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Abstract

I examine the postulates of the Fiscal Theory of the Price Level (FTPL) under a nominal interest rate peg. First, I show that the usual definition of a non-Ricardian plan involves a number of government’s non-credible policy commitments, thus confuting the interpretation of the FTPL as a policy-based equilibrium selection device. The main novelty of this criticism is that it is based on the same core assumptions maintained by this theory: there is a positive stock of government-issued assets at the beginning of the history owned by the households, flow of funds constraints must be respected in every contingency, although transversality conditions may be violated at off-equilibrium prices.

Then I investigate some additional necessary conditions that allow the government to implement non-Ricardian fiscal plans that result in a unique equilibrium under an interest rate peg. A critical necessary condition for the credibility of such a fiscalist plan is that the equilibrium level of seigniorage must be non-positive. I argue that the fiscalist stock-analogy, under this monetary rule, is only meaningful, precisely, when money enters into the government constraint as a destination of funds, rather than as a source.

**Keywords:** Fiscal-monetary interactions, Fiscal Theory of the Price Level, Interest rate pegging, government commitment.

**JEL classification:** E31, E42, E61.
1 Introduction

Sargent and Wallace (1981), in a pathbreaking article, showed a natural way in which fiscal and monetary policies are related to each other by highlighting the role played by the government’s intertemporal budget constraint. One of the most important messages from the “Unpleasant Monetarist Arithmetic” is that, regardless of the policy regime at work, a certain degree of coordination between fiscal and monetary decisions is always needed, provided the government is willing to honor its policy announcements. Another critical observation drawn from Sargent and Wallace’s story is that fiscal variables, such as taxes or public debt, can only affect nominal variables, such as prices, money supply or nominal interest rates, as long as the central bank accommodates its policy to satisfy some fiscal requirements, printing as much money (i.e. collecting seigniorage) as needed to cover a portion of the government’s outlays, i.e., when the fiscal-monetary plan is conducted under a fiscal-dominance regime. Overall, in their economy the monetarist dictum goes on: the price level is always a monetary phenomenon, in spite of the eventual fiscal roots of the observed monetary stance.

Over the last decade, a number of economists\footnote{Some of the seminal works on the FTPL include the following: Leeper (1991), Sims (1994) and Woodford (1995). Other contributions supporting its theoretical foundations are contained in Bassetto (2002), Cochrane (1998, 2003a, 2003b), Daniel (2003, 2004), Davig \textit{et al.} (2004), Gordon and Leeper (2002), Sims (1999, 2002) and Woodford (1998, 2001). Christiano and Fitzgerald (2000), and Kocherlakota and Phelan (1999) contain clear explanations of the basic FTPL’s arguments.} have challenged the above arguments concerning both the necessity of some degree of monetary and fiscal coordination and the monetary nature of the price level, developing the so-called Fiscal Theory of the Price Level (FTPL, henceforth). The cornerstone of this novel theory is the assumption that the government can commit to implement non-Ricardian policies (to be defined later). Taking this assumption as valid forces an important reconsideration of the main results of Sargent and Wallace’s. First, under a non-Ricardian policy, the policy coordination problem vanishes: fiscal and monetary policies can be designed in a completely uncoordinated fashion without it meaning that the government violates any budget constraint in equilibrium. Second, the previous monetarist dictum breaks down: under the fiscalist postulates the price level and inflation are, fundamentally, fiscal phenomena, with money playing a secondary role.

The fact that under a non-Ricardian policy the price level is directly influenced by fiscal variables (e.g. the stock of government debt and the sequence of primary surpluses/deficits) can be exploited to design policies which are supposed to remove the classic nominal indeterminacy problem associated with a pure nominal interest rate peg and the multiplicity of equilibria under an exogenous money supply rule. Over the last few years, a growing number of papers have proliferated in which the intellectual framework of the FTPL is employed to provide answers to some practical macroeconomic questions such as liquidity traps, hyperinflations, currency crisis, international monetary policy coordination problems, questions related to the fiscal policy design in monetary unions, the effectiveness of the independent central bank paradigm, etc.\footnote{Benhabib \textit{et al.} (2002) study alternative policies, including non-Ricardian ones, to avoid liquidity traps.}
Given the long-reaching implications of the FTPL, the validity of its basic postulates has attracted much attention. Among the advocates of the FTPL, it is usually argued that the government is not limited in its actions by any intertemporal budget constraint, that the real value of fiat government-issued money can be determined in a fiscalist world as the price of a private firm’s equity (this is the so-called stock-analogy, defended by Cochrane (2003a, 2003b) and Sims (1999)) and, in general, that a non-Ricardian policy is theoretically as valid as it is a Ricardian one, with the assumptions underlying the former being neither unfeasible nor unrealistic (e.g. Daniel (2003, 2004), Woodford (1998, 2001)). Some critiques of the FTPL point towards the implausibility of non-Ricardian policies, since these policies imply that the government is allowed to violate its intertemporal budget constraint at off-equilibrium prices (Buiter (1999, 2002)), the incompatibility of non-Ricardian policies with the requirements imposed by the Walrasian competitive equilibrium concept (Marimón (2001)), the possibility of government default under a non-Ricardian fiscal policy (Cushing (1999)) or the unrealistic assumption on the existence of a non-zero stock of nominal government-issued assets at the “beginning of the world” (Niepelt (2004)).

Bassetto (2002) suggests that much of the current confusion about the FTPL is mainly due to two particular reasons. First, he argues that the Walrasian framework employed in all the previous papers in this area is not the appropriate one to deal with a theory which critically depends on some assumptions about the off-equilibrium dynamics of the economy. Second, he makes the point that the usual way in which non-Ricardian policies are defined is not a “correct” one. Bassetto frames a simple non-monetary model, exploiting the so-called cash-less economy assumption, into a non-Walrasian economy. His main results are summarized in the following passage: “I show that there exist government strategies that lead to a version of the fiscal theory, in which the price level is determined by fiscal variables alone. However, these strategies are more complex than the simple budgetary rules usually associated with the fiscal theory [...]”.

In this paper, I identify a minimum set of conditions that guarantee the internal consistency of the FTPL’s arguments, under the assumption that the government commits to maintain a nominal interest rate peg, supplying as much money as demanded by the private sector, a policy that has received much attention in earlier works in this area. Some of the arguments developed here resemble one of the observations made by Bassetto: the design of a fiscal-monetary program in the spirit of the FTPL’s non-Ricardian policies is not as straightforward as it usually assumed. However, this paper departs from Bassetto’s one in two main respects. First, I allow individuals to hold money, by introducing real balances as an additional argument in the utility function and, second, I cast my arguments in a standard Walrasian framework widely employed by an

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3In a related paper, Arce(2004), I study the validity of the FTPL when the central bank directly fixes the nominal supply of money.
important number of previous authors (both proponents and opponents of the FTPL). The first departure is rooted on an obvious observation: in an economy in which money does not exist, one cannot get any meaningful result on how the real value of money is determined. The second departure is of technical nature. Firstly, I argue that the analysis of the fundamental “weak points” of the FTPL can be carried out within a Walrasian framework, for some critical questions in this analysis, such as the fulfilment of flow of funds constraints and individual rationality, are not specific of the equilibrium concept employed or of the particular assumptions about how prices are formed. Secondly, I show that, under some conditions, the government can credibly commit to maintain some exogenously targeted fiscal-monetary policy which is consistent with a unique equilibrium price sequence, in the spirit of the FTPL, by implementing a price-contingent fiscal plan in a standard Walrasian economy.

I first argue that the validity of the FTPL’s argument needs a mechanism relating the individual’s optimal decisions to the price level and that the standard definition of a non-Ricardian policy assumes that such a mechanism exists without proving it. Moreover, I show that, according to the most basic arguments of the standard FTPL, such a theory can never perform as a policy-based equilibrium selection device in the context of the economy and the policy rule studied here. In doing so, I add a number of new arguments to the existing critiques against the FTPL. Then I define a minimum set of conditions that allow the government to credibly commit to a fiscal-monetary plan which results in a unique equilibrium by creating a link between prices, policies and individual optimal behavior. A critical necessary condition for a credible government policy, that includes a monetary plan not involving a direct control of a monetary aggregate, to yield a unique equilibrium price sequence is that the equilibrium level of seigniorage cannot be positive, i.e., except in the particular case in which seigniorage is equal to zero, the government must redeem a fraction of the existing stock of money in exchange for consumption goods. This observation implies that the fiscalist arguments under a nominal interest rate peg, as they are usually presented in the literature, are not generally consistent with the notion of fiat (i.e. non-convertible) money, or, equivalently, a properly defined non-Ricardian plan requires dollars (rather than bonds) to be net wealth. This is, perhaps, the main result of this paper.

Some other questions approached here include the following. I show that, contrary to the standard FTPL’s fiscal-monetary independence proposition, a credible fiscalist policy must always involve a certain degree of coordination. I also provide a critical revision of the general validity of the comparison between the FTPL’s arguments and the standard financial asset-pricing theory. In particular, I argue that there is no analogy between the standard FTPL and the asset-pricing theory, although there is a similarity between the mechanism underlying the determination of the real value of a firm’s stock and the one underlying the determination of the equilibrium price sequence under a credible (i.e. with convertible money) non-Ricardian fiscal-monetary plan.

The paper is structured as follows. Section 2 describes the general set-up of the monetary economy studied here and presents a basic “canonical” example of a non-Ricardian fiscal-monetary program. Section 3 explores in detail the problems associated with the standard definition of a non-Ricardian policy using a simple one-period model.
Section 4 describes a set-up in which the government can credibly commit to implement non-Ricardian policies. Section 5 deals with the fiscalist stock-analogy. Section 6 contains the extension of the main results to a multi-period economy. Section 7 summarizes the main conclusions of the paper.
2 The canonical non-Ricardian argument

In this paper I consider a simple representative agent model with exogenous and constant endowments of consumption goods. Time is discrete and there is no uncertainty. Money is introduced as an argument in the household’s utility function. Thus, in its basic aspects, the economy studied here is similar to the ones considered by some advocates (e.g. Daniel (2003, 2004), Leeper (1991) and Woodford (1995, 1998, 2001)), and opponents (e.g. Buiter (2001, 2002), Cushing (1999), Niepelt (2002) and McCallum (2001)) of the FTPL.

2.1 The model

2.1.1 The households

The households form their expectations rationally and try to maximize the following objective function

\[ \max_{\{c,M\}} \sum_{t=1}^{\infty} \beta^{t-1} [u(c_t) + v(M_t/P_t)] \] (1)

where \( u \) and \( v \) are increasing and concave functions in their respective arguments and satisfy the Inada conditions. The function \( u \) captures the utility enjoyed by the household from consuming a certain amount of goods, \( c_t \), and \( v \) stands for the liquidity services provided by real money holdings, i.e. nominal balances, \( M_t \), deflated by the general price level, \( P_t \). The parameter \( \beta \) is the subjective discount factor and satisfies \( 0 < \beta < 1 \).

