ASYMMETRIC SHOCKS, RISK SHARING, AND THE LATTER MUNDELL

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* Departamento de Economía, Universidad Carlos III de Madrid, 28903 Getafe (Madrid), Spain. Email: **desmet@eco.uc3m.es**. This paper is part of a larger research program with Marco Celentani and J. Ignacio Conde-Ruiz on the effects of fiscal and monetary policy on risk sharing. Much of this research was carried out while I was visiting the Bank of Spain and Stanford University, and I wish to thank the hospitality of both institutions. I am furthermore indebted to Jordi Galí, Berthold Herrendorf, Ignacio Ortuño, Javier Vallés, and especially Ronald McKinnon for helpful comments and discussions. Financial support from the Commission for Cultural, Educational and Scientific Exchange between the United States of America and Spain (Project 7-42) is gratefully acknowledged.

Abstract

This paper analyzes optimal monetary policy in a two-country model with asymmetric shocks. Agents insure against risk through the exchange of Arrow-Debreu securities. Although central banks commit to the policy that maximizes domestic welfare, this does not lead to price stability. In an attempt to improve their country's terms of trade of securities, central banks may choose an inflationary policy rule in good states. If both central banks do so, the effects on the terms of trade wash out, leaving both countries worse off. Countries facing asymmetric shocks may therefore gain from monetary cooperation.

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1 Introduction

Mundell's theory of optimum currency areas has dominated the scholarly debate on monetary unions since its publication forty years ago (Mundell, 1961). Its rule-of-thumb says that countries should keep separate currencies if they face asymmetric shocks and labor is immobile across borders. If one country is in a boom and the other in a recession, a common monetary policy is unable to react adequately. The failure of the euro zone to pass the Mundell test has led to widespread skepticism about the prospects of European Monetary Union (Eichengreen, 1997).

However, a decade after his celebrated 1961 paper the same Mundell made a very different argument: countries experiencing asymmetric shocks could actually gain from adopting a common currency because of its beneficial effect on international risk sharing (Mundell, 1973). Recently, several authors have drawn attention to this "latter Mundell" (McKinnon, 2000, 2001; Ching and Devereux, 2000a, 2000b). They informally argue that if countries hold claims on each other's currencies to insure themselves against asymmetric shocks, central banks will be tempted to inflate prices whenever those claims are redeemed. This may end up jeopardizing risk sharing, and it may be necessary to adopt a common currency to solve the problem.

That the temptation to create surprise inflation may be self-defeating has been well known since the seminal work of Barro and Gordon (1983a,b). If agents understand the incentives of central banks, inflation ceases to come as a surprise, so that we end up with the worst of two worlds: incurring the cost of inflation without benefiting from its surprise effect. Only if central banks can credibly commit to a policy rule can we improve upon this outcome. In the absence of commitment problems, the optimal policy rule in Barro and Gordon (1983a,b) would prescribe zero inflation: since rational expectations render surprise inflation impossible, the best central banks can do is to maintain price stability.

Applying this argument to our problem, we may think that credible commitment could do away with the temptation of creating inflation, thus ensuring the optimal solution without the need of adopting a common currency. Contrary to this intuition, our paper suggests that credible commitment may not be enough. As we will show, inflation may improve a country's terms of trade in the risk sharing market, so that the central bank may optimally commit to creating inflation. Of course, if both central banks try to distort relative security prices, their attempts fail, leaving everyone worse off. This implies that the Mundell (1973) argument in favor of monetary cooperation does not disappear when allowing for commitment à la Barro and Gordon (1983a,b).

To understand the intuition behind this result, we briefly lay out the mechanics of the model. Consider two countries with perfectly negatively correlated supply shocks. Risk sharing happens through the exchange of nominal Arrow-Debreu securities. Countries have separate currencies and separate central banks. When deciding on the optimal level of inflation, central banks weigh off costs and benefits. On the one hand, inflation reduces welfare due to, for instance, menu costs or inefficient government spending. On the other hand, inflation — if it comes as a surprise — increases welfare because it lowers the real value of nominal payments to the other country. Allowing for rational expectations should do away with the benefits of inflation, since nominal contingent claims become equivalent to real contingent claims. Surely in that case we would expect central banks to stick to price stability, as long as they have the capacity to commit.

Not necessarily. There is a potentially important secondary effect which may overturn our previous argument: central banks can use inflation to affect the terms of trade of Arrow-Debreu securities. More particularly, central banks may commit to creating inflation in states of nature in which their country is a supplier of securities. Since inflation lowers welfare, agents become less willing to supply securities, causing their price to go up. If the welfare gain from an increase in the relative price is enough to compensate for the welfare loss from inflation, central banks will optimally choose to create inflation.

Given the symmetric setup, both central banks follow this strategy. As a result, the corresponding non-cooperative Nash equilibrium leaves everyone worse off compared to the case of monetary stability: the positive effect on the terms of trade of contingent claims washes out, and we are left with the welfare cost of positive inflation. To reach the optimal outcome, some form of cooperation is therefore necessary. As is well known, cooperation could be sustained in a repeated game framework without the need for an institutional or a formal arrangement. However, if for whatever reason this is not possible, a full-fledged monetary union with a common currency may well be the answer. This suggests that, contrary to conventional wisdom, countries facing asymmetric shocks may stand to gain from adopting a common currency.

Note that our model is related to the literature on monetary policy coordination (see Persson and Tabellini, 2000, for a survey and references). The standard textbook example is the case of competitive devaluations. In the wake of a negative demand shock, the central bank devalues the currency to improve the terms of trade: this stabilizes demand at the cost of higher inflation. However, if the country's trading partner also faces a negative demand shock, and reacts in the same way, the gain vanishes, while the welfare cost of higher inflation remains. These models of monetary policy coordination have a distinctly pre-rational expectations Keynesian flavor: a currency devaluation can only boost domestic demand if agents fail to foresee the future or if there are nominal rigidities.

This is what makes our model different: individuals have rational expectations; monetary policy is announced in advance; and prices are flexible. Even though monetary policy loses its capacity to surprise, central banks may optimally commit to creating inflation in an attempt to improve their country's terms of trade in securities. The same mechanism has been studied in the context of fiscal policy by Celentani, Conde-Ruiz and Desmet (2002), where they show how voters may choose the level of government spending to manipulate the relative price of securities.

2 Setup of the model

Consider an infinite horizon model with two countries, A and B. Both countries produce the same freely transportable non-storable good. This rules out shocks to the relative price of exportables and asymmetric demand shocks.[1] Time is discrete. In each period

[1] Shocks to the relative price of exportables are impossible because we are focusing on a single-good economy. Asymmetric demand shocks are excluded because openness acts as a complete automatic stabilizer. Consider, for instance, a negative consumption shock in A and a positive consumption shock of the same size in B. These shocks would simply be absorbed through the current account: in A the drop in

the economy's total production is 2. Although there is no aggregate uncertainty, countries are subject to perfectly correlated asymmetric supply shocks of size α, where $0 < \alpha < 1$. Which region gets the positive shock is uncertain, so that in each period there are two possible states of nature: in state $s = 1$ country A experiences a positive shock, and B a negative shock; in state $s = 2$ country B receives a positive shock, and A a negative shock. Focusing on the perfectly symmetric case, each state occurs with probability 1/2.[2] Country-level production in any given period is then:

$$Q^A = \begin{cases} 1+\alpha & \text{if } s=1 \text{ (with probability } 1/2) \\ 1-\alpha & \text{if } s=2 \text{ (with probability } 1/2) \end{cases} \tag{1}$$

$$Q^B = \begin{cases} 1-\alpha & \text{if } s=1 \text{ (with probability } 1/2) \\ 1+\alpha & \text{if } s=2 \text{ (with probability } 1/2) \end{cases} \tag{2}$$

Each country is populated by a unit mass of homogeneous risk-averse agents. To keep the problem analytically tractable, we focus on CRRA preferences, so that the period expected utilities of the representative agents can be written as:

$$U^A = \frac{1}{2}\frac{(C_1^A)^{1-\rho}}{1-\rho} + \frac{1}{2}\frac{(C_2^A)^{1-\rho}}{1-\rho} \tag{3}$$

$$U^B = \frac{1}{2}\frac{(C_1^B)^{1-\rho}}{1-\rho} + \frac{1}{2}\frac{(C_2^B)^{1-\rho}}{1-\rho} \tag{4}$$

where ρ is the coefficient of relative risk aversion (where $\rho > 0$ and $\rho \neq 1$), and C_s^A and C_s^B denote state s consumption in A and B. Given the existence of asymmetric shocks, there is obvious room for risk sharing. To insure risk, agents trade in nominal Arrow-Debreu securities.

