MONEY IN AN ESTIMATED BUSINESS CYCLE MODEL OF THE EURO AREA

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Money in an Estimated Business Cycle Model of the Euro Area*

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Abstract

We present maximum likelihood estimates of a small scale dynamic general equilibrium model for the Eurozone. We pay special attention to the role of money, both through its direct effect upon private agents' decisions and as a component of the monetary policy rule. Our results can be summarized as follows. First, we find no direct effect of money upon inflation and output but money growth plays a significant role in the interest rate rule. Second, money demand shocks mainly help to forecast real balances while real shocks explain the bulk of price, output and interest rates fluctuations. Third, the estimated model predicts sensible conditional correlations among those variables both to demand and supply disturbances. Finally, the systematic response of interest rates to money growth does not seem to have affected the output-inflation variability trade-off.

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1. Introduction

This paper presents a maximum likelihood estimation of a small scale dynamic general equilibrium model of the business cycle for the euro area. We pay special attention to the role of money in shaping the joint evolution of output, interest rates and prices. An essential feature of this approach is that we make use of all the cross-restrictions implied by the theoretical model, which affect the dynamics of the endogenous variables and the structural sources of fluctuations. Thus, we go one step further with respect to the more conventional method of estimation, on an equation-by-equation basis, of which the *Area Wide Model* of Fagan, Henry and Mestre (2001) is the best example to date. To some extent our work is akin to that of Coenen and Wieland (2000), although these authors only focus on the inflation-output dynamics through the estimation of an overlapping wage contract à la Taylor, leaving the demand side and the money market unrestricted. A much closer approach to ours is that in Smets and Wouters (2001), who estimate a dynamic general equilibrium model to analyze the business cycle in the euro area, although with no explicit role for money.

Our model is an extension of Ireland's (2001) and can be described in terms of four building blocks. Three of these come from the optimal agent's decisions: the intertemporal allocation of consumption, the Phillips curve and a money demand equation. The model is augmented with an interest rate rule, which does not admit a truly structural interpretation (since the European Central Bank was inexistent before 1999) but that is a necessary device to allow for the existence of a unique rational expectations stationary equilibrium.

Optimal choices made by households and firms result in forward-looking demand and supply schedules, which makes it difficult to account for the inertia observed in output and prices. This issue has recently been subject to substantial empirical research from two different perspectives. On the one hand, authors like Fuhrer and Moore (1995) advocate some ad-hoc mechanisms in both demand and supply to match the persistence and the dynamic cross correlations observed in the data. Other papers, like Rotemberg and Woodford (1997, 1999), emphasize the importance of sticking to solid microfoundations to understand economic relationships, allowing for higher order autocorrelations in the unobservable as well as for cross correlations among them instead. In this paper we extend the latter approach by considering the role of habit formation in consumer preferences (Fuhrer (2000) and Boldrin, Christiano and Fisher (2000)) but we also allow for a subset of backward-looking price setters (Galí and Gertler (1999)).

There is a lively debate about the merits of the ECB's "two pillars" strategy for conducting its policy, in which monetary aggregates are given a large weight as indicators of future inflation. Several papers have used the P* model to look at this issue (see, for example, Gerlach and Svensson (2000)), while ours can be viewed as complementary to those in that the role of real balances is assessed in a model with well-rooted microfoundations. In this respect our main result is that the business cycle influence of money balances seems to have been very limited during the sample period. Preferences are separable between consumption and real balances and the policy rule, although with a significant response of the nominal rate to money growth, can be written as a flexible targeting inflation rule. We also obtain that money demand shocks explain very little of the variability of output, inflation and nominal interest rates, while account for most of the real balances fluctuations.

These results are obtained in models whose estimated parameters are in general both reasonable and similar to others available in the literature. Preferences are (near to) logarithmic in consumption, display high habit dependence and are strikingly robust across specifications. The supply side parameters differ across specifications. In models without consumption persistence the estimated Phillips is close to the one obtained by Galí, Gertler and López-Salido (2001) in a partial equilibrium framework. In particular, we find supporting evidence of an important forward-looking component in driving inflation dynamics as well as of some non-negligible price inertia. When the model allows for habit formation the estimated supply side is consistent with much less price stickiness and with a high labor-supply elasticity. Finally, the conditional correlations (impulse-responses) and the contribution of the estimated sources of fluctuations to the forecast error of the main variables are consistent with those found in the literature.

Section 2 presents the model economy. Section 3 describes the maximum likelihood estimates of alternative models. Section 4 analyses the quantitative implications of the estimated models in terms of the volatility and the persistence of the variables as well as in terms of both impulse-responses and forecast variance decompositions of the variables to structural shocks. Section 5 presents two counterfactual policy exercises to further assess the role of money in the model. Section 6 concludes.

2. Money in a Sticky Price Model

In this section we set out the basic equations of the model, whose predecessors can be found in McCallum and Nelson (1999), Woodford (1999), and more recently Ireland (2001), among others. The economy consists of a representative household, a continuum of producing firms indexed by $j \in [0,1]$ and a monetary authority. The model features enough symmetry so that the analysis can be focused on the behavior of a representative goods-producing firm.

2.1. Households

The representative household of the economy maximizes the following expected stream of utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \; a_t \left[U(C_t, \frac{M}{e_t P_t}) - \frac{N_t^{1+\varphi}}{1+\varphi} \right] \tag{2.1}$$

where C_t is the CES aggregator of the quantities of the different goods consumed:[1]

$$C_t = \left(\int_0^1 C_t(j)^{\frac{\varepsilon-1}{\varepsilon}} \; dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

M_t/P_t and N_t represent real balances and hours, respectively; a_t is a preference shock and e_t is a shock to the demand for real balances. The parameter $\beta \in (0,1)$ is a discount factor and $\varphi \geq 0$ represents the inverse of the Frisch labor supply elasticity.[2] The marginal utility of consumption depends upon real balances but it is independent of labor supply decisions. In addition, the assumption of separability between a consumption-real balances basket and hours implies that aggregate demand relationships are invariant to the specification of the firm's problem (Driscoll (2000)).

The budget constraint is:

$$\frac{M_{t-1} + B_{t-1} + W_t N_t + T_t + D_t}{P_t} = C_t + \frac{B_t/r_t + M_t}{P_t} \tag{2.2}$$

[1] $P_t = \left(\int_0^1 P_t(j)^{1-\varepsilon} \; dj \right)^{\frac{1}{1-\varepsilon}}$ is the aggregate price index that is consistent with the first order conditions of the producing firms that face the differenciated demand and $P_t(j)$ is the price of good j.

[2] When $\varphi = 0$ preferences are linear in labor (Hansen (1985)) and the labor-supply elasticity is infinite.

Households enter period t with money holdings M_{t-1} and bonds B_{t-1}. At the beginning of the period they receive lump sum nominal transfers T_t, labor income $W_t N_t$, where W_t denotes the nominal wage, and a nominal dividend D_t from the firms. They use some of these funds to purchase new bonds at nominal cost B_t/r_t, where r_t denotes the gross nominal interest rate between t and $t+1$. The household carries M_t units of money into the period $t+1$.

2.2. Firm Behavior and Price Setting

The production function for firm j is,

$$Y_t(j) = z_t N_t(j)^{1-\alpha} \tag{2.3}$$

where $Y_t(j)$ is output $N_t(j)$ represents the number of hours hired from the household (i.e., $N_t = \int_0^1 N_t(j)\, dj$), $z_t$is a common technology shock and $(1-\alpha)$ is the elasticity of labor with respect to output. Letting $Y_t = \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$ the market clearing condition implies $Y_t = C_t$.

The representative firm sells its output in a monopolistically competitive market and sets nominal prices on a staggered basis, as in Calvo (1983). Each firm resets its price with probability $1-\theta$ each period, independently of the time elapsed since the last adjustment. Thus, each period a measure $1-\theta$ of producers reset their prices, while a fraction θ simply adjust prices at the pace of steady-state inflation, π, (i.e. non-adjusting firms simply set: $P_t(j) = P_{t-1}(j)\pi$). Hence, θ^k will be the probability that the price set at time t will still hold at time $t+k$. Notice that, if there were no constraints on the adjustment of prices the typical firm would set a price according to the rule $P_t(j) = (\frac{\varepsilon}{\varepsilon-1})MC_t(j)$, where $MC_t(j)$ is the nominal marginal cost, $\frac{\varepsilon}{\varepsilon-1}$ is the steady-state price markup and

$$MC_t(j) = \frac{W_t}{\frac{\partial Y_t(j)}{\partial N_t(j)}}$$

As emphasized by Galí and Gertler (1999), this framework implies that inflation is a purely forward-looking variable. Nevertheless, recent research has pointed out the importance of allowing for a hybrid specification in which part of the inflation dynamics is explained by some backward looking component in order to account for the inertia observed in inflation time series. To formally account for this, we follow Galí and Gertler (1999) by assuming that only a fraction $(1-\omega)$ of firms behave on a staggered basis when setting prices at each point of time.

We denote by P_t^f the prices set by these forward looking firms. The remaining firms, of measure ω, use instead a simple rule of thumb (backward looking) when setting prices (P_t^b). In logs, the price index of newly set prices is:

$$p_t^* = (1-\omega)\ p_t^f + \omega\ p_t^b \tag{2.4}$$

The aggregate price level evolves as follows:

$$P_t = \left[\theta\ P_{t-1}^{1-\varepsilon}\ +\ (1-\theta)(1-\omega)\ \left(P_t^f\right)^{1-\varepsilon}\ +\ (1-\theta)\omega\ \left(P_t^b\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

and we shall further assume that the backward looking firms set their prices according to the following rule of thumb:

$$P_t^b = P_{t-1}^* \Pi_{t-1}$$

where $\Pi_{t-1} = P_{t-1}/P_{t-2}$.

