

**TESTING THE FORECASTING  
PERFORMANCE OF IBEX 35  
OPTION-IMPLIED RISK-NEUTRAL  
DENSITIES**

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## **Abstract**

The main objective of this paper is to test whether the risk-neutral densities (RNDs) implied in the prices of the future options contract on the Spanish IBEX 35 index accurately predict the distribution of future outcomes of the underlying asset. We estimate RNDs using both parametric and nonparametric procedures. We find that between 1996 and 2003 we cannot reject the hypothesis that the RNDs provide accurate predictions of the distributions of future realisations of the IBEX 35 index at four-week horizon. However, this result is not robust by subperiods. In particular, from October 1996 to February 2000, we find that RNDs are not able to consistently predict the actual realisations of returns. In this period, option prices assign a low risk-neutral probability to large rises compared with realisations. Tests based on the tails of the distribution show that RNDs significantly understate the right tail of the distribution for both the whole period and the first subperiod.

## 1 Introduction

Prices of European exchange-traded options on stock indices implicitly contain the risk-neutral density (RND hereafter) which is a key component for risk-neutral valuation. In this context, prices are the present value at the risk-free rate of their expected payoffs calculated under the RND. When the market is dynamically complete it is well known that the RND can be recovered from the corresponding option prices using the insights on Breeden and Litzenberger (1978). In particular, the RND is proportional to the second derivative of the option pricing function with respect to the exercise. In practice, however, there is no a continuum of exercise prices. Neither very low nor high exercises are available and, in any case, they are set at discrete intervals by market officials. This complicates the estimation of RND and, not surprisingly, numerous alternative methods have been proposed in literature which can be divided into parametric and nonparametric procedures.

Regarding the parametric methods that rely on specific assumptions on the data generating process, we may recall the generalized beta distribution employed by Anagnou, Bedendo, Hodges and Tompkins (2003) (ABHT hereafter); the two-lognormal mixture used by Melick and Thomas (1997), Bliss and Panigirtzoglou (2002), ABHT, Syrdal (2002) and Craig, Glatzer, Keller and Scheicher (2003); the Normal inverse Gaussian, as a special case of the generalized hyperbolic densities, suggested by Barndorff-Nielsen (1998) and ABHT; the expansion methods of Jarrow and Rudd (1982) and Rubinstein (1998), and of course the models for the stochastic process of Black and Scholes (1973), Heston (1993), Bates (1996), and Wu and Huang (2004).

On the other hand, we have the flexible, data-driven nonparametric methods of implied trees suggested by Rubinstein (1994); the smoothing techniques based on either kernel estimation employed by Ait-Sahalia and Lo (1998) and Rosenberg and Engle (2002), or the splines methods for implied volatility suggested by Campa, Chang and Reider (1998), Jackwerth (2000), Weinberg (2001), Bliss and Panigirtzoglou (2002), Syrdal (2002), Bliss and Panigirtzoglou (2004) (BP hereafter), and the positive convolution approximation of Bondarenko (2003).

The papers by Bliss and Panigirtzoglou (2002) and Bondarenko (2003) compare several competing procedures and conclude that nonparametric methods based on either the smoothed (spline) implied volatility smile and the positive convolution approximation seem to dominate the two-lognormal approach and other parametric techniques when estimating RNDs.

Noting that option prices should capture forward-looking distributions of the underlying assets, some researchers<sup>1</sup> and central banks<sup>2</sup> have used implied RNDs to proxy the market expectations of the distribution of the underlying asset or to forecast future outcomes<sup>3</sup>. They have the advantage relative to other historical time-series data that they are taken from a single point in time when looking toward expiration. Hence, they should be more responsive to changing expectations than competing alternatives. However, the existence of risk aversion means that RNDs will differ from the actual density from which realisations of returns are drawn.

Little is known about the ex-post assessment of the implied RNDs as a way of forecasting the actual realisations of the underlying asset at expiration. Surprisingly the only papers analyzing systematically the predictive ability of RNDs are Weinberg (2001),

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1. See Melick and Thomas (1997) or Campa, Chang and Reider (1998).

2. In the Financial Stability Review of the Bank of England this approach has been used to proxy the probability of large falls in equity prices.

3. Note that an assumption of rational expectations is behind this reasoning. If agents are in fact rational, their subjective density forecasts should be, on average, the distribution of realisations.

ABHT (2003), Craig, Glatzer, Keller and Scheicher (2003), and BP (2004). They reject that the observed return observations are realisations drawn from the implied RNDs. This may not be surprising given the risk-neutrality embedded in these estimates. In other words, these papers suggest that the forecasting differences arise from the risk aversion of the representative investor. In fact, by imposing a stationary utility function (a stationary risk aversion parameter), ABHT and BP test whether either power or exponential utility functions are improved forecasters of future values of the underlying<sup>4</sup>. In general, they are not able to reject the null that implied risk-adjusted densities are equal to the true density functions that has generated the data.

Against this background, this paper tests whether the RNDs implied in the prices of the future options contract on the Spanish IBEX 35 index accurately predict the distribution of future outcomes of the underlying asset. We focus on the four-week horizon, which is the longer non-overlapping horizon that allows us to maximise the number of observations, and use both parametric and non-parametric procedures. The results of this paper show that between 1996 and 2003 we cannot reject the hypothesis that the RNDs provide accurate predictions of the distributions of future realisations of the IBEX 35 index at four-week horizon. However this result is not robust by subperiods. More specifically, we find that RNDs are not able to consistently predict the realisations of returns from October 1996 to February 2000. In this period, option prices assign a low risk-neutral probability to large rises compared with realisations. Tests based on the tails of the distribution show that RNDs significantly understated the right tail of the distribution for both the whole period and the first subperiod. These results suggest that the ability of RNDs to forecasts future realisations might possibly be improved if risk preference adjustments were introduced.

This paper is organized as follows. Section 2 discusses how we estimate RNDs, while in Section 3 we present the testing procedures to assess the forecasting ability of our RNDs to check if they conform to the actual densities from which realisations are drawn. Section 4 contains the description of the data set used in the paper, and Section 5 reports the empirical results using RNDs. Conclusions follow in Section 6.

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4. Weinberg (2001) also studies this issue but he only adjusts the mean of the distribution to incorporate the average risk premium.

## 2 Estimating risk-neutral densities

As discussed in the introduction, there are several well developed methods to estimate RNDs. Given the previous empirical evidence, this paper employs two alternative approaches to estimate RNDs which are quite popular among the available possibilities in the parametric and nonparametric techniques: the two-lognormal mixtures of Melick and Thomas (1997) and the smoothed implied volatility smile of Bliss and Panigirtzoglou (2002).