Agents cannot consume their own output; to consume, they need to buy goods in the marketplace. To give a meaningful role to money, we follow Lucas (1982) and assume that money is needed to buy and sell goods. Morever, in each country goods must be paid for in the national currency. If countries have separate currencies, they are denoted by $\$_A$ and $\$_B$; if, instead, there is a common currency, it is denoted by a simple \$ sign.

At the beginning of each period central banks set the money supply. To increase the money supply relative to the previous period, they distribute extra cash to the population;

consumption would be compensated by an increase in exports, and in B the rise in consumption would be resolved through an increase in imports, leaving demand (and supply) unchanged in both countries.

[2] We will later discuss different forms of asymmetries between countries.

to decrease the money supply, they confiscate part of the cash holdings. By setting the money supply, central banks are able to determine the price level.[3] For instance, if central bank A creates a money supply M^A, then applying the quantity equation of money gives us a price level of $\frac{M^A}{Q^A}\$_A$.[4]

We assume away intertemporal trade in goods and assets. This seems reasonable: since goods are non-storable and since in each period countries are ex ante identical, agents only trade in Arrow-Debreu securities to insure against risk within periods. Moreover, since all periods are identical, it will suffice to focus on one period to solve our infinite horizon problem. Note that introducing a more explicit intertemporal structure through, for instance, interest-bearing bonds, would further complicate the model without changing any of the relevant results.

There is a cost attached to inflation. For our conclusions to hold, it is essential that inflation decreases welfare; it is irrelevant, however, why this is so. One possibility is to introduce menu costs. Following Barro and Gordon (1983a,b), we take menu costs to be increasing in the rate of inflation. We furthermore assume that the cost of adjusting prices is proportional to output. To be more concrete, in country A an inflation rate π^A leads to a welfare cost equivalent to a decrease in output by:

$$\gamma\pi^A Q^A \tag{5}$$

where $\gamma \in [0, 1]$; the greater γ, the bigger the cost of inflation.

Rather than focusing on menu costs, we could alternatively assume that inflation reduces welfare through inefficient government spending. More specifically, if central banks use the printing press to buy up part of their country's output, and if this leads to inefficiencies, there will be a welfare loss. Since central banks actually *need* inflation to be welfare decreasing if they want to manipulate the relative price of securities, we can think of these inefficiencies as being more of a decision than an undesirable side-effect. If so, we could simply assume that central banks throw away a fraction γ of the goods they buy.

[3] We assume away interest-bearing bonds and currency trading, so that agents are willing to hold all the money they receive.

[4] We assume a velocity of money of 1, since time periods do not have an explicit duration.

To see how this would work, consider the case of central bank A. Normalize the price level of the previous period to $1\$_A$. In the absence of inflation, central bank A prints $Q^A\$_A$ units of money, which it hands out to the population. To create inflation of π^A, it prints an additional $\pi^A Q^A \$_A$ units of money, which it uses to buy $\frac{\pi^A}{1+\pi^A}Q^A$ of the country's goods. Assuming a fraction γ of those goods is thrown away, inflation π^A leads to a loss equivalent to a drop in output by $\gamma\frac{\pi^A}{1+\pi^A}Q^A$. For small enough levels of inflation this output loss can be approximated by $\gamma\pi^A Q^A$, an expression identical to the loss due to menu costs in (5).

The goal of this paper is to analyze optimal policy rules under different monetary arrangements. Before entering into details, we must be more specific about the timing of events. At the beginning of time central banks announce a monetary policy — and thus an inflation rate — for each of the two states of nature. Monetary policy rules are credible,[5] and hold for all periods. Once these policy rules have been announced, time starts running. Each period consists of an identical sequence of events:

1. Agents from both countries meet to buy and sell nominal Arrow-Debreu securities referring to the given period. These securities are simply exchanged; no money is used in these transactions.

2. The state of nature is revealed. Central banks adjust the money supply, with the goal of setting inflation consistent with the policy they committed to.

3. Arrow-Debreu securities are redeemed for money.

4. All money is spent, and agents consume.

In the non-cooperative case, each central bank maximizes the welfare of its representative agent when setting inflation. The optimal policy rules will be given by the Nash equilibrium of the non-cooperative game between both central banks. In spite of maximiz-

[5] It is well known that policy rules may be dynamically inconsistent, if the benefit from deviating is greater than the cost. Since this is not the main focus of our paper, we abstract from these issues of credibility and commitment.

ing domestic welfare, central banks may set policy rules that deviate from price stability in an attempt to improve the terms of trade of securities.

This possibility of tinkering with the relative prices of securities is based on two crucial assumptions: central banks announce monetary policy before agents trade in security markets; and inflation is welfare decreasing. The first assumption allows central banks to affect outcomes in security markets. The second assumption gives them a means of doing so: by creating inflation when their country receives a positive shock, they reduce welfare in good states, lowering the supply of securities, and thus pushing up their price. While there exists a certain consensus that inflation is costly, it may seem somewhat arbitrary to assume that policy rules are announced before security markets open. In fact, this assumption is less restrictive than it seems. For our results to hold, security markets need not be closed before policy is announced; the only thing we require is that there is some residual uncertainty which has not been traded away at the time policy is set.[6]

Since both central banks try to improve their terms of trade in securities, the non-cooperative outcome generally reduces welfare, compared to a situation where both central banks commit to price stability. This leads us to consider monetary cooperation, where policy is decided through a bargaining process. This cooperation may take place between separate central banks, or through the formation of a monetary union.

3 Optimal monetary policy in the absence of cooperation

In this section we focus on the case where countries use separate currencies, managed by separate central banks that do not cooperate. We solve the model backwards. First the security markets equilibrium is determined, taking monetary policy in both countries as given. This information is then used by each central bank to choose the monetary policy that maximizes domestic welfare, taking the policy of the other central bank as given. The equilibrium of the model is defined as the Nash equilibrium of the non-cooperative game between both central banks.

We start by looking at the optimization problem of country A's representative

[6] See Celentani et al. (2002) for a further discussion of this point.

agent.[7] Given the absence of intertemporal issues, we focus on one period, and we normalize the price level in the previous period to $1\$_A$. Let x_s^A be country A's purchase — or sale, if x_s^A is a negative number — of nominal Arrow-Debreu securities in state of nature s. If country A buys x nominal securities, it receives x units of country B's currency when the securities are redeemed; if, on the other hand, A sells x nominal Arrow-Debreu securities, it pays x units of its own currency when the securities are redeemed. How many consumption goods can be bought with the proceeds of these securities depends on the price level. For instance, if country A receives x units of B's currency, it can acquire $\frac{x}{1+\pi^B}$ goods from B. Put differently, a purchase of x nominal securities (claims on x units of B's currency) is equivalent to a purchase of $\frac{x}{1+\pi^B}$ real securities (claims on $\frac{x}{1+\pi^B}$ consumption goods from B).

