THE EURO AREA INEFFICIENCY GAP

Documento de Trabajo n.º 0302

Jordi Galí, Mark Gertler and J. David López-Salido
THE EURO AREA INEFFICIENCY GAP

Documento de Trabajo nº 0302

Jordi Galí
CREI, UPF, BANCO DE ESPAÑA, CEPR and NBER

Mark Gertler
NEW YORK UNIVERSITY and NBER

J. David López-Salido
BANCO DE ESPAÑA and CEPR

(*) We thank the comments of the participants in the BIS Autumn 2002 Central Bank Economist's Meeting on Monetary policy in a changing environment, Basel 14-15 October 2002. The views presented in this paper are those of the authors and do not represent those of the Banco de España.
The Working Paper Series seeks to disseminate original research in economics and finance. All papers have been anonymously refereed. By publishing these papers, the Banco de España aims to contribute to economic analysis and, in particular, to knowledge of the Spanish economy and its international environment.

The opinions and analyses in the Working Paper Series are the responsibility of the authors and, therefore, do not necessarily coincide with those of the Banco de España or the Eurosystem.

The Banco de España disseminates its main reports and most of its publications via the INTERNET at the following website: http://www.bde.es

© Banco de España, Madrid, 2003

ISSN: 0213-2710 (print)
ISSN: 1579-8666 (online)
Depósito legal: M. 19714-2003
Imprenta del Banco de España
Abstract

We construct a measure of Euro area cyclical efficiency, following the approach developed in Galí, Gertler and López-Salido (2002). Our measure — which we call the “gap” — corresponds to the inverse of price over social marginal cost. Here we present a time series of this gap for the Euro area, as well as its two components, the price and wage markups. As with U.S. data, the inefficiency gap is highly procyclical, and driven largely by countercyclical movements in the wage markup. We are also able to use our gap variable to derive a theory-based measure of the output gap for the Euro area, which we can compare to other measures often used in applications. We also show that the wage markup moves closely with the unemployment rate, as theory would suggest. Finally, we discuss briefly the implications for monetary policy of alternative interpretations of our evidence.

JEL Classification: E3

Key Words: Business Cycles, Countercyclical Markups, Monetary Policy.
1 Introduction

In this paper we construct measures of cyclical aggregate inefficiency for the Euro area based on the approach we proposed in an earlier paper (Gali, Gertler, and López-Salido (2002), henceforth GGL). Our measure of inefficiency is given by the wedge between the marginal product of labor and the household’s marginal rate of substitution between consumption and leisure. That wedge is equivalent, to put it more graphically, to the vertical distance between the labor supply and demand curves, whereas the area between those curves measures the lost surplus due to deviating from the first best. To the extent this area fluctuates with the business cycle, there will be cyclical movements in the efficiency of resource allocation. By constructing a time series measure of the Euro area inefficiency gap (or “Euro Gap” for short), we are able to obtain some insight into the nature and sources of business cycles in the Euro area.

As we show, the inefficiency gap corresponds to the inverse of the markup of price over social marginal cost. Hence, procyclical movements in the inefficiency gap are associated with countercyclical movements in this markup. We differ from the conventional literature on markups and business cycles, however, by allowing for the possibility of labor market frictions that introduce a wedge between the wage and the household’s marginal rate of substitution, the true social cost of labor. Cyclical movements in the overall markup, accordingly, may be the result not only of the movement of the markup of prices over marginal costs, but also of the wage markup over the marginal rate of substitution. Our approach will allow us to sort out the relative importance of the price versus the wage markup in the fluctuation of the overall markup, and hence of the inefficiency gap. In this respect, we are able to ascertain not only the degree of cyclical inefficiency but also the extent to which product market versus labor market rigidities may be responsible.

In addition, from our measure of the inefficiency gap, we are able to construct
a theory-based output gap, a more conventional indicator of cyclical inefficiency. Traditional measures of the output gap may be suspect because they typically model the unobserved, frictionless equilibrium level of output (i.e. the “natural level,”) in some ad hoc fashion. With our approach, the measure of the natural level of output is theory-based. As we show, our theory-based measure bears some correspondence to a more traditional output gap measure. Finally, we are able to draw a connection between our measure of the wage markup - the key factor underlying the inefficiency gap - and the unemployment rate, another key cyclical indicator. The underlying theory suggests a close correspondence. We show in fact that this is the case.

The paper is organized as follows. Section 2 introduces our proposed gap measure and its components. Section 3 presents the evidence on the behavior of the Euro gap. We discuss its cyclical properties, and its relationship with standard cyclical indicators in section 4. Section 5 discusses issues of interpretation and policy implications.

