SEARCH FRICTIONS, REAL RIGIDITIES AND INFLATION DYNAMICS

Carlos Thomas

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SEARCH FRICTIONS, REAL RIGIDITIES AND INFLATION DYNAMICS(*)
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Carlos Thomas(**)

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(**) DG Economics, Statistics and Research, Banco de España, Alcalá 48, 28014 Madrid, Spain. E-mail: carlos.thomas@bde.es. Tel.: +34913386280.

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Abstract

The literature on New Keynesian models with search frictions in the labor market commonly assumes that price-setters are not actually subject to such frictions. Here I propose a model where firms are subject both to infrequent price adjustment and search frictions. This interaction gives rise to real price rigidities, which have the effect of slowing down the adjustment of the price level to shocks. This has a number of consequences for equilibrium dynamics. First, inflation becomes less volatile and more persistent. More importantly, the model’s empirical performance improves along its labor market dimensions, such as the size of unemployment fluctuations and the relative volatility of the two margins of labor.

Keywords: search and matching, real rigidities, New Keynesian Phillips curve, labor market fluctuations.

JEL classification: E32, J60.
1 Introduction

The search and matching model has become a popular treatment of labor market dynamics in New Keynesian models of the monetary transmission mechanism.\(^1\) One of the main advantages of this kind of framework is that it makes it possible to analyze the joint dynamics of unemployment and inflation in a relatively simple way. Due to search frictions in the labor market, it takes time for unemployed workers to find jobs. This, together with recurrent job destruction, gives rise to unemployment in equilibrium. On the other side of the labor market, search frictions imply that firms must spend time and resources before they can find suitable workers. To the extent that firms have monopoly power on the goods they sell, this naturally raises the question as to how pricing decisions are affected by the fact that firms cannot costlessly and instantaneously adjust the size of their workforce.

In fact, the existing literature has paid very little attention to the latter question. The reason is an assumption commonly made in previous studies, namely that the firms setting prices are different from the firms that are subject to search frictions.\(^2\) These two subsets of firms are sometimes called ‘retailers’ and ‘producers’, respectively, whereby the latter sell an intermediate good to retailers at a perfectly competitive price. This assumption is very convenient, as it allows one to disentangle forward-looking vacancy-posting and pricing decisions and thus simplify the analysis. However, it eliminates from the outset the possibility of analyzing the effect of search frictions on the pricing decisions of individual firms. The aim of this paper is to build a model where price-setters do face such frictions, and analyze the resulting implications for equilibrium dynamics. In particular, I consider a framework in which firms reset their prices at random intervals. In order to meet a sudden change in demand for its product, each firm can immediately adjust the number of hours worked by its employees. However, in order to adjust employment the firm must first incur the cost of posting vacancies and then wait for the

\(^1\)For a simple exposition of the search and matching model, see Pissarides (2000, Ch. 1).

latter to be filled.

I find that the interaction of price-setting decisions and search frictions slows down the adjustment of the price level to shocks. The reason is the following. Due to search frictions, firms’ short-run marginal costs depend on the cost of increasing production along the intensive margin of labor. Wage bargaining between the firm and its workers implies that the latter must be compensated for the disutility of work. If the latter is realistically convex in hours worked, firms’ marginal cost curves become upward-sloping. This gives price-setting firms an incentive to keep their prices in line with the overall price level. That is, search frictions give rise to real rigidities in prices, using Ball and Romer’s (1990) terminology. For example, suppose that following an aggregate shock that decreases marginal costs, each price-setter considers a certain reduction in its nominal price. Given the prices of other firms, the reduction in the firm’s nominal price represents a reduction in its real price. This leads the firm to anticipate stronger sales and therefore higher marginal costs for the duration of the price contract. As a result, the firm ends up choosing a smaller price cut than the one initially considered. Because all price-setters follow the same logic, real rigidities slow the adjustment of the overall price level in response to the same fluctuations in average real marginal costs. This effect is reflected in a flatter slope of the New Keynesian Phillips curve.

The real rigidity mechanism just described is absent in New Keynesian search-and-matching models with a producer-retailer structure, because each retailer’s marginal cost (the price of the intermediate good) is independent of its own output. Interestingly, I show that the log-linear equilibrium conditions in a producer-retailer model with identical preferences and technology are exactly the same as in the model with real rigidities, except for the slope of the New Keynesian Phillips curve. This allows me to use the producer-retailer model as a ‘control’ for isolating the effect of real rigidities on equilibrium dynamics. Two main results arise. First, inflation becomes less volatile and more persistent for a given frequency of nominal

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3The notion that the marginal wage (and hence the marginal cost of production) is increasing in average hours per employee goes back to Bils (1987). See also Rotemberg and Woodford (1999).
price adjustment. Through this endogenous mechanism, real rigidities help the model match inflation dynamics without the need of assuming unrealistically long price contracts. This is a well-known property of real rigidity mechanisms, as exemplified by models with firm-specific capital (Sveen and Weinke, 2005; Woodford, 2005; and Altig et al., 2004) and models with industry-specific labor markets (Woodford, 2003).

Second, and perhaps more importantly, real rigidities improve the empirical performance of the New Keynesian search-and-matching model along those labor market dimensions that the standard New Keynesian model is not designed to address. For a plausible calibration, and relative to the producer-retailer specification, I show that the model with real rigidities comes closer to US data in two important dimensions: the volatility of unemployment (both in absolute terms and relative to output), and the volatility of the extensive margin of labor (employment) relative to the intensive margin (hours per employee). At the heart of these results is again the interaction between infrequent price adjustment and search frictions, together with the long-term nature of employment relationships in this framework. Once the firm sets its price, its output is demand-determined and its revenue is independent of its number of employees. Therefore, job creation decisions are driven, not by the marginal revenue product of new hires (as in flexible price models), but by the fact that hiring additional workers allows the firm to satisfy its future demand with less hours per employee and thus reduce its wage bill. This implies that, when firms expect hours per employee to be higher in future periods, they have an incentive to create more jobs. By making the price level more sluggish, real rigidities amplify fluctuations in aggregate demand and hence in hours per employee; as a result, job creation becomes more volatile and so does unemployment. Crucially, since what matters for job creation

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*This part of the analysis has some connection with the literature on unemployment volatility in search models initiated by Shimer (2005). The latter author emphasized the inability of the canonical search and matching model to produce realistic unemployment fluctuations (the so-called 'unemployment volatility puzzle'). An extensive literature has subsequently suggested a number of mechanisms aimed at amplifying unemployment fluctuations in search models. While this paper is not aimed at addressing the unemployment volatility puzzle, it does illustrate that real rigidities provide an amplification mechanism in the context of New Keynesian models with search frictions.*
is the entire expected path of hours per employee (due to the on-going nature of jobs), a certain increase in the volatility of hours per employee produces a more than proportional increase in the volatility of employment; the same effect implies that the increase in (un)employment volatility takes place also relative to output. Notice that these labor market effects are indirect, as they operate through the increased sluggishness of prices produced by the flattening of the New Keynesian Phillips curve. I conclude that integrating search frictions and staggered price adjustment at the firm level has important payoffs in terms of labor market fluctuations, while increasing only slightly the model’s complexity.

Other papers have departed from the producer-retailer assumption in the context of New Keynesian models with search and matching frictions. A notable example is Krause and Lubik (2007). These authors assume quadratic costs of price adjustment, rather than staggered price adjustment. As a result, price decisions are symmetric across firms. Since all real prices are one, the real rigidity effect is absent in such a framework. More closely related is the independent work of Sveen and Weinke (2009) and Kuester (2010). Sveen and Weinke (2009) identify a real rigidity mechanism similar to the one analyzed here. Our papers differ mainly in focus: whereas Sveen and Weinke (2009) emphasize the implications of strategic complementarities in price setting and of real wage stickiness for inflation dynamics, I stress the consequences of real price rigidities for the cyclical behavior of the labor market. Kuester’s (2010) model features firm-worker pairs where both nominal prices and wages are bargained in a staggered fashion. This gives rise to real rigidities in prices as well as in wages, the latter effect amplifying fluctuations in unemployment. The mechanism is therefore different from the one presented here, which does not rely on staggered wage bargaining. Our papers also differ in terms of scope. Kuester incorporates a number of additional frictions (habit formation, price indexation and wage indexation) and estimates the model using a number of US macroeconomic time series.

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5Krause and Lubik (2007) focus their analysis on the relevance of real wage rigidity for inflation dynamics.

6Notice that, in the model proposed here, firms are assumed to employ many workers, which realistically allows them to change both margins of labor over time.
Instead, I compare two calibrated search and matching models where monopolistic competition and staggered price-setting are the only additional frictions, with the purpose of isolating the effect of the real price rigidities on equilibrium dynamics.

This paper is also related to previous analyses of how the specificity of labor can give rise to real rigidities in New Keynesian models. In particular, Woodford (2003, Ch. 3) considers a setup of industry-specific labor markets where firms in each industry hire labor at that industry’s perfectly competitive wage. This generates upward-sloping marginal cost curves at the industry level and hence real rigidities. This paper considers instead a framework in which the search frictions that characterize the labor market give rise endogenously to long-run employment relationships, thus making labor specific to each firm.

There also exists a parallel between the real rigidity mechanism studied in this paper and the one arising in models of firm-specific capital (Sveen and Weinke, 2005; Woodford, 2005; and Altig et al., 2004). Here, the employment stock plays an analogous role to the capital stock in the latter class of models, namely that of being a firm-specific endogenous state variable. This implies similar complications in firms’ price-setting decisions, because the latter interact with forward-looking hiring decisions (here) or investment decisions (in firm-specific capital models) in a non-trivial manner. In order to solve for pricing decisions, I use Woodford’s (2005) solution method, which he develops in the context of a model with firm-specific capital. Regarding the nature of the real rigidity mechanism, in both cases the latter arises because the firm’s marginal cost curve becomes upward-sloping, although for different reasons. In the case of firm-specific capital, marginal costs slope upwards due to decreasing marginal returns to labor for a given capital stock. Here, it is due to increasing marginal disutility of labor and hence increasing marginal wages.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 shows how to solve for individual pricing decisions in this framework, using Woodford’s (2005) methodology. I then derive the New Keynesian Phillips curve and analyze the effect of real
rigidities on equilibrium dynamics. Section 4 calibrates the model and provides a quantitative assessment of the theoretical mechanisms. Section 5 concludes.

2 The model

I now present a New Keynesian model with search and matching frictions in the labor market. The model therefore brings together two frameworks that have become the standard for analyzing the monetary transmission mechanism and the cyclical behavior of the labor market, respectively. The main difference with respect to previous models of this type is that I do not separate the firms making the pricing decisions from the firms that face search frictions. Instead, I consider a single set of firms which set prices and post vacancies in a labor market characterized by search frictions.