The representative household’s flow budget constraints are

\[ \frac{B_t + M_t}{P_t} \leq y - \tau_t - c_t + \frac{(1 + i_{t-1}) B_{t-1} + M_{t-1}}{P_t} \quad (t \geq 1) \] (2)

where \( y \) is the (positive) exogenous household’s endowment of consumption goods, \( \tau_t \) is a lump-sum tax payable to the government; \( B_t \) is the amount of dollar-denominated government bonds which are assumed to mature one period after issued. Each bond issued at \( t - 1 \) is sold at a price of one dollar and redeemed at a value equivalent to \( 1 + i_{t-1} \) dollars at \( t \). The household may also be endowed with an initial stock of financial wealth: some dollar-denominated government bonds, inclusive of interest, \( (1 + i_0) B_0 \), and some money balances, \( M_0 \). The value of each of these two terms is given exogenously and assumed to be non-negative. Also, the households are precluded from issuing money, i.e. \( M_t \geq 0 \). For the ease of the exposition, it is assumed that the households do not issue debt either\(^5\), i.e. \( B_t \geq 0 \). In choosing a path for lifetime consumption and money holdings, the household is restricted by the following transversality condition

\(^4\)In some of the following sections I use some restricted versions of the infinite horizon model described in this section (e.g. one-period and two-periods economies). Those restricted versions can be readily derived as special cases of this more general framework.

\(^5\)This assumption does not affect the generality of the arguments in this paper. See Bassetto (2002) for a similar constraint.
\[
\lim_{T \to \infty} \left( M_{T-1} + (1 + i_{T-1}) B_{T-1} \right) / \prod_{s=1}^{T-1} (1 + i_s) \geq 0
\]  

(3)

I define the household’s utility maximization problem as his choice of a lifetime consumption and portfolio paths\(^6\) \(\{c_t, M_t, B_t\}\) satisfying (2), the non-negativity constraints on assets holdings, given an initial stock of financial wealth, the price and interest rate sequences \(\{P_t, i_t\}\), and the income and tax sequences \(\{y, \tau_t\}\). The following conditions are necessary and sufficient for an optimal choice of consumption, money, and bonds holdings plans.

When the household behaves optimally, i.e., not leaving any intrinsically valuable resources unconsumed, both the set of budget constraints (2) and the limiting condition (3) hold as equalities, in which case we learn that the following present-value constraint necessarily holds

\[
(1 + i_0) B_0 + \sum_{t=1}^{\infty} \frac{P_t (y - \tau_t)}{\prod_{s=1}^{t-1} (1 + i_s)} = \sum_{t=1}^{\infty} \frac{P_t c_t + M_t - M_{t-1}}{\prod_{s=1}^{t-1} (1 + i_s)}
\]  

(4)

This optimality condition states that the total value of the household’s wealth at any period \(t \geq 1\), inclusive of maturing government bonds plus current and future endowments net of taxes be equal to the total value of the household’s expenditure which includes consumption and seigniorage (i.e. purchases of government-issued money).

The following standard first order conditions complete the characterization of the solution for this maximization problem

\[
\frac{v'(m_t) - u'(c_t)}{P_t} + \beta \frac{v'(c_{t+1})}{P_{t+1}} = 0
\]  

(5)

\[
\beta (1 + i_t) \frac{w'(c_{t+1})}{P_{t+1}} - \frac{w'(c_t)}{P_t} = 0
\]  

(6)

where \(m_t \equiv \frac{w'}{v'}\) is the household’s demand for real balances. Unless otherwise stated, it is also assumed that the felicity function \(v\) satisfies the following inequality

\[
\lim_{m \to 0} mv'(m) > 0
\]  

(7)

In view of the above inequality, we learn that speculative hyperinflations are not possible equilibrium outcomes\(^7\).

\(^6\) Notice that there is another choice faced by the household at every period: whether to redeem his entire stock of maturing bonds in exchange for the goods and assets (money and/or newly issued bonds) given by the government. Since maturing bonds are assumed to expire after their maturity date, it will be assumed throughout that the household always redeems his entire stock of initial bonds.

\(^7\) See Obstfeld and Rogoff (1983). This assumption is made here for simplicity as the main arguments in this paper do not depend on the possibility of rational speculative paths. In Arce (2004) I analyze some speculative solutions in the context of the FTPL.
2.1.2 The government

The government in this economy sets the level of taxes and public consumption and manages the public debt, issuing and redeeming bonds, fixing the nominal interest rate and supplying money endogenously in the amount required by the private sector so as to accommodate the “needs of trade”. The consolidated government’s sequence of flow of funds constraints is given by

\[
\begin{align*}
E_v^w + P_v^w S_w = j_w, \\
E_v^w + P_v^w S_w = (1 + i) E_v^w + P_v^w S_w (w) (8)
\end{align*}
\]

where \(E_v^w\) and \(P_v^w\) are, respectively, the government supply of bonds and money and \(j\) stands for government consumption and satisfies \(j_t < y\). Iterating forward the constraint (8) and imposing the following transversality condition

\[
\lim_{T \to \infty} \left( M^w_{T-1} + (1 + i_{T-1}) B^w_{T-1} \right) / \prod_{s=1}^{T-1} (1 + i_s) = 0 \quad (9)
\]

gives the government intertemporal budget constraint

\[
(1 + i_0) B_0 + \sum_{t=1}^{\infty} \frac{P_t g_t}{\prod_{s=1}^{t-1} (1 + i_s)} = \sum_{t=1}^{\infty} \frac{P_t \tau_t + M_t^w - M_{t-1}^w}{\prod_{s=1}^{t-1} (1 + i_s)} \quad (10)
\]

which holds if and only if (8) and (9) are simultaneously satisfied.

2.2 Ricardian and Non-Ricardian policies

Bellow I introduce a standard definition of Ricardian and non-Ricardian policies and examine the implications of alternative fiscal-monetary programs for the equilibrium of the model, using the paradigm of the Walrasian competitive equilibrium. The aim here is to make a description of the implications of these two alternative classes of policies in terms of the (in)determinacy of the equilibrium price sequence, as they are usually presented in the previous literature. I use the term “canonical” here to refer to the class of solutions of the model in which it is explicitly assumed that the government always honors its policy-announcements, (i.e. redeeming its eventual outstanding obligations at their contractual value and meeting its targeted sequences of taxes and government consumption). Whether this a sensible assumption to be held under any circumstance or not is at the heart of the current discussion on the validity of the FTPL’s postulates, an issue which is analyzed in detail in the following sections.

Definition 1 (Canonical non-Ricardian vs Ricardian distinction) A policy is Ricardian if it is formulated in such a way that the government intertemporal constraint (10) is satisfied for any price sequence \(\{P_t\}_{t=1}^\infty\). It is non-Ricardian if it is only satisfied in equilibrium. Equivalently, a policy is Ricardian if the transversality condition (9) is satisfied for any price sequence and non-Ricardian if it is only satisfied in equilibrium\(^8\).

\(^8\)This definition corresponds to the one given by Woodford (1995). Although Woodford explicitly refers to equation (10), the fact that the flow of funds constraint (8) holds as an identity allows for an equivalent definition for a non-Ricardian policy based on the transversality condition (9).
In examining the cases presented below, I focus on the following particular, although widely used in the literature, Ricardian and non-Ricardian policies:

1. Ricardian interest rate peg (RP): the government sets a price-invariant level of public consumption for each period, \( \{g_t\} \), and commits to a sequence of non-negative interest rates \( \{i_t\} \), which is also assumed to be independent of the observed price-level.

2. Non-Ricardian interest rate peg (NRP): the government chooses the same policy-instruments and money supply rule as in RP and also commits to a price-invariant sequence of taxes, \( \{\tau_t\} \), without any feedback from the observed price level.

The definition of a competitive equilibrium for this economy is the following:

**Definition 2** A perfect foresight competitive equilibrium in this economy is a set of allocations \( \{c_t\}, \{g_t\}, \{B_t\}, \{M_t\} \), price and interest rate sequences \( \{P_t, i_t\} \), and a sequence of taxes \( \{\tau_t\} \) such that the following conditions are satisfied:

1. The government satisfies its budget constraint (8) as an equality in every single period and the transversality condition (9).

2. Households maximize their utility subject to subject to the constraints (2) and (3) and the non-negativity constraints \( M_t \geq 0 \) and \( B_t \geq 0 \), given the sequences \( \{P_t, i_t\} \) and the government policy.

3. All markets clear in every period, i.e., \( y = c_t + g_t, M_t = M^*_t, \) and \( B_t = B^*_t \).

### 2.2.1 The Ricardian canonical solution

Here I study how equilibrium prices are determined under the hypothesis that the government intertemporal budget constraint (10) is always binding, regardless of the value taken by the endogenous variables of the model. The following proposition contains the main results with respect to the (in)determinacy of the equilibrium price sequence.

**Proposition 1** Under the Ricardian policy RP, there are a continuum of equilibrium price sequences.

**Proof.** Given the set of government’s commitments, a competitive equilibrium is characterized by the following conditions:

(i) The first order conditions (5) and (6).

(ii) The household’s flow of funds constraint (2) and the transversality condition (3) hold as equalities.

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9The main arguments of this paper do not hinge on the particular design of Ricardian and non-Ricardian policies, as it will become clear later. Focusing on the simple policy rules described here, however, adds to the clarity of the exposition. All the sequences are understood to be defined for \( t \geq 1 \).
(iii) The government implements price-invariant sequences of public consumption and nominal interest rates, according to the definition of the policy RP given above, adjusting the sequence of taxes so as to satisfy (10) and (8) for any price sequence.

(iv) All the markets (consumption goods, money and bonds) clear.

Let’s consider an arbitrary positive and finite initial price level \( P^*_1 \). It can be verified that there exists a competitive equilibrium associated with that initial price level. As the government sets the sequence of public consumption without any feedback from the price level, we learn that in equilibrium private consumption satisfies \( f_w = |j_w| \) for all \( w \). Then, the government’s commitment to an exogenous sequence of nominal rates implies that, according to the first order conditions (5) and (6), the optimal household’s demand for nominal balances in the initial period \( w=1 \) in equilibrium, satisfies

\[
M_1(P^*_1) = v'^{-1} \left[ \frac{\tau_1}{1 + i_1} v' (y - g_1) \right] P^*_1 \tag{11}
\]

Given that particular \( P^*_1 \), the government sets a sequence of taxes satisfying (10), i.e. the whole tax-sequence satisfies

\[
\frac{(1 + i_0) B_0}{P^*_1} + \sum_{t=1}^{\infty} \frac{g_t}{\prod_{s=1}^{t-1} (1 + r_s)} = \sum_{t=1}^{\infty} \frac{\tau_t + \frac{M^*_t - M^*_{t-1}}{P_t}}{P_{t+1}} \tag{12}
\]

where \( r_s = (1 + i_s) \left( p_s / p_{s+1} \right) - 1 \) is the real rate of return on bonds. Then, for a particular first element of the tax sequence, call it \( \tau_1(P^*_1) \), there is a unique end-of-period stock of government bonds satisfying the budget constraints (2) and (8) and the market clearing conditions at \( t = 1 \), given by

\[
B_1(P^*_1) = P^*_1 (g_1 - \tau_1(P^*_1)) - M_1(P^*_1) + M_0 + (1 + i_0) B_0
\]

The equilibrium price level for the second period can then be solved uniquely from (6), and applying the same steps above, the entire collection of the equilibrium sequences for the endogenous variables (i.e. private consumption, stock of nominal money, stock of government bonds, taxes and prices) can be solved recursively for an arbitrary \( P^*_1 \).

The main message from this Ricardian fiscal-monetary program is that, provided we assume that the present-value constraint (10) must hold regardless of the particular values of the endogenous variables, the government must react to the real value of the initial stock of debt by adjusting the sequence of taxes as required (by (12)), so as to avoid default or supersolvency premium on the initial stock of debt. In other words, according to the SW’s “game of the chicken” parable, as the fully elastic money supply rule at work leaves the equilibrium price sequence undetermined, fiscal policy must “blink”.

