RATIONAL UNDERDEVELOPMENT

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Abstract

We propose a two-region two-sector model of uneven development, where technological change benefits either the lagging or the leading region. In this framework inter-regional transfers may lead to persistent underdevelopment; by raising wages without changing productivity, transfers reduce the chance of the backward region adopting a new technology and taking off. Due to uncertainty about which region benefits from technological change, the backward region may rationally choose to remain underdeveloped, while the advanced region continues to pay transfers. The Model provides a rationale for cases, such as Italy's Mezzogiorno, where the same rich region subsidizes the same poor region on a continuous basis.
1 Introduction

In many countries regional disparities are important and persistent, often in spite of — or maybe due to — large-scale public intervention. The standard textbook example is Italy, where the post-war convergence of the Mezzogiorno with the rest of the country grinded to a halt and reversed at the beginning of the 1970s. Since then GDP per capita in the North relative to the South has crept up steadily, from a minimum of 1.55 in 1971 to 1.74 in 1990. Explanations point to the South's loss of competitiveness, with relative unit labor costs increasing 23% between 1971 and 1990 (Attanasio and Padoa Schioppa, 1991).

Boltho, Carlin and Scaramazzino (1997) have related this erosion in competitiveness to a change in public policy, which shifted away from investment incentives to income support. This view is borne out by the data (Desmet, 2001). During the 1970s and the 1980s unemployment benefits as a share of value added in the South increased about twice as fast as in the North. Public employment shows much the same picture: starting off at similar levels in the beginning of the 1970s, over the next two decades the share of public employment grew double as fast in the South. The disproportionate share of public employment in the South is an important channel for regional redistribution: Alesina, Danninger and Rostagno (1999) estimate that about half of the Mezzogiorno's public wage bill can be viewed as a subsidy.

By raising equilibrium wages without improving productivity, these transfers have turned the South into a less attractive investment location, and have thus contributed to the persistent underdevelopment of the Mezzogiorno.¹ This, in its turn, has led to further transfers from North to South. But why would the North

¹Eastern Germany has followed a similar path since re-unification: wages have been increasing faster than productivity, with GDP per capita convergence stopping in its tracks (Sinn and Westermann, 2001). Union wage bargaining is one reason for Eastern Germany's labor costs being out of line with productivity; the other main culprit are transfers — which in 1999 accounted for over 30% of Eastern Germany's GDP — pushing up reservation wages.
be willing to subsidize the South on a continuous basis? And why would the South accept transfers which keep it trapped into underdevelopment?

In the literature inter-regional transfers have typically been justified as an insurance device between regions facing asymmetric shocks (Alesina and Perotti, 1995; Persson and Tabellini, 1996a, 1996b). Asymmetric shocks imply two-way transfers: in some periods region $A$ subsidizes region $B$, and in other periods $B$ subsidizes $A$. “Taking turns” is essential to sustain such risk sharing. This mechanism is therefore unable to explain the Mezzogiorno problem: one-way transfers, where the same rich region subsidizes the same poor region on a continuous basis. Our paper rationalizes such persistent one-way transfers between North and South.

Following Brezis, Krugman and Tsiddon (1993) and Desmet (2001), we propose a two-period model of uneven regional development. In the first period the economy is divided into a rich manufacturing North and a poor agricultural South. In the second period we introduce a new manufacturing technology, which can either locate in the backward South — attracted by its low wages — or in the advanced North — if spillovers from the previous technology are sufficiently strong to compensate for the higher wages. Inter-regional transfers raise wages in the backward region, reducing its chance to adopt the new technology and take off. With high enough transfers, the backward South never adopts the new technology, and is doomed to remain backward.

Levels of transfers that condemn the backward region to underdevelopment may Pareto dominate (lower) levels of transfers that give the backward region a chance to take off. This is what we call rational underdevelopment. The advanced region gives transfers to protect itself against low-wage competition, thus keeping the backward region from taking off and ensuring its dominant position; the backward region accepts those transfers — and rationally chooses to remain backward — because even without transfers it is not sure to benefit from the new technology. In
this framework transfers serve a two-fold purpose: they provide insurance against the uncertain effects of technological change, and they lead to consumption smoothing between the first and the second period.

