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EXPECTATIONS ABOUT INFLATION
AND INTEREST RATES
FROM A LIQUID MARKET** **2009**

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Abstract

In this paper we propose an affine model that uses as observed factors the Nelson and Siegel (NS) components summarising the term structure of interest rates. By doing so, we are able to reformulate the Diebold and Li (2006) approach to forecast the yield curve in a way that allows us to incorporate a non-arbitrage opportunities condition and risk aversion into the model. These conditions seem to improve the forecasting ability of the term structure components and provide us with an estimation of the risk premia. Our approach is somewhat equivalent to the recent contribution of Christensen, Diebold and Rudebusch (2008). However, not only does it seem to be more intuitive and far easier to estimate, it also improves that model in terms of fitting and forecasting properties. Moreover, with this framework it is possible to incorporate directly the inflation rate as an additional factor without reducing the forecasting ability of the model. The augmented model produces an estimation of market expectations about inflation free of liquidity, counterparty and term premia. We provide a comparison of the properties of this indicator with others usually employed to proxy the inflation expectations, such as the break-even rate, inflation swaps and professional surveys.

Keywords: Interest Rate Forecast, Inflation Expectations, Affine Model, Diebold and Li.

JEL Classifications: G12, E43, E44, C53.

1 Introduction

The forecasting properties of the yield curve have been the focus of recent attention. In this respect Diebold and Li (2006) propose using the shape of the yield curve, captured by the parameters of the Nelson and Siegel (1987) term structure model, as predictors of the future value of interest rates.

In this paper we depart from the model originally introduced by Diebold and Li (2006) by rewriting it with an arbitrage-free specification following Vasicek (1977) and Cox *et al.* (1985). In these models it is possible to improve the consistency of the model by adding the non-arbitrage opportunities condition together with risk aversion in order to compute additionally the risk premia. Christensen *et al.* (2007) proposed another approach to introduce non-arbitrage opportunities into the Diebold and Li model based on an unobserved component. In their model, the underlying Nelson and Siegel (1987) parameters are used as a subjacent structure of the latent factors in the affine models along the lines of Duffie and Kan (1996). On the contrary, our model relies on the Nelson and Siegel (1987) factors as being completely exogenous to the affine specification, reducing the complexity of the Kalman filter that is usually required in the standard latent models¹ [other example of this latent-factor approach can be found in Ang *et al.* (2008)]. By using the Nelson and Siegel parameters we can guarantee a good fit of the whole term structure of nominal interest rates² and, by taking these factors as exogenous, the results are much more robust (i.e. they are not dependent on initial conditions or on the selection of the interest rates observed without error). Moreover, the risk premia could be easily obtained simply by comparing the value obtained with the estimated price of risk and those based on a price of risk equal to zero.

Our empirical results for the United States and the Euro Area suggest that the introduction of non-arbitrage opportunities and risk aversion presents several advantages when compared with the original Diebold and Li model. Firstly, we obtained a slightly better forecast and it is possible to provide an estimation for the risk premia, something that was not possible under the original model. Secondly, the affine formulation makes it easier to include other variables and, therefore, to obtain estimations for these variables that are compatible with the term structure. This is especially useful when dealing with macroeconomic variables, as has been highlighted in the literature [i.e. Carriero *et al.* (2006), Dewatcher *et al.* (2006), Dewatcher and Lyrio (2006) or Diebold *et al.* (2004)]. Nevertheless, previous approaches that pointed out the importance of macroeconomic variables were hindered by the latent factors used to describe the term structure, which exhibit poor forecasting properties [Duffee (2002) and several optima [Kim and Orphanides (2005)].

Regarding the comparison with the approach of Christensen *et al.* (2007), our model seems to be easier to estimate. The empirical results for the same sample period suggest that the fitting accuracy of our model is superior, while in terms of forecasting we attain far better results over the long-term horizon.

1. There is an extensive literature that uses the latent factor in order to estimate the real interest rate and inflation expectations. See for example Dai and Singleton (2000) or Laubach and Williams (2003).

2. Most of the models based on endogenous factors typically focus only on a selection of four interest rates that have to be replicated. By contrast, the model used here relies on the parameters that replicate the whole term structure of interest rates and, by definition, replicate all the bonds used to estimate these parameters.

Another important feature of our model is that it is possible to introduce the inflation rate into the components of the VAR in the affine model while maintaining the forecasting accuracy of the model with respect to the interest rate. Other approaches to the estimation of inflation expectations from the yield curve include those of Ang *et al.* (2008) or García and Werner (2008); however, these exercises are usually based on latent factors and their estimations conditional upon several ad-hoc assumptions, and they normally present robustness problems. Our empirical result suggests that the term structure of the interest rate contains useful information for forecasting the inflation rate compared with a simple AR model.

In this paper we do not perform a comparison of our results with those obtained with affine models based on unobserved components; this kind of exercise can be found in Gimeno and Marqués (2008). Instead, we focus on the comparison of our estimations for inflation expectations with other indicators usually taken by analysts as proxies for inflation expectations. Theoretically, our measure of expected inflation should contain better properties than other alternatives given that this estimation rate was not affected by risk premia, counterparty risk or liquidity premia (given that the nominal public debt market could be considered as liquid). Our results for long-term inflation rates are clearly in line with the inflation expectations of analyst surveys. Moreover, the evolution of our estimates of expected inflation seems to be less volatile and more plausible both for the euro area and the United States than other market measures such as inflation-linked bonds or inflation swaps, which could be affected by risk premia, liquidity premia or counterparty risk. In particular, during the last part of the sample, a period characterised by significant shifts in liquidity premia, the indicators calculated as a comparison of different markets, such as inflation-linked and nominal bonds, could be misleading.

Finally, with this model it is possible to perform an impulse-response analysis in order to evaluate how an unexpected movement in one of these variables changes market expectations about the others. In particular, we will focus on the effects on inflation expectations after a permanent increase in short-term interest rates. Our results suggest that monetary policy seems to have a bigger effect in the United States than in the euro area.

The rest of the paper is structured in five additional sections. The second section analyses the forecasting properties of the Diebold and Li (2006) approach if we incorporate the non-arbitrage opportunities condition, risk aversion and inflation rates. In the third section, we compare the results on inflation expectations and risk premia with the surveys of analysts and professional forecasters and the information content in other markets, such as inflation-linked bonds or inflation swaps. In Section four, we analyse the reaction of inflation expectations after a monetary policy decision. Finally, section 5 concludes.

2 Forecasting Interest Rates

2.1 Forecasting Interest Rates: The Set-Up

Diebold and Li (2006) addressed the problem of interest rate expectations indirectly by forecasting the values of the parameters of the yield curve. For this purpose, they used the Nelson and Siegel (1987) model for the term structure,

$$y_{t,t+k} = L_t + S_t \frac{1 - e^{-k/\tau}}{k/\tau} + C_t \left(\frac{1 - e^{-k/\tau}}{k/\tau} - e^{-k/\tau} \right) + u_{t,t+k} \quad u_t \sim N(0, \sigma^2 I) \quad (1)$$

where $y_{t,t+k}$ is the interest rate at time t maturing in k periods; L_t is the long-term interest rate (both forward and spot); S_t is the spread (difference between long-term and short-term interest); C_t is a measure of the term structure curvature; τ is a parameter determining the speed of transition between the short and the long-term interest rates; while $u_{t,t+k}$ is an error term. Diebold and Li (2006) proposed fixing the value of τ in the mean observed value throughout the original sample³, so interest rates becomes a linear model (equation 2),

$$y_{t,t+k} = \begin{pmatrix} 1 & \frac{1 - e^{-k/\tau}}{k/\tau} & \left(\frac{1 - e^{-k/\tau}}{k/\tau} - e^{-k/\tau} \right) \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} + u_{t,t+k} \quad (2)$$

In this way, parameters L_t , S_t and C_t can be easily estimated by an OLS regression. Diebold and Li (2006) showed that then it is possible to forecast these parameters by VAR equations (equation 3),

$$X_t = \mu + \Phi X_{t-1} + \Sigma \varepsilon_t \quad \varepsilon_t \sim N(0, I) \quad (3)$$

where $X_t = (L_t \ S_t \ C_t)'$, μ is a vector of the constant drifts, Σ is the variance-covariance matrix of the noise term, Φ is a matrix of the autoregressive coefficients and the i.i.d. Gaussian noise vector ε_t represents the uncertainty in the future values of interest rates. Once we have forecasts of the parameters, it is possible to recover good projections of the whole term structure of interest rates.

