} > \tau_{0.95})$ that equals one if the test is rejected, and zero otherwise. As the sample size grows, ι_{js} will converge in distribution to a Bernoulli variate with a 5% probability of being 1. Hence, the distribution of the proportion of test rejections can be obtained from ι_{js}. Importantly, this method does not make any assumption about the distribution of the hedge fund returns. Thus, it accounts for the fact that not only correlation but also non-linear dependence between the funds can increase the proportion of rejections even when non-linear dependence with respect to the market does not exist.

I compute the critical values for the tests based on polynomial alternatives with 1,000 simulations. I also use these critical values to assess the statistical significance of the piecewise linear tests, because of their computational burden.

C The GH distribution

The GH distribution is a multivariate family of asymmetric and leptokurtic distributions. It can be generated as a location-scale mixture of normals, in which the mixing variable is Generalised Inverse Gaussian variable (see Jørgensen, 1982). Following the parametrisation of Mencía and Sentana (2009b), one can parametrise this distribution in terms of its mean vector, covariance matrix, a vector $\mathbf{b}$ of asymmetry parameters and two additional scalar shape parameters $\eta \in (-\infty, \infty)$ and $\psi \in [0, \infty)$. If $\mathbf{b} = \mathbf{0}$, then the distribution becomes symmetric. If in addition $\psi = 1$ and $\eta > 0$, then one obtains a symmetric Student t with η^{-1} degrees of freedom. This is why a GH with $\psi = 1$, $\eta > 0$ and $\mathbf{b} \neq \mathbf{0}$ is known as an asymmetric t distribution. In addition, normality is obtained when $\eta\psi \to 0$, regardless of $\mathbf{b}$. Other interesting particular cases are the asymmetric Normal-Gamma ($\psi = 1$, $\eta < 0$), the Normal Inverse Gaussian ($\eta = 1$), the Hyperbolic ($\eta = -1/2$) and the asymmetric Laplace ($\eta = -1/2$, $\psi = 0$).

C.1 Form of the conditional mean and variance

Let $\xi = \psi^{-1} - 1$ and $\nu = -.5\eta^{-1}$, and consider the joint conditional covariance matrix of $\mathbf{y}_{mt}$ and $\mathbf{y}_{ht}$,

$$\mathbf{\Sigma} = \begin{bmatrix} \mathbf{\Psi} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Gamma} \end{bmatrix}.$$

Then, the form of the conditional mean vector and covariance matrix under this distribution can be expressed as

$$
\begin{aligned}
E\left(\mathbf{y}_{ht}\middle|\,\mathbf{y}_{mt}, I_{t-1};\boldsymbol{\theta},\eta,\psi,\mathbf{b}\right) &= \boldsymbol{\alpha}+\sum_{i=1}^{p}\beta_{hi}\mathbf{y}_{ht-i}+\sum_{j=0}^{q}\beta_{mj}\boldsymbol{\varepsilon}_{mt-j}+\boldsymbol{\kappa}_h(\mathbf{b},\eta,\psi)\\
&\quad +\boldsymbol{\Upsilon}_{hm}(\mathbf{b},\eta,\psi)\boldsymbol{\Upsilon}_{mm}^{-1}(\mathbf{b},\eta,\psi)\left[\boldsymbol{\varepsilon}_{mt}-\boldsymbol{\kappa}_m(\mathbf{b},\eta,\psi)\right]\\
&\quad +\frac{\xi g(\boldsymbol{\varepsilon}_{mt})R_{\nu-\frac{N_m}{2}}\left(g(\boldsymbol{\varepsilon}_{mt})\xi^{*}\right)}{\xi^{*}R_{\nu}(\xi)}\boldsymbol{\Upsilon}_{h|mt}(\mathbf{b},\eta,\psi)\mathbf{b}_h,
\end{aligned} \tag{C8}
$$