The permanent one-way transfers between the rich North and the poor South are a central feature of rational underdevelopment. One-way transfers have been studied before in the literature. Spilimbergo (1999), for instance, proposes a model where the North pays unemployment benefits to the South in an attempt to raise wages and stem immigration. The transfers in Spilimbergo do not contribute to the South’s backwardness though. Our model therefore goes one step further by rationalizing the widely held view that public policy, rather than solving the “Southern question”, forms an integral part of the Mezzogiorno’s failure (Boltho et al., 1997; Alesina et al., 1999). In the absence of transfers the South has a chance of taking off; by accepting transfers, the South gives up this opportunity.

Another justification for one-way transfers, often invoked by federal governments, is the need for inter-regional solidarity and social cohesion to keep countries together. In Canada, for instance, equalization transfers between provinces are explicitly mentioned in the federal constitution (Bayoumi and Masson, 1995). In our model we limit ourselves to economic motives driving inter-regional transfers; we do not enter the political economy debate of what defines countries.

2 The model

The basic structure of the model follows closely Brezis, Krugman and Tsiddon (1993) and Desmet (2001). Consider two regions, North and South. Variables referring to the South are denoted by an asterisk. Labor is the only factor of production and is

\[^2\text{Likewise, Eastern Germany's unsuccessful bid to catch up with the rest of the country has been largely blamed on misplaced government policy (Sinn, 2000; Sinn and Westermann, 2001).}\]

\[^3\text{For an overview, see Alesina, Perotti and Spolaore (1995).}\]
geographically immobile. The size of the labor force is the same in both regions:

\[ L = L^* = 1 \]

Let \( Q_F (Q_M) \) denote output of food (manufactures), and \( L_F (L_M) \) labor employed in food (manufactures). Food technologies are identical in North and South:

\[ Q_F = L_F \]
\[ Q_F^* = L_F^* \]

Manufacturing production, however, is subject to region-specific learning externalities, so that manufacturing technologies differ across regions:

\[ Q_M = A L_M \]
\[ Q_M^* = A^* L_M^* \]

All consumers have identical preferences:

\[ U(C_M, C_F) = v(C_M^{\mu} C_F^{1-\mu}) \]

where \( v \) is a strictly increasing and strictly concave function; \( C_F \) and \( C_M \) denote the consumption of food and manufactures; and \( \mu > 1/2 \). Consumers, therefore, are risk averse, and spend a fraction \( \mu \) of their income on manufactures. After normalizing the food price to 1, the inverse demand function of manufactures relative to food is:

\[ \frac{C_M}{C_F} = \frac{\mu}{1 - \mu p_M} \]

where \( p_M \) denotes the manufacturing price.

---

4 Since there are no skill differences between North and South, uneven development would disappear if labor were perfectly mobile. In that sense our results are more pertinent for countries with limited labor mobility. A similar outcome could be obtained though in a model with skill differences and labor mobility (see Desmet, 2000).

5 This ensures uneven development between both regions. Without this assumption, both regions would produce food, and wages would equalize.
The model consists of two periods: in period 1 productivity differences lead the economy to diverge into a rich manufacturing North and a poor agricultural South; in period 2 a new manufacturing technology is introduced, which either reinforces or reverses this pattern of uneven development. In what follows we study in turn each period.

2.1 Period 1

In period 1 the economy is fully specialized: the North produces manufactures, and the South produces food. Plugging production into (6) gives us the equilibrium manufacturing price:

\[ p_M = \frac{\mu}{1 - \mu A} \]  

(7)

For full specialization to be an equilibrium, nobody in the South should have an incentive to switch to manufacturing production, so that:

\[ 1 > \frac{\mu A^*}{1 - \mu A} \]  

(8)

In other words, the productivity advantage of the North needs to be sufficiently large compared to the relative importance of manufacturing consumption, so that the North is able to satisfy on its own all of the economy’s manufacturing demand. Likewise, no workers in the North should have an incentive to switch to food; this result is immediate, since \( \mu > 1/2 \). Given the manufacturing price (7), relative wages are:

\[ \frac{w}{w^*} = \frac{\mu}{1 - \mu} \]  

(9)

The economy is therefore divided into a rich manufacturing North and a poor agricultural South.

