

**WAGE INEQUALITY, SEGREGATION  
BY SKILL AND THE PRICE  
OF CAPITAL IN AN ASSIGNMENT  
MODEL**

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BANCO DE ESPAÑA

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## **Abstract**

Some pieces of empirical evidence suggest that in the U.S., over the last few decades, (i) wage inequality between-plants has risen much more than wage inequality within-plants and (ii) there has been an increase in the segregation of workers by skill into separate plants. This paper presents a frictionless assignment model in which these two features can be explained simultaneously as the result of the decline in the relative price of capital. Additional implications of the model regarding the skill premium and the dispersion in labor productivity across plants are also consistent with the empirical evidence.

# 1 Introduction

It is a well-documented fact that wage inequality in the U.S. labor market has increased substantially over the last few decades.<sup>1</sup> In the manufacturing sector, one important feature of this increase is that it has come almost exclusively from an increase in the wage inequality *between-plants*. In this sense, Dunne et al. (2004) find that "virtually the entire increase in overall dispersion in hourly wages for U.S. manufacturing workers from 1975 to 1992 is accounted for by the between-plants components". More specifically, their decomposition of the overall wage inequality into the *within-plants* and *between-plants* wage inequality shows that the former only increased slightly during this period, while the latter increased in a similar manner as the overall wage inequality.<sup>2</sup> As a consequence, the *between-plants* wage inequality increased its share in the overall wage inequality from 53% in 1977 to 64% in 1992.

Assuming that wages are closely related to skill, one could interpret this evidence as an indication that the composition of U.S. plants has changed over the last few decades in a way that has increased the segregation of high- and low-skilled workers into separate plants. In this sense, greater segregation would reduce the skill heterogeneity within the workers of a plant and would make more likely to have plants composed of *either* high- or low-skilled workers rather than of *both* high- and low-skilled workers. In other words, it would make more likely to have high-skilled (low-skilled) individuals together with other high-skilled (low-skilled) individuals in the same plants. Thus intuitively, if wages are closely related to skill, an increase in the segregation of workers by skill into separate plants would increase the wage inequality *between-plants* and would reduce the wage inequality *within-plants* (or, when the overall wage inequality is increasing, would increase more the former than the latter).

Some empirical evidence supports this idea of increasing segregation. For instance, using worker classification (production/non-production) as a proxy for skill, Kremer and Maskin (1996) find that, in the U.S. manufacturing sector, production workers are now more likely to be together in the same plants. In particular, they consider a segregation index that lies between 0 (no segregation) and 1 (maximal segregation) and find that the segregation of these workers rose from 0.195 to 0.228 between 1976 and 1987.<sup>3</sup> They also obtain similar evidence for Britain and France using this and

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<sup>1</sup>For instance, Acemoglu (2002) reports that, while in 1971 a worker at the 90<sup>th</sup> percentile of the wage distribution earned 266% more than a worker at the 10<sup>th</sup> percentile, in 1995 this number was 366%.

<sup>2</sup>Davis and Haltiwanger (1991) provide similar evidence.

<sup>3</sup>One could argue that this evidence could be the result of firms reorganizing their tasks across their plants in such a way that non-production workers and production workers are located in different plants. This paper does not consider multi-plant firms. However, as long as the purpose of this intra-firm reallocation of tasks is to improve productivity, this idea would be consistent with the essence of this paper.

other proxies for skill (seniority, experience, and other forms of worker classification) and this leads them to conclude that "wages, experience and worker classification are all imperfect indicators of skill, but together they paint a consistent picture of increasing segregation".<sup>4</sup>

In light of this evidence this paper proposes that both (i) the larger increase in the wage inequality *between-plants* than in the wage inequality *within-plants*, and (ii) the increase in the segregation of workers by skill observed in the U.S. over the last few decades can be explained simultaneously by the decline in the relative price of capital.<sup>56</sup> In this sense, Krusell et al. (2000) report that the relative price of capital equipment (relative to consumption of nondurables and services) fell in the U.S. at an average rate of about 4.5% per year between 1954 and 1992.<sup>7</sup>

To illustrate this connection this paper presents a frictionless assignment model in which the relative price of capital decreases exogenously. In the model, individuals with different skill are imperfect substitutes in production and they must assign themselves to plants and to occupations within those plants. Specifically, the model assumes that plants are composed of one manager, one worker and a stock of capital. In production, the skill of the manager and the skill of the worker are complementary, but they play a non-symmetric role, and there is a form of capital-skill complementarity. In this set up, for a given price of capital, the shape of the equilibrium assignment (the equilibrium composition of the plants) depends on how strong is the complementary in production between skills relative to their asymmetry. In particular, stronger complementarity pushes the equilibrium towards greater segregation of individuals by skill into different plants.

Then, intuitively, the connection in the model between the decline in the relative price of capital and (i) and (ii) works in the following way. When the relative price of capital declines the overall wage inequality increases (due to the capital-skill complementarity in the economy) but also the complementarity in production between the skill of the manager and the skill of the worker becomes stronger relative to their asymmetry. This modifies the equilibrium composition of the plants in a way that the

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<sup>4</sup>They rely on Kramarz, Lollivier and Pele (1996) for the French evidence.

<sup>5</sup>To the best of my knowledge, only for the manufacturing sector there is evidence regarding the evolution over time of the *between-plants* and the *within-plants* wage inequality and of the segregation of workers by skill. The same happens regarding the dispersion of labor productivity across plants discussed below. This paper tries to explain this evidence and, in this sense, it concentrates in an economy with only one sector, the manufacturing sector. Thus, "economy" and "manufacturing sector" are equivalent in this paper.

<sup>6</sup>Certainly, many other factors, besides the decline in the relative price of capital, may have contributed to (i) and (ii) too. For instance, the changes over time in the economy's skill distribution, in the level of plant outsourcing, in the role of unions or in the pressure from international trade. Future work should integrate some of these factors into the analysis.

<sup>7</sup>Gordon (1990) and Cummins and Violante (2002) provide similar evidence.

segregation of individuals by skill increases what, in turn, produces a larger increase in the wage inequality *between-plants* than in the wage inequality *within-plants*.<sup>8</sup>

The model has other interesting predictions. In particular, the model also predicts that, when the relative price of capital declines, the skill premium and the labor productivity dispersion across plants increase. These predictions are broadly consistent with the empirical evidence. In particular, a large empirical literature documents an overall increase in the skill premium over the last few decades in the U.S. For instance, Autor, Katz and Krueger (1998) report that the log relative wage of college and post-college workers to high-school workers went from 0.465 in 1970 to 0.557 in 1996.<sup>9,10</sup> Moreover, Dunne et al. (2004) find that the 90-10 differential of the log of labor productivity across U.S. manufacturing plants increased from around 1.7 to around 1.9 during the period 1975-92.

This paper is related to the literature in several ways. To begin with, there is a large literature that, as in this paper, connects the decline in the relative price of capital with increases in the overall wage inequality and in the skill premium through the existence of a form of capital-skill complementarity in production.<sup>11</sup> See, for instance, Krusell et al. (2000) that try to account for the recent evolution of the skill premium in the U.S. There is also a large literature that, as in this paper, links the decline in the relative price of capital (or, in many instances, the decline in the price of the Information Technology) to changes in the internal organization of plants and/or in their demand of skills. Some examples of this literature are Bresnahan et al. (2002), Autor et al. (2003), Cremer, Garicano and Prat (2004), and Garicano and Rossi (2005). This paper contributes to these two strands of the literature by showing how the decline in the relative price of capital, not only increases the overall wage inequality and modifies the equilibrium composition of the plants, but also affects both the *between-* and the *within-plants* wage inequality and the extent of individuals' segregation by skill.

Moreover, this paper can be thought as complementary with Kremer and Maskin (1996). In this sense, while this paper analyzes the evolution of individuals' segregation by skill and wage inequality in a context of a constant skill distribution and a

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<sup>8</sup>This model simultaneously obtains (i) and (ii). Of course, it is possible to obtain (i) without (ii), and many models in the literature could potentially do so, but that would miss part of the evidence mentioned above.

<sup>9</sup>The evidence in Autor, Katz and Krueger (1998) is for all workers in the economy and not only for manufacturing workers. However, both Davis and Haltiwanger (1991) and Dunne et al. (2004) find that the behavior of the overall wage inequality and of the skill premium for manufacturing workers closely tracks that for all the workers in the economy.

<sup>10</sup>Similar evidence is found in many other studies. In particular, Beaudry and Green (2005) find that the skill premium also increased from the mid-1990s through 2000.

<sup>11</sup>There is a broad consensus in the empirical literature about the existence of capital-skill complementarity in the economy. See, for instance, Griliches (1969), Hamermesh (1993) or Goldin and Katz (1998).



declining relative price of capital, Kremer and Maskin (1996) do the same in a context of a constant price of capital (in fact, they do not have capital in their model) and a changing skill distribution. There are some differences though. For instance, Kremer and Maskin (1996) do not have results regarding the evolution of the *between-* and the *within-plants* wage inequality or the skill premium, and they consider a discrete skill distribution (it is continuous in this paper). Moreover, the framework proposed here is rich enough to consider simultaneously both changes in the economy's skill distribution and in the relative price of capital.<sup>12</sup>

Finally, this paper obviously relates to (and benefits from) the assignment literature, especially to that one studying one-to-one matchings.<sup>13</sup> In this paper, individuals must endogenously divide themselves between managers and workers and (simultaneously) pair themselves creating plants. Then, in a sense, the assignment problem here could be considered a *two-step assignment*.<sup>14</sup> Two well-known papers in the literature with this kind of *two-step assignment* problem are Kremer (1993) and Lucas (1978).<sup>15</sup> In Kremer (1993) the equilibrium assignment involves complete segregation of individuals by skill into different plants (the best are paired with best and the worst with the worst). Instead, in Lucas (1978) in equilibrium there is a cutoff level of skill such that everybody above that level is a manager and everybody below it is a worker. A nice feature of the general framework presented in this paper is that, as illustrated in section 5.4, these two kinds of equilibrium can appear as extreme cases (when the price of capital is free or infinitely expensive, respectively) and then it is possible to see the implications for the economy of the transition from one to the other.