Prices of European call options at time  $t$  on the underlying asset  $P$  with expiration at  $t+\tau$  and strike prices  $K$  are given by the well known expression<sup>5</sup>:

$$c(t, \tau, K) = e^{-r\tau} \int_K^{\infty} q(P_{t+\tau}) (P_{t+\tau} - K) dP_{t+\tau} \quad (1)$$

where  $q(P_{t+\tau})$  is the risk-neutral probability density function for the value of the underlying asset at time  $t+\tau$ . As pointed out by Breeden and Litzenberger (1978), if we differentiate (1) with respect to  $K$  we obtain

$$\frac{\partial c(t, \tau, K)}{\partial K} = -e^{-r\tau} \int_K^{\infty} q(P_{t+\tau}) dP_{t+\tau} \quad (2)$$

while differentiating twice we obtain the risk-neutral probability density function

$$\frac{\partial^2 c(t, \tau, K)}{\partial K^2} = e^{-r\tau} q(P_{t+\tau}) \quad (3)$$

In the parametric case, we assume that the RND function is given by a mixture of two-lognormal density functions. In particular

$$q(P_{t+\tau}) = \theta \log N(\alpha_1, \beta_1; P_{t+\tau}) + (1 - \theta) \log N(\alpha_2, \beta_2; P_{t+\tau}) \quad (4)$$

where  $\log N(\alpha_i, \beta_i; P_{t+\tau})$  is the  $i^{\text{th}}$  lognormal density with parameters  $\alpha_i$  and  $\beta_i$ :

$$\alpha_i = \ln P_t + \left( \mu_i - \frac{1}{2} \sigma_i^2 \right) \tau ; \quad \beta_i = \sigma_i \sqrt{\tau} ; \quad i = 1, 2 \quad (5)$$

and where  $\mu_i$  and  $\sigma_i$  are, respectively, the mean and standard deviation of associated normal distributions, and the stochastic process is based on two states with different first and second moments, governed by the weights  $\theta$  and  $1 - \theta$  for  $0 \leq \theta \leq 1$ . Thus, this is a flexible specification for the RND that is able to capture skewness and excess kurtosis and allows for a rich and wide range of shapes including bi-modal distributions, which would appear if, for example, market participants are placing a high weight on an extreme move in the underlying price but are unsure of its direction [Bahra (1997)].

Then, equation (1) and the corresponding put expression for alternative strike prices can be written as

$$c_j(t, \tau, K_j) = e^{-r\tau} \int_{K_j}^{\infty} [\theta \log N(\alpha_1, \beta_1; P_{t+\tau}) + (1 - \theta) \log N(\alpha_2, \beta_2; P_{t+\tau})] (P_{t+\tau} - K_j) dP_{t+\tau} \quad (6a)$$

5. The same reasoning can be done in term of put options.



$$p_j(t, \tau, K_j) = e^{-r\tau} \int_{K_j}^{\infty} [\theta \log N(\alpha_1, \beta_1; P_{t+\tau}) + (1-\theta) \log N(\alpha_2, \beta_2; P_{t+\tau})] (K_j - P_{t+\tau}) dP_{t+\tau} \quad (6b)$$

The numerical estimation of the five parameters,  $\alpha_1, \beta_1, \alpha_2, \beta_2, \theta$ , is obtained by minimising the squared pricing error as defined by the difference between the theoretical and observed option prices:

$$\min_{\{\alpha_1, \alpha_2, \beta_1, \beta_2, \theta\}} \left\{ \sum_{j=1}^{N_j} [c_j(t, \tau, K_j) - c_j^m]^2 + \sum_{h=1}^{N_h} [p_h(t, \tau, K_h) - p_h^m]^2 \right\} \quad (7)$$

subject to  $\beta_1, \beta_2 > 0$  and  $0 \leq \theta \leq 1$ , and where  $N_j$ ,  $N_h$ ,  $c_j^m$  and  $p_h^m$  stand respectively for number of calls, number of puts, market price of call  $j$  and market price of put  $h$ <sup>6</sup>.

The nonparametric method is based on the smoothing spline for fitting implied volatility curves introduced by Campa, Chang and Reider (1998), and studied in detail by Bliss and Panigirtzoglou (2002). They use a weighted natural spline which is a piece-wise cubic polynomial in order to fit a smoothing function to data. More precisely, the method developed by Bliss and Panigirtzoglou consists of smoothing the implied volatilities from the Black-Scholes formula through a cubic spline using delta rather than strike price as the independent variable. They argue that the transformation from strike space into delta space gives more relevance to the most liquid contracts of options which trade at strikes near the current spot price of the underlying asset. This property played a key role in the results reported by Bliss and Panigirtzoglou (2002) in terms of stability of the RND functions estimated using nonparametric methodology relative to the mixture of log-normals.

For a given smoothing parameter  $\lambda \geq 0$ , the smoothing spline is obtained by minimising the following objective function<sup>7</sup>:

$$\min_{f(\Delta, \Phi)} \lambda \sum_{j=1}^N \omega_j [\sigma_j^{\text{imp}} - f(\Delta_j, \Phi)]^2 + (1-\lambda) \int_0^{\infty} f''(\Delta, \Phi)^2 d\Delta \quad (8)$$

where  $N$  is the number of non-repeated strikes,  $\sigma_j^{\text{imp}}$  is the implied volatility for option  $j$ ,  $\Phi$ , the parameters that define the smoothing spline and  $f$  is a piece-wise cubic polynomial with knots points  $\Delta_j$  at the observed deltas. The weights,  $\omega_j$ , are given by the option vegas,  $v \equiv \partial c / \partial \sigma$ , so that less weighting is given to away-from-the-money options and, consequently, more weight is concentrated on liquid trades. The smoothness is determined by the parameter  $0 \leq \lambda \leq 1$ , which controls how much to penalize departures from smoothness in the spline function  $f$ . The usual procedure to choose  $\lambda$  is the method of generalized cross validation, where we find a value of  $\lambda$  that minimises the error

$$\sum_{k=1}^N \omega_j [\sigma_k^{\text{imp}} - f_{\lambda}^k(\Delta_k, \Phi)]^2 \quad (9)$$

where  $f_{\lambda}^k$  is the minimisation of equation (8), for a given  $\lambda$ , with data point  $k$  omitted. Hence, this method finds an optimal  $\lambda$  by lowering the influence of outlying data points on the curve. In any case, BP imposes  $\lambda = 0.99$  and argue that the forecast results are insensitive to the choice of  $\lambda$ .

Finally, once the spline,  $f(\Delta, \Phi)$ , is fitted, 15,000 points along the function are converted back to price/strike space using Black-Scholes formula, and the same at-the-money implied volatility employed for the previous strike-to-delta conversion. All call price/strike data points are then used to numerically differentiate the call price function to obtain the estimated RND for each cross-section.

6. Note that the mean of a RND is the futures price. Some papers include in equation (7) the difference between the futures price and the expected value of the underlying asset at  $t+\tau$ . In our sample the impact on the estimated parameters of the introduction of this additional term is negligible.

7. It should be recalled that  $0 \leq \Delta \leq e^{r\tau}$ .

### 3 Testing the forecasting performance of risk-neutral densities

To study the predicting ability of the estimated RNDs, we first employ a method based on the relationship between the data generating process (the true density function),  $f_{t,\tau}(p_{t+\tau})$ , and the estimated sequence of density forecasts,  $q_t(p_{t+\tau})$ , as related through the probability integral transform,  $z_{t,\tau}$ , of the realisation of the process taken with respect to the density forecast, where  $\tau$  represents the forecasting horizon. In other words, each cross-section of options at time  $t$  for a given time-to-expiration  $\tau$  produces an estimated RND,  $q_t(p_{t+\tau})$ . We want to test the hypothesis that our estimated  $q_t(p_{t+\tau})$  are equal to  $f_{t,\tau}(p_{t+\tau})$ . Note of course that we have an estimated RND for a given expiration and only one realisation,  $P_{t+\tau}$ , is available on a given date and for a particular expiration. The probability integral transform is defined as

$$z_{t,\tau} = \int_{-\infty}^{P_{t+\tau}} q_{t,\tau}(u) du = Q_{t,\tau}(P_{t+\tau}) \quad (10)$$

Hence,  $z_{t,\tau}$  is equal to the probability value of the estimated cumulative density function,  $Q_{t,\tau}(\cdot)$ ,  $\tau$  days ahead at the realisation of the underlying on day  $t+\tau$ ,  $P_{t+\tau}$ .