2.1 The matching function

The search frictions in the labor market are represented by a matching function, \( m(v_t, u_t) \), where \( v_t \) is the total number of vacancies and \( u_t \) is the total number of unemployed workers. Normalizing the labor force to 1, \( u_t \) also represents the unemployment rate. The function \( m \) is strictly increasing and strictly concave in both arguments. Assuming constant returns to scale in the matching function, the matching probabilities for unemployed workers, \( m(v_t, u_t)/u_t = m(v_t/u_t, 1) \), and for vacancies, \( m(v_t, u_t)/v_t = m(1, u_t/v_t) \), are functions of the ratio of vacancies to unemployment, also known as labor market tightness. I denote the latter by \( \theta_t \equiv v_t/u_t \). In what follows, I let \( p(\theta_t) \equiv m(\theta_t, 1) \) denote the matching probability for unemployed workers. The latter is an increasing function of \( \theta_t \): in a tighter labor market, job-seekers are more likely to find jobs. Similarly, I let \( q(\theta_t) \equiv m(1, 1/\theta_t) \) denote the matching probability for vacancies. The latter is decreasing in \( \theta_t \): firms are less likely to fill their vacancies in a tighter labor market.

\footnote{See Petrongolo and Pissarides (2001) for empirical evidence of constant returns to scale in the matching function for several industrialized economies.}
2.2 Households

In the presence of unemployment risk, we may observe differences in consumption levels between employed and unemployed consumers. However, under the assumption of perfect insurance markets, consumption is equalized across consumers. This is equivalent to assuming the existence of a large representative household, as in Merz (1995). In this household, a fraction $n_t$ of its members are employed in a measure-one continuum of firms. The remaining fraction $u_t = 1 - n_t$ search for jobs. All members pool their income so as to ensure equal consumption across members. Household welfare is given by

$$H_t = u(c_t) - n_t b - \int_0^1 n_{it} \frac{h_{it}^{1+\eta}}{1+\eta} di + \beta E H_{t+1},$$

(1)

where $n_{it}$ and $h_{it}$ represent the number of workers and hours per worker respectively in firm $i \in [0, 1]$, $b$ is labor disutility unrelated to $h_{it}$ (forgone utility from home production, commuting time, etc.), and

$$c_t \equiv \left( \int_0^1 \frac{c_{it}}{P_{it}} di \right)^{\frac{1}{\gamma}}$$

is the Dixit-Stiglitz consumption basket, where $\gamma > 1$ measures the elasticity of substitution across differentiated goods. Cost minimization implies that the nominal cost of consumption is given by $P_t c_t$, where

$$P_t \equiv \left( \int_0^1 P_{it}^{1-\gamma} di \right)^{\frac{1}{\gamma}}$$

is the corresponding price index. The household’s budget constraint is given by

$$\frac{M_{t-1} + R_{t-1} B_{t-1} + T_t}{P_t} + \int_0^1 n_{it} w_{it}(h_{it}) di + \Pi_t \geq c_t + \frac{B_t + M_t}{P_t},$$

(2)

where $M_{t-1}$ and $B_{t-1}$ are holdings of money and one-period nominal bonds, respectively, $R_{t-1}$ is the gross nominal interest rate, $T_t$ are net cash transfers from the government, $w_{it}(h_{it})$ is the
wage income earned by workers in firm $i$ as a function of hours worked, and $\Pi_t = \int_0^1 \Pi_{i,t} di$ are aggregate real profits, which are reverted to households in a lump-sum manner.

Employed members separate from their jobs at the exogenous rate $\lambda$, whereas unemployed members find jobs at the rate $p(\theta_t)$. Therefore, the household’s employment rate evolves according to the following law of motion,

$$n_{t+1} = (1 - \lambda)n_t + p(\theta_t)(1 - n_t). \quad (3)$$

It is useful at this point to find the utility that the marginal worker in firm $i$ contributes to the household. Equations (1), (2) and (3), together with $n_t = \int_0^1 n_{i,t} di$, imply that

$$\frac{\partial H_t}{\partial n_{i,t}} = u'(c_t)w_{i,t}(h_{i,t}) - b - \frac{h_{i,t}^{1+\eta}}{1 + \eta} - p(\theta_t) \int_0^1 \frac{v_{j,t}}{v_t} \beta E_t \frac{\partial H_{t+1}}{\partial n_{j,t+1}} dj + (1 - \lambda)\beta E_t \frac{\partial H_{t+1}}{\partial n_{i,t+1}}, \quad (4)$$

where $p(\theta_t)v_{j,t}/v_t$ is the probability that an unemployed member is matched to firm $j \in [0, 1]$. The right hand side of equation (4) consists of the real wage (in utility units), minus labor disutility and outside opportunities, plus the continuation value of the job.

I assume the existence of a standard cash-in-advance (CIA) constraint on the purchase of consumption goods.\footnote{I use the CIA specification for aggregate demand in order to simplify the exposition of the main mechanisms. Section 4.3.2 considers alternative demand specifications, such as money in the utility function (MIU) or a Taylor rule for the nominal interest rate. All the main qualitative results regarding the effects of real rigidities on inflation and labor market volatility are invariant to these alternative specifications.} Assuming that goods markets open after the closing of financial markets, the household’s nominal expenditure in consumption cannot exceed the amount of cash left after bond transactions have taken place,

$$P_t c_t \leq M_{t-1} + T_t - B_t. \quad (5)$$

Cash transfers are given by $T_t = M_t^p - M_{t-1}^p$, where $M_t^p$ is exogenous money supply. The growth rate of money supply, $\zeta_t = \log(M_t^p/M_{t-1}^p)$, follows an AR(1) process, $\zeta_t = \rho_m \zeta_{t-1} + \varepsilon_t^m$, where
$\varepsilon_t^m$ is an iid shock. Assuming that the net nominal interest rate (i.e. the opportunity cost of holding money) is always positive, equation (5) holds with equality. In equilibrium, money demand equals money supply, $M_t = M_t^s$, which implies $M_{t-1} + T_t = M_t$. Combining this with (5) and the fact that bonds are in zero net supply ($B_t = 0$), I obtain

$$P_1c_t = M_t.$$  \hfill (6)

### 2.3 Firms

The value of firm $i \in [0, 1]$ in period $t$ is given by

$$V_{it} = \frac{P_{it}}{P_t} y_{it}^d - w_{it}(h_{it}) n_{it} - \frac{\chi}{u'(c_t)} v_{it} + E_t \beta_{t,t+1} V_{it+1},$$

where $P_{it}$ and $y_{it}^d$ are the firm’s nominal price and real sales, respectively, $v_{it}$ are vacancies posted in period $t$, $\chi$ is the utility cost for the management of posting a vacancy and $\beta_{t,T} \equiv \beta^{T-t} u'(c_T)/u'(c_t)$ is the stochastic discount factor between periods $t$ and $T \geq t$. Due to imperfect substitutability among individual goods, the firm faces the following demand curve for its product,

$$y_{it}^d = \left(\frac{P_{it}}{P_t}\right)^{-\gamma} y_t,$$  \hfill (7)

where aggregate demand is given by $y_t = c_t$. The firm’s production technology is given by

$$y_{it}^s = A_t n_{it} h_{it},$$

where $A_t$ is an exogenous labor productivity process. The log of the latter, $a_t \equiv \log A_t$, follows an AR(1) process, $a_t = \rho_t a_{t-1} + \varepsilon_t^a$, where $\varepsilon_t^a$ is an iid shock. Once the firm has chosen a price, it commits to supplying whatever amount is demanded at that price, $y_{it}^s = y_{it}^d$. This requires
the following condition to hold at all times,

\[
\left( \frac{P_t}{P_1} \right)^{-\gamma} y_t = A_t n_{it} h_{it}.
\]  

(8)

In each period, the individual firm posts a number \( v_{it} \) of vacancies. Assuming that firms are large, \( \lambda \) and \( q(\theta_t) \) are the fraction of workers that separate from the firm and the fraction of vacancies that the firm fills, respectively. Due to the time involved in searching for suitable workers and (possibly) training them, new hires become productive in the following period. Therefore, the firm’s workforce, \( n_{it} \), is given at the start of the period. The law of motion of the firm’s employment stock is given by

\[
n_{it+1} = (1 - \lambda)n_{it} + q(\theta_t)v_{it}.
\]  

(9)

Let \( mc_{it} \) and \( \phi_{it} \) denote the Lagrange multipliers with respect to constraints (8) and (9), respectively. Therefore, \( mc_{it} \) represents the real marginal cost of production. The firm chooses the state-contingent path \( \{h_{it}, v_{it}, n_{it+1}\}^\infty_{t=0} \) that maximizes

\[
E_0 \sum_{t=0}^\infty \beta_{0,t} \left\{ (P_{it}/P_1)^{-\gamma} y_t - w_{it}(h_{it})n_{it} - \chi v_{it}/u'(c_t) + mc_{it} \left[ A_t n_{it} h_{it} - (P_{it}/P_1)^{-\gamma} y_t \right] + \phi_{it} [(1 - \lambda)n_{it} + q(\theta_t)v_{it} - n_{it+1}] \right\}. 
\]

The first-order conditions are given by

\[
mc_{it} = \frac{w'_t(h_{it})}{A_t}, \quad \text{ (10)}
\]

\[
\frac{\chi}{u'(c_t)} = q(\theta_t)\phi_{it}, \quad \text{ (11)}
\]

\[
\phi_{it} = E_t \beta_{t,t+1} \left[ mc_{it+1} A_{t+1} h_{it+1} - w_{it+1}(h_{it+1}) + (1 - \lambda)\phi_{it+1} \right]. \quad \text{ (12)}
\]
According to equation (10), the real marginal cost is given by the ratio between the real marginal wage, \( w'_{it}(h_{it}) \), and the marginal product of labor, \( A_t \). Intuitively, since employment is predetermined, the firm needs to raise hours per employee in order to increase production. This comes at a marginal cost of \( w'_{it}(h_{it}) \) per employee. Equation (11) says that the marginal cost of posting a vacancy must equal the probability that the vacancy is filled times the expected value of an additional worker in the following period. The latter, from equation (12), is given by the expected marginal reduction in the firm’s cost, minus the expected wage to be paid to the new hire, plus her continuation value for the firm.