2 Measuring Cyclical Variations in Goods and Labor Market Inefficiencies

2.1 Introducing the Gap

Following GGL (2002) we introduce a measure of aggregate inefficiency which we refer to as the gap. Formally, the gap is defined as follows:

\[ gap_t = mrs_t - mpn_t \] (1)

where \( mpn_t \) and \( mrs_t \) denote, respectively, the (log) marginal product of labor and (log) marginal rate of substitution between consumption and leisure. As discussed in GGL (2002), the above gap measure has a simple relationship to measures of inefficiency wedges in goods and labor markets, which we refer to, respectively, as price and wage markups. Thus, we define the economy’s average price markup as:

\( \text{Note: See Appendix 1 for details of how to derive the expressions of this section.} \)
\[ \mu_t^p = p_t - (w_t - mpm_t) \]  
(2)

where \( p_t \) and \( w_t \) denote the (log) average price and wage levels, and where \( w_t - mpm_t \) can be interpreted as a measure of nominal marginal cost under the assumption of wage-taking firms and no labor adjustment costs.

Analogously, we define the economy’s average wage markup as:

\[ \mu_t^w = (w_t - p_t) - mrs_t \]  
(3)

i.e., the wedge between the wage and the marginal disutility of work, both expressed in terms of consumption.

We can combine equations (2), (3), and (1) to derive a fundamental relation linking the gap to the wage and price markups:

\[ \text{gap}_t = - (\mu_t^p + \mu_t^w) \]  
(4)

Notice that an increase in our gap measure is associated with lower markups, and hence a smaller distortion (i.e., an allocation closer to the efficient one).

Before we can construct measures of the gap and its components we need to make some assumptions on technology and preferences. Those are discussed next.

### 2.2 A Parametric Model

We assume that technology is characterized by a constant elasticity of output with respect to hours. In that case we have (up to an additive constant),

\[ mpm_t = y_t - n_t \]  
(5)

where \( y_t \) is output and \( n_t \) is hours (both in per capita terms). Combining equations (2) and (5) yields:
\[ \mu_t^p = (p_t + y_t) - (w_t + n_t) \]  
\[ \equiv - ulc_t \]  

Hence the price markup can be measured (up to an additive constant) as \textit{minus} the (log) real unit labor costs, denoted by \( ulc_t \).

Letting \( c_t \) denote per capita consumption, we assume that the (log) marginal rate of substitution takes the form (up to an additive constant) as:

\[ mrs_t = \sigma c_t + \varphi n_t \]  

where \( \sigma \) is the coefficient of relative risk aversion and \( \varphi \) measures the elasticity of the marginal disutility of labor. It follows from (3) that the wage markup is given by:

\[ \mu_t^w = (w_t - p_t) - (\sigma c_t + \varphi n_t) \]  

Finally, and given a measure of both the price and the wage markup, one can obtain a measure of the gap using equation (4).

### 3 Evidence for the Euro Area

We now use the simple relationships derived in the previous section to construct measures of the gap and its components for the Euro area. All the data are taken from Fagan, Henry and Mestre (2001), and are described in Table 1.\(^2\)

Constructing our measure of wage markup variations requires that we make an assumption on \( \sigma \) and on \( \varphi \). In our baseline calibration we set a unit value for both parameters, in a way consistent with much of the business cycle literature. Furthermore, since we are interested in the cyclical component of gap and markup variations

\(^2\)This data set has been recently used to estimate general equilibrium models with variable price and wage markups for the Euro area by Smeets and Wouters (2003a and b).
we detrend the variables constructed on the basis of (7), (9), and (4) by fitting a fifth order polynomial of time.3

Figure 1 shows our empirical measure of the Euro gap, under our baseline calibration. Our gap variable is seen to display large and persistent fluctuations, suggesting non-negligible variations in aggregate efficiency over the sample period considered. Furthermore, the timing of the observed variations, as captured by the main peaks and troughs, seems to match pretty well conventional accounts of the European business cycle: thus, the gap experiences a large decline in the late 70s, starts a long recovery in the mid 80s, reaches a peak in the early 90s, it declines until about 94, and stabilizes thereafter.

We next decompose the observed movements of the gap into wage and price markup variations. The wage markup measures were constructed using (9). The price markup corresponds to minus the log of real unit labor costs, as implied by (7). Figure 2 displays the behavior of those two variables over the sample period. Notice that, once we remove the low frequency movement by means of our detrending procedure, the wage markup still shows fluctuations of a magnitude and persistence similar to those of the gap. By way of contrast, the cyclical fluctuations in the price markup appear more tamed.

That visual impression is formalized in Table 1, which reports some basic statistics for the gap, its components, and detrended GDP. Note first that the standard deviation of the gap and the wage markup are similar, and large relative to detrended output and the price markup. The four variables display a high positive autocorrelation. Most interestingly, however, both the gap and the wage markup show a high correlation with detrended GDP. That correlation is positive in the case of the gap (confirming its procyclical nature) and negative in the case of the wage markup.