As shown by Diebold, Gunther and Tay (1998), under independence and if the forecasts and the true densities coincide, then the sequence of the probability integral transforms,  $z_{t,\tau}$ , is uniformly distributed as  $U(0,1)$ . Berkowitz (2001) proposes a parametric approach for jointly testing uniformity and independence. In particular, a further transformation,  $x_{t,\tau}$ , of the inverse probability transform,  $z_{t,\tau}$ , is defined using the inverse of the standard normal cumulative density function,  $N(\cdot)$ :

$$x_{t,\tau} = N^{-1}(z_{t,\tau}) = N^{-1}\left(\int_{-\infty}^{P_{t+\tau}} q_{t,\tau}(u) du\right) \quad (11)$$

under the null,  $q_{t,\tau}(p_{t+\tau}) = f_{t,\tau}(p_{t+\tau})$ ,  $x_{t,\tau} \approx$  i.i.d.  $N(0,1)$ . In order to estimate the independence and standard normality of the  $x_{t,\tau}$ , Berkowitz suggests the following autoregressive model<sup>8</sup>

$$x_{t,\tau} - \mu = \rho(x_{t-1,\tau} - \mu) + \varepsilon_{t,\tau} \quad (12)$$

which is estimated using maximum likelihood and then testing the corresponding restrictions by a likelihood ratio test. The log-likelihood function,  $L(\mu, \sigma^2, \rho)$ , associated with the model in (12) is given by<sup>9</sup>

$$L(\mu, \sigma^2, \rho) = -\frac{1}{2} \log(2\pi) - \frac{1}{2} \log\left[\frac{\sigma^2}{(1-\rho^2)}\right] - \frac{[x_{t,\tau} - \mu/(1-\rho)]^2}{2\sigma^2/(1-\rho^2)} - \frac{n-1}{2} \log(2\pi) - \frac{n-1}{2} \log(\sigma^2) \\ - \sum_{t=2}^T \left[ \frac{(x_{t,\tau} - \mu(1-\rho) - \rho x_{t-1,\tau})^2}{2\sigma^2} \right] \quad (13)$$

Note that, under the assumptions of the model, the parameters should be equal to  $\mu = \rho = 0$  and  $\sigma^2(\varepsilon_{t,\tau}) = 1$ . Then, the likelihood ratio statistic,  $LR = -2[L(0,1,0) - L(\hat{\mu}, \hat{\sigma}^2, \hat{\rho})]$ , is distributed as  $\chi^2(3)$  under the null hypothesis.

When the available data implies that we have to test overlapping forecasts, a potential rejection may be due from the overlapping nature of the data, which may produce autocorrelation. Berkowitz also proposes to test the independence assumption separately

<sup>8</sup> Berkowitz (2001) shows that higher order autoregressive processes results in increasing the number of parameters and reduced power. Also, BP (2004) compare alternative tests and conclude that the Berkowitz tests is more reliable in small samples.

<sup>9</sup> See ABHT (2003).

by the alternative likelihood ratio statistic given by  $LR(\mathbf{i}) = -2[L(\hat{\mu}, \hat{\sigma}^2, 0) - L(\hat{\mu}, \hat{\sigma}^2, \hat{\rho})]$  which is distributed as  $\chi^2(1)$  under the null hypothesis. In this paper we also test for autocorrelation of different powers of residuals to control for non-linear dependence.

As explained by BP (2004), if  $LR$  rejects the hypothesis, failure to reject  $LR(i)$  provides evidence that the estimated RNDs are not producing accurate forecasts of the true density. However, if both  $LR$  and  $LR(i)$  reject, it is not possible to conclude if there is lack of predicting ability or serial correlation. Finally, failure to reject both  $LR$  and  $LR(i)$  would be consistent with forecasting capacity.

Unlike most previous papers testing the forecasting ability of RNDs, we not only want to test the performance of the whole body of the distribution, but also analyse the performance of the tails of the distribution. We follow ABHT (2003) in employing the scoring rules based on the distance between the forecasted probability mass,  $q_{t,\tau}^{\text{tail}}$ , in a given tail and a binary variable,  $R_{t,\tau}$ , which takes the value of 1 if the actual realisation of the underlying falls in the tail, and 0 otherwise. The so called Brier score is given by

$$B = \frac{1}{T} \sum_{t=1}^T 2(q_{t,\tau}^{\text{tail}} - R_{t,\tau})^2 \quad (14)$$

which takes values between 0 and 2 and a better performance is captured by smaller values for the score. To test if it departs from its expected value,  $\sum_{t=1}^T q_{t,\tau}^{\text{tail}}(1 - q_{t,\tau}^{\text{tail}})$ , the following statistic, suggested by Seillier-Moisewisch and Dawid (1993), is employed:

$$ASN = \frac{\sum_{t=1}^T (1 - 2q_{t,\tau}^{\text{tail}})(R_{t,\tau} - q_{t,\tau}^{\text{tail}})}{\left[ \sum_{t=1}^T (1 - 2q_{t,\tau}^{\text{tail}})^2 q_{t,\tau}^{\text{tail}}(1 - q_{t,\tau}^{\text{tail}}) \right]^{\frac{1}{2}}} \quad (15)$$

which is asymptotically distributed as a standard normal.

## 4 Data

In this research, we employ the European-style Spanish equity option contract on the IBEX 35 futures which is one of the largest options equity market within the euro area. The Spanish IBEX 35 index is a value-weighted index comprising the 35 most liquid Spanish stocks traded in the continuous auction market system. The official derivative market for risky assets, which is known as MEFF, trades a futures contract on the IBEX 35, the corresponding option on the IBEX 35 futures contracts for calls and puts, and individual futures and option contracts for blue-chip stocks. The option contract on the IBEX 35 futures is a cash settled European option with trading over the three nearest consecutive months and the other three months of the March-June-September-December cycle. The expiration day is the third Friday of the contract month. The multiplier is 1 € and the exercise prices are given by 50 index point intervals. Our database is comprised of settlement IBEX 35 index futures prices, the associated settlement prices of all call and put options traded on each day, and the implied volatility for each option. Moreover, for each option we also have the expiration date and the associated strike. At expiration, the options settle to the exchange delivery settlement future price determined by MEFF by calculating the arithmetic average between 16:15 and 16:45 taking an index value per minute. This series is employed to compute the payoffs of the future in this work<sup>10</sup>.

The options prices employed throughout our research are the MEFF-reported settlement prices. The implied volatility for all at-the-money options reflects the closing market price of each option. For the rest of strikes, the implied volatility is linearly approximated by two segments. MEFF employs two different slopes for strikes corresponding to options in-the-money and out-of-the-money. The slopes are obtained according the closing market conditions of the market on each Friday which will be the day from which forecasts are made in our study. The settlement prices are calculated using Black's (1976) formula, the underlying settlement price and the previous volatilities. Therefore, by construction all option prices reflect closing market conditions and are synchronous with the underlying asset price. The data cover the period from October 1996 through December 2003, i.e. 87 months<sup>11</sup>.

Option settlement prices are available for expirations from one week to one year. It is very important to point out that a target observation date in the study is determined four weeks before every option expiration. The number of strikes ranges between 23 and 211 with an average of 103. As in BP (2004), we are particularly concerned with overlapping data. Options with expires of less than three months, expire at monthly intervals. Forecasts and realisations for horizons less than or equal to one month may be expected to be independent. However, for forecast beyond one month, the price path of the underlying asset begins to overlap and thus contain some common information which makes implausible the assumption of independence. For this reason, we keep our research to the maximum available number of non-overlapping weeks. The number of cross-sections is 87 for a forecast horizon of four weeks. This is similar to the cross-sections employed by BP (2004) in the case of their data on FTSE 100, and just half of the available cross-sections for options on the S&P 500.