### 2.2.2 The non-Ricardian canonical solution

The following proposition contains the main implications arising from the assumption that the government implements a plan without any degree of fiscal-monetary coordination, as indicated by the non-Ricardian program defined above.
Proposition 2 Under the non-Ricardian policy NRP, the equilibrium price sequence is unique provided a sign condition holds.

Proof. Combining the market clearing and the household’s optimality conditions, including the transversality condition (3) holding as an equality, we learn that in equilibrium the following equation must hold in the initial period

$$M_0 + (1 + i_0) B_0 P_1 = \sum_{t=1}^{\infty} \frac{\tau_t - g_t + \frac{i_t}{\pi_t} m_t}{\prod_{s=1}^{t}(1 + r_s)}$$

Notice that every term in the RHS of (13) is independent of the price level, $P_1$, under this policy. Some elements are directly fixed by the government ($\tau_t$ and $g_t$), $r_s$ can be derived by combining the exogenously set fiscal instruments together with the market clearing and individual optimization conditions, while $M_0$ and $(1 + i_0) B_0$ in the LHS are given at the beginning of period 1. It follows that under this set of policy-choices the price level in the initial period, $P_1$, is uniquely determined by equation (13). For this price to be positive and finite, the following sign condition must hold

$$0 < \sum_{t=1}^{\infty} \frac{\tau_t - g_t + \frac{i_t}{\pi_t} m_t}{\prod_{s=1}^{t}(1 + r_s)} < \infty$$

The above sign-condition is the only restriction placed upon the sequence of taxes and government consumption. Once the price level for the initial period is pinned down, the entire set of sequences of endogenous variables is uniquely determined too. The equilibrium money supply in the initial period, $M_1^S$, can be obtained from (5), then $P_2$ can be solved uniquely from (6), with the supply of government debt in real terms, $\frac{B_t}{\pi_t}$, obeying the sequence of budget constraints (8) and so on.

Clearly, assuming that the economic authorities can commit to a non-Ricardian policy like the one outlined in the previous proposition results in a considerable “gain” in terms of the (lower) number of potential equilibria. Treating the intertemporal constraint (10) (or the transversality condition (9)) as an independent equilibrium condition rather than as an identity, which must hold always, provides an “extra” equation that can be exploited through the implementation of a non-Ricardian policy so as to solve the “classical” nominal indeterminacy problem described in Proposition 1. The main practical implications, from the perspective of the policy-mix design, arising from Proposition 2 is that fiscal-monetary coordination is not longer needed: if neither player is blinking the price level will adjust.

In providing supportive arguments for the fiscalist (unique) equilibrium constructed before, the proponents of the FTPL justify the interpretation of (13) as an additional separate equilibrium condition either by accepting that the government is limited on its actions by an intertemporal budget constraint, but with the extra qualification that the government is a “big player”, so that it need not to take prices as given (see Woodford (1998, 2001)), or even by denying that such a constraint exists at all, arguing that (13)

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10 For the remaining of the paper, I will assume that this sign condition holds unless otherwise stated.
must be interpreted as a “government valuation equation” rather than as a constraint on its actions. Under this approach, the government can be viewed as a private firm whose equity is priced according to the future stream of profits, where an analogy is drawn between a firm’s equity and the nominal-denominated government’s obligations (the term \((1 + i_0) B_0 + M_0\)), and between the firm’s profit stream and the government’s total surpluses (the RHS of (13)). From this perspective, the plausibility of non-Ricardian policies is accepted as it is the general idea that the future profits of a firm need not to be influenced by the current price of its shares and, therefore, the transversality condition is only satisfied when the stock of the firm is “correctly” valued (i.e., at equilibrium prices). Further, if money and nominal debt can be understood as the firm’s stock and, hence, as residual claims to government total surpluses, the issue of default on government-issued assets is irrelevant, simply because it makes no sense to talk about default on shares. Thus, the non-default assumption embedded in the “canonical” case presented here would be plainly justified. This stock-analogy argument can be found, e.g., in Cochrane (2003a, 2003b), Christiano and Fitzgerald (2000) and Sims (1999).
3 Transversality conditions: Assumptions vs theory

This section explores the internal consistency of the assumptions underlying the non-Ricardian canonical case discussed in Proposition 2. Some other authors have shown their concern about the validity of the FTPL under an interest rate peg, as well. Buiter argues that, as the government commits to run a completely exogenous fiscal policy, there is no reason why the sign condition (14) will hold with generality: “The most the fiscal theory of the price level therefore could aspire to, when the arbitrary (sign condition) is satisfied, is to be a way of removing the price level indeterminacy characteristic of equilibria under a Ricardian nominal interest rule, when nominal prices are flexible” (Buiter (1999), parentheses added). Niepelt (2004) shows his concern about the assumption of a non-zero initial stock of nominal assets: “In this paper, I offer a resolution to this debate. The fundamental problem of the FTPL is that the feasibility of non-Ricardian policy hinges on the assumption of non-zero initial nominal government liabilities. This assumption is not well founded [...]”. Cushing (1999) argues that, in face of an initial stock of government debt, the possibility of default in the first period breaks down the fiscalist uniqueness result: “In this section, I point out that admitting the possibility of fiscal default shows the price level to be indeterminate under an interest rate rule”.

Notwithstanding the relevance of the above criticisms, the arguments given in this section point towards a different direction. Kocherlakota and Phelan (1999) argue that the FTPL, at its core, contains prescriptions about the government behavior at off-equilibrium prices, an issue which is not testable, thus rendering the question on the validity of the FTPL as an equilibrium selection device “a religious, not a scientific issue”. Below, I argue that it is indeed possible to analyze the arguments given by the FTPL regarding the behavior of the economic agents in those situations that this theory claims to be off-equilibrium outcomes, even after recognizing that this question is not testable. In doing so, I exploit a feature of a critical element in the model: flow of funds constraints must always hold (in- and off-equilibrium) regardless of the identity, size and behavior of the corresponding agent. Such a status for the flow of funds constraints, to the best of my knowledge, has not been challenged by any proponent of the FTPL. Interestingly, the analysis pursued here does not need to give such a status to the transversality conditions and conforms the fiscalist view that those conditions are, indeed, only equilibrium conditions. The role of the transversality conditions have been placed at the center of the debate. Treating them as “equilibrium” conditions is at the core of non-Ricardian policies, as stated in Definition 2. For some critics of this theory (e.g. Buiter), this is not an admissible step. By keeping on the interpretation given by the proponents of the FTPL, I aim to provide a clearer and less controversial exposition of the weak points of this theory.

In particular, I show that the equilibrium uniqueness result in Proposition 2 is not an outcome driven by government’s policies, but rather by some special assumptions about the households’ decisions. In this sense, it can be argued that the FTPL is not a policy-based equilibrium selection device. And this proposition goes on even when the

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economist does not observe the off-equilibrium implementation of the particular equilibrium selection device at work, something which is necessarily the case when in face of a successful device, like e.g. the one designed by Obstfeld and Rogoff (1983) to rule out speculative hyperinflations.

To keep the exposition as simple as possible, I first consider a one-period model\(^{12}\), in which the representative household holds an initial stock of financial wealth consisting on some dollars and bonds. As in the general model presented before, the household receives an endowment, \( y \), and tries to maximize his utility by choosing how much to consume, how many dollars to hold at the end of the period and how many bonds, maturing after death, to buy. The government sets the level of taxes, \( \tau_1 \) and public consumption, \( g_1 \). Notice, that within this narrow one-period framework there is no room for the government to commit to a non-zero effective redemption-value for bonds issued this period. However, for the arguments defended here, the critical fact concerning monetary policy is the commitment to elastically supply as much money as demanded, rather than the eventual announcement of a particular next period’s redemption value payable by government’s bonds\(^{13}\). The extension of the main results obtained within this one-period framework to a multi-period setting, where the government announces a particular interest rate is straightforward, as shown later.

The household is constrained in his choices by the following flow of funds and no-Ponzi games constraints

\[
\begin{align*}
  c_1 & \leq y - \tau_1 + \frac{M_0 + (1 + i_0) B_0 - M_1 - B_1}{P_1} \\
  \frac{B_1}{P_1} & \geq 0
\end{align*}
\]  

(15)  

(16)

The optimal behavior of the household is characterized by the following conditions:

\[
\begin{align*}
  u' (c_1) & = u' \left( \frac{M_1}{P_1} \right) \\
  \frac{B_1}{P_1} & = 0
\end{align*}
\]  

(17)  

(18)

Equation (17) implies that the individual optimally chooses consumption and real balances equating their respective marginal utilities. Condition (18) follows from the “end of the world” assumption: as bonds issued in period 1 mature one period after, no rational agent would be willing to give up some valuable resources for that debt. An additional necessary condition for optimization is that the budget constraint (15) holds as an equality.

\(^{12}\)This simplification can also be found in Christiano and Fitzgerald (2000). Cochrane (2003b) also provides some examples in one-period economies.

\(^{13}\)At this point, it is worth noticing that what drives the multiplicity of equilibria result in the Ricardian case discussed in Proposition 1 is the mathematical property of homogeneity of degree zero in the equilibrium demand for real balances. The one-period framework preserves that feature and, not surprisingly, yields similar “canonical” results as in Propositions 1 and 2.
The consolidated government’s budget constraint is

$$ \frac{B_1^f + M_1^f}{P_1} = g_1 - \tau_1 + \frac{M_0 + (1 + i_0) B_0}{P_1} $$

(19)

Regarding the monetary rule at work, it is assumed that the government commits to supply as many dollars as demanded at the ongoing price level.

According to Proposition 2, if the government commits to set \( \tau_1 \) and \( g_1 \) without any feedback from the observed price level, the following equation, which is derived by combining (17), (18), (19) and the market-clearing conditions, gives the unique fiscalist equilibrium price level, \( P_1^F \),

$$ \frac{M_0 + (1 + i_0) B_0}{P_1^F} = \tau_1 - g_1 + m_1 $$

(20)

where \( m_1 = \nu'^{-1}[\nu' (y - g_1)] \). The FTPL’s logic for this uniqueness result is, at first sight, simple. Any price different from \( P_1^F \) above will result in the violation of an individual optimal condition or/and a market clearing condition, for these are, in principle, the only ingredients included in the characterization of the fiscalist equilibrium (see proof of Proposition 2). In what follows I check the general validity of this argument. In doing so, I follow a simple strategy: pick up an arbitrary price \( S_1 \neq S_1^F \) and analyze how, if at all, such a price is inconsistent with the three equilibrium requirements listed in Definition 2: (i) fulfillment of government constraints, (ii) individual optimization problem and (iii) market-clearing.

To clear the desk, it is convenient to recall the following observation: when dealing with a failure of the market clearing conditions, we can concentrate just on situations in which the markets for goods and for new bonds fail to clear since, given the fully elastic monetary rule considered here, we learn that the supply of dollars is always identical to the demand.

Firstly, let’s consider several alternative interpretations about the violation of some of the equilibrium requirements listed above when considering an arbitrary price \( P_1 < P_1^F \). For the sake of clarity, it is convenient to consider the two central pieces of the intuitive fiscalist argument given above, market clearing and individual optimization, separately.