Nevertheless, the Diebold and Li (2006) approach lacks some properties that should be present in a dynamic modeling of interest rates: non-arbitrage opportunities and risk aversion. Christensen *et al.* (2007) addressed this problem by transforming the Diebold and Li (2006) model into an affine model.

Affine term structure models have been widely used in the financial literature to price fixed-income assets since the seminal works of Vasicek (1977) and Cox *et al.* (1985).

³ Gimeno and Nave (2009) showed that trying to estimate τ jointly with L_t , S_t and C_t produces non-trivial problems of identification.

An affine model assumes that interest rates can be explained as a linear function of certain factors,

$$y_{t,t+k} = \frac{-1}{k}(A_k + B'_k X_t) + u_{t,t+k} \quad u_t \sim N(0, \sigma^2 I) \quad (4)$$

where A_k and B'_k are coefficients. Changes in interest rates over time will be the outcome of changes in the factors, whereas differences in the term structure will be driven by the coefficients A_k and B'_k applied. In fact, this linear approach to interest rates is similar to the linearisation of the Nelson and Siegel (1987) model proposed by Diebold and Li (2006). Additionally, in an affine model, X_t factors have a dynamic behavior usually modelled as a VAR [see Diebold *et al.* (2004)] in the same vein as Diebold and Li (2006) did, we consider the parameters of the Nelson and Siegel (1987) model as the affine factors.

In an affine model, arbitrage opportunities are avoided by imposing equation 5. Christensen *et al.* (2007) proposed attaining this equality by modifying the X_t factors. By doing so, they maintain the original B'_k of the original Nelson and Siegel model and leave X_t to change according to equation 5.

$$e^{A_{k+1} + B'_{k+1} X_t} = E^Q_t \left[e^{A_k + B'_k X_{t+1}} \right] \quad (5)$$

The left-hand side of equation 5 represents the valuation of a zero-coupon bond maturing in $k+1$ which, under the non-arbitrage condition, should be equivalent to the expected value one period ahead of the same bond with maturity k discounted by the short-term interest rate.

Christensen *et al.* (2007) imply that in order to ensure that equation 5 holds, the value of X_t must be restricted. Therefore, if X_t values are taken as latent factors, they must be estimated by a Kalman filter, adding complexity to the rather difficult task of estimating the Nelson and Siegel model⁴. Additionally, the dynamics of the X_t variables are obscured by the restriction imposed.

In the present paper we propose an alternative approach to include non-arbitrage opportunities in the Diebold and Li (2006) model. It is possible to maintain the X_t obtained from the Nelson and Siegel estimation as exogenous variables, avoiding the need for the Kalman filter. Therefore, the non-arbitrage condition is reached by restricting the values of parameters A_k and B'_k according to equation 5. This could be done by solving forward equation 5, which would imply a recursive form for the A_k and B'_k coefficients.

The consideration of risk aversion in this affine model framework implies some compensation for the uncertainty about longer maturities⁵, in which the random shocks ε_t accumulate. In this respect, it is clear that the higher the variance of random shocks on VAR equation (3) [identified by matrix Σ which we will impose to be orthogonal in line with

4. See Gimeno and Nave (2009).

5. Bekaert and Hodrick (2001) reviewed the evidence which suggests that expected returns on long bonds are, on average, higher than on short bonds, reflecting the existence of a risk premium and that this premium is time-varying.

Ang *et al.* (2008) among others], the greater the uncertainty about future values of interest rates. So, in order to compensate investors for lending money at longer terms, some risk premium related to Σ should be embedded in the nominal interest rates. Coefficients that translate matrix Σ into the risk premium are called prices of risk (λ_t) and, following the literature, these coefficients can be set to be affine to the same factors X_t ,

$$\lambda_t = \lambda_0 + \lambda_1 X_t \quad (6)$$

where λ_0 is a vector and λ_1 a matrix of coefficients. If λ_1 is set to be equal to zero, then the risk premium will be constant, while if we leave it unrestricted, we will obtain a time-varying risk premium.

Taking together non-arbitrage opportunities and risk aversion, it is possible, after some algebra, to transform equation 5 into a recursive system of equations represented by equations 7 and 8.

$$A_{k+1} = A_1 + A_k + B'_k \mu - B'_k \Sigma \lambda_0 + \frac{1}{2} B'_k \Sigma \Sigma' B'_k \quad (7)$$

$$B'_{k+1} = B'_1 + B'_k \Phi - B'_k \Sigma \lambda_1 \quad (8)$$

In equations 7 and 8 the coefficients determining interest rates maturing in $k+1$ (A_{k+1} and B'_{k+1}) are the result of the aggregation of the determinants of the short-term interest rate (A_1 and B'_1), the difference between the actual short-term interest rate and its forecast value (reflected by $A_k + B'_k \mu$ and $B'_k \Phi$ terms, respectively) a compensation for risk ($B'_k \Sigma \lambda_0$ and $B'_k \Sigma \lambda_1$ terms, respectively), and a quadratic term consequence of the Jensen Inequality ($\frac{1}{2} B'_k \Sigma \Sigma' B'_k$). As can be seen, risk compensation depends on matrix⁶ Σ and the price of risk λ_t .

Summarising, the affine model to be estimated under our framework will consist of equations 3 and 4, with the coefficients of equation 4 being subject to restrictions 7 and 8. In order to compare our model with that proposed by Christensen *et al.* (2007), we have used the same sample they use⁷. In table 1, we present in-sample fitting statistics of Diebold and Li (2006) and the Christensen *et al.* (2007) reported root mean squared error (RMSE) for each maturity. These results are then compared with those obtained from our own model in the last columns. As can be seen, our model is clearly superior to those previously proposed. This is mainly a consequence of better performance at the long-term end of the yield curve, where the other models obtain poor results.

6. Although the Σ matrix is time-invariant, it appears jointly with the time-variant price of risk. Hence an increase in the volatility of any factor would be captured via λ_t .

7. Monthly data on U.S. Treasury security yields from January 1987 to December 2002. The data are end-of-month, unsmoothed Fama-Bliss (1987) zero-coupon yields at the following 16 maturities: 3, 6, 9, 12, 18, 24, 36, 48, 60, 84, 96, 108, 120, 180, 240, and 360 months.

Table 1: Summary Statistics of In-Sample Fit

Maturity in months	DNS		DNS		CDR		CDR		GMS	
	indep.-factor		corr.-factor		indep.-factor		corr.-factor			
	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
3	-1.64	12.26	-1.84	11.96	-2.85	18.54	-2.47	11.53	1.68	7.77
6	-0.24	1.09	-0.29	1.34	-1.19	7.12	-0.04	0.75	-0.43	3.23
9	-0.54	7.13	-0.51	6.92	-1.24	3.44	-0.35	6.86	-3.12	6.08
12	4.04	11.19	4.11	10.86	3.58	9.60	3.69	10.11	0.04	7.23
18	7.22	10.76	7.28	10.42	7.15	10.44	5.49	8.31	2.33	5.17
24	1.18	5.83	1.19	5.29	1.37	5.94	-1.20	4.37	-2.87	4.58
36	-0.07	1.51	-0.19	2.09	0.31	1.98	-1.10	3.16	-0.41	4.87
48	-0.67	3.92	-0.85	4.03	-0.39	3.72	0.94	4.14	2.65	5.70
60	-5.33	7.13	-5.51	7.31	-5.27	6.82	-1.99	5.20	0.15	4.56
84	-1.22	4.25	-1.30	4.25	-1.50	4.29	0.90	3.83	3.33	5.51
96	1.31	2.10	1.29	2.02	1.02	2.11	1.05	1.83	3.34	4.74
108	0.03	2.94	0.07	3.11	-0.11	3.02	-3.24	5.28	-1.19	5.51
120	-5.11	8.51	-5.01	8.53	-4.96	8.23	11.67	14.02	-9.96	10.72
180	24.11	29.44	24.40	29.66	27.86	32.66	3.76	16.50	2.71	9.43
240	25.61	34.99	26.00	35.33	35.95	42.61	4.20	23.93	2.10	10.10
360	-	-	-	-	1.37	22.04	-0.81	23.02	-0.95	10.84
All maturities	1.19	64.46	1.23	64.31	3.82	64.61	-0.18	45.19	-0.04	28.13

Source: Christensen *et al.* (2007) and authors' own data.