The rest of the paper is organized as follows. Section 2 presents the production technology in the economy. Then, section 3 describes the assignment problem in the paper and defines the equilibrium. Some basic properties of this equilibrium are presented in section 4. Then, the equilibrium is fully characterized in sections 5.1-5.3 for a particular version of the model, and section 5.4 shows how that equilibrium changes as the relative price of capital declines and the implications of that change in terms of wage inequality and of segregation by skill. Finally, section 6 concludes the paper.

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<sup>12</sup>Some preliminary work is pursuing this extension.

<sup>13</sup>Sattinger (1993) and Legros and Newman (2002) are some examples of this literature.

<sup>14</sup>This feature differentiates this paper from many in the assignment literature where the assignment problem consists on pairing individuals/things that belong to two *ex-ante defined groups*. For instance, Koopmans and Beckmann (1957) pair plants with locations, Becker (1973) pairs men with women and Caselli (1999) pairs workers with machines.

<sup>15</sup>The assignment problems in these two papers are not exactly the same to the one considered here. In particular, in Kremer (1993) plants have more than two occupations, and in Lucas (1978) a manager can hire more than one worker and skill is a two-dimensional variable (there is skill as a manager and skill as a worker although, in fact, he considers that everyone has the same skill as a worker).

## 2 The production technology

There is only one good in the economy and this is produced by plants. A plant is composed of one manager, one worker and a stock of capital, and its output is given by the following production function:<sup>16</sup>

$$f(x, z, k) = x^\mu [\theta k^\beta + (1 - \theta) z^\beta]^{\frac{1-\mu}{\beta}} \quad (1)$$

where  $x$  denotes the skill of the manager,  $z$  is the skill of the worker,  $k$  is the amount of capital in the plant, and  $\mu$ ,  $\theta$  and  $\beta$  are parameters. In particular, consider that  $\mu \in [\frac{1}{2}, 1)$ ,  $\theta \in (0, 1)$  and  $\beta < 0$ .

This characterization of the production technology has three crucial features that, as discussed in Kremer and Maskin (1996), are strictly required for the purposes of this paper:

- *Imperfect substitutability.* In the description above individuals with different skills are imperfect substitutes in production. One and only one person can be in charge of a given occupation within a plant, so it is impossible to substitute quality (skill) for quantity (number of persons) in that occupation. Imperfect substitutability is required in this paper in order to obtain implications about the composition of the plants. In particular, these implications could not be obtained if individuals with different skills were perfect substitutes in production, as they are in the classical efficiency units model. In that case, the output of a plant could be expressed as a function of an aggregate measure of skill in the plant. Then, plants with the same aggregate measure of skill would be observationally equivalent, even though they could have very different workforces.
- *Complementarity between skills.* In (1) the skill of the manager and the skill of the worker are complementary in production. This feature of the production technology is relevant in order to obtain sensible implications about the equilibrium composition of the plants. In this sense, the empirical evidence broadly supports that idea that there is positive sorting among managers and workers in the economy (the best managers hire the best workers).<sup>17</sup> The fact that  $\frac{\partial^2 f(x, z, k)}{\partial x \partial z} > 0$  delivers this result. Instead, if  $\frac{\partial^2 f(x, z, k)}{\partial x \partial z} < 0$  the equilibrium composition of the plants would involve negative sorting, and if  $\frac{\partial^2 f(x, z, k)}{\partial x \partial z} = 0$

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<sup>16</sup>This convention calls "manager" to one of the members of the plant and "worker" to the other. This is done just for simplicity. What is really important is that a plant is composed of 2 types of workers. In this sense, what the model means by workers' segregation by skill below is simply individuals' segregation by skill.

<sup>17</sup>See, for example, Doms et al. (1997) and Bartelsman and Doms (2000).

one could not establish any kind of relationship between the skills of the two individuals paired together in a plant.

- *Asymmetry between skills.* In (1) there is an asymmetry in production between the skill of the manager and the skill of the worker. Basically, they affect the output of the plant in different ways. Again, this asymmetry is needed to avoid compositional implications that are not interesting for the purposes of this paper. In this sense, with a symmetric production technology, in which the skill of the manager and the skill of the worker affect the output of the plant in the same way, the equilibrium would always imply, as in Kremer (1993), no skill heterogeneity within plants, perfect segregation of individuals by skill into different plants and zero wage inequality *within-plants*. Moreover, this asymmetry needs to be introduced in a sensible way. In particular, one could intuitively expect that, within a plant in equilibrium, the manager is more skilled than his worker. As it will be clear below, imposing that  $\mu \in [\frac{1}{2}, 1)$  delivers this result. For now, just note that when  $\mu \in [\frac{1}{2}, 1)$ , the output produced by any plant composed of two individuals with different skills is always larger when the most skilled individual is the manager. Specifically,  $\forall a > b$  and  $\forall k > 0$ ,<sup>18</sup>

$$f(a, b, k) > f(b, a, k) \quad (2)$$

In addition to these features, note that (1) also shows complementarity in production (i) between the skill of the manager and the amount of capital and (ii) between the skill of the worker and the amount of capital.<sup>19</sup> Thus, even though these conditions are not the conventional way in the literature of introducing capital-skill complementarity in production, one could say that this model exhibits a form capital-skill complementarity.<sup>20</sup> This is relevant for the results of this paper. In particular,

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<sup>18</sup>Equation (2) is satisfied if,  $\forall a > b$  and  $\forall k > 0$ ,  $a^\mu [\theta k^\beta + (1 - \theta) b^\beta]^{\frac{1-\mu}{\beta}} > b^\mu [\theta k^\beta + (1 - \theta) a^\beta]^{\frac{1-\mu}{\beta}}$ . This happens if  $\frac{\partial}{\partial s} \left( \frac{s^\mu}{[\theta k^\beta + (1 - \theta) s^\beta]^{\frac{1-\mu}{\beta}}} \right) > 0$ , and this is always the case when  $\mu \in [\frac{1}{2}, 1)$ .

<sup>19</sup>The fact that  $\beta < 0$  guarantees that there is always complementarity in production between the skill of the worker and the amount of capital.

<sup>20</sup>The most usual approach in the literature to introduce capital-skill complementarity is through a production function with three inputs, skilled labor ( $S$ ), unskilled labor ( $U$ ), and capital ( $K$ ), in which the direct elasticity of substitution (or, in other cases, the Allen-Uzawa partial elasticity of substitution) between skilled labor and capital ( $\sigma_{SK}$ ) is lower than between unskilled labor and capital ( $\sigma_{UK}$ ). This interpretation of capital-skill complementarity can not be considered here. In particular, note that in the production function (1) one can not associate  $x$ , the skill level of the manager (or the labor input in the managerial occupation), with the *skilled* labor input, nor  $z$ , the skill level of the worker (or the labor input in the workers' occupation), with the *unskilled* labor input (and, by extension, one can not associate  $\sigma_{xk}$  and  $\sigma_{zk}$  with  $\sigma_{SK}$  and  $\sigma_{UK}$ , respectively). The reason why this association is not possible is that, as it will be clear below, in many instances

given this complementarity, the decline in the relative price of capital will constitute a skill-biased technological change that will push wage inequality upwards.

Two additional comments about the particular functional form considered in (1). First, since there is not a standard production function involving capital in the assignment literature, this paper considers a functional form, the combination of a Cobb-Douglas and a CES, that has been used extensively in the literature to explain a wide variety of issues. Second, note that there are three possible ways of distributing  $x$ ,  $z$  and  $k$  in a functional form that combines a Cobb-Douglas and a CES: one could place  $x$  outside of the CES,  $z$  or  $k$ . The option adopted in (1) is not arbitrary. In particular, leaving  $k$  outside of the CES does not work for the purposes of this paper because, in that case, capital would affect  $x$  and  $z$  in the same way and, then, the equilibrium composition of the plants would not be affected by a change in the relative price of capital.<sup>21</sup> Leaving  $z$  outside of the CES is not a good choice either as it could produce some unappealing assignment implications.<sup>22</sup> Leaving  $x$  outside of the CES does not present any of these problems and that is why it is the option adopted in (1).

A final comment about this production technology. Considering that there are only two occupations within a plant and that skill is a one-dimensional variable is enough for the purposes of this paper. Assignment models are very demanding analytically and this strongly pushes for simplicity in the characterization of the production technology. This is why these two features are very common in the assignment literature. Obviously, more realism in these dimensions would be better but this would come at a great analytical cost.

## 2.1 The production function net of capital costs

The previous section characterized the plants' production function. However, in order to define and to characterize the equilibrium in the following sections, it is more useful to consider the plants' production function *net of the optimal capital costs*. In this sense, consider that plants do not have capital when they are created, but that they can buy any amount of it at an exogenously given price  $p$ . Then, the plants' production function net of the optimal capital costs can be defined as:

$$h(x, z, p) \equiv f(x, z, k^*) - pk^*$$

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it happens that in the equilibrium of the economy some plants have both the managerial and the workers' occupation filled by *unskilled* individuals while other plants have both of them filled by *skilled* individuals.

<sup>21</sup>For this same reason, it is not convenient for the purposes of this paper to consider a Cobb-Douglas production function with inputs  $x$ ,  $z$  and  $k$ .