When writing down country A's optimization problem, we can save on notation in two ways. First, note that in state $s = 1$ country A is a supplier of securities, whereas in state $s = 2$ it is a demander. This implies that in state $s = 1$ securities are denominated in A's currency (and thus deflated by the price level in A); whereas in state $s = 2$ they are denominated in B's currency (and deflated by the price level in B). Second, note that the incentive to create inflation only exists when a country experiences a positive shock. Indeed, creating inflation when hit by a negative shock does not make sense, because it worsens the country's terms of trade in securities. For instance, if central bank A creates inflation when A experiences a negative shock, it pushes up the demand (and thus the price) for securities in that state. Since A is a demander of those securities, welfare decreases. This implies that in state $s = 1$ only central bank A may have an incentive to create inflation, whereas in state $s = 2$ only central bank B may wish to do so.

The maximization problem of country A's representative agent can thus be written

[7] To simplify, we will often refer to the "representative agent of country A " as "country A".

as:

$$\max_{x_1^A, x_2^A} U^A = \frac{1}{2}\frac{((1+\alpha)(1-\gamma\pi_1^A) + \frac{x_1^A}{1+\pi_1^A})^{1-\rho}}{1-\rho} + \frac{1}{2}\frac{(1-\alpha+\frac{x_2^A}{1+\pi_2^B})^{1-\rho}}{1-\rho} \tag{6}$$

$$\text{s.t.} \quad \frac{x_1^A}{1+\pi_1^A} + p\frac{x_2^A}{1+\pi_2^B} = 0$$

where p is the price of real securities in state $s = 2$ relative to the price of real securities in state $s = 1$. In other words, p represents the number of consumption goods A gives to B in state $s = 1$, in return for each consumption good A receives from B in state $s = 2$. We therefore refer to p as country B's terms of trade in real securities (and to $1/p$ as country A's terms of trade in real securities).

One may have found it more natural to talk about the relative price of nominal — rather than real — securities in the constraint of maximization problem (6); after all, claims are eventually redeemed for money. However, since agents know the level of future inflation in each state of nature, agents also know the real value of any nominal security, so that there is no problem in writing the constraint in (6) in terms of real securities. This approach has the advantage of allowing us to interpret the relative price of real securities as the terms of trade, a concept which will come in handy once we analyze optimal monetary policy. In the same way, country B's maximization problem can be written as:

$$\max_{x_1^B, x_2^B} U^B = \frac{1}{2}\frac{(1-\alpha+\frac{x_1^B}{1+\pi_1^A})^{1-\rho}}{1-\rho} + \frac{1}{2}\frac{((1+\alpha)(1-\gamma\pi_2^B) + \frac{x_2^B}{1+\pi_2^B})^{1-\rho}}{1-\rho} \tag{7}$$

$$\text{s.t.} \quad \frac{x_1^B}{1+\pi_1^A} + p\frac{x_2^B}{1+\pi_2^B} = 0$$

Finally, the market clearing conditions require that:

$$x_s^B = -x_s^A \qquad \text{for } s = \{1, 2\} \tag{8}$$

Solving both maximization problems (6) and (7), and taking into account the market clearing conditions (8), give us the equilibrium quantities of real securities demanded and supplied:

$$\frac{x_1^A(\pi_1^A, \pi_2^B, \widehat{p})}{1+\pi_1^A} = \frac{[(1-\alpha)\widehat{p} - (1+\alpha)(1-\gamma\pi_1^A)\widehat{p}^{\frac{\rho-1}{\rho}}]}{\widehat{p}^{\frac{\rho-1}{\rho}} + 1} \tag{9}$$

$$\frac{x_2^A(\pi_1^A,\pi_2^B,\widehat{p})}{1+\pi_2^B} = \frac{[(1+\alpha)(1-\gamma\pi_1^A)\widehat{p}^{-\frac{1}{\rho}} - (1-\alpha)]}{\widehat{p}^{\frac{\rho-1}{\rho}}+1} \tag{10}$$

$$\frac{x_1^B(\pi_1^A,\pi_2^B,\widehat{p})}{1+\pi_1^A} = \frac{[(1+\alpha)(1-\gamma\pi_2^B)\widehat{p} - (1-\alpha)\widehat{p}^{\frac{\rho-1}{\rho}}]}{\widehat{p}^{\frac{\rho-1}{\rho}}+1} \tag{11}$$

$$\frac{x_2^B(\pi_1^A,\pi_2^B,\widehat{p})}{1+\pi_2^B} = \frac{[(1-\alpha)\widehat{p}^{-\frac{1}{\rho}} - (1+\alpha)(1-\gamma\pi_2^B)]}{\widehat{p}^{\frac{\rho-1}{\rho}}+1} \tag{12}$$

Again, there is no problem in considering the equilibrium quantities of real — rather than nominal — securities. Although we are assuming securities to be nominal, agents know their real value, because they know future contingent inflation. Note that the equilibrium price $\widehat{p}$ in (9)-(12) is itself a function of π_1^A and π_2^B:

$$\widehat{p} = (\frac{(1+\alpha)(1-\gamma\pi_1^A)+(1-\alpha)}{(1+\alpha)(1-\gamma\pi_2^B)+(1-\alpha)})^{\rho} \tag{13}$$

Substituting (13) into (9)-(12) therefore allows us to write the equilibrium quantities of real securities in function of the inflation rates: $\frac{\widehat{x}_1^A(\pi_1^A,\pi_2^B)}{1+\pi_1^A}$, $\frac{\widehat{x}_2^A(\pi_1^A,\pi_2^B)}{1+\pi_2^B}$, $\frac{\widehat{x}_1^B(\pi_1^A,\pi_2^B)}{1+\pi_1^A}$, and $\frac{\widehat{x}_2^B(\pi_1^A,\pi_2^B)}{1+\pi_2^B}$.

Each central bank now chooses the monetary policy that maximizes domestic welfare, taking as given the equilibrium quantities and prices (9)-(13), as well as the monetary policy of the other central bank. This gives us the following two maximization problems:

$$\begin{aligned} \max_{\pi_1^A} W^A &= \frac{1}{2}\frac{((1+\alpha)(1-\gamma\pi_1^A)+\frac{\widehat{x}_1^A(\pi_1^A,\pi_2^B)}{1+\pi_1^A})^{1-\rho}}{1-\rho} + \frac{1}{2}\frac{(1-\alpha+\frac{\widehat{x}_2^A(\pi_1^A,\pi_2^B)}{1+\pi_2^B})^{1-\rho}}{1-\rho} \\ \text{s.t.} \quad & 0 \le \pi_1^A \le 1 \end{aligned} \tag{14}$$

and

$$\begin{aligned} \max_{\pi_2^B} W^B &= \frac{1}{2}\frac{(1-\alpha+\frac{\widehat{x}_1^B(\pi_1^A,\pi_2^B)}{1+\pi_1^A})^{1-\rho}}{1-\rho} + \frac{1}{2}\frac{((1+\alpha)(1-\pi_2^B)+\frac{\widehat{x}_2^B(\pi_1^A,\pi_2^B)}{1+\pi_2^B})^{1-\rho}}{1-\rho} \\ \text{s.t.} \quad & 0 \le \pi_2^B \le 1 \end{aligned} \tag{15}$$

Note that we are restricting inflation to be weakly positive.[8] Moreover, if central banks create inflation to buy up part of domestic output, they can of course not buy more than

[8] If we assume menu costs, it is straightforward to allow for welfare decreasing deflation. In that case, the incentive on the part of central banks to create inflation becomes more generally an incentive to create monetary instability.

what their country produces, so that inflation must be smaller than 1.[9] The first order conditions to problems (14) and (15) give the reaction functions of the central banks. The intersection of both reaction functions defines the Nash equilibrium of this game, and gives us the equilibrium inflation rates.

Before entering into further analytical details, note that monetary policy (inflation) affects the demand and the supply — and thus the price — of securities. Central banks can therefore 'use' monetary policy to improve the terms of trade of securities. For instance, if country A experiences a positive shock, its central bank may have an incentive to create inflation: although the direct effect is a drop in domestic consumption, the indirect effect is that the lower consumption reduces the supply of real securities, thus pushing up their relative price. In other words, by creating inflation, the central bank reduces the magnitude of the positive shock, making agents in A less willing to supply claims on their (now smaller) production. This leads to an increase in the relative price of those securities.