$$
\begin{aligned}
V\left(\mathbf{y}_{ht}\middle|\,\mathbf{y}_{mt}, I_{t-1};\boldsymbol{\theta},\eta,\psi,\mathbf{b}\right) &= \frac{\xi g(\boldsymbol{\varepsilon}_{mt})R_{\nu-\frac{N_m}{2}}\left[g(\boldsymbol{\varepsilon}_{mt})\xi^{*}\right]}{\xi^{*}R_{\nu}(\xi)}\boldsymbol{\Upsilon}_{h|m}(\mathbf{b},\eta,\psi)\\
&\quad +R_{\nu-\frac{N_m}{2}+1}\left[g(\boldsymbol{\varepsilon}_{mt})\xi^{*}\right]R_{\nu-\frac{N_m}{2}}\left[g(\boldsymbol{\varepsilon}_{mt})\xi^{*}\right]\\
&\quad \times\left[\frac{\xi g(\boldsymbol{\varepsilon}_{mt})}{\xi^{*}R_{\nu}(\xi)}\right]^2\boldsymbol{\Upsilon}_{h|m}(\mathbf{b},\eta,\psi)\mathbf{b}_h\mathbf{b}_h'\boldsymbol{\Upsilon}_{h|m}'(\mathbf{b},\eta,\psi)\\
&\quad -\left[\frac{\xi g(\boldsymbol{\varepsilon}_{mt})R_{\nu-\frac{N_m}{2}}\left[g(\boldsymbol{\varepsilon}_{mt})\xi^{*}\right]}{\xi^{*}R_{\nu}(\xi)}\right]^2\boldsymbol{\Upsilon}_{h|m}(\mathbf{b},\eta,\psi)\mathbf{b}_h\mathbf{b}_h'\boldsymbol{\Upsilon}_{h|m}'(\mathbf{b},\eta,\psi),
\end{aligned} \tag{C9}
$$

where

$$
g(\boldsymbol{\varepsilon}_{mt})=\sqrt{\left(\boldsymbol{\varepsilon}_{mt}-\boldsymbol{\kappa}_m(\boldsymbol{\beta},\eta,\psi)\right)'\boldsymbol{\Upsilon}_{mm}^{-1}(\boldsymbol{\beta},\eta,\psi)\left(\boldsymbol{\varepsilon}_{mt}-\boldsymbol{\kappa}_m(\boldsymbol{\beta},\eta,\psi)\right)\xi^{-1}R_{\nu}(\xi)+1},
$$

$$
\xi^{*}=\sqrt{\begin{array}{c}\mathbf{b}'\left[\begin{array}{c}\boldsymbol{\Upsilon}_{mm}(\mathbf{b},\eta,\psi)\\ \boldsymbol{\Upsilon}_{hm}(\mathbf{b},\eta,\psi)\end{array}\right]\boldsymbol{\Upsilon}_{mm}^{-1}(\mathbf{b},\eta,\psi)\left[\begin{array}{cc}\boldsymbol{\Upsilon}_{mm}(\mathbf{b},\eta,\psi) & \boldsymbol{\Upsilon}_{hm}'(\mathbf{b},\eta,\psi)\end{array}\right]\mathbf{b}\frac{\xi}{R_{\nu}(\xi)}\\ +\xi^2,\end{array}}
$$

$$
\boldsymbol{\kappa}(\mathbf{b},\eta,\psi)=-c(\mathbf{b}'\boldsymbol{\Sigma}\mathbf{b},\eta,\psi)\boldsymbol{\Sigma}\mathbf{b}
$$

$$
\begin{aligned}
\boldsymbol{\Upsilon}(\mathbf{b},\eta,\psi) &= \boldsymbol{\Sigma}+\frac{c(\mathbf{b}'\boldsymbol{\Sigma}\mathbf{b},\eta,\psi)-1}{\mathbf{b}'\boldsymbol{\Sigma}\mathbf{b}}\boldsymbol{\Sigma}\mathbf{b}\mathbf{b}'\boldsymbol{\Sigma},\\
c(x,\eta,\psi) &= \frac{-1+\sqrt{1+4[D_{1-.5\eta^{-1}}(\psi^{-1}-1)-1]x}}{2[D_{1-.5\eta^{-1}}(\psi^{-1}-1)-1]x},\\
D_{\nu}(x) &= \frac{K_{\nu+1}(x)K_{\nu}(x)}{K_{\nu+1}^2(x)},\; R_{\nu}(x)=\frac{K_{\nu+1}(x)}{K_{\nu}(x)},
\end{aligned}
$$