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<sup>10</sup>. To the best of our knowledge this is the first paper which analyses the information content of IBEX 35 options. Manzano and Sánchez (1998) extract RNDs from short-term interest rates options traded at MEFF.

<sup>11</sup>. Before this date MEFF computed settlement prices using constant implied volatilities.

## 5 The empirical performance of risk-neutral density predictability

As described in Section 2, four weeks before each option expiration in our sample period, we estimate the RNDs using both the mixture of two lognormals and splines.

Chart 1 contains an example of the estimation of the RNDs employing a cross-section of available options for four expiration days in our sample period using the above procedures. The densities are evaluated in log returns using the future prices observed four weeks before. Note that the mean of the distribution is close to zero, reflecting the risk-neutrality. Panels A and B show four RNDs estimated for two distinct days. Panel A shows the estimations under both procedures just after the Russian crisis of August 1998, while Panel B contains the estimation for the peak of the bull market at the beginning of 2000. Both procedures show similar densities with a relatively pronounced left tail. They suggest that the market expectations were assigning a higher probability mass to falling prices in the near future than to rising prices. However, the left tail of the density estimated under the spline nonparametric method is less smooth than the one estimated using the mixture of two lognormals. At the same time, Panels C and D show four RNDs estimated using data before and after the shocking events of 11 September 2001. Generally speaking, as before, RNDs under both procedures are similar. However, it must be noted that nonparametric spline assigns a higher probability around the mean than the mixture of lognormals for 19 October 2001. Note also that the probability of large movements in a four week horizon, in particular falls, increases significantly between the last two dates.

Chart 2 displays the time series of the mean, standard deviation, skewness and excess kurtosis from the 87 RNDs using the two procedures. Table 1 reports the mean and the standard deviation of these moments for the whole sample and for two subperiods. It appears that all these moments, under both procedures, show a time-varying behaviour during the sample period. For example, skewness, which is consistently negative under both estimation procedures, changes from -0.04 to -0.83 with a mean value of -0.41 using the mixture of lognormals (under the spline methodology results are almost identical). These are similar to the findings of Craig, Glatzer, Keller and Scheicher (2003) for the DAX index, and considerably lower than the average skewness of -1.1 found by ABHT (2004) for the S&P 500. This evidence suggests that risk-neutral probability of large negative shocks to the Spanish stock market is higher than risk-neutral probability of large positive shocks. On the other hand, average excess kurtosis under the mixture of lognormals is 0.45 which is lower than the values reported by Craig, Glatzer, Keller and Scheicher, BP and ABHT for DAX, FTSE 100 and S&P 500 of 0.9, 4.3 and 5.4 respectively. Excess kurtosis is systematically lower when we estimate RNDs using splines, with average excess kurtosis of 0.25.

This time-varying behaviour of all moments is reflected in the allocation of the probability mass between the centre and the tails of the distribution. To understand the behaviour of RNDs over time, it is important to note that this allocation presents a time-varying behaviour reflecting the uncertainty incorporated into prices. This is easily appreciated in Panels A and B of Chart 3, where we plot the mean and the median together with the 5 percent and 95 percent estimated percentiles of the RNDs using, respectively, the mixture of lognormals and the spline methodology. Independently of the estimation procedure employed, at the beginning and the end of the sample, the distance is comparatively small so that densities have more probability mass around the median. The opposite evidence is observed in the middle of the sample. Along these lines, Panel C of Chart 3 shows the difference between the distances of 5 percent and 95 percent percentiles

relative to the median for both procedures. It turns out that, from December 1997 to November 1998, and also from January 2000 to March 2000, and for both procedures, the width of the interval increases asymmetrically quite a lot. It should be realised that not only the distance around the median increases, but it becomes particularly large at the left side of the distribution. Other increasing periods correspond to events around 11 September 2001 and fall during 2002. This captures the rising of uncertainty about the future behaviour of the Spanish stock market, and especially it suggests that the market is placing more probability mass into potential crashes than to potential large rises. This asymmetric behaviour is systematically more pronounced along the sample period when the nonparametric splines are used. On the other hand, extreme events seem to be equally identified by both estimation devices, although the impact is much larger with splines. Thus, independently of the estimation procedure, the conflicting fall of 1998 and the first months of 2000 have the highest asymmetric distance which indicates that the market was more concerned with larger falls of the IBEX 35 index. Hence, in principle, the shifts of allocation of probability mass of RNDs estimated from option prices provides to investors and analysts with an interesting device to understand expectations of the market regarding the immediate risks embedded in financial assets. However, of course, it is first necessary to study the forecasting ability of our estimated RNDs.

In order to determine whether there is evidence that the RNDs adequately forecast the distribution of ex-post realisations of the underlying indices, we first employ the Berkowitz test statistics discussed in Section 3. Table 2 shows the empirical results using both the mixture of two lognormals and splines as the estimation of RNDs for the Spanish stock IBEX 35 index. The results are practically identical under both procedures. For the whole sample period, we cannot reject the hypothesis that the RNDs provide accurate predictions of the distributions of future realisations of the IBEX 35 index at the four-week horizon. On the one hand, with a p-value of 0.300 (0.308) for the  $LR$  test statistic for lognormals (splines), we do not support that the RND forecasts poorly the actual realisations<sup>12</sup>. At the same time, by looking at the  $LR(i)$  statistics we cannot reject the hypothesis that the probability integral transforms are uncorrelated. Autocorrelation tests based on different power of residuals confirm the lack of dependence. These results contrast the empirical evidence found by ABHT (2003) and BP (2004) for FTSE 100 and S&P 500. Hence, we divide the whole sample period into two non-overlapping sub-periods from October 1996 to February 2000, and from March 2000 to December 2003. This allows us to check the robustness of the surprising results found for the complete sample. As reported in Table 2, in the first sub-period and independently of the procedure used, the Berkowitz test rejects the hypothesis that the RNDs are good forecasts of future realisations of the IBEX 35 index. Moreover, the  $LR(i)$  and the autocorrelation tests of power of residuals show that the reason for rejecting is not the violation of the independence assumption underlying the test statistic. This result is consistent with the intuition that RNDs are very unlikely to adequately capture the future behaviour of equity prices. It seems reasonable to expect that the stock market prices risks<sup>13</sup>. As in ABHT (2003) and BP (2004), this result also confirms that the Berkowitz test has sufficient power to reject the null hypothesis. Finally, as in the case of the complete sample, the  $LR$  statistic is not able to reject the good predictive performance of the RNDs during the second sub-period. This is interesting since the years of the second sub-period coincide with a continuous negative performance of the stock

<sup>12</sup>. Berkowitz (2001) approach is very sensitive to outliers, which can arise if the underlying uniform random variables are close to either 0 or 1. To check for this potential problem we have marginally perturbed observations close to 0 and 1 and qualitatively results remain unchanged. We also have checked the impact of parameter uncertainty of the estimated RNDs in our results. To do so we have used the Hessian at the maximum likelihood solution as the estimated parameter variance-covariance matrix and then carried out a Monte Carlo simulation assuming the parameters were multivariate normals to randomly perturb them, recomputing 100 series of RNDs. Qualitative results based on LR tests are unchanged in all 100 series. We thank a referee for suggesting this approach.