<sup>22</sup>In particular, under a production function of this type, it could happen that, for some prices of capital, in some plants with heterogeneous individuals the least skilled ones choose to be the managers.

where  $k^*$  is the solution to the following maximization problem:

$$\max_k f(x, z, k) - pk$$

This function  $h(x, z, p)$  replicates the most relevant properties of the function  $f(x, z, k)$  in equation (1). Specifically,  $h(x, z, p)$  also (i) increases with the skill of the manager and with the skill of the worker, (ii) exhibits complementarity in production between the skill of the manager and the skill of the worker, and (iii) has an asymmetry in production between the skill of the manager and the skill of the worker. In particular, similar to (2), the asymmetry in  $h(x, z, p)$  makes that the *net* output produced by any plant composed of two individuals with different skill levels is always larger when the most skilled individual is the manager. Formally,  $\forall a > b$  and  $\forall p > 0$ ,

$$h(a, b, p) > h(b, a, p)$$

Results (i) and (ii) can be easily obtained using the envelope theorem. As for the last result, it comes directly from equation (2). To see this simply note that (2) implies that  $f(a, b, k_{ba}^*) - pk_{ba}^* > f(b, a, k_{ba}^*) - pk_{ba}^*$ , where  $k_{ba}^*$  is the amount of capital that a plant composed of a manager with skill  $b$  and a worker with skill  $a$  would optimally buy. But since, by definition,  $h(a, b, p) = f(a, b, k_{ab}^*) - pk_{ab}^* \geq f(a, b, k_{ba}^*) - pk_{ba}^*$  and  $f(b, a, k_{ba}^*) - pk_{ba}^* = h(b, a, p)$ , this implies (iii). As it will be clear below, these properties of  $h(x, z, p)$  will be crucial in determining the features of the economy's equilibrium assignment.

### 3 The assignment problem and the equilibrium

Consider that the economy is populated by a continuum of individuals with different skill  $s$ . In particular, consider that skill is distributed across the population according to a continuous density function  $\phi(s)$  defined over the interval  $[s_{min}, s_{max}]$ . Furthermore, consider that the assignment of individuals to plants and to occupations is frictionless. Specifically, everybody's skill is public information and the movement of individuals across plants and occupations is costless and it does not require time. Then, for a given price of capital  $p$ , the assignment problem in this paper is to allocate individuals to plants and to occupations within those plants and to allocate net output (payoff) to individuals in a way that is feasible given the production technology and the skill distribution and that is stable. Formally, the equilibrium (solution) of this problem is the combination of:

- An occupational correspondence,  $\Omega : [s_{min}, s_{max}] \rightrightarrows \{manager, worker\}$ , that specifies, for each skill level, the occupational choice of the individuals with that

skill. This in turn defines the sets:

$$\begin{aligned} M &= \{s \in [s_{min}, s_{max}] : \Omega(s) = \textit{manager}\} \\ WO &= \{s \in [s_{min}, s_{max}] : \Omega(s) = \textit{worker}\} \end{aligned}$$

- A matching function,  $\psi : M \longrightarrow WO$ , that specifies the way managers are paired with workers to create plants.<sup>23</sup>
- A payoff function,  $W : [s_{min}, s_{max}] \rightarrow \mathbb{R}$ , that determines everybody's payoff.<sup>24</sup>

such that:

- The payoff structure is feasible. That is, in any plant, the combined payoff of its members is not greater than the net output they produce:<sup>25</sup>

$$W(s) + W(\psi(s)) \leq h(s, \psi(s), p) \quad \forall s \in M \quad (3)$$

- The assignments are feasible. That is, for any type of plant, the mass of managers is equal to the mass of workers:<sup>26</sup>

$$\int_{s \in A} \phi(s) ds = \int_{s \in \psi(A)} \phi(s) ds \quad \text{for every measurable set of managers } A \in M \quad (4)$$

- None has an incentive to deviate. That is:

$$\nexists a, b \in [s_{min}, s_{max}] : \max \{h(a, b, p), h(b, a, p)\} > W(a) + W(b) \quad (5)$$

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<sup>23</sup>To be precise,  $\psi$  could be a correspondence instead of a function. However, in the analysis that follows this is never the case and it is less intuitive to define the equilibrium when  $\psi$  is a correspondence. That is why  $\psi$  is considered to be a function here. See Legros and Newman (2002) for a definition of the equilibrium when  $\psi$  is a correspondence.

<sup>24</sup>This definition already incorporates one equilibrium result. In particular, the equilibrium in this model requires that individuals with identical skill obtain the same payoff. In other words,  $W$  is a function and not a correspondence.

<sup>25</sup>Due to condition (5) below, in equilibrium this equation is always satisfied with equality  $\forall s \in M$ .

<sup>26</sup>Since plants are composed of one manager and one worker, it is unfeasible to pair, for instance, a mass of managers of measure  $\frac{1}{3}$  to a mass of workers of measure  $\frac{2}{3}$ .

## 4 Basic properties of the equilibrium

The equilibrium assignment defined above has several interesting properties that do not depend on the particular skill distribution, price of capital or values of the parameters of the production function considered. These are formally stated in Lemma 1:<sup>27</sup>

*Lemma 1. Regardless of the economy's skill distribution, the price of capital and the values of the parameters of the production function, the equilibrium assignment:*

- (i) always exists,*
- (ii) maximizes the economy's aggregate net output among all the feasible assignments,*
- (iii) requires that the most skilled individual within any plant is the manager,*
- (iv) involves positive sorting between managers and workers, and*
- (v) requires a payoff function that is strictly increasing with respect to skill.*

The existence and efficiency of the equilibrium assignment should come at no surprise given the fact that the model does not contain any friction or imperfection. Instead, (iii) and (iv) come, respectively, from the asymmetry and the complementarity in production between the skill of the manager and the skill of the worker imposed in section 2. Finally, the equilibrium payoff function needs to be increasing with respect to skill because the net output of a plant strictly increases both with the skill of its manager and with the skill of its worker.

Given the results presented in Lemma 1, only one additional piece of information is needed to fully characterize the equilibrium assignment: who are managers and who are workers in equilibrium. Now note that there are two forces in this model that play a role in determining these sets in equilibrium: the *complementarity* and the *asymmetry force*.<sup>28</sup> The former is due to the complementarity in production between the skill of the manager and the skill of the worker and the latter is due to the different roles that these skills play in production, as imposed in section 2. These two forces push in different directions in determining who must be managers and who must be workers in equilibrium. To understand this, consider the case where there is complementarity in production between the skill of the manager and the skill of the worker but not asymmetry. In this case, the equilibrium assignment would be such that the best individuals would be paired with the best and the worst with the worst. Then, both low- and high-skilled individuals would be managers (and workers) in equilibrium and there would be maximal segregation of individuals by skill into different plants. This is the kind of equilibrium, for instance, in Kremer (1993). Instead, consider the case where there is asymmetry in production between

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<sup>27</sup>See the Appendix for the proofs of all the Lemmas in this paper.

<sup>28</sup>These two forces also appear simultaneously, for example, in Kremer and Maskin (1996) and in Davis (1997).

the skill of the manager and the skill of the worker (as stated in section 2) but not complementarity. In this case, the equilibrium assignment would be such that everybody with skill above the median skill in the population would be a manager while everybody else would be a worker. This equilibrium partition of individuals between of occupations is similar to one in Lucas (1978). Then, intuitively, the *complementarity force* pushes towards individuals' segregation by skill into different plants while the *asymmetry force* pushes high-skilled individuals into the managerial occupation inducing lower levels of skill segregation.

These two forces operate simultaneously in this model and, since they push in different directions in determining who must be managers and who must be workers in equilibrium, the exact shape of the equilibrium assignment depends on their relative strengths. These strengths, in turn, depend on the economy's skill distribution, on the price of capital and on the values of the parameters of the production function. Therefore, more structure is needed in order to know who are managers and who are workers in the equilibrium assignment and to completely characterize that assignment. In this sense, the two equilibrium assignments described above will only appear here as extreme cases while, in general, the equilibrium assignment will be a combination of both. That is, in general, in the equilibrium assignment in this model neither there will be complete segregation of individuals by skill nor everybody with skill below (above) the median skill in the population will be a worker (manager).

As stated in the Introduction, this paper claims that the decline in the relative price of capital, by changing the economy's equilibrium assignment, can produce simultaneously both (i) a larger increase in the wage inequality *between-plants* than in the wage inequality *within-plants* and (ii) an increase in the segregation of individuals by skill. Despite, as mentioned above, more structure needs to be introduced in the model to be able to completely characterize the economy's equilibrium assignment and its evolution, one can intuitively see already that the model presented so far can deliver this connection. The intuition is the following. On the one hand, since there is capital-skill complementarity in production, the decline in the relative price of capital increases the returns to skill increasing the overall wage inequality. On the other hand, that decline also increases the strength of the *complementarity force* relative to the *asymmetry force*.<sup>29</sup> Given the discussion above, this change in the strengths of the two forces pushes the economy's equilibrium assignment towards higher segregation of individuals by skill and this change in the equilibrium composition of the plants produces, in turn, larger increases in the wage inequality *between-plants* than in the wage inequality *within-plants*.

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<sup>29</sup>When the relative price of capital declines plants buy more capital. Now note that  $\frac{\partial^2 f(x,z,k)}{\partial x \partial z}$  increases with  $k \forall (x, z)$ . Then, intuitively, the strength of the *complementarity force* increases as  $p$  decreases. Alternatively,  $\frac{\partial}{\partial s} \left( \frac{s^\mu}{[\theta k^\beta + (1-\theta)s^\beta]^{\frac{1-\mu}{\beta}}} \right)$  decreases with  $k \forall s$ . Then, intuitively, the strength of the *asymmetry force* decreases as  $p$  decreases.



The following section illustrates this connection more clearly. In order to do that, it considers a particular version of the model described above for which it is possible to completely characterize the equilibrium assignment of the economy for any price of capital. Then, given that complete characterization of the equilibrium assignment, a simple comparative static exercise shows how the decline in the relative price of capital affects the equilibrium levels of wage inequality and of segregation by skill in the economy.

## 5 A particular case of the model

*Particular case.*- Consider that:

(A1)  $\mu = \frac{1}{2}$ .

(A2) Skill is distributed across the population according to a uniform distribution between  $[0, s_{max}]$ .

These two assumptions help characterizing the equilibrium assignment of the economy to a larger extent than in section 4. To begin with, assumption (A1) implies that the *complete assortative assignment*, the one in which individuals are perfectly segregated by skill into different plants (that is, all individuals with the same skill are paired among themselves), is the equilibrium assignment only when capital is free.<sup>30</sup> Lemma 2 presents this result.<sup>31</sup>

*Lemma 2.* When  $\mu = \frac{1}{2}$ , the complete assortative assignment is the equilibrium assignment of the economy if and only if  $p = 0$ .