---

3This procedure leads to virtually the same results as those obtained using a band-pass filter which discards fluctuations outside a frequency range between 2 and 60 quarters (as recommended by Staiger, Stock and Watson (2001)). See Appendix 2 and Appendix 3, for details.
(which is thus countercyclical). The price markup, on the other hand, does not show any significant cyclical behavior.

Figure 3 shows the behavior of the gap against the wage markup. To facilitate visual inspection, we plot the inverse of the wage markup ($-\mu_t^w$). The strong co-movement between the two variables is clear to the naked eye. Put differently, the evidence suggests that countercyclical variations in the wage markup account for the bulk of fluctuations in the Euro area gap.\footnote{Similar results can be found in GGL (2002) for the U.S. economy.}

We now turn to the analysis of the effects of alternative assumptions about the coefficient of relative risk aversion and the labor supply elasticity on both the gap and its components. Figures 4 and 5 display how changes in the labor supply elasticity affect the properties of the gap and the wage markup, while keeping the risk aversion at its baseline value of one. In Figure 4 we compare the baseline gap measure (corresponding to $\varphi = 1$) with two alternative calibrations corresponding to lower ($\varphi = 5$) and higher ($\varphi = 0.1$) labor supply elasticities.\footnote{While the micro-evidence suggests a small labor supply elasticity, the business cycle literature tends to assume a high elasticity, typically unity and above. See discussion in GGL (2002). Evidence for the Euro Area on the rationale for those parameter values can be found in Smeets and Wouters (2003a) and in Andrés, López-Salido and Vallés (2003).} Notice that the size of gap fluctuations is inversely related to the labor supply elasticity, and hence it increases with $\varphi$.

Finally, Figure 5 translates this results into the comparison between fluctuations in the gap and the wage markup (notice that, by construction, the difference between these components corresponds to the price markup). Overall, the countercyclical wage markup fluctuations are the dominant component of the time series properties of the gap, though, as noted above, the higher the labor supply elasticity the lower is the correlation between the gap and the wage markup.\footnote{Though we do not report the results here, it also clears that lower (higher) values of risk aversion ($\sigma$) tend to dampen (increase) the fluctuations of the gap. See GGL (2002).}
Finally, in Figures 6(a) and 6(b) we demonstrate that the basic properties of the Euro gap are not affected significantly when we use two alternative measures of the marginal cost. Thus, the Euro gap measure displayed in Figure 6(a) uses a measure of marginal cost that accounts for labor adjustment costs. In Figure 6(b) we use a measure of the marginal rate of substitution that allows for time non-separabilities in leisure (as advocated by Eichenbaum, Hansen and Singleton (1988)). The variation patterns in the modified Euro gap measures are largely undistinguishable from the baseline one, thus indicating the latter’s robustness to some plausible changes in specification of preferences and technology.

4 The Euro Gap, the Output Gap, and the Unemployment Rate

In this section we discuss the connection between our gap and wage markup measures and two traditional cyclical indicators: the output gap and the unemployment rate. We also compare our implied measure of the output gap with other measures available for the Euro area.

4.1 The Gap and the Output Gap

Usually, the output gap is defined as the deviation of output from its natural or frictionless level, defined as the equilibrium value in the absence of nominal rigidities. More formally we have,

\[ \hat{y}_t \equiv y_t - \bar{y}_t \]  

where \( \hat{y}_t \), and \( \bar{y}_t \) denote the output gap and the frictionless level of output, respectively. In this paper, we show that under certain assumptions it is not possible to derive an exact relation between the output gap and the inefficiency gap without

---

7In each case we follow the parametrization recommended by Rotemberg and Woodford (1999). See Appendix 4 for details.

8Though we do not report the results here, it is clear from Figure 6(a) that the movements in the gap are strongly associated with a countercyclical wage markup. See Appendix 4 for details.
specifying a complete model. In doing that, we proceed in several steps. First, we assume that the reduced form (log of) aggregate production function can be written as:

$$y_t = a n_t + z_t$$  \(11\)

where \(z_t\) is exogenous or, at least, invariant to the degree of nominal rigidities. Think of \(z_t\) as a technology shifter.\(^9\) Second, given equation (11), we can derive the following expression for the gap:

$$\text{gap}_t = \left(1 - a + \varphi\right) y_t + \sigma c_t - \left(1 + \varphi\right) z_t$$  \(12\)

Third, we assume that the only source of gap variation lies in the presence of nominal rigidities in labor and/or goods markets.\(^11\) Accordingly, if we let \(\bar{c}_t\) be the level of consumption in the absence of nominal rigidities, then it follows that \(\bar{y}_t\) satisfies

$$\bar{\text{gap}}_t = \left(1 - a + \varphi\right) \bar{y}_t + \sigma \bar{c}_t - \left(1 + \varphi\right) z_t$$  \(13\)

To obtain a relation between the output gap and our demeaned gap measure \(\tilde{\text{gap}}_t = \text{gap}_t - \bar{\text{gap}}_t\), first combine equations (12) and (13):

$$\tilde{\text{gap}}_t = \left(1 - a + \varphi\right) \tilde{y}_t + \sigma \tilde{c}_t$$  \(14\)

where \(\tilde{c}_t = c_t - \bar{c}_t\). If we disregard capital accumulation and other demand components (so that \(\tilde{c}_t \simeq \tilde{y}_t\)), then we can rearrange expression (14) as follows:

\(^9\)See Appendix 1 for details.