The central government now reduces regional inequality by limiting the rich region’s relative wage to \( \alpha \), where \( 1 \leq \alpha \leq \frac{\mu}{1 - \mu} \); the smaller \( \alpha \), the greater the degree
of redistribution. If \( \alpha = \frac{\mu}{1-\mu} \), we are in the laissez faire case with no inter-regional transfers; if \( \alpha = 1 \), we are at the other extreme, with maximum redistribution and wages equalizing across regions.

The value of \( \alpha \) is decided at the beginning of the first period, and lasts until the end of the second period. In a two-period model — unlike in an infinite horizon model — such a policy is time-inconsistent: the rich region has no incentive to subsidize the poor region in the second period. Although for reasons of simplicity we stick to a setup with two periods, we should think of our model as the reduced form of a time-consistent infinite horizon model. Appendix A discusses this point in further detail.

Inter-regional redistribution is implemented by taxing manufacturing workers in the rich region, and subsidizing food workers in the poor region. The redistribution policy does not affect production.\(^6\) Given homothetic preferences, the relative manufacturing price does not change either. Only wages are affected by redistribution.

Since in the first period Northern taxes subsidize Southern workers, wages drop in the North and rise in the South:

\[
\begin{align*}
\text{w} &= p_M A - t \\
\text{w}^* &= 1 + t^*
\end{align*}
\]

where the tax \( t \) and the subsidy \( t^* \) are such that:

\[
\frac{\text{w}}{\text{w}^*} = \alpha
\]

Assuming a balanced budget, we have:

\[
t = t^*
\]

\(^6\)To see this, note that nobody has an incentive to switch sectors: in the North manufacturing wages continue to be higher than food wages since \( \alpha \geq 1 \); in the South nobody had an incentive to switch to manufacturing in the absence of transfers, so that \( \text{a fortiori} \) nobody wants to switch once transfers are introduced.
Using (7) and (10)-(13), we can derive wages, as a function of $\alpha$, in both regions:

$$w = \frac{1}{1 - \mu} \frac{\alpha}{1 + \alpha}$$  \hspace{1cm} (14)

$$w^* = \frac{1}{1 - \mu} \frac{1}{1 + \alpha}$$  \hspace{1cm} (15)

This translates into the following utilities for North and South in period 1:

$$U_1(\alpha) = v\left(\frac{\alpha}{1 + \alpha} A^\mu\right)$$  \hspace{1cm} (16)

$$U_1^*(\alpha) = v\left(\frac{1}{1 + \alpha} A^\mu\right)$$  \hspace{1cm} (17)

Not surprisingly, redistribution increases welfare in the South, and decreases welfare in the North.

Higher wages make the South less attractive for new industries.\textsuperscript{8} Inter-regional transfers may thus contribute to persistent underdevelopment. The next section will develop this argument in detail. Though the focus will be on wage subsidies, we will claim that other types of transfers, such as unemployment benefits and public employment, have the same effect.

### 2.2 Period 2

In period 2 a new manufacturing technology is exogenously introduced. Although neither region has any direct experience with the new technology, the North benefits from learning spillovers coming from the old technology. These spillovers may be large or small, depending on the similarity between the two technologies.

As can be seen in Figure 1, with probability $p$ spillovers are large, and the North's productivity in the new technology is high: $a_n > A$; with probability $1 - p$...
spillovers are small, and the North’s productivity in the new technology is low: \( a_t < A \). Since the South does not benefit from learning spillovers,\(^9\) its productivity in the new technology is in either case below that of the North: \( a^* < a_t < a_h \).

\[
\begin{align*}
\text{(Large spillovers)} & \quad \text{North adopts: } (U_{2h}, U_{2n}^*) \\
\text{p} & \quad \text{South adopts: } (U_{2y}, U_{2y}^*) \\
\text{(Small spillovers)} & \quad \text{Nobody adopts: } (U_{2n}, U_{2n}^*)
\end{align*}
\]

\[\alpha > \frac{A}{\mu a^*} - 1\]

\[\alpha \leq \frac{A}{\mu a^*} - 1\]

Figure 1: Adoption of new technology.

