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CAPACITY UTILIZATION AND MONETARY POLICY

Documento de Trabajo nº 0306

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BANCO DE ESPAÑA

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Abstract

This paper presents a model featuring variable utilization rates across firms due to production inflexibilities and idiosyncratic demand uncertainty. Within a New Keynesian framework, we show how the corresponding bottlenecks and stock-outs generate asymmetries in the transmission mechanism of monetary policy. We derive an expression for the Phillips curve where the dynamics of inflation depend on real marginal costs and on a measure of resource underutilization.

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Key Words: Capacity Constraints, Nominal Rigidities Idiosyncratic Uncertainty, Asymmetries

1 Introduction

This paper addresses the following two questions: (i) What are the implications of production inflexibilities for the transmission mechanism of monetary policy?; (ii) How should monetary policy be conducted in this environment?.

Production inflexibilities arises in the economy when firms cannot immediately adapt their production possibilities to meet changes in their environment. Specifically, firms find it difficult to modify their capacity to produce goods in the short-run, giving rise to supply shortages and production bottlenecks. One important implication due to the existence of capacity constraints is that the response of macroeconomic variables to aggregate disturbances or to policy actions is asymmetric: similar actions or shocks are likely to generate quantitatively different macroeconomic effects. A significant number of empirical studies have found asymmetry in the real effects of monetary policy.[1] Cover (1992), Karras (1996) provide evidence of asymmetries between positive and negative monetary shocks on output and prices, while Weise (1999) and Lo and Piger (2002) find that monetary shocks have dramatically different effects over the business cycle.[2]

In this paper, we develop an analytical framework consistent with the aforementioned features of the monetary transmission mechanism. This framework consists of a dynamic stochastic general equilibrium model which displays the non-neutralities of money needed to perform policy analysis in the short run and production inflexibilities that are able to generate an asymmetric dynamic behavior of key macroeconomic variables. Specifically, the model developed in this paper has two basic ingredients: (i) It incorporates a real side with production inflexibilities that result in variable rates of utilization across firms; (ii) It considers nominal rigidities that create short-run real effects of monetary policy.

The first element of the model is due to three basic features: first, the limited possibilities of a short run substitutability between production factors; second, the existence of uncertainty at the time of capacity choices, which explains the presence of underutilized equipments; and third, the existence of idiosyncratic uncertainty which results in a non-degenerated distribution of utilization rates across firms. In equilibrium, a proportion of firms face demand shortages and have idle capacities, while others are at full capacity and are unable to serve any extra demand. The second basic element of the model is a monetary side with new-keynesian-type nominal rigidities. Specifically, it is assumed that it is costly for firms to increase the price level above a stable value. This may reflect costs of advertising or that erratic pricing causes consumers dissatisfaction.

The interaction between resource underutilization and monetary policy has recently been the object of attention in the monetary macroeconomics literature. For instance, Dotsey and King (2001) and Christiano, Eichenbaum and Evans (2001) have considered it a key element of their analysis. However, the description of the underutilization phenomenon in these papers is highly stylized. Their modeling approach is based upon the endogenous depreciation models of Greenwood et al (1989). By contrast, the issue of capacity utilization is modeled here under explicit microfoundations. In this regard, we follow Gilchrist

[1] Some authors postulate the existence of asymmetric nominal price and/or wage rigidities as a rationale in this regard -see Ball and Mankiw (1994) and Stiglitz (1986). Another strand in the literature emphasizes the role credit market imperfections in the monetary transmission mechanism. See, for instance, Bernanke, Gertler and Gilchrist (1999).

[2] Similar results are obtained in Peersman and Smets (2001) for several European countries.

and Williams (2000) and, particularly, Fagnart, Licandro and Portier (1999) by modeling very explicitly key features of the production sector, such as idiosyncratic uncertainty, firm heterogeneity and the absence of an aggregate production function. All these elements are embedded in a monopolistic competitive environment, where the endogenous behavior of mark-ups plays a crucial role in the transmission mechanism of monetary policy.

The model developed in this paper follows this latter approach. Equipped with it, we are able to deal with the first question posed above. Specifically, we show how the proportion of firms with idle capacities crucially influences the response of the economy to monetary policy stimulus. In particular, the existence of production inflexibilities due to a low short-run capital-labor substitutability induces a "capacity" effect of a tight link between changes in capacity and employment, output and prices. This feature of the model will ultimately determine the intensity of the real effects of monetary policy. Moreover, the model presented in this paper successfully replicates the dynamics of key variables observed in the aggregate data. In particular, the model produces a "hump-shaped" response of key macroeconomic variables to a monetary policy shock, a feature that is consistent with the main findings of the identified VAR research.[3]

The second question approached in this paper asks how the existence of production inflexibilities influences the appropriate conduct of monetary policy. In order to analyze this issue, we consider a version of the model with staggered price mechanism à la Calvo (1983); Albeit observationally equivalent to quadratic adjustment costs, it has received much of the attention in the recent New-Keynesian (NK) literature. In a model without capital accumulation and exogenous maximum productive capacity, we show how the Phillips curve depends on the distribution of the utilization rates across firms. Under the basic formulation of the NK Phillips curve, the dynamics of inflation are related to a measure of economic activity that is based on real marginal costs. Our results point to convenience of including, additionally, a measure of the dispersion of resource utilization. This paper provides a theoretically-based measure in this regard.

The paper proceeds as follows. Section 2 presents a formal description of the model's behavioral aspects. Section 3 offers a characterization of the general equilibrium of the economy and its qualitative properties. The implications for the transmission mechanism of monetary policy are analyzed in Section 4. Section 5 presents a version of the model with staggered pricing and studies the implications of production inflexibilities for the nature of the Phillips curve and the characterization of a simple zero-inflation monetary policy rule. Section 6, offers some concluding remarks and plausible lines for further research.

2 The Model Economy

The model economy consists of households, a central bank in charge of the conduct of monetary policy and two productive sectors: a competitive sector producing a final good and a monopolistic sector providing intermediate goods. These intermediate goods are the only inputs necessary for the production of the final good. The final good can be used either for consumption or for investment purposes. Capital and labor are used in the production of intermediate goods by means of a putty-clay technology. Capital and labor are substitutes *ex ante*, i.e., before investing, but complement *ex post*, i.e., when

[3] See Christiano, Eichenbaum and Evans (1999) for a survey.

equipment is installed. This implies that each firm makes a capacity choice when investing. This specification of the production function allows for the introduction of a simple, but realistic, concept of capacity. Each input firm makes its investment, pricing and employment decisions under idiosyncratic demand uncertainty, that is, before knowing the exact demand for its production. The particular specification in this regard, which follows Ireland (1997, 2001) assumes a quadratic function for these costs.

The structure of the model implies that intermediate goods firms can be either sales or capacity constrained; it also allows different firms to face different capacity constraints. Consequently, the idiosyncratic uncertainty is what explains the presence of heterogeneity between firms at equilibrium regarding the degree of utilization of their productive capacities.[4]

2.1 Final Good Firms

At time t, a single final good, denoted by Y, is produced by a representative firm which sells it in a perfectly competitive market. Such commodity can either be used for consumption or for investment. There is no fixed input, which implies that the optimization program of these firms remain purely static. The production activities are carried out by combining a continuum of intermediate goods, indexed by $j \in [0,1]$. The production technology is represented by a constant return-to-scale CES function defined as follows

$$Y_t = \left[\int_0^1 Y_{j,t}^{\frac{\epsilon-1}{\epsilon}} v_{j,t}^{\frac{1}{\epsilon}} \mathrm{d}j \right]^{\frac{\epsilon}{\epsilon-1}} , \tag{1}$$

with $\epsilon > 1$ being the elasticity of substitution of inputs and where $Y_{j,t}$ is the quantity of input j used in production at date t. Here, $v_{j,t} \geq 0$ is a productivity parameter corresponding to input j. It is assumed to be drawn from a stochastic process i.i.d. distributed across time and input firms, with a log normal distribution function $\mathrm{F}(v)$ that has unit mean and is defined over the support $[0,\infty)$. The representative firm purchases inputs to intermediate good firms taking into account that the supply of each input j is limited to an amount $\bar{Y}_{j,t}$. Assuming a uniform non-stochastic rationing scheme, the optimization program of the final firm can be written as follows

$$\max_{\{Y_t, Y_{j,t}\}} \mathbf{P}_t Y_t - \int_0^1 P_{j,t} Y_{j,t} \mathrm{d}j \ , \tag{2}$$

where $\mathbf{P}_t$ is the price of the final good which is taken as given by the firm. When maximizing profits, the final firm faces no uncertainty: it knows the input prices $\{P_{j,t}\}$, the input supply constraints $\{\bar{Y}_{j,t}\}$ and the productivity parameters $\{v_{j,t}\}$. The solution to (2 determines the quantity demanded by the final good firm of the goods produced by each intermediate firm. Under deterministic quantity constraints and a uniform rationing scheme, effective demands are not well defined. Realized transactions can be derived, however. The quantity of inputs used will be determined by the corresponding idiosyncratic productivity level of each intermediate firm as described in the next result:

[4] In order to keep the model tractable, it is assumed that the idiosyncratic shock is not serially correlated. Its realization influences exclusively contemporary production and employment decisions, but not investment decisions.

Lemma 1 (Realized Transactions) *The optimal allocation of inputs across intermediate good firms is given by the following system of equations*

$$Y_{j,t} = \begin{cases} Y_t v_{j,t} \left(\frac{P_{j,t}}{\mathbf{P}_t}\right)^{-\epsilon} & \text{if} \quad 0 \leq v_{j,t} \leq \tilde{v}_{j,t} \\ \bar{Y}_{j,t} & \text{otherwise} \end{cases} \tag{3}$$

with

$$\tilde{v}_{j,t} = \frac{\bar{Y}_{j,t}}{Y_t \left(\frac{P_{j,t}}{\mathbf{P}_t}\right)^{-\epsilon}} . \tag{4}$$

The variable $\tilde{v}_{j,t}$ determines the critical value of the productivity parameter $v_{j,t}$ for which the unconstrained demand equals the supply constraint $\bar{Y}_{j,t}$. The term $(P_{j,t}/\mathbf{P}_t)^{-\epsilon}$ appearing in the demand function of a firm with excess capacities represents, at given Y_t, the positive spillover effects an input producing firm with idle resources benefits from. This term is going to play a significant role in the model's behavior, as will be stressed later.

As mentioned above, for tractability purposes it will be assumed that all intermediate firms are *ex ante* equal. This symmetry means that input prices and capacities are the same across firms. Assuming that a law of large numbers applies in the present context, the final output supply can be expressed as follows

$$Y_t = \left[\int_0^\infty Y_{j,t}^{\frac{\epsilon-1}{\epsilon}} v_{j,t}^{\frac{1}{\epsilon}} \mathrm{dF}(v)\right]^{\frac{\epsilon}{\epsilon-1}} \tag{5}$$

or taking into account equation (3),

$$Y_t = \left\{ \left[\left(\frac{P_{j,t}}{\mathbf{P}_t}\right)^{-\epsilon} Y_t \right]^{\frac{\epsilon-1}{\epsilon}} \int_0^{\tilde{v}_t} v \mathrm{dF}(v) + \bar{Y}_t^{\frac{\epsilon-1}{\epsilon}} \int_{\tilde{v}_t}^\infty v^{\frac{1}{\epsilon}} \mathrm{dF}(v) \right\}^{\frac{\epsilon}{\epsilon-1}} . \tag{6}$$

Recall that $\mathrm{F}(v)$ is the distribution function of idiosyncratic shocks; thus, for a proportion $\mathrm{F}(\tilde{v})$ of intermediate firms, the realized value of the productivity parameter is below $\tilde{v}$.