\(^10\)More generally, equation (11) allows the possibility of variable capital utilization. Hence, as emphasized by Burnside and Eichenbaum (1995) and more recently King and Rebelo (1999). In particular, those authors have shown that variable capital utilization will raise the effective output elasticity of employment, \(a\).

\(^11\)Hence, we interpret the natural level of output as the level of output consistent with a constant gap (which corresponds to its steady state value, \(\text{gap}\)).
\[\tilde{y}_t = \left( \frac{a}{1 + \varphi + a(\sigma - 1)} \right) \text{gap}_t \] (15)

From the previous expression, several comments are in order. First, we have found a simple positive linear relationship between our measure of inefficiency and the output gap. Second, under our baseline calibration, i.e. when preferences satisfy a balanced growth condition \((\sigma = 1)\), the two parameters that determine the mapping between the gap and the output gap are the output elasticity to labor \((a)\) and the labor supply elasticity. To calibrate the former, we follow King and Rebelo (1999), who argue that the evidence is consistent with a value of \(a\) of roughly unity. Moreover, it is easy to see that when the labor supply elasticity is large enough, i.e. \(\varphi \to 0\), then our gap measure corresponds to the output gap.

Figure 7(a) plots the output gap corresponding to our baseline calibration of the gap jointly with a measure of the output gap constructed in the Area Wide Model of the ECB. As can be seen from the graph, starting in the mid 70’s our measure captures the main peaks and troughs of the AWM measure. As noted above, these results are largely robust to alternative choices of the labor supply elasticity parameter.

4.2 Unemployment and the Wage Markup

As noted in the previous section most of the variations in the Euro gap are accounted for by large variations in the wage markup. Here, and following Galí (1996), we show the existence of a tight connection, both theoretical and empirical, between wage markups and conventional unemployment measures. Let \(n_t^*\) denote the quantity of labor that the representative household is willing to supply at the currently prevailing real wage (taking the latter as given, and conditional on the current marginal utility of wealth), expressed in logs. We define the unemployment rate \(u_t\) in period \(t\) as

\[u_t = \frac{\tilde{y}_t}{n_t^*}\]

\(^{12}\text{In GGL (2002) we show that we can also express, without loss of generality, the consumption gap, }\tilde{c}_t, \text{ as a time varying proportion of the output gap, as follows: }\tilde{c}_t = \eta_t \tilde{y}_t. \text{ In the main text we are assuming that }\eta_t = 1. \text{ Changes in that baseline parameterization yields extremelly similar results. See GGL (2002).}\]
\[ u_t \equiv n_t^* - n_t \]

where \( n_t \) is actual employment (in logs).

The notional, “perfectly competitive” labor supply \( n_t^* \) satisfies

\[ \sigma c_t + \varphi n_t^* = w_t - p_t \]

On the other hand, from the definition of the wage markup we have

\[ \sigma c_t + \varphi n_t + \mu_t^w = w_t - p_t \]

Hence, it follows that the unemployment rate can be written, in terms of deviation from steady state values, as

\[ \hat{u}_t = \frac{1}{\varphi} \mu_t^w \]

i.e., fluctuations in the unemployment rate are proportional to those in the wage markup, with the coefficient of proportionality given by the (Frisch) elasticity of labor supply. Figure 7(b) illustrates the empirical validity of that predictions, by displaying both the cyclical unemployment rate and the wage markup. The positive comovement between the two series is quite striking, with the correlation coefficient being equal to 0.92.

5 Policy Implications

The results thus far suggest that the business cycle is associated with large co-incident movements in the efficiency gap. A decomposition of the gap, further, suggests that the countercyclical movement in the wage markup is by far the most important source of overall variations in the gap. This in turn suggests two possible interpretations of observed markup variations.
First, they might reflect desired changes in the markups by firms and workers in an environment where price and wages are flexibles. Examples of changes in desired price markups include the models discussed in Rotemberg and Woodford (1995), while non-Walrasian labor market dynamics have been discussed in Danthine and Donaldson (1990, 1995) and Galí (1996), among others. Alternatively, markup changes may be the unintended consequence of the existence of price and wage rigidities.