The intuition behind this result is the following. When capital is free, plants buy an infinite amount of it.<sup>32</sup> Then, if  $\mu = \frac{1}{2}$ , the asymmetry in production between the skill of the manager and the skill of the worker disappears. To see this, simply note that, in that case, the plants' production function of net output is  $h(x, z, 0) = (1 - \theta)^{\frac{1}{2\beta}} (xz)^{\frac{1}{2}}$ . Therefore, only the *complementarity force* operates in the economy and the *complete assortative assignment*, where there is maximal segregation by skill, becomes the equilibrium. However, when  $p \neq 0$ , even if  $\mu = \frac{1}{2}$ , the *asymmetry force* operates in the economy and it keeps the equilibrium away from perfect segregation.

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<sup>30</sup>Numerical simulations of the model (available from the author upon request) suggest that the evolution of wage inequality and of segregation by skill as the price of capital declines is qualitatively the same in the model regardless of  $\mu$  being equal to or greater than  $\frac{1}{2}$ . In this sense, considering that  $\mu = \frac{1}{2}$  simply helps illustrating that behavior. The only qualitative difference between the two cases is that when  $\mu > \frac{1}{2}$  the *complete assortative assignment* is never the equilibrium assignment in the economy (even when capital is free) and, therefore, when  $p = 0$  wage inequality *within-plants* is greater than zero.

<sup>31</sup>Obviously, when the *complete assortative assignment* is the equilibrium of the economy, the equilibrium payoff function is  $W(s) = \frac{1}{2}h(s, s, p)$ .

<sup>32</sup>To see this, just note that  $\lim_{k \rightarrow \infty} f_3(x, z, k) = 0 \forall (x, z)$ .

As for assumption (A2), it helps characterizing further the equilibrium assignment of this economy when  $p \neq 0$ . This is shown in the following three subsections. To begin with, section 5.1 obtains the exact shape of the equilibrium assignment of this economy for any  $p \neq 0$  departing from an initial guess about its shape, *that is assumed to be correct*. Then, using a discrete version of the economy, section 5.2 shows numerically that this guess is correct and section 5.3 obtains the equilibrium payoffs.

## 5.1 A guess about the equilibrium assignment

*Guess.-* For each price of capital  $p \neq 0$  there exists one value  $\lambda = \lambda(p) \in (0, 1)$  such that, in the equilibrium assignment, the individuals in the interval  $[\lambda s_{max}, m]$  are workers for the individuals in the interval  $[m, s_{max}]$  and they are paired according to the matching function  $\psi(s) = s - t$ , where  $m = s_{max} \left(\frac{1+\lambda}{2}\right)$  and  $t = s_{max} \left(\frac{1-\lambda}{2}\right)$ .

Graphically, such an assignment could be represented in the following way:



Figure 1. Guess about the equilibrium assignment.

where the arrow indicates that the individuals in the interval of origin hire the individuals in the interval of destination according to the matching function  $\psi(s)$ .

To begin with, note that this assignment within the interval  $[\lambda s_{max}, s_{max}]$ :

- involves positive sorting between managers and workers. This is so because  $\frac{\partial \psi(s)}{\partial s} > 0$ .
- is feasible given the uniform skill distribution assumed in (A2). To see this, note that  $m$  is the median skill level in the interval  $[\lambda s_{max}, s_{max}]$  so that the mass of managers in the interval  $[m, s_{max}]$  is equal to the mass of workers in the interval  $[\lambda s_{max}, m]$ . Note also that  $\psi(s)$  is such that equation (6) below is satisfied. It turns out that this implies that condition (4) is also satisfied.

$$\int_{\psi(s)}^m \phi(s) ds = \int_s^{\lambda s_{max}} \phi(s) ds \quad \forall s \in [m, \lambda s_{max}] \quad (6)$$

If the previous guess is correct, and the individuals in the interval  $[\lambda s_{max}, s_{max}]$  are paired among themselves in equilibrium, then the assignment of the individuals in the interval  $[0, \lambda s_{max}]$  in that same equilibrium must obviously coincide with the solution to the assignment problem that must allocate individuals with skills distributed uniformly between  $[0, \lambda s_{max}]$ . Now it turns out that the assignment problem in which skill is distributed uniformly between  $[0, s_{max}]$  and the assignment problem in which skill is distributed uniformly between  $[0, \lambda s_{max}]$  are isomorphic. To see this, simply note that the latter problem is just a redefinition of the former one in which  $\tilde{s} = \lambda s$ . This redefinition does not affect the basic structure of the problem for two reasons:

1.- A uniform distribution between  $[0, s_{max}]$  is identical to a uniform distribution between  $[0, \lambda s_{max}]$  except for a multiplicative term.<sup>33</sup>

2.- The plants production function of net output is homogeneous of degree one in the skill of the manager and the skill of the worker. That is,  $h(\lambda x, \lambda z, p) = \lambda h(x, z, p)$ .

Therefore, since the only difference between the original and the redefined problem is a multiplicative term, the shape of the solution to both problems must be the same. To be more specific, if the solution to the original assignment problem implies that the individuals in the interval  $[\lambda s_{max}, s_{max}]$  are paired among themselves according to the rules defined above, then the solution to the redefined assignment problem must require that the individuals in the interval  $[\lambda \tilde{s}_{max}, \tilde{s}_{max}]$  must also be paired among themselves according to the same rules. That is, individuals in the interval  $[\lambda^2 s_{max}, \lambda m]$  must be workers for the individuals in the interval  $[\lambda m, \lambda s_{max}]$  and they must be paired according to the matching function  $\psi(s) = s - \lambda t$ .

Taking this reasoning repetitively, if the guess at the beginning of this section is correct, it is possible to know completely the exact shape of the equilibrium assignment for an arbitrary price of capital  $p$  given  $\lambda(p)$ . In particular, the equilibrium assignment would be such that there is an infinite number of intervals of the form  $[\lambda^i s_{max}, \lambda^{i-1} s_{max}]$ ,  $i = 1, 2, 3, \dots$  and, in each one of these intervals,

- the individuals with skill  $s \in [\lambda^i s_{max}, m_i]$  are workers, where  $m_i = \lambda^{i-1} m$ .
- the individuals with skill  $s \in [m_i, \lambda^{i-1} s_{max}]$  are managers.<sup>34</sup>
- each manager with skill  $s$  is paired with a worker with skill  $\psi_i(s)$ , where  $\psi_i(s) = s - t_i$  and  $t_i = \lambda^{i-1} t$ .<sup>35</sup>

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<sup>33</sup>Here is where the uniformity of the skill distribution and the fact that  $s_{min} = 0$  (assumption (A2)) plays its role.

<sup>34</sup>In other words,  $\Omega(s) = \text{manager} \forall s \in [m_i, \lambda^{i-1} s_{max}]$ ,  $i = 1, 2, 3, \dots$  and  $\Omega(s) = \text{worker} \forall s \in [\lambda^i s_{max}, m_i]$ ,  $i = 1, 2, 3, \dots$

<sup>35</sup>It is immediate to see, given the discussion above for the assignment proposed within the interval  $[\lambda s_{max}, s_{max}]$ , that this assignment for the whole economy involves positive sorting between managers and workers and it is feasible given the skill distribution.

A graphical representation of this assignment could be the following:

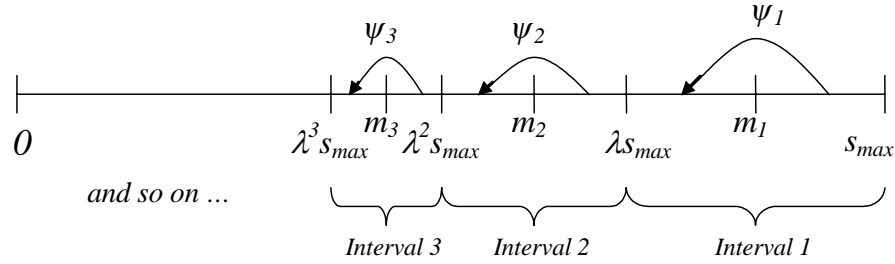


Figure 2. Proposed equilibrium assignment.

It is also possible to know the value of  $\lambda = \lambda(p)$  that characterizes this equilibrium assignment for a given price of capital  $p$ . The strategy is the following. As stated in Lemma 1, the equilibrium assignment maximizes the aggregate net output produced in the economy among all the feasible assignments. Then, if the guess at the beginning of this section is correct and the assignment proposed above is really the equilibrium assignment of the economy, it must necessarily maximize the economy's aggregate net output among (in particular) all the feasible assignments *with the same shape*. Now note that, given  $\lambda$ , the aggregate net output produced in the economy under the proposed assignment,  $Y(\lambda)$ , is equal to:<sup>3637</sup>

$$\begin{aligned}
 Y(\lambda) &= \sum_{i=1}^{\infty} Y_i(\lambda) = \sum_{i=1}^{\infty} \int_{m_i}^{\lambda^{i-1} s_{max}} h(s, \psi_i(s), p) \phi(s) ds = & (7) \\
 &= \sum_{i=1}^{\infty} \lambda^{2(i-1)} Y_1(\lambda) = Y_1(\lambda) \frac{1}{1 - \lambda^2}
 \end{aligned}$$

where  $Y_i(\lambda)$  is the net output produced within interval  $i$  and  $Y_1(\lambda)$  is the net output produced within the first interval. Thus, for a given price of capital  $p$ , the value of  $\lambda = \lambda(p)$  that characterizes the equilibrium assignment described above must be the one that solves:

$$\max_{\lambda} Y_1(\lambda) \frac{1}{1 - \lambda^2}$$

That is,  $\lambda = \lambda(p)$  is defined implicitly by the following equation:

$$Y_1'(\lambda) [1 - \lambda^2] + Y_1(\lambda) 2\lambda = 0 \tag{8}$$

<sup>36</sup>The first equality in the second line of (7) comes after making the following change of variable in all the integrals,  $s = \hat{s} \lambda^{i-1}$ .

<sup>37</sup>Although not shown explicitly, it must be clear that both  $Y(\lambda)$ ,  $Y_i(\lambda)$  and  $Y_1(\lambda)$  depend on  $p$ .