(A market clearing approach). Let’s assume that the households never demand a positive quantity of non-performing bonds, regardless of the price level, i.e. \( \frac{B_1}{P_1} = 0 \) \( \forall P_1 \). Notice that such a lending rule is always optimal. Given that lending rule, we can solve for the household’s goods and dollars demand functions combining the budget constraint (15), holding as an equality after imposing a zero-final stock of desired bonds, and the first order condition (17). Nothing up to this point presumes an underlying non-optimal behavior of the households. In face of \( P_1 < P_1^F \), we see from the government budget constraint (19) that the households refusal to purchase new bonds implies that the government cannot attain its fiscal objectives simultaneously, thus, unchaining a potential “crisis”. A natural next step is to consider the possibility that the government activates a crisis-resolution device so as to render \( P_1 \) a non-equilibrium outcome, hence, defeating the crisis. There are not many available options for a government who has
already committed to some no price-contingent monetary and fiscal policies, apart from printing bonds and trying to sell them. This, assuming is a costless activity, is always feasible for the government. Then, one could think that a positive supply of government bonds, when coupled with a zero demand, creates a problem of excess of supply in the newly-issued bonds market or, alternatively, an excess of demand in the goods market

Let’s treat these two markets (bonds and goods) separately.

In the bonds market, the government, as a monopolist, may take two courses of action: it may fix the quantity of bonds, $B_1^*$, or it may announce the price at which it will meet the demand. This latter form of government intervention is, perhaps, the most popular in macro models, specially when assuming that the government chooses the price of a dollar as the reference-price at which it supplies its bonds, say by setting the price of one bond supplied today equal to the price of one dollar (i.e. $\bar{P}_1$; this is the convention followed in this paper). In this case there exists a unique market-clearing quantity consistent with a zero real demand for bonds, $B_1 = B_1^* = 0$. On the other hand, if the government chooses the quantity of bonds to be supplied at each bond-price, say $\frac{1}{Q_1}$, then there is a unique market clearing bond-price, $\frac{1}{Q_1} = 0$. Alternatively, we might think of the government offering contracts specifying a given quantity of bonds to be delivered at a particular dollar-price, of the kind $(B_1^*, P_1^*)$, letting the “market forces” determine the price of each contract. Still, the market for debt-contracts may clear at a zero real value for each contract offered. To put it in plain words, the market for a valueless asset (from the perspective of the potential buyers) may always “clear”: either that asset is not traded at all or, if one wishes so, it is transacted at a zero real value.

Now, given that the household never wastes resources purchasing non-performing bonds when these are offered at a positive price, we learn that the government demand for consumption satisfies the following equality

$$g_1 = \tau_1 - \frac{M_0 + (1 + i_0) B_0 - M_1}{\bar{P}_1} \quad (21)$$

Actually, in writing (21), we do not need to assume that the bonds market would “clear” according to the argument in the paragraphs above, since, the fact that the households never lend resources to the government, implies that the government cannot posit a demand for goods greater than its available resources at that price (i.e. the terms in the RHS of (21)). That is, for writing a well-posed demand function, we need to consider the amount of wealth that the agent can seize in supporting its bids. In view of (21), when the government commits to a fixed $\tau_1$, total wealth available for government consumption increases with the price level and vice versa, i.e. a commitment to a given $g_1$ is only

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This “disequilibrium” outcome becomes apparent by summing up the household’s constraint (15), holding as an equality with $\frac{B_1^*}{\bar{P}_1} = 0$ and the government’s constraint (19) after imposing $\frac{B_1^*}{\bar{P}_1} > 0$ and $M_1 = M_1^*$.

The fact that the government is only choosing the price of reference at which it is willing to supply bonds and not choosing the exact value of that price directly, does not mean that the government is not choosing the supply-price. Private banks usually fix the interest rate at which they are willing to meet the demand for loans by using a reference rate (e.g. LIBOR), which, of course, cannot be understood as the private banks as having their hands free to simultaneously choose interest rates and transacted quantities.
possible if \( \tau_1 \) varies (inversely) with the price so as to satisfy (21) as an identity\(^{16}\). Once total wealth available for government consumption is computed correctly, i.e. accounting for the fact that the households will not purchase non-performing bonds, the ability of the government to preclude any \( \overline{P}_1 < P_1^F \) as an equilibrium outcome by printing bonds so as to induce an excess-of-demand in the goods market vanishes\(^{17}\).

In sum, a government’s plan to fight a crisis based upon its unlimited ability to print non-performing bonds will not preclude the crisis whenever the households behave in a contingent optimal behavior. Further, when printing bonds is the unique\(^{18}\) alternative government plan, a crisis is unavoidable and the non-Ricardian set of policy commitments becomes non-credible.

(An optimality principle approach). We may assume that for \( \overline{P}_1 < P_1^F \) the households demand a strictly positive real amount of government bonds. Then, from the definition of equilibrium (condition (ii)) we learn that such a price-contingent individual behavior can not be compatible with the existence of an equilibrium at that \( \overline{P}_1 \). That is, any crisis-price inducing households to demand non-performing bonds will never lead to an observed crisis: it is self-defeated by a households’ non-optimal behavior. Notice, however, that a non-optimal household’s lending behavior is not sufficient for the government’s original fiscal plan to be feasible, in the sense that out of the infinite possible combinations of prices and household’s demand for government bonds, only one is compatible with some exogenously targeted \( g_1 \) and \( \tau_1 \), namely the one satisfying the following equation

\[
\frac{B_1}{\overline{P}_1} = g_1 - \tau_1 + \frac{M_0 + (1 + i_0) B_0 - M_1}{\overline{P}_1} \tag{22}
\]

Should private lending behavior be rightly described by (22), the government would be in a position to meet all its targets (i.e. to formulate a well-posed consumption demand without abandoning its tax-target and its commitment to redeem the initial debt at its contractual value) at any price provided it satisfies the ongoing private demand for bonds\(^{19}\). If, for example, \( \frac{B_1}{\overline{P}_1} \) falls below the amount in the right side of (22), total resources available for government consumption would be insufficient to go ahead with its original plans. In this sense, (22) is a necessary condition to guarantee the solvency

\(^{16}\)One of Bassetto’s (2002) main conclusions is in close connection with this observation: “[...] the strategy I outline above forces the government to increase its taxes in response to a debt crisis; in such an occurrence, not enough resources would be available to pursue the original plan”. While he considers the occurrence of a crisis as the outcome of “any (possibly irrational) reason”, the crisis I consider here is unambiguously compatible with an individual (contingent) rational behavior.

\(^{17}\)This idea should be clear: I cannot influence the outcome of the market for diamonds by just saying that I want to buy many of them. As the rest of the agents in that market understand that my resources are not in line with my desires, one shouldn’t expect any effect in the price of diamonds from my words. Properly speaking, I cannot demand any diamond at a positive price if a do not have a cent and nobody is (perhaps, rationally) lending me. The same is true for the government. The demand versus desire distinction is clearly captured in Cochrane’s words: “you can’t double your demand for Porsches, counting on the price to halve.” (Cochrane (2003b)).

\(^{18}\)Whether there are alternative government’s plans to prevent crisis and the conditions under which may be implemented is the subject of the next sections.

\(^{19}\)As noticed by Cochrane (2003b), “If the consumers were willing to lend ever increasing amounts, the government budget constraint must allow them to do so” (italics as in original).
of the government’s plans, notwithstanding the fact that it is also a sufficient condition for rendering $P_1$ a non-equilibrium outcome. That is, government solvency requires a particular non-optimal individual behavior. Still, the following are natural questions when analyzing an equilibrium selection device designed by the government: (i) what is the precise mechanism that renders most prices incompatible with the existence of an equilibrium?, and (ii) how is that mechanism implemented by government? In answering the first question (what is the precise mechanism that makes $P_1 < P_1^F$ incompatible with the existence of an equilibrium?), the “optimization principle approach” offers a simple resolution: when the households face a price $P_1 > P_1^F$, they make a suboptimal lending decision, this behavior being the “true” equilibrium rejection mechanism. However, this argument remains silent with respect to the second query (how is that mechanism implemented by government?). All we know is that a lending decision described by (22) is consistent with the government plans and inconsistent with the existence of equilibrium but we do not know the underlying process leading the household to demand useless bonds when the money-price is “too low” and, critically, we can never deduce a mechanism linking the set of feasible government actions to such a private decision for, as argued above, a rule prescribing a zero demand for government bonds is always feasible, no matter what the price level would happen to be.

Taking as valid this “optimization principle approach” also forces a reconsideration of the cause-effect relationships postulated by the FTPL: what allows the government to formulate a price-invariant demand for goods while insisting on an exogenous level of taxation, a commitment to honor its debts at par and an elastic money-supply rule when faced with a price $P_1 < P_1^F$ is the households’ willingness to demand a particular amount of non-performing bonds (dictated by (22)) while the contrary is not true, i.e. it is not possible to combine the set of policy instruments considered in this economy in such a way that the government forces the households to demand worthless bonds. In this precise sense, an equilibrium selection device that rejects some price vectors as potential equilibria by ex ante assuming rather than proving the existence of a conflict between individual rational behavior and prices cannot be considered a policy-based device, but, rather, a psychological-based one.

Finally, the case of a potential crisis-price $P_1$, such that $P_1 > P_1^F$, can be handled exploiting the reasonings followed above for the opposite kind of deviations from the fiscalist unique equilibrium, with an added qualification. Now, when considering eventual violations of the transversality condition associated with positive private borrowing (i.e. situations in which the households find a positive demand for their own bonds), one cannot invoke an argument based on sub-optimal household’s lending decisions since, indeed, not taking advantage of an opportunity for selling bonds, maturing after death, at a positive price cannot be optimal. Further, if we were to allow the government to passively purchase any amount of bonds supplied by the households paying an strictly positive price, these would optimally supply infinite amounts of debt, thus precluding any economically meaningful equilibrium\footnote{This argument can be found in Woodford (1998).}. In view of these reflections, it seems natural to impose the no-Ponzi games condition $B_1 \geq 0$ and, similarly, $B_1^* \geq 0$, with both inequalities holding for any $P_1$, so that there is never a market for privately-issued bonds. This
modelling device simplifies the previous analysis to a great extent, as now we do not need to worry about potential violations of the bonds-market clearing condition along which the government tries to implement its original plan by demanding private bonds when faced with an arbitrary $\overline{P}_1 > P_1^F$: we are ruling out those violations by assumption and for any price level, not just for one, very much like in the Ricardian case in Proposition 1.

As argued by Weil (2002), imposing $B_1 \geq 0$ (“consumers are forbidden to die in debt”) amounts to impose a feasibility constraint, rather than an optimality constraint. But all we know about feasibility constraints is that they can never be violated, no matter what the price level is or whether such a price can be supported as an equilibrium or not. Hence, a mechanism designed by the government to rule out a crisis-price $\overline{P}_1 > P_1^F$ based upon the promise to transfer any resources above the (unique) amount compatible with the original fiscal plan to the private sector by purchasing private debt is unfeasible and, hence, non-credible.