Since monetary policy has very limited role to play on the first scenario, our discussion will focus on the second environment. Recently, Erceg, Henderson and Levin (1999) has developed a simple theoretical framework in which the analysis of monetary policy in the presence of price and wage stickiness can be cast. In that framework, maintaining price stability is still the main task for monetary policy design, but as a result of the different distortions affecting the allocation of resources in the economy, several additional findings emerge from their analysis. First, their framework allows one to derive an approximation to the welfare of the representative consumer, which naturally can be taken as the objective function of the central bank. It can be shown that the relevant objective function for a benevolent central bank (i.e., one that seeks to maximize the utility of the representative household) depends on the variances of the output gap, price inflation, and wage inflation. In that context, when both nominal price and wage rigidities coexist, it can be shown that is not possible for the central bank to attain the first best or efficient outcome; instead the central bank faces a trade-off involving the stabilization of these variables.

In particular, the relative weights of output gap, price and wage inflation on the loss function are functions of the relative degree of stickiness. Hence, if price stickiness is dominant then is optimal for the central bank to focus predominantly on price inflation, while allowing wages to carry much of the burden necessary for real adjustment. If rigidities are mainly concentrated in the labor market, and so on wages, then it might be desirable, at least theoretically, to attempt to also mitigate
variations in the wage markup and so output fluctuations.

The large and dominant fluctuations in the wage markup detected in our empirical analysis point to the importance of wage rigidities. However, the evidence on relative small cyclical fluctuations in the price markup has a more ambiguous interpretation. On the one hand, it might reflect the fact that prices are not very sticky, making it possible for firms to keep the markups close to its desired levels most of the time. On the other hand, the low cyclical volatility of price markups may just be capturing the deliberate (and successful) attempt by the central bank to stabilize price markups as a way to achieve price stability.

Although our analysis does not allow us to discriminate between those two hypotheses, two lessons emerge from the previous discussion and the evidence above. First, we have uncovered a key role of labor market frictions as a source of cyclical variations in aggregate inefficiency. To the extent that those frictions are mainly associated to nominal wage rigidity, a stronger focus on wage markups, in addition to price stabilization, can be called for, since this will be crucial to mitigate business fluctuations in the economy. But, if real rigidities dominate the observed fluctuations in wage markups, then the role of monetary policy to moderate inefficient output fluctuations is much more limited. Perhaps surprisingly, not until recently, labor market frictions have been incorporated by macroeconomics in the models they have developed for monetary policy analysis. Further research is needed to assess how the relative importance of nominal/real wage and price rigidities affect the design of an optimal monetary policy for the Euro area.
### Table 1. Description of the Variables

<table>
<thead>
<tr>
<th>Mnemonics</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>lfn</td>
<td>labor force</td>
</tr>
<tr>
<td>lnn</td>
<td>total employment</td>
</tr>
<tr>
<td>lint</td>
<td>trend employment</td>
</tr>
<tr>
<td>pcr</td>
<td>real private consumption</td>
</tr>
<tr>
<td>urx</td>
<td>unemployment rate</td>
</tr>
<tr>
<td>urt</td>
<td>trend unemployment</td>
</tr>
<tr>
<td>win</td>
<td>compensation to employees</td>
</tr>
<tr>
<td>yed</td>
<td>GDP deflator</td>
</tr>
<tr>
<td>yen</td>
<td>nominal GDP</td>
</tr>
<tr>
<td>yer</td>
<td>real GDP</td>
</tr>
<tr>
<td>yet</td>
<td>Potential output (GDP)</td>
</tr>
<tr>
<td>popemu</td>
<td>Total population</td>
</tr>
</tbody>
</table>

### Table 2. Basic Statistics

*Baseline Calibration (σ = 1, φ = 1)*

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>s.d.(%)</td>
</tr>
<tr>
<td></td>
<td>Autocorrelation</td>
</tr>
<tr>
<td>GDP</td>
<td>1.83</td>
</tr>
<tr>
<td>Gap</td>
<td>2.48</td>
</tr>
<tr>
<td>Price Markup</td>
<td>1.18</td>
</tr>
<tr>
<td>Wage Markup</td>
<td>2.20</td>
</tr>
</tbody>
</table>
Appendix 1
Variable Markups and the GAP: A simple model

*Households*

The economy consists of a continuum of households indexed by \( i \in [0, 1] \) with preferences represented by the following expected utility function:

\[
\max_{C_t, N_t, B_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t(i)^{1-\sigma(i)}}{1-\sigma(i)} - a_t \frac{N_t(i)^{1+\varphi}}{1+\varphi} \right]
\]

where \( C_t(i) \) and \( N_t(i) \) represent consumption and hours of household \( (i) \), respectively; \( 1 > \beta > 0 \) is the household discount factor, \( \sigma > 0 \) represents the relative risk aversion (i.e. the inverse of the elasticity of intertemporal substitution), and \( \varphi \geq 0 \) represent the inverse of the Frisch labor supply elasticity (when \( \varphi = 0 \) preferences are linear in labor (Hansen (1985)) and the labor supply elasticity is infinite). The variable \( a_t \) represents exogenous medium run preference shocks. The budget constraint is:

\[
\frac{B_{t-1} + W_t(i)N_t(i) + T_t + D_t}{P_t} = C_t(i) + \frac{B_t(i)/r_t}{P_t}
\]

The households enter period \( t \) with risk-free bonds \( B_{t-1} \). At the beginning of the period the households receive lump sum nominal transfers \( T_t \) and labor income \( W_t N_t \), where \( W_t \) denotes the nominal wage. They also collect a nominal dividend \( D_t \) from the firms. The households consume and use some of these funds to purchase new bonds at nominal cost \( B_t/r_t \), where \( r_t \) denotes the gross nominal interest rate between \( t \) and \( t+1 \).