The means and the root mean squared errors for 16 different maturities. All numbers are measured in basis points. DNS represents the original Diebold and Li (2006) model, with both assuming independent and correlated X_t ; CDR is for the Christensen *et al.* (2007) approach; finally GMS represents the model proposed in this paper

A second analysis performed in Christensen *et al.* (2007) is to assess the out-of-sample forecast properties of each model. We have replicated this analysis⁸, and the results are presented in table 2. As can be seen, in terms of forecast accuracy, too, our proposal outperforms the other models. This is more evident in the 12-month forecast. This better outcome than that of Christensen *et al.* (2007) may be a consequence of the way non-arbitrage conditions are imposed on the model. By freeing the X_t variables from the burden of these restrictions, the VAR equation is then able to produce better estimations in terms of forecast.

Another consequence is that this VAR can be modified for adding all the variables that you may consider necessary without affecting the non-arbitrage conditions that would fall on the recursive form of the A_k and B'_k coefficients. This possibility will be explored in a later section, where inflation rates would be added to the model. An additional advantage of a non-restricted VAR equation is that we are able to perform impulse-response exercises on the model, giving us interesting information on market reactions to change in some relevant variables such as monetary policy short-term interest rates or inflation rates.

⁸ One-, six-, and twelve-month-ahead forecasts are constructed for all the models and for six yields with maturities of 3 months and 1, 3, 5, 10, and 30 years, with a recursive procedure. For the first set of forecasts, the model is estimated from January 1987 to December 1996; then, one month of data are added, the models are re-estimated, and another set of forecasts is constructed. The largest estimation sample for the one-month-ahead forecasts ends in November 2002 (72 forecasts in all). For the six- and 12-month horizons, the largest samples end in June 2002 and December 2001 (67 and 61 forecasts), respectively.

Table 2: Forecast RMSE for the Five Models

Model	Forecast horizon		
	One month	Six months	Twelve months
	3-month yield		
DNSindep	22.93	96.87	173.39
DNScorr	20.43	87.43	166.91
CDRindep	22.84	91.60	164.97
CDRcorr	20.56	88.67	162.33
GMS	22.89	78.18	137.96
	1-year yield		
DNSindep	29.41	103.25	170.85
DNScorr	27.06	102.71	173.14
CDRindep	29.12	98.58	164.01
CDRcorr	33.89	98.87	165.99
GMS	30.90	93.10	146.81
	3-year yield		
DNSindep	30.64	92.22	135.24
DNScorr	30.59	99.55	145.82
CDRindep	30.29	87.23	127.78
CDRcorr	36.95	91.00	136.44
GMS	32.40	87.63	120.80
	5-year yield		
DNSindep	30.77	87.87	122.09
DNScorr	31.23	94.95	132.40
CDRindep	30.13	82.68	113.83
CDRcorr	32.37	88.46	125.42
GMS	30.50	80.52	104.07
	10-year yield		
DNSindep	28.35	74.71	105.02
DNScorr	29.06	79.48	112.37
CDRindep	27.18	67.72	93.36
CDRcorr	35.08	90.42	124.28
GMS	30.25	69.82	87.12
	30-year yield		
DNSindep	38.42	71.35	96.90
DNScorr	38.73	72.71	99.68
CDRindep	30.42	48.82	63.50
CDRcorr	38.30	71.35	96.86
GMS	24.68	54.53	66.32

Source: Christensen *et al.* (2007) and authors' own computation.

2.2 Forecasting Interest Rates: Empirical Results

In order to estimate the equations we will focus on the Euro Area and the United States. For the Euro Area we take the parameters for the yield curve provided by the ECB in its web page while for the United States we have to estimate the Nelson and Siegel parameters based on the observation of 11 yields of treasuries in the secondary markets with terms from 1 month to 30 years, taken from Datastream. In both cases we use monthly observations from January 1999 to May 2008. First, we are interested in comparing the forecasting properties of the model against two benchmarks that have been widely used in the literature: the implicit forecast for interest rates that we can obtain from the yield curve using the forward rates, and the forecast that can be obtained from the Diebold and Li model without imposing the non-arbitrage opportunities.

Table 3: Mean Squared Error on 1-year interest rates forecast 1-year ahead using forward interest rates, Diebold and Li (1986) VAR model, and the affine model

	Forward	VAR model	Affine model
Euro Area	0.938	0.537	0.526
United States	2.665	1.279	1.272

As can be seen in Table 3, there are clear forecasting improvements on the VAR model proposed by Diebold and Li (2006) over forward interest rates⁹. Nevertheless, restrictions imposed by the affine model presented in this paper do not reduce the forecast accuracy but even marginally improve it. This is a clear sign that this restriction does not have any cost in terms of the ability to forecast. Other important finding on table 1 is that it seems easier to forecast 1 year interest rate in the euro area than in the United States. This feature, that could be observed also when using forward interests, could be due to the low degree of smoothness of monetary policy in the United States during the sample period. Moreover since the ECB has less ambiguity on its mandate than the Federal Reserve it seems more plausible that financial markets have been more successful forecasting interest rates in the euro area than in the US.

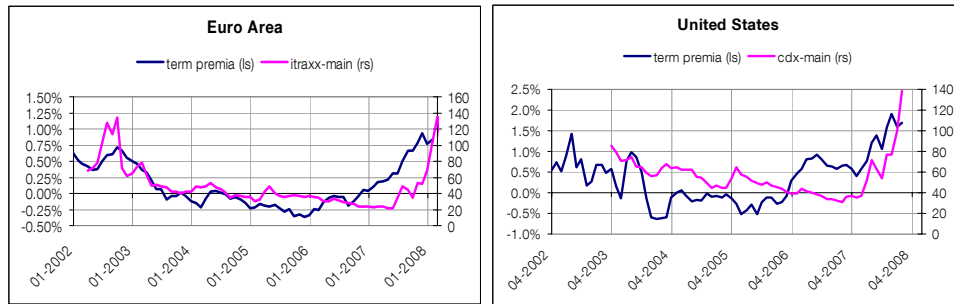
An additional advantage of the affine model over the unrestricted VAR of Diebold and Li (2006) is that a clean measure of the risk premia can be extracted from the model as was shown in Annex 2. Risk or term premia are usually blamed for the forward interest rates' poor forecasting performance. This can be seen in Table 4, where it is shown that forward interest rates overestimate spot interest rates (by 71bp in the EA and 30bp in the US). This forecasting skewness is quite lower in the case of the VAR models. Nevertheless, the assumption that the difference between forward interest rates and forecast interest rates with the VAR model is the consequence of the term premia does not take into account the measurement error in the estimation of the VAR. By contrast, in the affine model, the term premia can be obtained as the difference between the estimated nominal interest rate and a computed interest rate where the price of risk is set to be equal to zero,

$$\gamma_{t,t+k} = \hat{y}_{t,t+k} - \hat{y}(\lambda_t = 0)_{t,t+k} \quad (9)$$

This term premium will be a mixture of the uncertainty about the future course of short-term interest rates and the price of risk. Although the uncertainty is originated by this specific market, the price of risk should be generally the same across financial markets. So, a similar pattern should be found if compared with other risk measures. This similarity can be seen in figure 1, where the term premia extracted from equation 9 are compared with credit default swaps. Obviously, these two measures of risk premia are of a different magnitude since the yield curve has to reflect a premium related to the uncertainty over the broad economy (and therefore to monetary policy, the growth rate and inflation shocks) and the indices of CDS were based mainly on firms' default probability. However, macroeconomic uncertainty is an important determinant even in the case of individual CDS [see, for example, Alonso *et al.* (2006)]. Moreover, in the case of a CDS index, the importance of the individual factors of each CDS is diluted while common factors, such as inflation or real growth rate uncertainty, have an important role to play. In fact, Figure 1 shows a significant relationship between our measure of risk premia uncertainty from the yield curve and the premia of the indices of CDS.