Once a region adopts the new technology, it starts accumulating experience and moves up the new technology’s learning curve. Productivity increases until it reaches a maximum \( \bar{A} > A \), at which point learning gets exhausted. For reasons of simplicity — and without loss of generality — we assume that learning happens instantaneously. This allows us to ignore the transition phase, so that the model boils down to a simple discrete two-period problem.\(^10\)

The effect of the new technology on the development of the North and the South depends on where the new technology locates. In what follows we distinguish between three cases: the North adopts the new technology, the South adopts the

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\(^9\)There are no spillovers between the food technology and the new manufacturing technology.

\(^10\)For a full-blown continuous-time approach, which explicitly takes into account the learning dynamics, see Brezis, Krugman and Tsiddon (1993) and Desmet (2001).
new technology; or neither region adopts the new technology.

**Case 1: the North adopts the new technology**

In the case of large spillovers, which occurs with probability $p$, the North adopts the new technology (since $a_N > A$), whereas the South remains stuck in agriculture.\(^{11}\) The specialization pattern has not changed; the only difference is that the North now uses a superior manufacturing technology. Given learning happens instantaneously, period 2 utilities for North and South are, by analogy with (16) and (17):

$$U_{2N}(\alpha) = v\left(\frac{\alpha}{1 + \alpha} A^\mu\right)$$  \hspace{1cm} (18)

$$U_{2S}^*(\alpha) = v\left(\frac{1}{1 + \alpha} A^\mu\right)$$  \hspace{1cm} (19)

Spillovers have allowed the North to attract the new technology in spite of its higher wages. The North reinforces its dominant position, whereas the South remains trapped in underdevelopment.

**Case 2: the South adopts the new technology**

In the case of small spillovers, which occurs with probability $1 - p$, the North does not adopt the new technology (since $a_N < A$). The South, however, does adopt the new technology if its workers can earn higher wages by switching from food to the new technology. As can be seen in Figure 1, this happens if $p_M a^* > \frac{1}{1 - \mu \frac{1}{1 + \alpha}}$, or,\(^{11}\) Of course the South may also adopt the new technology if, by doing so, its workers earn higher wages, i.e., if $\mu a^*_N > \frac{1}{1 + \alpha}$. We will assume this condition is never satisfied. See Desmet (2001) for a more detailed discussion of this possibility.
equivalently, if redistribution is relatively limited:  

\[ \alpha > \frac{A}{\mu a^*} - 1 \]  

(20)

Once the South adopts the new technology, it starts moving up its learning curve. Since the new technology is superior to the old technology, the South overtakes the North and the specialization pattern reverses: the South becomes rich and industrialized, and the North poor and agricultural. Assuming learning happens instantaneously, period 2 utilities in North and South are:

\[ U_{2x}(\alpha) = v \left( \frac{1}{1 + \alpha} \tilde{A}^\nu \right) \]  

(21)

\[ U_{2y}(\alpha) = v \left( \frac{\alpha}{1 + \alpha} \tilde{A}^\nu \right) \]  

(22)

The new technology has located in the low-wage South, helping that region to take off and leapfrog. This picture fits the example of Belgium, where during the 1960s the rural North overtook the industrialized South. As the traditional heavy industries of the South declined, new activities, such as plastics and lighter metals, located in the North, attracted by its low wages. This eventually led to leapfrogging: whereas in 1950 GDP per capita in the North was still 14% lower than in the South, by 1990 it was 31% higher.

Though not profitable from an individual agent’s point of view, adopting the new technology in the North may be welfare-improving for that region. Intervention

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12Note that condition (20) takes the manufacturing price as given. It therefore represents the incentive for a worker in the South to adopt the new technology, assuming nobody else does. Although the manufacturing price would of course change if expectations were that others would adopt the new technology too, this would not strengthen the incentive to adopt. If others would switch to the new technology, the manufacturing price would drop, making adoption of the new technology less attractive. Therefore, for at least one worker to adopt the new technology in equilibrium, condition (20) must hold.

13We assume the condition for full specialization is satisfied.