2.3. Central Bank Reaction Function

Whereas the assumption of a representative agent in the private sector poses no particular problems, it is harder to think of a single monetary policy rule for the euro area, in which a variety of central bank policy rules coexisted before 1999. Still, we cannot proceed to estimate the model without an explicit rule to shape the endogenous behavior of the nominal interest rates, since otherwise a unique rational expectations equilibrium, which the maximum likelihood method pins down, would not be guaranteed.

We assume that the central bank sets the nominal interest rate following an augmented Taylor-type monetary policy rule. In particular, the nominal rate responds not only to the interest rate in the previous period and to deviations of output and inflation from their steady-state but also to *money growth*:

$$\ln(r_t/r) = \rho_r \ln(r_{t-1}/r) + (1-\rho_r)\rho_\pi \ln(\pi_t/\pi) + (1-\rho_r)\rho_y \ln(y_t/y) + (1-\rho_r)\rho_\mu \ln(\mu_t/\mu) + \varepsilon_{r_t}$$

where the innovation ε_{r_t} is normally distributed with standard deviation σ_r; and $\mu_t = M_t/M_{t-1}$ is the rate of money growth.[3]

[3] The presence of nominal money growth in the rule makes it possible to interpret the rule as a flexible monetary targeting one (see Rudebusch and Svensson (1999)).

The rationale for this rule is that, given the potential importance of money to affect the equilibrium output and price allocations, it may also have some value as a monetary policy indicator of those variables. More generally, we can test whether money plays an independent role in setting interest rates within this class of rules.

2.4. Equilibrium

The symmetric equilibrium can be log-linearized to yield the following set of equations (see Appendix A):[4]

$$\widehat{y}_t = E_t\widehat{y}_{t+1} - \psi_1 \, [\widehat{r}_t - E_t\widehat{\pi}_{t+1}] + \psi_2 \, [(\widehat{m}_t - \widehat{e}_t) - E_t(\widehat{m}_{t+1} - \widehat{e}_{t+1})] + \psi_1(1-\rho_a)\,\widehat{a}_t \tag{2.5}$$

$$\widehat{m}_t = \gamma_1 \, \widehat{y}_t - \gamma_2 \, \widehat{r}_t + \gamma_3 \, \widehat{e}_t \tag{2.6}$$

$$\widehat{r}_t = \rho_r \, \widehat{r}_{t-1} + (1-\rho_r)\rho_y \, \widehat{y}_t + (1-\rho_r)\rho_\pi \, \widehat{\pi}_t + (1-\rho_r)\rho_\mu \, \widehat{\mu}_t + \varepsilon_{r_t} \tag{2.7}$$

$$\widehat{\mu}_t = \widehat{m}_t - \widehat{m}_{t-1} + \widehat{\pi}_t \tag{2.8}$$

$$\widehat{\pi}_t = \gamma_f \, E_t\{\widehat{\pi}_{t+1}\} + \gamma_b \, \widehat{\pi}_{t-1} + \lambda \, \widehat{mc}_t \tag{2.9}$$

$$\widehat{mc}_t = \left(\chi + \frac{1}{\omega_1}\right) \widehat{y}_t - \frac{\psi_2}{\psi_1} \, (\widehat{m}_t - \widehat{e}_t) - \chi \, \widehat{z}_t \tag{2.10}$$

$$\widehat{a}_t = \rho_a \widehat{a}_{t-1} + \varepsilon_{a_t} \tag{2.11}$$

$$\widehat{e}_t = \rho_e \widehat{e}_{t-1} + \varepsilon_{e_t} \tag{2.12}$$

$$\widehat{z}_t = \rho_z \widehat{z}_{t-1} + \varepsilon_{z_t} \tag{2.13}$$

where the following relationships hold between structural parameters, the steady-state, and the reduced form parameters of equations (2.5)-(2.10): $\psi_1 = \left(-\frac{U_c}{CU_{cc}}\right)$,

[4] The symbol $\widehat{}$ represent per cent deviations of a variable from its steady-state value.

$\psi_2 = \left(-\frac{U_{cm}}{\overline{C}U_{cc}}\right)\left(\frac{\overline{m}}{\overline{e}}\right)$, $\gamma_1 = \left(\overline{r}\frac{\overline{C}}{\overline{m}}\frac{\psi_2}{\psi_1} + (r-1)\frac{1}{\psi_1}\right)\gamma_2$, $\gamma_2 = \frac{\overline{r}}{(\overline{r}-1)}\frac{\overline{e}}{\overline{m}}\left(\frac{U_m}{(\overline{r}-1)\overline{e}U_{cm}-\overline{r}U_{mm}}\right)$, $\gamma_3 = 1-(\overline{r}-1)\gamma_2$, $\lambda \equiv (1-\theta)(1-\beta\theta)(1-\omega)\ \xi\ \phi^{-1}$, $\gamma_f \equiv \beta\theta\ \phi^{-1}$, $\gamma_b \equiv \omega\ \phi^{-1}$, $\xi \equiv \frac{(1-\alpha)}{1+\alpha(\varepsilon-1)}$, $\phi \equiv \theta + \omega[1-\theta(1-\beta)]$, and $\chi = \frac{\varphi+\alpha}{1-\alpha}$.[5]

Equation (2.5) represents the optimal intertemporal allocation of wealth. The assumption of non-separability between consumption and real balances makes the marginal utility of consumption a function of the amount of real balances optimally demanded by the households. Hence, in equilibrium, output will depend on the current and expected real balances after accounting for the money demand shock. The effect of real balances on demand will vanish when the parameter $\psi_2 = 0$, i.e. as long as the cross derivative between consumption and real balances is zero in the utility function. As stressed by Ireland (2001), non-separability is allowed for and tested as one of the potential direct channels through which real balances can exert real effects in the economy. Demand depends upon the present discounted value of future real balances and short-term interest rates; this forward-looking character is inherited from the dynamics of consumption, so the sensitivity of output to interest rate movements depends upon the coefficient ψ_1, which is related to the inverse of risk aversion (i.e. intertemporal substitution attitudes).

Expressions (2.6), (2.7) and (2.8) describe the money market. Equation (2.6) is a standard money-demand equation, where the coefficients γ_1 and γ_2 are the money-income and money-interest rate elasticities. Equation (2.8) is an identity that specifies nominal money growth in terms of real balances and inflation.[6]

The supply side of the model is characterized by two equations: first, a New Keynesian Phillips curve, (2.9), that allows for both expected and past inflation terms as well as real marginal costs to affect current inflation; and second, a linear relationship between the real marginal costs with detrended output, real balances and the technology shock (2.10). Notice that, if we assume that all new prices (p_t^*) are set on a staggered basis, i.e. $\omega = 0$, then inflation becomes a purely forward-looking variable. The non-separability in preferences between real balances and consumption generates a direct effect of the former variable on marginal costs and then on inflation. Moreover, the assumption of decreasing returns to labor implies that the effect of output on inflation depends not only on the degree of nominal rigidities, but also upon the elasticity of output to employment $(1-\alpha)$ and the labor supply elasticity (φ) through the coefficient χ.

[5] The upper bar refers to the steady state value of the variable.

[6] Some authors refer to these equations as to a version of an optimizing IS-LM model (e.g. McCallum and Nelson (1999)).

As noted above, allowing for non-separability between real balances and consumption leads real balances to play a direct role as a determinant of both output and inflation equilibrium relationships. In this sense, the model resembles somewhat the reduced form P* model.[7] In fact, our model provides a well-rooted microfoundation for the direct effect of real balances in inflation, a key equation of the P* setup. Notwithstanding, this model imposes cross-parameter restrictions that should be tested in order to assess the empirical relevance of such hypotheses. Finally, notice that what matters for the dynamics of output and inflation is the change in real balances once the money demand shock has been taken into account (i.e., $\widehat{m}_t - \widehat{e}_t$). To close the model we include the AR(1) distribution for the aggregate demand shock (2.11), the money demand shock (2.12) and the technology shock (2.13), with innovations ε_{a_t}, ε_{e_t} and ε_{z_t} respectively.

The estimation procedure follows Hansen and Sargent (2000), Kim (2000) and Ireland (2001) who propose a maximum likelihood method to exploit the cross-equation restrictions implied by the stationary solution of the model (2.5)-(2.13). To that end the model is expressed in a state-space form as explained in Appendix B and it is estimated using a recursive Kalman filter procedure. We use aggregate euro area quarterly data from 1980:1 to 1999:4 for the logs of detrended output, detrended real balances, inflation and gross nominal interest rates. The output, inflation and nominal interest rate data comes from the Area Wide Model data set (see Fagan et al. (2001)). Real output is measured through real GDP, inflation is defined as the change in the log of GDP deflator, and the interest rate is the three month money market rate. Real balances are measured dividing M3 by the GDP deflator, and are obtained from the work by Brand and Cassola (2000).[8] Figure 1 displays the data used in the estimation process.

3. Maximum-Likelihood Estimates

3.1. Testing the Real Balances Effect

Table 1 reports the maximum likelihood estimates of alternative models. In the first column we present the parameter estimates corresponding to our benchmark model. We test for the existence of a backward- looking component among the

[7] Hallman et al. (1991) provide evidence of how that model may explain the observed US inflation.

[8] We thank Nuno Cassola for supplying us that variable.

firms setting their price at t ($\omega \geq 0$).[9] The unconditional means of the observable variables ($\ln(y), \ln(m), \ln(\pi), \ln(r)$) are rather precisely estimated and imply a reasonable quarterly discount factor ($\beta = \frac{\pi}{r} = 0.988$).[10] Most of the inertial behavior of supply and demand is inherited through the high persistence presented especially in both preferences (ρ_a) and supply (ρ_z) shocks.

Turning now to the model parameters, the most interesting result concerns ψ_2 that governs the separability of the utility function on real balances. The estimation implies that real balances effect in the dynamics of demand (2.5) and supply (2.9) is positive but not significant. Ireland (2001) also obtains a non-significant effect for the US although he estimates a simpler model and finds a negative value for the point estimate. The other parameter of the output equation, the interest rate elasticity (ψ_1), is significantly positive.