⁹ For the model estimations we have used the parameters of the yield curve obtained through the Nelson and Siegel (1987) methodology from nominal interest rates at different maturities compiled from Datastream.

Figure 1: Term premia for the 2-year interest rates (blue) and indices of CDS (pink) for both the Euro Area and the United States



The risk premia we obtain follow a similar pattern to the inflation risk premia estimated by Garcia and Werner (2008) for the euro area based on an unobserved component approach and on the information contained in the Survey of Professional Forecasters and the Break-Even Inflation Rates.

3 Forecasting Inflation

3.1 Forecasting Inflation: The Set-Up

The model proposed by Diebold and Li (2006) focused only on forecasting interest rates. However, by reformulating the affine version of the model it seems natural to incorporate the inflation rate as an additional factor in the X_t vector. On one hand, the inflation rate could help to forecast the term structure and, therefore, the Nelson and Siegel parameters. The main reason behind this resides on the role that this variable plays in the reaction function of monetary policymakers and, consequently, on expected interest rates. In fact, the ECB and other central banks have as their primary target the achievement of price stability measured through an appropriate level of the inflation rate. In other countries, like the United States, inflation rates are not an explicit target although these central banks are clearly involved in maintaining price stability. In this sense, Estrella and Mishkin (1997) show that term structure was clearly affected not only by the monetary policy rate but also by central bank credibility.

On the other hand, the importance of the term structure in predicting future inflation changes has been extensively documented in the finance literature. In particular, Mishkin (1990) and Jorion and Mishkin (1991) used term spreads to predict future inflation changes in the United States and found that, although term spreads are not useful in the very short term, their accuracy increases as predictions extend beyond a year. In the same vein, Estrella and Mishkin (1997) found a similar relationship not only for the United States but also for most of the European countries.

In this respect, we analyse the relationship between the term structure and inflation rate from a more general perspective. By including the inflation rate in the X_t vector, we consider that this variable is not only correlated with the slope of the yield curve but also, and simultaneously, with other characteristics such as the long-term level or curvature. This approach is in line with Ang *et al.* (2007) or Ang and Piazzesi (2003), although they use an affine model with latent components instead of the Nelson and Siegel factors of Diebold and Li (2006). In a previous paper [see Gimeno and Marqués (2008)] we showed how, if we have to deal with an economy with a significant structural shift, the affine model with the parameters of the term structure seems to perform much better than traditional affine models based on unobserved components in order to obtain some estimation of ex-ante real interest rates.

3.2 Does the term structure contain any information on the inflation rate?

To ascertain the relationship between the interest rate and the inflation rate we can compare the results of the model with the inflation rate with those previously obtained without this variable. In order to properly compare both areas, we have used the same measure of inflation rate based on core inflation instead of the headline over the CPI. In the case of the Federal Reserve, several papers highlighted the fact that the core PCE is the main variable for price stability in the setting of interest rates¹⁰. However, given that in the euro area there is no equivalent to the PCE, we will use core inflation rate (referred to the CPI) for both areas, since its evolution does not differ significantly from the PCE and has a more comparable variable for the euro area. For the euro area, the official objective in the ECB mandate for

¹⁰. A good review of the importance of core inflation in the monetary policy of the Federal Reserve and other central banks can be found at Wynne (2008.)

price stability established headline inflation computed with the HICP as the official reference for monetary policy. Nevertheless, the reduced sample available since the beginning of the third phase in the euro area has coincided with a period where expected and repeated shocks in both oil and food prices has pushed up inflation. So, if we try to estimate the model with the HICP, we would have upward biased estimations of inflation expectations. Moreover, Gali *et al.* (2004) show that monetary policy decisions in the euro area are better explained by means of a Taylor rule based on a sentiment index and core inflation rather than the HICP. Given these caveats, it seems natural to expect the term structure of interest rates to be based more on core inflation than on the headline inflation rate.

We compare in table 4 the prediction of the 1-year interest rate 1 year ahead using the traditional Diebold and Li (2006) VAR model with the forecast once we incorporate the information on the inflation rate. Based on the mean squared error the inclusion of the inflation rate seems to clearly improve the forecast of the interest rate both in the Euro area and the United States. This result confirms previous findings in the literature that established that the inflation rate contains relevant information for forecasting interest rates [see for example Estrella and Mishkin (1997)]. Moreover, the inclusion of the inflation rate does not change the difference in the forecast accuracy of financial markets for interest rates between both areas.

Table 4: Mean Squared Error on 1-year interest rates forecast 1-year ahead using VAR model and the joint estimation with the inflation term

	Forward	VAR model	Joint Estimation
Euro Area		0.537	0.331
United States		1.279	0.995

In table 5 we compare the forecasts of the inflation rate using an AR(1) model (estimating the inflation rate only with its lagged values) and the VAR model for the interest rates augmented with the inflation rate (considering, therefore, not only the lagged values of the variable but also the dynamics of the Nelson and Siegel factors). Results clearly suggest that in both areas the term structure provides some information that improves the forecast of the inflation rate. As was the case for the nominal interest rate, in the euro area was easier to predict the core inflation rate than US, either with a simple AR model or with the information contained in the bond market, although the evidence was not so clear as for the nominal interest rate. This difference in the forecasting errors between both areas could be simply reflecting the fact that prices in the euro area have a higher degree of persistence.

Table 5: Mean Squared Error on 1-year inflation rate forecast 1-year ahead using AR model and the Joint estimation with Nelson and Siegel factors

	Inflation	AR model	Joint Estimation
Euro Area		0.164	0.110
United States		0.190	0.102

Nevertheless, results in table 4 and 5 are not market-consistent given that the non-arbitrage opportunity condition and risk aversion were not imposed. We could introduce these conditions in a similar way as in section 2.1 to the augmented VAR, given that both conditions have been derived independently from the factors contained in the X_t vector. Table 6 compares the forecast performance of this augmented model with arbitrage opportunities (labelled as affine model) with the model that considers jointly the Nelson and Siegel factors and the inflation rate (labelled as Joint estimation) and the model that considers the Nelson and Siegel factors and the inflation rate separately (labelled as VAR/AR model). The mean squared errors show that the inclusion of the non-arbitrage opportunity condition does not lessen the improvement in forecasting ability for the inflation rate and for the nominal interest rates that we had obtained by combining the term structure and the inflation rate. Estimated parameters for both models are presented in tables 7 and 8.

These results are in contrast with Ang *et al.* (2007) who, using an affine model with latent factors, show that while inflation is a very important determinant of yield curve movements, the term structure appears to provide little marginal forecasting ability for the dynamics of future inflation over simple time series models. This apparent contradiction could be related to the need for affine models with latent factors to incorporate some restrictions on the VAR that are not necessary in our case.

Table 6: Mean Squared Error on 1-year interest rates forecast 1-year ahead using VAR/AR model, Joint Estimation and the affine model with the inflation term

Interest rates	VAR model	Affine Model	
Euro Area	0.537	0.343	
United States	1.279	0.967	

Inflation	AR model	Joint Estimation	Affine Model
Euro Area	0.164	0.110	0.107
United States	0.190	0.102	0.099

3.3 Estimation of expected inflation rate

The results in the previous section reveal that the term structure of interest rates is closely related to the inflation rate. Thus, this framework could allow us to obtain a forecast for the inflation rate that could be considered as the market participants' expectations that underlie the bond markets. Therefore, we can compare the results obtained with other measures of inflation expectations that are also available from the financial markets.