If this indirect (positive) effect — an improvement in the terms of trade — dominates the direct (negative) effect — a drop in consumption —, central bank A optimally chooses to create inflation. Of course, if the other country behaves in a similar way whenever it is hit by a positive shock, the effect on the relative price of real securities will wash out, and we get a suboptimal outcome: both countries suffer the welfare losses of inflation, and neither benefits from an improvement in the terms of trade of its securities.

The following proposition gives the optimal levels of inflation when a country experiences a positive shock.

Proposition 1. *If countries are perfectly symmetric, the non-cooperative Nash equilibrium corresponding to the solution to (14) and (15) gives the following levels of inflation:*

$$\widehat{\pi}_1^A = \begin{cases} \frac{2\alpha\rho-4}{(1+\alpha)\gamma(\rho-2)} & \text{if } \alpha\rho > 2 \\ 0 & \text{else} \end{cases} \tag{16}$$

[9] In reality, a central bank is never able to buy up all of the country's production as long as the population holds part of the money supply and inflation is finite. As discussed before, for an inflation of π a central bank can buy a proportion $\pi/(1+\pi)$ of the country's output. In spite of this, we still need to restrict inflation to be smaller than 1 in optimization problems (14) and (15) because we are approximating $\pi/(1+\pi)$ by π. Although this linear approximation is only reasonable for small values of inflation, we stick with it for reasons of algebraic simplicity and to keep the expression equivalent to that of menu costs.

and

$$\widehat{\pi}_2^B = \begin{cases} \frac{2\alpha\rho-4}{(1+\alpha)\gamma(\rho-2)} & \textit{if } \alpha\rho > 2 \\ 0 & \textit{else} \end{cases} \tag{17}$$

Proof. See Appendix A.1.

In other words, in order for the positive terms of trade effect to compensate the negative welfare effect from inflation, the coefficient of relative risk aversion ρ and/or the size of the supply shock α need to be sufficiently large.

This makes sense. The stronger risk aversion, the greater the positive effect on the terms of trade: if risk aversion is high, the demand for securities is relatively inelastic, so that the drop in supply has a large positive effect on the terms of trade of the country creating inflation. As for the supply shock, if its size is small, the scope for using inflation is limited. As we have argued, the welfare cost of inflation is equivalent to a reduction in output. Therefore, if the positive supply shock is small, the country cannot afford to have output drop too much; this limits the optimal level of inflation.

Whether the condition for positive inflation — i.e., $\alpha\rho > 2$ — is empirically relevant depends essentially on whether we are willing to accept a relatively high level of relative risk aversion: for instance, $\rho = 40$ and $\alpha = 0.05$ will do the job; so will $\rho = 20$ and $\alpha = 0.1$. Though such high degrees of risk aversion may seem unacceptable, they should not necessarily be ruled out.[10] In any event, the goal of this paper is limited to pointing out a novel mechanism; to get a feel of whether this mechanism is empirically relevant, surely a more complete model of the economy would be needed.

Proposition 2 provides further insights into the optimal level of inflation by considering the following comparative statics results:

Proposition 2. *If countries are perfectly symmetric and if $\alpha\rho > 2$, then the following comparative statics results hold for the optimal level of inflation in country A and B:*

[10] Although Mehra and Prescott (1985) consider a ρ of 10 to be the maximum acceptable level, Kandel and Stambaugh (1991) argue that a ρ as high as 30 should not be ruled out as unrealistic. Moreover, it is well known that much higher values of ρ are needed to explain the equity premium puzzle: as pointed out by Mankiw and Zeldes (1991), the equity premium for postwar data requires a ρ of 100.

1. $\frac{\partial \widehat{\pi}_1^A}{\partial \rho} = \frac{(1+\alpha)(4(1-\alpha))}{((1+\alpha)\gamma(\rho-2))^2} > 0$ and $\frac{\partial \widehat{\pi}_2^B}{\partial \rho} = \frac{(1+\alpha)(4(1-\alpha))}{((1+\alpha)(\rho-2))^2} > 0$

2. $\frac{\partial \widehat{\pi}_1^A}{\partial \alpha} = \frac{(\rho-2)(2\rho+4)}{((1+\alpha)\gamma(\rho-2))^2} > 0$ and $\frac{\partial \widehat{\pi}_2^B}{\partial \alpha} = \frac{(\rho-2)(2\rho+4)}{((1+\alpha)(\rho-2))^2} > 0$

Proof. Left to the reader.

These results are related to what we said before. On the one hand, as the coefficient of relative risk aversion increases, the demand for securities becomes more inelastic, thus strengthening the incentive to raise inflation. On the other hand, the smaller the supply shock, the smaller the scope to use inflation to affect security prices.

The following proposition summarizes what happens to risk sharing and welfare when the optimal level of inflation is positive:

Proposition 3. *If countries are perfectly symmetric and $\alpha\rho > 2$ then:*

1. *Compared to a situation of price stability, the equilibrium relative price of real securities $\widehat{p}$ remains unchanged at 1.*

2. *Although risk sharing is complete, it is not efficient: welfare in both countries is lower compared to a situation of price stability.*

Proof. See Appendix A.2.

These results are intuitive. If countries are perfectly symmetric, their attempts at using inflation to improve the terms of trade of their securities fail, as they cancel each other out completely. Moreover, since in each state of nature one of the two countries creates inflation, and since that level of inflation is identical, aggregate production *net* of the losses from inflation remains constant across states of nature. This implies that risk sharing continues to be complete: each country fully smooths consumption. However, risk sharing ceases to be efficient: creating inflation is equivalent to cutting back production. This makes both countries worse off compared to a situation of price stability, as both suffer the negative welfare effects of inflation, while neither benefits from an improvement in their terms of trade.

Before talking about the need for monetary cooperation, we briefly discuss if and how our results change when we move away from our perfectly symmetric setup. There are of course many forms of asymmetries we might want to consider; we will focus on differences in country size.[11] More specifically, assume the population of A increases to $(1+\sigma)$, without affecting per capita output, so that the representative agent in A still produces $(1+\alpha)$ if $s=1$, and $(1-\alpha)$ if $s=2$.[12] The maximization problems of the representative agents of A and B — expressions (6) and (7) — are therefore unchanged. The market clearing condition (8), however, now looks different as a consequence of A's greater population:

$$x_s^B = -(1+\sigma)x_s^A \qquad \text{for } s=\{1,2\} \tag{18}$$

This, in turn, affects the expression of the equilibrium price of securities (13):

$$\widehat{p} = \left(\frac{(1+\alpha)(1+\sigma)(1-\gamma\pi_1^A)+(1-\alpha)}{(1+\alpha)(1-\gamma\pi_2^B)+(1-\alpha)(1+\sigma)}\right)^{\rho} \tag{19}$$

To see what happens when the size of A increases, we use a numerical example. As parameter values, we set the coefficient of relative risk aversion ρ equal to 25; we choose a supply shock α of 0.1; and we set $\gamma=0.5$.[13] Fig. 1 and 2 plot the equilibrium inflation rates for countries A (when $s=1$) and B (when $s=2$) in function of $1+\sigma$. The basic mechanism continues to apply: to improve the terms of trade of their securities, central banks may have an incentive to create inflation whenever their country is faced with a positive shock. However, as soon as we introduce size asymmetries, the optimal level of inflation starts to differ across countries. More particularly, the greater the size of A, the higher inflation in A, and the lower inflation in B. The intuition is the following. Because

[11] Other types of asymmetries, such as differences in the probabilities of receiving a positive shock, are discussed in Appendix B.