The estimates for the supply side of the economy reveals the importance of the forward-looking component of inflation (i.e. γ_f). The slope coefficients of the Phillips curve (λ and χ) are both significant and reflect that marginal costs are an important determinant of inflation. The point estimates of λ (0.14) and γ_f (0.62), are in the range of the ones estimated using only information about euro area marginal costs by Galí, Gertler and Lopez-Salido (2001). Nevertheless, assuming an average labour share for the euro area of $(1 - \alpha) = 3/4$, the point estimate of $\chi = \frac{\varphi + \alpha}{1 - \alpha}$ (10.4) implies a value of the labor supply elasticity ($1/\varphi$) of 0.13, considerably lower than the one used in the business cycle literature, though in line with microeconometric evidence.

Besides the direct real balance effects on demand and supply, money is also related to the behavior of output and prices through the money demand and the policy reaction function equations. The elasticity of money demand with respect to output (γ_1) and interest rate (γ_2) are poorly estimated. As regards the interest rate rule, the response to output (ρ_y) is nil, the inflation response (ρ_π) is above one and the smoothing parameter (ρ_r) is imprecisely estimated. Moreover, there is a significant response to the rate of growth of money ($\rho_\mu = 0.53$).

These results might be interpreted as if the (aggregate of) European monetary authorities had followed a flexible monetary targeting strategy or inflation target-

[9] We impose $\gamma_b = 1 - \gamma_f$. This constraint is not exactly satisfied in the theoretical model. As noted by Boivin and Gianonni (2001) it helps in estimating the model and the error is very small for values of β close to one. This restriction makes it impossible to recover the parameters ω and θ from the reduced form parameters γ_b, γ_f and λ.

[10] In order to restrict the steady state values for y and m some functional form for the utility function would be needed.

ing in which current money growth was a leading indicator of future inflation.[11] This is striking given the poor role assigned to money in our estimates of output and inflation equations; we will turn to this issue below. Notwithstanding, the previous results are consistent with recent general equilibrium model estimates for the US (i.e.Ireland (2001) and Keen (2001)), but slightly depart from the single equation estimates provided by Clarida, Galí and Gertler (1998) for the Bundesbank during a similar sample period.[12] While, those authors found a significant response of nominal interest rates to output, the response to monetary aggregates was negligible.

In order to assess the quantitative importance of the estimated parameter ψ_2, measuring the real balance effect, we have estimated the model imposing separability, i.e. $\psi_2 = 0$. The likelihood value, in column (2) of Table 1, falls from 1425.0 to 1423.7. Thus, a likelihood ratio test does not reject the null of non-separability at the 95% probability. Moreover, most of the remaining parameters remain at a similar estimated value. In particular, notice that under separability the parameter ψ_1 identifies intertemporal substitution attitudes (i.e., $\sigma = \frac{1}{\psi_1}$). As it can be seen from the second column of Table 1, the estimated value is slightly above one, close to the values used in the business cycle literature.

The most significant changes affect to the parameters of the money demand equation and the smoothing of the interest rate rule. The interest rate elasticity, $\gamma_2 = 0.31$, remains very much in line with other estimated values,[13] but the income elasticity takes a very small value, $\gamma_1 = 0.008$, which contrasts with the usual calibration of this parameter at its long-run value of $\gamma_1 = 1.0$. It should be noticed though that γ_1 is a short run elasticity, since we are working at bussiness cycle frequency; in fact this estimate is not far from the values obtained in single equation estimations that do not restrict that elasticity in the short-run (e.g.,Coenen and Vega (1999) for the euro area). Later, we will discuss the implications that changes in the properties of the money demand have on the business cycle properties of the model. The smootihing parameter $\rho_r = 0.25$ in the monetary policy rule becomes significant.

[11] Gerlach and Svensson (2000) find that the real balances gap is a reliable leading indicator for future inflation. Our model also includes a real money gap but makes explicit the market structure and the agents' decisions.

[12] Notice that the monetary policy of the current euro area countries became progressively more influenced by that of the Bundesbank through the European Monetary System.

[13] See, for example, Chari et al. (2000) for the US.

3.2. Money under Separability: Habit formation

From the previous results we cannot reject the hypothesis of separability in preferences between consumption and real balances. This implies that money has no direct effect on output and inflation; still, money appears to have a significant role through the policy rule. Hence, fluctuations in money cannot account for the observed output and inflation inertia. In this section we impose $\psi_2 = 0$, though we further enrich the structure allowing not only for backward-looking firms but also for habit formation in consumption aimed as capturing both output and inflation persistence. This additional assumption has been emphasized by Fuhrer (2000) and Christiano, Eichenbaum and Evans (2001) as a potentially important component of the monetary-transmission mechanism. In particular, it helps to explain the existence of serial correlation in the response of output to monetary policy shocks. We consider that the households preferences take the following functional form:

$$U(C_t, \frac{M}{e_t P_t}) = \left(\frac{1}{1-\sigma}\right)\left[\frac{C_t}{C_{t-1}^h}\right]^{1-\sigma} + \frac{1}{1-\delta}\left[\frac{M_t}{e_t P_t}\right]^{1-\delta} \tag{3.1}$$

where the parameter of the curvature of the utility function, σ, is the inverse of the intertemporal elasticity of substitution $h > 0$ reflects dependence of current utility on past consumption and δ is related to the interest rate elasticity of the money demand. This changes the intertemporal demand, the money demand and the marginal cost equations as follows:

$$\begin{aligned} \widehat{y}_t \;=\;& \frac{\phi_1}{\phi_1+\phi_2}\widehat{y}_{t-1} + \frac{\beta\phi_1+\phi_2}{\phi_1+\phi_2}E_t\widehat{y}_{t+1} - \frac{1}{\phi_1+\phi_2}\left[\widehat{r}_t - E_t\widehat{\pi}_{t+1}\right] - \\ & \frac{\beta\phi_1}{\phi_1+\phi_2}E_t\widehat{y}_{t+2} + \frac{1-\beta h\rho_a}{(1-\beta h)}\frac{(1-\rho_a)}{\phi_1+\phi_2}\widehat{a}_t \end{aligned} \tag{3.2}$$

$$\widehat{m}_t = -\frac{\phi_1}{\delta}\widehat{y}_{t-1} + \frac{\phi_2}{\delta}\widehat{y}_t - \frac{\beta\phi_1}{\delta}E_t\widehat{y}_{t+1} - \frac{1}{\delta(r-1)}\widehat{r}_t + \frac{\beta h(1-\rho_a)}{(1-\beta h)\delta}\widehat{a}_t - \frac{\delta-1}{\delta}\widehat{e}_t \tag{3.3}$$

$$\widehat{mc}_t = -\phi_1\widehat{y}_{t-1} + (\chi+\phi_2)\,\widehat{y}_t - \beta\phi_1 E_t\widehat{y}_{t+1} - (1+\chi)\,\widehat{z}_t - \left(\frac{\beta h(1-\rho_a)}{1-\beta h}\right)\widehat{a}_t \tag{3.4}$$

where $\phi_1 = \frac{(\sigma-1)h}{1-\beta h}$, $\phi_2 = \frac{\sigma+(\sigma-1)\beta h^2-\beta h}{1-\beta h}$

Allowing for habit formation makes the demand equation (3.2) a function relating current output to past and future output as well as to real interest rates and preference shocks. Notice that as $h \to 0$, expression (3.2) approaches to the usual Euler equation for consumption under time-separable preferences. By the same token, the existence of habits also induces higher dynamic relationships between output and marginal costs. In addition, demand shocks have a direct impact effect on both real balances and marginal costs. Again, when $h \to 0$ we recover the previous expression (2.10) under $\psi_2 = 0$. Finally, the demand side of the model present a much more complex dynamics. Moreover, the separability of real balances implies a tight restriction between the interest rate elasticity and the output elasticity of money demand trough the parameter δ.

Column (3) in Table 1 reports the estimated values for the parameters of this augmented model. We find clear evidence of habit formation, which further improves the fit of the model with a significant increase in the likelihood to 1428.8. Thus, the strong significance of the parameter measuring habit persistence h is not surprising. This is estimated at around 0.9, slightly higher than the value of 0.8 obtained by Fuhrer (2000) for the US. The parameter of intertemporal substitution (σ) is robust and remains close to one. The parameter δ is precisely estimated implying strongly significant elasticities of the demand for money. The implied point estimate of the interest rate elasticity is around 0.4 with a standard error of 0.09. Notwithstanding, these estimates continues yielding an extremely low value for the income elasticity (i.e. $\gamma_1 = \frac{\phi_2}{\delta}$) of 0.018 with a standard error of 0.0058.

The presence of habits in consumption affects the relationship between the marginal cost and output too. On the one hand, we find that allowing for habit formation, the backward-looking component of inflation becomes negligible. Thus, we present the results under the restriction that $\gamma_f = \beta$, (i.e. $\omega = 0$). On the other hand, since the firms are not backward-looking, we may recover the degree of price stickiness (θ), from the estimated slope coefficient, $\lambda = 1.19$. Given the values for the average labour share ($(1-\alpha) = 3/4$) and the steady state markup ($\frac{\varepsilon}{\varepsilon - 1} = 1.20$), we obtain $\theta = 0.2$, which implies that prices changes every 1.2 quarters. This very small degree of price inertia contrasts with the results found in the model without habit formation corresponding with a much lower slope of the Phillips curve. Moreover, the low value of χ implies that the implicit labor supply elasticity is around 6, quite close to the assumption of a perfectly elastic labor supply.

Aside from the interest rate smoothing parameter (now higher $\rho_r = 0.5$), the policy rule is largely unaltered. The main driving variables of the anticipated part of the rule continue to be inflation and the money growth rate, although now output has a significant but still small role in line with most of the existing estimates of monetary policy rules. Finally, considering richer dynamics in the output behavior brings the estimated persistence of the shocks to lower values.