2.3.1 Wage bargaining

I assume Nash wage bargaining between the firm and each individual worker. Both the firm and the worker enjoy an economic surplus from their employment relationship. The worker’s surplus in consumption units, which I denote by \( S^{w}_{it} \equiv (\partial H_t/\partial n_{it}) / u'(c_i) \), is given by equation (4) divided by \( u'(c_i) \), that is,

\[
S^{w}_{it} = w_{it}(h_{it}) - \frac{b + \eta^1 + \eta}{u'(c_i)} - p(\theta_i) \int_0^1 \frac{v_{it}}{v_t} E_t^\beta_{t,t+1} S^{w}_{it+1} dt + (1 - \lambda) E_t^\beta_{t,t+1} S^{w}_{it+1}. \quad (13)
\]

On the firm’s side, the surplus obtained from the marginal worker is given by

\[
S^{f}_{it} = mc_{it} A_t h_{it} - w_{it}(h_{it}) + (1 - \lambda) E_t^\beta_{t,t+1} S^{f}_{it+1}. \quad (14)
\]

The term \( mc_{it} A_t h_{it} \) is the marginal increase in costs that the firm would have to incur if the employee walked away from the job. Since the firm is demand-constrained, it would have to make up for the lost production, \( A_t h_{it} \), by raising working hours for all other employees, which comes at a cost of \( mc_{it} A_t h_{it} \). Therefore, the contribution of the marginal worker to flow profits is given, not by the marginal revenue product of the worker (as in standard RBC models), but
by the marginal reduction in the wage bill.\(^9\)

Let \(\xi\) denote the firm’s bargaining power. Nash bargaining implies the following surplus-sharing rule,

\[
(1 - \xi)S^f_{it} = \xi S^w_{it}.
\]  

(15)

Combining this with (13) and (14), I obtain the following wage agreement,

\[
w_{it}(h_{it}) = (1 - \xi)mc_{it}A_t h_{it} + \xi \left[\frac{b + h_{it}^{1+\eta}/(1 + \eta)}{u'(c_t)} + p(\theta_t) \int_0^1 \frac{v_{jt}}{v_t} E_t \beta_{l,t+1} S^w_{jt+1} dj\right].
\]  

(16)

Therefore, the worker receives a weighted average of her contribution to cost reduction and the opportunity cost of holding the job (the sum of labor disutility and outside options). It is possible to simplify the equation (16). Notice first that, from equations (12) and (14), it follows that \(\phi_{it} = E_t \beta_{l,t+1} S^f_{it+1}\). This, together with equations (11) and (15), implies that

\[
\int_0^1 \frac{v_{jt}}{v_t} E_t \beta_{l,t+1} S^w_{jt+1} dj = \frac{1 - \xi}{\xi} \int_0^1 \frac{v_{jt}}{v_t} E_t \beta_{l,t+1} S^f_{jt+1} dj = \frac{1 - \xi}{\xi} \frac{\chi}{q(\theta_t)u'(c_t)}.
\]

Inserting this into equation (16), and using the fact that \(p(\theta_t)/q(\theta_t) = \theta_t\), I finally obtain

\[
w_{it}(h_{it}) = (1 - \xi) \left[mc_{it} A_t h_{it} + \frac{\chi}{u'(c_t)} \theta_t\right] + \xi \left[\frac{b + h_{it}^{1+\eta}/(1 + \eta)}{u'(c_t)}\right].
\]

(17)

2.3.2 Vacancy posting

Combining the first-order conditions (11) and (12), and the real wage schedule, equation (17), I obtain the following expression for the vacancy-posting decision,

\[
\frac{\chi}{q(\theta_t)} = \beta E_t \left\{\xi \left[u'(c_{t+1})mc_{it+1} A_{t+1} h_{it+1} - b - \frac{h_{it+1}^{1+\eta}}{1 + \eta}\right] - (1 - \xi)\chi \theta_{t+1} + (1 - \lambda) \frac{\chi}{q(\theta_{t+1})}\right\}.
\]  

(18)

\(^9\)This result is analogous to the one in Woodford’s (2005) model of firm-specific capital, where the marginal contribution of capital to flow profits is given by the marginal reduction in the wage bill, rather than the marginal revenue product of capital.
The real wage schedule, equation (17), implies the following real marginal wage,

$$w'_{it}(h_{it}) = (1 - \xi)mc_{it}A_t + \xi \frac{h^n_{it}}{u'(c_t)}.$$

Using this to substitute for $w'_{it}(h_{it})$ in equation (10), I can express the real marginal cost in terms of the marginal rate of substitution between consumption and labor,

$$mc_{it} = \frac{h^n_{it}/u'(c_t)}{A_t}.$$ (19)

Using this in equation (18), I finally obtain

$$\frac{\chi}{q(\theta_t)} = \beta E_t \left[ \xi \left( \frac{\eta}{1 + \eta}h^{1+\eta}_{it+1} - b \right) - (1 - \xi)\chi\theta_{t+1} + (1 - \lambda)\frac{\chi}{q(\theta_{t+1})} \right].$$ (20)

According to equation (20), the firm’s incentives to hire are driven by fluctuations in the expected path of hours per employee. Intuitively, if the firm expects hours to be higher in the future, it also expects larger reductions in its wage bill from having additional workers. This leads the firm to post more vacancies today, up to the point in which the expected marginal benefit of hiring equals its marginal cost, $\chi/q(\theta_t)$.10

### 2.3.3 Pricing decision

As is standard in the New Keynesian literature, I use the Calvo (1983) model of staggered price setting. Each period, a randomly selected fraction $\delta$ of firms cannot change their price. Therefore, $\delta$ represents the probability that a firm is not able to change its price in the following

---

10 Notice that, if one were to assume instantaneous hiring instead of time-to-hire (hence replacing equation 9 by $n_{it} = (1 - \lambda)n_{it-1} + q(\theta_t)c_{it}$), the resulting job creation condition would feature current hours, rather than expected hours (as in equation 20). Under the maintained assumption of linear hiring costs, hours per worker would then be equalized across firms, $h_{it} = h_t$ for all $i$. Since marginal costs under instantaneous hiring would still be given by equation (19), the latter would also be equalized across firms, $mc_{it} = mc_t$ for all $i$, thus implying the absence of real rigidities.
At any time \( t \), the part of the firm’s value that depends on its current price is given by

\[
E_0 \sum_{T=t}^{\infty} \delta^{T-t} \beta_{1,T} \left\{ \left( \frac{P_{it}}{P_T} \right)^{1-\gamma} y_T - mc_{i|T|t} \left( \frac{P_{it}}{P_T} \right)^{-\gamma} y_T \right\},
\]

where the subscript \( T|t \) denotes period-\( T \) values conditional on the firm not having reset its price since period \( t \). Therefore, \( mc_{i|T|t} \) is the firm’s real marginal cost in period \( T \) conditional on the price \( P_{it} \) being still in place. When a firm has the chance to reset its price, it chooses \( P_{it} \) so as to maximize (21). The first order condition is given by

\[
E_t \sum_{T=t}^{\infty} \delta^{T-t} \beta_{1,T} P_T^{\gamma} y_T \left( \frac{P_{it}^*}{P_T} - \frac{\gamma}{\gamma - 1} mc_{i|T|t} \right) = 0,
\]

where \( P_{it}^* \) is the pricing decision. Using the expression for real marginal cost, equation (19), and the fact that hours must adjust in order for the firm to meet demand, \( h_{it} = \frac{y_{it}}{A_t n_{it}} \), I can express \( mc_{i|T|t} \) as a function of the firm’s output in period \( T \),

\[
mc_{i|T|t} = \left( \frac{y_{iT|t}}{A_T n_{iT|t}} \right)^{\eta} \frac{1}{u'(c_T) A_T},
\]

where \( y_{iT|t} = (P_{it}^*/P_T)^{-\gamma} y_T \). Equation (23) implies that, under the assumption of convex labor disutility (\( \eta > 0 \)), the firm’s marginal cost curve is an increasing function of its own output level.

### 3 Log-linear equilibrium dynamics

Following standard practice in the New Keynesian literature, I now perform a log-linear approximation of the equilibrium conditions around a zero-inflation steady state. This will allow me to obtain the law of motion of inflation, also known as the ‘New Keynesian Phillips curve’. At this point, I assume the following functional forms for the utility function and the matching
function,

\[ u(c) = \frac{c^{1-\sigma^{-1}}}{1-\sigma^{-1}}, \]

\[ m(v, u) = \zeta v^\epsilon u^{1-\epsilon}, \]

where \( \sigma, \zeta > 0 \) and \( \epsilon \in (0, 1) \). In terms of notation, I will use 'hats' to denote log-deviations of a certain variable from its steady-state value, and 'tildes' to denote log-deviations of that variable from its cross-sectional average.

### 3.1 Relative dynamics of the firm

Log-linearization of the firm’s pricing decision, equation (22), yields

\[ \log P^*_t = (1 - \delta \beta)E_t \sum_{T=t}^{\infty} (\delta \beta)^{T-t} [\widehat{mc}_{iT|t} + \log P_T]. \tag{24} \]

Equation (23) implies that the real marginal cost in period \( T \geq t \) of a firm that has not changed its price since period \( t \) can be expressed as

\[ \widehat{mc}_{iT|t} = \widehat{mc}_T + \eta (\hat{y}_{iT|t} - \hat{y}_T) - \eta \hat{n}_{iT|t}, \tag{25} \]

where

\[ \hat{y}_{iT|t} = \hat{y}_T - \gamma (\log P^*_t - \log P_T) \tag{26} \]

and \( \widehat{mc}_T \) is the average real marginal cost. Notice that a firm’s relative marginal cost is decreasing in its relative stock of workers, \( \hat{n}_{iT|t} \). Having more workers allows the firm to produce a certain amount of output with a smaller number of hours per worker, which reduces the marginal labor disutility of its workers and hence the marginal real wage. I now combine
(24), (25) and (26) to obtain

\[(1 + \eta \gamma) \log P^*_t = (1 - \delta \beta) E_t \sum_{T=t}^{\infty} (\delta \beta)^{T-t} \left[ \hat{\mu} c_T + (1 + \eta \gamma) \log P_T - \eta \hat{n}_{IT|\hat{\tau}} \right]. \tag{27}\]

This expression for a firm’s pricing decision is very similar to the one produced by a standard New Keynesian model. The only difference is the presence of the \(E_t \hat{n}_{IT|\hat{\tau}}\) terms, which reflect the fact that a firm’s marginal cost is decreasing in its stock of workers. These additional terms complicate the analysis in the following way. In order to determine \(\log P^*_t\), we need to compute the expected path of \(\hat{n}_{IT|\hat{\tau}}\). The latter however depends on the firm’s current and future expected vacancy posting decisions, which in turn depend on the price chosen today. Solving for the firm’s pricing decision therefore requires that one considers the effect of a firm’s relative price on the evolution of its relative employment stock.