2.2 Intermediate Good Firms

In this sector, each intermediate good is produced by a monopolistically competitive firm making use of capital and labor, which are combined for production through a *putty-clay* technology. Firms are *ex ante* identical, thus, the notation is simplified by omitting index j. These firms start period t with a predetermined level of capacity. Such a production plan cannot be adapted to the needs of the firm within the period. Hence, investment achieved during period $t-1$ becomes productive at date t. Investment consists of the design of a production plan by simultaneously choosing a quantity of capital goods K_t and employment capacity N_t according to the following Cobb-Douglas technology:

$$\bar{Y}_t = A_t K_t^\alpha N_t^{1-\alpha} \tag{7}$$

where $0 < \alpha < 1$ and $A_t \equiv \exp(z_t)$ is a productivity parameter, which evolves over time according to

$$z_t = \rho_z z_{t-1} + \varepsilon_{z,t}, \tag{8}$$

where $\varepsilon_{z,t}$ is an i.i.d. technology shock and $0 < \rho_z < 1$. The variable N_t represents the maximum number of available work-stations in the firm. Hence, the firm is at full capacity when all these work-stations are operating full-time. As it is common in models featuring a *putty-clay* technology, it is convenient to express investment decision as the choice of both K_t and a capital-labor ratio $X_t \equiv K_t/N_t$. Consequently, the expression in (7) can be rewritten as

$$\bar{Y}_t = A_t X_t^{\alpha-1} K_t,$$

from where the technical productivity of the installed equipments can be deduced. For the case of capital, it is given by $A_t X_t^{\alpha-1}$, whereas $A_t X_t^{\alpha}$ represents that of labor, so that this production function displays constant returns-to-scale in the within-period labor input. In particular, if the firm j uses a quantity of labor $L_{j,t}^d$ smaller than N_t, it then produces $A_t X_t^{\alpha} L_{j,t}^d$ units of intermediate good. Once the idiosyncratic (demand) shock $v_{j,t}$ is revealed, the firm instantaneously adjusts its labor demand $L_{j,t}^d$ to cover the needs of its production plan, $Y_{j,t}$, that is,

$$L_{j,t}^d = \frac{Y_{j,t}}{A_t X_t^{\alpha}} = \frac{1}{A_t X_t^{\alpha}} \min \left\{ Y_t v_{j,t} \left(\frac{P_t}{\mathbf{P}_t} \right)^{-\epsilon}, \bar{Y}_t \right\} . \tag{9}$$

2.2.1 The intertemporal decisions

Each intermediate goods firm chooses a price and a capacity level in order to maximize the stream of expected future nominal profits, D, that is,

$$\max_{P_t, K_{t+1}, X_{t+1}} E_t \left\{ \sum_{s=t}^{\infty} \rho_s D_s \right\}, \tag{10}$$

where ρ_s is the stochastic discount factor of the firm, which represents a pricing kernel for contingent claims. Assuming that households and firms have access to a complete set of frictionless securities markets, one obtains the following equilibrium expression

$$\rho_s = \beta^s \Delta_s,$$

where Δ_s corresponds to the representative household's relative valuation of cash across time, that is, the marginal utility value to the household of an additional dollar in profits received during period s, measured in terms of consumption, that is,

$$\Delta_s = \frac{U_{c,s}}{\mathbf{P}_s}. \tag{11}$$

Alternatively, the maximization problem can be written in a recursive manner as

$$V\left(P_{t-1}, K_t, X_t, \Omega_t\right) = \max_{P_t, K_{t+1}, X_{t+1}} \left\{ D_t + \beta E_t \left\{ \frac{\Delta_{t+\tau+1}}{\Delta_{t+\tau}} V\left(P_t, K_{t+1}, X_{t+1}, \Omega_{t+1}\right) \right\} \right\},$$

where $V(\cdot)$ is the value function satisfying the Bellman-equation, in which Ω_t is the informational set of the typical input firm j. The *ex ante* nominal dividends for such a firm in period t is given by

$$D_t = P_t E_v \{Y_t\} - W_t L^d_{j,t} - \mathbf{P}_t \left(I^A_t + \mathcal{A}_{p,t} Y_t \right).$$

where

$$I^A_t = I_t \left(1 + \mathcal{A}_{k,t}\right)$$

is investment in capital goods plus adjustment costs in capital, that is,

$$I_t = K_{t+1} - (1-\delta) K_t$$

and

$$\mathcal{A}_{k,t} = \frac{\phi_k}{2} \left(\frac{I_t}{K_t} - \delta \right)^2$$

with δ being the capital's depreciation rate and $\phi_k$is the adjustment cost scale parameter for capital. This functional form produces zero steady-state adjustment costs.[5] The term $\mathcal{A}_{p,t}$ is the cost of adjusting the nominal price, and it is specified as a quadratic function

$$\mathcal{A}_{p,t} = \frac{\phi_p}{2} \left(\frac{P_t}{\pi P_{t-1}} - 1 \right)^2,$$

where π is the steady state inflation rate. Notice that these costs are measured in terms of the final good. Firms' maximization problem is subject to the expected demand resulting from the realization of the idiosyncratic shock, which can be derived directly from expression (3),

$$E_v \{Y_t\} = \left(\frac{P_t}{\mathbf{P}_t} \right)^{-\epsilon} Y_t \int_0^{\tilde{v}_t} v \mathrm{dF}(v) + \bar{Y}_t \int_{\tilde{v}_t}^{\infty} \mathrm{dF}(v). \tag{12}$$

2.2.2 The price setting problem

After observing the aggregate shocks, but before knowing the idiosyncratic one, input producing firms take their price decisions. Input prices are announced on the basis of (rational) expectations, before the exact value of the demand for their production is realized. This price-setting assumption has the advantage of giving a symmetric equilibrium in prices, avoiding in this manner price aggregation difficulties. The price decision is dynamic and the same rule will be followed by all firms given that, *ex ante,* all of them are identical, that is, $P_t = P_{j,t}$. Taking into account these considerations, the optimal price decision can be characterized by the following result:

Lemma 2 (Intermediate-Goods Pricing) *The price decision of any input firm j at date t satisfies the following relation:*

$$P_t = \frac{\epsilon \Gamma(\tilde{v}_t)}{(\epsilon \Gamma(\tilde{v}_t) - 1)} \left[\frac{W_t}{A_t X^\alpha_t} - \phi_p E_t \{\Delta_t \Upsilon_t - \beta \Delta_{t+1} \Upsilon_{t+1}\} \right], \tag{13}$$

where

$$\Upsilon_{t+h} = \left(\frac{P_{t+h}}{\pi P_{t+h-1}} - 1 \right) \left(\frac{P_{t+h}}{\pi P_{t+h-1}} \right) \frac{Y_{t+h} \mathbf{P}_{t+h}}{E_v \{Y_{t+h-1}\}} \quad \textit{for } h = 0, 1$$

[5]Quadratic costs are justified on the ground that it is easier to absorb new capacity into the firm at a slow rate -see Kim (2001).

and where $\Gamma(\tilde{v}_t)$ *represents the probability of excess capacity in the economy, that is,* $\Gamma(\tilde{v}_t)$ *is a weighted measure of the proportion of firms for which demand is smaller than their productive capacity,*

$$\Gamma(\tilde{v}_t) = \frac{(P_t/\mathbf{P}_t)^{-\epsilon} Y_t}{E_v\{Y_t\}} \int_0^{\tilde{v}_t} v dF(v).$$

Notice that $\Gamma(\tilde{v})$ depends only on $\tilde{v}$, as becomes clear from the combination of equations (12) and (4) above,

$$\Gamma(\tilde{v}_t) = \frac{\int_0^{\tilde{v}_t} v\mathrm{dF}(v)}{\int_0^{\tilde{v}_t} v\mathrm{dF}(v) + \tilde{v}_t \int_{\tilde{v}_t}^{\infty} \mathrm{dF}(v)}. \tag{14}$$

The pricing mechanism resulting from (13) implies that intermediate firms set their price as a mark-up over expected future demand and marginal costs.[6] The precise nature of the mark-up will be discussed later. The above solution satisfies also the following transversality condition

$$\lim_{\tau\to\infty} E_t\left\{\beta^{t+\tau}\frac{\Delta_{t+\tau+1}}{\Delta_{t+\tau}}\frac{\partial V_{t+\tau+1}}{\partial P_{t+\tau}}P_{t+\tau}\right\} = 0.$$

2.2.3 The Capacity and Investment Choice

The next step in the description of the behavior of intermediate goods firms corresponds to the choice of the productive capacity to be installed. Firms choose a contingency plan $\{K_{t+1}, X_{t+1}\}_{t=o}^{\infty}$ to maximize the expected discounted value of the dividend flow given in (10). The first order conditions are summarized in the following result

Lemma 3 (Capacity Choice) *The optimal decision of investment in capital* K_{t+1} *and capital-labor ratio* X_{t+1} *is given, respectively, by the following Euler equations*

$$E_t\{\Delta_t(\Psi_t + \mathcal{W}_t)\mathbf{P}_t\} - E_t\left\{\Delta_{t+1}\left((1-\delta)\Psi_{t+1} + \left(\frac{K_{t+2}}{K_{t+1}}\right)\mathcal{W}_{t+1}\right)\mathbf{P}_{t+1}\right\} = \tag{15}$$
$$E_t\left\{\Delta_{t+1}(1-F(\tilde{v}_{t+1}))\Phi_{t+1}\left(\frac{\bar{Y}_{t+1}}{K_{t+1}}\right)\mathbf{P}_{t+1}\right\}$$

and

$$E_t\left\{\Delta_{t+1}\Phi_{t+1}\left(\frac{\bar{Y}_{t+1}}{X_{t+1}}\right)\left[\left(\frac{\alpha(\epsilon-1)}{\tilde{v}_{t+1}}\right)\int_0^{\tilde{v}_{t+1}} v dF(v) - \int_{\tilde{v}_{t+1}}^{\infty} dF(v)\right]\right\} = 0, \tag{16}$$

where

$$\Psi_{t+h} = 1 + \frac{\phi_k}{2}\left(\frac{I_{t+h}}{K_{t+h}} - \delta\right)^2 \qquad \text{with } h=0,1$$

the term

$$\mathcal{W}_{t+h} = I_{t+h}\left(\frac{K_{t+1+h}}{K_{t+h}} - 1\right)\frac{\phi_k}{K_{t+h}} \qquad \text{with } h=0,1$$

[6] The derivation of this condition supposes that each monopolistic firm only considers the direct effect of its price decision on demand and neglects all indirect effects (e.g. the effects through $\mathcal{Y}_t$). This approximation is reasonable in a context where there is a continuum of firms.

and

$$\Phi_{t+1} \equiv \left(P_{t+1} - \frac{W_{t+1}}{A_t X_{t+1}^{\alpha}} \right).$$

The first equation states that the optimal capital stock is such that the expected user cost of capital, including adjustment costs, is equal to its expected revenue, which is given by the discounted increase in profits generated by an additional unit of capital corrected by the probability of operating such unit. Notice that in the absence of adjustment costs, $\Psi_{t+h} = 1$ and $\mathcal{W}_{t+h} = 0$. Also notice that in the steady state, there are not adjustment costs of investment. From the second equation one can observe the trade-off faced by the intermediate firm when choosing the optimal capital-labor ratio. When increasing the capital-labor ratio, the firm increases its labor productivity, which is given by $A_t X_t^{\alpha}$, something that has a favorable effect on its competitive position in case of excess capacities. However, increasing X_t means that the maximum level of employment available in period t will be lower, and likewise the maximum volume of sales of the firm. The optimal capital-labor ratio will be such that the two opposite effects on expected profits are equal in the margin. In solving the problem, we have imposed the transversality conditions for the stock of capital

$$\lim_{\tau \to \infty} E_t \left\{ \beta^{t+\tau} \frac{\Delta_{t+1+\tau}}{\Delta_{t+1+\tau}} \frac{\partial V_{t+1+\tau}}{\partial K_{t+1+\tau}} K_{t+1+\tau} \right\} = 0,$$

as well as for the capital-labor ratio

$$\lim_{\tau \to \infty} E_t \left\{ \beta^{t+\tau} \frac{\Delta_{t+1+\tau}}{\Delta_{t+\tau}} \frac{\partial V_{t+1+\tau}}{\partial X_{t+1+\tau}} X_{t+1+\tau} \right\} = 0.$$

2.3 Households

The economy is populated by a continuum of homogeneous households of unit measure. These agents value alternative stochastic streams of a (composite) consumption good C_t, real balances $M_t^d/\mathbf{P}_t$ and labor L_t^s, according to the following lifetime expected utility function

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[u\left(C_t^*, \frac{M_t^d}{\mathbf{P}_t}; \xi_t \right) + V\left(1 - L_t^s\right) \right] \right\} \tag{17}$$

where $\beta > 0$ represents households' intertemporal discount factor.[7] Household's preferences take into account past consumption as well as present consumption. Thus, $C_t^* = C_t + bC_{t-1}$ so that whenever $b > 0$, household's value habit formation in consumption. ξ_t represents a preference shock that follows an autoregressive process:

$$\ln \xi_t = \left(1 - \rho_\xi\right) \xi + \rho_\xi \ln \xi_{t-1} + \varepsilon_{t,\xi},$$

with $1 > \rho_\xi > -1$, $\xi > 0$ and $\varepsilon_{\xi,t}$ a zero-mean serially uncorrelated innovation normally distributed with standard deviation σ_ξ.

The household begins period t holding an amount M_{t-1}^d of liquid assets that represent the economy's stock of money (a non interest bearing asset) and an amount B_{t-1} of

[7] The assumption of separability between the basket formed by consumption and real balances and hours implies that aggregate demand relationships are invariant to the specification of the firm's problem.

discount bonds with a nominal return of R_t. It receives a wage payment of $W_t L_t^s$, a lump-sum transfer, T_t, from the central bank and firm profits, D_t.

$$\mathbf{P}_t C_t + \left(M_t^d - M_{t-1}^d\right) + R_t^{-1} B_t - B_{t-1} = W_t L_t^s + D_t + T_t. \tag{18}$$

Thus, the representative household's budget constraint states that consumption expenditures plus assets accumulation must equal disposable income. A No-Ponzi-Game condition is imposed on households' borrowing: it requires debt not to increase asymptotically faster than the interest rate. The optimal behavior of the household is characterized as follows:

Lemma 4 *The optimal conditions for consumption and real balances are given by*

$$\frac{u_{C,t}}{\mathbf{P}_t} = \beta E_t \left\{ R_t \frac{u_{C,t+1}}{\mathbf{P}_{t+1}} \right\}, \tag{19}$$

and

$$\frac{u_{C,t}}{\mathbf{P}_t} = \beta E_t \left\{ \frac{u_{C,t+1}}{\mathbf{P}_{t+1}} \right\} + \frac{u_{M/\mathbf{P},t}}{\mathbf{P}_t}, \tag{20}$$

where U_C and $U_{M/\mathbf{P}}$ denote, respectively, the partial derivatives of U with respect to C and $M/\mathbf{P}$. The first order necessary condition for labor supply is characterized by

$$-\frac{V_{L_t}}{u_{C_t}} = \frac{W_t}{\mathbf{P}_t}, \tag{21}$$

where V_L is the derivative of V with respect to L.