We conclude that the habit formation model (under separability in real balances) gives a slightly different picture of the structural properties driving output, inflation, money and the interest rate in Euroland, as compared to that in a model without habit formation. On the demand side, while some crucial parameters, like the elasticity of intertemporal substitution and the elasticities in the demand for real balances, remain largely unchanged; it appears that consumption preferences are more closely associated with the change in consumption rather than just its level (i.e. habits). The supply side parameter estimates point towards little nominal price stickiness, and high labor supply elasticity. We view these results as compatible with low nominal inertia and high real rigidity (i.e. the existence of a low response of real wages to large employment fluctuations). More explicitly, given our assumptions about the labor market (i.e. frictionless), the only source of real rigidities is a high labor supply elasticity. However, we do not claim that intertemporal substitution is the dominant mechanism to understand the European labor market dynamics. Rather, we interpret these results as calling for an extension of the model with non-competitive labor market features which might be behind the observed real wage rigidity.[14]

A closer look at the estimated parameters reveals the likely cause of the discrepancy among models in columns (2) and (3) as far as the degree of price inertia is concerned. Notice that the estimated value of $\sigma \approx 1$ makes $\phi_1 \approx 0$ and $\phi_2 \approx 1$, thus rendering the anticipated part of the model without habit formation similar to that of the model with habit formation regardless of the value of h. What a high and significant value of h induces is a substantial difference across models in the unaticipated part, since in the habit augmented model the innovation ε_{a_t} directly affects money demand and the real marginal costs (and thus inflation). This suggests that the model without habit formation might be inadequately specified since it excluded relevant sources of cross-correlations among the variables. Once we allow for this cross-correlation, the source of inertia of inflation changes and becomes partly explained by demand shocks. Consider the Phillips curves implied by the model without habit formation,

[14]For example, Christiano, Eichenbaum and Evans (2001) contains an estimation with that feature.

$$\widehat{\pi}_t = \gamma_f \; E_t\{\widehat{\pi}_{t+1}\} + \gamma_b \; \widehat{\pi}_{t-1} + \lambda \left(\chi + \frac{1}{\omega_1} \right) \; \widehat{y}_t - \lambda\chi \; \widehat{z}_t \tag{3.5}$$

and with habit formation,

$$\begin{aligned} \widehat{\pi}_t = \gamma_f \; E_t\{\widehat{\pi}_{t+1}\} + \gamma_b \; \widehat{\pi}_{t-1} \qquad & -\lambda\phi_1\widehat{y}_{t-1} + \lambda\left(\chi + \phi_2\right)\widehat{y}_t - \lambda\beta\phi_1 E_t\widehat{y}_{t+1} \\ & -\lambda\left(1+\chi\right)\widehat{z}_t - \lambda\left(\frac{\beta h(1-\rho_a)}{1-\beta h}\right)\widehat{a}_t \end{aligned} \tag{3.6}$$

If (3.6) were the true model, after a positive innovation ε_{a_t} we would expect a rise in inflation (due to the increase in y_t) partially compensated by the negative direct effect of a_t. This low response of inflation is captured in the estimation of (3.5) by a low elasticity with respect to y_t, since it is not allowed a direct effect of a_t.

4. A Quantitative Assessment of the Alternative Models

In this section we explore the implications of the estimated models in two directions. First we analyze the ability of the alternative models to match some of the unconditional moments that appear in the data. This exercise does not need to identify the sources of fluctuations in the economy. Second, we carry out standard conditional exercises (i.e. variance decomposition and impulse responses) using for that the estimated process of the structural supply and demand shocks.

4.1. Unconditional Moments

Table 2 shows the standard deviations of the variables, both in the data and those estimated in the three models presented in the previous section. The benchmark model that allows for non-separability in the utility function overpredicts the variability of output, being more than twice the observed variance. The model that imposes separability does even worse since the estimated output variability is more than three times the observed one. By contrast, incorporating habit formation in the model brings the variability of output down though it is still above the observed value. Moreover, the estimated variability of real balances and inflation are robust across models and close to that observed in the data.

Figure 2 depicts the autocorrelation for output and inflation implied by an estimated fourth order VAR for the euro area.[15] Both variables display a high persistence that remains significantly different from zero until approximately lag ten for output and lag sixteen for inflation.[16] Figure 2 also shows the simulated autocorrelation function for the estimated models with separable real balances ($\psi_2 = 0$), with and without habit formation in consumption. Both models overestimate the observed persistence in the data, especially in the model without habit formation.[17] Clearly, these results are very much dependent of the distribution of the shocks. The estimated high persistence of the three shocks and the assumption that they are uncorrelated lies behind these weak results in terms of the unconditional moments.

4.2. Conditional Properties

Figure 3 compares the responses of output, inflation, nominal interest rates and money growth in the two models estimated under separable preferences to both a demand and a technology shock.[18] Figure 3 (a) displays the response of these variables to an innovation (of one standard deviation size) in the demand shock. Under both specifications, after a positive demand shock, there is a clear inflationary effect that calls for an instantaneous tightening of the monetary policy. The effects of this shock implies very different output and money growth responses between the two models but similar inflation and nominal interest rate responses. The difference in the impact effect and the subsequent adjustment in output (hump-shaped) observed in the second model (line with circles) is explained by the large habit formation parameter and the lower shock persistence.

Figure 3 (b) displays the responses of the variables in both models to a negative technology shock. In this case the estimated process for the $\widehat{z}_t$ shock in the model without habit formation has a higher autocorrelation and standard error, thus implying a very long-lasting effect of the shock on output. Under this shock, though, the difference in output persistence produces a much bigger inflationary

[15] We estimated a 4-lag VAR model with detrended output and real balances, inflation and nominal interest rates, all the variables in logs.

[16] Coenen and Wieland (2000) estimate a lower persistency for inflation than for output. We suspect this might reflect that they are using detrended inflation as oposed to observed inflation.

[17] The estimated persistence of the cross-correlations between output and inflation and the remaining variables is also significantly higher than that observed in the data. These estimates are not shown to save space.

[18] All the numbers in the impulse responses are in annual terms.

response effect under the habit formation model. Moreover, in this model the impact fall in output, the subsequent recovery and the persistent increase in inflation are compatible with an important increase at the same time in the nominal interest rates and the money growth.

Next we analyze quantitatively the importance of the structural shocks. Table 3 presents the forecast error variance decomposition of the variables implied by those two estimated models.[19] Money demand shocks mainly help to forecast real balances but they have almost no predictive power power as far as the movements of output, inflation and interest rates is concerned. Both real demand and supply shocks are the most important sources of fluctuations in output and inflation and therefore in interest rates.

Do these simulated impulse responses and forecast error variance decompositions resemble in any way the available VAR evidence on the dynamic effects of both supply and demand shocks? To answer this we compare the simulated conditional properties with the empirical results of the VAR literature. We use the evidence provided by Gerlach and Smets (1995) for the G-7 countries and by Monticelli and Tristani (1999) for the euro area.[20] Both VAR studies found, first, that supply and IS shocks account for most of the observed output volatility over the business cycle. Second, that real demand shocks are the main force behind the inflation and the interest rate volatility. Third, monetary policy shocks have a minor role in explaining macroeconomic fluctuations, basically explaining the long-run volatility of inflation. These quantitative results are close to the ones reported in Table 3, especially for the model with habit formation.

As regards the quantitative responses to demand and supply shocks, Gerlach and Smets report that on average a positive demand shock expands output and prices immediately and calls for an increase in interest rates. The size of the impact effect for the euro area obtained by Monticelli and Tristani is of around 0.2% for output, 0.1% for inflation and of around 10 basis points for interest rates. The results in Figure 3 for the model with habit formation (line with circles) are of the same order of magnitude, especially for nominal interest rates and inflation: a positive demand shock produces a contemporaneous increase in output of 0.1

[19] The non-separable preferences model has a variance decomposition of the shocks similar to the model with separable preferences.

[20] These VARs studies do not use money in their information set, so they don't separate between money supply and money demand shocks.The identification criteria is that real demand and monetary shocks do not have a long-run effect on output and that monetary shocks have no contemporaneous effect on output.

percentage points, an immediate increase of inflation of 0.12% and an interest rate rise of around 12 basis points. The only caveat of the demand shock is that our estimated models seem to overestimate the persistence of output, which in the VAR dies away after 8-12 quarters.

With respect to a supply shock the VAR evidence is that the output effect is very persistent and that inflation jumps on impact. Gerlach and Smets find that the sign of the impact effect on interest rates varies across countries, whereas for the euro area Monticelli and Tristani report an average positive increase in interest rates after a negative supply shock. This coincides with our results in Figure 3 for the model with habit formation and the size of the remaining effects are also similar: a fall in output of between 0.3% and 0.4%, an inflation increase of between 0.2% and 0.3% and an interest rate rise of 25 basis points.

5. Counterfactual exercises

As we have discussed above, the data does not support a model in which there is a real balance effect. Thus, the limited influence of money on inflation and output stems from the demand and supply interactions in the money market. In this section we pursue this issue further. In particular, we carry out two counterfactual exercises. The first one focuses on the role played by the income elasticity of real balances. An income elasticity of one implies that velocity only responds to movements in the interest rates. By contrast, in the estimated models, a lower than one value for such an elasticity makes velocity dependent on income too. We have analyzed the differences between those two elasticities in terms of the impulse responses for the model without habit formation. The persistency and the sign of the effects remains equal, what changes is the impact response of the variables. As expected, the response of output both to a demand and to a supply shock is much lower in the model with a unit income elasticity. This is so because the rise in output leads (*caeteris paribus*) in this case to a major reaction of interest rates, thus compensating somewhat the effect of a given shock on output.