Unfortunately, it is not possible to derive analytically a close form solution for the function  $\lambda(p)$ . However, one can still characterize this function numerically. In this sense, (i) it decreases continuously as  $p$  increases, (ii) it approaches to 1 as  $p$  goes to zero and (iii) it approaches to 0 as  $p$  goes to infinite. This behavior is robust to the specific values of  $s_{max}$ ,  $\theta$  and  $\beta$  considered. For completeness, Figure 3 represents  $\lambda(p)$  using arbitrarily  $\beta = -1.5$ ,  $\theta = 0.5$  and  $s_{max} = 100$ .<sup>38</sup>

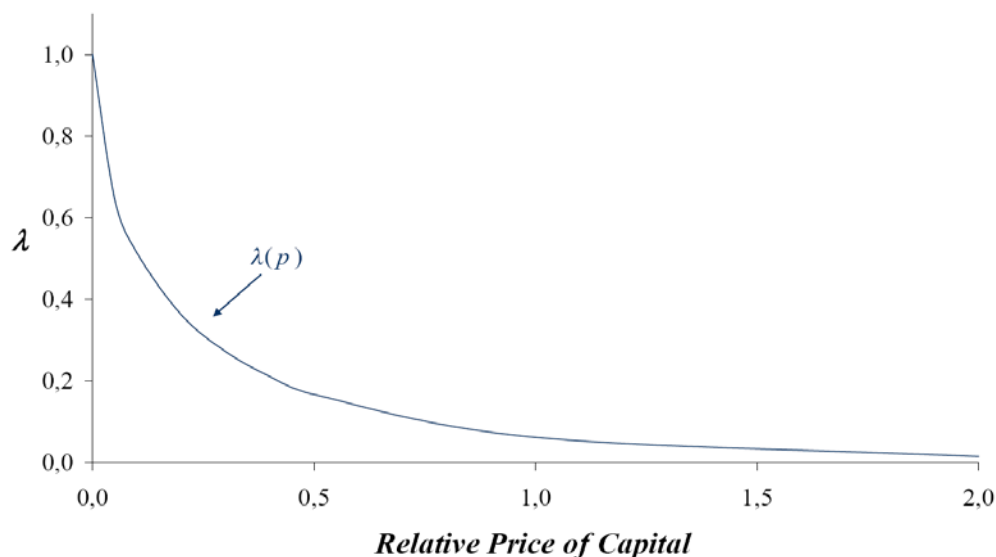


Figure 3. Function  $\lambda(p)$ .

## 5.2 Verifying that the guess is correct

Section 5.1 derived, for any price of capital  $p \neq 0$ , the exact shape of the equilibrium assignment of the economy described above under the assumption that an initial guess about its shape was correct. Obviously, one still needs to check that this guess is correct. This section proposes a simple numerical strategy to check if this is the case.

Consider arbitrary values for  $p$ ,  $\beta$ ,  $\theta$  and  $s_{max}$ . Then, consider  $I$  different *skill types* evenly distributed over the interval  $(0, s_{max})$  and the corresponding  $I$  different *skill types* that are paired with them according to the proposed equilibrium assignment for those arbitrary values of  $p$ ,  $\beta$ ,  $\theta$  and  $s_{max}$ . This produces a discrete economy with  $N = 2I$  different *skill types*.<sup>39</sup> For notational convenience, denote by  $S = \{s_1, s_2, \dots, s_N\}$

<sup>38</sup>The numerical strategy used to characterize the function  $\lambda(p)$  is available from the author upon request. Basically, it involves solving numerically for the function  $h(x, z, p)$  and approximating the definite integrals contained in  $Y_1(\lambda)$ , which has been done using the extrapolated Simpson's rule.

<sup>39</sup>Since (A2) imposes that skill is distributed uniformly across the population, this description implicitly considers that all skill types have the same mass of individuals (mass 1, without loss of generality).

the set of sorted skill types.

If the guess at the beginning of section 5.1 is correct, then the assignment described in that section maximizes the aggregate net output produced in the economy among all the feasible assignments. This implies, among other things, that the aggregate net output produced under such an assignment by the  $N$  skill types defined above must be equal to the maximum they could produce in isolation. Therefore, it must not exist an alternative feasible way of pairing those  $N$  skill types that produces more aggregate net output. Checking if this is the case is a way of verifying whether the guess is correct or not. This can be done very easily with the following linear programming problem.

*Linear programming problem.*- The aggregate net output produced in the discrete economy defined above when the price of capital is  $p$  can be expressed as:<sup>40</sup>

$$Y = \sum_{l,j} a_{lj}^p e_{lj} \quad (9)$$

where:

- $a_{lj}^p$  denotes the net output optimally produced by a plant composed of one individual with skill  $s_l$  and one individual with skill  $s_j$  when the price of capital is  $p$ . That is,  $a_{lj}^p \equiv \max \{h(s_l, s_j, p), h(s_j, s_l, p)\}$ .
- $e_{lj}$  denotes the fraction of individuals with skill  $s_l$  that are paired with individuals with skill  $s_j$ . For instance,  $e_{lj} = 1$  when all the individuals with skill  $s_l$  are paired with individuals with skill  $s_j$ , and  $e_{lj} = 0$  when no individual with skill  $s_l$  is paired with an individual with skill  $s_j$ .

Obviously, the assignments described by the  $e_{lj}$ 's must be feasible. In particular, they must satisfy the following easy-to-interpret conditions:

$$e_{lj} \in [0, 1] \quad \forall l, j = 1, \dots, N \quad (10)$$

$$\sum_l e_{lj} = 1 \quad \forall l = 1, \dots, N \quad (11)$$

$$\sum_j e_{lj} = 1 \quad \forall j = 1, \dots, N \quad (12)$$

$$e_{lj} = e_{jl} \quad \forall l, j = 1, \dots, N \quad (13)$$

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<sup>40</sup>See Koopmans and Beckmann (1957) for a somehow similar problem.

Then, for any price of capital  $p$ , the assignment among the  $N$  skill types defined above that maximizes the aggregate net output that they can produce in isolation is simply the solution to the linear programming problem that looks for the  $e_{lj}$ 's that maximize (9) subject to conditions (10)-(13). Now, it turns out that the optimal assignment of those  $N$  skill types obtained solving this maximization problem always coincides with the one disposed by the proposed equilibrium assignment derived in section 5.1.<sup>41,42</sup> This result is robust to the specific values of the parameters of the model and of  $I$  considered.<sup>43</sup> Then, one could argue that the guess is really correct and that the assignment proposed in section 5.1 is really the equilibrium assignment of the economy.

### 5.3 Wages in the equilibrium assignment

Section 5.1 derived the exact shape of the equilibrium assignment departing from an initial guess that the strategy proposed in section 5.2 showed to be correct. Then, the only thing left to completely characterize the equilibrium assignment of the economy is to determine everybody's payoff in that equilibrium. This section proposes a numerical strategy that can deliver the equilibrium payoff of the  $N$  different skill types described in section 5.2.<sup>44</sup> In this sense, it is reasonable to expect that, when  $I$  is a large number, this strategy provides a good approximation to the actual equilibrium payoff function.

In the discrete economy described in section 5.2, denote by  $s_l^*$  the skill type that is optimally paired with the skill type  $s_l \in S$  according to the solution to the linear programming problem proposed in that section. As already explained, for each  $s_l$  that  $s_l^*$  is equal to the one proposed by the equilibrium assignment described in section 5.1. Then, conditions (3) and (5) together impose the following restrictions on the equilibrium payoffs:

$$W(s_l) + W(s_l^*) = \max \{h(s_l, s_l^*, p), h(s_l^*, s_l, p)\} \quad \forall l = 1, 2, \dots, N \quad (14)$$

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<sup>41</sup>It is important to mention that, although condition (10) allows *fractional assignment* (the  $e_{lj}$ 's are allowed to take any value in the interval  $[0, 1]$ , and not only 0 or 1), the solution to this maximization problem always involves corner solutions (that is, the optimal  $e_{lj}$ 's are always either 0 or 1) as in the proposed equilibrium assignment.

<sup>42</sup>This solution can be found numerically using the linear programming tools in Matlab. The exact Matlab program employed to obtain it is available from the author upon request. Basically, it solves the problem using a Matlab's large scale algorithm based on interior point methods.

<sup>43</sup>Obviously, the larger  $I$  the more guarantees this procedure provides.

<sup>44</sup>Alternatively, and since the exact shape of the equilibrium assignment is already known, one could use rather standard techniques in the assignment literature to derive analytically an expression for the equilibrium payoff function. That expression, however, is almost intractable (mainly due to the complexity of the functional form adopted in (1)) and, eventually, one still needs to resort to numerical analysis to show the behavior of wages in the model.

$$W(s_l) = \begin{cases} \max_{s_j} [\max \{h(s_l, s_j, p), h(s_j, s_l, p)\} - W(s_j)] \\ \text{s.t. } s_j \in S \end{cases} \quad \forall l = 1, 2, \dots, N \quad (15)$$

Together, conditions (14) and (15) impose some bounds on each  $W(s)$  but do not completely define it. In other words, it is possible to find different equilibrium payoff functions associated to the same equilibrium assignment. This non-uniqueness is due to the discreteness of the economy (this does not happen in a continuous economy) but, fortunately, it is not a big problem for the purposes of this paper. On the one hand, the bounds imposed by conditions (14) and (15) on each  $W(s)$  for a given equilibrium assignment are tighter the greater the number of skill types in the economy. Thus, by considering sufficiently large values of  $I$  one can make the range of variation (across all the equilibrium payoff functions associated to the same equilibrium assignment) of any  $W(s)$  very small. On the other hand, even if a given  $W(s)$  varies a little bit across the different equilibrium payoff functions consistent with a given equilibrium assignment, this has very little effect on the variables that are relevant for this paper since they are aggregate variables (skill premium, wage inequality *between-* and *within-plants*, ...). These two claims are confirmed by the following iterative procedure that, departing from an initial guess about everybody's equilibrium payoff, can deliver a set of equilibrium payoffs for everybody consistent with a given equilibrium assignment:

1.- Make a guess about everybody's equilibrium payoff associated with a given equilibrium assignment.