With this purpose, I now follow Woodford’s (2005) method to solve for the firm’s relative dynamics. I start by noticing that, in a log-linear approximation, the firm’s pricing decision is a linear function of the state of the economy and its individual state, \(\hat{n}_{it}\). On the other hand, since price-setters are randomly chosen, their average employment stock coincides with the economy-wide average employment stock. Therefore, it is plausible to guess that a firm’s pricing decision, relative to the average pricing decision, is proportional to its relative employment stock,

\[\log P^*_t = \log P^*_t - \tau^* \hat{n}_{it}. \tag{28}\]

I now log-linearize the vacancy posting decision, equation (20), and rescale the resulting expression by \(u'(c)y/n\) to obtain

\[\frac{S_v}{\lambda} (1 - \epsilon) \hat{\theta}_t = \beta E_t \left[ \xi \frac{\eta}{\mu} \hat{h}_{it+1} + \left( 1 - \lambda - \frac{1 - \xi}{1 - \epsilon} p(\theta) \right) \frac{S_v}{\lambda} (1 - \epsilon) \hat{\theta}_{t+1} \right], \tag{29}\]

11 See e.g. Walsh (2003, chap. 3).
12 Woodford (2005) develops his method in the context of a model where capital, rather than labor, is specific to each individual firm.
where \( s_e \equiv [x/u'(c)] v/y \) is vacancy posting costs over GDP in the steady state and \( \mu \equiv \gamma/(\gamma-1) \) is the monopolistic mark-up.\(^{13}\) Notice that the only idiosyncratic term in equation (29) is \( E_{t+1} \hat{h}_{it+1} \). The latter depends on \( P_{it} \) (by affecting the firm’s demand in \( t + 1 \) should it not reset its price) as well as on its stock of workers at the beginning of \( t + 1 \). It is now possible to obtain the following result.\(^{14}\)

**Proposition 1** Let relative pricing decisions be given by equation (28), up to a log-linear approximation. Then the relative employment stock of any firm evolves according to

\[
\hat{n}_{it+1} = -\tau^n \left( \log P_{it} - \log P_t \right),
\]

where

\[
\tau^n = \frac{\gamma \delta}{1 - \gamma(1 - \delta)\tau^*}.
\]

Intuitively, firms with a higher price in the current period also expect to have a higher price in the next period, which means that they also expect lower demand. Anticipating this, such firms post a number of vacancies that leaves them with a smaller workforce than the average firm in the following period. Proposition 1 allows me to write

\[
E_t \sum_{T=t}^{\infty} (\delta \beta)^{T-t} \hat{n}_{iT|t} = \hat{n}_{it} + \delta \beta E_t \sum_{T=t}^{\infty} (\delta \beta)^{T-t} \hat{n}_{iT+1|t}
\]

\[
= \hat{n}_{it} - \delta \beta \tau^n E_t \sum_{T=t}^{\infty} (\delta \beta)^{T-t} \left( \log P_{it} - \log P_T \right).
\]

Using (32) in equation (27), I can write the firm’s pricing decision as

\[
(1 + \phi) \log P_{it}^* = (1 - \delta \beta) E_t \sum_{T=t}^{\infty} (\delta \beta)^{T-t} [\hat{m} \hat{c}_T + (1 + \phi) \log P_T] - (1 - \delta \beta) \eta \hat{n}_{it},
\]

\(^{13}\) In the derivation of equation (29), I use the steady-state relations \( h^n = u'(c)m c A \) (equation 19 in the steady-state), \( m c = 1/\mu \) (derived from equation 22) and \( h = y/(An) \). I have also used the fact that, in the steady state, \( q(\theta)v = \lambda \).

\(^{14}\) The proofs of all propositions are in the Appendix.
where $\phi \equiv \eta \gamma - \delta \beta \eta \tau^n$. Averaging (33) across price-setters, and using the fact that the latter are randomly chosen, I obtain

$$(1 + \phi) \log P^*_t = (1 - \delta \beta) E_t \sum_{T=t}^{\infty} (\delta \beta)^{T-t} [\tilde{m} c_T + (1 + \phi) \log P_T]. \quad (34)$$

Subtracting (34) from (33) yields $(1 + \phi)(\log P^*_t - \log P^*_w) = -(1 - \delta \beta)\eta \tilde{n}_t$. This is consistent with my initial guess, equation (28), only if

$$\tau^* = \frac{(1 - \delta \beta)\eta}{1 + \eta \gamma - \delta \beta \eta \tau^n}. \quad (35)$$

Therefore, if relative pricing decisions and relative employment stocks are to have a solution, the latter is given by equations (28) and (30), respectively, where the parameters $\tau^*$ and $\tau^n$ must satisfy equations (31) and (35). The following result establishes that such a solution exists and is unique.

**Proposition 2** The firm’s relative employment stock evolves according to equation (30), where the parameter $\tau^n > 0$ is given by equation (31). A price-setter’s price decision, relative to the average price decision, is given by equation (28), where

$$\tau^* = \frac{-b - \sqrt{b^2 - 4ac}}{2a} > 0,$$

$$a \equiv (1 + \eta \gamma)\gamma(1 - \delta) > 0,$$

$$b \equiv -[1 + \gamma(2 - \delta - \delta \beta)\eta] < 0,$$

$$c \equiv (1 - \delta \beta)\eta > 0.$$
3.2 Real rigidities and inflation dynamics

I am now ready to discuss the presence of real rigidities in this framework and how they affect inflation dynamics. The average pricing decision, equation (34), can be written as

\[(1 + \phi) \log P_t^* = (1 - \delta \beta) [\hat{m}c_t + (1 + \phi) \log P_t] + \delta \beta E_t(1 + \phi) \log P_{t+1}^*.\] \hspace{1cm} (36)

In the Calvo model of staggered price-setting, the law of motion for the price level is given by \(P_t^{1-\gamma} = \delta P_{t-1}^{1-\gamma} + (1 - \delta)(P_t^*)^{1-\gamma}\). The latter admits the following log-linear approximation, \(\log P_t^* - \log P_t = [\delta / (1 - \delta)] \pi_t\), where \(\pi_t \equiv \log(P_t/P_{t-1})\) is the inflation rate. Combining this with (36), I obtain the following New Keynesian Phillips curve,

\[\pi_t = \kappa \hat{m}c_t + \beta E_t \pi_{t+1},\] \hspace{1cm} (37)

where

\[\kappa \equiv \frac{(1 - \delta \beta)(1 - \delta)}{\delta} \frac{1}{1 + \phi},\] \hspace{1cm} (38)

\[\phi \equiv \eta \gamma - \delta \beta \eta \tau^n.\] \hspace{1cm} (39)

The parameter \(\phi\) has two components, \(\eta \gamma\) and \(\delta \beta \eta \tau^n\). The term \(\eta \gamma\) reflects the existence in this framework of strategic complementarities in price-setting, also known as real rigidities after Ball and Romer (1990). This mechanism has the effect of slowing down the adjustment of the overall price level in response to fluctuations in average real marginal costs. To see this, take a price-setter that is considering a reduction in its price. Given the prices of other firms, a reduction in the firm’s nominal price represents also a reduction in its real price. This increases its sales (with elasticity \(\gamma\)) and therefore, given its employment stock, the required amount of hours per worker. This increases the firm’s marginal costs through the increase in workers’ marginal disutility of labor (with elasticity \(\eta\)). The anticipated rise in its current and future
expected marginal costs leads the firm to choose a smaller price cut than the one initially considered. Therefore, the fact that some firms keep their prices unchanged leads price-setters to change theirs by little, hence the 'strategic complementarity' in price-setting. Equivalently, price-setters have an incentive to keep their prices in line with the overall price level, hence the 'real rigidity' in prices. Because all price-setters follow the same logic, the price level and therefore inflation become less sensitive to changes in average real marginal costs.

The term $\delta \beta \eta \tau^n$ reflects the fact that the position of a firm’s marginal cost curve depends on its stock of workers, by affecting how many hours per worker are needed to produce a certain amount of output. This has the effect of accelerating price adjustment. To see this, take the same firm considering a price cut. From Proposition 1, today’s price cut leads the firm to expect a larger relative employment stock and, by equation (25) a lower marginal cost in future periods. Holding everything else constant, this would lead the firm to choose an even larger price cut than initially considered. It is possible to show however that this latter effect is dominated by the real rigidity effect. Using the definition of $\tau^n$, equation (31), I can write

$$\eta \gamma - \delta \beta \eta \tau^n = \eta \gamma - \delta \beta \eta \left( \frac{\gamma \delta}{1 - \gamma (1 - \delta) \tau^*} \right) = \eta \gamma \left( 1 - \frac{\delta^2 \beta}{1 - \gamma (1 - \delta) \tau^*} \right).$$

The latter expression is positive only if the expression in brackets is. The Appendix shows that $\tau^*$ must be smaller than $1/\gamma$ in order for the model to have convergent dynamics. This implies that

$$1 - \frac{\delta^2 \beta}{1 - \gamma (1 - \delta) \tau^*} > 1 - \frac{\delta^2 \beta}{1 - \gamma (1 - \delta) \frac{1}{\gamma}} = 1 - \delta \beta > 0.$$  

It follows that $\phi > 0$. Therefore, the real rigidities in price-setting that arise under search frictions unambiguously flatten the New Keynesian Phillips curve.

It is interesting to note the parallelism between the real rigidity mechanism just explained
and the one arising in models of firm-specific capital (Sveen and Weinke, 2005; Woodford, 2005; and Altig et al., 2004). In the present framework, the firm’s employment stock plays an analogous role to the capital stock in models of firm-specific capital, namely that of being a firm-specific endogenous state variable. In the latter class of models, firms are able to adjust production in the short run by varying their labor input; here, they adjust production by varying the number of hours worked by their employees. In both cases, real rigidities arise because the firm’s marginal cost curve becomes upward-sloping. In the case of firm-specific capital, this is due to decreasing marginal returns to labor for a given capital stock, although such an effect may be reinforced by the existence of industry-specific labor markets (see e.g. Woodford, 2005). Here, it is due to increasing marginal disutility of labor and hence increasing marginal wages.\textsuperscript{15}

### 3.3 Aggregate equilibrium

Equilibrium in the search model with real rigidities is characterized by the AR(1) processes for exogenous money growth and labor productivity, together with the following six equations,

\[
\pi_t = \kappa \hat{m} c_t + \beta E_t \pi_{t+1}, \tag{E1}
\]

\[
\hat{m} c_t = \eta \hat{h}_t + \sigma^{-1} \tilde{y}_t - \alpha z_t, \tag{E2}
\]

\textsuperscript{15}It is interesting to compare the slope of the NKPC in this framework with the one that arises in a model of firm-specific capital. As shown by Woodford (2005), the slope coefficient in such a model can be expressed again as in equation (38), with the parameter $\phi$ replaced by

\[
\phi_{fs} \equiv \left( \frac{\eta}{\alpha} + \frac{1 - \alpha}{\alpha} \right) \gamma - \frac{\delta \beta}{1 - \delta \beta \lambda} \left( \frac{\eta}{\alpha} + \frac{1 - \alpha}{\alpha} - \eta \right) \tilde{\tau},
\]

where $1 - \alpha$ is the capital elasticity in a Cobb-Douglas production function, and both $\tilde{\tau}$ and $\tilde{\lambda}$ are functions of structural parameters. The interpretation of $\phi_{fs}$ is analogous to that of $\phi$: the first component, $(\eta/\alpha + (1 - \alpha)/\alpha) \gamma$, captures the real rigidity effect, where decreasing returns to labor ($\alpha < 1$) add to convex labor disutility ($\eta > 0$) as a source of strategic complementarities. The second term captures the fact that the position of the marginal cost curve depends on the capital stock. While a comparative analysis of firm-specific capital and search frictions as sources of real rigidities would be an interesting exploration, such a comparison is however beyond the scope of this paper.
\[ \hat{y}_t = \hat{y}_{t-1} + \xi_t - \pi_t, \quad (E3) \]
\[ \frac{s_v}{\lambda} (1 - \epsilon) \hat{\theta}_t = \beta E_t \left[ \xi \eta h_{t+1} + \left( 1 - \lambda \frac{1 - \xi}{1 - \epsilon} p(\theta) \right) \frac{s_v}{\lambda} (1 - \epsilon) \hat{\theta}_{t+1} \right], \quad (E4) \]
\[ \hat{k}_t = \hat{y}_t - a_t - \hat{n}_t \quad (E5) \]
\[ \hat{n}_{t+1} = (1 - \lambda - p(\theta)) \hat{n}_t + \lambda \epsilon \hat{\theta}_t. \quad (E6) \]