The second exercise is designed to assess the importance of the other channel through which money may have a direct effect in this model: the response of interest rates to money growth as estimated in the policy rule. Rudebusch and Svensson (1999) have shown, in the context of a backward looking model, that the difference between a monetary targeting and a flexible inflation targeting is that the interest rate policy rule in the first case depends on current and lagged real money stocks. Similarly, our estimated reaction function can be rewritten as a inflation targeting rule with a real balances term. Consider, for example, the estimated model under separability and without habit formation, substituting money growth in (2.7) by (2.8):

$$\widehat{r}_t = 0.25\widehat{r}_{t-1} + (1-0.25)(1.05+0.55)\widehat{\pi}_t + (1-0.25)0.55(\widehat{m}_t - \widehat{m}_{t-1}) + \varepsilon_{r_t} \quad (5.1)$$

How important is that reaction function response to real balances? To answer that question we repeated the conditional exercises from the previous section imposing that in (5.1) the real balances coefficient is zero. Under this restriction the responses of output, nominal interest rates, inflation and money growth to both demand and supply shocks remain the same than the ones displayed in Figure 3. Consistently, the variance decomposition of the shocks under that restriction is also equal to the one presented in Table 3. Therefore the outcome under a policy rule that resembles a monetary targeting strategy is very similar to the one followed under a flexible inflation targeting strategy (i.e. a Taylor rule); the reason is that in our model the movements in real balances are mainly driven by money demand shocks. Although money growth is statistically significant in the policy rule the counterfactual shows that this variable is capturing some effect linked to the inflation rate, rather than a true role for the quantity of money in setting the interest rate.

The exercises in this section highlighted the importance of the specification of money demand in the propagation of structural shocks in the Eurozone. We also confirm that the level of real balances in the policy reaction function plays a limited role in shapping the responses of output and prices to structural shocks.

6. Conclusions

We estimate a small scale dynamic general equilibrium model for the euro area specified at the level of preferences and technology, that can be expressed in terms of four building blocks: an intertemporal demand equation, a New Phillips curve, a money demand equation, and a Taylor rule for monetary policy. A distinctive feature of our model is the consideration of a direct effect of real balances in private agents' decisions and also on the central bank's policy rule. Non-separability between consumption and real balances and habit dependence in consumption generate a rich dynamic structure for the marginal utility of consumption, which extends to marginal costs. The estimation is carried out by a maximum likelihood method, exploiting the cross equation restrictions implied by the stationary solution of the model.

The demand side is reasonably well estimated, implying logarithmic preferences in consumption. When allowing to habit formation we find similar estimates to those existing in the literature. With respect to the monetary block, the data accepts the separability between consumption and real balances; the policy rule resembles the now familiar interest rate rules which assign a large weight to inflation and money growth and a minor role to output. On the supply side, we estimate a New Phillips Curve that is very much in line with the single equation estimations. In particular, we find supporting evidence of an important forward looking component in driving inflation dynamics. Nevertheless, once we enlarge the model with habits in consumption, our estimates are consistent with a high labor-supply elasticity and a low degree of price stickiness. Fluctuations in output and inflation are mainly driven by real shocks whereas money demand shocks affect only real balances. Finally, the implied impulse-response functions broadly match those reported in the VAR literature.

Two results have potential interest for further research. First, no direct effect of money upon inflation and output is found, since preferences appear to be separable in consumption and real balances. Interestingly, money growth plays a significant role in the policy rule meaning that the European monetary authorities may have paid special attention to a monetary aggregate either on its own, to attain price stability, or because they consider it a leading indicator of future inflation. However, this result is conditioned by the lack of a truly common monetary policy during the sample period analyzed in the paper.

The supply side of the model is less precisely estimated and it changes somewhat as we move from our benchmark model to a model with habits in consumption. The latter is consistent with a low degree of nominal inertia, but a high elasticity of labor supply suggesting a non-negligible amount of real (wage) rigidity. This difference may be explained on the basis of an inadequate specification of the simpler model, but it might also be argued that intertemporal substitution attitudes in leisure can hardly be behind a flat labor supply in economies with high unemployment, like those in the Eurozone. Since our model allows neither for labor market imperfections nor for adjustment costs in capital, this high elasticity is about the only channel through which real rigidities can show up.

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Appendix A: Equilibrium conditions

Households

In the model with non-separability, the first order conditions of the household problem are given by:

$$\lambda_t = a_t U_{c_t} \tag{A1}$$

$$a_t N_t^{\varphi} = \lambda_t \left(\frac{W_t}{P_t}\right) \tag{A2}$$

$$\lambda_t = \beta r_t E_t \frac{\lambda_{t+1}}{\pi_{t+1}} \tag{A3}$$

$$\lambda_t - \beta E_t \lambda_{t+1} = (a_t/e_t) U_{m_t} \tag{A4}$$

and the budget constraint (2.2), with $m_t = M_t/P_t$, $\pi_{t+1} = P_{t+1}/P_t$, and λ_t the Lagrange multiplier on (2.2). Using expressions (A3) and (A4) we can obtain the following expression for the money demand:

$$(a_t/e_t) U_{m_t} = \lambda_t \left(\frac{r_t - 1}{r_t}\right) \tag{A5}$$

Firms

They choose the price $P_t(j)$ as to maximize the expected present discounted value of future dividends:

$$\max_{P_t(j)} \; E_t \sum_{k=0}^{\infty} (\beta\theta)^k \; \lambda_{t+k} \left\{ Y_{t,t+k}(j) \pi^k P_t(j) - \Psi_{t,t+k}(j) Y_{t,t+k}(j) \right\}$$

where $\Psi_{t,t+k}(j)$ is the nominal marginal cost at $t+k$ of the firm j setting its price at t. Where $Y_{t,t+k}(j) = \left(\frac{\pi^k P_t(j)}{P_{t+k}}\right)^{-\varepsilon} \; Y_{t+k}$.The first order condition is:

$$E_t \sum_{k=0}^{\infty} (\beta\theta)^k \; \lambda_{t+k} \left(\frac{P_t^*(j)}{P_{t+k}}\right)^{-\varepsilon} \; Y_{t+k} \left\{ (1-\varepsilon) - \frac{\Psi_{t,t+k}(j)}{P_t^*(j)} \right\} = 0 \tag{A6}$$

where $P_t^*(j)$ represents the optimal price of the firms that change prices at time t. The previous expression can be rearranged as follows:

$$E_t \sum_{k=0}^{\infty} (\beta\theta)^k \ \lambda_{t+k} \left(\frac{\pi^k P_t^*(j)}{P_{t+k}} \right)^{-\varepsilon} \ Y_{t+k} \left\{ \pi P_t^*(j) - \frac{\varepsilon}{\varepsilon - 1} \Psi_{t,t+k}(j) \right\} = 0 \qquad \text{(A7)}$$

notice that under $k = 0$ the firm's problem becomes a static one, and the firm set prices as a constant markup over marginal costs: $P_t^*(j) = \pi^{-1} \frac{\varepsilon}{\varepsilon-1} \Psi_{t,t+k}(j)$, with $\frac{\varepsilon}{\varepsilon-1}$ the firm's desired gross markup.

The model is better expressed in terms of real marginal costs $MC_{t,t+k}$. Thus,

$$\Psi_{t,t+k}(j) = MC_{t,t+k}(j) P_{t+k} \qquad \text{(A8)}$$

Using expression (A8) in (A7) we obtain:

$$\pi P_t^*(j) = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{E_t \sum_{k=0}^{\infty} (\beta\theta)^k \ \lambda_{t+k} \left(\frac{\pi^k P_t^*(j)}{P_{t+k}} \right)^{-\varepsilon} \ \Psi_{t,t+k}(j)}{E_t \sum_{k=0}^{\infty} (\beta\theta)^k \ \lambda_{t+k} \left(\frac{\pi^k P_t^*(j)}{P_{t+k}} \right)^{-\varepsilon}}$$

This equation can be interpreted as a dynamic markup equation, so firms set prices using the expected future evolution of demand and marginal costs.

As stressed by Galí, Gertler and López-Salido (2001a), the goal now is to find an expression for inflation in terms of an observable measure of aggregate marginal cost instead of the firm-specific marginal cost. To do this we proceed in three steps. First, cost minimization implies that the firm's real marginal cost will equal the real wage divided by the marginal product of labor. Given Cobb-Douglas technology, the real marginal cost of $t + k$ for a firm that optimally sets its price at t is given by:

$$MC_{t,t+k}(j) = \frac{(W_{t+k}/P_{t+k})}{(1 - \alpha) \ (Y_{t,t+k}(j)/N_{t,t+k}(j))} \qquad \text{(A9)}$$

where $Y_{t,t+k}(j)$ and $N_{t,t+k}(j)$ are output and employment for a firm that has set its price at t at the optimal value P_t^*. Second, following Woodford (1996) we exploit the assumptions of a Cobb-Douglas production technology and the isoelastic demand curve introduced to obtain the following log-linear relationship between $MC_{t,t+k}$ and the actual real marginal cost in $t + k$ (MC_{t+k}) as:

$$MC_{t,t+k}(j) = MC_{t+k} \left(\frac{\pi^k P_t^*(j)}{P_{t+k}} \right)^{\frac{-\varepsilon\alpha}{1-\alpha}} \qquad \text{(A10)}$$

Where:

$$MC_{t+k} = \frac{(W_{t+k}/P_{t+k})}{(1-\alpha)(Y_{t+k}/N_{t+k})}$$

The additional equations needed for the equilibium are the definition of the price level, inflation and real balances and the government budget constraint:

$$P_t = \left[\theta \ P_{t-1}^{1-\varepsilon} \ + \ (1-\theta) \ (P_t^*)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{A11}$$

$$\pi_t = \frac{P_t}{P_{t-1}} \tag{A12}$$

$$m_t/m_{t-1} = (M_t/P_t)/(M_{t-1}/P_{t-1}) = \mu_t/\pi_t \tag{A13}$$

$$M_t - M_{t-1} = T_t \tag{A14}$$

In a symmetric equilibrium all firms that reset their price at time t choose the same price. Thus, $P_t^*(j) = P_t^*$. Aggregating across firms (and assuming $B_t = B_{t-1} = 0$) we get the equilibrium conditions expressions (2.5)-(2.13) that are already in log-deviations from the steady-state.