[12] Note that this setup introduces aggregate uncertainty: when the larger country receives the positive shock (in state 1) aggregate output is greater than when the smaller country gets the positive shock (in state 2). Appendix B.2 discusses size asymmetries between countries that do not lead to aggregate uncertainty.

[13] If the cost of inflation takes the form of inefficient government spending, $\gamma=0.5$ corresponds to the government throwing away half of what it buys (and redistributing the rest back to the population). If the cost of inflation takes the form of menu costs, we would probably want a lower value of γ. That simply increases the optimal inflation rate: dividing γ by x corresponds to multiplying the optimal inflation rate by x.

of its greater size, A finds it easier to manipulate its terms of trade: a given increase in inflation in A has a larger effect on relative prices than a same increase in inflation in B. In other words, for a same marginal cost in terms of output per capita, the marginal benefit of inflation is bigger in the larger country, so that in equilibrium the larger country distorts more.

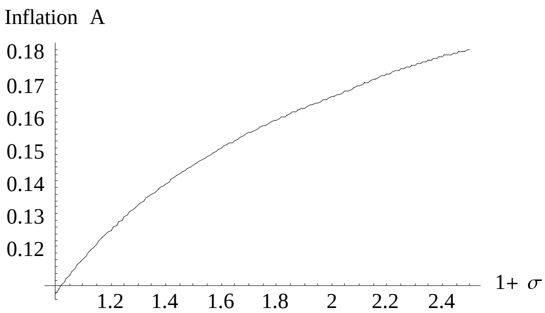

Fig. 1: Inflation in A.

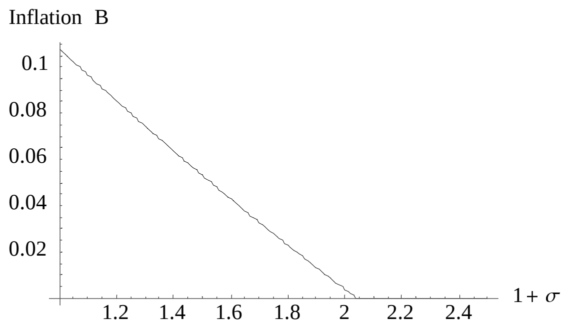

Fig. 2: Inflation in B.

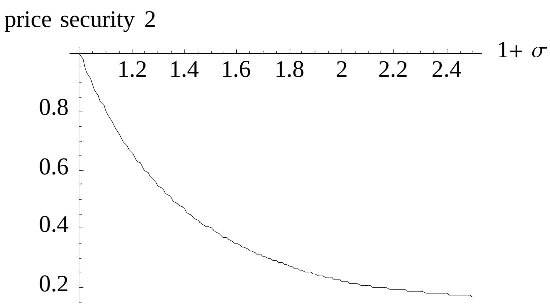

Fig. 3: Relative price security 2.

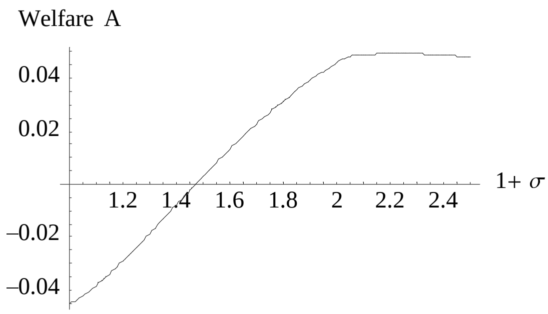

Fig. 4: Welfare in A.

Fig. 1 through 4: Asymmetries in country size ($1 + \sigma$ is the relative size of A).

Although inflation in A and B continue to have compensating effects on the relative price of securities, they cease to cancel out completely. Because A now has a greater distortive capacity than B, its terms of trade actually improve as the country grows in size. Fig. 3 illustrates this point by plotting p in the non-cooperative outcome, relative to p in the absence of distortions: a ratio below 1 indicates an improvement in A's terms of trade. Since A experiences an increase in its terms of trade as the country grows in size, it is not clear anymore that it becomes worse off compared to the benchmark case of price stability. If the gains from the improvement in the terms of trade more than compensate the losses from inflation, A may actually become better off. Fig. 4 confirms this argument by plotting the change in A's welfare when moving from price stability to the non-cooperative Nash solution: when A becomes sufficiently large, its welfare improves. This suggests that large countries may be less keen on monetary cooperation.

The result that larger countries distort more will be mitigated — or even disappear — if we incorporate the stylized fact that larger countries experience proportionally smaller shocks (Head, 1995). Although it is easier for a larger country to distort prices, the smaller shocks give it less scope to do so. Appendix B.2 discusses this point in further detail.

4 Monetary Cooperation

As stated in Proposition 2, a system with two separate central banks setting monetary policy in a non-cooperative manner leads to a suboptimal outcome if countries are perfectly symmetric: positive inflation decreases welfare in both countries, without improving their terms of trade. In this section we will show that monetary cooperation brings us back to the optimal solution.

Cooperation can take on different forms: separate central banks may continue to exist, with the difference that they now (explicitly or implicitly) coordinate their monetary policies; or a common central bank may emerge that sets a common monetary policy for both regions. Which form of cooperation is implemented may have to do with enforcement issues. As is well known, in a repeated game setting cooperation can be sustained without the need for an institutional framework, as long as the punishment for deviating is sufficiently harsh. However, if for whatever reason such an arrangement is not possible, formal cooperation, or even a monetary union, may well be the answer.

We start by looking at the case of monetary cooperation between separate central banks; we will later say something more about monetary unions. By cooperation we mean that each country's monetary policy is now the outcome of a bargaining process between both central banks. Following the axiomatic bargaining approach, we impose individual rationality: any outcome must be welfare-improving for both countries, compared to the non-cooperative solution. If not, countries would have no interest in cooperating. For the case of Nash bargaining, the outcome will be monetary stability:

Proposition 4. *If countries are perfectly symmetric, the solution to the Nash bargaining process between both central banks prescribes zero inflation in both states of nature.*

Proof. See Appendix A.3.

The intuition is straightforward. Given the perfectly symmetric setup, the solution must also be perfectly symmetric: in other words, it must satisfy the condition $\tilde{\pi}_1^A = \tilde{\pi}_2^B$. Following (13), this implies that the relative price of securities remains unchanged (and equal to 1), independently of the level of inflation. Given that neither country benefits from an improvement in its terms of trade, it follows that the unique Pareto optimal solution is to have zero inflation in both states of nature.

We now turn to the case of a monetary union, where representatives from both countries decide on a common monetary policy through bargaining. The difference with simple cooperation is that inflation is now set at the level of the union. How union-wide inflation affects welfare in both countries depends on how we interpret the cost of inflation. In the case of menu costs, the effect of inflation is equivalent to a proportional drop in output in both countries by $\gamma\pi$. In the case of the central bank throwing away part of output, the overall loss will still be a proportional decrease in aggregate output by $\gamma\pi$, but the central bank can now choose how to divide that loss between both countries.[14] Either way, the Nash bargaining solution in a monetary union also leads to price stability:

Proposition 5. *If countries are perfectly symmetric, the solution to the Nash bargaining process between representatives of both countries in a monetary union gives us zero inflation in both states of nature.*

Proof. See Appendix A.4.

To see this, it suffices to show that no matter how the cost of inflation is divided between countries, the relative price of securities remains equal to 1. The rest of the argument

[14] Depending on where the central bank decides to buy up output, it also decides how it allocates the cost of inflation.

is identical to that of Proposition 4: since once again neither country benefits from an improvement in its terms of trade, the Pareto optimal solution is zero inflation.

We now discuss how our problem changes when we deviate from perfect symmetry. We again start by focusing on the case of monetary cooperation between central banks. Even though analytically solving for the Nash bargaining solution is too complex, we can show that monetary cooperation will generally lead to a Pareto superior outcome:

Proposition 6. *If countries are asymmetric and if in the non-cooperative equilibrium inflation is strictly positive in both countries ($\widehat{\pi}_1^A > 0$ and $\widehat{\pi}_2^B > 0$), monetary cooperation between central banks allows for a Pareto superior outcome.*

Proof. See Appendix A.5.