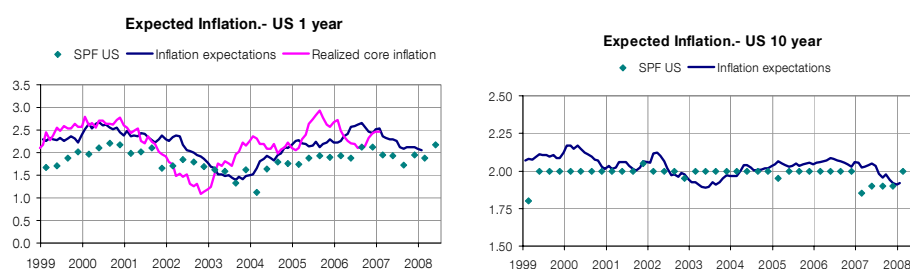
3.3.1 SURVEY EXPECTATIONS

The most straightforward measure of inflation expectations is to directly ask agents in the market. This is precisely what the *Consensus Forecast* and the *Survey of Professional Forecasters* (SPF) do¹¹. However, some caveats have to be considered before making

¹¹ The comparison of these series has to be viewed with caution given that *Consensus Forecast* and the SPF ask about the arithmetic average for these years and therefore do not consider the compound effect of inflation over the years. The plotted observed inflation and expected inflation refer to geometric average means that consider this compound effect.

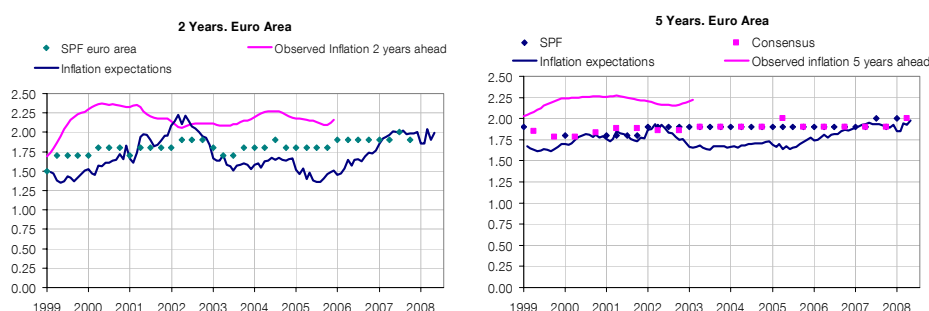
this comparison. Firstly, we take the Consensus Forecast and the SPF as indicators of expectations for the headline inflation rate, given the short sample of the statistics relating to core inflation. Although the difference between the expectations for headline inflation and core inflation in the long run has to be reduced in the short run (1 year), there could be significant differences. Secondly, the surveys refer in general to average inflation over the sample period, a measure that coincides with that which we reported from our estimation. However, in the case of the SPF for the euro area, the expected inflation for short-term periods (i.e. 2 years) refers to the point estimation.

Figure 2: Expected core inflation for the US compared with the Survey of Professional Forecasters for the 1-year (left) and 10-year (right) horizon



In the case of the United States, reported in Figure 2, results show that the forecast did not differ substantially from the expectations implicit in the nominal bond markets. In fact, the 1-year-ahead forecast follows a course that is more closely related to the SPF than with the final inflation outturn. We consider that this is clear evidence that our model is capturing inflation expectations and not simply an inflation forecast.

Figure 3: Expected Inflation for the Euro Area compared with the Survey of Professional Forecasters and the Consensus Forecast for the 2-year (left) and 5-year (right) horizon



In the euro area it is possible to compare the inflation expectations reported by Consensus Forecast and the SPF for the 2- and 5-years horizon with the expectations we have estimated from the term structure (Figure 3). As can be seen, model-implied inflation expectations are in line with those obtained from the surveys. Garcia and Werner (2008) showed that traditional affine models that try to estimate inflation expectations from the term structure for the euro area generally overestimate them. In their model they link inflation

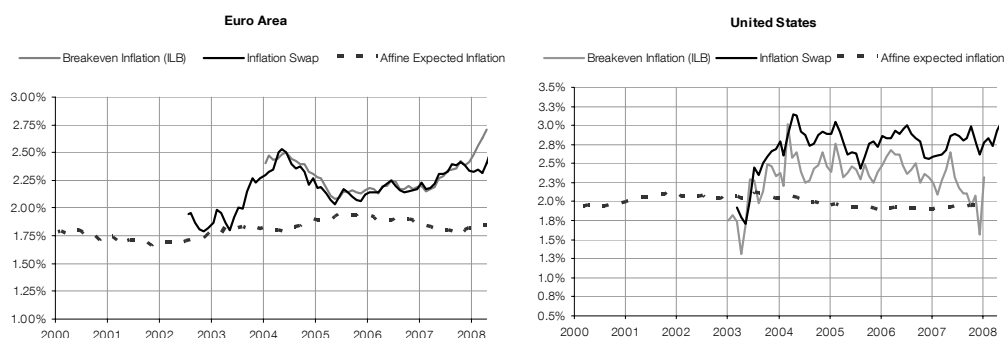
expectations to the SPF in a model similar to that of Ang *et al.* (2008), obtaining results not too different from those presented here.

3.3.2 EXPECTATIONS DRAWN FROM OTHER FINANCIAL ASSETS

Another possibility for obtaining inflation expectations is to use the prices of other financial assets whose return is somehow linked to inflation, i.e. inflation swaps or inflation-linked bonds. An inflation swap is a contract between two investors in which one of them agrees to receive an amount that is linked to future inflation in return for a fixed amount from the other party. This amount would be a signal of the expected inflation of both investors. Inflation-linked bonds (ILB) are similar to nominal bonds, but in the former case the principal is regularly updated with the evolution of the CPI. Therefore, the difference in the return on the nominal bond and that on the ILB is called break-even inflation and can be considered a measure of expected inflation, since this will be the inflation required to close the gap between both bonds.

These inflation expectations, like those obtained from the affine model of nominal interest rates, have an advantage over surveys in that they can be obtained daily and calculated for any horizon. In order to compare the results of the affine model with these market measures, we will focus on the forward 1 year inflation rate 4 years ahead that is reported in figure 4 for both the euro area and the United States. This indicator has been widely used in this market in order to obtain an indicator for the long-term inflation rate that partially removes some of the problems related to liquidity premia, which we will discuss later. The inflation expectations extracted from the model are substantially more stable than those obtained from inflation swaps or ILB. In the euro area, market measures incorporate risk premia and show values above those reported by the affine model. For the United States, inflation-linked assets are highly volatile and present such discrepancies that they are difficult to consider as an acceptable tracker of long-term inflation expectations.

Figure 4: Expected Inflation for the Euro Area (left) and the United States (right) compared with that derived from inflation-linked bonds and inflation swaps for 1 year on a 4 years-ahead horizon



Sources: ECB, Federal Reserve and Barclays Capital Inc.

In fact, neither inflation swaps nor inflation-linked bonds give clean measures of inflation expectations. Neither asset is free from term premia, which would mean that the longer the term of the asset, the higher the compensation required by the investor to buy the asset. Additionally, they are not remotely as liquid as nominal bonds, so investors would also ask for compensation for the difficulties in reducing their position if needed. In the case of Inflation swaps there other problems, such as counterparty risk, since the source of payment

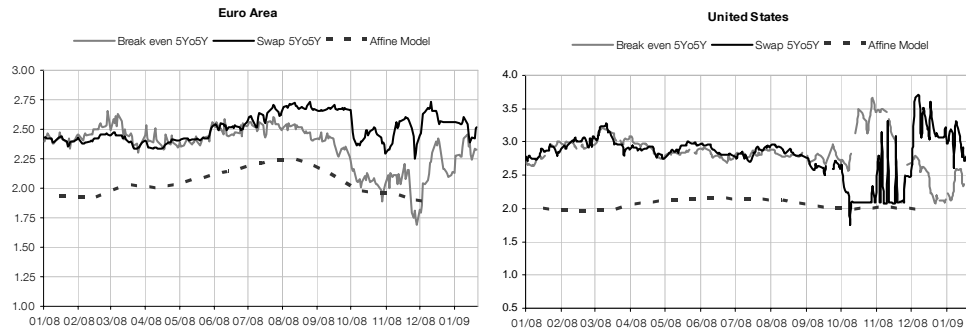
would not be any government but just another investor. Furthermore, swaps are traded over the counter, so information about prices is not as reliable as that obtained from a regulated market. Moreover, both ILB and Inflation swaps refer to the overall index of consumer prices (CPI), while in our model we refer to the core inflation rate. Both price indices should have similar expected behavior for long horizons, though in the short run they could present sizeable discrepancies.