Steady State

In the absence of shocks, the economy converges to the steady state, which is characterized by the following set of identities:

$$Y = C \tag{A15}$$

$$r = \pi/\beta \tag{A16}$$

$$\pi = \mu \tag{A17}$$

$$Y = Z(N)^{1-\alpha} \tag{A18}$$

$$\left(\frac{r-1}{r}\right) U_c = (\frac{1}{e})U_M \tag{A19}$$

$$U_c = \left(\frac{\varepsilon}{\varepsilon-1}\right) \frac{N^{\alpha+\varphi}}{(1-\alpha)Z} \tag{A20}$$

$$MC = \left(\frac{\varepsilon - 1}{\varepsilon} \right) \tag{A21}$$

Extended Model: Habit Formation

Considering the household's preferences (3.1), we have to replace expression for λ_t (A1) by:

$$\lambda_t = a_t \left[\frac{C_t}{C_{t-1}^h} \right]^{-\sigma} \frac{1}{C_{t-1}^h} - \beta h E_t \; a_{t+1} \left[\frac{C_{t+1}}{C_t^h} \right]^{-\sigma} C_{t+1} \frac{C_t^{h-1}}{\left(C_t^h \right)^2} \tag{A22}$$

Appendix B: Estimation procedure, model with separable preferences and habit formation.

This follows Ireland (2000)'s estimation procedure. The three optimality conditions (3.2), (3.3), (2.9) (taking into account that the marginal costs satisfy (3.4), the decision rule (2.7), the defintion of money growth (2.8), five expectational equations (for $\widehat{m}_t$,$\widehat{r}_t$,$\widehat{y}_t$,$\widehat{\pi}_t$ and $E_t\widehat{y}_{t+1}$) and the three distribution processes for the unobservables ((2.11), (2.12), (2.13)) form a system of thirteen equations that may be written as

$$Af_t^0 = Bs_t^0 + Cv_t \tag{B1}$$

$$DE_t s_{t+1}^0 + FE_t f_{t+1}^0 = Gs_t^0 + Hf_t^0 + Jv_t \tag{B2}$$

$$v_t = Pv_{t-1} + \varepsilon_t \tag{B3}$$

where:
$f_t^0 = [\widehat{y}_t \; \widehat{r}_t \; \widehat{\mu}_t]'$,
$s_t^0 = [\widehat{r}_{t-1} \; \widehat{m}_{t-1} \; \widehat{\pi}_{t-1} \; \widehat{y}_{t-1} \; \widehat{m}_t \; \widehat{\pi}_t \; E_t\widehat{y}_{t+1}]'$,
$v_t = [\widehat{a}_t \; \widehat{e}_t \; \widehat{z}_t \; \varepsilon_{rt}]'$,
$\varepsilon_t = [\varepsilon_{at} \; \varepsilon_{et} \; \varepsilon_{zt} \; \varepsilon_{rt}]'$
The model is solved along the following steps:
1) From (B1) to (B3),

$$E_t s_{t+1}^0 = Ks_t^0 + Lv_t$$

where:
$K = [D + FA^{-1}B]^{-1} [G + HA^{-1}B]$,
$L = [D + FA^{-1}B]^{-1} [J + HA^{-1}C - FA^{-1}CP]$,
2) Define the matrices:
$K = M^{-1}NM = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}^{-1} \begin{bmatrix} N_1 & 0 \\ 0 & N_2 \end{bmatrix} \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}$

Where, N_1 is diag(4) with the 4 eigenvalues of K within unit cicle (for the predetermined variables in s_t^0: $\widehat{r}_{t-1}$, $\widehat{m}_{t-1}$, $\widehat{\pi}_{t-1}, \widehat{y}_{t-1}$) and N_2 is diag(3) with the 3 eigenvalues of K outside the unit cicle (for the non-predetermined variables in s_t^0: $\widehat{m}_t$, $\widehat{\pi}_t$, $E_t\widehat{y}_{t+1}$)

3) Now let:

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} E_t s^0_{t+1} = \begin{bmatrix} N_1 & 0 \\ 0 & N_2 \end{bmatrix} \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} s^0_t + \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} v_t$$

or

$$E_t s^1_{1t+1} = N_1 s^1_{1t} + Q_1 v_t$$

$$E_t s^1_{2t+1} = N_2 s^1_{2t} + Q_2 v_t$$

where:

$$s^1_{1t+1} = M_{11} \begin{bmatrix} \widehat{r}_{t-1} \\ \widehat{m}_{t-1} \\ \widehat{\pi}_{t-1} \\ \widehat{y}_{t-1} \end{bmatrix} + M_{12} \begin{bmatrix} \widehat{m}_t \\ \widehat{\pi}_t \\ E_t \widehat{y}_{t+1} \end{bmatrix},$$

$$s^1_{2t+1} = M_{21} \begin{bmatrix} \widehat{r}_{t-1} \\ \widehat{m}_{t-1} \\ \widehat{\pi}_{t-1} \\ \widehat{y}_{t-1} \end{bmatrix} + M_{22} \begin{bmatrix} \widehat{m}_t \\ \widehat{\pi}_t \\ E_t \widehat{y}_{t+1} \end{bmatrix}$$

$Q_1 = M_{11} L_1 + M_{12} L_2, Q_2 = M_{21} L_1 + M_{22} L_2$

4) Solving for s^1_{2t+1} forward:

$$s^1_{2t} = -N_2^{-1} Q_2 v_t + N_2^{-1} E_t s^1_{2t+1} = -N_2^{-1} \sum_{j=0}^{\infty} N_2^{-j} Q_2 P^j v_t = -N_2^{-1} R v_t$$

5) Thus we may write,

$$\begin{bmatrix} \widehat{m}_t \\ \widehat{\pi}_t \\ E_t \widehat{y}_{t+1} \end{bmatrix} = -M_{22}^{-1} N_2^{-1} R v_t - M_{22}^{-1} M_{21} \begin{bmatrix} \widehat{r}_{t-1} \\ \widehat{m}_{t-1} \\ \widehat{\pi}_{t-1} \\ \widehat{y}_{t-1} \end{bmatrix} = S_1 \begin{bmatrix} \widehat{r}_{t-1} \\ \widehat{m}_{t-1} \\ \widehat{\pi}_{t-1} \\ \widehat{y}_{t-1} \end{bmatrix} + S_2 v_t$$

$$s^1_{1t} = M_{11} \begin{bmatrix} \widehat{r}_{t-1} \\ \widehat{m}_{t-1} \\ \widehat{\pi}_{t-1} \\ \widehat{y}_{t-1} \end{bmatrix} + M_{12} S_1 \begin{bmatrix} \widehat{r}_{t-1} \\ \widehat{m}_{t-1} \\ \widehat{\pi}_{t-1} \\ \widehat{y}_{t-1} \end{bmatrix} + M_{12} S_2 v_t$$

or

$$
\begin{aligned}
&[M_{11}+M_{12}S_1]\begin{bmatrix}\widehat{r}_t\\ \widehat{m}_t\\ \widehat{\pi}_t\\ \widehat{y}_t\end{bmatrix}+M_{12}S_2Pv_t\\
=\ & N_1\left[M_{11}+M_{12}S_1\right]\begin{bmatrix}\widehat{r}_{t-1}\\ \widehat{m}_{t-1}\\ \widehat{\pi}_{t-1}\\ \widehat{y}_{t-1}\end{bmatrix}+\left[N_1M_{12}S_2+Q_1\right]v_t
\end{aligned}
$$

thus,

$$
\begin{bmatrix}\widehat{r}_t\\ \widehat{m}_t\\ \widehat{\pi}_t\\ \widehat{y}_t\end{bmatrix}=N_1\begin{bmatrix}\widehat{r}_{t-1}\\ \widehat{m}_{t-1}\\ \widehat{\pi}_{t-1}\\ \widehat{y}_{t-1}\end{bmatrix}+\left[M_{11}+M_{12}S_1\right]^{-1}\left[N_1M_{12}S_2+Q_1-M_{12}S_2P\right]v_t
$$

or in compact form:

$$
\begin{bmatrix}\widehat{r}_t\\ \widehat{m}_t\\ \widehat{\pi}_t\\ \widehat{y}_t\end{bmatrix}=S_3\begin{bmatrix}\widehat{r}_{t-1}\\ \widehat{m}_{t-1}\\ \widehat{\pi}_{t-1}\\ \widehat{y}_{t-1}\end{bmatrix}+S_4v_t
$$

6) Now we can stack these expressions:

$$
\begin{bmatrix}\widehat{r}_{t-1}\\ \widehat{m}_{t-1}\\ \widehat{\pi}_{t-1}\\ \widehat{y}_{t-1}\\ \widehat{m}_t\\ \widehat{\pi}_t\\ E_t\widehat{y}_{t+1}\end{bmatrix}=\begin{bmatrix}\widehat{r}_{t-1}\\ \widehat{m}_{t-1}\\ \widehat{\pi}_{t-1}\\ \widehat{y}_{t-1}\\ S_1\begin{bmatrix}\widehat{r}_{t-1}\\ \widehat{m}_{t-1}\\ \widehat{\pi}_{t-1}\\ \widehat{y}_{t-1}\end{bmatrix}+S_2v_t\end{bmatrix}=\begin{bmatrix}I_{(4,4)}\\ S_1\end{bmatrix}\begin{bmatrix}\widehat{r}_{t-1}\\ \widehat{m}_{t-1}\\ \widehat{\pi}_{t-1}\\ \widehat{y}_{t-1}\end{bmatrix}+\begin{bmatrix}0_{(4,4)}\\ S_2\end{bmatrix}v_t
$$