Let \( N_t(i) \) denote the household \( i \) labor supply (in hours) with \( W_t(i) \) the corresponding nominal wage. The household is a monopolistically competitive supplier of labor. A labor aggregator combines labor services from all households and sells a bundle of such services in the same proportion as firms would choose according to the CES production technology defined above \( N_t = \left[ \int_0^1 N_t(i) \frac{\pi w_t^{-1}}{\pi w_t} \, di \right]^{\pi w_t^{-1}} \). The aggregator minimizes the cost of producing a given amount of aggregate labor taking
$W_t(i)$ as given. The household $i$ labor demand is given by:

$$N_t(i) = \left( \frac{W_t(i)}{W_t} \right)^{-\varepsilon w_t} N_t$$

where $N_t$ is aggregate per capita hours and, from the zero profit condition of the labor aggregator we obtain that $W_t \equiv \left( \int_0^1 W_t(i)^{1-\varepsilon w_t} \, di \right)^{1-\varepsilon w_t}$ is the aggregate wage index. Notice that we allow for the elasticity of labor demand to vary over time. This can capture either real or nominal rigidities in the labor market.

The first order conditions of this problem are given by:

$$\lambda_t(i) = C_t^{-\sigma}(i)$$  \hspace{1cm} (18)

$$a_t \ MRS_t(i) \ (1 + \mu_t^W) = \left( \frac{W_t(i)}{P_t} \right)$$  \hspace{1cm} (19)

$$\lambda_t = \beta r_t E_t \frac{\lambda_{t+1}}{\pi_{t+1}}$$  \hspace{1cm} (20)

where $\pi_{t+1} = P_{t+1}/P_t$, $\lambda_t$ the Lagrange multiplier on (17), $MRS_t(i) = N_t^\sigma(i) C_t^\sigma(i)$ and $\mu_t^W = \frac{1}{\varepsilon w_t - 1}$ represents the optimal wage markup.

The Representative Finished Goods-Producing Firm

Each period $t = 0, 1, 2, \ldots$ the firm uses $Y_t(j)$ units of each intermediate good $j \in [0, 1]$, purchased at at nominal price $P_t(j)$. Following Dixit and Stiglitz (1977), the production function of this firm is defined according to the following constant-returns to scale aggregator:

$$Y_t = \left( \int_0^1 Y_t(j) \frac{\varepsilon w_t - 1}{\varepsilon w_t} \, dj \right)^{\frac{\varepsilon w_t}{\varepsilon w_t - 1}}$$

where $\varepsilon_t > 1$ will vary over time due to several reasons related to entry-exit dynamics of the market (see e.g. Rotemberg and Woodford (1999)). The representative firm demands the differentiated goods in an efficient manner to maximize its profits:
\[ P_t \left( \int_0^1 Y_t(j) \frac{\epsilon_t}{1-\epsilon_t} dj \right)^{\frac{\epsilon_t}{1-\epsilon_t}} - \int_0^1 P_t(j)Y_t(j) \, dj \]

The first order conditions of this problem generate the following isoelastic demand function for intermediate inputs:

\[ Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon_t} Y_t \]

In equilibrium, a zero profit condition implies that the aggregate price is given by the following expression:

\[ P_t = \left( \int_0^1 P_t(j)^{1-\epsilon_t} \, dj \right)^{\frac{1}{1-\epsilon_t}} \]

**Intermediate Goods-Producing Firms**

The production function for firm \( j \) is,

\[ Y_t(j) = Z_t N_t(j)^a \]

(21)

where \( N_t(j) \) represents the number of hours hired from the households

\[ N_t(j) = \left[ \int_0^1 N_t(i) \frac{e_{W_t-1}}{e_{W_t}} \, di \frac{e_{W_t}}{e_{W_t-1}} \right] \]

and \( Z_t \) is a common technology parameter, and \( a \) is the output elasticity to hours.