Again, this result is quite easy to understand. Assume the non-cooperative outcome leads to strictly positive inflation whenever a country receives a positive shock: $\widehat{\pi}_1^A > 0$ and $\widehat{\pi}_2^B > 0$. Following expression (19), these inflation rates give us a relative price of real securities in the non-cooperative game equal to $\widehat{p}$. Since both $\widehat{\pi}_1^A$ and $\widehat{\pi}_2^B$ are strictly positive, it is clear that we can obtain the same relative price $\widehat{p}$ for lower values of inflation $\tilde{\pi}_1^A$ and $\tilde{\pi}_2^B$, so that welfare improves in both countries. In other words, cooperation can lead to an outcome which is Pareto superior to the non-cooperative solution.

Looking at our numerical example of asymmetric country sizes, Fig. 1 and Fig. 2 show that optimal inflation remains strictly positive in both countries as long as A does not become too big, so that according to Proposition 6 it pays off to cooperate. However, if A continues to grow in size, optimal inflation in B goes to zero, but remains positive in A. This causes the argument in our proof to break down: we cannot anymore lower inflation in both countries, while leaving the relative price of securities unchanged, because inflation in the smaller country is already zero. Under those circumstances it is not clear whether the larger country would still have an interest in cooperating through simple bargaining. Of course this picture would change if we allowed for non-distortionary side payments. In that case monetary stability would always be optimal, since it maximizes aggregate consumption. It would then suffice to bargain over the size of non-distortionary transfers to ensure that both parties gain.

Before concluding, we analyze the case of a monetary union when we deviate from perfect symmetry. If the cost of inflation takes the form of the central bank throwing away part of output, all results go through. If, on the contrary, we prefer the menu cost interpretation, this is not obvious anymore. In a monetary union, menu costs benefit from one less degree of freedom, because the common central bank loses the capacity to allocate the cost of union-wide inflation between the two countries. The following proposition therefore only refers to the interpretation of the central bank throwing output away:

Proposition 7. *If countries are asymmetric and if in the non-cooperative equilibrium inflation is strictly positive in both countries ($\widehat{\pi}_1^A > 0$ and $\widehat{\pi}_2^B > 0$), a monetary union allows for a Pareto superior outcome, as long as the central bank can decide how to allocate the cost of inflation between countries.*

Proof. See Appendix A.6.

The proof essentially shows that the Pareto superior solution of Proposition 6 can be replicated for the case of a monetary union.

5 Concluding Remarks

If countries are subject to asymmetric shocks, the absence of monetary cooperation may lead to a suboptimal outcome. This happens even if central banks can commit in advance to the monetary policy that maximizes social welfare. The reason is that central banks may want to use inflation to affect the terms of trade in the risk sharing market. However, given that both central banks have the same incentive, the terms of trade effect washes out, and both countries end up worse off. To get rid of this externality problem, some form of monetary cooperation is necessary: if cooperation between separate central banks is difficult, forming a monetary union may be the way out.

Contrary to the conventional wisdom of Mundell (1961), this paper therefore makes a case for countries facing asymmetric shocks to adopt a common currency. By doing so, it revives the Mundell (1973) argument in favor of monetary unions. Note that by giving central banks the capacity to commit, we are biasing our results against Mundell (1973). Even so, the "latter Mundell" proves to be alive and well.

One last comment: in our model optimal monetary policy prescribes creating inflation in booms and keeping prices stable in recessions. While this fits the empirical regularity of pro-cyclical inflation, it goes against the the standard textbook recommendation of contractionary monetary policy during booms and expansionary monetary policy during recessions. This discrepancy is not surprising. To isolate the specific mechanism of interest, we switched off any other link between inflation and output. In a more fully specified model our policy recommendation could thus be rephrased as making monetary policy less contractionary during booms and less expansionary during recessions, compared to standard practice.

A Proofs of Propositions

A.1 Proof of Proposition 1

The equilibrium is defined as a solution to (14) and (15), i.e., a combination of monetary policies $(\widehat{\pi}_1^A, \widehat{\pi}_2^B)$, from which neither central bank has an incentive to deviate. Since the problem is perfectly symmetric, any solution must satisfy $\widehat{\pi}_1^A = \widehat{\pi}_2^B$. Interior solutions are characterized by the following first order and second order conditions: $\frac{\partial W^A}{\partial \pi_1^A} = 0$, $\frac{\partial^2 W^A}{\partial (\pi_1^A)^2} < 0$, $\frac{\partial W^B}{\partial \pi_2^B} = 0$, and $\frac{\partial^2 W^B}{\partial (\pi_2^B)^2} < 0$. Corner solutions can arise in two cases: on the one hand, monetary policies $(\widehat{\pi}_1^A, \widehat{\pi}_2^B) = (0,0)$ constitute an equilibrium if at that point $\frac{\partial W^A}{\partial \pi_1^A} < 0$ and $\frac{\partial W^B}{\partial \pi_2^B} < 0$; and on the other hand, monetary policies $(\widehat{\pi}_1^A, \widehat{\pi}_2^B) = (1,1)$ form an equilibrium if at that point $\frac{\partial W^A}{\partial \pi_1^A} > 0$ and $\frac{\partial W^B}{\partial \pi_2^B} > 0$. We now distinguish between the following cases:

1. Case 1: $\alpha\rho \geq 2$.

   As a preliminary remark, note that since $\alpha \leq 1$, this condition implies that $\rho > 2$. The first order condition of maximization problem (14) of central bank A is:

$$\frac{\partial W_1^A}{\partial \pi_1^A}|(\pi_1^A = \pi_2^B) = -2^{\rho-2}(1+\alpha)\gamma \frac{4 + (1+\alpha)\gamma\widehat{\pi}_1^A(\rho-2) - 2\alpha\rho}{(2-(1+\alpha)\gamma\widehat{\pi}_1^A)^{1+\rho}} = 0 \qquad (20)$$

Solving out for $\widehat{\pi}_1^A$ gives us:

$$\widehat{\pi}_1^A = \frac{2\alpha\rho - 4}{(1+\alpha)\gamma(\rho-2)} \tag{21}$$

Note that this solution is within the acceptable range: $0 \leq \widehat{\pi}_1^A \leq 1$. To check whether (21) corresponds to a local maximum, we check the second order condition:

$$\frac{\partial^2 W^A}{\partial(\pi_1^A)^2}|(\pi_1^A = \widehat{\pi}_1^A = \pi_2^B) = -0.1875(1+\alpha)^2\gamma^2(\rho-4/3)(\frac{\rho-2}{(1-\alpha)\rho})^{1+\rho} < 0 \tag{22}$$

Given that $\rho > 2$, the second order condition is negative, so that we are indeed in presence of a local maximum. The monetary policies $(\widehat{\pi}_1^A, \widehat{\pi}_2^B) = (\frac{2\alpha\rho-4}{(1+\alpha)\gamma(\rho-2)}, \frac{2\alpha\rho-4}{(1+\alpha)\gamma(\rho-2)})$ therefore satisfy the equilibrium conditions. It can easily be checked that there are no corner solutions.

2. Case 2: $\alpha\rho < 2$ and $\rho > 2$.

Taking the first order condition of maximization problem (14), and solving out for $\widehat{\pi}_1^A$ gives us the same expression as in (21). Note, however, that in this case $\widehat{\pi}_1^A < 0$, so that the solution is outside the acceptable range. This leaves us with possible corner solutions. For monetary policies $(\pi_1^A, \pi_2^B) = (0,0)$ we have:

$$\frac{\partial W^A}{\partial \pi_1^A} = -2^{\rho-2}(1+\alpha)\gamma\frac{4-2\alpha\rho}{2^{1+\rho}} < 0 \tag{23}$$

By symmetry, $\frac{\partial W^B}{\partial \pi_2^B} < 0$, so that monetary policies $(\widehat{\pi}_1^A, \widehat{\pi}_2^B) = (0,0)$ constitute an equilibrium. It can easily be checked that there do not exist other corner solutions.