2.- Define a new set of equilibrium payoffs,  $\tilde{W}(s)$ , as:

$$\tilde{W}(s_l) = \begin{cases} \max_{s_j} [\max \{h(s_l, s_j, p), h(s_j, s_l, p)\} - W(s_j)] \\ \text{s.t. } s_j \in S \end{cases} \quad \forall l = 1, 2, \dots, N$$

3.- Check if  $\tilde{W}(s)$  satisfies conditions (14) and (15).

4.- If  $\tilde{W}(s)$  satisfies condition (14) but not condition (15), then begin the process again using  $\hat{W}(s)$  as the initial guess, where  $\hat{W}(s)$  is defined as:

$$\hat{W}(s_l) = \begin{cases} \max_{s_j} [\max \{h(s_l, s_j, p), h(s_j, s_l, p)\} - \tilde{W}(s_j)] \\ \text{s.t. } s_j \in S \end{cases} \quad \forall l = 1, 2, \dots, N$$

5.- If  $\tilde{W}(s)$  satisfies condition (15) but not condition (14), then begin the process again using  $\hat{W}(s)$  as the initial guess, where  $\hat{W}(s)$  is defined as:



$$\hat{W}(s_l) = \tilde{W}(s_l) + \frac{1}{2} \left[ \max \{h(s_l, s_l^*, p), h(s_l^*, s_l, p)\} - \tilde{W}(s_l) - \tilde{W}(s_l^*) \right] \quad \forall l = 1, 2, \dots, N \quad (16)$$

6.- If  $\tilde{W}(s)$  fails to satisfy both conditions (14) and (15), then begin the process again using  $\hat{W}(s)$  as the initial guess, where  $\hat{W}(s)$  is defined as in equation (16).

As mentioned above, it turns out that:

- departing from an initial guess, this iterative procedure always converge to a set of payoffs satisfying conditions (14) and (15) and, therefore, consistent with the equilibrium assignment being considered.
- by changing the initial guess, this iterative procedure produces different sets of equilibrium payoffs associated to the same equilibrium assignment.
- when  $I$  is large, there is very little difference between the different sets of equilibrium payoffs systems associated to a given equilibrium assignment. In fact, they all behave almost identically in terms of the skill premium, and the overall, *between-plants* and *within-plants* wage inequality, that are the most relevant variables for this paper.

## 5.4 Evolution of the equilibrium as $p$ decreases

From the results derived in the previous sections it is possible to know, for any price of capital, who is paired with whom in the equilibrium assignment and everybody's payoff. This information is enough to obtain the predictions of the model in terms of wage inequality and of segregation by skill as the price of capital decreases. It turns out that these predictions are *qualitatively* the same regardless of the specific parameter values considered. In this sense, all the results presented in this section were obtained using arbitrarily  $\beta = -1.5$ ,  $\theta = 0.5$ ,  $s_{max} = 100$  and  $I = 50$ .

To begin with note, from Figure 3, that the smaller the price of capital the higher the value of  $\lambda = \lambda(p)$ . Therefore, as  $p$  decreases, the equilibrium assignment in the economy changes qualitatively as in Figure 4.<sup>45</sup>

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<sup>45</sup>In each one of the equilibrium assignments depicted in Figure 4 there is an infinite number of intervals but only the first intervals are shown.

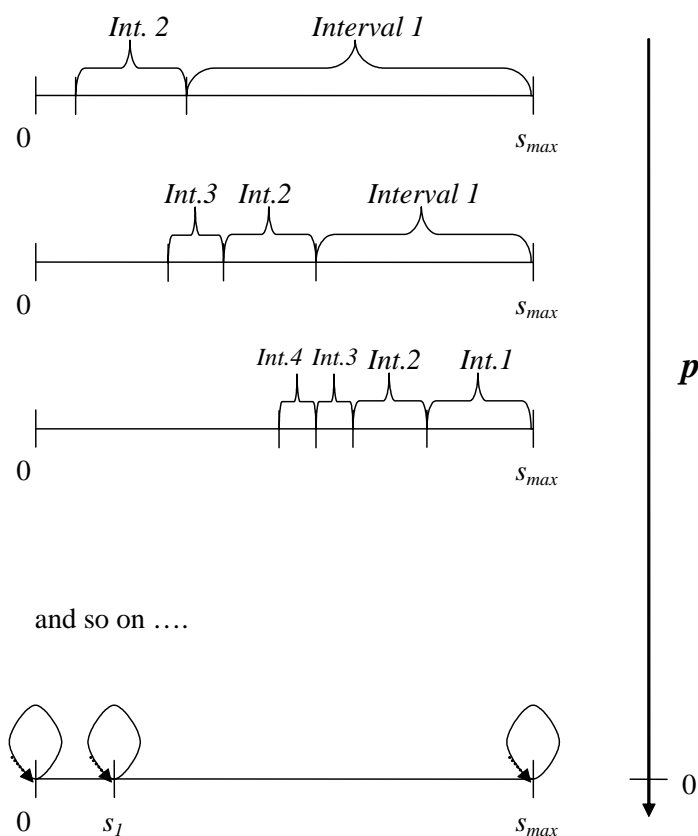


Figure 4. Evolution of the equilibrium assignment as  $p$  decreases

Obviously, this change in the equilibrium assignment affects the extent of individuals' segregation by skill in the economy. In this sense, Lemma 3 shows that individuals' segregation by skill increases in the model as the price of capital decreases.

*Lemma 3. As  $p$  decreases, the difference between the skill of the manager and the skill of the worker within plants in equilibrium decreases in average.*

Note also how this model integrates the equilibrium assignments in Kremer (1993) and in Lucas (1978) within the same framework as two extreme cases. In particular, when  $p = 0$  ( $\lambda = 1$ ) the equilibrium presents complete segregation of individuals by skill as in Kremer (1993). Instead, as  $p$  goes to infinite ( $\lambda$  goes to 0) the equilibrium assignment moves towards the one in Lucas (1978) in the sense that there is a cutoff level of skill such that everybody above that level is a manager and everybody below it is a worker. The essence of this paper comes from the transition from one kind of equilibrium to the other.

As for the predictions of the model regarding the evolution of wage inequality, consider the following decomposition of the overall wage inequality ( $\sigma_T^2$ ) into the *between-plants* ( $\sigma_{BP}^2$ ) and the *within-plants* ( $\sigma_{WP}^2$ ) wage inequality:<sup>46</sup>

$$\begin{aligned}\sigma_T^2 &= \frac{\sum_{l=1}^N [W(s_l) - \bar{W}]^2}{N} = \\ &= \frac{\sum_{j=1}^J 2 [\bar{W}^j - \bar{W}]^2}{N} + \frac{\sum_{j=1}^J \left( [W(s_1^j) - \bar{W}^j]^2 + [W(s_2^j) - \bar{W}^j]^2 \right)}{N} = \\ &= \sigma_{BP}^2 + \sigma_{WP}^2\end{aligned}\tag{17}$$

where  $\bar{W}$  is the average wage in the economy's equilibrium assignment and  $\bar{W}^j$  is the average wage in the  $j^{\text{th}}$  plant,  $j = 1, 2, \dots, J = N/2$ , that in the equilibrium assignment is composed of two individuals, one with skill  $s_1^j$  and another one with skill  $s_2^j$ . In the second line of equation (17), the first term is the variance in average wage across plants and the second term averages the wage inequality within each type of plant in equilibrium. Thus, one could consider the former a measure of wage inequality *between-plants*,  $\sigma_{BP}^2$ , and the latter a measure of wage inequality *within-plants*,  $\sigma_{WP}^2$ .

Figures 5 and 6 show that both  $\sigma_{BP}^2$ ,  $\sigma_T^2$  and the ratio  $\frac{\sigma_{BP}^2}{\sigma_T^2}$  increase continuously as  $p$  decreases. Instead,  $\sigma_{WP}^2$  first increases slightly when  $p$  decreases but eventually decreases towards 0 as  $p$  goes to 0.<sup>47</sup>

<sup>46</sup>This decomposition is similar to the one in Davis and Haltiwanger (1991).

<sup>47</sup>Recall that when  $p = 0$  the economy's equilibrium assignment is the *complete assortative assignment*. Therefore, when  $p = 0$  there is not skill heterogeneity within plants,  $\sigma_{WP}^2 = 0$  and  $\sigma_{BP}^2 = \sigma_T^2$ .

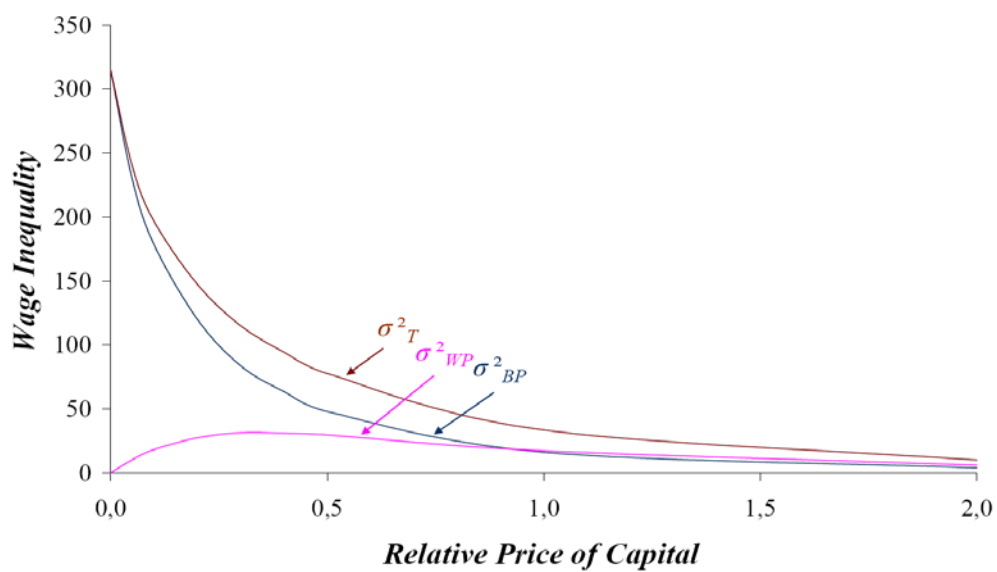


Figure 5. Evolution of the overall, *between-plants* and *within-plants* wage inequality.