and $K_{\nu}(\cdot)$ is the modified Bessel function of the third kind (see Abramowitz and Stegun, 1965). Note that I have partitioned $\boldsymbol{\kappa}(\mathbf{b},\eta,\psi)$ as $(\boldsymbol{\kappa}_m'(\mathbf{b},\eta,\psi),\boldsymbol{\kappa}_h'(\mathbf{b},\eta,\psi))'$, $\mathbf{b}$ as $(\mathbf{b}_m',\mathbf{b}_h')'$,

$$
\boldsymbol{\Upsilon}(\mathbf{b},\eta,\psi)=\left[\begin{array}{cc}\boldsymbol{\Upsilon}_{mm}(\mathbf{b},\eta,\psi) & \boldsymbol{\Upsilon}_{hm}'(\mathbf{b},\eta,\psi)\\ \boldsymbol{\Upsilon}_{hm}(\mathbf{b},\eta,\psi) & \boldsymbol{\Upsilon}_{hh}(\mathbf{b},\eta,\psi)\end{array}\right]
$$

and I have defined $\boldsymbol{\Upsilon}_{h|m}(\mathbf{b},\eta,\psi)=\boldsymbol{\Upsilon}_{hh}(\mathbf{b},\eta,\psi)-\boldsymbol{\Upsilon}_{hm}(\mathbf{b},\eta,\psi)\boldsymbol{\Upsilon}_{mm}^{-1}(\mathbf{b},\eta,\psi)\boldsymbol{\Upsilon}_{hm}'(\mathbf{b},\eta,\psi)$.

C.2 Test of linear dependence

It is possible to test linear vs. GH dependence by using (C8) and (C9) instead of (3) and (4). The null in this case would be obtained when $\eta\psi \to 0$. In practice, this implies that there are three different paths in the parameter space that yield linear dependence: $\eta \to 0^+$, $\eta \to 0^-$ or $\psi \to 0^+$. Since $\mathbf{b}$ is not identified under the null, I first construct a test for a fixed value of $\mathbf{b}$, which has the following closed form expression

$$\tau_d(\tilde{\boldsymbol{\theta}}_T, \mathbf{b}) = \frac{\left[\mathbf{b}^{+\prime}\frac{\sqrt{T}}{T}\sum_t \mathbf{v}_{\eta t}\left(\mathbf{y}_{ht}|\,\mathbf{y}_{mt};\tilde{\boldsymbol{\theta}}_T\right)\right]^2}{\mathbf{b}^{+\prime}\mathcal{F}\left(\tilde{\boldsymbol{\theta}}_T\right)\mathbf{b}^{+}}, \tag{C10}$$

for $\mathbf{b}^+ = (1, \mathbf{b}^{+\prime})'$. Interestingly, the formula of the test is the same for the three testing directions. Its asymptotic distribution is a chi-square with one degree of freedom. Since there is no a priori value of $\mathbf{b}$ that is likely to prevail under the alternative of GH dependence, I compute (C10) for the whole range of values of the unidentified vector of parameters. Then I combine these values to construct an overall test statistic (see Andrews, 1994). In particular, I follow Hansen (1996) and consider the supremum of (C10) with respect to $\mathbf{b}$. Interestingly, this supremum can be obtain in closed form and the result coincides with (7).

Table 1

Median descriptive statistics of individual Hedge funds

(a) Hedge funds excess returns (1993-2003)

Strategies	# of funds		Mean	Std. dev.	Correlation with		
	Total	Dead			S&P 500-r_f	SMB	HML
Market neutral	92	26	2.80	8.24	0.02	0.00	0.00
Equity hedge	412	98	9.45	15.86	0.38	0.26	-0.27
Event driven	84	15	7.36	9.15	0.39	0.28	-0.18
Fund of funds	404	121	5.19	7.58	0.31	0.31	-0.25
Equity non hedge	79	24	9.68	19.76	0.55	0.23	-0.29

(b) Market factors (1993-2003)

Factors	Mean	Std. dev.	Correlation with		
			S&P 500-r_f	SMB	HML
S&P 500-r_f	4.45	15.18	1.00	-0.04	-0.44
SMB	2.61	14.48	-0.04	1.00	-0.51
HML	4.60	13.58	-0.44	-0.51	1.00

Notes: Only funds with at least 30 observations are considered. SMB and HML are the "small minus big" and "high minus low" Fama-French factors. Excess returns computed with respect to the one-month Euro-Dollar rate (r_f). Dead funds are assumed to be those that dropped out of the database prior to the end of the sample. The moments are expressed in percent annualised terms.