<sup>13</sup>. See the recent evidence of Ghysels, Santa-Clara and Valkanov (2004).

market, and the opposite occurred from October 1996 to February 2000. Craig, Glatzer, Keller and Scheicher (2003) also split their sample, which basically coincides with ours, and find similar results for the DAX index<sup>14</sup>. Therefore it seems that the difference between our results and those reported by ABHT (2003) and BP (2004) has more to do with the sample period (their sample ends in mid-2001) than the market<sup>15</sup>.

Our results suggest that during the first sub-period RNDs are unable to place enough probability on the right tail of the distribution relative to actual realisations. Simultaneously, it seems that RNDs contain too much probabilistic mass in the left tail of the distribution, which suggests that option prices assigned a high risk-neutral probability to potential crashes which were not confirmed by realisations. This would explain the poor performance of the RNDs during the first half of the sample. The bear market of the second sub-period may have alleviated the overpricing of out-of-the-money (in-the-money) puts (calls), and underpricing of in-the-money (out-of-the-money) puts (calls). We now turn to investigate this potential explanation by analyzing the behaviour of the tails.

Table 3 presents the results from the tests designed to analyse the misspecification of the estimated RNDs on the tails under both the mixture of two lognormals and splines. For the right tail, the left tail and the combination of both tails we compare the frequency with which realisations lie on those areas with the probability mass assigned by the estimated RNDs. We also report the test statistic given by equation (15). These tests indicate that, for the second sub-period, the probability mass left in both tails are a good prediction of the actual frequency with which realisations of the IBEX 35 index at expiration lie into those tails. As before, however, during the first sub-period the results do not seem to be as favourable as at the end of our sample. In particular, from October 1996 to February 2000, the probability mass assigned by both our parametric and nonparametric specifications of the RND to the right tail significantly underestimates the frequency of actual realisations. In other words, the performance of the stock market during the first sub-period is not adequately forecasted by the RNDs estimated from option prices. In a surprising 22 percent of realisations the IBEX 35 future rose more than 10 percent from the levels four weeks before the expiration of the option, which contrasts with the 8 percent probability mass assigned by the estimated RNDs. The RNDs also assigned a 10 percent probability to the left tail (a fall more than 10%) compared to 4.9 percent of actual realisations. Again, it seems that option prices are assigning a high risk-neutral probability to potential crashes which are not confirmed by realisations. However, this difference is not statistically significant. For the right tail of the distribution the same results are observed for the full sample period.

These results are consistent with the evidence reported for the full body of the implied RNDs. There seems to be a good performance of RNDs from March 2000 to December 2003, and a relatively bad performance of our parametric and nonparametric estimates during the bull market of the first sub-period which may be explained by both the high frequency of realisations on the right tail of the distribution and the mean of the risk-neutral distribution which seems to be understating the mean of the actual distribution. Of course, this would be expected since those differences may be arising from the risk aversion of the representative investor. In any case, it is important to note that a risk premium adjustment may not be sufficient to adequately capture actual realisations. The evidence from the tails suggests that the risk aversion adjustment should be time-varying to reflect the business cycle behaviour embedded in the stock market.

The empirical results for the complete sample and both sub-periods are also presented in Chart 4. It represents the deviations of the empirical density from

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<sup>14</sup> Craig, Glatzer, Keller and Scheicher (2003) note that using data up to mid-2001 they reject the hypothesis that the RNDs are good forecasts of future realisations of the DAX index. Then their results and possibly ours are consistent with those reported by both ABHT (2003) and BP (2004), who use a sample that ends at around mid-2001.

<sup>15</sup> As a matter of fact, using a sample ending in mid-2001 we can reject the null that RNDs are good forecasts of the futures outcomes of the IBEX 35 index.

individual quantiles. In each of the three panels we plot the actual density of the integral-transformed realisations,  $z_{t,t}$ , against the theoretical density, which is a uniform function between 0 and 1 and, therefore its cumulative distribution function is the 45 degree line. Our purpose is to provide a visual and more intuitive way of understanding our empirical results. The vertical axis represents the cumulative probabilities and the horizontal axis is some bin number  $n$ , where  $n$  is between 0 and 200. Hence, the  $n^{\text{th}}$  bar in Chart 4 is the sum of the observed  $z_{t,t}$ 's that are equal to or less than  $n/200$ . Under the null hypothesis the number of  $z_{t,t}$ 's in bin  $n$  is always equal to  $n/200$ . Moreover, to assess whether deviations of the theoretical values are significant we show +/- two standard deviations confidence intervals. These charts contain both the results for the mixture of two lognormals and for splines. As before, in our empirical applications, the results obtained under both procedures are indistinguishable.

Once again, for the complete sample, the empirical distribution approximates reasonably well the theoretical distribution lying into the confidence intervals. However, the behaviour of both sub-samples is very different. The empirical distribution function tends to be below the theoretical counterpart from October 1996 to February 2000, while for the second sub-period, the empirical function is generally above the theoretical distribution. It is clear that the RNDs of the first sub-period contain more forecasting errors since some segments of the theoretical function are even outside the confidence intervals. This evidence suggests that during the bull market of the first sample period, the true distribution assigned more probability to high returns than the estimated RNDs. This is of course consistent with results reported in Tables 2 and 3. Over the bull year market participants seem to be less optimistic about the future behaviour of the market as compared with realisations. The reverse case is observed during the second sub-period, although less intensity is found. Either the mixture of lognormal densities or the splines do not place enough probability mass at the left side. This evidence is also found by Craig, Glatzer, Keller and Scheicher (2003) for the DAX index and a period similar to ours.

Finally, Table 4 contains a summary statistics for the moments of the actual and forecasted time series distributions of 4 weeks forecast horizon based on log returns of the IBEX 35 future for the complete sample from October 1996 to December 2003, and the two sub-periods. From the estimated RNDs, we generate 10,000 series of log-returns, where the return on a given date is obtained from the RND of that particular period. For each of those series, we calculate the moments of the distribution over time. Lastly, from these 10,000 realisations we calculate its mean and standard deviation and obtain a confidence interval of +/- two standard deviations. If the theoretical distributions are correct, we should expect that the realised moments from the actual sample lie on the confidence interval. Once again, the two sub-samples show different estimated moments relative to the actual values. As expected, given our previous results and for both estimation procedures, the realised mean over the first sample is much higher than the upper bound obtained from the RNDs. For all other moments there is enough variability to capture realised moments. These results suggest that, as expected, risk premium adjustments are necessary, at least during the first subperiod.



## 6 Conclusions

Option prices provide information about how investors assess the likelihood of alternative outcomes for future market prices of underlying assets. More specifically they contain the RND of the price of the underlying asset, which have the advantage relative to other historical time-series data that they are taken from a single point in time when looking toward expiration. Hence, they should be more responsive to changing expectations than competing alternatives. However, the existence of risk aversion means that RNDs will differ from the actual density from which realisations of returns are drawn.

The main objective of this paper is to analyse the value of information of implied RNDs contained in prices of options on the IBEX 35 index at the Spanish Stock Exchange Market and, more specifically, to test their forecasting ability to predict the distribution of the futures outcomes of the IBEX 35 index. The RNDs are estimated by both parametric and nonparametric procedures. Our results show that the moments estimated under any of the two techniques have a time-varying behaviour, although the estimates of both skewness and kurtosis are more pronounced under the spline methodology. Moreover, our estimations seem to capture the rising uncertainty about the future behaviour of the Spanish stock market during distress time periods, and it suggests that the market places more probability mass into potential crashes than the mass placed into large rises. This asymmetric behaviour and the forecasting ability of RNDs is practically the same under both procedures. In particular, between 1996 and 2003, we cannot reject the hypothesis that the RNDs provide accurate predictions of the distributions of future realisations of the IBEX 35 index at four-week horizon. However, this result is not robust to the sample period chosen. More specifically, when the whole period is divided into two subperiods, we find that RNDs are not able to consistently predict the outcomes of the price of the underlying asset from October 1996 to February 2000. In this period, option prices assign a low risk-neutral probability to large rises compared with realisations. This is confirmed by the analysis of the tails of the distributions and by comparing the averages statistics for the moments of the actual and forecasted time series distributions. These results tend to confirm the necessity of risk premium adjustments with a (probably) countercyclical risk aversion parameter, which seems to be especially relevant for bull markets. Another extension of this paper would be the analysis of the forecasting ability of RNDs for other horizons. These are key aspects of our future research agenda.