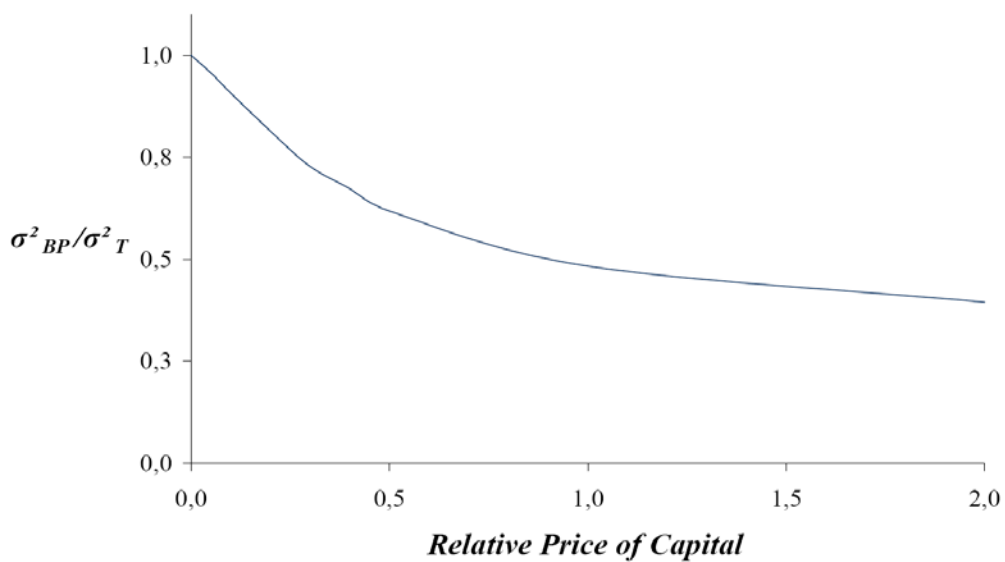


Figure 6. Evolution of  $\frac{\sigma_{BP}^2}{\sigma_T^2}$ .

Another measure of wage inequality frequently used in the literature is the skill premium. In this sense, one could define the skill premium in this model as the average wage in equilibrium for individuals with skill above  $s_{median}$  over the same measure for individuals with skill equal or lower than  $s_{median}$ . As Figure 7 shows, the model clearly predicts an increase in this measure of the skill premium when  $p$  decreases.<sup>48</sup>

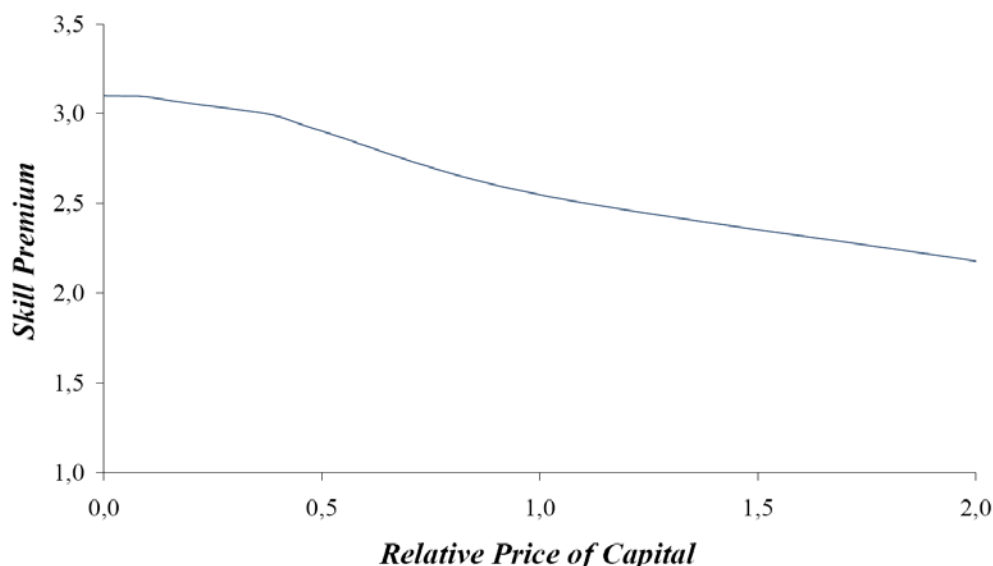


Figure 7. Evolution of the skill premium.

Finally, the model has another interesting prediction. In this sense, if one defines a plant's labor productivity as half the net output it produces, then the variance of labor productivity across plants coincides in this model with the variance of average wage across plants, that is, with  $\sigma_{BP}^2$ . Thus, according to Figure 5, the model also predicts an increase in the dispersion in labor productivity across plants when  $p$  decreases.<sup>49</sup> The same conclusion can be obtained considering the 90-10 differential of the log of labor productivity across plants, that also increases in the model when  $p$  decreases.

All these predictions of the model when the economy faces a declining relative price of capital (as observed empirically in the U.S.) are broadly consistent with the empirical evidence.<sup>50</sup> In this sense, as mentioned in the Introduction, the empirical evidence suggests that, *overall*, over the last few decades in the U.S. there has been:

<sup>48</sup>The prediction of the model that the skill premium increases as  $p$  decreases is robust to the cutoff level of skill used to compute the premium.

<sup>49</sup>This model considers that skill is perfectly observable. Alternatively, if someone who does not observe skill perfectly analyzes this economy, he would conclude that there are TFP differences across plants, and that the dispersion in TFP across plants increases when  $p$  decreases.

<sup>50</sup>Although not shown here, the model also predicts that both the aggregate output and the aggregate amount of capital in the economy increase smoothly as the price of capital declines. The

- an increase in the segregation of workers by skill into separate plants.<sup>51</sup>
- a large increase in the overall and the between-plants wage inequality and a slight increase in the within-plants wage inequality, and therefore an increase in the ratio  $\frac{\sigma_{BP}^2}{\sigma_T^2}$ .<sup>52,53</sup>
- an increase in the skill premium.<sup>54</sup>
- an increase in the dispersion of labor productivity across plants.<sup>55</sup>

These are *overall* patterns and the model, admittedly stylized, succeeds in capturing them. Of course, one would need to introduce additional elements into the model to be able to account for more detailed aspects of the evolution of these variables. For instance, to account for the decline in the skill premium observed in the U.S. during the second half of the 1970s (although it is clear in the literature that, *overall*, the skill premium has increased over the last few decades).

## 6 Conclusion

Empirical evidence for the U.S. suggests that, over the last few decades, (i) wage inequality *between-plants* has risen much more than wage inequality *within-plants* and (ii) there has been an increase in the segregation of workers by skill into separate plants. This paper presents a frictionless assignment model that is able to produce simultaneously these two features as the result of the decline in the relative price of

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same happens with the dispersion in capital holdings across plants. These predictions are also broadly consistent with the empirical evidence.

<sup>51</sup>As mentioned in the Introduction, Kremer and Maskin (1996) provide several pieces of evidence in this direction. For instance, they find that, in the U.S. manufacturing sector, production workers are now more likely to be together in the same plants (the correlation of a dummy variable for being production worker in the same plant rose from 0.195 to 0.228 between 1976 and 1987).

<sup>52</sup>This is documented in Dunne et al. (2004). In particular they find that both the overall and the *between-plants* wage inequality increased monotonically between 1975 and 1992 in the U.S. manufacturing sector, while the *within-plants* wage inequality only increased slightly and it even declined at some moments. As a consequence, the ratio  $\frac{\sigma_{BP}^2}{\sigma_T^2}$  increased from around 0.53 in 1977 to around 0.64 in 1992.

<sup>53</sup>As seen in Figure 5, the exact impact on wage inequality *within-plants* of a given decline in the relative price of capital is ambiguous. The model needs to be evaluated quantitatively in order to solve that ambiguity. This is the purpose of Gavilan (2006).

<sup>54</sup>A large empirical literature documents this feature. As an example, Autor, Katz and Krueger (1998) report that the log relative wage of college and post-college workers to high-school workers went from 0.465 in 1970 to 0.557 in 1996.

<sup>55</sup>This is documented in Dunne et al. (2004). In particular they find that the 90-10 differential of the log of labor productivity across U.S. manufacturing plants increased from around 1.7 to around 1.9 during the period 1975-92.

capital, while still obtaining results consistent with the evidence regarding the skill premium and the dispersion in labor productivity across plants.

It is important to mention that the connection shown in this paper between wage inequality, individuals' segregation by skill and the relative price of capital is not an exclusive feature of the particular functional form in (1) or of the particular version of the model considered in section 5. With these elements one can illustrate the connection more clearly but the same connection can be found numerically using different functional forms or skill distributions.<sup>56</sup>

In the model presented above a decline in the relative price of capital always increases wage inequality. However, empirically, while in the U.S. the relative price of capital has fallen at least since the 1950's, only after the 1970's wage inequality has increased substantially. In defence of the model one could argue two things. First, although the relative price of capital has fallen at least since the 1950's, Krusell et al. (2000) report that its rate of decline accelerated considerably in the period 1975-92 relative to the period 1954-75. This is precisely when wage inequality increased the most. And second, as mentioned in the Introduction, other factors besides the relative price of capital may have affected the extent of wage inequality in the economy too. Considering some of these factors (and their evolution over time) would certainly increase the empirical fit of the model. For instance, a natural extension of the general framework presented here would be to consider simultaneously both changes in the economy's skill distribution (as in Kremer and Maskin (1996)) and in the relative price of capital. In fact, a careful calibration of this extension of the model could be very informative about what fraction of the recent increase in wage inequality is due to changes in the economy's skill distribution and what is due to the decline in the relative price of capital.