Equation (E2) is obtained by log-linearizing (19), averaging across all firms and using the fact that \( \hat{c}_t = \hat{y}_t \). Equation (E3) is obtained by log-linearizing (6), taking first differences and using again \( \hat{c}_t = \hat{y}_t \). Equation (E4) is obtained by averaging equation (29) across firms.\(^{16}\) Equation (E5) is the log-linear version of the firm’s production function, after averaging across firms. Finally, equation (E6) is the log-linear approximation of equation (3), where I also use the steady-state condition \( \lambda n = p(\theta)(1 - n) \). This log-linear representation allows to understand easily the effect of shocks on the economy. In response to a positive monetary shock (an increase in \( \xi_t \) in equation E3), aggregate demand increases, which puts upward pressure on real marginal costs and inflation. To the extent that the increase in demand is persistent, firms anticipate longer hours per employee in the future. In order to avoid large pay rises for existing employees, firms post more vacancies. This results in a tightening of the labor market (equation E4) and an increase in total employment (equation E6). In response to a positive productivity shock (an increase in \( a_t \)), real marginal costs fall and so does inflation. For a constant level of nominal GDP, the fall in prices produces an expansion in aggregate demand (equation E3). The effect on employment is however ambiguous. If the expansion in output is strong enough relative to the increase in \( a_t \), firms expect hours per employee to be higher, which leads them to post more vacancies and thus increase employment. If the increase in output is weak enough, the opposite will be true.

\(^{16}\) As shown in the Proof of Proposition 1, in the Appendix, \( E_t \hat{h}_{t+1} \) can be written as \( E_t \hat{h}_{t+1} = \gamma \hat{P}_t - [1 - \gamma (1 - \delta) \tau^+ \hat{n}_{t+1} ] \), which averages to \( E_t \hat{h}_{t+1} \).
3.3.1 Comparison to a search model with a producer-retailer structure

Most of the literature on New Keynesian models with search and matching frictions separates vacancy-posting and pricing decisions by assuming a producer-retailer structure, in which the former are subject to search frictions and the latter to staggered price-setting. While simplifying the analysis, this assumption eliminates from the outset the possibility of analyzing price-setting decisions in an environment in which price-setters cannot adjust employment costlessly and instantaneously. In such models, producers produce a homogenous intermediate good that is sold to retailers at a perfectly competitive real price. We may denote the latter by $mc_t$. Each retailer then transforms the intermediate good into a differentiated final good using a linear technology. Therefore, $mc_t$ represents the real marginal cost common to all retailers.

It is relatively straightforward to construct a model with this kind of producer-retailer structure that is otherwise equivalent to the model presented in section 2. In particular, I may assume that household preferences and the production function of producers are the same as in the model with real rigidities. The Appendix derives the equilibrium conditions in such a model. Once the producer-retailer model is log-linearized, the New Keynesian Phillips curve is given by

$$\pi_t = \kappa_{pr} \hat{mc}_t + \beta E_t \pi_{t+1},$$

where

$$\kappa_{pr} \equiv \frac{(1 - \delta \beta)(1 - \delta)}{\delta} > \kappa.$$ 

Therefore, the New Keynesian Phillips curve in the model with a producer-retailer structure is steeper than in the model with real rigidities. Because retailers can buy as much intermediate input as they need at the perfectly competitive price $mc_t$, their pricing decisions have no effect on their own marginal costs. As a result, the real rigidity effect disappears.

As shown in the Appendix, the remaining log-linear equilibrium conditions in the producer-retailer model are exactly the same as in the model with real rigidities, equations (E2) to (E6).
The producer-retailer model thus serves as a 'control' that allows me to isolate the effect of real rigidities in models with search frictions and staggered price-setting. One important dimension of this comparison is the difference in inflation dynamics between both models. Real rigidities have the property of slowing down the adjustment of the price level in response to different shocks, for a given degree of nominal price rigidity ($\delta$). As a result, the inflation response to shocks will be smaller on impact and more persistent in the model with real rigidities. This effect is shared in general by models that incorporate a real-rigidity mechanism, such as the New Keynesian model with firm-specific capital introduced by Sveen and Weinke (2005) and Woodford (2005), and further analyzed by Altig et al. (2004).17

More importantly, this comparison allows to measure the extent to which real rigidities affect the model’s behavior along those labor market dimensions that the standard New Keynesian model is not designed to address, such as unemployment, employment and hours per employee. Take for instance a positive money growth shock. *Ceteris paribus*, the slower response of the price level in the presence of real rigidities leads to a larger increase in aggregate demand. To the extent that the economic expansion is expected to persist, firms expect larger increases in hours per employee. Since the latter are the driving force of job creation, real rigidities generate a larger rise in job creation and therefore a larger drop in unemployment. The next section will show that real rigidities also amplify the unemployment response to productivity shocks. Therefore, real rigidities amplify the unconditional fluctuations in employment and unemployment. Furthermore, because employment relationships have a long-term nature in this framework, firms base their hiring decisions on the entire expected path of hours per employee, such that a small increase in the latter’s volatility is enough to generate a large increase in employment volatility. Therefore, real rigidities also have the property of increasing the volatility of employment relative to that of hours per employee. The same effect implies that

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17 As argued by Woodford (2005) and Altig et al. (2004), estimates of New Keynesian models (or New Keynesian Phillips curves) that abstract from real rigidities imply average durations of nominal price contracts that are too long when compared with actual micro data. These authors propose firm-specific capital, and the resulting real rigidities, as a way for econometric estimates to imply more realistic durations of price contracts.
the amplification of (un)employment fluctuations takes place not just in absolute terms, but also relative to those in output. I now turn to the quantitative assessment of these mechanisms.

4 Quantitative analysis

4.1 Calibration

Following most of the literature on search and matching models, I calibrate the model to monthly US data. As in most of the RBC literature, I set the discount factor to 4% per quarter, or $\beta = 0.99^{1/3}$. I also choose a standard value for the intertemporal elasticity of substitution, $\sigma = 1$. It is customary to set $1/\eta$ on the basis of micro estimates of labor supply elasticities. According to Card’s (1994) review of the empirical literature, this elasticity is surely no higher than 0.5. I therefore set $\eta$ to 2 as a conservative choice.\(^{18}\)

Regarding the New Keynesian side of the model, following the evidence in Bils and Klenow (2004) I assume that firms change prices every 1.5 quarters, or 4.5 months, which implies $\delta = (4.5 - 1)/4.5 = 0.78$. As in Woodford (2005), I choose a monopolistic mark-up of $\mu = 1.15$, which implies an elasticity of substitution among differentiated goods of $\gamma = \mu / (1 - \mu) = 7.67$. Given the values of $\beta$, $\eta$, $\gamma$ and $\delta$, the parameters governing relative firm dynamics are given by $\tau^* = 0.08$ and $\tau^n = 6.90$. From (39), the parameter measuring (net) real rigidity equals $\phi = 4.63$. From (38), the slope of the New Keynesian Phillips curve equals $\kappa = 0.011$. This compares with a slope of $\kappa_{rr} = 0.064$ in the producer-retailer model.

The parameters that describe the labor market are calibrated as follows. Following Shimer (2005), I set the monthly separation probability, $\lambda$, to 0.035. Shimer (2008) and Pissarides (2009) calculate an average job-finding probability and an average vacancies-to-unemployment ratio of 0.286 and 0.72, respectively, for the US. I therefore target the steady-state values

\(^{18}\)Notice that a lower labor supply elasticity (that is, a higher $\eta$) would actually strengthen the real rigidity effect, by reducing further the slope of the inflation equation. It is in this sense that $\eta = 2$ is chosen as a conservative calibration.
\( p(\theta) = 0.30 \) and \( \theta = 0.72 \). The elasticity of the matching function with respect to vacancies, \( \epsilon \), is set to 0.6, following the evidence in Blanchard and Diamond (1989). This, together with \( p(\theta) = \zeta \theta^c \) and the targets for \( p(\theta) \) and \( \theta \), imply \( \zeta = 0.365 \). Following standard practice in the literature, I set the bargaining power parameter, \( \xi \), equal to \( \epsilon \). Notice however that \( \xi \) does not affect the slope of the New Keynesian Phillips curve. Finally, following Andolfatto (1996), Gertler and Trigari (2009) and Blanchard and Galí (2010), I target a steady-state ratio of vacancy-posting costs to GDP, \( s_v \), of 1 per cent. With log utility, this requires setting the utility cost of posting a vacancy to \( \chi = 0.133 \).\(^{19}\) One can then use the steady-state job creation condition to solve for the fixed component of labor disutility, obtaining \( b = 0.563 \).\(^{20}\)

The shock parameters are calibrated as follows. Following Shimer (2005), the parameters of the labor productivity process are calibrated in a model-consistent way. Aggregating equation (8) across firms, we have that \( A_t = y_t \Delta_t / (n_t h_t) \), where \( h_t = n_t^{-1} \int n_t h_t \, dt \) are average hours per employee and \( \Delta_t = \int (P_t / P_t)^{-\gamma} \, dt \) is a price-dispersion term that experiences second-order fluctuations around one (Woodford, 2003, Ch. 6). We thus have \( \log A_t = \log y_t - \log (n_t h_t) \), up to a first-order approximation. Using BLS data for real output \( (y_t) \) and total hours \( (n_t h_t) \) to construct a quarterly series for log labor productivity, I obtain an autocorrelation coefficient and a standard deviation for the corresponding monthly process of \( \rho_a = 0.86 \) and \( \sigma_a = 0.56\% \), respectively.\(^{21}\)

As in Krause and Lubik (2007), I set the autocorrelation coefficient of the money growth process to 0.49 on a quarterly basis, or \( \rho_m = 0.49^{1/3} \) on a monthly basis. Finally, the

\(^{19}\)From the definition of \( s_v \), we have that \( \chi = s_v y u'(c) / y = s_v / v = s_v / (\theta u) \), where I have used \( u'(c) = 1 / c \), \( c = y \), and \( v = \theta u \). The values of \( p(\theta) \) and \( \lambda \) imply a steady-state unemployment rate of \( u = \lambda / (\lambda + p(\theta)) = 0.1045 \), with together with \( s_v = 0.01 \) and \( \theta = 0.72 \) implies \( \chi = 0.133 \).