The formulation and results in this section are rather standard within the monetary DSGE literature. Equation (19) is a standard Euler consumption relation, while (21) governs the household's labor supply decision. Combining (19) with (20), it is possible to obtain the following relation

$$\frac{u_{M/\mathbf{P},t}}{u_{c,t}} = 1 - \frac{1}{R_t}, \tag{22}$$

which implicitly defines a money demand function that depends positively on consumption and negatively on the nominal interest rate. This latter result is attributable to the opportunity cost of holding money.

2.4 The Monetary Authority

At each period of time, the central bank decides the amount of money to introduce into the economy in order to achieve a given objective or target. The stock of money is given through a transfer, T_t, to the households. Specifically,

$$T_t = M_{t+1} - M_t,$$

with the money supply growing at a gross rate μ_t

$$\frac{M_{t+1} - M_t}{M_t} = \mu_t. \tag{23}$$

We consider two possible implementations of monetary policy: In the first one, the central bank follows an exogenous money growth rule of the form

$$\mu_t = \left(1 - \rho_\mu\right) \mu + \rho_\mu \mu_{t-1} + \varepsilon_{\mu,t}. \tag{24}$$

where ρ_μ is the persistence parameter and the monetary policy shock $\varepsilon_{\mu,t}$is assumed to be i.i.d. with zero mean and standard deviation σ_μ. A second implementation of monetary policy is based on an endogenous nominal interest rate rule. In this case, the monetary authority allows μ_t to adjust in a way which ensures that an endogenous interest rate rule of the following form

$$R_t = f\left(\Omega_t\right) + \varepsilon_{R,t} \tag{25}$$

holds, where $f(\cdot)$ is a function of the variables tracked by the central bank which are given in the information set Ω. The term $\varepsilon_{R,t}$ represents exogenous policy actions by the monetary authority. One can interpret this rule as a relationship that the monetary authority sustains in equilibrium by appropriately manipulating the money supply. To do so, μ_t has to respond in a particular way to the current and past values of all the fundamental shocks hitting the economy. Below, we consider different variants of the monetary policy rule in (25) and analyze their implications in the present framework

2.5 Dynamic General Equilibrium

An equilibrium for this model can be defined in the usual way. We first consider the case in which the central banks follow an exogenous money growth rule: given the initial productive equipments K_0 and X_0, the initial state of the technology A_0, with its corresponding stochastic process (8), the initial monetary growth rate μ_0 with the exogenous process (24), the initial input goods price P_0, an equilibrium for the model economy described above is stated as follows:

Definition 5 (Equilibrium with Exogenous Policy) *The general equilibrium of the economy during any period $t \geq 0$ is given by a stochastic process for prices $\{\mathbf{P}_t, P_t, R_t, W_t, \Delta_t\}_{t=0}^{\infty}$, a quantity vector $\{K_t, X_t, C_t, B_t, L_t, Y_t, M_t\}_{t=0}^{\infty}$ and a proportion of firms $\{F(\tilde{v}_t)\}_{t=0}^{\infty}$ that result from the optimal choices (consistent with the available information) of the central bank, the households and the firms. In an equilibrium, these choices are required to be made under rational expectations and consistent with the market-clearing conditions in the labor market, $L_t^s = L_t^d$, the money market, $M_t^d = M_t^s$, the bonds market, $B_t = 0$, and the final goods market*

$$Y_t = C_t + I_t^N + \mathcal{A}_{p,t} Y_t. \tag{26}$$

Alternatively, we can define an equilibrium with an interest rate monetary policy rule. In this case, the central bank adjusts the rate of growth of money supply μ_t to satisfy the interest rate given by the rule (25). Hence, equations (20) and (23) are redundant. The equilibrium can be defined, in this case, as follows:

Definition 6 (Equilibrium with Endogenous Policy) *Given the endogenous monetary policy rule (25) for the nominal interest rate R_t, the general equilibrium of the economy during any period $t \geq 0$ is given by a stochastic process for prices $\{\mathbf{P}_t, P_t, W_t, \Delta_t\}_{t=0}^{\infty}$, a quantity vector $\{K_t, X_t, C_t, B_t, L_t, Y_t, M_t\}_{t=0}^{\infty}$ and a proportion of firms $\{F(\tilde{v}_t)\}_{t=0}^{\infty}$ that result from the agents' optimal choices and such that all markets clear.*

Notice that in this model $\mathrm{F}(\tilde{v}_t)$ represents the proportion of firms that, at equilibrium, underuse their productive capacities, i.e., those for which $v_t \in [\underline{\mathrm{v}}, \tilde{v}_t]$. The variable $\Gamma(\tilde{v}_t)$ weights this proportion of firms by the relative importance of their production in total output. An important feature of this equilibrium is its symmetry: all input firms j choose the same capacity level and take the same pricing decisions. With all prices identical, aggregate employment, denoted by L_t, is equal to individual expected employment levels (up to a scaling factor):

$$L_t = \frac{(P_t/\mathbf{P}_t)^{-\epsilon} Y_t}{A_t X_t^{\alpha}} \int_0^{\tilde{v}_t} v_t \mathrm{dF}(v_t) + \frac{K_t}{X_t} \int_{\tilde{v}_t}^{\infty} \mathrm{dF}(v_t), \tag{27}$$

where K_t and X_t stand for aggregate capital and capital/labor respectively at time $t-1$ and available at time t, and

$$\tilde{v}_t = \frac{\bar{Y}_t}{(P_t/\mathbf{P}_t)^{-\epsilon} Y_t}$$

represents the ratio of productive capacity to expected demand for intermediate inputs. Notice that, as $v_t < \tilde{v}_t$, the aggregate productive capacity is underutilized at equilibrium.

3 Asymmetries

This section explores the main insights and qualitative implications derived from the model economy presented above. We first characterize the dynamic general equilibrium associated to this economy, introducing next the important concept of aggregate capacity utilization rate. This section concludes with an analysis of the implications that different values of this variable has on the response of key macroeconomic variables to monetary policy shocks.

3.1 Qualitative Analysis

The individual capacity utilization rates are given by:

$$\mathcal{C}_t = \begin{cases} \left(\frac{P_t}{\mathbf{P}_t}\right)^{-\epsilon} Y_t v_t / \bar{Y}_t & \text{if} \quad v_t \leq \tilde{v}_t \\ 1 & \text{if} \quad v_t > \tilde{v}_t \end{cases}$$

which introduced into (1) yields the aggregate capacity utilization rate,

$$\mathcal{C}_t \equiv \frac{Y_t}{\bar{Y}_t}. \tag{28}$$

For a given distribution $\mathrm{F}(v_t)$, and thus given σ_v^2, there is a decreasing relationship between the capacity utilization rate, $\mathcal{C}_t$, and the weighted proportion of firms with idle resources, $\Gamma(\tilde{v}_t)$, which subsequently determines the mark-up rate. The aggregate capacity utilization rate is directly linked to the proportion of firms that produce at full capacity, $1 - \Gamma(\tilde{v}_t)$. An important feature of the model is that the relative price of the input producing firms, $P_t/\mathbf{P}_t$, is less than one. Some manipulation of (6) allows one to write relative prices as a function of $\tilde{v}_t$, the proportion of firms with excess capacities:

$$\frac{P_t}{\mathbf{P}_t} = \left(\int_0^{\tilde{v}_t} v \mathrm{dF}(v) + \tilde{v}_t^{\frac{\epsilon-1}{\epsilon}} \int_{\tilde{v}_t}^{\infty} v^{\frac{1}{\epsilon}} \mathrm{dF}(v) \right)^{\frac{1}{\epsilon-1}}. \tag{29}$$

The right hand side of this expression is increasing in $\tilde{v}$ and bounded above by one. As a result, the spillover term $(P_t/\mathbf{P}_t)^{-\epsilon}$is larger than one. Recall that the optimal price is given by

$$P_t = \frac{\epsilon\Gamma\left(\tilde{v}_t\right)}{\left(\epsilon\Gamma\left(\tilde{v}_t\right)-1\right)}\left[\frac{W_t}{A_t X_t^{\alpha}} - \frac{\phi_p}{\epsilon}\left(\Upsilon_t - \beta E_t\left\{\frac{\Delta_{t+1}}{\Delta_t}\Upsilon_{t+1}\right\}\right)\right],$$

which can be rewritten as

$$P_t = \left(1-\frac{1}{\Theta_t}\right)^{-1}\frac{W_t}{A_t X_t^{\alpha}}. \tag{30}$$

where the term

$$\left(1-\frac{1}{\Theta_t}\right)^{-1} \tag{31}$$

represents the mark-up of the price over the marginal cost; in this expression, the term Θ_t is defined as

$$\Theta_t = \epsilon\Gamma\left(\tilde{v}_t\right)\left(1-\phi_p\left[\Upsilon_t - \beta E_t\left\{\frac{\Delta_{t+1}}{\Delta_t}\Upsilon_{t+1}\right\}\right]\right)^{-1},$$

where

$$\Upsilon_{t+h} = \left(\frac{P_{t+h}}{\pi P_{t+h-1}}-1\right)\left(\frac{P_{t+h}}{\pi P_{t+h-1}}\right)\frac{Y_{t+h}\mathbf{P}_{t+h}}{E_v\left\{Y_{t+h-1}\right\}}, \qquad \text{for} \quad h=0,1.$$

Notice that in the case of flexible prices, that is when $\phi_p = 0$, the mark-up collapses to

$$\Theta_t^{flex} = \left(1-\frac{1}{\epsilon\Gamma\left(\tilde{v}_t\right)}\right)^{-1}.$$

The mark-up rate depends negatively on the (absolute) value of the price elasticity of expected sales, which is defined as the elasticity of expected sales to expected demand, $\Gamma\left(\tilde{v}_t\right)$, times the price elasticity of expected demand, ϵ. This means that when $\Gamma\left(\tilde{v}_t\right)$, the probability of a sales constraint, is large, that is, when more input firms are likely to produce under their full capacity level, firm's actual market power is reduced, implying a smaller mark-up rate. At given price elasticity of demand, ϵ, this implies a positive relationship between the capacity utilization and mark-up rates. Notice also that when no firm is capacity constrained, that is, no firm is producing at full capacity which implies that $\Gamma\left(\tilde{v}\right) = 1$, the pricing rule is given a constant mark-up over the marginal cost as in the standard monopolistic competition model.

$$\Theta = \left(1-\frac{1}{\epsilon}\right)^{-1}.$$

In order to understand why the response of key macroeconomic variables to a monetary policy shock depends on the degree of utilization of productive resources, a diagrammatic representation of the labor and final goods market equilibrium, at given capacity level, is presented. Figure 1 corresponds to the short-run labor market, where the upward sloping curve represents the aggregate labor supply schedule, as given in equation (21). The other curve, concave and sloping downwards, represents the macroeconomic labor demand curve given in equation (27). In the very short run, at given capacity, the labor demand curve intersects both axes. The intersection with the horizontal axis is due to the fact that even at zero real wage rates, the short-run demand for labor is bounded above by the maximum number of work stations corresponding to the full employment of installed capacities.

Notice that when $\tilde{v} \rightarrow \underline{v}$, equation (27) reduces to the following expression:

$$L_t = \frac{K_t}{X_t}.$$

In the opposite case, when all firms have idle resources, and thus underutilize their productive capacities, the proportion of firms $\Gamma(\tilde{v}_t) = 1$ and the real wage rate given in (30) becomes,

$$\frac{W_t}{P_t} = \left(1 - \frac{1}{\tilde{\Theta}_t}\right) A_t X_t^{\alpha},$$

with

$$\tilde{\Theta}_t = \epsilon \left[1 - \phi_p \left(\Upsilon_t - \beta E_t \left\{ \frac{\Delta_{t+1}}{\Delta_t} \Upsilon_{t+1} \right\}\right)\right]^{-1}.$$

It must be pointed out that along the short-run labor demand curve there is a negative relationship between the demand elasticity of sales, $\Gamma(\tilde{v}_t)$, and employment, L_t. Also, a downward shift along the short-run labor demand curve increases the mark-up, since the proportion of firms at full capacity is larger and so is the spill-over effect from constrained to unconstrained firms. The implications of a monetary policy shock on the response of the labor market are the following: when a monetary policy shock occurs, the marginal revenue curve will shift upwards. As a result, output will increase and prices will also increase, but less than in the fully flexible case, so that the markup will decrease. This implies an upwards shift in the labor demand curve as shown in Figure 1. The maximum feasible real wage rate increases; the short-run labor demand curve intersects now the vertical axis at a higher value, consequently, real wages increase as well as the equilibrium level of employment. At given capacity, output also increases. The real interest rate decreases, what stimulates investment and consumption. It is important to notice that the effects of the monetary disturbance will depend crucially on the capacity utilization rate of the economy at the time of the shock. Hence, further expansionary policies will have less impact on employment and a higher effect on prices.

3.2 Numerical Analysis

We proceed by numerically illustrating the main results derived from the model economy presented above. The objective here is to analyze the implications of production inflexibilities on the behavior of key macroeconomic variables in response to a monetary policy shock. To that end, the model's parameters are chosen in order to match the properties of US data. Moreover, given the importance of non-linearities, we solve the model using a numerical method that preserves these features of the model.