Table 2

Correlation and normality of hedge fund excess returns

(a) Proportion of funds with non-zero correlations with market factors

Strategies	S&P 500-r_f	SMB	HML	Joint
Market neutral	41.3*	28.3*	38.0*	46.7*
Equity hedge	69.2*	60.2*	63.8*	81.1*
Event driven	82.1*	64.3*	44.0*	89.3*
Fund of funds	63.6*	65.3*	59.4*	74.8*
Equity non hedge	84.8*	57.0*	59.5*	89.9*

(b) Proportion of funds with test rejections of the Jarque-Bera normality test

Strategies	Skewness	Kurtosis	Joint
Market neutral	20.7*	45.7*	45.7*
Equity hedge	43.7*	51.5*	53.9*
Event driven	57.1*	76.2*	72.6*
Fund of funds	46.5*	56.9*	59.2*
Equity non hedge	36.7*	49.4*	55.7*

Notes: Only funds with at least 30 observations are considered. The reported numbers indicate the proportion of funds with test rejections at the 95% level. Asterisks are used for those proportions of test rejections that are statistically significant, also at the 95% level. SMB and HML are the "small minus big" and "high minus low" Fama-French factors. Excess returns computed with respect to the one-month Euro-Dollar rate (r_f).

Table 3

Descriptive statistics of Hedge funds indices excess returns (1990-2009)

	Mean	Std. dev.	Correlation with			Skewness	Kurtosis	Jarque-Bera
			S&P 500-r_f	SMB	HML			
Hedge fund indices								
Market neutral	3.25	3.11	0.18*	0.10	0.02	-0.22	4.18*	15.15*
Equity hedge	9.24	9.26	0.71*	0.47*	-0.39*	-0.10	4.06*	12.33*
Event driven	7.37	7.01	0.68*	0.43*	-0.18*	-1.01*	6.55*	158.26*
Fund of funds	3.65	6.00	0.50*	0.35*	-0.27*	-0.62*	6.08*	109.58*
Market factors								
S&P 500-r_f	1.72	14.99	1.00	0.06	-0.24*	-0.58*	4.17*	25.55*
SMB	2.22	12.24	0.06	1.00	-0.38*	0.88*	10.63*	588.82*
HML	3.29	11.57	-0.24*	-0.38*	1.00	0.16	5.98*	84.28*

Note: Asterisks in the correlation estimates indicate statistical significance at the 95% confidence level. SMB and HML are the "small minus big" and "high minus low" Fama-French factors. Excess returns computed with respect to the one-month Euro-Dollar rate (r_f). The moments are expressed in percent annualised terms.

Table 4

Proportion of test rejections by strategy in the tests of non-linear dependence of individual hedge funds

(a) Dependence with respect to the S&P 500-r_f

	Mean					
	Polynomial		Piecewise			
Strategies	LM	Wald	LM	Wald	Variance	Joint
Market neutral	7.6	8.7	4.3	7.6	16.3*	17.4*
Equity hedge	5.8	9.2	4.1	8.3	14.6*	16.0*
Event driven	38.1*	39.3*	19.0*	28.6*	25.0*	34.5*
Fund of funds	24.8*	27.7*	11.6*	20.0*	21.0*	29.7*
Equity non hedge	3.8	8.9	10.1	12.7	10.1	11.4

(b) Dependence with respect to the S&P 500-r_f, SMB and HML

	Mean					
	Polynomial		Piecewise			
Strategies	LM	Wald	LM	Wald	Variance	Joint
Market neutral	7.6	19.6	14.1*	26.1*	27.2*	26.1*
Equity hedge	8.0*	19.4*	8.7*	19.4*	24.8*	21.8*
Event driven	7.1	13.1	6.0	11.9	19.0*	15.5*
Fund of funds	2.5	9.7	5.0	14.1	18.3*	14.9*
Equity non hedge	8.9	19.0	11.4*	12.7	20.3*	21.5*

Note: Only funds with at least 30 observations are considered. The reported numbers indicate the proportion of funds with test rejections at the 95% level. Asterisks are used for those proportions of test rejections that are statistically significant, also at the 95% level. SMB and HML are the "small minus big" and "high minus low" Fama-French factors. Excess returns computed with respect to the one-month Euro-Dollar rate (r_f).