**Cost Minimization**

The firm \( j \) chooses factor (i.e. labor) demand in a perfectly competitive market. Hence, it chooses \( N_t(j) \) to minimize total costs:

\[ \min_{\{N_t(j)\}} \frac{W_t}{P_t} N_t(j) \]

(22)

subject to

\[ Y_t(j) - Z_t N_t(j)^a = 0 \]
The demand for labor is given by:

\[ MC_t(j) \cdot MPN_t(j) = \frac{W_t}{P_t} \]  

(23)

where \( MPN_t(j) = \frac{\partial Y_t(j)}{\partial N_t(j)} = F_{N,t} \), and \( MC_t(j) \) is the Lagrange multiplier that can be interpreted as the real marginal cost of producing an additional unit of output. Notice that, given cost minimization, the firm will take \( MC_t(j) \) as given when choosing its output price. Alternatively, the inverse of the \( MC_t(j) \), i.e. \( MC_t(j)^{-1} = 1 + \mu_{p_t}(i) \) can be considered as a firm price-markup. Notice that under our assumption of \( a = 1 \), the real marginal costs, i.e. the price markup is equal across firms, i.e. \( \mu_{p_t} = \frac{1}{e_t} \).

The GAP

From expressions (19) and (23) it is easy to obtain that, in a symmetric equilibrium

\[ \frac{MPN_t}{1 + \mu_{p_t}} = \frac{W_t}{P_t} = a_t \cdot \frac{MRS_t}{(1 + \mu_W)} \]

Rearranging terms

\[ \frac{MRS_t}{MPN_t} = GAP_t = \frac{1}{a_t \cdot (1 + \mu_{p_t}) \cdot (1 + \mu_W)} \]

which under the assumption of no preference shocks can be log-linearized to get the expression in the text.
Appendix 2
Alternative Detrending and the Gap

In this appendix we check the robustness of the cyclical properties of the gap to alternative detrending methods. In particular, we compare our baseline case with three alternative cases. The first one consists on a 3rd order polynomial in trend, the second is calculated using a standard HP filter with smoothing parameter 1600, and finally we also construct a cyclical measure based on the BP filter selecting the cyclical frequencies between 2 to 60 quarters. As can be seen from the figure below, the alternative measures behave on a relatively similar way, specially from mid 70’s onward, as our baseline cyclical component.

![Figure 1: Alternative detrending and the Euro area Gap](image-url)

Figure 1: Alternative detrending and the Euro area Gap
Appendix 3

The Gap and Technology Shocks

As we have discussed in the main text, the cyclical movements in the gap can be contaminated by the presence of preference shocks. In this Appendix we show that the high frequency movements in our baseline gap cannot be simply due to preference shocks.

Under the null hypothesis of preference shocks our gap measure should be exogenous, i.e. unaffected by other sort of shocks in the economy. We next present a simple test that reject the null of exogeneity, since we show that the gap and its components react quite strongly to technology shocks.

We have estimated the dynamic response of our gap variable to an identified exogenous productivity shock. The identification scheme is the one recently proposed by Galí (1999). We estimate a four lag VAR over the sample period 1970-2000 including the growth rate of labor productivity, the cyclical measure of employment and marginal costs, the consumption output ratio and the changes in the GDP deflator. From our estimated VAR we can recover the time series properties of the gap as well as the wage markup for our baseline calibration. In the next Figure we show the estimated responses to a positive productivity of selected variables. Interestingly, the inefficiency gap increases significantly in response to a permanent positive productivity shock, in line with employment and output. This endogenous reaction, of course, is inconsistent with the preference shock hypothesis, but fully consistent with our hypothesis that countercyclical markups.

\footnote{We identify the productivity shock as the only one having permanent effects on labor productivity.}
Appendix 4
Alternative Price and Wage Markups

A) Price Markup with Labor Adjustment Costs

The real marginal costs, $mc_t$, (i.e. the inverse of the markup) is given by: $mc_t = \frac{wp_t}{F_{N_t}}$, where $wp_t$ is the real wage and $F_{N_t}$ is the partial derivative of the production function (i.e. of output) with respect to labor. Under the previous assumptions, the real marginal costs can be expressed as follows:

$$mc_t = \frac{w_t}{F_{N_t}} = \frac{s_t}{\gamma_t}$$

where $s_t$ is the labor income share, and $\gamma$ is the elasticity of output with respect to
labor. In log-deviations from steady state \((mc = \frac{1}{\mu} = \frac{\xi}{\gamma})\), the previous expression is just:

\[
\overline{mc}_t = \hat{s}_t - \hat{\gamma}_t
\]  

(24)

The benchmark case used in this paper is based upon the assumption of no adjustment costs, and a constant elasticity of output with respect to labor. Hence, the expression (24) collapses to: \(\overline{mc}_t = \hat{s}_t\).