3.2.1 Parameter Values

The time period is one quarter. Table 1 summarizes the values of the calibrated parameters which are described in the sequel. The parameter for preferences and technology are assigned values that are standard in the DSGE literature; the discount factor is set at

$(\beta) = (1.03)^{-0.25}$; the utility function is assumed to be:

$$U\left(C_t^*, L_t\right) \equiv \xi_t u\left(C_t^*\right) + V\left(L_t^s\right) = \xi_t \frac{C_t^{*1-\sigma}}{1-\sigma} + \eta \frac{\left(1-L_t\right)^{1-\varphi}}{1-\varphi},$$

where the risk aversion $(\sigma) = 1$, the consumption/leisure share in utility $(\eta) = 0.35$ so that one third of the time endowment in the steady state corresponds to time engaged in market activity; the inverse of the labor supply elasticity $(\varphi) = 0.8$, implying a value of this elasticity of 1.25; the parameter corresponding to habit persistence is set equal to $(h) = 0.7$ which is the value estimated by Boldrin, Christiano and Fisher (2001). Following the estimates of Ireland (2001) the AR(1) preference shock, ξ_t has a coefficient $\left(\rho_\xi\right) = 0.9$ a mean value $(\xi) = 1$ and an innovation with standard deviation $(\sigma_\xi) = 0.03$.

Capital's share on aggregate income $(\alpha) = 0.33$; the annual depreciation rate of 10% implies a value $(\delta) = 0.018$; next, the parameter of the price adjustment function $\left(\phi_p\right) = 4.05$ as suggested by Ireland (1997); the parameter for the adjustment costs of capital is set equal to $(\phi_k) = 318$, which is the value estimated by Kim (2001); estimates of average mark-up by Fernald and Basu (1997) suggest a value of 1.2, however, with the standard value for the elasticity of intermediate goods $(\epsilon) = 6$, our model produces a high mark-up of 1.7; in order to reduce this magnitude, we have followed an argument similar to the one in Rotemberg and Woodford (1992) and considered the possibility of introducing a fixed cost of production; denoting by Ψ this fixed cost, the expression for the mark-up (31) becomes

$$\left(1 - \frac{1}{\Theta_t}\right)^{-1},$$

where now

$$\Theta_t = \epsilon \Gamma\left(\tilde{v}_t\right) \left[1 - \phi_p \left(\Upsilon_t - \beta E_t \left\{\frac{\Delta_{t+1}}{\Delta_t} \Upsilon_{t+1}\right\}\right) - \Psi\right]^{-1},$$

which, evaluated at the non-stochastic steady state, is

$$\Theta = \epsilon \Gamma\left(\tilde{v}\right) \left(1 - \Psi\right)^{-1};$$

thus, setting $(\eta) = 0.3$ and taking into account that the distribution of firms with idle resources $\Gamma(\tilde{v})$ is chosen is in order to reproduce the average capacity utilization rate measured by the Board of Governors of the Federal Reserve System by setting the variance of the idiosyncratic shock $\left(\sigma_v^2\right) = 0.75$, we obtain the desired level for the steady-state mark-up.

Notice that the magnitude of the firm level uncertainty is measured through the variance of the idiosyncratic shock, denoted here by σ_v^2. In order to show the influence of σ_v^2 on the capacity utilization rate of the economy, recall from (14) that the distribution of firms with idle resources is given by the following expression:

$$\Gamma\left(\tilde{v}_t\right) = \frac{\int_0^{\tilde{v}_t} v \mathrm{dF}(v)}{\int_0^{\tilde{v}_t} v \mathrm{dF}(v) + \tilde{v}_t \int_{\tilde{v}_t}^{\infty} \mathrm{dF}(v)},$$

which, given the properties of the log normal distribution, becomes:[8]

$$\Gamma\left(\tilde{v}_t; \sigma_v^2\right) = \frac{\mathcal{Z}\left(w_t - n_t\right)}{\mathcal{Z}\left(w_t - n_t\right) + \tilde{v}_t \left[1 - \mathcal{Z}\left(w_t\right)\right]},$$

[8] See Johnson-Kotz-Balakrishnan (1994) for a detailed description of such properties.

where $\mathcal{Z}(\cdot)$ is the standard-normal cumulative distribution function and

$$w_t = \frac{\ln(\tilde{v}_t) + 0.5n_t^2}{[\ln(1+\sigma_v^2)]^{1/2}}.$$

Figure 2 illustrates, by means of numerical arguments, the existence of a negative relationship between σ_v^2 and the weighted proportion of firms at full capacity $1-\Gamma(\cdot)$ and, consequently, with the average capacity utilization rate of the economy.

3.2.2 Solution Method

In order to solve and simulate the model, a numerical procedure is needed. Given the important non-linearities that are inherent to the model, we adopt a solution method that preserves these features. Specifically, we have considered a non-linear implementation of the method of Eigenvalue-Eigenvector Decompositions proposed by Sims (2000). In this solution approach, each conditional expectation is treated as an additional endogenous variable and an equation is added to the model defining the expectation error. The numerical solution is in the form of a set of time series for all the variables in the model, including all the conditional expectations and the associated expectation errors. This series are obtained from the original non-linear model which has been augmented with some appropriate stability elements needed to guarantee that transversality conditions will hold. These stability conditions are given by the left eigenvectors corresponding to the unstable eigenvalues of the linear approximation to the model economy about the non-stochastic steady-state.[9]

3.2.3 Results

This section analyses the model through a discussion of the dynamic behavior of selected variables to temporary exogenous shocks. Specifically, Figure 3 shows the impulse response functions of the model corresponding to a 1 % technology shock; Figure 4 illustrates the dynamic responses due to a monetary policy action consisting of an unexpected increase in the rate of growth of the money supply; and, finally, Figure 5 displays the marginal responses of selected model variables to a series of cumulated monetary policy shocks. This later exercise illustrates how such responses will differ as the economy moves away from its steady state position.

Regarding technology shocks, z_t, is assumed to follow an AR(1) stochastic process with a coefficient $(\rho_z) = 0.95$; the associated disturbance term, $\varepsilon_{z,t}$, is a white noise with standard deviation $(\sigma_z) = 0.0065$. The features of the model economy discussed in this paper has several interesting implications regarding the response to a technology shock: The first panel in Figure 3 shows the impulse response function of output. It is noticeable the resulting hump-shape of this response. It must be pointed out that this result is obtained also in the absence of habit formation and adjustment costs of investment. To illustrate this, we computed the impulse responses for a baseline model in which these two features were absent. The resulting impulse responses clearly show a hump-shape. For instance the output reaction to the monetary policy shock in the period after the shock is noticeably greater than that of the impact period -see Figure 7 below.

[9] Novales et al. (2000) offer a practical exposition of this numerical solution method. Appendix 2 describes in detail the implementation of this approach to the model economy presented in this paper.

One key finding is the negative impact response of employment to the positive technology shock. The intuition is that the existence of nominal price rigidities imply that aggregate demand increases less proportionately than the increase in productivity induced by the technology shock. As Figure 3.b illustrates, aggregate employment must decline accordingly. Such a decline is only on impact. Moreover, consumption smoothing and the structure of preferences imply that real wages follow only partially the initial productivity increase. Firms invest initially in less capital intensive technology, that is, the capital-labor ratio decreases, making investment choices more favorable to employment. In this manner, the following period employment increases so that the response of this variable is greater than the instantaneous one. The technology shock increases the productive capacity of input producing firms which is, however, of a higher magnitude than the increase in output, so that the capacity utilization rate of the economy decreases as shown in Figure 3.c. Figure 3.d shows, next, the response of investment. The existence of adjustment costs of capital makes the magnitude of such a response consistent with the pattern suggested in the literature. Specifically, these costs reduce the elasticity of investment demand to the real interest rate and restrain the increase in investment caused by the technology shock. Figure 3.e displays the response of consumption to the technology disturbance. The consideration of habit formation reduces the impact response of consumption, also rendering the model consistent with the dynamics observed in aggregate data.[10] Finally, Figure 3.f displays the dynamic response of the mark-up. One can observe that the average mark-up is procyclical. To understand this, notice that when a positive technology shock shifts the marginal cost curve downward, input producing firms do not adjust their price fully and so the mark-up increases.

Regarding the response to a monetary policy shock, the stock of money in the economy grows stochastically as following the process

$$\mu_t = \left(1 - \rho_\mu\right) \mu + \rho_\mu \mu_{t-1} + \varepsilon_\mu;$$

the persistence parameter $\left(\rho_\mu\right) = 0.50$ and the monetary policy shock $\varepsilon_{R,t}$is assumed to be i.i.d. with zero mean and standard deviation $(\sigma_\mu) = 0.003$. As shown in Figure 4, a number of results are worth noting here. The model is able to reproduce the stylized facts of monetary policy claimed in many studies of the identified VAR literature, such as Christiano, Eichenbaum and Evans (2001). In particular, an expansionary money supply shock leads to an increase of output and employment. As illustrated in Figure 4.a and Figure 4.b, the model clearly generates "hump-shaped" responses in these two variables, a feature strongly supported in the empirical literature. As in the case of a technology shock, an explanation in this regard is that firms invest initially in less capital intensive technology due to the fact that real wages follow only partially the initial demand (monetary) increase. Hence, firms make investment choices more favorable to employment, with the period ahead response of those variables greater than the corresponding to the impact period. Figure 4.c shows that the aggregate capacity utilization rate increases, since the monetary policy shock does not impact on the productive capacity of input-producing firms. The consideration of adjustment costs in investment and habit formation in consumption make the model's impulse responses closer to the results in the empirical

[10] A complete set of results comparing this specification of the model with respect to alternative cases is avalible from the author on request.

literature, with Figure 4.d and Figure 4.e illustrating this issue. In particular, the reduction in the elasticity of investment to the real interest rate and the persistence in consumption, put downward pressure on the real and nominal interest rates. Thus, the model is able to generate a liquidity effect of monetary policy as shown in Figure 4.f and Figure 4.g. The behavior of the mark-up is displayed in Figure 4.h, where one can observe that this variable moves counter-cyclically. The positive demand (monetary) shock moves marginal revenue upward and, given that prices increase less than in the fully flexible case, the mark-up decreases. Finally, Figure 4.i illustrates the dynamic behavior of inflation. Interestingly, we can observe a "hump-shape" response of this variable. Altogether, the model provides a good approximation to underlying consumer and firm behavior over the monetary policy horizon, that is, the short-run.

Figure 5 displays the impact responses of selected model variables to a series of cumulated monetary policy shocks, illustrating in this manner the asymmetric behavior of the model economy. The experiment consists of a moving the economy away from the steady-state through a series of money supply shocks. Figure 5 shows the results of such an experiment for selected model variables. One can observe that as the economy departs from its initial situation corresponding to a long-run average level of the capacity utilization rate of 82%, the impact of the similar monetary policy shock has a lower effect on quantity variables such as output, utilization, employment and investment. The intuition is the following: when a monetary policy shock occurs, the marginal revenue curve of input producing firms shifts upwards, leading to an increase in the equilibrium level of output and prices; the mark-up decreases due to the fact that nominal price stickiness; altogether, this implies an upwards shift in the labor demand curve, resulting in an increase in the real wage rate as well as in the equilibrium level of employment; to the extent that the monetary policy shock does not affect firms' productive capacity, output also increases; the liquidity effect induces a reduction in the nominal and real interest rate, what stimulates investment and consumption; Due to the existence of constraints in firms' productive capacity, further expansionary policies will have less impact on employment and a higher effect on prices. In particular, Figure 5.f shows the cumulative response of the wholesale price level, that features a remarkable non-linear convex shape. Certainly, this experiment illustrates the fact that the same policy actions have significantly different effects depending on the extent to which productive resources are being used in the economy.

In order to illustrate the non-linearities in a more clear manner, we present the impulse responses to a monetary policy shock for different degrees of capacity utilisation. Specifically, we show how similar policy actions can have significantly different impact effects depending on the state of the economy. We consider two states: "high-capacity" and "low-capacity", where each one corresponds to a long-run level of the aggregate capacity utilization rate. As it is shown later, these two states implicitly depend on the degree of idiosyncratic uncertainty, that is, on σ_v^2. Accordingly, the magnitude of firm-level uncretainty should not be viewed as a fixed structural parameter, but a stochastic component of the economy. Indeed, there is strong empirical evidence suggesting the counter-cyclically nature of idiosyncratic risk.[11] Hence, we compute impulse response functions with respect

[11] Campbell et al. (2001) show that a wide range of disaggregated firm-level volatility measures move together counter-cyclically. Higson et al. (2002) found evidence of a negative correlation between the rate of growth of gdp and the cross sectional variance of growth rates of sales.

to a first-order approximation about two states of the economy is performed. To justify the validity of this procedure, one can think of our exercise as an approximation to the computation of "generalized" or "state-dependent" impulse response functions.[12] Figures 6 show the different responses of output and inflation in the two capacity regimes.

4 Capacity Utilization and Staggered Pricing

The previous section looked at the effects of exogenous changes in the money supply and technology level in the context of a model featuring nominal price adjustment costs and production inflexibilities. In this section, we proceed to analyze the implications of the kind of production inflexibilities discussed above with a different source of nominal price stickiness. The results obtained from this analysis shown how changes, for instance, in monetary policy are transmitted to a number of macroeconomic variables and, more importantly, how the restrictions on firms' ability to adapt their production processes to a changing environment generates an asymmetric response of those macroeconomic variables.