14The subscript \( y \) stands for "yes, the South adopts".
by the regional government may thus be called for. We discard such intervention arguing that the government cannot possibly know the future value of a new technology. This may seem to contradict our assumption that redistribution is decided knowing the new technology's productivity \( \hat{A} \). However, think of our model as the reduced form of a more realistic setup where many new technologies emerge, most of which flounder. The government knows that one of those new technologies will have productivity \( \hat{A} \), but it does not know which one. In that case subsidies across the board may be too expensive, whereas “picking winners” may be impossible because of informational limitations;\(^{15}\) attempts suggest that government failure typically outweighs market failure.\(^{16}\)

**Case 3: neither region adopts the new technology**

The third possibility consists of neither region adopting the new technology. As suggested by Figure 1, this occurs when spillovers are small (in the North the old technology remains more productive) and when redistribution is relatively high (in the South wages continue to be higher in the food sector):

\[
\alpha \leq \frac{A}{\mu a^*} - 1
\]

(23)

In that case nothing changes; period 2 utilities in North and South coincide with period 1 utilities:\(^{17}\)

\[
U_{2n}(\alpha) = v\left(\frac{\alpha}{1 + \alpha} A^n\right)
\]

(24)

\[
U_{2n}^*(\alpha) = v\left(\frac{1}{1 + \alpha} A^n\right)
\]

(25)

\(^{15}\)For a survey, see Grossman (1990). Similar problems arise when trying to implement policies to support infant industries in developing countries. See Grubel (1993) for theoretical arguments, and Krueger and Tuncer (1982) for one of the few empirical tests.

\(^{16}\)We borrow this turn of phrase from Krueger (1990).

\(^{17}\)The subscript \( n \) stands for “no, the South does not adopt”.
Whether the economy is in Case 2 or in Case 3 depends on whether condition (20) or (23) is satisfied. The degree of redistribution therefore determines whether the South has a chance to develop and leapfrog. If redistribution is too high (Case 3), the South is sure to remain backward; at lower levels of redistribution (Case 2), the South may take off. This allows us to define the following two subsets of $\alpha$:

**Definition 1.** For a given $\mu$, $a^*$ and $A$, we define $\Gamma$ as the set of $\alpha$ for which the South never adopts the new technology (condition (23) holds); and we define $\Gamma^c$ as the set of $\alpha$ for which the South has a chance of adopting the new technology (condition (20) holds).

The effect of subsidies to food workers is straightforward: by raising wages in the South, subsidies make the region less attractive for new industries, thus leading to persistent underdevelopment. This suggests, for instance, that the European Union’s income support system for farmers contributes to keeping poor agricultural regions poor. Our results are not limited to the specific example of wage subsidies though. Any transfers that increase wages without improving productivity — such as unemployment benefits or public employment — have a similar effect (Desmet, 2001). This of course suggests that the South would prefer transfers that do not diminish its chance of attracting new technologies. The North may be uninterested though: one of the North’s incentives in paying transfers is to keep the South from taking off, and thus protect itself against low-wage competition.\(^{18}\) In the case of German reunification, the West was willing to put up with high transfers to the East

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\(^{18}\) As an example, compare the transfer policy in this paper (wage subsidies) to a transfer policy that does not distort the South’s incentive to adopt new technologies (e.g., consumption subsidies). Assume, as above, that transfers are set to reduce income (or consumption) inequality to a level which remains constant for the two periods. For all the parameter values used in the numerical exercises in Section 4, it turns out that the North always prefers transfers leading to rational underdevelopment to any level of transfers which does not affect the South’s incentive to switch to new technologies.
as long as such policies protected Western jobs (Sinn and Sinn, 1992).\textsuperscript{19}

2.3 Putting the two periods together

Let $\beta$ be the common discount factor for both regions.\textsuperscript{20} The discounted sum of expected utility in the North depends on whether the South leapfrogs or not:

$$
U_\gamma(\alpha) = U_1(\alpha) + \beta[pU_{2\gamma}(\alpha) + (1 - p)U_{2\gamma}(\alpha)]
$$

(26)

$$
U_n(\alpha) = U_1(\alpha) + \beta[pU_{2n}(\alpha) + (1 - p)U_{2n}(\alpha)]
$$

(27)

Similarly, the discounted sum of expected utility in the South depends on whether it takes off or not:

$$
U^*_\gamma(\alpha) = U^*_1(\alpha) + \beta[pU^*_{2\gamma}(\alpha) + (1 - p)U^*_{2\gamma}(\alpha)]
$$

(28)

$$
U^*_n(\alpha) = U^*_1(\alpha) + \beta[pU^*_{2n}(\alpha) + (1 - p)U^*_{2n}(\alpha)]
$$

(29)