In sum, the standard FTPL’s interpretation of the government transversality condition (or, equivalently, its intertemporal constraint) as a separate equilibrium condition does not shed any light into the question of how the government may combine its fiscal instruments so as to implement a successful equilibrium selection device able to solve the classical nominal indeterminacy associated with an elastic money supply rule. A government strategy of “keeping on the original plan” in view of a crisis will imply, in most cases, a non-credible “threat”, for a well-posed demand function cannot be conceived without respecting an underlying budget set. Altering the government budget set by printing non-performing bonds is not always possible, for someone else must voluntarily stand on the other side of the market ready to rescue a insolvent government plan. However, accepting such a possibility amounts to assume that the households behave in a non-optimal way when nothing precludes them from making the (contingent) optimal decisions. This is not a good axiom to start with for any theory claimed to be an equilibrium selection device founded on a “suitable” design of fiscal rules.
4 The limits of well-defined non-Ricardian policies

Using the simple one-period framework presented above, in this section I argue that a suitable re-design of the class of fiscal policies analyzed in the preceding section, which differs from a completely exogenous fiscal policy rule, can effectively rule out some potential equilibria, very much in the spirit of the standard FTPL. However, price and equilibrium uniqueness requires some extra conditions not considered by the standard FTPL. Specifically, a critical (and rather strong) necessary condition for the government to be able to implement an exogenously targeted equilibrium primary surplus while committing to a fully-elastic money-supply rule is that the fiscal plan must involve a non-positive level of seigniorage.

As shown below, implementation of “genuine” non-Ricardian policies (i.e. policies that create a link between prices and private wealth that break the Ricardian Equivalence Theorem) resolves only a part of the indeterminacy problem associated to an elastic supply of money, although this requires a fiscal policy that reacts at off-equilibrium prices, for the presence of non-Ricardian elements is not a sufficient condition for equilibrium uniqueness. Roughly speaking, non-Ricardian fiscal policies solve “half of the problem”, and they do so in a way compatible with the arguments given by the advocates of the FTPL, namely, by using fiscal policy to back money with valuable resources collected by the government, i.e. taxes. By contrast, the other half of the problem, which has been systematically neglected, is related to situations in which money injections should back an insufficient amount of taxes. A solution to this latter problem cannot be given by invoking non-Ricardian fiscal-effects. This requires a much more drastic assumption: the government must plan a fiscal policy sufficiently solvent so as to never need new positive, in net terms, monetary injections.

4.1 Ruling out government supersolvency

Below I argue that there is a simple policy rule which removes some potential equilibrium price sequences, namely those associated with prices satisfying $\mathcal{P}_1 > P^F_1$ in (20), which, otherwise, would force the government to increase its consumption, to reduce taxes or to pay a supersolvency premium on the initial stock of debt (i.e. redeeming bonds above par). I first show the basic insights of this argument by using a simplified version of the one-period model presented above in which the households do not have government bonds at the beginning of the period\footnote{This is consistent with Woodford’s (1995) analysis of an economy without initial nominal-bonds holdings.}. Then I extend the analysis to a more general case with a positive initial stock of government debt and, in Section 6, I consider a multiple-period economy.

Consistently with the arguments given in the previous section, I assume that individuals never demand nor supply bonds at the end of the period, i.e. $B_1 = 0$, so that the argument does not give any special role to the transversality condition. Let’s consider a
price level, $P_1$, such that $P_1 > P_1^F$ (with $P_1^F$ given in (20) after imposing $(1 + i_0) B_0 = 0$) and assume that the government sets some exogenous targets for $g_1$ and $\tau_1$, labelling these targets as $g_1$ and $\tau_1$. As (20) holds as an identity, a price $P_1 > P_1^F$ implies that the fiscal targets cannot be attained. For example, if the government insists on consuming $g_1$ regardless of the price level then, given that $B_1 = 0 \forall P_1$, taxes must adjust downwards so as to satisfy (20). The equilibrium associated with $P_1$ would entail the following allocations:

$$g_1 = g_1, \quad c_1 = y - g_1, \quad M_1(P_1) = v^{-1}[u'(y - g_1)]P_1, \quad B_1 = 0$$

with the equilibrium tax, $\tau_1$, satisfying

$$\tau_1 = g_1 - \frac{M_1(P_1) - M_0}{P_1}$$

Labelling seigniorage as $s(P_1)$, we may express deviations of the actual tax from the target as

$$\tau_1 - \tau_1 = s(P_1) - s(P_1^F) > 0 \iff P_1 > P_1^F$$

This adjustment is plainly consistent with the Ricardian argument in Proposition 1: as the government is receiving more seigniorage, $\frac{M_1 - M_0}{P_1}$, than needed to implement its original consumption plan, its constraint forces a tax reduction, since, this is the only alternative for the government to dispose of its “extra” income from the Ricardian perspective. In what follows, I study the effects of allowing the fiscal branch of the government to consume that extra income through an alternative mechanism, along the lines of the following assumption.

Assumption 1. The fiscal branch of government can participate in the money market buying money and giving consumption goods in exchange.

I do not try to claim any realism in this assumption, for the aim here is just to uncover the necessary conditions under which fiscal policy may reduce the number of potential equilibria. The necessity of this assumption is discussed later. For the moment, it is important to notice that the action by the fiscal authority described in that assumption is not incompatible with the simultaneous central bank’s commitment to supply as much money as demanded so it is not at odds with the view of the two authorities, fiscal and monetary, implementing their policy instruments in a decentralized fashion.

Now, suppose that in an eventual situation in which the fiscal branch of the government would receive an amount of real resources (i.e. taxes plus seigniorage) above the one compatible with its exogenous targets, it commits to inelastically supply a positive quantity of goods equal to the “excess” of resources in exchange for any strictly positive amount of dollars. Then any $P_1 > P_1^F$ cannot be part of an equilibrium. The proof of this claim is based on a simple arbitrage argument. Let’s suppose that when $s(P_1) > s(P_1^F)$ the government keeps its targeted tax, so that its constraint now reads as

$$g_1 + w_1 = g_1 = \tau_1 + s(P_1)$$  \hspace{1cm} (23)

\[^{22}\text{Henceforth, I assume that the households behave in a contingent optimal way, i.e. their demands for goods and dollars obey the optimality condition (17) for any price.}\]
where \( w_1 \) stands for the excess of budget available for government consumption (with respect to its target \( \overline{\tau}_1 \)). The commitment to bid \( w_1 \) in exchange for any arbitrary strictly positive amount of dollars implies that any individual could buy, e.g., a single dollar from the central bank at a real cost\(^{23} \) of \( \frac{1}{\overline{p}_1} \) and then supply that dollar in exchange for the (strictly positive) amount of goods supplied by the fiscal authority, \( w_1 + \frac{1}{\overline{p}_1} \). Such an operation gives that individual a windfall gain of \( w_1 \) units of consumption\(^{24} \). Clearly, the existence of such a free-lunch opportunity is not compatible with the existence of an equilibrium. Thus, given the commitment to dispose of any eventual excess of seigniorage, by accepting dollars in exchange for it, a new necessary condition must be added to the definition of an equilibrium for this economy: for \( \overline{P}_1 \) to be part of an equilibrium, \( \overline{P}_1 - P_1^F \), cannot be positive.

It is worth noticing that this condition implies that at off-equilibrium prices government consumption, \( \overline{\tau}_1 + w_1 \), varies with the price, which, in turn, means that available private wealth and, hence, consumption demand, depend on the price, thus creating a link between prices and optimal decisions. In other words, this fiscal price-contingent strategy creates well-defined price-based non-Ricardian wealth effects\(^{25} \).

It remains to check that such a government’s commitment is a credible one. Clearly, this is the case here, as this commitment only implies that the government makes a “gift” to the households in case these were to pay a level of seigniorage above the (unique) one compatible with the fiscal targets. Further, as such a commitment can be made in a credible way regardless of the particular off-equilibrium price, it provides a “valid” extra condition which helps to rule out some equilibria.

The extension of the previous argument to the case in which the households have a positive stock of government debt at the beginning of the period is straightforward. In that case, the government does not only target a particular value for taxes and government consumption but also aims to redeem its existing debts at their contractual value. According to the arbitrage argument given above, any commitment to give up the excess of seigniorage with respect to the targeted level will be credible, where the targeted level of seigniorage now satisfies,

\[
s(P_1^F) = v^{-1}[u'(y - \overline{\tau}_1)] - \frac{M_0}{P_1^F} = \frac{(1 + i_0)b_0}{P_1^F} + \overline{\tau}_1 - \overline{\tau}_1
\]

Hence, such a price-contingent fiscal policy will remove any potential equilibrium price in which the government would be forced either to pay a supersolvency-premium on its initial debt or to decrease the level of taxes, i.e. any \( \overline{P}_1 > P_1^F \). However, notice that

\(^{23}\)Let’s assume, for simplicity, that this marginal increase in the total nominal demand for dollars does not have any significant effect on the price level, so that this additional dollar can be purchased at the original price, \( \overline{P}_1 \).

\(^{24}\)Notice that the original “excess of seigniorage” in hands of the fiscal authority, \( s(P_1) - s(P_1^F) \), is augmented by \( \frac{1}{\overline{p}_1} \), i.e. the extra seigniorage received by the sale of the additional dollar used in this arbitrage operation.

\(^{25}\)Notice that the assumption of an economy populated by identical households does not automatically implies the fulfillment of the Ricardian Equivalence Theorem, for such a result breaks down as long as government consumption is not constant (see, e.g., Blanchard and Fischer (1989)).
the wealth effect induced by this fiscal strategy does not arise directly from government bonds, rather, it is associated with the initial dollar holdings: government dollars are net wealth. Not surprisingly, implementation of the non-Ricardian rule described here require a positive initial stock of dollars rather than government bonds.

Why is Assumption 1 necessary in this argument? In this economy, there are four ways for the government to dispose of excess “unwanted” resources: paying a supersolvency premium on the initial stock of debt (i.e. violating its commitment to redeem bonds at par), consuming above the target, reducing taxes below the target and purchasing money paying a price above the market one26. Except the last one, any other distribution policy will imply the violation of a fiscal commitment, however without necessarily violating any equilibrium condition, as in the Ricardian case. It is worth noticing that a policy that makes total private outside wealth dependant on the price level need not deliver price uniqueness (e.g. a passive adjustment of government consumption). In this sense breaking the Ricardian Equivalence result is not sufficient to yield a unique equilibrium, for that policy must also generate arbitrage opportunities27.

4.2 The asymmetry between government default and supersolvency

Can we apply a similar arbitrage argument to rule out equilibrium prices, \( P_1 \), such that \( P_1 < P^F_1 \)? The answer depends critically on the sign of \( s(P^F_1) \), i.e. the targeted level of seigniorage required to avoid an upward adjustment in the primary surplus.

Firstly, assume that \( s(P^F_1) > 0 \), and, as before, let’s first consider the case with zero initial debt, so that a positive level of targeted seigniorage, \( s(P^F_1) \), goes in hand with a targeted primary fiscal deficit, i.e. \( \pi_1 - \tau_1 > 0 \). Now, a price \( P_1 < P^F_1 \) cannot be ruled out as a candidate equilibrium price exploiting the non-Ricardian argument outlined above. To see this, notice that when the price \( P_1 < P^F_1 \) and the government consumes \( \bar{\pi}_1 \), the level of seigniorage actually paid by the households falls below the targeted one, i.e. \( s(P_1) - s(P^F_1) < 0 \). The government can only meet its fiscal targets by inducing individuals to pay the targeted level of seigniorage which, in turn, implies that in face of \( P_1 \), it should be able to engineer a policy aimed at depreciating the real value of money, something which is only possible if it manages to increase money supply. In the previous case, for a price \( P_1 > P^F_1 \), the government was able to “defeat” the ongoing market price, \( P_1 \), by inelastically supplying an amount of goods equal to \( s(P^F_1) - s(P^F_1) > 0 \) in exchange for money. Symmetrically, to rule out a market price \( P_1 < P^F_1 \), the government should be able to inelastically supply an amount of money equal to \( M(P^F_1) - M(P^F_1) > 0 \), in exchange for consumption goods, where \( M(P^F_1) \) and \( M(P^F_1) \) are the stocks of money.