Much of recent research in monetary economics aims at analyzing monetary policy as an endogenous component of the modeling framework. An important question in this context, that we explore here, asks how should the monetary authority respond to different shocks. Given the fact that the behavior of the economy depends on the magnitude of the production inflexibilities, the response of the monetary authority should also depend on these issues.

In the previous section, we have considered the existence of quadratic price adjustment costs as a plausible source of monetary non neutralities. However, a large body of the so called New Keynesian literature has considered the modelling approach of Calvo's (1983) staggered price contracts. The underlying idea in this context is that each (intermediate) firm resets its price in any given period only with some probability, independently of other (intermediate) firms and of the time elapsed since the last adjustment.[13] In the sequel, we consider a simplified version of the model presented above equipped with this kind of nominal friction. Specifically, we abstract from capital and capacity accumulation and just focus on the implications of variable capacity utilization for the performance of monetary policy. To that end, we, first, present a log-linear version of the model; Then, we introduce staggered prices á la Calvo in order to analyze the dynamics of inflation..Importantly, the exact form of the equation describing such dynamics depends on the manner sticky prices are modelled -we derive a Phillips curve relation that nests the conventional New-Keynesian Phillips Curve (NKPC).

4.1 The New Keynesian Phillips Curve

Following the Calvo (1983) setup, we assume that firms adjust their price infrequently and that the opportunity to adjust follows an exogenous Poisson process. Each period there is a constant probability $(1-\varphi)$ that the firm will be able to adjust its price, independently of past history. Now, there are two sources of idiosyncratic uncertainty: one related to

[12] This concept of impulse response function has recently been proposed in the econometric literature to analyze non-linear economic dynamics. See, for instance, Potter (2000) and the references therein.

[13] It can be shown that there is an isomorphism between the two pricing mechanisms cited above. See Roemberg (1987) for details.

the productivity (input-demand) of each intermediate firm and another one related to the ability of a firm to change its price. We maintain the assumption that firms choose their price before the realization of the idiosyncratic demand shock. Hence, the timing is the following: each period begins with the realization of the aggregate (technology) shock, then, the idiosyncratic price shock is realized and a proportion φ of firms re-optimize their price; Finally, the productivity (input-demand) shock is realized and the production and employment take place.

Let P_t^{new} denote the price of an intermediate-goods producing firm that can change its price in a given period t. Notice that our notation does not allow the price to depend on the specific firm j. We do this following well known results in the literature of staggered price models that claim that all firms who can re-optimize their price at time t choose the same price. Thus, the optimization problem associated with this pricing mechanism is the following:

$$\max E_t \sum_{k=0}^{\infty} (\beta\varphi)^k \Delta_{t+k} E_v \{Y_{t+k}\} [P_t^{new} - MC_{t+k}], \tag{32}$$

where $MC_{t+k} \equiv W_{t+k}/(A_{t+k}X^\alpha)$ is the nominal marginal cost. This problem is subject to the expected demand of the intermediate good in each period which is given by:

$$E_v \{Y_{t+k}\} = \left(\frac{P_t^{new}}{\mathbf{P}_{t+k}}\right)^{-\epsilon} Y_{t+k} \int_0^{\tilde{v}_{t+k}^t} v\mathrm{dF}(v) + \bar{Y} \int_{\tilde{v}_{t+k}^t}^{\infty} \mathrm{dF}(v).$$

Notice that P_t^{new} influences firms profits only as long as it cannot re-optimize its price and this happens for k periods with probability φ^k, hence the discount factor$(\beta\varphi)^k$ in (32). Notice also that the critical value of the idiosyncratic productive (demand) shock in any given period $t+k$ depends on the price chosen at period t

$$\tilde{v}_{t+k}^t = \frac{\bar{Y}}{Y_{t+k}\left(\frac{P_t^{new}}{\mathbf{P}_{t+k}}\right)^{-\epsilon}}. \tag{33}$$

Once the problem has been introduced, its solution is characterized by the following expression:

$$P_t^{new} = \frac{E_t \sum_{k=0}^{\infty} (\beta\varphi)^k \Delta_{t+k} E_v \{Y_{t+k}\} MC_{t+k} \Gamma\left(\tilde{v}_{t+k}^t\right) \lambda}{E_t \sum_{k=0}^{\infty} (\beta\varphi)^k \Delta_{t+k} E_v \{Y_{t+k}\} \left[1 - \lambda\left(1 - \Gamma\left(\tilde{v}_{t+k}^t\right)\right)\right]}, \tag{34}$$

where $\Gamma(\tilde{v}_{t+k})$ is as defined in (14), that is,

$$\Gamma(\tilde{v}_{t+k}) = \frac{\int_0^{\tilde{v}_{t+k}^t} v\mathrm{dF}(v)}{\int_0^{\tilde{v}_{t+k}^t} v\mathrm{dF}(v) + \tilde{v}_{t+k}^t \int_{\tilde{v}_{t+k}^t}^{\infty} \mathrm{dF}(v)} \tag{35}$$

and $\lambda \equiv \epsilon/(\epsilon - 1)$. We can see that when $\varphi = 0$, the price (34) reduces to

$$P_t = \left(\frac{\epsilon\Gamma\left(\tilde{v}_t^t\right)}{\epsilon\Gamma\left(\tilde{v}_t^t\right) - 1}\right) \frac{W_t}{A_t X_t^\alpha},$$

which is similar to (13) when $\phi_p = 0$, that is, the flexible price under capacity constraints; Moreover, when no input producing firm is supply constrained, $\Gamma\left(\tilde{v}_t^t\right) = 1$ and the optimal price is the standard constant markup over marginal costs:

$$P_t = \left(\frac{\epsilon}{\epsilon - 1}\right) \frac{W_t}{A_t X_t^{\alpha}}.$$

In order to find an intuitive reading of the optimal price (34), we log-linearize it about the steady state. This leads to the following expression, where symbol "$\widehat{\ }$" represents the percentage deviation of the variable with respect to its steady state,

$$\widehat{P}_t^{new} = (1 - \beta\varphi) \sum_{k=0}^{\infty} (\beta\varphi)^k \left[\widehat{\Theta}_{t+k}^c + \widehat{MC}_{t+k}\right], \tag{36}$$

where $\widehat{\Theta}_{t+k}^c$ is a log-linear approximation of the distribution of output across firms, that is, a log-linear approximation of

$$\frac{\Gamma\left(\tilde{v}_{t+k}^t\right)\lambda}{1 - \lambda\left(1 - \Gamma\left(\tilde{v}_{t+k}^t\right)\right)}.$$

Hence, when the proportion of firms at capacity is large, that is, when $1 - \Gamma\left(\tilde{v}_t^t\right)$ is close to one, any increase in the level of economic activity will produce an additional effect on the inflation rate due to the presence of capacity constraints in firms' production processes. Additionally, we can write (36) as

$$\widehat{P}_t^{new} = (1 - \beta\varphi)\left(\widehat{\Theta}_t^c + \widehat{mc}_t + \widehat{P}_t\right) + (\beta\varphi)\,\widehat{P}_{t+1}^{new} \tag{37}$$

where mc_t is the log of the real marginal cost (in terms of intermediate-goods), that is,

$$\widehat{mc}_t = \widehat{W}_t - \widehat{P}_t - \widehat{z}_t, \tag{38}$$

where $\widehat{z}_t = \log\left(A_t/A\right)$.

If the law of large numbers holds, a fraction $(1 - \varphi)$ of firms will reset the price at each point in time. The evolution of the aggregate input price index therefore is

$$P_t = \left[\varphi P_{t-1}^{1-\epsilon} + (1 - \varphi)\left(P_t^{new}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}},$$

which in log-linear terms reads:

$$\widehat{P}_t = \varphi\widehat{P}_{t-1} + (1 - \varphi)\,\widehat{P}_t^{new}.$$

Combining the aggregate price index with equation (37) results in the following version of the NKPC in terms of wholesale inflation:

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1 - \varphi)(1 - \varphi\beta)}{\varphi}\left(\widehat{\Theta}_t^c + \widehat{mc}_t\right). \tag{39}$$

Notice that when no firm is capacity constrained, equation (39) results in the conventional NKPC. The latter is a stochastic difference equation describing the dynamics of inflation, with marginal costs as the only driving force. However, when production inflexibilities are present, the behavior of inflation depends also upon the term $\widehat{\Theta}_t^c$. This term measures the tightness of the production inflexibilities in the economy, that is, a measure of the proportion of firms at capacity.

5 Concluding Remarks and Extensions

This paper provides a formal approximation to the issue of asymmetries within the quantitative monetary macroeconomics literature. The overall message arising from the present analysis is that the same policy actions have significantly different effects depending on the extent to which productive resources are being used in the economy. These results have been obtained under a framework that considers the interaction of endogenous capacity utilization (derived from productive constraints and firm heterogeneity) and market power, together with a monetary structure that assumes nominal price stickiness. The source of the asymmetry is directly linked to the bottlenecks and stock-outs that emerge from the existence of capacity constraints in the real side of the economy. These constraints act as a source of amplification of monetary shocks and generates asymmetries in the response of key macroeconomic variables. These effects interact additionally with those emerging from the imperfectly-competitive environment that characterizes the intermediate-good sector through optimal mark-up changes.

There are two immediate extensions to the present analysis: One would involve the computation of optimal monetary policy in this context. An important implication of the capacity constraint hypothesis is that the Phillips curve displays a convex shape. This is a feature of the economy that has relevant consequences for the performance of a monetary policy aimed at controlling inflation and it provides support for arguments favoring a tough anti-inflationary stance by the monetary authority.[14] With a convex Phillips curve, the more stable output is, the higher the level of output will be in the economy on average. The thinking behind this result is that given the lags in the effects of monetary policy, there is an incentive for pre-emptive tighten responses to inflationary pressure. Specifically, if central bankers act in this way, it will prevent the economy from moving too far up the level where inflation begins to rise more rapidly, thereby avoiding the need for a larger negative output gap in the future to reverse this large rise in inflation.[15] As a result, the response of the monetary authority to shocks should be asymmetric. In this regard, Schaling (1999) and Dolado et al. (2003) show that a convex Phillips curve leads to an non-linear interest rate rule, where the weighting coefficients are state-dependent. Sims (2001) finds a clear improvement of a state-dependent interest rate rule over a fixed-parameter linear specification. Deriving the optimal policy rule and comparing its performance with respect to standard formulations constitutes an interesting line of future research.

A second extension of this work involves an econometric estimation of the capacity-based Phillips curve derived above. In this manner, we could quantify the dynamics of inflation that cannot be appropriately captured with the specifications considered in the standard literature.

[14] Nobay and Peel (2000) have shown that the analysis of optimal discretionary monetary policy under a non-linear Phillips curve yields results that are in marked contrast with those obtained under a conventional linear paradigm.

[15] Several recent paper have analyzed this issue by means of stochastic simulations in a model with an asymmetric output-inflation nexus. See, for instance, Clark, Laxton and Rosen (2001) and Tambakis (2002).

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Appendix 1

The Transformed Model Equations

Notice that in equilibrium, most of the model nominal variables grow at the rate of growth of money supply. Hence, all nominal variables, except the nominal interest rate, are divided by the final-goods price level, $\mathbf{P}_{t-1}$,.in order to induce stationarity. Accordingly, we define $m_t \equiv M_t/\mathbf{P}_{t-1}$ and $\pi_t \equiv \mathbf{P}_t/\mathbf{P}_{t-1}$. The stationary model for the case of an exogenous monetary policy rule consists of eleven equations corresponding to the optimal choices of households, final-good firms, intermediate-good firms, as well as the monetary authority, there is one equilibrium condition corresponding to the final-goods market. Altogether, these equations determine a process for prices $\{\boldsymbol{\pi}_t, p_t, R_t, w_t\}_{t=0}^{\infty}$ a quantity vector $\{K_t, X_t, C_t, L_t, Y_t, m_t\}_{t=0}^{\infty}$ and a proportion of firms $\{F(\tilde{v}_t)\}_{t=0}^{\infty}$. Additionally, there are three exogenous processes $\{z_t, \mu_t\}_{t=0}^{\infty}$ related to technology and money supply shocks $\{\varepsilon_{z,t}, \varepsilon_{\mu,t}\}_{t=0}^{\infty}$. Finally, the idiosyncratic shock, v_t, follows a log-normal distribution function with unit mean and variance equal to σ_v^2. The model equations are the following:

Final-Goods Firms:

$$\frac{P_t}{\mathbf{P}_t} = \left\{ \int_0^{\tilde{v}_t} v\mathrm{dF}(v) + \tilde{v}_t^{\frac{\epsilon-1}{\epsilon}} \int_{\tilde{v}_t}^{\infty} v^{\frac{1}{\epsilon}} \mathrm{dF}(v) \right\}^{\frac{1}{\epsilon-1}}. \qquad \text{(A.1.1)}$$

$$\tilde{v}_t = \frac{\bar{Y}_t}{p_t{}^{-\epsilon} Y_t} \qquad \text{(A.1.2)}$$

Intermediate-Goods Firms:

$$p_t = \frac{\epsilon\Gamma(\tilde{v}_t)}{(\epsilon\Gamma(\tilde{v}_t) - 1)} \left\{ \frac{w_t}{A_t X_t^{\alpha}} - \phi_p E_t \left\{ \tilde{\Upsilon}_t - \beta\tilde{\Upsilon}_{t+1} \right\} \right\}, \qquad \text{(A.1.3)}$$

with

$$\tilde{\Upsilon}_{t+h} = \tilde{\Delta}_{t+h} \left(\frac{p_{t+h}}{p_{t+h-1}} \frac{\pi_{t+h}}{\pi} - 1 \right) \left(\frac{p_{t+h}}{p_{t+h-1}} \frac{\pi_{t+h}}{\pi} \right) \frac{Y_{t+h}}{E_v(Y_{t+h-1})} \quad \text{for } h = 0, 1$$

and

$$\tilde{\Delta}_{t+h} \equiv \frac{U_{c,t+h}}{\boldsymbol{\pi}_{t+h}}$$

$$E_t\{u_{c,t}(\Psi_t + \mathcal{W}_t)\} - \beta E_t \left\{ u_{c,t+1} \left((1-\delta)\Psi_{t+1} + \left(\frac{K_{t+2}}{K_{t+1}} \right) \mathcal{W}_{t+1} \right) \right\} = \qquad \text{(A.1.4)}$$
$$\beta E_t \left\{ u_{c,t+1}(1 - F(\tilde{v}_{t+1})) \left(\frac{\phi_{t+1}}{\boldsymbol{\pi}_{t+1}} \right) \left(\frac{\bar{Y}_{t+1}}{K_{t+1}} \right) \right\},$$

$$E_t \left\{ u_{c,t+1}\phi_{t+1} \left(\frac{\bar{Y}_{t+1}}{X_{t+1}} \right) \left[\left(\frac{\alpha(\epsilon-1)}{\tilde{v}_{t+1}} \right) \int_0^{\tilde{v}_{t+1}} v\mathrm{dF}(v) - \int_{\tilde{v}_{t+1}}^{\infty} \mathrm{dF}(v) \right] \right\} = 0, \qquad \text{(A.1.5)}$$

with

$$\phi_{t+1} \equiv \left(p_{t+1} - \frac{w_{t+1}}{A_t X_{t+1}^{\alpha}} \right),$$

$$L_t = \frac{p_t^{-\epsilon} Y_t}{A_t X_t^{\alpha}} \int_0^{\tilde{v}_t} v_t \mathrm{dF}(v_t) + \frac{K_t}{X_t} \int_{\tilde{v}_t}^{\infty} \mathrm{dF}(v_t) \tag{A.1.6}$$

Households:

$$u_{C,t} = \beta E_t \left\{ \frac{u_{C,t+1}}{\boldsymbol{\pi}_{t+1}} \right\} + u_{m,t}, \tag{A.1.7}$$

$$E_t \left\{ u_{C,t} - \beta R_t \frac{u_{C,t+1}}{\boldsymbol{\pi}_{t+1}} \right\} = 0, \tag{A.1.8}$$

$$-\frac{V_{L_t}}{u_{C_t}} = w_t, \tag{A.1.9}$$

Final-Goods Market Equilibrium:

$$\mathcal{Y}_t = C_t + I_t^N + \tilde{\mathbf{A}}_{p,t} \mathcal{Y}_t, \tag{A.1.10}$$

with

$$\tilde{\mathbf{A}}_{p,t} = \frac{\phi_p}{2} \left(\frac{p_t}{p_{t-1}} \frac{\pi_t}{\pi} - 1 \right)^2,$$

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$$\frac{m_{t+1}}{m_t} = \frac{1 + \mu_t}{\pi_t}. \tag{A.1.11}$$

that determine the equilibrium vector of prices $\{\pi_t, p_t, R_t, w_t\}$, the equilibrium vector of quantities $\{K_t, X_t, C_t, L_t, \mathcal{Y}_t, m_t\}$ and a proportion of firms $\{F(\tilde{v}_t)\}$. The two exogenous processes

$$\mu_t = (1 - \rho_\mu) \mu + \rho_\mu \mu_{t-1} + \varepsilon_{R,t}, \tag{A.1.12}$$

$$z_t = \rho_z z_{t-1} + \varepsilon_{z,t}, \tag{A.1.13}$$

characterize the evolution of $\{\mu_t, z_t\}$.

Appendix 2

Implementation Numerical Solution Method

The solution approach involves the consideration of each conditional expectation as an endogenous variable. The associated expectation error, which is also treated as endogenous variable, is added to the model. Hence, define:

$$W_{1,t} \equiv E_t \left\{ u_{C,t} - \beta R_t \frac{u_{C,t+1}}{\boldsymbol{\pi}_{t+1}} \right\}$$

and substitute the expectation in equation (A.1.8) by $W_{1,t}$. Moreover, add to the model the following equation:

$$W_{1,t-1} = u_{C,t-1} - \beta R_{t-1} \frac{u_{C,t}}{\boldsymbol{\pi}_t} + \xi_{1,t},$$

where $\xi_{1,t}$ is the expectation error. Next, define

$$W_{2,t} \equiv E_t \left\{ \begin{array}{l} u_{c,t} \left(\Psi_t + \mathcal{W}_t \right) - \\ \beta u_{c,t+1} \left((1-\delta) \Psi_{t+1} + \left(\frac{K_{t+2}}{K_{t+1}} \right) \mathcal{W}_{t+1} - \left(1 - F\left(\tilde{v}_{t+1}\right)\right) \left(\frac{\phi_{t+1}}{\boldsymbol{\pi}_{t+1}} \right) \left(\frac{\bar{Y}_{t+1}}{K_{t+1}} \right) \right) \end{array} \right\}$$

and substitute the expectation in equation (A.1.4) by $W_{2,t}$. Moreover, add to the model the following equation:

$$W_{2,t-1} = \left\{ \begin{array}{l} u_{c,t-1} \left(\Psi_{t-1} + \mathcal{W}_{t-1} \right) - \\ \beta u_{c,t} \left((1-\delta) \Psi_t + \left(\frac{K_{t+1}}{K_t} \right) \mathcal{W}_t - \left(1 - F\left(\tilde{v}_t\right)\right) \left(\frac{\phi_t}{\boldsymbol{\pi}_t} \right) \left(\frac{\bar{Y}_t}{K_t} \right) \right) \end{array} \right\} + \xi_{2,t}.$$

Next, define

$$W_{3,t} \equiv E_t \left\{ u_{c,t+1} \phi_{t+1} \left(\frac{\bar{Y}_{t+1}}{X_{t+1}} \right) \left[\left(\frac{\alpha (\epsilon - 1)}{\tilde{v}_{t+1}} \right) \int_{\underline{\mathrm{v}}}^{\tilde{v}_{t+1}} v \mathrm{dF}(v) - \int_{\tilde{v}_{t+1}}^{\bar{\mathrm{v}}} \mathrm{dF}(v) \right] \right\}$$

and substitute the expectation in equation (A.1.5) by $W_{3,t}$. Moreover, add to the model the following equation:

$$W_{3,t-1} = \left\{ u_{c,t} \phi_t \left(\frac{\bar{Y}_t}{X_t} \right) \left[\left(\frac{\alpha (\epsilon - 1)}{\tilde{v}_t} \right) \int_{\underline{\mathrm{v}}}^{\tilde{v}_t} v \mathrm{dF}(v) - \int_{\tilde{v}_t}^{\bar{\mathrm{v}}} \mathrm{dF}(v) \right] \right\} + \xi_{3,t}.$$

Finally, define

$$W_{4,t} \equiv E_t \left\{ \tilde{\Upsilon}_t - \beta \tilde{\Upsilon}_{t+1} \right\}$$

and substitute the expectation in equation (A.1.3) by $W_{4,t}$. Moreover, add to the model the following equation:

$$W_{4,t-1} = \left\{ \tilde{\Upsilon}_{t-1} - \beta \tilde{\Upsilon}_t \right\} + \xi_{4,t}.$$

Notice that now there are four new variables corresponding to the expectation errors, namely, $\left\{ \xi_{1,t}, \xi_{2,t}, \xi_{3,t}, \xi_{4,t} \right\}$; thus we need four extra equations.[16] These equations are the

[16] When considering a forward-looking interest rate rule, we have to consider another expectation variable related to future inflation.

stability conditions calculated in the algorithm based on a linear approximation of the model to a non-stochastic steady state. Specifically, the linearized system can be written as follows:

$$\Gamma_0 Y_t = \Gamma_1 Y_{t-1} + \Psi \varpi_t + \Pi \eta_t,$$

where the vector Y_t consists of fifteen equations corresponding to the four conditional expectation variables $\{W_{1,t}, W_{2,t}, W_{3,t}, W_{4,t}\}$ and the eleven decision-equilibrium variables $\{p_t, R_t, w_t, K_t, X_t, C_t, L_t, \mathcal{Y}_t, m_t, \pi_t, \tilde{v}_t\}$; the vector ϖ_t consists of the two exogenous state variables $\{z_t, \mu_t\}$; and the vector η_t consists of the four expectation errors $\{\xi_{1,t}, \xi_{2,t}, \xi_{3,t}, \xi_{4,t}\}$.

Next, premultiplying by the inverse of Γ_0, one obtains a transformed system with an identity matrix in Y_t

$$Y_t = \Gamma_0^{-1}\Gamma_1 Y_{t-1} + \Gamma_0^{-1}\Psi \varpi_t + \Gamma_0^{-1}\Pi \eta_t. \tag{40}$$

Characterizing the equilibrium dynamics of the model under the optimal commitment policy implies solving the set of difference equations. Typically, the matrix Γ_s^0 is singular, hence the "QZ" decomposition is a convenient way of solving such a system of equations. In this regard, the square matrices Γ_s^0 and Γ_s^1 can be decomposed as

$$\Gamma_s^0 = M_s \Lambda_s Z_s^H \quad \text{and} \quad \Gamma_s^1 = M_s \Omega_s Z_s^H,$$

where Z_s^H is the complex conjugate of Z_s; the matrices M_s and Z_s are unitary and Λ_s and Ω_s are upper triangular. Letting $w_t = Z_s^H Y_t$, we can multiply (40) by M_s to get

$$\Lambda_s w_t = \Omega_s w_{t-1} + M_s \Theta_s + M_s \Pi \eta_t + M_s \Psi_s \varepsilon_t \tag{41}$$

The "QZ" decomposition is reordered so that the largest of the generalized eigenvalues $\{\omega_{ii}/\lambda_{ii}\}$ in absolute value appear at the lower right. Accordingly, the system (41) is partitioned so that $|\omega_{ii}/\lambda_{ii}| \geq \bar{\varphi}$ for all $i > m$ and $|\omega_{ii}/\lambda_{ii}| < \bar{\varphi}$ for all $i \leq m$, where $\bar{\varphi}$ is a single bound on the maximal growth rate of any component of Y_t. The resulting system is

$$\begin{bmatrix} \Lambda_s^{11} & \Lambda_s^{12} \\ \mathbf{0} & \Lambda_s^{22} \end{bmatrix} \begin{bmatrix} w_{1,t} \\ w_{2,t} \end{bmatrix} = \begin{bmatrix} \Omega_s^{11} & \Omega_s^{12} \\ \mathbf{0} & \Omega_s^{22} \end{bmatrix} \begin{bmatrix} w_{1,t-1} \\ w_{2,t-1} \end{bmatrix} + \begin{bmatrix} M_{s,1\bullet} \\ M_{s,2\bullet} \end{bmatrix} (\Theta_s + \Psi_s \varepsilon_t + \Pi \eta_t), \tag{42}$$

where the second block corresponds to the unstable eigenvalues[17]. We can write the $((2(n+n_1)+k)-m) \times 1$ vector $w_{2,t}$ as

$$w_{2,t} = \left(\Lambda_s^{22}\right)^{-1} \Omega_s^{22} w_{2,t-1} + \left(\Lambda_s^{22}\right)^{-1} M_{s,2\bullet} (\Theta_s + \Psi_s \varepsilon_t + \Pi \eta_t), \tag{43}$$

which letting $\chi_t \equiv M_{s,2\bullet} (\Theta_s + \Psi_s \varepsilon_t + \Pi \eta_t)$ and defining $H_s = \left(\Lambda_s^{22}\right)^{-1} \Omega_s^{22}$, becomes

$$w_{2,t} \equiv Z_{s,2\bullet}^H Y_t = H_s w_{2,t+1} + \left(\Lambda_s^{22}\right)^{-1} \chi_{t+1}.$$

Because this set of equations corresponds to the unstable eigenvalues, it must be solved towards the future, which makes $w_{2,t}$ dependent on the whole future path of χ_t. Proceeding in this manner and under the assumption that $(H_s)^t w_{2,t} \to 0$ as $t \to \infty$, we obtain

$$w_{2,t} = -\sum_{l=1}^{\infty} (H_s)^{l-1} \left(\Lambda_s^{22}\right)^{-1} \chi_{t+1}.$$

[17] Note that some diagonal elements of Λ_s^{11}, but not of Λ_s^{11}, may be zero. Moreover, if Λ_s and Ω_s have a zero in the same position, then the system is incomplete, since some equation is a linear combination of the others.