\(^{20}\)In using equation (20) in the steady state to solve for \( b \), I also use the fact that \( (\gamma - 1) / \gamma = mc = h^\gamma y / A = h^{1+n} n \), which implies \( h = [(\gamma - 1) / (\gamma n)]^{1/(1+n)} \), where \( n = 1 - u = 0.8955 \).

Notice that there are no unemployment benefits in the model (consumers are perfectly insured as a result of the large household assumption). However, the steady-state ratio between the real value of total labor disutility, \( [b + h^{1+n} / (1 + \eta)] / u'(c) \), and worker output, \( y / n \), can be interpreted as an (endogenous) counterpart to the unemployment payoff parameter in the standard search and matching model. Under my calibration, the latter ratio equals 0.794, which is in between the values of the above-mentioned parameter used by Shimer (0.40) and Hagedorn-Manovskii (0.955), and similar to the value 0.71 used by Hall and Milgrom (2008) and Pissarides (2009).

\(^{21}\)See section 4.3 for details about the data sources, the sample period and the detrending procedure.
standard deviation of money growth shocks, $\sigma_m$, is calibrated to match the standard deviation of real output in the data.

Table 1. Parameter values

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99$^{1/3}$</td>
<td>discount factor</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1</td>
<td>intertemporal elast. of subs.</td>
</tr>
<tr>
<td>$\eta$</td>
<td>2</td>
<td>convexity of labor disutility</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.78</td>
<td>fraction of sticky prices</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>7.67</td>
<td>elasticity of demand curves</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.035</td>
<td>job separation rate</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.365</td>
<td>scale of matching fct.</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>0.6</td>
<td>elasticity of matching fct.</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.6</td>
<td>firm’s bargaining power</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.133</td>
<td>vacancy posting cost</td>
</tr>
<tr>
<td>$b$</td>
<td>0.56</td>
<td>labor disutility parameter</td>
</tr>
<tr>
<td>$\rho_u$</td>
<td>0.86</td>
<td>AC of productivity shock</td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>0.56%</td>
<td>SD of productivity shock</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>0.49$^{1/3}$</td>
<td>AC of monetary shock</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>0.47%</td>
<td>SD of monetary shock</td>
</tr>
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Reduced-form parameters

<table>
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<tr>
<th>Value</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\phi$</td>
<td>4.63 net real rigidity</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.011 slope of NKPC, model with real rigidities</td>
</tr>
<tr>
<td>$\kappa_{pr}$</td>
<td>0.064 slope of NKPC, producer-retailer model (no real rigidities)</td>
</tr>
<tr>
<td>$s_v$</td>
<td>0.01 vacancy posting costs/GDP</td>
</tr>
</tbody>
</table>
4.2 Impulse responses

In order to illustrate graphically the effects of real rigidities on equilibrium dynamics, I now compare the economy’s response to shocks in the model with and without real rigidities.

4.2.1 Monetary shocks

Figure 1 displays the impulse-responses of prices, inflation, output and both labor margins to a one-standard-deviation positive shock to money growth. The real rigidity mechanism slows down the adjustment of the price level. This is reflected in an inflation response that is both smaller on impact and more persistent afterwards. Given the exogenous expansion in nominal GDP, the more sluggish price adjustment leads to a larger expansion in output in the model with real rigidities. As a result, the dynamic path of hours per employee experiences a larger increase. Since hiring incentives are driven by fluctuations in expected hours per employee, job creation increases more strongly under real rigidities, which leads to a larger employment expansion.
4.2.2 Productivity shocks

Figure 2 displays the economy’s response to a one-standard-deviation positive shock to labor productivity. Again, the inflation response is more muted and more persistent under real rigidities. Since nominal GDP remains unchanged in this simulation, the extra sluggishness in the price level leads to a weaker expansion in aggregate demand. In both models, the expansion in aggregate demand is not strong enough to compensate for the fact that firms now need less labor to produce the same output. As a result, total hours worked fall, the adjustment being shared by both labor margins (hours per employee and employment). However, the weaker output increase under real rigidities produces a stronger fall in total hours. Notice that the drop in the dynamic path of hours per employee is just slightly larger when real rigidities are present. However, because of the long-term nature of jobs in this framework, such a drop is enough to generate a substantially larger drop in job creation and therefore in employment.

---

22 Such an effect of productivity shocks on total hours is consistent with a large body of empirical evidence starting with Galí (1999). It is also consistent with the predictions of estimated medium-scale DSGE models of the US economy, such as Smets and Wouters’ (2007).
4.3 Labor market volatility

I now analyze the extent to which real rigidities affect the empirical performance of the New Keynesian model with search and matching frictions regarding labor market aggregates. The second column of Table 2 displays a set of business cycle statistics in the United States. I use seasonally-adjusted data on real output in the nonfarm business sector ($y_t$, in model notation), hours of all persons in the nonfarm business sector ($n_t h_t$, where $h_t \equiv n_t^{-1} \int_0^1 n_{id} h_{id} di$ are average hours per employee), total nonfarm payroll employment ($n_t$), number of unemployed ($u_t$), average hourly earnings in the private sector ($w^h_t \equiv (n_t h_t)^{-1} \int_0^1 n_{id} w_{it}(h_{it}) di$) and quarter-on-quarter inflation in the Consumer Price Index ($\pi^q_t \equiv \log P_t - \log P_{t-3}$), all of them from the Bureau of Labor Statistics; I also use the Conference Board’s Help-Wanted Advertising Index as a proxy for vacancies ($v_t$). The sample runs from 1964:Q1 to 2008:Q2. Since output and total hours are only available quarterly, and the data on employment, unemployment, inflation, hourly wages and vacancies are monthly, I take quarterly averages of the latter five series. I then use the identities $h_t = (n_t h_t)/n_t$ and $\theta_t = v_t/u_t$ in order to obtain quarterly series for average hours per employee and labor market tightness, respectively. All data except inflation are logged and HP-filtered with a conventional smoothing parameter (1600). The remaining columns of table 2 display the corresponding statistics generated by the model with and without real rigidities.23

---

23 Model moments are calculated as follows. I simulate 624 months of artificial data. I take quarterly averages and discard the first 30 observations so as to eliminate the effect of initial conditions, which leaves me with 178 observations (the sample size). I then calculate the relevant second moments. I repeat this operation 200 times and finally take averages for each vector of moments.
Two main messages can be extracted from Table 2. First, real rigidities amplify the size of fluctuations in employment and unemployment, both in absolute terms and relative to those of output.\(^{24}\) For instance, the relative unconditional standard deviation of unemployment is 95\% of its data counterpart, compared to 63\% in the model without real rigidities. Similarly, relative employment volatility is 94\% of the corresponding value in the data, which contrasts with 63\% in the absence of real rigidities. As can be seen in the fifth row, the effect of real rigidities operates through the amplification of fluctuations in labor market tightness, which is the single driving force of (un)employment fluctuations. Both models however overstate the relative standard deviation of hours per employee by a factor of about one and a half. As a result, the relative standard deviation of total hours is somewhat higher in the model with real rigidities than in the data.

Table 2. Business cycle statistics

<table>
<thead>
<tr>
<th></th>
<th>model with real rigidities</th>
<th>model without real rigidities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>US data</td>
<td>monetary</td>
</tr>
<tr>
<td>(\sigma(y_t), %)</td>
<td>2.03</td>
<td>2.00</td>
</tr>
<tr>
<td>(\sigma(u_t), %)</td>
<td>10.70</td>
<td>9.90</td>
</tr>
<tr>
<td>(\sigma(u_t)/\sigma(y_t))</td>
<td>5.28</td>
<td>4.95</td>
</tr>
<tr>
<td>(\sigma(v_t)/\sigma(y_t))</td>
<td>6.52</td>
<td>5.86</td>
</tr>
<tr>
<td>(\sigma(\theta_t)/\sigma(y_t))</td>
<td>11.59</td>
<td>9.88</td>
</tr>
<tr>
<td>(\sigma(n_t)/\sigma(y_t))</td>
<td>0.62</td>
<td>0.58</td>
</tr>
<tr>
<td>(\sigma(h_t)/\sigma(y_t))</td>
<td>0.35</td>
<td>0.44</td>
</tr>
<tr>
<td>(\sigma(n_t h_t)/\sigma(y_t))</td>
<td>0.86</td>
<td>1.00</td>
</tr>
<tr>
<td>(\sigma(h_t)/\sigma(n_t))</td>
<td>0.57</td>
<td>0.76</td>
</tr>
<tr>
<td>(\rho(w_t h_t, y_t))</td>
<td>0.54</td>
<td>0.99</td>
</tr>
<tr>
<td>(\sigma(\pi_t^2), %)</td>
<td>0.74</td>
<td>1.45</td>
</tr>
</tbody>
</table>

\(^{24}\) Notice that amplification holds unconditionally, as well as conditional on each type of shock (to money growth and productivity), reflecting our earlier analysis of impulse responses. Most of the amplification in absolute volatility, \(\sigma(u)\), is due to monetary shocks, whereas most of the amplification in relative volatility, \(\sigma(u)/\sigma(y)\), is due to productivity shocks.
Second, real rigidities increase the volatility of employment relative to that of hours per employee. According to the model without real rigidities, the intensive margin of labor is about one third more volatile than the extensive margin. This is clearly at odds with the data, according to which the intensive margin is roughly half as volatile as the extensive one. By amplifying employment fluctuations substantially, real rigidities manage to make employment more volatile than hours per employee, even though the ratio of standard deviations is still 30 percentage points higher than in the data.25

Finally, it is interesting to discuss the implications of real rigidity for the cyclical behavior of inflation and real wages. As shown by the last line of the table, both models overpredict the volatility of quarter-on-quarter inflation. This is not surprising however, given the model’s simplicity and the fact that the shock processes have been calibrated to match the cyclical volatility of labor productivity and output.26 The relevant insight is that, for a given frequency of price adjustment calibrated to match micro data, real rigidities work towards reducing inflation volatility and hence bring it closer to the data. The flip-side of this coin is that, if one were to estimate both models with macro data, the average duration of price contracts implied by the real-rigidity specification would be shorter and hence closer to the micro data.27 Similarly, both models overpredict the cyclicality of average real wages, as measured by their contemporaneous correlation with output. This is of course a consequence of wages being perfectly

25While the paper’s focus is to understand the effects of search frictions and the resulting real rigidities on equilibrium dynamics, it is also interesting to ask how other model features (such as two labor margins, monopolistic competition and sticky prices, and monetary shocks in addition to productivity shocks) contribute to (un)employment volatility. First, it can be showed that, in a stripped down version of the model with flexible prices and without variations in hours per worker, unemployment is invariant to productivity shocks. This neutrality result is analogous to the one found by Blanchard and Galí (2010) and further analyzed by Shimer (2010). Second, the latter result continues to hold after introducing variations in the intensive margin. Third, introducing monopolistic competition and sticky prices (while abstracting from real rigidities and monetary shocks) produces a standard deviation of unemployment of 0.93%, as shown in the second-to-last column of Table 2. Fourth, adding monetary shocks raises unemployment volatility to 3.14% (last column). Finally, adding real rigidities increases unemployment volatility to 10.25%. It is therefore clear from this analysis that real rigidities are the most important factor contributing to unemployment fluctuations.