## REFERENCES

- AÏT-SAHALIA, Y. and A. LO (1998). "Nonparametric estimation of state-price densities implicit in financial asset prices", *Journal of Finance*, N.º 53, pp. 499-547.
- ANAGNOU, I., M. BEDENDO, S. HODGES and R. TOMPKINS (2003). *The relation between implied and realized probability density functions*, Working Paper, Financial Options Research Centre, University of Warwick.
- BARNDORFF-NIELSEN, O. (1998). "Processes of Normal inverse Gaussian type", *Finance and Stochastic*, N.º 2, pp. 41-68.
- BATES, D. (1996). "Jumps and stochastic volatility: exchange rate processes implicit in deutsch mark options", *Review of Financial Studies*, N.º 9, pp. 69-107.
- BERKOWITZ, J. (2001). "Testing density forecasts with applications to risk management", *Journal of Business and Economic Statistics*, N.º 19, pp. 465-474.
- BHARA, B. (1997). *Implied risk-neutral probability density functions from option prices: theory and application*, Working Paper N.º 66, Bank of England.
- BLACK, F. (1976). "The pricing of commodity contracts", *Journal of Financial Economics*, N.º 3, pp. 167-179.
- BLACK, F. and M. SCHOLES (1973). "The pricing of options and corporate liabilities", *Journal of Political Economy*, N.º 81, pp. 637-654.
- BLISS, R., and N. PANIGIRTZOGLU (2002). "Testing the stability of implied probability density functions", *Journal of Banking and Finance*, N.º 26, pp. 381-422.
- (2004). "Option-implied risk aversion estimates", *Journal of Finance*, N.º 59, pp. 407-446.
- BONDARENKO, O. (2003). "Estimation of risk-neutral densities using positive convolution approximation", *Journal of Financial Econometrics*, N.º 116, pp. 12.
- BREEDEN, D., and R. LITZENBERGER (1978). "Prices of state contingent claims implicit in option prices", *Journal of Business*, N.º 51, pp. 621-652.
- CAMPA, J., K. CHANG and R. REIDER (1998). "Implied exchange rate distributions: evidence from OTC option markets", *Journal of International Money and Finance*, N.º 17, pp. 117-160.
- CRAIG, B., E. GLATZER, J. KELLER and M. SCHEICHER (2003). *The forecasting performance of German stock option densities*, Discussion Paper N.º 17, Studies of the Economic Research Centre, Deutsche Bundesbank.
- DIEBOLD, F., T. GUNTHER and A. TAY (1998). "Evaluating density forecasts, with applications to financial risk management", *International Economic Review*, N.º 39, pp. 863-883.
- GHYSELS, E., P. SANTA-CLARA and R. VALKANOV (2004). "There is a risk-return tradeoff after all", forthcoming in the *Journal of Financial Economics*.
- HESTON, S. (1993). "A closed-form solution for options with stochastic volatility with applications to bond and currency options", *Review of Financial Studies*, N.º 6, pp. 327-343.
- JARROW, R., and A. RUDD (1982). "Approximate option valuation for arbitrary stochastic processes", *Journal of Financial Economics*, N.º 10, pp. 347-369.
- MANZANO, Mª C., and I. SÁNCHEZ (1998). *Indicators of short-term interest rate expectations. The information contained in the options market*, Documento de Trabajo N.º 9816, Banco de España.
- MELICK, W., and C. THOMAS (1997). "Recovering an asset's implied PDF from option prices: an application to crude oil during the Gulf crisis", *Journal of Financial and Quantitative Analysis*, N.º 32, pp. 91-115.
- ROSENBERG, J., and R. ENGLE (2002). "Empirical pricing kernels", *Journal of Financial Economics*, N.º 64, pp. 341-372.
- RUBINSTEIN, M. (1994). "Implied binomial trees", *Journal of Finance*, N.º 49, pp. 771-818.
- (1998). "Edgeworth binomial trees", *Journal of Derivatives*, N.º 5, pp. 20-27.
- SEILIER-MOISEWISCH, F., and PH. DAWID (1993). "On testing the validity of sequential probability forecasts", *Journal of the American Statistical Association*, N.º 88, pp. 355-359.
- SYRDAL, S. A. (2002). *A study of implied risk-neutral density functions in the Norwegian option market*, Working Paper N.º 13, Norges Bank, Norway.
- WEINBERG, S. (2001). *Interpreting the volatility smile: an examination of the informational content of option prices*, International Finance Discussion Paper N.º 706, Federal Reserve Board, Washington, DC.
- WU, L., and J. HUANG (2004). "Specification analysis of option pricing models based on time-changed Levy processes", *Journal of Finance*, N.º 59, pp. 1405-1439.

Panel A: Mixture of two lognormals						
	Oct. 1996-Dec. 2003		Oct. 1996-Feb. 2000		Mar. 2000-Dec. 2003	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Mean	8.324,30	1.919,47	8.420,55	2.095,92	8.236,60	1.738,59
Standard deviation	635,46	216,61	664,80	269,30	608,73	148,64
Skewness	-0,41	0,17	-0,47	0,18	-0,36	0,14
Excess kurtosis	0,45	0,27	0,46	0,33	0,45	0,21

Panel B: Splines						
	Oct. 1996-Dec. 2003		Oct. 1996-Feb. 2000		Mar. 2000-Dec. 2003	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Mean	8.323,48	1.919,20	8.419,46	2.095,40	8.236,02	1.738,62
Standard deviation	632,58	215,59	661,30	267,93	606,41	148,28
Skewness	-0,41	0,17	-0,48	0,17	-0,35	0,13
Excess kurtosis	0,25	0,17	0,26	0,21	0,24	0,12

BERKOWITZ STATISTICS P-VALUES FOR ESTIMATED RISK NEUTRAL DENSITIES (a)

TABLE 2

Panel A: Mixture of two lognormals					
Oct. 1996-Dec. 2003		Oct. 1996-Feb. 2000		Mar. 2000-Dec. 2003	
LR (p-value)	LR(i) (p-value)	LR (p-value)	LR(i) (p-value)	LR (p-value)	LR(i) (p-value)
3,664 (0,300)	1,813 (0,178)	9,451 (0,024)	0,830 (0,362)	1,627 (0,653)	0,064 (0,801)

Panel B: Splines					
Oct. 1996-Dec. 2003		Oct. 1996-Feb. 2000		Mar. 2000-Dec. 2003	
LR (p-value)	LR(i) (p-value)	LR (p-value)	LR(i) (p-value)	LR (p-value)	LR(i) (p-value)
4,139 (0,247)	1,799 (0,180)	10,253 (0,017)	0,787 (0,375)	1,607 (0,658)	0,076 (0,782)

a. The reported LR value is the Berkowitz likelihood ratio test for i.i.d. normality of the inverse-normal transformed inverse probability transforms of the realisations as given by  $LR = -2[L(0,1,0) - L(\mu, \sigma, \rho)]$  which is distributed as  $\chi^2(3)$ . The LR(i) statistic is the Berkowitz likelihood ratio test for independence. Rejection of the test for independence suggests that rejection of the RNDs as a good forecast may be due to serial correlation rather than poor forecasting performance.