Given all these characteristics, it is usual to find some anomalies in the behavior of these indicators. In Figure 5 we compare the evolution of the break-even inflation rate, Inflation Swaps and our measure of inflation expectation during the whole of 2008. In order to partially eliminate the liquidity problem we will compare these indicators by using the implicit expected average inflation rate for 5 years but 5 years ahead¹². However, as we can see in the Figure, even if using this implicit measure of the long-term inflation rate, the information contained in ILB and in swap inflation rates could be particularly misleading.

During the third quarter of this year the inflation rate trend changed suddenly, prompting some significant shifts in the demand for ILB and inflation swaps. It is not the purpose of this paper to go into detail on the technicalities surrounding the trading of this kind of product, but it should be pointed out that institutional investors, such as pension funds and insurance companies, usually seek inflation protection with customised inflation swaps offered by banks. These banks hedge their position by buying portfolios of inflation-linked bonds. Therefore, there is a clear arbitrage between the ILB market and inflation swaps, and, as a consequence, the expected inflation path derived from both markets is usually very similar. During the second half of 2008 the current inflation rate changed sharply. Most of this decline was related to oil price developments and will not necessarily affect the long-term inflation rate (which is more related to the core price index). However, this reversal in the inflation rate trend makes a hyperinflation scenario more implausible, and this produces a dramatic decline in the demand for inflation hedges. This event brings about a decoupling between the break-even rate and the swap inflation and a considerable increase in the volatility of this indicator (especially so in the United States). The evolution of this indicator during the early months of 2009 and the degree of volatility suggest that most of this change could not be attributed to a genuine update of inflation expectations and should rather be related to the liquidity positions of market participants. In fact, our measure of inflation expectations (for the core inflation rate) during this period shows a moderate decline that may be compatible with the new scenario once the prolonged increase in oil prices turns around.

12. During the last quarter of 2008 the liquidity premia component became so important in these markets that the break-even spot rate and the spot swap rate had negative values for certain periods. This mistakenly gave rise to some analysts explaining that financial markets were discounting a deflationary scenario. As can be seen in Figure 4, this scenario was not reflected once consideration was given to the implicit forward expected inflation rate, which partially removed the liquidity component.

Figure 5: Expected Inflation for the Euro Area (left) and the United States (right) during 2008 compared with that derived from inflation-linked bonds and inflation swaps for 5 year on 5 years-ahead horizon



Sources: ECB, Federal Reserve and Barclays Capital Inc

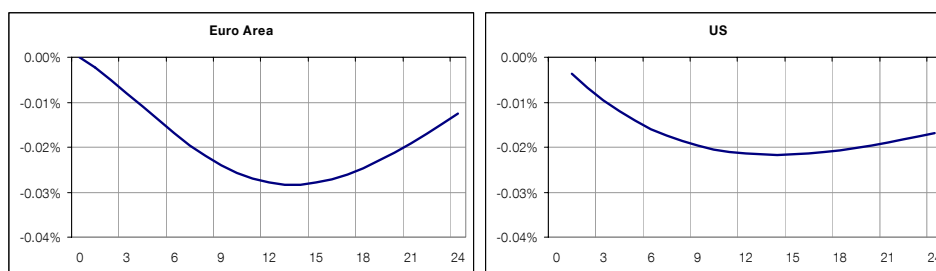
4 How do inflation expectations react to changes in monetary policy?

The estimated affine models give us a VAR equation that relates the parameters of the term structure to expected inflation. This equation allows us to perform an analysis of the reaction of inflation expectations to a shock in short-term interest rates by means of an impulse-response exercise.

To approach the effect of a 25 basis points increase in the monetary policy rate (the Fed Fund Rate in the US and the Main refinancing operations interest rate of the ECB) we have considered an equivalent increase at the shorter end of the term structure. This effect is achieved by reducing the slope factor (S_t) that was defined as the difference between the interest rate at both ends of the yield curve. In the affine model, the VAR equation has been defined to include an orthogonal Σ matrix, so an impulse-response exercise can be performed by simply adding a permanent shock to the random variable associated with the S_t variable.

The result of this shock on inflation is shown in Figure 6. As can be seen, such a movement would reduce expected inflation, although the speed of transmission of such a shock is quite slow, the full impact of such a measure being received more than a year after the movement. This transition is prolonged for somewhat longer than the actual perception of more than 6 months required for a change in monetary policy to take effect on the real side of the economy.

Figure 6: Response of expected inflation to a 25bp increase in short-term interest rates for both the Euro Area (left) and the United States (right).



When comparing the effect on the two models for the euro area and the US, it is possible to see that the impact of the movement in short-term interest rates on inflation expectations is higher in Europe than in the United States. Additionally, the shock is reversed earlier in the euro area. This higher sensitivity is consistent with the fact that the ECB requires fewer movements in its monetary policy rate than the Fed [see Adjemian *et al.* (2008)].

5 Conclusions

In this paper we reformulate the Diebold and Li (2006) model as a traditional affine model in order to incorporate the non-arbitrage opportunities condition and risk aversion. By doing so, we improve the consistency of the model while maintaining, and even increasing, the forecasting ability of the Diebold and Li (2006) methodology. This framework provides a tool for extracting risk premia implicit in the term structure. This variable seems to be the main driver of the variation of nominal interest rates in the United States and Europe over the last decade.

Moreover, with the affine formulation it is possible to introduce the inflation rate as an additional factor. Based on this augmented model we could obtain market expectations about the inflation rate that outperform those computed with an autoregressive approach. Moreover, the inflation rate seems to contain useful information for forecasting the interest rate. But the main advantage of this methodology for obtaining inflation rate expectations resides on the fact that it is based on a liquid market and that we can isolate inflation expectations from the risk premia component, which was not possible with other measures such as inflation-linked bonds or inflation swaps.

Lastly, the VAR equation of the affine model could be used to perform an impulse response analysis to assess the connection between short-term interest rates and inflation expectations. Our results indicate that the impact of a short-term interest rate on inflation expectations is higher in the 18-month horizon.

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Table 7: Estimated model for the Euro Area

$$\begin{pmatrix} L_{t+1} \\ S_{t+1} \\ C_{t+1} \\ \pi_{t+1} \end{pmatrix}_t = \begin{pmatrix} .000042 \\ .000172 \\ .000671 \\ .000317 \end{pmatrix} + \begin{pmatrix} .997 & .017 & .057 & .000 \\ .038 & .950 & -.047 & -.147 \\ -.235 & -.104 & .843 & -.012 \\ -.014 & -.046 & -.018 & .884 \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \\ \pi_t \end{pmatrix} + \begin{pmatrix} .00010 & & & \\ & .00026 & & \\ & & .00079 & \\ & & & .00018 \end{pmatrix} \begin{pmatrix} \varepsilon^1_{t+1} \\ \varepsilon^2_{t+1} \\ \varepsilon^3_{t+1} \\ \varepsilon^4_{t+1} \end{pmatrix}$$

$$\lambda_t = \begin{pmatrix} .091 \\ .423 \\ 1.119 \\ 3.924 \end{pmatrix} + \begin{pmatrix} -12.11 & 53.59 & 206.66 & -177.57 \\ 160.18 & 113.94 & -19.73 & -660.50 \\ -312.97 & -343.76 & -170.60 & 149.58 \\ 0.00 & -4466.82 & -1239.52 & 2683.68 \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \\ \pi_t \end{pmatrix}$$

$$y_{t,t+1} = -.00005 + (1.0031 \quad .0614 \quad .0233 \quad -.0258) \begin{pmatrix} L_t \\ S_t \\ C_t \\ \pi_t \end{pmatrix} + u_{t,t+1}$$