Thus:

$$f_t^0 = A^{-1}B \begin{bmatrix} I \\ S_1 \end{bmatrix} \begin{bmatrix} \widehat{r}_{t-1} \\ \widehat{m}_{t-1} \\ \widehat{\pi}_{t-1} \\ \widehat{y}_{t-1} \end{bmatrix} + A^{-1}B \begin{bmatrix} 0 \\ S_2 \end{bmatrix} v_t = S_5 \begin{bmatrix} \widehat{r}_{t-1} \\ \widehat{m}_{t-1} \\ \widehat{\pi}_{t-1} \\ \widehat{y}_{t-1} \end{bmatrix} + S_6 v_t$$

7) The state-space solution of the model can be represented as:

$$\begin{bmatrix} \widehat{r}_t \\ \widehat{m}_t \\ \widehat{\pi}_t \\ \widehat{y}_t \\ \widehat{a}_{t+1} \\ \widehat{e}_{t+1} \\ \widehat{z}_{t+1} \\ \varepsilon_{rt+1} \end{bmatrix} = \begin{bmatrix} S_3 & S_4 \\ 0 & P \end{bmatrix} \begin{bmatrix} \widehat{r}_{t-1} \\ \widehat{m}_{t-1} \\ \widehat{\pi}_{t-1} \\ \widehat{y}_{t-1} \\ \widehat{a}_t \\ \widehat{e}_t \\ \widehat{z}_t \\ \varepsilon_{rt} \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \begin{bmatrix} \varepsilon_{at} \\ \varepsilon_{et} \\ \varepsilon_{zt} \\ \varepsilon_{rt} \end{bmatrix}$$

$$\begin{bmatrix} \widehat{y}_t \\ \widehat{r}_t \\ \widehat{\mu}_t \\ \widehat{m}_t \\ \widehat{\pi}_t \\ E_t\widehat{y}_{t+1} \end{bmatrix} = \begin{bmatrix} S_5 & S_6 \\ S_1 & S_2 \end{bmatrix} \begin{bmatrix} \widehat{r}_{t-1} \\ \widehat{m}_{t-1} \\ \widehat{\pi}_{t-1} \\ \widehat{y}_{t-1} \\ \widehat{a}_t \\ \widehat{e}_t \\ \widehat{z}_t \\ \varepsilon_{rt} \end{bmatrix}$$

or

$$s_{t+1} = \Pi s_t + W\varepsilon_t \tag{B4}$$

$$f_t = Us_t \tag{B5}$$

The model in this state-space format is used to construct the likelihood function (see Hansen and Sargent, (2000)).

Table 1. ML Estimates

	Benchmark Model	Separable Preferences ($\psi_2 = 0$)	
	($\psi_2 \neq 0$)	No Habits	Habit Formation
Estimated Parameters	(1)	(2)	(3)
β	0.9881 (0.0038)	0.9878 (0.0014)	0.9876 (0.0031)
ψ_1	0.9947 (0.0111)	0.9931 (0.0025)	-
ψ_2	0.9120 (1.8674)	-	-
σ	-	-	1.0573 (0.0306)
h	-	-	0.9025 (0.0286)
γ_1	0.1971 (0.1117)	0.0083 (0.0007)	-
γ_2	0.2503 (0.5438)	0.3130 (0.0622)	-
γ_3	1.4290 (0.7348)	0.8323 (0.3657)	-
δ	-	-	108.76 (3.4942)
γ_f	0.6150 (0.2241)	0.6685 (0.0458)	0.9876 (0.0031)
χ	10.4338 (3.9166)	10.6670 (0.2097)	0.5432 (0.0034)
λ	0.1423 (0.0734)	0.2505 (0.0092)	1.1939 (0.0286)
ρ_r	0.1649 (0.1939)	0.2523 (0.0497)	0.5058 (0.0181)
ρ_y	0.0000 (0.0000)	0.0000 (0.0000)	0.0555 (0.0005)
ρ_π	1.0525 (0.0946)	1.0487 (0.0621)	1.1796 (0.1482)
ρ_μ	0.5397 (0.0800)	0.5491 (0.0123)	0.6845 (0.0795)
y	4.2533 (0.0450)	4.2576 (0.0720)	4.2529 (0.0243)
m	4.0830 (0.0133)	4.0828 (0.0109)	4.0822 (0.0161)
π	0.0089 (0.0062)	0.0096 (0.0022)	0.0049 (0.0028)
r	0.0209 (0.0099)	0.0219 (0.0034)	0.0224 (0.0058)
ρ_a	0.9906 (0.0091)	0.9906 (0.0115)	0.9835 (0.0220)
ρ_e	0.9600 (0.0254)	0.9638 (0.0203)	0.9625 (0.0302)
ρ_z	0.9947 (0.0082)	0.9977 (0.0033)	0.9701 (0.0221)
σ_a	0.0551 (0.0509)	0.0558 (0.0661)	0.0495 (0.0660)
σ_e	0.0046 (0.0004)	0.0046 (0.0004)	0.0047 (0.0004)
σ_z	0.0050 (0.0004)	0.0052 (0.0005)	0.0046 (0.0005)
σ_r	0.0025 (0.0005)	0.0023 (0.0003)	0.0018 (0.0002)
Log-Likelihood	1425.01	1423.7	1428.8

Table 2. Standard Deviations (%)

		Benchmark Model	Separable Preferences ($\psi_2 = 0$)	
	Data	($\psi_2 \neq 0$)	No Habits	Habit Formation
		(1)	*(2)*	*(3)*
Variable				
y	1.81	5.03	7.50	3.04
m	1.96	1.88	1.72	1.73
π	0.43	0.66	0.65	0.55
r	0.70	1.00	0.99	0.83

Table 3. Variance Decomposition (one year ahead) (%)

	Output (y)				Real Balances (m)				Inflation (π)				Interest Rate (r)			
Model	ε_{a_t}	ε_{e_t}	ε_{z_t}	ε_{r_t}	ε_{a_t}	ε_{e_t}	ε_{z_t}	ε_{r_t}	ε_{a_t}	ε_{e_t}	ε_{z_t}	ε_{r_t}	ε_{a_t}	ε_{e_t}	ε_{z_t}	ε_{r_t}
(2)	0.2	0.2	99.4	0.2	0.9	99.0	0.0	0.1	48.1	19.8	0.0	32.1	92.6	4.7	0.1	2.5
(3)	50.3	0.1	49.4	0.2	1.6	97.9	0.4	0.0	45.2	14.7	15.0	25.1	79.6	3.6	14.6	2.1