Following Rotemberg and Woodford (1999), we consider the effect of having cost of adjusting labor. These costs take the form: \(U_t/N_t\phi(N_t/N_{t-1})\), where \(U_t\) is the price of the input required to make the adjustment. In this case, the real adjustment costs associated with hiring an additional worker for one period is given by:

\[
(U_t/P_t)\{(N_t/N_{t-1})\phi(N_t/N_{t-1}) - \hat{E}_t[q_{t+1}((U_{t+1}/P_{t+1})(N_{t+1}/N_t)^2\phi(N_{t+1}/N_t))]\}
\]

letting \(\zeta_t \equiv \frac{\eta_{t-1}(U_t/P_t)}{(U_{t-1}/P_{t-1})}\), and \(g_{N_t} \equiv (N_t/N_{t-1})\), we can approximate the previous expression by:

\[
(U_t/P_t)\phi''(1)\{\hat{g}_{N_t} - \hat{E}_t[\hat{g}_{N_{t+1}}]\}
\]

Assuming that the ratio \(U_t/W_t\) is stationary, the real marginal costs are given by:

\[
mct = \left(\frac{\hat{s}_t}{\hat{\gamma}}\right)[1 + (U/W)\phi''(1)\{\hat{g}_{N_t} - \hat{E}_t[\hat{g}_{N_{t+1}}]\}]
\]

which, in terms of deviations from steady state yields

\[
\overline{mc}_t = \hat{s}_t + \xi\{\hat{g}_{N_t} - \hat{E}_t[\hat{g}_{N_{t+1}}]\}
\]  

(25)

where \(\xi = \mu^{-1}(U/W)\phi''(1)\). Under the assumption that the employment follows a random walk, then

\[
\overline{mc}_t = \hat{s}_t + \xi\{\hat{g}_{N_t}\}
\]  

27
**B) Wage Markup with Time-Non Separabilities in Hours**

Let assume that household preferences are given by:

\[ U(C_t, N_t) = \log C_t + \psi \frac{(N_t^*)^{1+\varphi}}{1 + \varphi} \]

where \( \psi \) is constant parameter and

\[ N_t^* = N_t + bN_{t-1} \]

with \( 0 \leq b < 1 \). This specification incorporates time dependence in hours as emphasized by Kydland and Prescott (1982), Eichenbaum, Hansen and Singleton (1988) among others.

In this model the (log linear approximation around steady state of the) marginal rate of substitution between consumption and income can be described as follows:

\[ mrs_t = c_t + \varphi \left[ \gamma_n n_t^* + (1 - \gamma_n)n_{t+1}^* \right] \]

\[ n_t^* = \beta_n n_t + (1 - \beta_n)n_{t-1} \]

with \( \gamma_n = (1 + \beta_n)^{-1} \), and \( \beta_n = (1 + b)^{-1} \).

Notice that the previous expressions can be rewritten as follows:

\[ mrs_t = c_t + \varphi n_t + \varphi E_t \{ \Psi_{t+1} \} \]

where

\[ E_t \{ \Psi_{t+1} \} = \gamma_n(1 - \beta_n)n_{t-1} + [(\gamma_n(\beta_n - 1) + (1 - \gamma_n)(1 - \beta_n))] n_t + (1 - \gamma_n)\beta_n E_t n_{t+1} \]

Notice that under intertemporal separability \( \beta_n = \gamma_n = 1 \), and \( \Psi_{t+1} = 0 \), so we obtain the baseline specification for the \( mrs_t \). Thus, the wage markup displays time dependence as follows:
\[ (\mu_t^w)^{NS} = (w_t - p_t) - mrs_t = \]
\[ (w_t - p_t) - [c_t + \varphi m_t] - \varphi E_t \{ \Psi_{t+1} \} = \]
\[ \mu_t^w - \varphi E_t \{ \Psi_{t+1} \} \]

where \((\mu_t^w)^{NS}\) represents the wage markup under time non-separabilities as a function of the baseline wage markup, \(\mu_t^w\), and the variable \(E_t \{ \Psi_{t+1} \}\). We set \(b = 0.5\) as a benchmark case.
References


Figure 1

The Euro area Gap
Baseline Specification
Figure 2

The Euro area Gap Components

Baseline Specification

wage markup

price markup
Figure 3

The Gap and the Wage Markup in the Euro area
Figure 4

The Euro area Gap
*Alternative Specifications*

![Graph of the Euro area Gap with baseline and alternative specifications.]
Figure 5

The Gap and the Wage Markup in the Euro area

Baseline

\[
\begin{array}{c}
\Phi = 5.0 \\
\Phi = 0.1
\end{array}
\]

- \[1970 \quad 1972 \quad 1974 \quad 1976 \quad 1978 \quad 1980 \quad 1982 \quad 1984 \quad 1986 \quad 1988 \quad 1990 \quad 1992 \quad 1994 \quad 1996 \quad 1998 \quad 2000
\]

- \[-6.4 \quad -4.8 \quad -3.2 \quad -1.6 \quad -0.0 \quad 1.6 \quad 3.2 \quad 4.8 \quad 6.4\]

- \[-15 \quad -10 \quad -5 \quad 0 \quad 5 \quad 10 \quad 15 \quad 20\]

- \[-4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4\]
Figure 7(a)

Output Gaps: Euro area

Figure 7(b)

Unemployment and Wage Markup Fluctuations

Cyclical Components