\footnote{In the argument developed here, the government does not commit directly to pay a particular price when redeeming money but, by committing to sell a given amount of consumption goods in exchange of any quantity of dollars, it is indirectly allowing for the possibility of selling goods at a zero net price.}

\footnote{This observation resembles one of the conclusions of Cushing (1999). He allows for non-Ricardian households in an OLG model concluding that such a departure is not sufficient for equilibrium uniqueness under a pure interest rate peg.}
satisfying (17) for $P_1^F$ and $\overline{P}_1$, respectively. Such a policy announcement, however, is not consistent with the assumed commitment to meet the ongoing demand for nominal balances in a completely elastic fashion. Equivalently, a negative $u_1$ in (23), holding the target $\overline{y}_1$, can only be corrected by increasing $\gamma_1$ or abandoning the monetary rule.

Notice that the inability of the government to put upward pressure on the price level in the present case is not a consequence of the strict separation of competencies between the fiscal authority and the central bank, rather, it is a natural result arising from the specified money supply rule. Even if we think of a central bank fully subordinated to the dictates of a fiscal authority facing financial difficulties, there is not any arrangement capable of providing individuals with the right incentives to pay a higher level of seigniorage under this monetary rule. Therefore, an arbitrary price $\overline{P}_1 < P_1^F$ can be an equilibrium outcome, with the fiscal authority being forced to either increase taxes or decrease government consumption (or a combination of both). It follows that in this simple setting without initial debt holdings, the government cannot credibly commit to run an arbitrary primary deficit under any circumstance.

As before, extending this nominal indeterminacy result to the more general case in which there is a positive stock of initial government’s debts is straightforward. When the standard fiscalist solution in (20) calls for a positive level of seigniorage, i.e. $s \left(P_1^F\right) > 0$, there is no guarantee that the government will always be able to avoid default if it insists on a particular level of primary surplus.

The reason for the asymmetry between the default and supersolvency cases when $s \left(P_1^F\right) > 0$ is very intuitive. Ruling out default would need a violation of the money supply rule, that is, a critical departure from the context within which the FTPL claims to yield price uniqueness. Ruling out a supersolvency premium only requires a commitment from the government to distribute any excess of seigniorage by purchasing dollars and, as argued before, such a commitment can always be made in a credible way. Moreover, such a fiscal strategy is not incompatible with the commitment to sell as many dollars as demanded at the market price. Put it other way, the ability of the government to rule out supersolvency is possible because for a price such that $\overline{P}_1 > P_1^F$, it enjoys a strong financial position, in the sense that it is collecting more resources than needed to go ahead with its original plans. On the other hand, when $\overline{P}_1 < P_1^F$, the government faces a weak financial position in that it will be unable to meet its plans at that price. For a particular agent, whether a household or an economic authority, to be able to “defeat” a potential market equilibrium outcome, that agent must be able to generate arbitrage opportunities at the ongoing market price, which is only possible if that agent is willing to face losses in trading with the different assets and goods. The later, using the above terminology, requires that agent to be in a strong financial position. In the next section, I provide further intuition on this asymmetry in the context of a private firm.

4.3 The necessity of convertible (non-fiat) money

In this simple one-period model, an obvious way to circumvent the multiplicity of equilibria problem arising when $s \left(P_1^F\right) > 0$ is to further restrict the fiscal-monetary program so
that it satisfies \( s(P_1^F) \leq 0 \). This non-positive seigniorage condition can then be exploited to rule out prices satisfying \( P_1 < P_1^F \), as potential equilibria, provided the government sets an upper bound to the (negative) volume of seigniorage, as stated in the following assumption\(^{28}\).

Assumption 2. The central bank commits to supply any amount of money at the ongoing market price level but does not commit to symmetrically purchase any amount of money at any market price.

Then, only if \( s(P_1^F) \leq 0 \) and Assumptions 1 and 2 hold, it follows that there is unique equilibrium price, which coincides with the fiscalist’s standard solution, \( P_1^F \) in (20), provided the government commits to the appropriate price-contingent fiscal rule. The proof for this claim is simple given the previous results. Given that the government can credibly commit to distribute any excess of seigniorage by accepting dollars in exchange for consumption goods, any price \( P_1 > P_1^F \) can be ruled out as an equilibrium price. On the other hand, any price \( P_1 < P_1^F \) would be associated with a negative level of seigniorage higher, in absolute value, than the targeted one, \( s(P_1^F) \). But given Assumption 2, for such a price to be part of an equilibrium, the households must expect a transfer or resources from the government greater than the one compatible with the fiscal objectives. As the government is not obliged to redeem money for an arbitrary amount of consumption goods, such individual’s beliefs are not rational and, hence, prices consistent with those beliefs can be ruled out. The necessity of Assumption 2 for this uniqueness result is clear, as well.

Yet, it must recognized that the latter class of deviations from the equilibrium price level (i.e. prices below \( P_1^F \)) are not corrected by means of a any non-Ricardian wealth effect. Indeed, government consumption may be kept at the targeted value along those deviations. Instead, the driving force to rule out those prices is a pure convertible-asset valuation argument. It is worth providing some intuition about the necessary condition for the above uniqueness result, \( s(P_1^F) \leq 0 \), relating it to the economic concepts of convertible and fiat money. Condition \( s(P_1^F) \leq 0 \) implies that, except in the particular case in which \( s(P_1^F) = 0 \), money must be a convertible asset, in the sense that the government is actually retiring money from circulation, i.e. \( M_1(P_1^F) < M_0 \), something which is only possible by giving up some valuable resources (i.e. consumption goods) in exchange for it. Taken literally, that condition forces a reconsideration of the nature of money. When \( s(P_1^F) < 0 \), money is not longer a purely fiat asset whose real value is exclusively driven by individual expectations on the future acceptability of such an asset by other private agents, but rather, as the fiscal branch of the government repurchases money, its equilibrium real value must also reflect the value of those consumption goods being given in exchange for it. Further, this “dual-value of money” argument also applies in the limiting case in which the government “gives” zero units of consumption goods in exchange for a dollar (i.e. \( s(P_1^F) = 0 \) and money is neutral from a fiscal perspective). Notice that price uniqueness in this particular case follows from the assumption of no-speculative

\(^{28}\)Again, the aim here is not to claim the realism of this assumption but rather to impose a minimum set of conditions characterizing the institutional framework within which the FTPL’s basic postulates are consistent. As argued later, this assumption is necessary for equilibrium uniqueness.
hyperinflations. If \( \lim_{m \to 0} mv'(m) = 0 \), then speculative hyperinflationary paths cannot be ruled out unless the fiscal authority commits to a price-contingent real backing scheme as in Obstfeld and Rogoff (1983), i.e. even in this case money must be convertible when the Walrasian auctioneer were to dictate a infinite price level. Such a commitment can be readily included in the fiscal authority’s set of contingent actions without altering substantially the characterization of the unique equilibrium just described, since that commitment implies that convertibility never takes place in equilibrium.

The role of the necessary conditions for the uniqueness result (Assumptions 1 and 2) is a very intuitive one (as also discussed in the next section). Assumptions 1 and 2, when considered jointly, imply that turning money into a convertible asset is at the full discretion of the government through its fiscal choices. This condition is nothing but a reflection of the high degree of “fiscal dominance” that is necessary for a theory which tries to give fiscal policy a first order importance role in the determination of the price level, as the FTPL does.

The following proposition summarizes the main previous results.

**Proposition 3** In the one-period economy described above, when the government targets a particular primary surplus, \( \bar{\tau}_1 - \bar{\gamma}_1 \) and aims to redeem the initial stock of debt at the contractual value, and simultaneously follows a completely elastic money supply rule, the equilibrium price, \( P^*_1 \), satisfies the following conditions:

(i) If Assumption 1 holds and the government commits to inelastically supply any excess of seigniorage, \( s(P_1) - s(P^F_1) > 0 \), in exchange for money, then \( P_1 \) cannot be part of an equilibrium. Any equilibrium price must satisfy \( P^*_1 \leq P^F_1 \), where \( P^F_1 \) is defined as

\[
P^F_1 = \frac{M_0 + (1 + i_0) B_0}{\bar{\tau}_1 - \bar{\gamma}_1 + \nu^{-1}[w'(y - \bar{\gamma}_1)]}.
\]

(ii) If the targeted level of seigniorage satisfies \( s(P^F_1) > 0 \) and \( P_1 < P^F_1 \), then the original fiscal-monetary plan cannot be implemented.

(iii) If Assumptions 1 and 2 hold, the targeted level of seigniorage satisfies \( s(P^F_1) \leq 0 \) and the government commits to inelastically supply any excess of seigniorage in exchange for money, then there is a unique equilibrium price, \( P^*_1 \), satisfying \( P^*_1 = P^F_1 \).

The message of this proposition is also useful to understand one of the center claims of the FTPL: the non-necessary cooperative behavior between the fiscal and monetary authorities. As only fiscal plans that involve a non-positive level of seigniorage are credible in face of a commitment to an elastic money supply rule, the fiscal authority does not need any financial help from the central banker, for it is sufficiently solvent so as to never need seigniorage or, perhaps, even to purchase (i.e. destroy) money.
5 Learning from the fiscalist stock-analogy

In this section, I discuss to what extent the results of the previous sections force a reconsideration of the fiscalist stock-analogy mentioned earlier. The main claim here is that a strategy based on applying a standard asset valuation approach a la Lucas (1978) to determine the value of money through the “government valuation equation” (13) may be misleading, at least, for two reasons.

The first reason is an obvious one: the RHS of (13) is not a correct measure of the total discounted amount of resources, collected by the government, i.e. that term cannot be understood as the “profits” of the government. To see this, notice that net income gained by the government because of its monopolist production of money, according to the RHS of (13), is given by the so-called inflation-tax term, \( \frac{j}{1+j}M_t \), which is not the correct measure of those monopolistic profits, as these correspond to the amount of seigniorage, \( \frac{M_t - M_{t-1}}{P_t} \). The inflation-tax referred above is a measure of the opportunity cost borne by an individual who accepts to hold a share of his wealth in the form of monetary balances rather than bonds. But such an opportunity cost cannot be thought as of being net income going to the government\(^{29}\). Logically, if the RHS (13) is not a correct measure of the firm’s profits then the LHS of that equation cannot be thought as of being the number of outstanding shares of the firm, if one is still willing to accept an analogy between a firm and the government. Thus, when the “government valuation equation” is written so as to contain the total discounted value of the total net real government’s income in its RHS, what appears on the LHS is the initial stock of government’s debts, if any, i.e.,

\[
\frac{(1 + i_0)B_0}{P_1} = \sum_{i=1}^{\infty} \frac{\tau_t - g_t + \frac{M_t - M_{t-1}}{P_t}}{\prod_{i=1}^{\infty} (1 + r_s)}
\]

Equation (24) makes clear that the true “profits” depend on the initial period’s price level, \( P_1 \), through the term \( \frac{M_0}{P_1} \), so the firm’s stock-analogy loses its attractiveness here, as we cannot longer assume that total profits are fully independent of the price level.