Figure 1. Time Series Plots

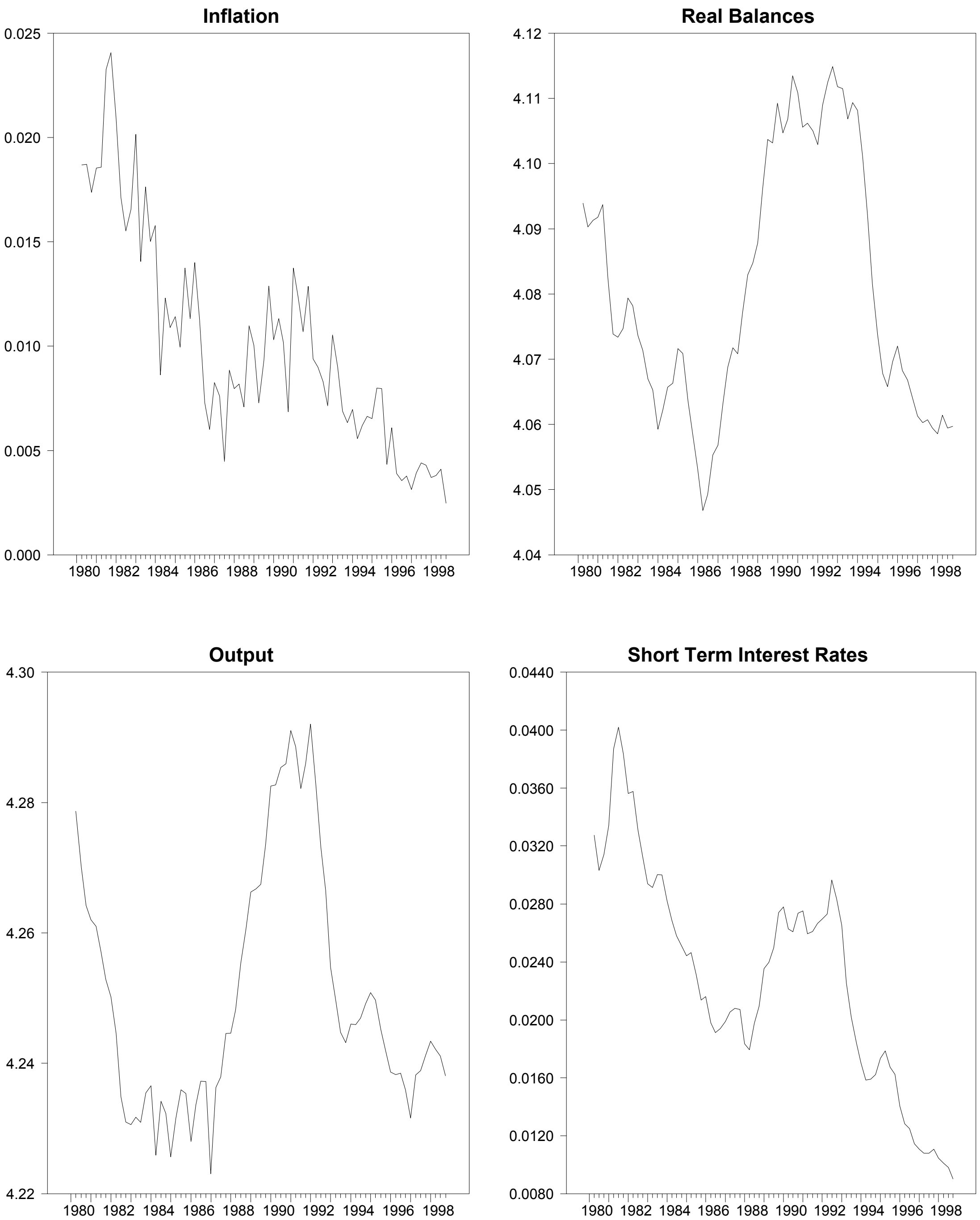
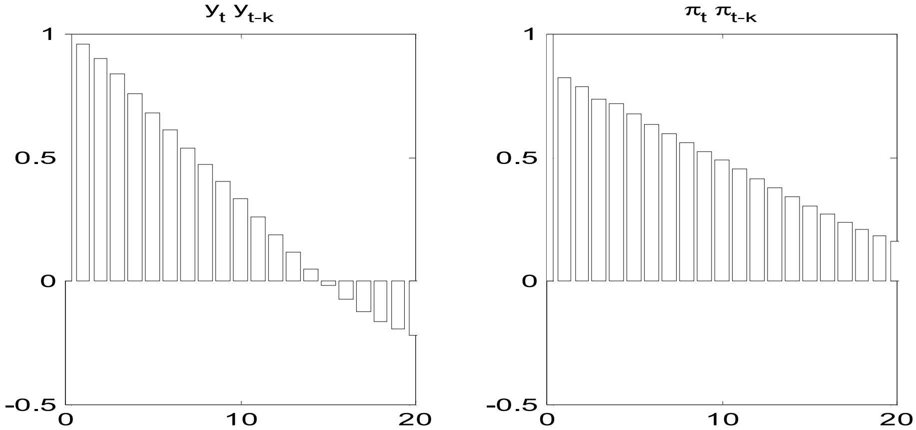
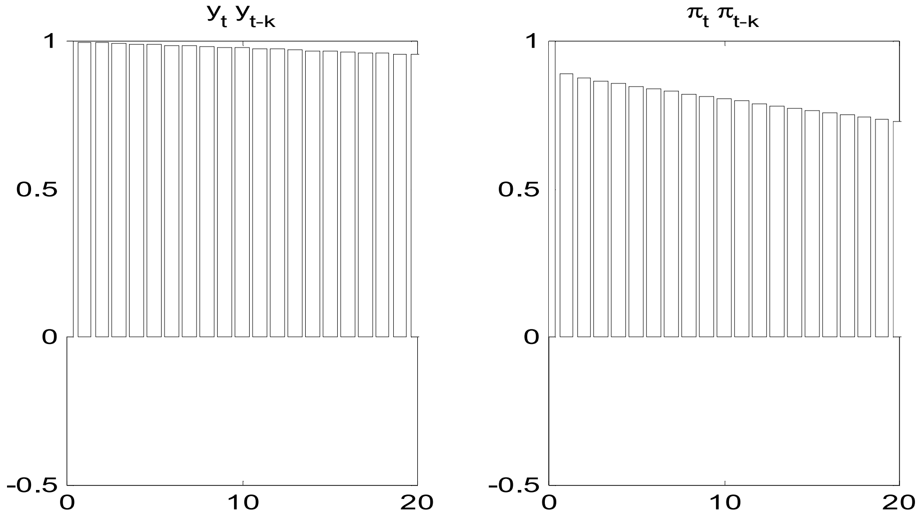


Figure 2. Autocorrelation Function of output and inflation

(a) Euro area



(b) Model without Habit Formation



(c) Model with Habit Formation

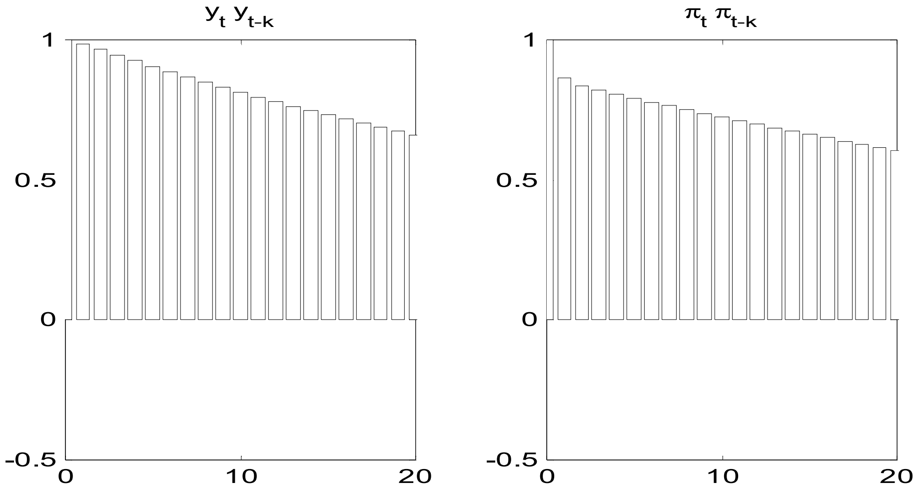
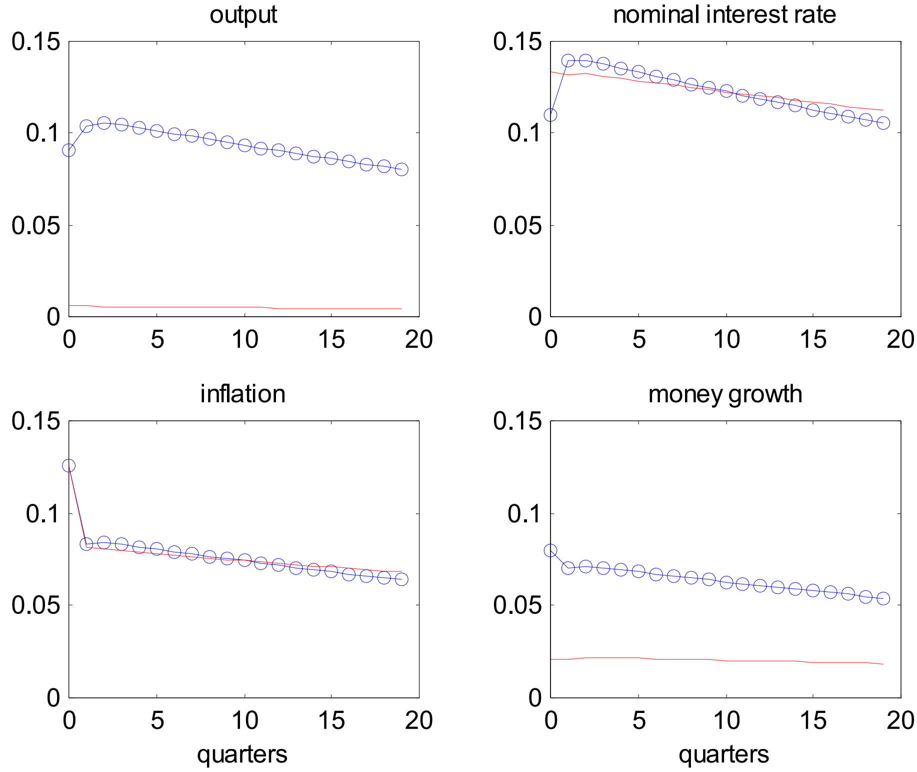
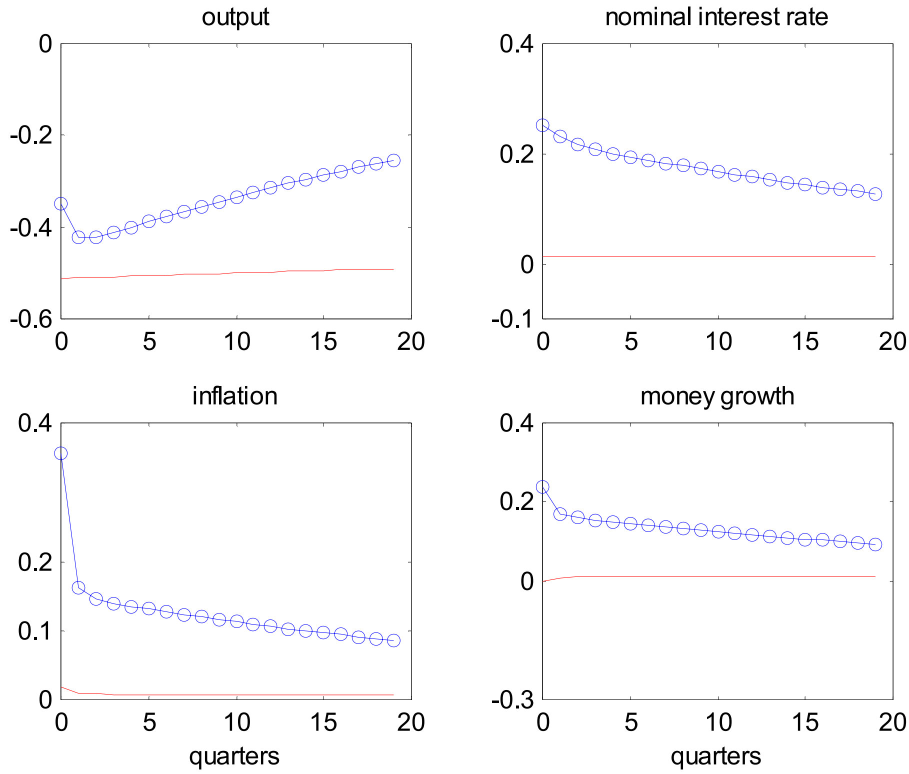


Figure 3. Impulse-Responses: Model without Habit Formation vs. Model with Habit formation

(a) Responses to a Preference Shock



(b) Responses to a Technology Shock



Note: circle line (model with habits) vs. continuous line (model without habits)