3. Case 3: $\alpha\rho < 2$ and $\rho \leq 2$.

Again, the first order condition of maximization problem (14) is given by (20). It is clear that the corresponding value $\widehat{\pi}_1^A$ is outside the allowed range: if $\rho = 2$, $\widehat{\pi}_1^A$ would be minus infinity; if $\rho < 2$, $\widehat{\pi}_1^A = \frac{2\alpha\rho-4}{(1+\alpha)\gamma(\rho-2)}$, which is greater than 1. This again leaves us with possible corner solutions. By analogy with Case 2, it can easily be shown that monetary policies $(\widehat{\pi}_1^A, \widehat{\pi}_2^B) = (0,0)$ constitute an equilibrium, and that there are no other corner solutions. ∎

A.2 Proof of Proposition 3

1. Given the problem is perfectly symmetric, we know that $\widehat{\pi}_1^A = \widehat{\pi}_2^B$, so that according to (13) the equilibrium price $\widehat{p} = 1$.

2. In state $s = 1$ aggregate production (of both regions together) *net* of inflation is $(1+\alpha)(1-\gamma\widehat{\pi}_1^A) + (1-\alpha)$; similarly, in state $s = 2$ aggregate production *net* of inflation is $(1-\alpha) + (1+\alpha)(1-\gamma\widehat{\pi}_2^B)$. Since $\widehat{\pi}_1^A = \widehat{\pi}_2^B$, aggregate production *net* of inflation is constant across states. This implies complete risk sharing; in each country consumption is fully smoothed across states. Given that countries are completely symmetric, each country will consume exactly half of aggregate production *net* of inflation. Since $\alpha\rho > 2$, inflation is strictly positive, so that consumption will be lower compared to the case of price stability. ∎

A.3 Proof of Proposition 4

The Nash bargaining solution maximizes

$$(U^A(\pi_1^A, \pi_2^B) - \widehat{U}^A)(U^B(\pi_1^A, \pi_2^B) - \widehat{U}^B) \tag{24}$$

with respect to π_1^A and π_2^B, where $\widehat{U}^A$ and $\widehat{U}^B$ refer to the utilities of the non-cooperative Nash equilibrium. Given complete symmetry, the two first order conditions corresponding to the above maximization problem will also be completely symmetric, so that the equilibrium levels of inflation corresponding to the Nash bargaining solution will be identical: $\tilde{\pi}_1^A = \tilde{\pi}_2^B$. This implies that the equilibrium relative price of securities will always be equal to 1, independently of the level of inflation. Since inflation does not affect the relative price of securities, the maximization problem boils down to maximizing production net of inflation. Not surprisingly, this implies setting inflation equal to zero in both countries: $\tilde{\pi}_1^A = \tilde{\pi}_2^B = 0$. ∎

A.4 Proof of Proposition 5

If the welfare loss from inflation is due to throwing away part of output, in a monetary union the central bank can decide how to divide that cost between both countries. Of the total loss in output $2\gamma\pi$, it allocates a share δ to the country experiencing the positive shock, and a share $1-\delta$ to the other. (Given the perfectly symmetric setup, δ does not depend on which region gets the positive shock.) Defining $\gamma_1 = \frac{2\delta}{1+\alpha}$ and $\gamma_2 = 2\frac{1-\delta}{1-\alpha}$, we can re-write optimization problems (6) and (7) in the following manner:

$$\max_{x_1^A, x_2^A} U^A = \frac{1}{2}\frac{((1+\alpha)(1-\gamma_1\pi_1)+\frac{x_1^A}{1+\pi_1})^{1-\rho}}{1-\rho} + \frac{1}{2}\frac{((1-\alpha)(1-\gamma_2\pi_2)+\frac{x_2^A}{1+\pi_2})^{1-\rho}}{1-\rho} \tag{25}$$

$$\text{s.t.} \quad \frac{x_1^A}{1+\pi_1} + p\frac{x_2^A}{1+\pi_2} = 0$$

and

$$\max_{x_1^B, x_2^B} U^B = \frac{1}{2}\frac{((1-\alpha)(1-\gamma_2\pi_1)+\frac{x_1^B}{1+\pi_1})^{1-\rho}}{1-\rho} + \frac{1}{2}\frac{((1+\alpha)(1-\gamma_1\pi_2)+\frac{x_2^B}{1+\pi_2})^{1-\rho}}{1-\rho} \tag{26}$$

$$\text{s.t.} \quad \frac{x_1^B}{1+\pi_1} + p\frac{x_2^B}{1+\pi_2} = 0$$

Solving these maximization problems, while taking into account the market clearing conditions (8), gives us the equilibrium price level:

$$\tilde{p} = (\frac{(1+\alpha)(1-\gamma_1\pi_1)+(1-\alpha)(1-\gamma_2\pi_1)}{(1+\alpha)(1-\gamma_1\pi_2)+(1-\alpha)(1-\gamma_2\pi_2)})^\rho \tag{27}$$

The value that δ takes (and, therefore, also the values that γ_1 and γ_2 take) does not change the perfectly symmetric nature of the problem. Following the argument in Proposition 4, the equilibrium levels of inflation must therefore be identical: $\tilde{\pi}_1 = \tilde{\pi}_2$. This implies that $\tilde{p}$ will be equal to 1, independently of the values of δ, γ_1 and γ_2. The rest of the proof follows by analogy with Proposition 4. Showing the same result if we focus on menu costs is even easier, since in that case $\gamma_1 = \gamma_2 = \gamma$. ∎

A.5 Proof of Proposition 6

We prove the proposition for the case of asymmetries in country sizes; other types of asymmetries can be dealt with in an analogous manner. Following the notation of Section 3, (19) gives the equilibrium price of the non-cooperative equilibrium:

$$\widehat{p} = (\frac{(1+\alpha)(1+\sigma)(1-\gamma\pi_1^A)+(1-\alpha)}{(1+\alpha)(1-\gamma\pi_2^B)+(1-\alpha)(1+\sigma)})^\rho \tag{28}$$

where $\widehat{\pi}_1^A > 0$ and $\widehat{\pi}_2^B > 0$. In that case it is obvious that we can lower inflation in both countries to $\tilde{\pi}_1^A < \widehat{\pi}_1^A$ and $\tilde{\pi}_2^B < \widehat{\pi}_2^B$, while maintaining the value of the relative price (28). If inflation is lower in both countries, and the relative price of securities is unchanged, welfare in both countries has increased. ∎

A.6 Proof of Proposition 7

The starting point (the non-cooperative solution) is equal to that in Proposition 6. It is easy to show that the Pareto superior outcome in Proposition 6 can be replicated in the framework of a monetary union. Using the same notation as in Proposition 5, it suffices to set $\delta = 1$, $\pi_1^* = \frac{\tilde{\pi}_1^A}{\gamma_1}$ and $\pi_2^* = \frac{\tilde{\pi}_2^B}{\gamma_1}$, where π_1^* and π_2^* refer to the union-wide inflation rates in state 1 and state 2, and $\tilde{\pi}_1^A$ and $\tilde{\pi}_2^B$ refer to the inflation rates under monetary cooperation in Proposition 6. ∎

B Different types of asymmetries

B.1 Different probabilities of positive shocks across countries

In this section we assume that state 1 happens with probability q and state 2 with probability $1-q$. Fig. 5 and Fig. 6 plot the equilibrium inflation rates for countries A (when $s=1$) and B (when $s=2$) in function of q. As can be seen, the lower the probability of getting a positive shock, the higher the optimal inflation rate. The intuition is the following. Inflation leads to a drop in output whenever the country experiences a positive shock. Therefore, the lower the probability of receiving a positive shock, the lower the expected cost of inflation, and the higher the optimal inflation rate. The different inflation rates in A and B leads to uncertainty in aggregate output *net* of inflation losses, so that risk sharing becomes incomplete: as soon as we deviate from $q = 1/2$, there is less-than-full consumption smoothing in each country.