Summarizing, total expected utility in North and South can be written as:

$$
U(\alpha) = \begin{cases} 
U_\gamma(\alpha) & \text{if } \alpha \in \Gamma^c \\
U_n(\alpha) & \text{if } \alpha \in \Gamma
\end{cases}
$$

(30)

$$
U^*(\alpha) = \begin{cases} 
U^*_\gamma(\alpha) & \text{if } \alpha \in \Gamma^c \\
U^*_n(\alpha) & \text{if } \alpha \in \Gamma
\end{cases}
$$

(31)

3 Rational underdevelopment

High enough transfers ensure the South remains backward. We now want to show that such transfers may be Pareto superior to any other level of transfers where the South takes off. In other words, the South chooses to remain poor, and the North chooses to continue paying transfers. This is what we call rational underdevelopment.

\textsuperscript{19}Op. cit, pages 164-168. Setting union wages in the East equal to those in the West caused widespread unemployment in the new Eastern Länder, prompting important flows of transfers from the richer West.

\textsuperscript{20}Taking a common discount factor tends to bias the results against our favor. If the South were to have a lower discount factor than the North — a reasonable assumption since the South is poorer — it would be more willing to give up development tomorrow for income today.
Definition 2. A redistribution policy $\alpha \in \Gamma$ leads to rational underdevelopment if $U(\alpha) \geq U(\alpha')$ and $U^*(\alpha) \geq U^*(\alpha')$ for all $\alpha' \in \Gamma^c$. We denote by $\Gamma^{RU}$ the set of such redistribution policies.

Our goal in this section is to show that the set of redistribution policies leading to rational underdevelopment is not empty. Such transfers serve as an insurance device for both regions. By giving transfers, the North keeps the South backward, thus making sure it remains in the lead. The South, on the other hand, prefers the certainty of transfers — and relative backwardness — to the uncertainty of taking off. In addition to risk sharing, transfers lead to consumption smoothing: the North gives up income in the first period for higher expected income in the second period, and the opposite happens in the South. Although both motives cannot be neatly separated in our particular setup, arguably risk sharing is the dominant factor. In the absence of uncertainty, consumption smoothing would most likely occur through private capital markets, without affecting the incentives to adopt new technologies.21

Figure 2 gives a graphical example of rational underdevelopment. It shows utility in function of redistribution. For high levels of redistribution — $\alpha \in \Gamma$ — the South never adopts the new technology. In that case the North wants to minimize redistribution, whereas the South wants to maximize redistribution; this shows up in the North’s utility increasing in $\alpha$ and the South’s utility decreasing in $\alpha$. For low levels of redistribution — $\alpha \in \Gamma^c$ — both the North and the South have a chance of attracting the new industry. At the cut-off point between $\Gamma$ and $\Gamma^c$ the utility functions experience a discrete jump. Since the North prefers the South not

\[21\text{Moreover, we could easily generate rational underdevelopment in a framework without consumption smoothing. To see this, consider a two-period model with a new technology arriving in each period. In order to pool risk, regions decide at the beginning of the first period (before the first technological shock) on a redistribution scheme for the two periods. If redistribution affects wages, it will also affect technology adoption.}\]
adopting the new technology, the North’s utility at the cut-off point between $\Gamma$ and $\Gamma^c$ is such that $U_n(\alpha) > U_S(\alpha)$. By analogy, the opposite happens in the South.

Rational underdevelopment occurs if there are levels of redistribution where the South is sure to remain backward, and which are Pareto superior to any other level of redistribution where the South has a chance of taking off. This corresponds to set $\Gamma^{RU} \subset \Gamma$ in Figure 2: for all $\alpha \in \Gamma^{RU}$, utilities in North and South are superior to utilities associated to any $\alpha \in \Gamma^c$.