Table 5

p-values of the tests of non-linear dependence for individual hedge fund indices

(a) Dependence with respect to the S&P 500-r_f

	Mean					
	Polynomial		Piecewise			
Strategies	LM	Wald	LM	Wald	Variance	Joint
Market neutral	0.36	0.31	0.48	0.47	0.09	0.16
Equity hedge	0.08	0.04	0.23	0.20	0.79	0.22
Event driven	0.00	0.00	0.01	0.00	0.00	0.00
Fund of funds	0.01	0.04	0.16	0.11	0.00	0.00

(b) Dependence with respect to the S&P 500-r_f, SMB and HML

	Mean					
	Polynomial		Piecewise			
Strategies	LM	Wald	LM	Wald	Variance	Joint
Market neutral	0.38	0.36	0.25	0.19	0.34	0.34
Equity hedge	0.34	0.24	0.33	0.30	0.11	0.13
Event driven	0.18	0.28	0.03	0.01	0.00	0.00
Fund of funds	0.74	0.72	0.47	0.40	0.16	0.26

Note: SMB and HML are the "small minus big" and "high minus low" Fama-French factors. Excess returns computed with respect to the one-month Euro-dollar rate (r_f).

Table 6

Non-linear dependence multivariate tests for hedge fund indices

Market factors	Mean	Variance	Joint
S&P 500-r_f	20.93 (0.00)	22.69 (0.00)	37.71 (0.00)
S&P 500-r_f, SMB and HML	6.37 (0.17)	13.13 (0.01)	16.94 (0.03)

Note: "Mean" refers to a LM test of the null hypothesis of joint linear dependence on the mean for the market neutral, equity hedge, event driven and fund of funds HFR indices. Similarly, the column labeled "Variance" considers linear dependence on the same four indices and column "Joint" tests linear dependence on the mean and variance. Polynomial quadratic alternative are considered in all cases. p-values are reported in parentheses. SMB and HML are the "small minus big" and "high minus low" Fama-French factors. Excess returns computed with respect to the one-month Euro-Dollar rate (r_f).

Table 7

Proportion of test rejections by strategy in the tests of non-linear dependence of individual hedge funds. Robustness checks