Moreover, the asymmetry implies that the terms of trade of one of the two countries may improve. Fig. 7 illustrates this point: it plots p in the non-cooperative outcome, relative to p in the absence of distortions. If the ratio in Fig. 7 rises above 1, B's terms of trade in securities improve; if the ratio drops below 1, A's terms of trade improve. Fig. 7

shows an improvement in A's terms of trade as q drops below $1/2$. This is consistent with our findings in Fig. 5 and 6: for q below $1/2$, inflation is higher in A than in B, so that the distortive capacity of A is greater than that of B.

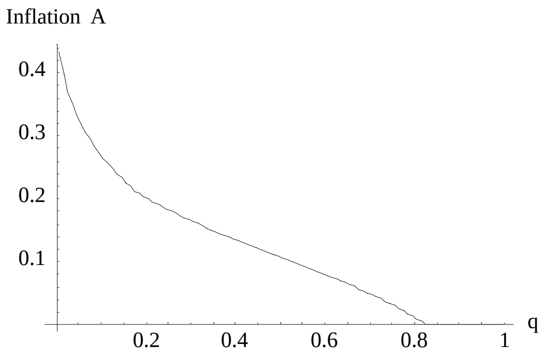

Fig. 5: Inflation in A.

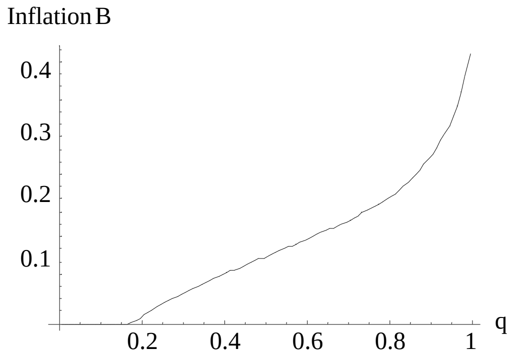

Fig. 6: Inflation in B.

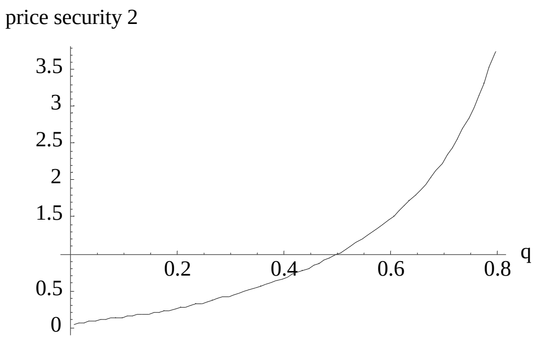

Fig. 7: Relative price security 2.

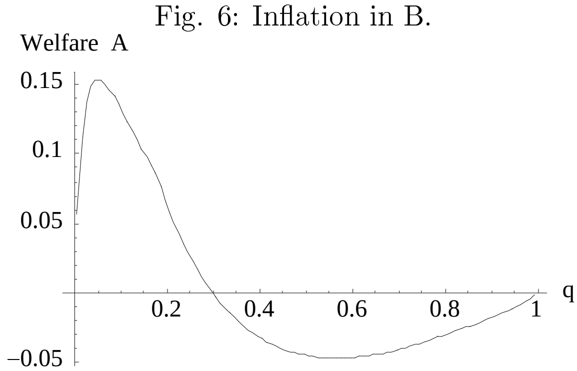

Fig. 8: Welfare in A.

Fig. 5 through 8: Asymmetry in the probability of receiving a positive shock.

Since the terms of trade of one of the two countries improves as we move away from $q = 1/2$, that country may become better off compared to the benchmark case of price stability. Fig. 8 confirms this argument by plotting the change in A's welfare when moving from price stability to the non-cooperative Nash solution: for q low enough, welfare improves. This is not surprising. As q drops below $1/2$, A's terms of trade improve, whereas the cost of inflation falls. The non-monotone shape of the welfare change has to do with the degree of uncertainty: as q approaches 0 or 1, uncertainty becomes very small, so that the gains from risk sharing — and the gains from distorting — become negligible. Our results therefore suggest that the country experiencing positive shocks with a low probability may have less to gain from monetary cooperation.

B.2 Different country sizes and different shock sizes

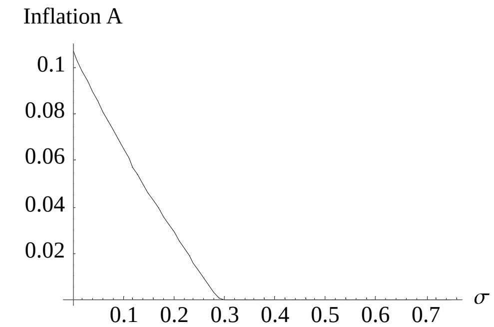

Fig. 9: Inflation A.

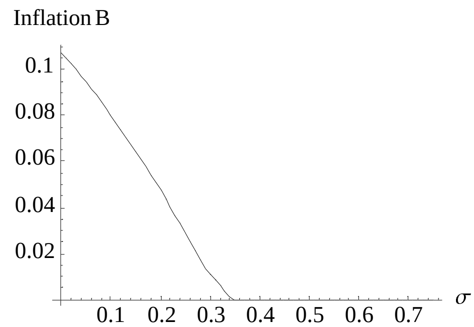

Fig. 10: Inflation B.

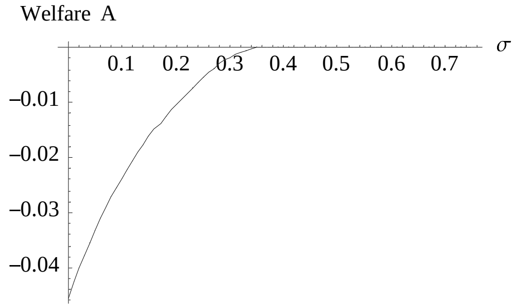

Fig. 11: Welfare A.

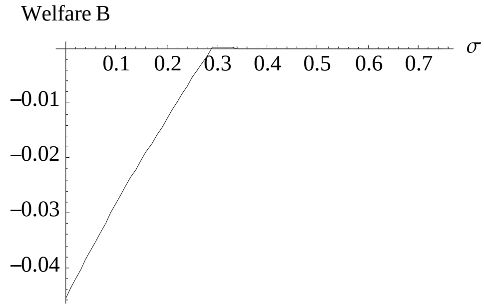

Fig. 12: Welfare B.

Fig. 9 through 11: Asymmetries in country size with bigger country receiving smaller shock ($1+\sigma$ is the size of A and $1-\sigma$ the size of B)

Rather than considering asymmetric country sizes with identical shocks, as we do in the main text, we now take into account the stylized fact that larger countries experience proportionally smaller shocks (Head, 1995). More specifically, suppose country A annexes a fraction σ of country A's population.[15] To keep things simple, assume agents in A continue to be homogeneous.[16] Per capita output in A is then $(1+\frac{1-\sigma}{1+\sigma}\alpha)$ if $s=1$, and $(1-\frac{1-\sigma}{1+\sigma}\alpha)$ if $s=2$. Contrary to the case we studied in the main text, the size of shocks in A therefore decrease with country size.

As shown in Fig. 9 and 10, the incentive to create inflation now also decreases

[15] Note that, contrary to the example in the main text, this type of size asymmetry does not introduce aggregate uncertainty. See footnote (12).

[16] This is equivalent to assuming complete redistribution across agents within countries.

in the larger country. The intuition is the following: although it is easier for the bigger country to distort prices, the smaller shock size gives it less room to do so. Since price distortions decrease with size asymmetry, so does the welfare loss due to distortions (Fig. 11 and 12).

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