Finally, a couple of extensions would increase the realism of the model and would make it more suitable for a quantitative examination. First, it would be nice to drop the restriction that plants need to have size two (in terms of individuals) and to introduce endogenous plant size. This could be done, for instance, within a hierarchical framework like the one in Garicano and Rossi (2005) and could produce interesting implications regarding the distribution of plant sizes in the economy and its evolution over time. And second, it would be nice to consider more than one sector in the economy. Then, individuals would have to allocate themselves across plants, occupations *and* sectors. This set up would be convenient, for instance, to understand how the assignment across sectors changes depending on their degrees of skill-biased technological change.

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<sup>56</sup>In the case of alternative functional forms, they must still satisfy the crucial properties established in section 2. Results under some alternative specifications are available from the author upon request.

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## Appendix

### Proof of Lemma 1

(i) *Existence.* The results in Gretsky et al. (1992) and in Kaneko and Wooders (1986), for a more general assignment problem than the one considered here, guarantee the existence of the equilibrium in this paper.

(ii) *Efficiency.* Consider that the equilibrium assignment for an arbitrary price of capital  $p$  is given by  $\{\Omega, \psi, W\}$  and produces an aggregate net output equal to  $Y$ . Now, by contradiction with statement (ii) of this Lemma, assume that there is another feasible assignment,  $\{\hat{\Omega}, \hat{\psi}, \hat{W}\}$ , that produces an aggregate net output  $\hat{Y} > Y$ .

We know that:

$$\begin{aligned}
 Y &= \int_{s \in M} h(s, \psi(s), p) \phi(s) \, ds = \\
 &= \int_{s \in M} [W(s) + W(\psi(s))] \phi(s) \, ds = \\
 &= \int_{s \in \hat{M}} [W(s) + W(\hat{\psi}(s))] \phi(s) \, ds
 \end{aligned} \tag{18}$$

In equation (18), the equality in the first line comes by definition. Instead, the equality in the second line comes from the equilibrium conditions (3) and (5). Finally, the equality in the third line holds since the assignments in  $\{\hat{\Omega}, \hat{\psi}, \hat{W}\}$  are also feasible given the skill distribution. Basically, the second and third lines in this equation are simply two different ways of aggregating the equilibrium wages under  $\{\Omega, \psi, W\}$ .

Analogously, we know that:

$$\hat{Y} = \int_{s \in \hat{M}} h(s, \hat{\psi}(s), p) \phi(s) ds \quad (19)$$

Now, combining equations (18) and (19), and using the fact that  $\hat{Y} > Y$ , one can easily see that there must exist at least one skill value  $s \in \hat{M}$  for which  $h(s, \hat{\psi}(s), p) > W(s) + W(\hat{\psi}(s))$ . But this contradicts condition (5) of the equilibrium assignment. Therefore, the assumption above that there exists a feasible assignment different from the equilibrium one that produces more aggregate net output is incorrect. This proves statement (ii) of this Lemma.

(iii) *Within plant assignment.* Consider that two individuals with skills  $a$  and  $b$ ,  $a \neq b$ , are together in the same plant in the equilibrium assignment for an arbitrary price of capital  $p$ . Without loss of generality, consider that  $a > b$ . Now, by contradiction with statement (iii) of this Lemma, assume that, in that plant, the individual with skill  $a$  is the worker and the individual with skill  $b$  is the manager. But this contradicts efficiency of the equilibrium assignment. This is so because, in that plant, the output could be larger by changing the assignment of occupations within the plant. That is,

$$h(a, b, p) > h(b, a, p)$$

Therefore, if this plant exists in the equilibrium assignment, the individual with skill  $a$  must always be the manager.

(iv) *Positive Sorting.* Consider that in the equilibrium assignment for an arbitrary price of capital  $p$  two individuals with skills  $a$  and  $b$ , such that  $a > b$ , are managers. Now, by contradiction with statement (iv) of this Lemma, assume that they are paired with workers of skills  $c$  and  $d$ , respectively, such that  $c < d$ .

By statement (iii) in this Lemma, we know that  $a \geq c$  and  $b \geq d$ . Combining these inequalities with the previous ones, we know that  $a > b \geq d > c$ . Under these circumstances, the fact that  $\frac{\partial^2 h(x, z, p)}{\partial x \partial z} > 0$  implies that:

$$h(a, d, p) + h(b, c, p) > h(a, c, p) + h(b, d, p)$$

This implies that the economy's aggregate net output is not maximized when individuals with skills  $a$  and  $b$  (such that  $a > b$ ) are paired with individuals of skills  $c$  and  $d$  (such that  $d > c$ ), respectively. Therefore, by statement (ii) in this Lemma, that cannot constitute an equilibrium assignment. This proves statement (iv) of this Lemma.

(v) *Payoff increasing with skill.* Consider, against statement (v) of this Lemma, that in the equilibrium assignment for an arbitrary price of capital  $p$  it happens that  $a > b$  but  $W(a) \leq W(b)$ . In this equilibrium, one of the following two cases must happen:

- Case 1: The individual with skill  $b$  is paired with an individual with skill  $a$ .
- Case 2: The individual with skill  $b$  is paired with an individual with skill  $c \neq a$ .

In both cases, someone has an incentive to deviate from the equilibrium assignment. This is a contradiction and, therefore, it must always happen that, under the equilibrium assignment,  $W(a) > W(b)$  whenever  $a > b$ .

In Case 1, the individual with skill  $a$  is better off by leaving the individual with skill  $b$  and matching with another individual with skill  $a$ . This is so because:

$$W(a) + W(a) \leq W(a) + W(b) = h(a, b, p) < h(a, a, p)$$

In Case 2, the individual with skill  $c$  is better off by leaving the individual with skill  $b$  and matching with one individual with skill  $a$ . This is so because:

$$W(a) + W(c) \leq W(b) + W(c) = \max \{h(b, c, p), h(c, b, p)\} < \max \{h(a, c, p), h(c, a, p)\} \blacksquare$$

## Proof of Lemma 2

(i) When  $\mu = \frac{1}{2}$ , the *complete assortative assignment* is the equilibrium assignment of the economy if  $p = 0$ .

Assume that in the equilibrium assignment when  $p = 0$  individuals with an arbitrary skill  $a$  are paired with individuals with skill  $b \neq a$ . In this case, since  $h(x, z, 0) = (1 - \theta)^{\frac{1}{2\beta}} (xz)^{\frac{1}{2}}$ , it is very easy to show that:

$$h(a, a, 0) + h(b, b, 0) > 2h(a, b, 0)$$

But this implies that the economy's aggregate net output is not maximized when individuals with skill  $a$  are paired with individuals with skill  $b \neq a$ . Therefore, by

Lemma 1, that can not constitute an equilibrium assignment, and the equilibrium assignment when  $p = 0$  requires that all the individuals of a given skill level are paired among themselves. That is,  $\Omega(s) = \{worker, manager\}$  and  $\psi(s) = s \quad \forall s \in [s_{min}, s_{max}]$  in the equilibrium.

(ii) When  $\mu = \frac{1}{2}$ , the *complete assortative assignment* is the equilibrium assignment of the economy *only if*  $p = 0$ .

Assume, by contradiction, that  $p \neq 0$  and the *complete assortative assignment* is the equilibrium assignment of the economy. Because in equilibrium none has incentive to move, it must happen (in particular) that,  $\forall a \in [s_{min}, s_{max}]$  and  $\forall \lambda \in [\frac{s_{min}}{a}, 1]$ :

$$F(a, \lambda a, p) \equiv h(a, \lambda a, p) - \frac{1}{2} [h(a, a, p) + h(\lambda a, \lambda a, p)] \leq 0 \quad (20)$$

Now note that  $F(a, \lambda a, p)|_{\lambda=1} = 0$  and that:

$$F_\lambda(a, \lambda a, p) = a \left[ h_2(a, \lambda a, p) - \frac{1}{2} [h_1(\lambda a, \lambda a, p) + h_2(\lambda a, \lambda a, p)] \right] \quad (21)$$

Evaluating (21) at  $\lambda = 1$  one gets that:

$$\begin{aligned} F_\lambda(a, \lambda a, p)|_{\lambda=1} &= \frac{a}{2} [h_2(a, a, p) - h_1(a, a, p)] = \\ &= \frac{a}{2} [f_2(a, a, k_{aa}^*) - f_1(a, a, k_{aa}^*)] \end{aligned}$$

where  $k_{aa}^*$  is the amount of capital that a plant composed of two individuals with skill  $a$  optimally buys and last the equality comes from applying the envelope theorem.

Now note that, for the production function in (1), since  $\mu = \frac{1}{2}$  it happens that:

$$f_2(a, a, k_{aa}^*) - f_1(a, a, k_{aa}^*) < 0 \quad \forall a$$

Therefore,  $F_\lambda(a, \lambda a, p)|_{\lambda=1} < 0$  which contradicts (20), as there must exist a  $\lambda$  sufficiently close to 1 for which  $F(a, \lambda a, p) > 0$ . This proves this part of the Lemma. ■

### Proof of Lemma 3

The difference between the skill of the manager and the skill of the worker for all plants in interval  $i$  in the equilibrium assignment for an arbitrary price of capital  $p$  is equal to:

$$s - \psi_i(s) = t_i = \lambda^{i-1} s_{max} \left( \frac{1 - \lambda}{2} \right)$$

Moreover, given the economy's skill distribution, the relative weight of interval  $i$  in the whole economy is equal to:

$$\int_{\lambda^i s_{max}}^{\lambda^{i-1} s_{max}} \phi(s) ds = \lambda^{i-1} (1 - \lambda)$$

Therefore, the average difference between the skill of the manager and the skill of the worker within the plants that exit in the equilibrium assignment for an arbitrary price of capital  $p$ ,  $dif(p)$ , is equal to:

$$dif(p) = \sum_{i=1}^{\infty} \left( \frac{s_{max}}{2} \right) (1 - \lambda)^2 \lambda^{2i-2} = \left( \frac{s_{max}}{2} \right) \frac{(1 - \lambda)^2}{1 - \lambda^2} = \left( \frac{s_{max}}{2} \right) \frac{1 - \lambda}{1 + \lambda}$$

Now, since  $\frac{\partial \lambda(p)}{\partial p} < 0$  (just see Figure 3), it is immediate to show that  $\frac{\partial dif(p)}{\partial p} > 0$ . ■

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