The second reason is related to a more fundamental observation. The standard FTPL argues that a policy mix resulting in a unique equilibrium sequence of discounted government’s total surpluses (i.e. primary surpluses plus seigniorage) will result in a unique equilibrium whenever there is an initial stock of government’s nominal obligations. Such an argument would be correct (put aside, for a while, the previous observation on the incorrect measure of total surpluses) whenever the only reason for holding government nominal bonds and, critically, money, is the expectation of redeeming these assets for government-provided valuable resources, i.e., according to the stock-analogy what gives value to money is the amount of resources that the government is going to employ in purchasing money or in distributing dividends on it. This simple and rather strong conclusion should not be surprising since, after all, this is the basic assumption underlying

\(^{29}\)For example, let’s consider an infinite-horizon economy in which the level of private consumption, the real interest rate and the supply of money are constant. Constant money supply implies that both seigniorage and inflation (ruling out bubble-solutions) are zero but the inflation-tax will be positive as long as the real interest rate is positive. Identifying this last concept with seigniorage (i.e. the net effective gain of the central bank) only leads to an erroneous calculation of total government’s surpluses.
the standard model of equity valuation which the FTPL looks at: the only reason for holding equity, and what gives that equity a real value, is the expectation of receiving profits from the issuer firm via distributed dividends or share repurchases. As the government does not pay any dividend on money (i.e. zero nominal interest on money), the only way in which the stock-analogy can work here is by allowing the government to repurchase money. Not surprisingly, the class of policies studied in the previous section yield the uniqueness result supported by the stock analogy precisely when the policy-mix involves a non-positive level of seigniorage money, i.e. when money effectively gets a share of the government’s “profits”.

Nevertheless, the fiscalist stock-analogy is still useful to shed some light on several issues discussed before. The asymmetry between government default and supersolvency arising in the “modified” FTPL developed in the previous section is something natural in the context of a firm. To see this, let’s write the following equation, reminiscent of the one-period economy government’s budget constraint (19),

\[ \frac{S_0}{Q_1} = \pi_1 + \frac{S_1}{Q_1} \]

where \( S_0 \) and \( S_1 \) are the initial and the final number of outstanding shares of the firm, respectively, \( \pi_1 \) is the net operational profit of the firm and \( \frac{1}{Q_1} \) is the real value of each share (measured in units of profits). A non-zero terminal portfolio, \( \frac{S_1}{Q_1} \), can be justified if the share-holders find this firm’s paper useful even after the firm has been liquidated, i.e. these \( S \)-shares are “tasty” for the share-holders\(^{30}\). Let’s also assume that the share-holders want to hold a particular terminal portfolio, measured in real terms, i.e. \( \frac{S_1}{Q_1} = \kappa \), for some given (positive) \( \kappa \), and that the firm commits to supply as many shares as demanded at the ongoing market price, \( Q_1 \).

Then, if \( \pi_1 > 0 \), the firm offers the investors the possibility of exchanging some shares for profits up to the point at which total profits are fully distributed. When the share-holders behave optimally (i.e. not leaving any valuable profit unexploited), the equilibrium price of each share satisfies

\[ \frac{S_0}{Q_1} = \pi_1 + \kappa \]  \hspace{1cm} (25)

and the number of shares repurchased by the firm is \( S_0 - Q_1^F \kappa \).

However, if the firm plans to suffer a loss, i.e. \( \pi_1 < 0 \), there is no mechanism to force investors to provide the firm with the required resources, notwithstanding the fact that investors may be willing to buy some new shares, just because they like them. But nothing guarantees that this later form of funding will be enough to allow the firm to go ahead with its plans, even if the RHS of (25) is strictly positive. The firm’s commitment to consume no more than \( \pi_1 \) rules out any price \( Q_1 > Q_1^F \), but there are multiple equilibria in which actual losses, in absolute value, are below \( \pi_1 \) and the equilibrium price level satisfies \( Q_1 \leq Q_1^F \).

\(^{30}\)For a similar “tasty-share” story, see, e.g., Marimón (2001).
We can extend this simple analogy by introducing “nominal bonds”, that is, a new type of shares that are not perceived as intrinsically useful by the holders and that are denominated in terms of the tasty ones (call them N-shares). Let’s focus on the case in which the firm plans a positive level of operational profits, i.e. $\pi_1 > 0$, and tries to repurchase the outstanding no-tasty shares, $N_0$, at a price equal to $Q^f_1$ in the following equation

$$\frac{S_0 + N_0}{Q^f_1} = \pi_1 + \kappa$$

Also, let’s assume that $S_0$, $N_0$, $\pi_1$ and $\kappa$ are such that the firm needs to sell new S-shares to meet its plans, i.e. $S_1 - S_0 > 0$. Then, at any price $Q_1 > Q^f_1$, the firm is not distributing all its profits even after having repurchased the entire initial stock of N-shares. A commitment to distribute any remaining profits via repurchases of S-shares renders such a price $Q_1 > Q^f_1$ a non-equilibrium outcome: not accepting such a repurchase offer amounts to give up a profitable opportunity for the investors. On the other hand, if $Q_1 < Q^f_1$, new net capital injections, as measured by $\kappa - \frac{S_0}{Q^f_1}$, are not sufficient to repurchase the outstanding $N_0$ shares at the ongoing price $Q_1$. What can the firm do? It cannot directly sell more newly printed S-shares, given the commitment to a completely elastic supply rule. It cannot increase its operational profits, as they are assumed to be given exogenously. There is nothing the firm can do to provide investors with the right incentives to purchase new V-shares, promising, at the same time, that this extra income will be rebated to those same investors through a higher redemption value for the Q-shares and, as a result, multiple equilibria along which each N-share takes a value below a S-share cannot be ruled out.

An interesting example shows up when assuming that the investors do not find S-shares useful \textit{per se} anymore (i.e. $\kappa = 0 \forall Q_1$). Under this assumption, the initial claims on the firm’s profits, $S_0$ and $N_0$ may always have a unique equilibrium common value since nothing may preclude the firm from committing to distribute its profits across the different types of shares evenly (i.e. redeeming both types of shares at the same value). Of course, if the only outstanding claims are N-shares ($S_0 = 0$), there will be a unique equilibrium price for those shares and, naturally, the price for a security that is not traded at all (S-shares) is not defined. Thus, what makes the problem of pricing S-shares a non-trivial one is the possibility of an existing demand for these securities which is not exclusively driven by the investors’ expectations on the firm’s stream of profits or losses (in this simple example, the existence of a positive portfolio $\kappa$, and in a monetary economy, the public’s perception of money as an object which provides utility \textit{per se}, helps to overcome a cash-in-advance constraint or reduces the cost of making transactions). In such a case, a new kind of profit, different form the operational one, emerges, for net sales of these shares increase the firm’s net worth. Further, when the firm commits to an elastic supply of S-shares, the amount of this later form of profit will not be uniquely determined, unless the firm actually uses a portion of its operational profits to purchase those shares, in which case, in equilibrium, S-shares are not longer an additional source of (indeterminate) profits but a destination in the distribution process of the exogenous (and uniquely determined) operational gains. In the same fashion, what makes the problem of pricing money a non-trivial one and, potentially, a “difficult” one is the possibility that private individuals find money useful even if it is well understood that the government
will never redeem that money for consumption goods, i.e. the possibility of money being a fiat asset.

Also, as the examples above show, the problem of default is linked to the existence of an underlying commitment to redeem the entire stock of $N$-shares at the price of each $S$-share. In this sense, default is a true possibility for certain firm’s claims as it is a true possibility for government dollar-denominated bonds. Were this commitment be absent (as in Bassetto (2002)), a concern on a possible debt-default would no longer be justified.

Comparing the above examples with the economy analyzed in the previous section yields, indeed, an interesting analogy. In particular, Assumptions 1 and 2 presented above, which are necessary for equilibrium uniqueness, can be also seen as the necessary elements to go from the standard institutional framework of a firm to the one described in the preceding section. Notice that Assumption 2 implies that money, while being potentially a convertible asset, is just a residual claim to the eventual government surpluses, that is, there is nothing forcing the government to redeem any arbitrary quantity of dollars at any arbitrary price, very much like the $S$-shares above. Assumption 1 plays a crucial role in removing potential equilibria in which the government, having committed to an exogenous primary surplus, would be forced to redeem the existing debt at a value above the contractual one. Indeed, this assumption can be seen as of capturing the autonomy of a solvent firm in deciding how to distribute its profits across different classes of shares (money and bonds).

Thus, once we assume that a firm’s total profits may depend on the seigniorage gained by the sales of tasty shares, there is an analogy between the mechanism of determination of the equilibrium value of those tasty shares and the one governing the value of government-issued money when a correctly specified (i.e. credible) fiscal-monetary plan that involves a non-positive level of seigniorage is at work.
6 A multiple-period economy

In this section I extend the arguments developed before to an economy with multiple periods. This scenario allows for an explicit consideration of the nominal rate as a policy instrument. Before developing the formal arguments, it is useful to briefly discuss the nature of the commitment associated to an interest rate peg and its influence on private expectations. Proposition 1 states that, in a perfect-foresight economy, under a Ricardian policy the ability of the government to commit to a particular nominal return paid by its debt regardless of the initial price level implies that, while the total present value of surplus depends on the particular realization of $P_1$, the sequence of equilibrium inflation rates is independent of $P_1$. Thus, at least regarding the determination of inflation, Ricardian policies in a perfect foresight setting do not pose important difficulties. However, as stressed by Sargent and Wallace (1975), and more recently by Woodford (2003), as we consider a scenario with uncertainty, a policy of pegging the nominal interest rate will involve price-indeterminacy at each date, and hence, the actual inflation rate will be indeterminate even if it is unique in expectation. The basis for a formal argument is simple. Let’s consider a linear version of a stochastic Fisher equation\(^{31}\),

\[
i_t = \tilde{r}_t + E_t \pi_{t+1}
\]

(26)

where $\tilde{r}_t$ is the targeted real return paid by a bond issued at $t$ and maturing at $t+1$, assumed to depend upon some fundamentals (e.g. preferences, endowments and government consumption) and $E_t \pi_{t+1}$ is the expectation as of $t$ of the inflation rate prevailing between $t$ and $t+1$. For given $i_t$ (assumed to be set directly by the government) and $\tilde{r}_t$, (26) can be solved for a unique $E_t \pi_{t+1}$. However, (26) does not have a unique finite solution for the stochastic process $\{\pi_{t+1}\}$, as any finite $\pi_{t+1}$ satisfying

\[
\pi_{t+1} = i_t - \tilde{r}_t + \psi_{t+1}
\]

(27)

is a solution of (26), provided $\psi_{t+1}$ is not forecastable at $t$, i.e. $E_t \psi_{t+1} = 0$. We can not say much about $\psi_{t+1}$, apart from this latter condition, as a non-zero $\psi_{t+1}$ could arise as the result of an underlying process linking prices to fundamentals but critically, as stressed by Woodford (2003; sec. 2.2.1), also as a pure speculative component in the process generating prices (sunspot states), totally unrelated to fundamentals. In a Ricardian world, the distinction of fundamental vs. non-fundamental elements in the solution for the realized inflation rate (i.e. in the price $P_{t+1}$) is unimportant regarding the private rational expectation formed one period earlier about $P_{t+1}$, for the public’s perception that the government will adjust its fiscal instruments so as to honor its maturing debt at the contractual value implies that for a given $\tilde{r}_t$, $E_t \psi_{t+1} = 0$ must hold always. In this sense, fiscal commitment rules out arbitrary self-fulfilled prophecies on $E_t \psi_{t+1}$, and hence a policy of fixing the nominal interest rate paid by government bonds, coupled with a Ricardian fiscal rule will induce a unique inflation expectation. If, on the other hand, the public understands that such a fiscal response at $t+1$ will not be observed, say because the government announces a sequence of primary surpluses totally independent of the price

\[^{31}\] A detailed derivation of a similar equilibrium condition as a local approximation in an optimizing model can be found in Woodford (2003, C. 2).
level, the previous reasoning will not apply. Under such a fiscal rule, the announcement of a particular nominal interest may not be credible and arbitrary expectations on $\psi_{t+1}$ consistent with individual optimization can not be ruled out in advance, even if there is not any other source of uncertainty beyond the random process that produces a particular expectation on $\psi_{t+1}$. In what follows I argue that such a possibility must not be disregarded, unless the fiscal plan involves the clear commitment to devote a fraction of the targeted primary surpluses to repurchase a fraction of the beginning-of-period stock of money, as in the one-period economy studied before.