26Another way to interpret these results is that the data seems to be calling for an even longer average duration of price contracts than the one assumed in the baseline calibration (1.5 quarters).

27See Altig et al. (2004) and Woodford (2005) for early discussions on the effects of real rigidities on the inference of price adjustment frequencies.
flexible in the model. While introducing some form of wage rigidity would improve the model’s performance along this dimension, the assumption of wage flexibility allows me to isolate the effects of real rigidities in the absence of any other amplification mechanisms.

4.3.1 Robustness: the Great Moderation period
As shown by a large number of studies, the United States seems to have experience a significant break in the size of its aggregate fluctuations starting approximately in 1984.\textsuperscript{28} To the extent that such a break has affected the size of fluctuations in labor market aggregates relative to those of output, the baseline results shown in Table 2 may be affected. For this reason, I now calculate the same set of statistics as in Table 2 using the sample period 1984:Q1-2008:Q2, and compare them with the corresponding statistics generated by the models. The results are displayed in Table 3.\textsuperscript{29}


<table>
<thead>
<tr>
<th></th>
<th>model with real rigidities</th>
<th>model without real rigidities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>US data</td>
<td>monetary</td>
</tr>
<tr>
<td>$\sigma(y_t),%$</td>
<td>1.13</td>
<td>1.10</td>
</tr>
<tr>
<td>$\sigma(u_t),%$</td>
<td>8.07</td>
<td>5.45</td>
</tr>
<tr>
<td>$\sigma(u_t)/\sigma(y_t)$</td>
<td>7.12</td>
<td>4.95</td>
</tr>
<tr>
<td>$\sigma(v_t)/\sigma(y_t)$</td>
<td>9.18</td>
<td>5.90</td>
</tr>
<tr>
<td>$\sigma(\theta_t)/\sigma(y_t)$</td>
<td>15.79</td>
<td>9.90</td>
</tr>
<tr>
<td>$\sigma(n_t)/\sigma(y_t)$</td>
<td>0.78</td>
<td>0.58</td>
</tr>
<tr>
<td>$\sigma(h_t)/\sigma(y_t)$</td>
<td>0.56</td>
<td>0.44</td>
</tr>
<tr>
<td>$\sigma(n_t h_t)/\sigma(y_t)$</td>
<td>1.22</td>
<td>1.00</td>
</tr>
<tr>
<td>$\sigma(h_t)/\sigma(n_t)$</td>
<td>0.71</td>
<td>0.76</td>
</tr>
<tr>
<td>$\rho(w_t, y_t)$</td>
<td>0.11</td>
<td>0.99</td>
</tr>
<tr>
<td>$\sigma(\pi_t^f)$</td>
<td>0.38</td>
<td>0.79</td>
</tr>
</tbody>
</table>

\textsuperscript{28}See e.g. Kim and Nelson (1999) and McConnell and Pérez-Quirós (2000).

\textsuperscript{29}The quarterly series of labor productivity since 1984 now implies an autocorrelation coefficient and standard deviation of the corresponding monthly process of $\rho_a = 0.84$ and $\sigma_a = 0.45$, respectively. The standard deviation of the shock to money growth is recalibrated to match again the standard deviation of output (1.13% in 1984-2008), yielding $\sigma_m = 0.26\%$. 
A quick comparison between Tables 2 and 3 reveals that the labor market has actually become more volatile in relative terms in the Great Moderation period. That is, the decline in output volatility that has been extensively documented seems not to have been accompanied by a proportional decline in the volatility of the labor market. As a result, both models now find it harder to match the observed labor market volatility. However, the model with real rigidities again performs much better in this regard. For instance, the relative standard deviation of unemployment is now 71% of that in the data, compared to 41% when real rigidities are abstracted from. The corresponding figures for the relative standard deviation of employment are 76% and 44%, respectively. This allows the model with real rigidities to come closer to the data also in terms of the volatility of total hours.

Also, the intensive margin of labor seems to have become more volatile relative to the extensive margin in the Great Moderation period, although the former is still less than three quarters as volatile as the latter. Both models actually hit the target for the relative standard deviation of hours per employee. However, the low volatility of employment in the model without real rigidities leads to the implausible prediction that hours per employee are about two thirds more volatile than employment. In contrast, the model with real rigidities predicts slightly smaller fluctuations in the intensive margin than in the extensive one.

4.3.2 Robustness: alternative demand specifications

The analysis so far has assumed a simple cash-in-advance specification for aggregate demand, in order to simplify the explanation of the main mechanisms at play. I now check the robustness of the baseline results to alternative specifications for aggregate demand. First, I consider a "money in the utility function" (MIU) specification, in order to introduce an interest-sensitive money demand equation. Following Krause and Lubik (2007), I assume that households enjoy a utility flow \( \chi \log (M_t/P_t) \) from their holdings of real money balances, \( M_t/P_t = m_t \). The first order condition for money, \( M_t \), can be expressed as \( \chi/m_t = u'(c_t) (R_t - 1)/R_t \), which together
with the standard consumption Euler equation, \( u'(c_t)/P_t = \beta R_t E_t u'(c_{t+1})/P_{t+1} \), represent the
new aggregate-demand block (replacing equation 6). Log-linearizing the latter two equations
and the identity \( m_t/m_{t-1} = (M_t P_{t-1})/(P_t M_{t-1}) = \exp(\zeta_t)/\pi_t \), and using \( \dot{c}_t = \dot{y}_t \), we obtain

\[
\hat{m}_t = \sigma^{-1}\dot{y}_t - \frac{\beta}{1-\beta}\hat{R}_t,
\]

\[
\hat{y}_t = E_t\hat{y}_{t+1} - \sigma \left( \hat{R}_t - E_t\pi_{t+1} \right),
\]

\[
\hat{m}_t = \hat{m}_{t-1} + \zeta_t - \hat{\pi}_t,
\]

respectively, which replace equation (E3) in the log-linear representation of the model. The
third and fourth columns of Table 4 report unconditional second moments in the MIU speci-
cation, both for the case with and without the real rigidity mechanism. Notice first that, relative
to the results from the baseline CIA specification in Table 2, the MIU specification generates
less labor market volatility. This reflects the fact that part of the variation in real money
balances is now absorbed by nominal interest rates and not just by output, hence dampening
the effects on job creation and (un)employment. However, I find again that the version with
real rigidities performs better in terms of labor market volatility.\(^{30}\) Relative unemployment
volatility in the model equals 76% of the empirical measure, compared to 44% in the model
without real rigidities; the volatility of vacancies, labor market tightness and employment are
also better captured when real rigidities are at work. Both models now counterfactually predict
larger fluctuations in employment than in hours per employee, although the model with real
rigidities again performs better in this respect.

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\(^{30}\)Real rigidities amplify employment and unemployment fluctuations also conditional on each type of shock
(monetary and productivity shocks). Conditional results are available upon request.
Second, following most of the recent New Keynesian literature I consider a cashless environment in which the central bank directly sets the nominal interest rate. In particular, I assume that the monetary authority follows a standard Taylor rule,

\[
\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) \left( \phi_x \pi_t + \phi_y \Delta \tilde{y}_t \right) + \varepsilon_t^R, \tag{42}
\]

where \( \rho_R, \phi_x \) and \( \phi_y \) capture the degree of interest-rate smoothing and the response to inflation and output growth, respectively, and \( \varepsilon_t^R \) is an iid shock that replaces the monetary shock \( \varepsilon_t^m \). Equation (42), together with (41), replace now equation (E3) as the aggregate-demand block of
the log-linearized model. The last two columns of Table 4 report unconditional second moments generated by the Taylor rule specification with and without real rigidities. Overall, this specification produces less labor market volatility than the MIU or CIA specifications. Following for instance a shock to the policy rate, the systematic component of the Taylor rule moves in the opposite direction, hence dampening the effects of the exogenous policy impulse on the economy. Once again, however, real rigidities act towards increasing labor market volatility and hence improve the model’s performance. For instance, the relative volatilities of unemployment and employment are both 61% of their respective empirical counterparts, compared to 34% in the absence of real rigidities. Indeed, the weaker inflation response under real rigidities dampens the fluctuations in the systematic component of the policy rule. Following e.g. a negative shock to the policy rate, the actual rate falls by more, hence amplifying the expansion in output and employment. Following a positive productivity shock, the policy rate falls by less and output increases by less; as a result, labor demand falls by more.

5 Conclusion

This paper has studied the effect of search and matching frictions in the labor market on firms’ pricing decisions, in a model where price-setters are actually subject to such frictions. In doing so, it departs from most of the literature on New Keynesian models with search and matching frictions, which separates the firms making the pricing decisions from the firms that face search frictions. The framework presented here therefore helps understand how price decisions are made in a context in which firms cannot costlessly and immediately adjust employment.

The main theoretical result is that search frictions give rise to real rigidities (or ’strategic complementarities’) in price-setting. This mechanism leads each individual price-setter to make

\footnote{For the purpose of this exercise, I set the policy rule coefficients to standard values: \( \rho_R = 0.8, \phi_x = 1.5 \) and \( \phi_y = 0.5 \). The standard deviation of the interest rate shock, \( \sigma(R_t) = 0.37\% \), is chosen to match the standard deviation of output in the data. Results conditional on each shock type (monetary and productivity) are available upon request.}
smaller price changes in response to the same macroeconomic fluctuations. On the aggregate, real rigidities slow the adjustment of the overall price level. This is reflected in a smaller sensitivity of inflation to average real marginal costs, that is, in a flatter New Keynesian Phillips curve. The increased sluggishness in the price level makes inflation less volatile and more persistent for a given average frequency of price adjustment, a feature that is common to other real-rigidity mechanisms such as firm-specific capital. More importantly, real rigidities improve the model’s performance along those labor market dimensions that the standard New Keynesian model is not designed to address. In particular, real rigidities bring both the size of unemployment fluctuations and the relative volatility of the two labor margins closer to the data. The corollary is that having firms make both hiring and pricing decisions within the popular New Keynesian search-and-matching model does not merely represent an increase in its realism, but can also help it match those labor market facts that constitute its raison d’être.
6 Appendix