Before analytically proving that rational underdevelopment may exist, we put some more structure on the problem. First, for the North to have any interest in paying transfers, it should be uncertain to attract the new technology, and it should care about this second period uncertainty:

**Condition 1.** $p < 1$ and $\beta > 0$

---

$^{22}$We are thus assuming that the new technology’s productivity increase is not too large; Condition 2 discusses this point in further detail.
Second, if the new technology’s productivity gain is too large, the North may experience welfare gains from the South switching to the new technology; although the North would lose out on the new technology, it would be more than compensated by the decline in the relative manufacturing price. Rational underdevelopment would not occur because the North would prefer the South adopting the new technology to nobody adopting the new technology. To avoid this case, we put an upper limit on the new technology’s productivity gain:

**Condition 2.** \( \left( \frac{\mu}{1-\mu} \right)^{\alpha} < \frac{\mu}{1-\mu} \)

Third, focusing on the range of redistribution policies for which neither region adopts the new technology, the North’s maximum level of redistribution should be higher than the South’s minimum level of redistribution. The North’s maximum level of redistribution \( \hat{\alpha} \) is the level above which it prefers zero transfers; the South’s minimum level of redistribution \( \hat{\alpha}^* \) is the level below which it prefers zero transfers. Therefore, \( \hat{\alpha} \) is the solution to \( U_n(\hat{\alpha}) = U_y\left(\frac{\mu}{1-\mu}\right) \); and \( \hat{\alpha}^* \) is the solution to \( U_n^*(\hat{\alpha}^*) = U_y\left(\frac{\mu}{1-\mu}\right) \). This gives us the third condition:

**Condition 3.** \( \hat{\alpha} < \hat{\alpha}^* \)

We do not provide the complete set of parameters which satisfy Condition 3, because that set depends in a non-trivial way on \( p, \mu, A, A^*, \beta \) and the specific utility function \( v \).

It is, however, easy to see that higher risk aversion increases the demand for transfers in both regions, and thus favors Condition 3. To illustrate this point, compare the cases of no risk aversion and very high risk aversion. In the case of no risk aversion, transfers never increase the economy’s total utility, so that they cannot

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\[ \text{If } U_n(\alpha) > U_y\left(\frac{\mu}{1-\mu}\right) \text{ for all } \alpha \in \Gamma \text{ we take } \hat{\alpha} = 1; \text{ and if } U_n^*(\alpha) > U_y\left(\frac{\mu}{1-\mu}\right) \text{ for all } \alpha \in \Gamma \text{ we take } \hat{\alpha}^* = \frac{\Delta^*}{\beta} - 1. \text{ Likewise, if } U_n(\alpha) < U_y\left(\frac{\mu}{1-\mu}\right) \text{ for all } \alpha \in \Gamma \text{ we take } \hat{\alpha} = \frac{\Delta}{\beta} - 1; \text{ and if } U_n^*(\alpha) < U_y\left(\frac{\mu}{1-\mu}\right) \text{ for all } \alpha \in \Gamma \text{ we take } \hat{\alpha}^* = 1. \text{ Since } U_n(\alpha) \text{ is strictly increasing and } U_n^*(\alpha) \text{ is strictly decreasing } \hat{\alpha} \text{ and } \hat{\alpha}^* \text{ are well defined.} \]
make both regions better off. One region’s gain is always the other region’s loss, so that Condition 3 is never satisfied. As risk aversion goes up, the possibility of a “bad” outcome starts to weigh heavier in a region’s utility: the North becomes increasingly concerned about falling behind, and the South becomes increasingly worried about being backward. Since transfers mitigate the “bad” outcome in both regions, at high levels of risk aversion it becomes easier to find a set of transfer policies which are welfare improving for both North and South, so that Condition 3 becomes more likely to hold.24

We are now ready to prove analytically that under certain conditions rational underdevelopment exists:

**Theorem 1.** Let Conditions 1, 2 and 3 hold. Let all parameters but $a^*$ be fixed. Then, there exists $k \leq A$, such that for all $a^* \leq k$ we have rational underdevelopment, i.e. the set $\Gamma^{RU}$ is not empty.

**Proof.** See Appendix B.

Summing up, rational underdevelopment tends to occur if productivity gains deriving from the new technology are modest (Condition 2); risk aversion is high (Condition 3); and the new technology’s initial productivity in the South is low (Theorem 1). All these conditions have been discussed before, except the last one, which has an easy interpretation: for a low value of $a^*$ the South only adopts the new technology if its wages are low too. In that case a modest transfer scheme may be enough to