$$u_{t,1} \sim N(0, .00001I)$$

$$\varepsilon_t \sim N(0, I)$$

Table 8: Estimated model for the United States

$$\begin{pmatrix} L_{t+1} \\ S_{t+1} \\ C_{t+1} \\ \pi_{t+1} \end{pmatrix}_t = \begin{pmatrix} .000695 \\ -.000309 \\ -.000129 \\ .000105 \end{pmatrix} + \begin{pmatrix} .904 & -.068 & .063 & -.103 \\ .012 & .968 & -.054 & -.152 \\ .046 & .052 & .871 & -.166 \\ .019 & .004 & .006 & .911 \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \\ \pi_t \end{pmatrix} + \begin{pmatrix} .00018 & & & \\ & .00038 & & \\ & & .00010 & \\ & & & .00010 \end{pmatrix} \begin{pmatrix} \varepsilon^1_{t+1} \\ \varepsilon^2_{t+1} \\ \varepsilon^3_{t+1} \\ \varepsilon^4_{t+1} \end{pmatrix}$$

$$\lambda_t = \begin{pmatrix} 3.206 \\ -1.325 \\ 0.594 \\ -0.633 \end{pmatrix} + \begin{pmatrix} -525.35 & 537.61 & 101.36 & -296.85 \\ 47.93 & 68.02 & -63.34 & 577.03 \\ 31.44 & -16.37 & -51.73 & -448.35 \\ -91.72 & 0.00 & -36.59 & 424.56 \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \\ \pi_t \end{pmatrix}$$

$$y_{t,t+1} = -.00002 + (1.0029 \quad .0383 \quad .0095 \quad .0810) \begin{pmatrix} L_t \\ S_t \\ C_t \\ \pi_t \end{pmatrix} + u_{t,t+1}$$

$$u_{t,1} \sim N(0, .00001I)$$

$$\varepsilon_t \sim N(0, I)$$

Annex 1: Recursive expression of term structure parameters

RISK AVERSION AND NON-ARBITRAGE CONDITIONS

A non-arbitrage condition guarantees the existence of a risk-neutral measure (noted as Q) that allows interest rates to be expressed in terms of future term structure outcomes,

$$e^{A_{k+1}+B'_{k+1}X_t} = E_t^Q \left[e^{A_1+B'_1X_t} e^{A_k+B'_kX_{t+1}} \right] \quad (\text{A.1})$$

Risk-neutral measures (Q) are usually converted into natural probabilities using the Radon-Nikodym derivative, as in Ang and Piazzesi (2003), denoted by ξ_t ,

$$e^{A_{k+1}+B'_{k+1}X_t} = E_t \left[e^{A_1+B'_1X_t} e^{A_k+B'_kX_{t+1}} \frac{\xi_{t+1}}{\xi_t} \right] \quad (\text{A.2})$$

Usually, ξ_t in equation A.2 is assumed to follow a log-normal process,

$$\xi_{t+1} = \xi_t e^{\left(-\frac{1}{2} \lambda_t' \lambda_t - \lambda_t' \varepsilon_{t+1} \right)} \quad (\text{A.3})$$

where λ_t is a time-varying vector that incorporates the concept of risk aversion into the valuation framework. The first part of the exponent ($\lambda_t' \lambda_t$) is the Jensen Convexity component that ensures that $E_t \left[\frac{\xi_{t+1}}{\xi_t} \right] = 1$, while in the second, λ_t multiplies the perturbation vector ε_{t+1} , scaling the uncertainty in the random variables. This second term is responsible for the introduction of the risk premium in the valuation framework, whereby λ_t can be considered as a price of risk. Time-variant risk premia [Bekaert and Hodrick (2001)] will be the consequence of changes in this price of risk that is modelled assuming it to be also affine to the same factors X_t ,

$$\lambda_t = \lambda_0 + \lambda_1 X_t \quad (\text{A.4})$$

Finally, substituting A.3 into A.2, we arrive at a modified non-arbitrage condition that now takes into account investors' risk aversion.

$$e^{A_{k+1}+B'_{k+1}X_t} = E_t \left[e^{A_1+B'_1X_t} e^{A_k+B'_kX_{t+1}} e^{-\frac{1}{2} \lambda_t' \lambda_t - \lambda_t' \varepsilon_{t+1}} \right] \quad (\text{A.5})$$

Only X_{t+1} and ε_{t+1} of expression (A.5) are not already known in period t , while the other terms in the exponents can be extracted from the expectations operator,

$$e^{A_{k+1}+B'_{k+1}X_t} = e^{A_1+A_k+B'_1X_t-\frac{1}{2}\lambda'_t\lambda_t} E_t \left[e^{B'_kX_{t+1}-\lambda'_t\varepsilon_{t+1}} \right] \quad (\text{A.6})$$

Nevertheless, vector X_{t+1} can be forecast using VAR equation (2),

$$e^{A_{k+1}+B'_{k+1}X_t} = e^{A_1+A_k+B'_1X_t+B'_k\mu+B'_k\Phi X_t-\frac{1}{2}\lambda'_t\lambda_t} E_t \left[e^{(B'_k\Sigma-\lambda'_t)\varepsilon_{t+1}} \right] \quad (\text{A.7})$$

The exponent left in the expectations operator of expression (A.7) is solved taking into account the Jensen inequality.

$$e^{A_{k+1}+B'_{k+1}X_t} = e^{A_1+A_k+B'_k\mu+\frac{1}{2}B'_k\Sigma\Sigma'B'_k+(B'_1+B'_k\Phi)X_t+B'_k\Sigma\lambda_t} \quad (\text{A.8})$$

Finally, replacing the price of risk λ_t in A.8 by its definition (equation A.4), we arrive at expression (A.9),

$$e^{A_{k+1}+B'_{k+1}X_t} = e^{A_1+A_k+B'_k\mu+\frac{1}{2}B'_k\Sigma\Sigma'B'_k+(B'_1+B'_k\Phi)X_t+B'_k\Sigma(\lambda_0+\lambda_1X_t)} \quad (\text{A.9})$$

This last expression allows us to recover the recursive expression of coefficients A_{k+1} and B'_{k+1} in the affine representation as a function of the shorter terms,

$$A_{k+1} = A_1 + A_k + B'_k\mu - B'_k\Sigma\lambda_0 + \frac{1}{2}B'_k\Sigma\Sigma'B'_k \quad (\text{A.10})$$

$$B'_{k+1} = B'_1 + B'_k\Phi - B'_k\Sigma\lambda_1 \quad (\text{A.11})$$

A1.1 Valuation without risk compensation

The risk neutrality valuation framework used in (A.1) allowed us to incorporate the risk premium into the term structure. In order to recover risk-free rates we should consider a framework where agents are not concerned about risk, so expectations derived from the non-arbitrage condition are evaluated under a natural measure,

$$e^{\tilde{A}_{k+1}+\tilde{B}'_{k+1}X_t} = E_t \left[e^{\tilde{A}_1+\tilde{B}'_1X_t} e^{\tilde{A}_k+\tilde{B}'_kX_{t+1}} \right] \quad (\text{B.1})$$

where \tilde{A}_j and \tilde{B}'_j are the coefficients of equation 1, that meet non-arbitrage conditions. Using the same reasoning as in annex 1.1, replacing X_{t+1} by its forecast and applying Jensen inequality to solve the expectations operator, we arrive at expression (B.2),

$$e^{\tilde{A}_{k+1} + \tilde{B}'_{k+1} X_t} = e^{\tilde{A}_1 + \tilde{A}_k + \tilde{B}'_k \mu + \frac{1}{2} \tilde{B}'_k \Sigma \Sigma' \tilde{B}_k + (\tilde{B}'_1 + \tilde{B}'_k \Phi) X_t} \quad (\text{B.2})$$

As can be seen, expression (B.2) is equivalent to (A.8), the only difference being that once risk aversion is avoided, the term $B'_k \Sigma \lambda_t$ is no longer needed. In fact, this was the term that added a risk premium for each extra period of investment. A risk-neutral individual would have a null price of risk, with both expressions becoming equivalent. Under this assumption, the term structure recursive expression would now be,

$$\tilde{A}_{k+1} = \tilde{A}_1 + \tilde{A}_k + \tilde{B}'_k \mu + \frac{1}{2} \tilde{B}'_k \Sigma \Sigma' \tilde{B}_k \quad (\text{B.3})$$

$$\tilde{B}'_{k+1} = \tilde{B}'_1 + \tilde{B}'_k \Phi \quad (\text{B.4})$$

A1.2 Exogenous model estimation

Prior to the estimation of the affine model we have to determine the factors related to the term structure. Following Diebold and Li (2006), we use the Nelson and Siegel (1987) formula of the term structure.