(a) Dependence with respect to the S&P 500-r_f

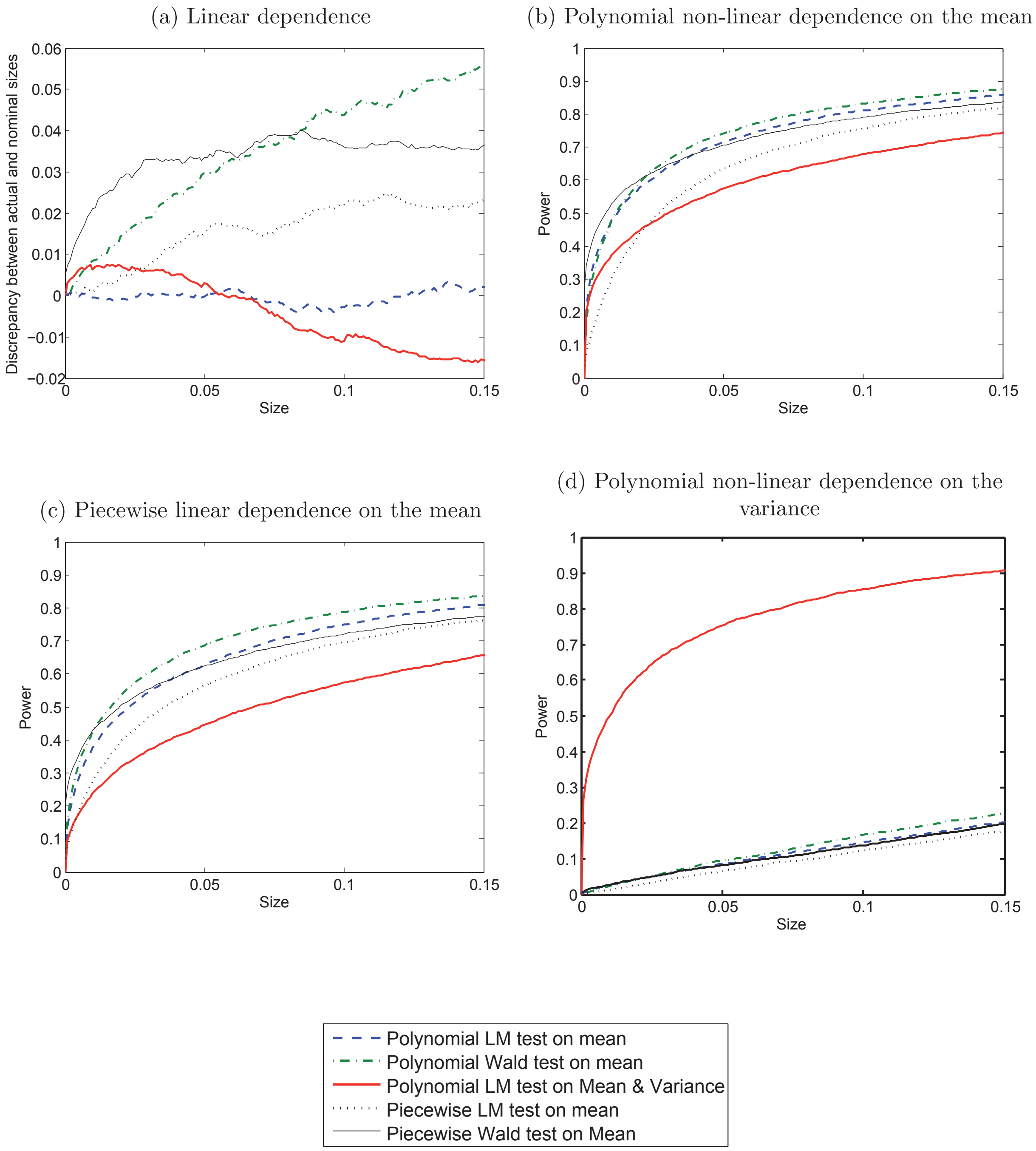
	End-game			Backfill			Live			Dead		
Strategies	Mean	Variance	Joint	Mean	Variance	Joint	Mean	Variance	Joint	Mean	Variance	Joint
Market neutral	5.9	15.3*	20.0*	6.8	18.9*	18.9*	6.1	18.2*	18.2*	11.5	11.5	15.4*
Equity hedge	6.6	16.0*	17.3*	6.5	16.1*	13.4*	6.7	13.7*	16.2*	3.1	17.3*	15.3*
Event driven	40.5*	26.6*	36.7*	36.0*	30.7*	34.7*	37.7*	26.1*	34.8*	40.0*	20.0*	33.3*
Fund of funds	27.2*	22.3*	30.8*	18.8*	19.7*	24.2*	24.7*	20.1*	29.3*	24.8*	23.1*	30.6*
Equity non hedge	4.0	13.3*	10.7	4.2	15.3*	13.9*	5.5	9.1	10.9	0.0	12.5	12.5

(b) Dependence with respect to the S&P 500-r_f, SMB and HML

	End-game			Backfill			Live			Dead		
Strategies	Mean	Variance	Joint	Mean	Variance	Joint	Mean	Variance	Joint	Mean	Variance	Joint
Market neutral	7.1	23.5*	22.4*	8.1	23.0*	24.3*	6.1	25.8*	25.8*	11.5	30.8*	26.9*
Equity hedge	8.1*	24.4*	23.1*	8.0*	24.1*	22.6*	8.3*	26.1*	22.6*	7.1	20.4*	19.4*
Event driven	7.6	13.9*	15.2*	6.7	20.0*	16.0*	8.7	21.7*	17.4*	0.0	6.7	6.7
Fund of funds	2.5	17.4*	15.0*	2.4	16.7*	14.6*	2.8	16.6*	13.1*	1.7	22.3*	19.0*
Equity non hedge	8.0	20.0*	24.0*	9.7	23.6*	23.6*	12.7*	23.6*	23.6*	0.0	12.5	16.7*