Panel A: Mixture of two lognormals									
	Oct. 1996-Dec. 2003			Oct. 1996-Feb. 2000			Mar. 2000-Dec. 2003		
	Freq.	Prob. Forecast (b)	ASN	Freq.	Prob. Forecast (b)	ASN	Freq.	Prob. Forecast (b)	ASN
Rise more than 10%	0.140	0.079	2.104	0.220	0.079	3.609	0.067	0.080	-0.503
Fall more than 10%	0.058	0.098	-1.188	0.049	0.102	-1.161	0.067	0.094	-0.532
Fall or rise more than 10%	0.198	0.177	0.558	0.268	0.181	1.580	0.133	0.174	-0.713
Fall or rise more than 15%	0.047	0.066	-0.690	0.049	0.069	-0.440	0.044	0.063	-0.534

Panel B: Splines									
	Oct. 1996-Dec. 2003			Oct. 1996-Feb. 2000			Mar. 2000-Dec. 2003		
	Freq.	Prob. Forecast (b)	ASN	Freq.	Prob. Forecast (b)	ASN	Freq.	Prob. Forecast (b)	ASN
Rise more than 10%	0.140	0.079	2.149	0.220	0.078	3.699	0.067	0.080	-0.505
Fall more than 10%	0.058	0.102	-1.296	0.049	0.105	-1.210	0.067	0.099	-0.636
Fall or rise more than 10%	0.198	0.181	0.474	0.268	0.183	1.525	0.133	0.179	-0.777
Fall or rise more than 15%	0.047	0.065	-0.633	0.049	0.067	-0.386	0.044	0.062	-0.508

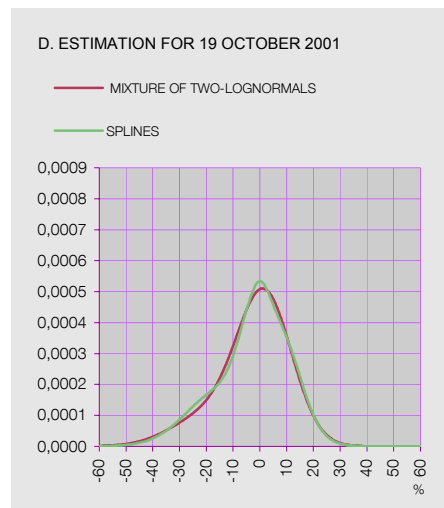
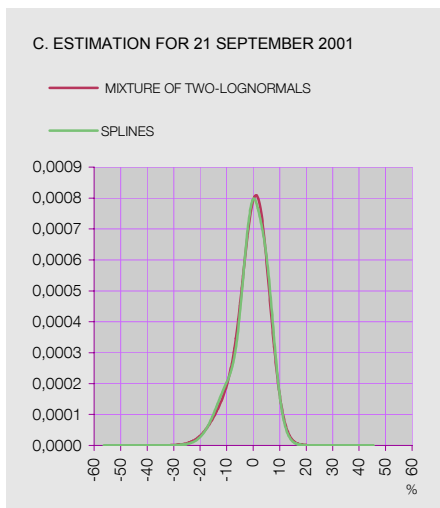
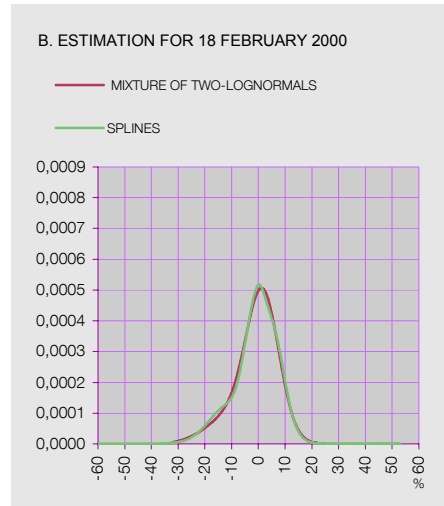
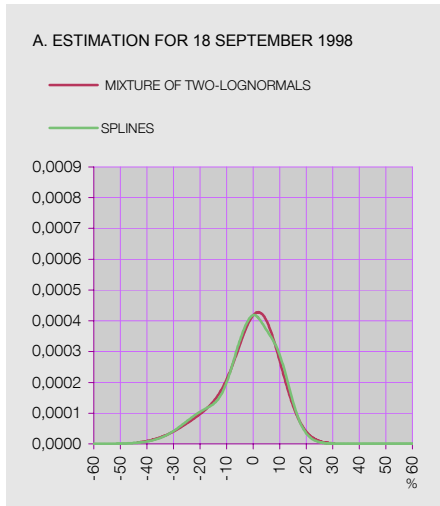
a. Tests of misspecification for tails of estimated RNDs. For the right tail, the left tail and the combination of both tails, the frequency with which actual observations fall in those areas and the probability mass assigned by the mixture of lognormals and splines are reported. The values of the ASN test statistic based on the Brier's score are also reported. The statistic is asymptotically distributed as a standard normal distribution.

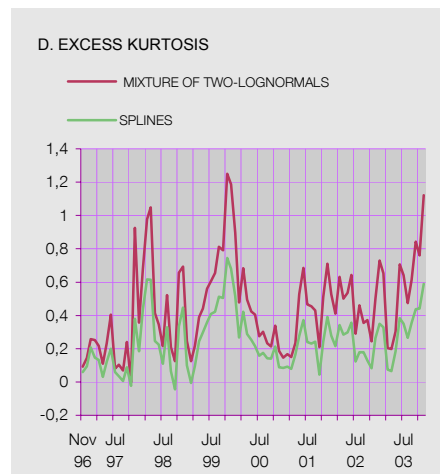
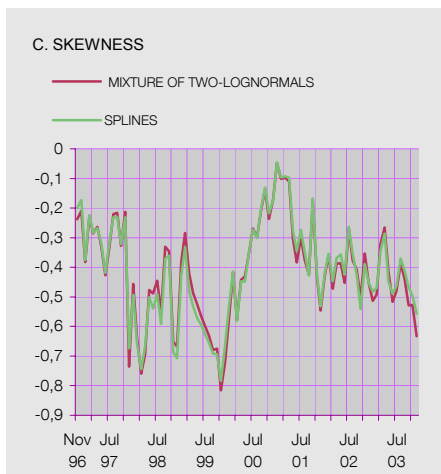
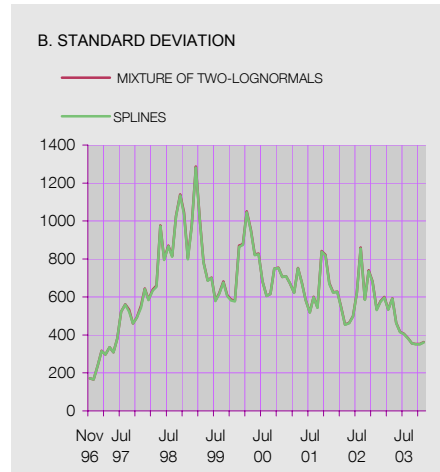
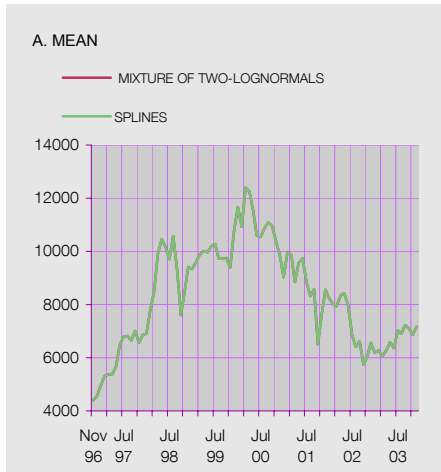
b. The probability forecast is obtained as the average of probabilities from the series of estimated RNDs