$$y_{t,t+k} = L_t + S_t \frac{1 - e^{-k/\tau}}{k/\tau} + C_t \left(\frac{1 - e^{-k/\tau}}{k/\tau} - e^{-k/\tau} \right) \quad (\text{C.1})$$

Diebold and Li (2006) fixed the value of τ to be the mean throughout the sample. Once τ is constant, equation C.1 can be estimated by OLS for each period, regressing interest rates for different terms (k) against matrix Z_t .

$$Z_k = \left[1 \quad \frac{1 - e^{-k/\tau}}{k/\tau} \quad \frac{1 - e^{-k/\tau}}{k/\tau} - e^{-k/\tau} \right] \quad (\text{C.2})$$

Once L_t , S_t and C_t are estimated as the parameters of these regressions for each period, they can be used as factors for the affine model. As vector X_t is completely determined, we no longer need to fix any interest rate as observed without error. In fact, it is now quite easy to recover initial values of the parameters via OLS estimations in three steps.

Since vector X_t is exogenously determined, we can estimate the VAR equation via OLS, which allows initial values to be obtained of μ , Φ and Σ

$$X_t = \mu + \Phi X_{t-1} + \Sigma \varepsilon_t \quad \varepsilon_t \sim N(0, I) \quad (C.3)$$

We can also use vector X_t to regress it against nominal interest rates for different terms using the term structure equation, in order to estimate consecutive values of A_k and B'_k ,

$$-k \cdot y_{t,t+k} = A_k + B'_k X_t + u_{t,t+k} \quad u_t \sim N(0, \sigma^2 I) \quad (C.4)$$

Finally, in order to incorporate a non-arbitrage condition and risk aversion we go further than Diebold and Li (2006) and use \hat{A}_k and \hat{B}'_k estimations to regress them against shorter term values, rearranging equations 5 and 6. Once we have tentative values from C.3 and C.4, then equations 5 and 6 become,

$$(\hat{A}_{k+1} - \hat{A}_k) - \hat{A}_1 - \hat{B}'_k \mu - \frac{1}{2} \hat{B}'_k \Sigma \Sigma' \hat{B}_k = -\hat{B}'_k \Sigma \lambda_0 \quad (C.5)$$

$$(\hat{B}'_{k+1} - \hat{B}'_k \Phi) - \hat{B}'_1 = -\hat{B}'_k \Sigma \lambda_1 \quad (C.6)$$

Equations C.5 and C.6 are linear with respect to λ_0 and λ_1 , and therefore, these parameters can also be estimated by OLS.

Once we have estimated separately C.3, C.4, C.5 and C.6 equations, we have tentative initial values of the affine model that allow for the swift computation of the joint maximum likelihood estimation of the affine model given by,

$$y_{t,t+k} = \frac{-1}{k} (A_k + B'_k X_t) + u_{t,t+k} \quad u_t \sim N(0, \sigma^2 I)$$

$$X_t = \mu + \Phi X_{t-1} + \Sigma \varepsilon_t \quad \varepsilon_t \sim N(0, I)$$

subject to (C.7)

$$A_{k+1} = A_1 + A_k + B'_k \mu - B'_k \Sigma \lambda_0 + \frac{1}{2} B'_k \Sigma \Sigma' B'_k$$

$$B'_{k+1} = B'_1 + B'_k \Phi - B'_k \Sigma \lambda_1$$

Annex 2: Nominal interest rate decomposition

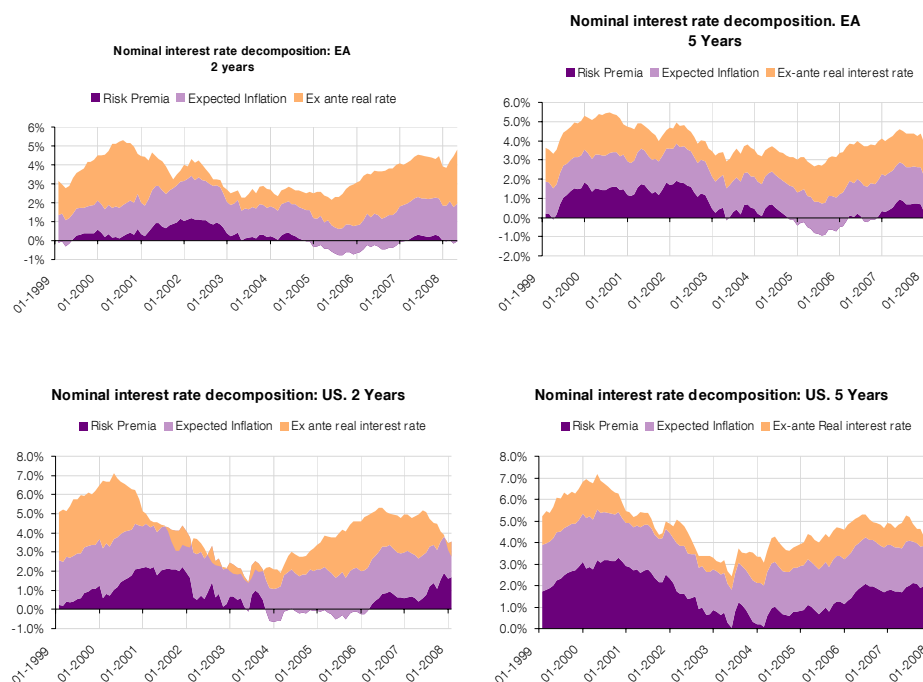
Another advantage of the proposed model is that we are able to decompose nominal interest rates into their three underlying components: the risk (term) premia, expected inflation and the ex-ante real rate.

$$Y_t^k = r_t + E_t[\pi_{t,t+k}] + \gamma_{t,t+k} \quad (D1)$$

The term premia, as stated in section 2.1, are computed following equation 9. Once we have an interest rate ($\hat{y}(\lambda_t = 0)_{t,t+k}$) that is free from the term premia, this variable could also be further decomposed by subtracting the expected inflation discussed in section 2.2. The remaining value would be an ex-ante real rate.

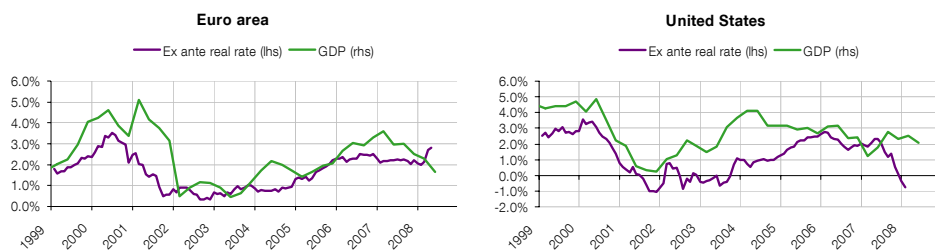
The results of this decomposition for both the United States and euro area are shown in Figure 7. As can be seen, most of the variability in nominal interest rates is derived from the risk premia. By contrast, Inflation expectations remain almost constant throughout the sample. Moreover, the risk premia seem to have played a more important role in explaining the evolution of the interest rate in the United States than in the euro area. This could be due to the relevant shifts of monetary policy in this area during the sample period.

Figure 7: Nominal interest rate decomposition for the Euro Area (above) and the United States (bottom) for 2-year (left) and 5-year (right) horizon



The evolution of ex-ante real rates should be closely related to the expected cost of financing for entrepreneurs. In this respect, this variable has to be connected with the growth of GDP. This evolution is shown in Figure 8, where 2-year ex-ante real interest rates for both the euro area and the United States are compared with the GDP posted at the same time. As can be seen, the real interest rates were higher in periods when strong growth of GDP was recorded, while they declined in periods of weaker economic performance.

Figure 8: Ex ante real interest rate for the Euro Area (left) and the United States (right) for 2 years compared with GDP growth



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