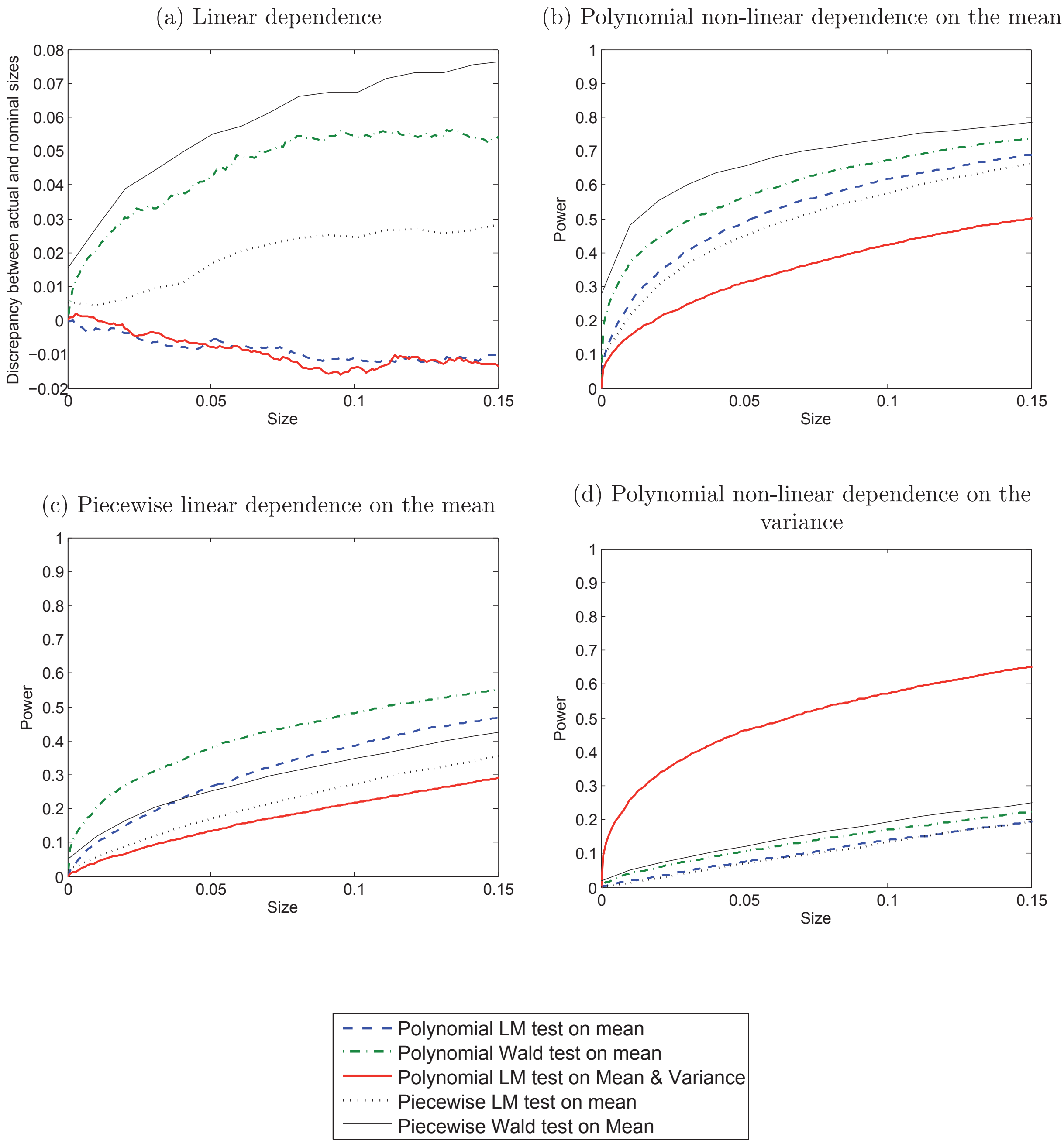
Note: End-game behaviour is analysed by dropping the last 6 observations on each fund. The backfill bias is analysed by dropping the first 12 observations from each fund. Dead funds are assumed to be those that dropped out of the database prior to the end of the sample. Only funds with at least 30 observations are considered. The reported numbers indicate the proportion of funds with test rejections at the 95% level. Asterisks are used for those proportions of test rejections that are statistically significant, also at the 95% level. SMB and HML are the "small minus big" and "high minus low" Fama-French factors. Excess returns computed with respect to the one-month Euro-Dollar rate (r_f).

Figure 1. Size and power of univariate non-linear dependence tests



Notes: Monte Carlo study with 5,000 simulations with T=100.

Figure 2. Size and power of multivariate non-linear dependence tests



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