6.1 Proof of Proposition 1

From equation (29) in the text, I can write the firm’s vacancy posting decision as

$$\frac{s_w}{\lambda}(1 - \epsilon)\hat{\theta}_t = \beta E_t \left[ \frac{\eta}{\mu} \left( \hat{h}_{it+1} + \hat{h}_{it+1} \right) + \left( 1 - \lambda - \frac{1 - \xi}{1 - \epsilon} p(\theta) \right) \frac{s_w}{\lambda}(1 - \epsilon)\hat{\theta}_{t+1} \right], \quad (A1)$$

where $\hat{h}_{it+1} = \hat{h}_{it+1} - \hat{h}_{it+1}$ is the firm’s relative number of hours per worker. Hours per worker admit the exact log-linear representation $\hat{h}_{it} = \tilde{y}_{it}^d - \tilde{n}_{it}$. Therefore, I can write $\hat{h}_{it} = \tilde{y}_{it}^d - \tilde{n}_{it}$. This becomes

$$\hat{h}_{it} = -\gamma \tilde{P}_{it} - \tilde{n}_{it} \quad (A2)$$

once I use the fact that $\tilde{y}_{it}^d = -\gamma \tilde{P}_{it}$. The firm’s expected relative price is given by

$$E_t \tilde{P}_{it+1} = \delta E_t \left( \log P_{it} - \log P_{t+1} \right) + (1 - \delta)E_t \left( \log P_{it+1}^* - \log P_{t+1} \right)$$

$$= \delta E_t \left( \tilde{P}_{it} - \pi_{t+1} \right) + (1 - \delta)E_t \left( \log P_{it+1}^* - \log P_{t+1}^* + \frac{\delta}{1 - \delta} \pi_{t+1} \right)$$

$$= \delta \tilde{P}_{it} - (1 - \delta)\tau^* \tilde{n}_{it+1}. \quad (A3)$$

In the second equality I have used the fact that $\log P_{t+1}^* - \log P_{t+1} = [\delta/(1 - \delta)] \pi_{t+1}$, where $\pi_{t+1} \equiv \log(P_{t+1}/P_t)$ is the inflation rate. In the third equality I have used $\log P_{it+1}^* - \log P_{t+1}^* = -\tau^* \tilde{n}_{it+1}$. Using (A2) and (A3), expected relative hours are given by

$$E_t \tilde{h}_{it+1} = -\gamma E_t \tilde{P}_{it+1} - \tilde{n}_{it+1}$$

$$= -\gamma \delta \tilde{P}_{it} - [1 - \gamma(1 - \delta)\tau^*] \tilde{n}_{it+1}. \quad (A4)$$
This implies that $E_t \tilde{h}_{it+1}$ averages to zero. Averaging (A1) across all firms and subtracting the resulting expression from (A1) yields $E_t \tilde{h}_{it+1} = 0$. Combining this with (A4), I finally obtain

$$\tilde{n}_{it+1} = -\frac{\gamma \delta}{1 - \gamma(1 - \delta)\tau^*} \tilde{P}_{it}.$$  

### 6.2 Proof of Proposition 2

Using (31) in the text to substitute for $\tau^n$ in (35), I obtain the following equation for $\tau^*$,

$$\tau^* = \frac{(1 - \delta \beta) \eta}{1 + \eta \gamma - \delta \beta \eta \left(\frac{\gamma \delta}{1 - \gamma(1 - \delta)\tau^*}\right)}.$$  

This can be written as

$$a(\tau^*)^2 + b\tau^* + c = 0,$$  

where

$$a \equiv (1 + \eta \gamma) \gamma (1 - \delta) > 0,$$  

$$b \equiv -[1 + \gamma (2 - \delta - \delta \beta) \eta] < 0,$$  

$$c \equiv (1 - \delta \beta) \eta > 0.$$  

The quadratic equation (B1) has two solutions. The latter are real numbers if and only if $b^2 - 4ac > 0$. Using the definitions of $a$, $b$ and $c$, the inequality $b^2 - 4ac > 0$ can be written as

$$[1 + \gamma (2 - \delta - \delta \beta) \eta]^2 > 4(1 + \eta \gamma) \gamma (1 - \delta) (1 - \delta \beta) \eta.$$  

After some algebra, it is possible to express the latter inequality as

$$1 + (\eta \gamma)^2 \delta^2 (1 - \beta)^2 + 2\eta \gamma \delta [1 - \delta \beta + \beta (1 - \delta)] > 0,$$
which holds for any $\delta \in [0,1)$ and $\beta \in [0,1)$. Equation (B1) has therefore two real solutions, given by

$$(\tau_1^*, \tau_2^*) = \left( \frac{-b - \sqrt{b^2 - 4ac}}{2a}, \frac{-b + \sqrt{b^2 - 4ac}}{2a} \right).$$

It is possible to show that the solutions for both $\tau^*$ and $\tau^n$ have to be positive. To see this, define

$$\tau_1^n(\tau^*) = \frac{1 + \eta \gamma - (1 - \delta \beta)\eta / \tau^*}{\delta \beta \eta},$$

$$\tau_2^n(\tau^*) = \frac{\gamma \delta}{1 - \gamma (1 - \delta) \tau^*}.$$

The function $\tau_1^n(\tau^*)$ is obtained by solving for $\tau_1^n$ in equation (35) in the text. The solutions for $\tau^n$ and $\tau^*$ are given by the two points of intersection of both functions in $(\tau^*, \tau^n)$ space. Both functions are strictly increasing in $\tau^*$. For $\tau^* < 0$, $\tau_1^n(\tau^*) > (1 + \eta \gamma) / \delta \beta \eta$ and $\tau_2^n(\tau^*) < \gamma \delta$. Since $(1 + \eta \gamma) / \delta \beta \eta > \gamma \delta$, there can be no solution for $\tau^* < 0$. For $\tau^* > 0$, the signs of $\tau_1^n$ and $\tau_2^n$ coincide only in the non-empty interval $((1 - \delta \beta)\eta (1 + \eta \gamma), \gamma^{-1}(1 - \delta)^{-1})$, in which both functions are strictly positive.

Finally, it is possible to show that $\tau_2^n$ implies explosive dynamics. To see this, notice that a firm’s relative price and employment stock evolve according to

$$\begin{bmatrix} \tilde{E}_{it+1} \\ \tilde{n}_{it+1} \end{bmatrix} = \begin{bmatrix} \delta + (1 - \delta)\tau^* \tau^n & 0 \\ -\tau^n & 0 \end{bmatrix} \begin{bmatrix} \tilde{P}_{it} \\ \tilde{n}_{it} \end{bmatrix}.$$

This system implies convergent dynamics only if the eigenvalues of the 2x2 matrix are inside the unit circle. These eigenvalues are 0 and $\delta + (1 - \delta)\tau^* \tau^n$. Since $\delta + (1 - \delta)\tau^* \tau^n > 0$ (as a result of both $\tau^*$ and $\tau^n$ being positive), a non-explosive solution must satisfy $\delta + (1 - \delta)\tau^* \tau^n < 1$, or equivalently $\tau^* \tau^n < 1$. Using equation (31) in the text, this requires in turn

$$\tau^* < \frac{1}{\gamma}.$$  (B5)
I now define $F(\tau^*) \equiv a(\tau^*)^2 + b\tau^* + c$, where $a$, $b$ and $c$ are given by equations (B2), (B3) and (B4), respectively. Since $F(\tau^*)$ is a convex function, it follows that $F(\tau^*) < 0 \iff \tau^* \in (\tau_1^*, \tau_2^*)$, where $\tau_1^*$, $\tau_2^*$ are the two roots of $F(\tau^*)$. Evaluating $F(\cdot)$ at $1/\gamma$, I obtain

$$
F\left(\frac{1}{\gamma}\right) = (1 + \eta\gamma)^\frac{1}{\gamma}(1 - \delta) - \left[\frac{1}{\gamma} + (2 - \delta - \delta\beta)\eta\right] + (1 - \delta\beta)\eta
$$

$$
= -\frac{\delta}{\gamma} < 0.
$$

It follows that $\tau_1^* < 1/\gamma < \tau_2^*$, which means that $\tau_2^*$ violates (B5) and therefore implies explosive dynamics. As emphasized by Woodford (2005), in order for a log-linear approximation around the steady state to be an accurate approximation of the model’s exact equilibrium conditions, the dynamics of firms’ relative prices and employment stocks must remain forever near enough to the steady state. Since $\tau_2^*$ violates this condition, I set $\tau^*$ equal to $\tau_1^*$.

### 6.3 A search model with a producer-retailer structure

Consider an economy where technology and preferences are the same as in the model presented in section 2, but with a different goods-market structure. In particular, a continuum of identical producers produce a homogenous intermediate good that is sold to retailers at the perfectly competitive price $mc_t$. Profits of an individual producer are given by

$$
\Pi_{it} = mc_t A_t n_t h_{it} - w_{it}(h_{it}) n_{it} - \frac{\chi}{u'(c_{it})} v_{it} + E_t \beta_{t,t+1} \Pi_{it+1}.
$$

The surplus of worker and firm are given respectively by

$$
S_{it}^w = w_{it}(h_{it}) - \frac{b + h_{it} + \eta}{u'(c_{it})} - p(\theta_t) \int_0^1 \frac{v_{jt}}{v_t} E_t \beta_{t,t+1} S_{jt+1}^w dj + (1 - \lambda) E_t \beta_{t,t+1} S_{it+1}^w,
$$

$$
S_{it}^f = mc_t A_t h_{it} - w_{it}(h_{it}) + (1 - \lambda) E_t \beta_{t,t+1} S_{it+1}^f.
$$

(C1)
Hours per employee are chosen in a privately efficient way, that is, so as to maximize the joint match surplus, $S_{it}^w + S_{it}^f$. The resulting first order condition is given by

$$mc_tA_t = \frac{h_{it}^\eta}{w'(c_t)}.$$  \hfill (C2)

which implies that hours are equalized across firms, $h_{it} = h_t$. Since all producers behave symmetrically, I can drop the subscript $i$. Log-linearization of equation (C2) produces equation (E2) in the text.

Nash-bargaining implies that $(1 - \xi) \log S_i^f = \xi \log S_i^w$. The solution for the real wage is given by

$$w_t(h_t) = (1 - \xi) mc_tA_th_t + \xi \left[ \frac{b + h_t^{1+\eta}/(1 + \eta)}{w'(c_t)} + p(\theta_t) \beta_t \delta_{t+1} S_t^w \right]$$

$$= (1 - \xi) \left[ mc_tA_th_t + \frac{\chi}{w'(c_t)} \theta_t \right] + \xi \frac{b + h_t^{1+\eta}/(1 + \eta)}{w'(c_t)}.$$  \hfill (C3)

The first-order conditions with respect to vacancies and employment are given by equations (11) and (12) in the text, without $i$ subscripts. Combining the latter with (C2) and (C3), we have that the job creation condition is given again by equation (20), without $i$ subscripts, which becomes (E4) when log-linearized.

Finally, retailers buy the intermediate input at the real price $mc_t$ and transform it into differentiated final goods with a linear technology. Therefore, $mc_t$ is also the real marginal cost of retailers and is independent of their pricing decisions. The optimal pricing decision common to all price-setting retailers is given by

$$E_t \sum_{T=t}^{\infty} \delta^{T-t} \beta_{t,T} P_T^y r_T \left( \frac{P_t^*}{P_T} - \frac{\gamma}{\gamma - 1} mc_T \right) = 0.$$

Log-linearizing the previous equation and combining it with $\pi_t = [(1 - \delta) / \delta] (\log P_t^* - \log P_t)$, I obtain equation (40) in the text.
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