To prove this claim in the simplest possible environment, let’s consider a two-period economy, similar in every other respect to the one presented before. Along the lines of the non-Ricardian policy described in Section 2, it is assumed that the government commits to implement a sequence of price-invariant primary surpluses. The government also announces the nominal interest rate prevailing between periods 1 and 2, $l_1$. Since the aim here is to show that such a plan involving sequences of exogenously targeted primary surpluses and nominal interest rates may not be always credible, it is convenient to distinguish between government’s targeted values for its policy instruments and households’ beliefs on those values. For this purpose I introduce a new notation to accommodate the eventual differences between government-announced and household-expected nominal interest rates, the former being denoted by $l_d$ and the later by $l_h$. Thus, the difference between these rates can be understood as the presence of a sunspot component in the expectation on the actual rate (i.e. using the notation above, I am considering here $E_t \psi_{t+1} = \psi_{t+1} \neq 0$, where the equality is imposed to preserve the certainty-hypothesis).

The household’s budget constraints are now given by (15) and

$$c_2 \leq y_2 - \tau_2 + \frac{(1 + i_1) B_1 + M_1 - M_2 - B_2}{P_2}$$

where $i_1$ is the observed (actual) interest rate paid by government bonds. As before, it is assumed that the household follows a bonds demand rule in the terminal period which is independent of the observed price level, i.e. $B_2 = 0$ for any $P_2$. Following a similar argument, the households’ demand for bonds in the first period is assumed to be optimal given their beliefs about the government’s resources backing its debt, i.e.,

$$E_1(i_h) S_1 = 1 + u_1$$

$$E_2(l_h) P_1$$

The LHS of (28) represents the demand for government bonds expressed in real terms and the RHS measures the discounted value of the resources available for debt redemption in the following period, i.e. the primary surplus plus seigniorage. The discount factor is defined, according to (6), as $\frac{1}{1+\tau_1} = \frac{\beta u(c_2)}{u(c_1)}$. In (28) I am representing the demand for real balances in the first period as $m_1(i_h) P_1$ to explicitly account for the fact that it depends directly on the nominal interest rate the individuals expect (see (5)). As before, $m_2$, being the demand for real balances in the terminal period, satisfies a first order condition similar to (17).

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32 Considering a non-degenerate distribution function for $\psi_{t+1}$ will not alter the argument.
First, let’s suppose that \( i_1^* > i_1 \), i.e. the households expect a nominal return on bonds higher than the one announced by the government. Then, using (6) we can express the level of seigniorage in period 2 for an interest rate \( i_1^* \) as

\[
s_2(i_1^*) = m_2 - \frac{1 + r_1}{1 + i_1^*} m_1(i_1^*)
\]

In order to learn whether those beliefs can be self-confirmed in equilibrium, I consider a vector of private consumption demand \( \{c_1, c_2\} \) compatible with the goods-market clearing condition. Then, for \( i_1 > i_1^* \), the following inequality must hold

\[
s_2(i_1^*) > s_2(i_1)
\]

In face of (29), the government may commit to follow a policy of rebating the excess of resources, \( s_2(i_1^*) - s_2(i_1) \), giving it in exchange for any arbitrarily positive amount of dollars (Assumption 1), thus creating an arbitrage opportunity which cannot be compatible with an equilibrium. As individuals anticipate that the government will distribute the “excess of profits” in exchange for dollars rather than for bonds, the expected effective return for bonds cannot be above the announced one. That is, if the government is determined to defend the announced nominal rate there cannot be any equilibrium with \( i_1 > i_1^* \).

Second, let’s consider the case in which \( i_1^* < i_1 \), so that \( s_2(i_1^*) < s_2(i_1) \). Then, as long as \( s_2(i_1^*) > s_2(i_1) > 0 \), we cannot rule out equilibria in which the government defaults. Given that the government commits to run an exogenous primary surplus, individuals correctly anticipate that the level of seigniorage in period 2 will be insufficient to redeem the stock of bonds issued one period before at their contractual value, increasing the relative demand for real balances and decreasing real lending to the government. Notice that in this economy the fact that (partial) nominal default is anticipated does not necessarily eliminate the incentives of the household to lend real resources to the government by purchasing bonds in period 1 at the price of one dollar. Formally, the relevant condition governing the optimal household’s decision on how much to lend can be discomposed into two separate decisions. First, the decision of whether to lend or not at all. The household will be willing to lend some valuable resources, in a positive and finite amount, if the following first order condition holds,

\[
(1 + i_1^*) \frac{P_1}{P_2} = \frac{u'(c_1)}{\beta u'(c_2)}
\]

Therefore, as long as the expected inflation rate, \( \frac{P_1}{P_2} \), satisfies (30), the fact that \( i_1^* < i_1 \) is irrelevant for this decision. All that matters is that each unit of consumption goods invested in bonds at period 1 yields the required real return. Thus, as discussed above, the government’s lack of commitment to respond to deviations of the effective nominal rate from the announced one allows for beliefs on expected inflation to become self-confirmed in equilibrium\(^{33}\). Second, as an expected positive rate of default is associated in equilibrium

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\(^{33}\)The trade-off between a standard Ricardian policy and the exogenous fiscal policy considered here becomes clear. The former is successful in pinning down inflation expectations, however at the cost of leaving taxes indeterminate. The opposite is true for the latter.
with a lower rate of inflation (see (30)) rational individuals will reduce the real amount of debt purchased in period 1. But this reduction is not a direct consequence of the rational expectation of nominal default, instead it is a reflection of the fact that seigniorage in period 2 and, hence, the amount of resources available to payback government’s debt falls.

Finally, when \( i_1^* < i_1^a \) and \( s_2(i_2^a) < s_2(i_1^a) \leq 0 \), under Assumptions 1 and 2 and an excess seigniorage-rebating policy, \( i_1^* \) cannot be a rational belief, so that the unique equilibrium effective nominal interest is the announced one, \( i_1^a \), i.e. the standard fiscalist non-default assumption holds. Thus, the necessary condition for the whole non-Ricardian program to be credible in the second period derived here, \( s_2(i_2^a) \leq 0 \), is similar to the one derived before for the one-period economy (Proposition 3 (iii)). Notice, however, that the credibility of the whole fiscal-monetary program requires condition \( s_t(i_{t-1}^o) \leq 0 \) to hold for any \( t \), and not just for the (eventual) terminal period. In the two-period economy analyzed here, we must impose \( s_1(i_1^a) \leq 0 \) and \( s_2(i_2^a) \leq 0 \) in order to obtain a unique equilibrium. For example, if \( s_2(i_2^a) \leq 0 \) holds, so that \( i_1^* = i_1^a \), we learn from (28) that the household’s demand for government bonds at the end of period 1, in equilibrium, is unique, given the credible government announcements for period 2. It follows that the problem faced by the household in period 1 is identical to the one faced in the one-period economy, as we only need to augment the end-of-period 1 household’s equilibrium portfolio with that extra debt-term\(^{34}\). Thus, the set of fiscal announcements for period 1 (exogenously targeted primary surplus and redemption of the initial debt, if any, at par) will only be credible if \( s_1 \leq 0 \).

The recursive nature of the optimization problem faced the household implies that we can extend this argument to economies with longer (and infinite) horizons in a straight-forward way. The following proposition summarizes these findings.

**Proposition 4** A fiscal-monetary plan in which the government commits to maintain exogenously targeted sequences of taxes, government consumption and nominal interest rates is only credible if Assumptions 1 and 2 hold and, at the targeted policy-instruments sequences, the following condition holds for every \( t \geq 1 \)

\[ M_t \leq M_{t-1} \]  \hspace{1cm} (31)

**Corollary** When Assumptions 1 and 2 and condition (31) hold and the government can commit to maintain its targets (i.e. the announced fiscal-monetary program is credible), the equilibrium is unique.

\(^{34}\)The fact that the expected amount of resources backing government bonds issued at the end of 1 is given uniquely implies that households’ demand for bonds cannot differ from that quantity when behaving optimally.
7 Concluding comments

In this paper, I first examine the off-equilibrium behavior of the economic agents prescribed by the FTPL, showing that the way in which non-Ricardian policies are usually defined by the advocates of the FTPL involves a non-credible commitment by the government whenever the assumed fiscal-monetary program contains the announcement for maintaining a completely exogenous sequence of government consumption and a fully elastic money supply rule consistent with the central bank announcing an arbitrary nominal interest rate\textsuperscript{35}, thus showing that the standard FTPL is never a policy-based equilibrium selection device. The main novelty of this criticism is that it is based on the same core assumptions maintained by the proponents of this theory: there is a positive stock of government-issued assets at the beginning of the history owned by the households, flow of funds constraints must be respected in every contingency, although transversality conditions may be violated at off-equilibrium prices.

Then, using a standard Walrasian model in which money is introduced as an argument in the household’s utility function, I identify the minimum set of conditions guarantying the credibility of a fiscalist policy close in spirit to the standard FTPL’s non-Ricardian policies. Firstly, I show that when the fiscal authority targets a particular (exogenously determined) sequence of primary surpluses, any equilibrium in which the government would be forced to increase the transfer of resources to the private sector above the targeted level can be ruled out only by assuming that the fiscal authority may participate in the money market purchasing dollars in exchange for real consumption goods. This assumption can then be incorporated into a fiscal strategy that breaks down the Ricardian Equivalence Theorem. Further, it is shown that such a fiscal strategy must necessarily be price-contingent for it to be credible. Secondly, I argue that the previous equilibrium-rejection device is not always implementable to symmetrically rule out equilibria in which the government is forced to reduce the transfer of resources to the private sector below the targeted level, as such a device would be inconsistent with the assumed monetary policy rule. Thirdly, I show that a fiscal-monetary plan resulting in a unique equilibrium under a nominal interest rate peg is only credible if that plan involves a non-positive level of seigniorage. That is, the main arguments of this modified FTPL are only valid for non-fiat money. This is one of the most interesting conclusions of this paper.

Finally, I argue that the so-called stock-analogy exploited by some advocates of the FTPL, according to which there exist fiscal strategies that turn the government intertemporal constraint into a “government valuation equation” requires an important qualification for it to be a useful analogy to explain how the equilibrium value of money is determined, namely, that money must be a convertible asset, a result which is plainly consistent with the above arguments about the credibility of a non-Ricardian policy.

\textsuperscript{35}The terms “exogenous” and “arbitrary” are used here in the precise sense of the corresponding policy-instruments not being chosen according to a Ricardian rule.
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