Sims (2001) shows that this discounted sum of future values of linear combinations of χ_t that defines $w_{2,t}$ must be equal to its conditional expectation. As a result, a stable solution exists only if the column space of $M_{s,2\bullet}\Psi_s$ is contained in that of $M_{s,2\bullet}\Pi$. If $M_{s,2\bullet}\Psi_s$ has full column rank, one manner of testing the condition for the existence of a solution is by regressing the columns of $M_{s,2\bullet}\Psi_s$ on the columns of $M_{s,2\bullet}\Pi$ to see if the resulting residuals are all equal to zero. If $M_{s,2\bullet}\Psi_s$ has full row rank, then its column space automatically includes any other space of the same dimension.[18]

The condition for the existence of a stable solution becomes also sufficient when the innovations of the structural shocks are serially uncorrelated, that is, $\mathbf{E}_t(\varepsilon_{t+1}) = \mathbf{0}$ for all t. In this case, a non-explosive solution for Y_t simply becomes

$$w_{2,t} \equiv Z^H_{s,2\bullet} Y_t = \mathbf{0} \quad \text{for all} \ \ t, \tag{44}$$

where $Z^H_{s,2,\bullet}$ is the appropriate submatrix of Z^H_s. Moreover, the stability conditions imply a set of relationships between the vector of rational expectation errors η_t and the vector of innovations ε_t; Specifically, taking into account (43), the stability condition requires that for every vector ε_t one can find, at least, a vector η_t that offsets the impact of ε_t on $w_{2,t}$. This means that

$$M_{s,2\bullet}\Psi_s\varepsilon_t + M_{s,2\bullet}\Pi\eta_t = \mathbf{0} \qquad \text{for all } t. \tag{45}$$

The vector η_t, however, need not be unique. A necessary and sufficient condition for uniqueness is that the row space of $M_{s,1\bullet}\Pi$.be contained in that of $M_{s,2\bullet}\Pi$. Moreover, whenever a solution exists, it is possible to write

$$M_{s,1\bullet}\Pi = \Phi_s M_{s,2\bullet}\Pi,$$

for some matrix Φ_s computed using a single value decomposition procedure.

Taking into account the precedent result, and the conditions (44) and (45), the system in (42) becomes

$$\Lambda^{11}_s w_{1,t} = \Omega^{11}_s w_{1,t-1} + (M_{s,1\bullet} - \Phi_s M_{s,2\bullet})(\Theta_s + \Psi_s\varepsilon_t),$$

which, by the definition of $w_{1,t}$, results in

$$Y_t = \alpha_s + \beta_s Y_{t-1} + \phi_s\varepsilon_t, \tag{46}$$

with

$$\alpha_s \equiv Z_{s,1\bullet}\left(\Lambda^{11}_s\right)^{-1}(M_{s,1\bullet} - \Phi_s M_{s,1\bullet})\Theta_s,$$
$$\beta_s \equiv Z_{s,1\bullet}\left(\Lambda^{11}_s\right)^{-1}\Omega^{11}_s Z^H_{s,1\bullet},$$

and

$$\phi_s \equiv Z_{s,1\bullet}\left(\Lambda^{11}_s\right)^{-1}(M_{s,1\bullet} - \Phi_s M_{s,1\bullet})\Psi_s.$$

and This is a complete set of equations for Y_t satisfying the condition that its solution grows at a slower rate than $\bar{\varphi}^t$. If the solution is not unique, the system (46) generates one of the multiple stable solutions to (40).

[18] If $M_{s,2\bullet}\Psi_s$ has neither full row nor column rank, one has to use other methods, such as the single value decomposition -Sims (2000) offers details in this respect.

Notice that uniqueness does not hold, for instance, when the number of expectation errors n_2 exceeds the number of explosive eigenvalues. In such a case, it is possible to introduce expectation errors that are unrelated to the fundamental uncertainty ε_t without destabilizing the system. Lubik and Shorfheide (2002) show that the full set of solutions to (45) is characterized by

$$\eta_t = -P_{s,\bullet 1}\left(F_{s,11}\right)^{-1} L_{s,\bullet 1} M_{s,2\bullet} \Psi_s \varepsilon_t + P_{s,\bullet 2} \tilde{\eta}_{2,t}, \tag{47}$$

where P_s and L_s are orthonormal matrices and F_s is a diagonal matrix, that corresponds to the singular decomposition of $M_{s,2\bullet}\Pi$, that is,

$$M_{s,2\bullet}\Pi = P_s F_s L_s' = P_{s,\bullet 1} F_{s,11} L_s';$$

the vector $\tilde{\eta}_{2,t}$ forms what is called reduced form sunspot shocks, and it is defined through the decomposition of the forecast errors η_t as

$$\eta_t = L_{s,\bullet 1}\tilde{\eta}_{1,t} + L_{s,\bullet 2}\tilde{\eta}_{2,t},$$

where $\tilde{\eta}_{1,t}$ are the component of the forecast errors due to the fundamental shocks. Recall that if the number of expectation errors equals the number of explosive eigenvalues, the solution is unique and the second term in (47) drops out. If the latter are lower than the former, the system (45) does not provide sufficient restrictions to identify the elements of η_t. In this case, the solution is not unique.

TABLE 1: PARAMETER VALUES
Costly Price Adjusmtent Model

Risk Aversion	σ	1
Consumption/leisure share	η	0.35
Inverse Labor Supply Elasticity	φ	0.8
Habit Persistence	h	0.7
Intertemporal Discount Rate	β	$(1.03)^{-0.25}$
Capital Share	α	0.33
Elasticity Intermediate Goods	ϵ	6
Depreciation Rate	δ	0.018
Fixed Cost of Production	Ψ	0.3
Price Adjustment Costs	ϕ_p	4.05
Capital Adjustment Costs	ϕ_k	318
Variance Idiosyncratic Shock	σ_v^2	0.75
Persistence preference shock	ρ_ξ	0.9
Standard deviation preference shock	σ_ξ	0.03
mean preference shock	ξ	1
Persistence technology	σ_z	0.95
Standard deviation technology	σ_z	0.0065
Persistence monetary growth rate	ρ_μ	0.5
Mean monetary growth rate	μ	1.016
Standard deviation monetary shock	σ_μ	0.003

Figures

Figure 1: Labor and Real Wage Response to Monetary Policy Shock

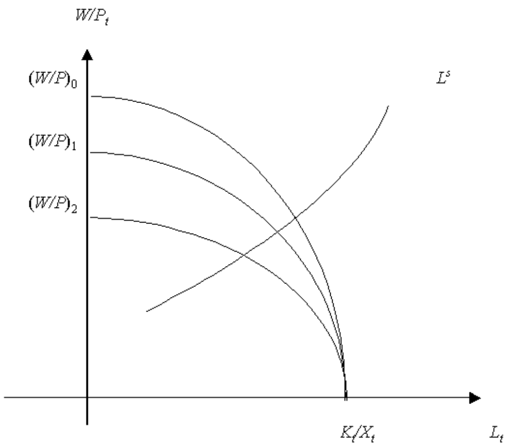


Figure 2: Idyosincratic Risk and Capacity Utilization

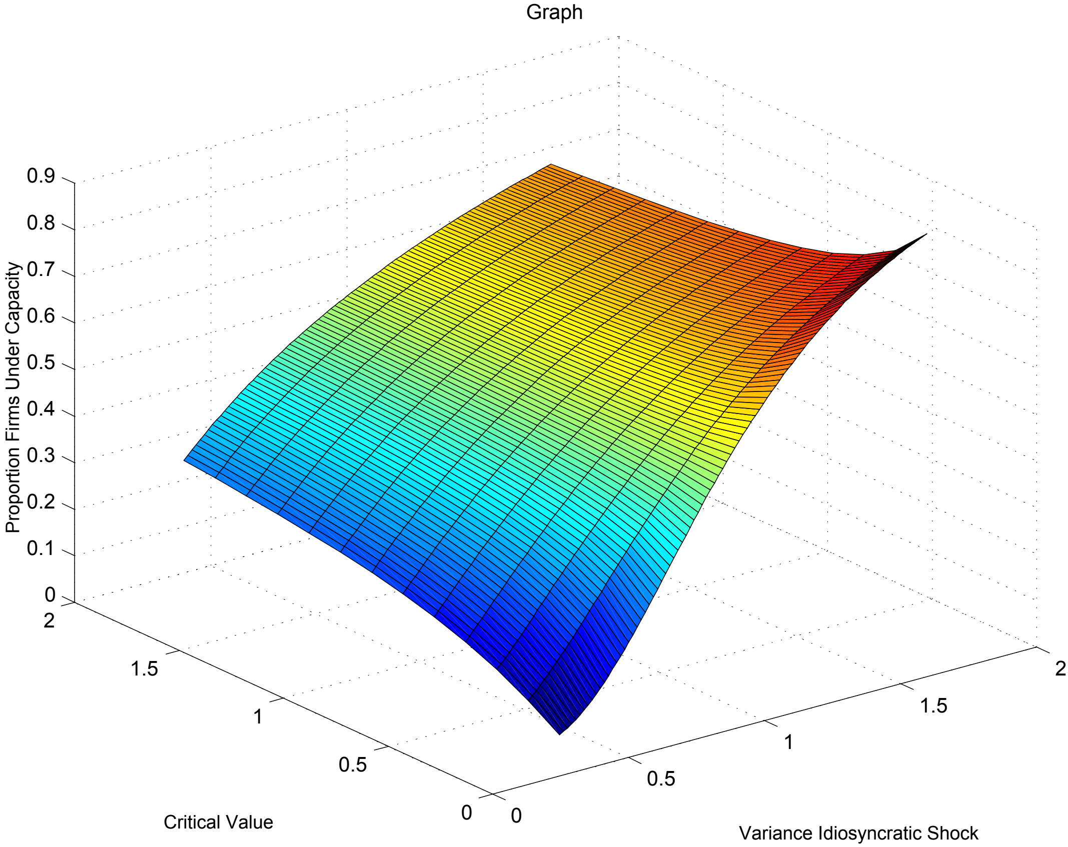


Figure 3: Impulse Responses to a Technology Shock

Figure 3.a

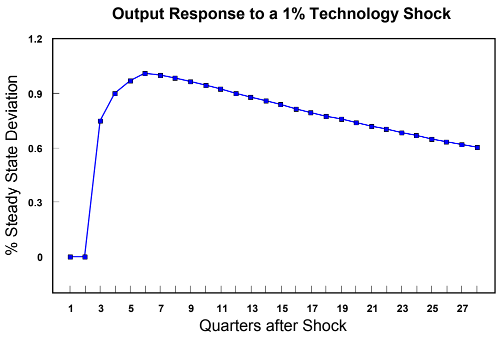


Figure 3.b

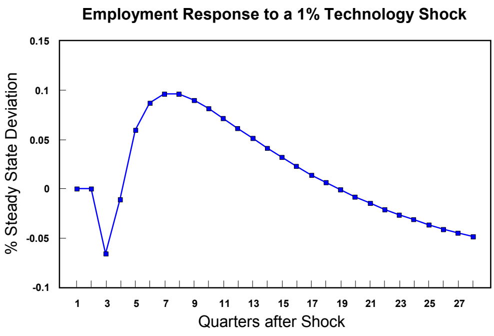


Figure 3.c

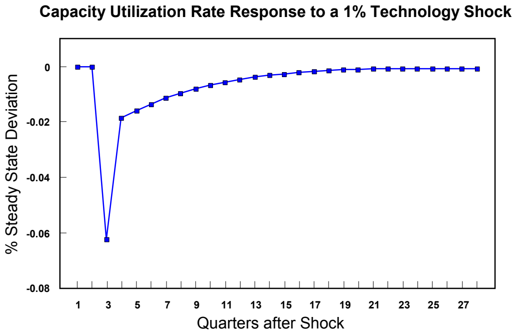


Figure 3 (continued)

Figure 3.d

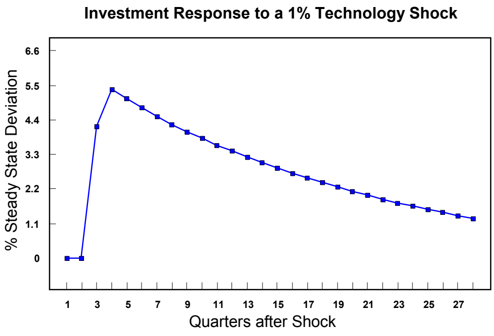


Figure 3.e

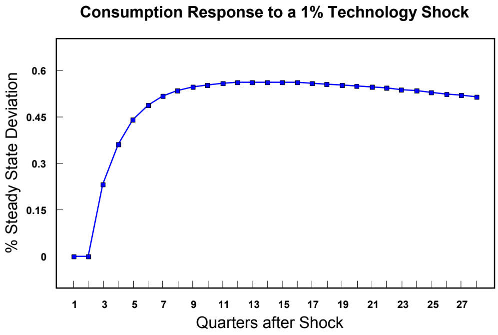


Figure 3.f

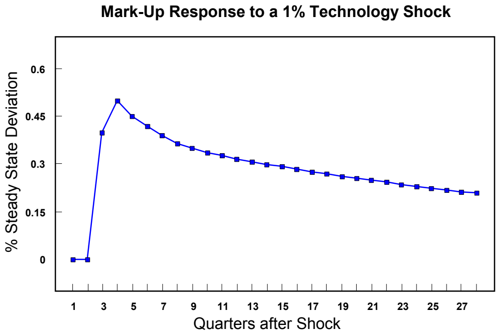


Figure 3 (continued)