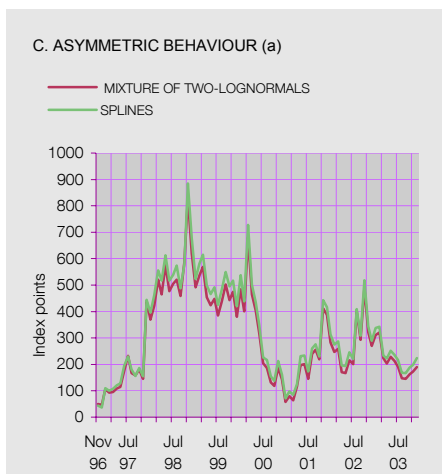
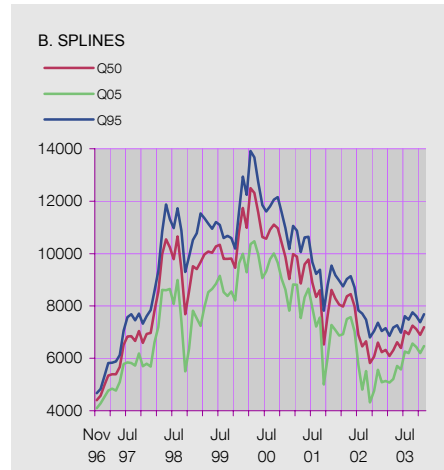
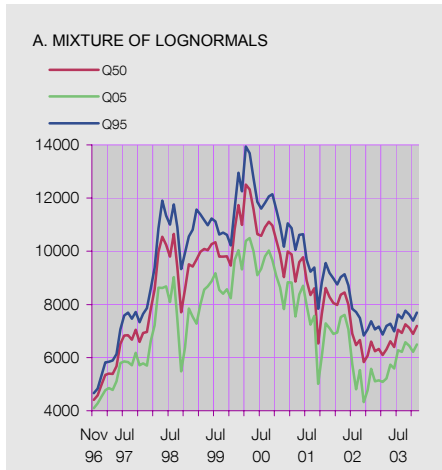
SUMMARY STATISTICS FOR ACTUAL AND FORECASTED DISTRIBUTIONS BASED ON LOG RETURNS OF THE IBEX 35 FUTURES (a)

	Oct. 1996-Dec. 2003				Oct. 1996-Feb. 2000				Mar. 2000-Dec. 2003			
	Mean	Vol.	Skew.	Kurt.	Mean	Vol.	Skew.	Kurt.	Mean	Vol.	Skew.	Kurt.
Actual Sample	0.69	7.43	-0.77	2.14	2.50	7.16	-0.77	1.56	-0.96	7.27	-0.89	2.98
Panel A: Mixture of two lognormals												
Upper Bound	1.43	9.92	0.32	6.69	2.25	10.78	0.58	5.70	2.08	10.42	0.65	5.65
Lower Bound	-2.09	6.31	-1.90	-2.54	-2.91	5.51	-2.04	-2.78	-2.74	5.56	-1.92	-2.86
Panel B: Splines												
Upper Bound	1.42	9.80	0.22	5.83	2.27	10.64	0.50	5.48	2.05	10.14	0.55	4.98
Lower Bound	-2.09	6.33	-1.78	-2.27	-2.94	5.53	-2.00	-2.69	-2.73	5.58	-1.83	-2.46

a. From the estimated RNDs using both mixture of lognormals and splines, we generate 10,000 series of log-returns. The return on a given date is obtained from the RND of that particular period. For each of those series, we calculate the moments of the distribution over time. Finally, from these 10,000 realizations we calculate its mean and standard deviation and obtain a confidence interval of +/- two standard deviations. If the theoretical distributions are correct, we should expect that the realized moments from the actual sample lie on the confidence interval.

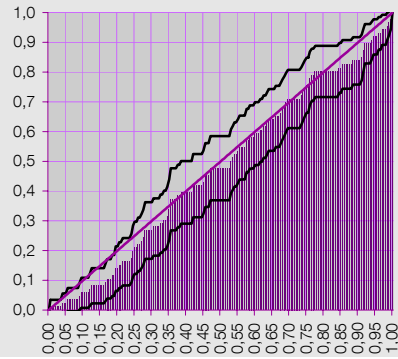




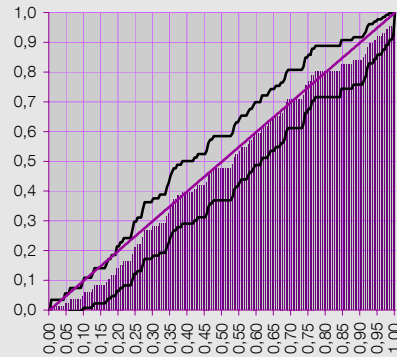


a. Computed as  $(Q50-Q05)-(Q95-Q50)$ .

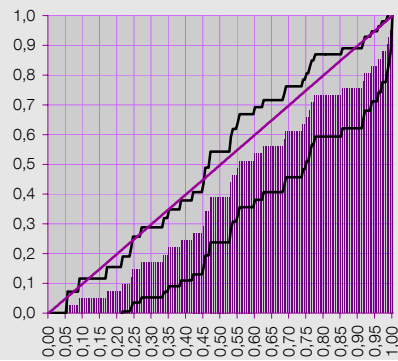
A1. MIXTURE OF LOGNORMAL DENSITIES  
October 1996-December 2003



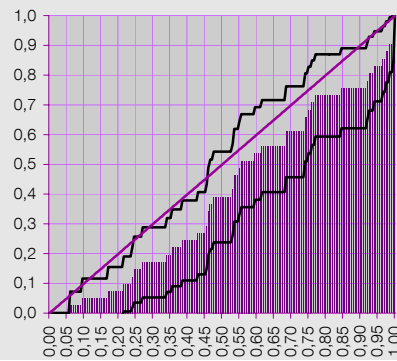
A2. SPLINES  
October 1996-December 2003



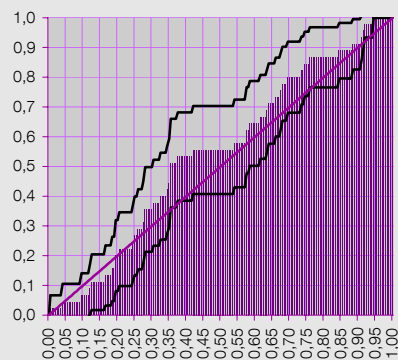
B1. MIXTURE OF LOGNORMAL DENSITIES  
October 1996-February 2000



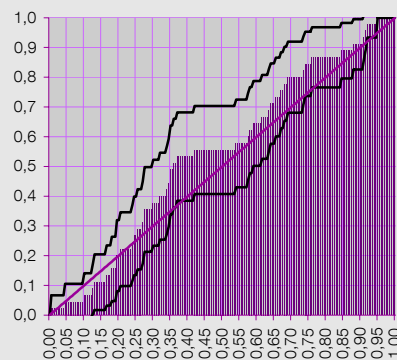
B2. SPLINES  
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C1. MIXTURE OF LOGNORMAL DENSITIES  
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C2. SPLINES  
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