Figure 3.g

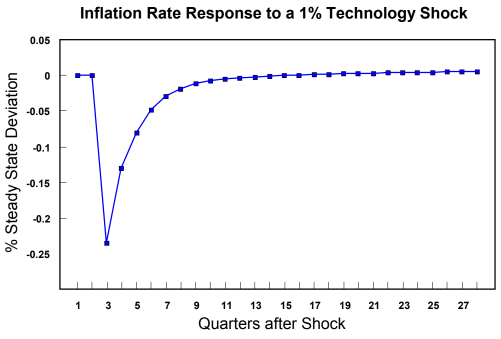


Figure 3.h

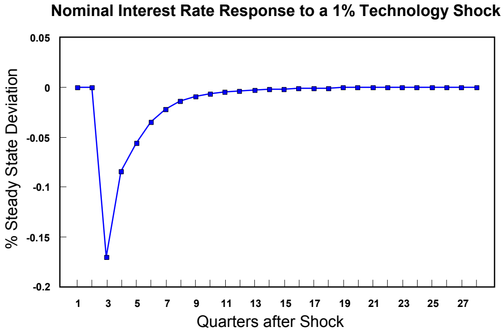


Figure 3.i

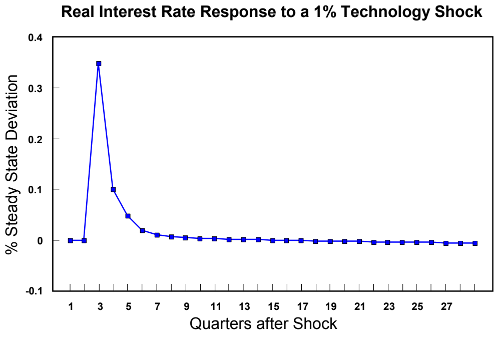


Figure 4: Impulse Responses to a Monetary Policy Shock

Figure 4.a

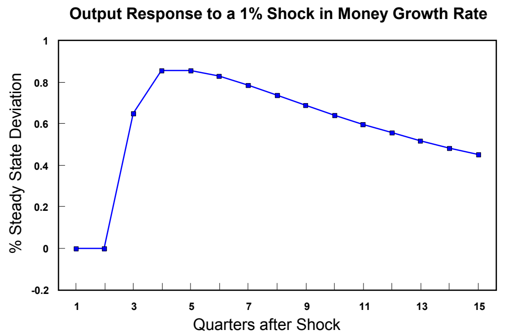


Figure 4.b

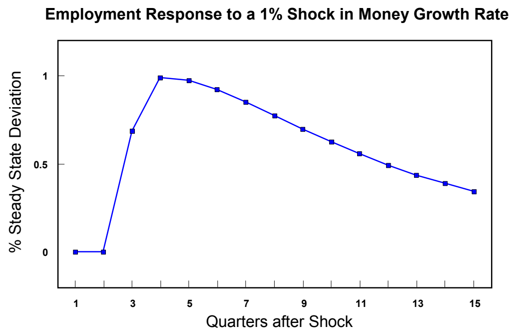


Figure 4.c

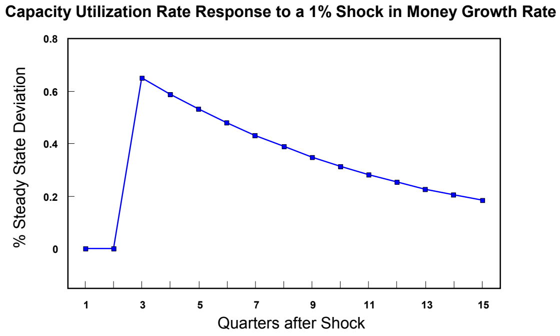


Figure 4 (continued)

Figure 4.d

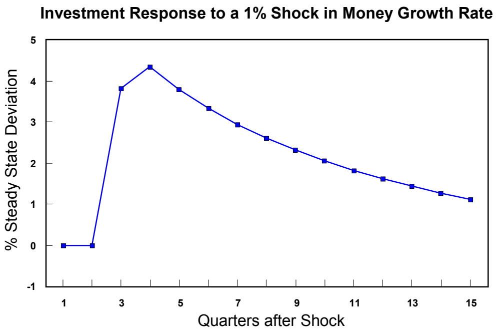
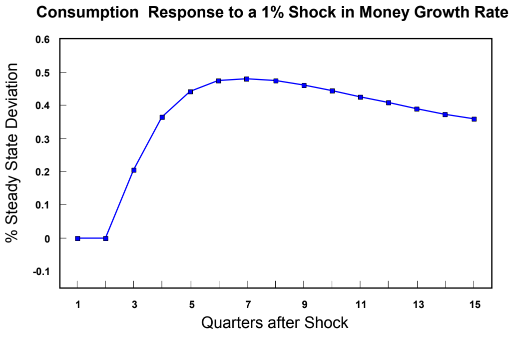


Figure 4.e



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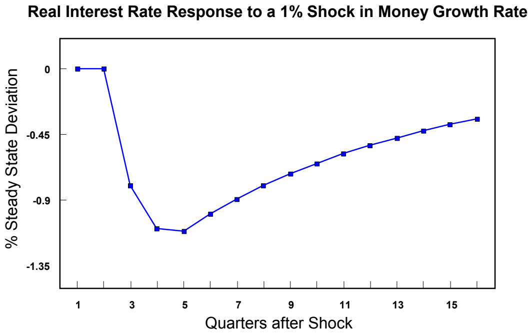
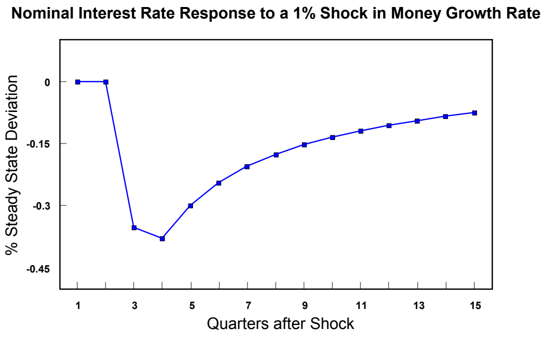


Figure 4 (continued)

Figure 4.g



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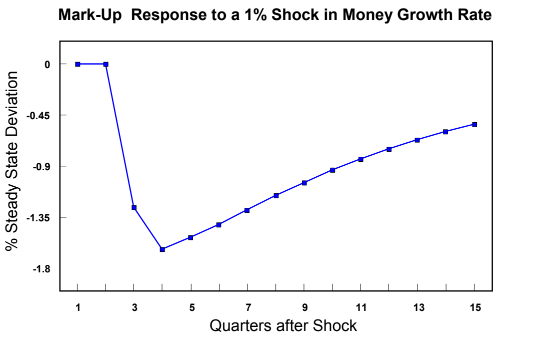

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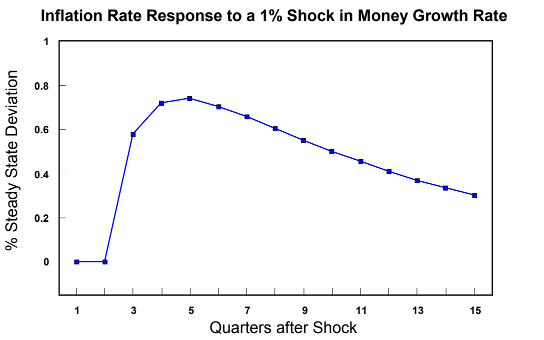


Figure 5: Responses to Cumulated Monetary Policy Shock

Figure 5.a

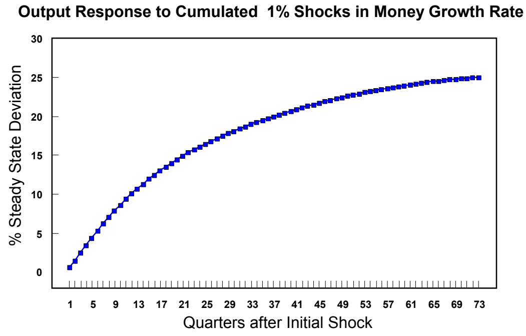


Figure 5.b

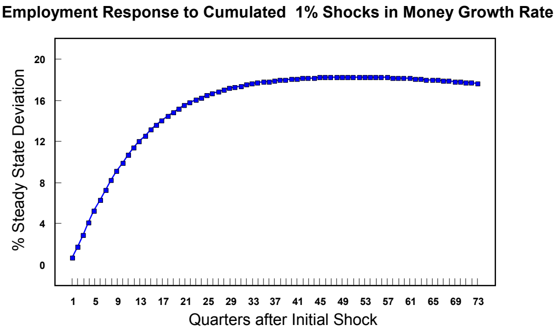


Figure 5.c

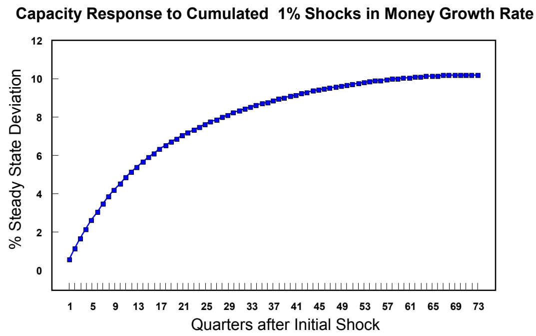


Figure 5 (continued)

Figure 5.d

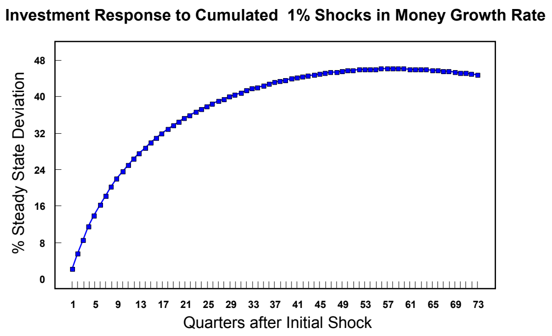
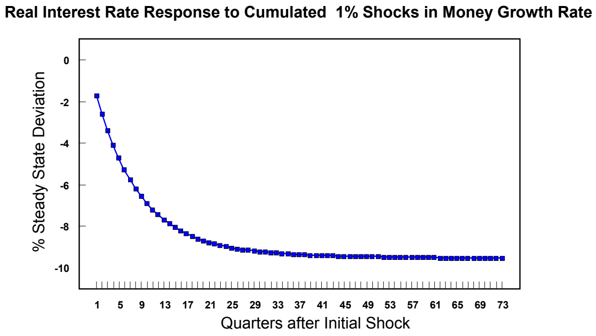


Figure 5.e



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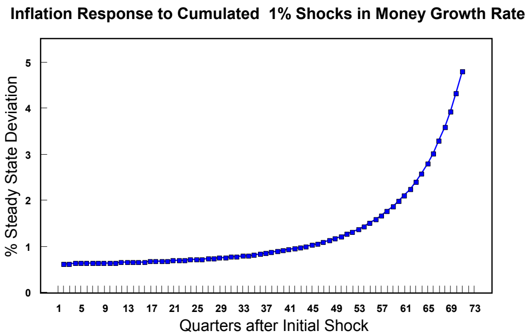

Figure 6: Responses to Monetary Policy Shock: Two-Capacity Regimes

Figure 6.a

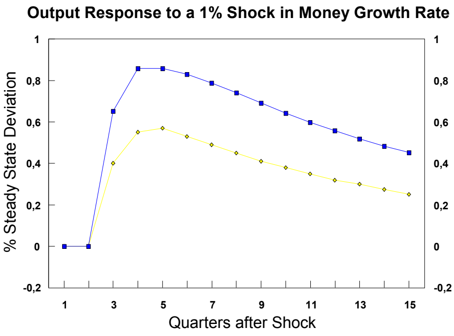

Figure 6.b

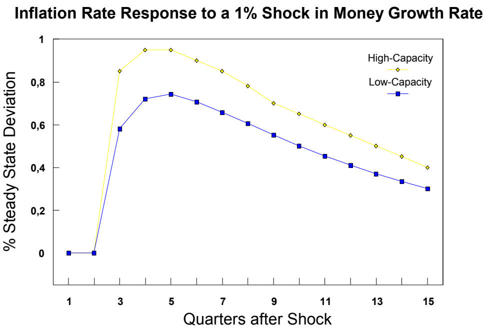

Figure 7: Responses to Monetary Policy Shock: Baseline Model

Figure 7.a

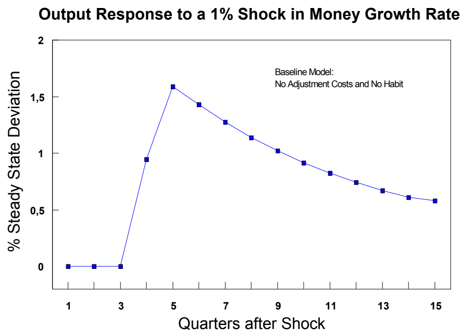

Figure 7.b

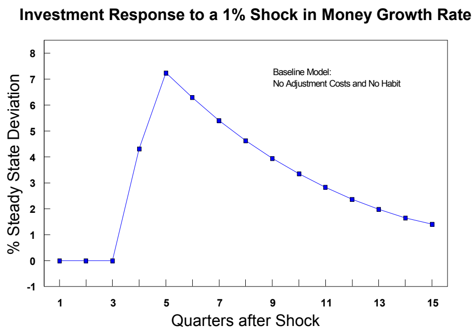

Figure 7.c