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24We now show this result for the particular case where $a^* = \frac{1-\mu}{\mu} A$. (The result can easily be generalized.) Take the South and determine $\bar{\sigma}$ by solving $v\left(\frac{1}{1+\delta} A^\mu\right) + \beta\left[pv\left(\frac{1}{1+\delta} \bar{A}^\mu\right) + (1-p)v(\mu A^\mu)\right] = v((1-\mu)A^\mu) + \beta\left[pv((1-\mu)\bar{A}^\mu + (1-p)v(\mu A^\mu)\right]$. As risk aversion increases, agents become more worried about the “bad” outcome: the left hand side of the above equation tends to $(1+\beta)v\left(\frac{1}{1+\delta} A^\mu\right)$ and the right hand side tends to $(1+\beta)v((1-\mu)A^\mu)$. As risk aversion goes to infinity, the problem boils down to solving $\frac{1}{1+\delta} A^\mu = (1-\mu)A^\mu$, so that $\bar{\sigma} = \frac{\mu}{1+\delta}$. In other words, any level of transfers makes the South better off. Following the same methodology for the North, as risk aversion goes to infinity, $\bar{\sigma}$ is the solution to $\frac{\mu}{1+\delta} A^\mu = (1-\mu)\bar{A}^\mu$, so that $\bar{\sigma} = \frac{\mu}{1+\delta}$. In other words, any level of transfers makes the North better off. Following the same methodology for the North, as risk aversion goes to infinity, $\bar{\sigma}$ is the solution to $\frac{\mu}{1+\delta} A^\mu = (1-\mu)\bar{A}^\mu$, so that $\bar{\sigma} < \frac{\mu}{1+\delta}$, so that $\bar{\sigma} < \bar{\sigma}^*$. 

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keep the South from attracting the new technology. Some of these points will now be discussed using numerical examples.

4 Some numerical illustrations

Consider the following baseline case. Take the CRRA utility function \( v = \frac{e^{1-r}}{1-r} \). Following Mehra and Prescott (1985) we restrict values for \( \rho \) to be less than 10\(^{-2}\); as a starting point we take \( \rho = 5 \). We choose GDP per capita in the rich region to be 50% higher than in the poor region, implying \( \mu = 0.6 \). This is reasonable: using data of 1996, in Italy GDP per capita in the North was 79% higher than in the South; in Spain the East had an advantage of 53% over the South; and in Belgium the difference was of 29% between the North and the South (Eurostat, 2000). U.S. figures are similar: in 1998 GSP per capita in Massachusetts stood 72% above that of Mississippi (Bureau of Economic Analysis).

We normalize \( A = 1 \) and set \( \hat{A} = 1.1 \). Comparing the South adopting the new technology with the North adopting the new technology, these figures imply an annual real productivity growth rate over a period of 25 years which is on average 1.6 percentage points higher in the South. This is roughly in line with differences in TFP growth rates across countries (Young, 1995). For the new technology's initial productivity in the South we take \( a^* = 0.7 \); this implies an annual productivity increase of around 2% if the South adopts the new technology.

Following standard practice, we assume an annual real interest rate of 4%, which leads to an annual discount factor of around 0.96. If we think of new technologies arising once every 25 years, this gives a value for \( \beta \) of 0.375. Finally, for want of a specific prior, we set the probability of the North adopting the new technology \( p = 0.5 \).

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\(^{25}\)Kandel and Stambaugh (1991) claim that larger values of \( \rho \) should not be ruled out. See Campbell (1999) for a summary of the empirical literature on the coefficient of risk aversion.
Table 1: Rational underdevelopment for different parameter values

<table>
<thead>
<tr>
<th>(1) Risk aversion</th>
<th>(2) Discount factor</th>
<th>(3) New techn. adopts</th>
<th>(4) North Productivity</th>
<th>(5) Rational tax rate</th>
<th>(6) Rational underdev't North</th>
<th>(7) Tax rate North</th>
</tr>
</thead>
<tbody>
<tr>
<td>ρ</td>
<td>β</td>
<td>A</td>
<td>p</td>
<td>α*</td>
<td>Γ^{RU}[α_1, α_2]</td>
<td>[\frac{f(α_1)}{u(α_1)}, \frac{f(α_2)}{u(α_2)}]</td>
</tr>
<tr>
<td>1. Baseline case</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.375</td>
<td>1.10</td>
<td>0.5</td>
<td>0.7</td>
<td>1.218, 1.312</td>
<td>0.085, 0.054</td>
</tr>
<tr>
<td>2. Risk aversion</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.375</td>
<td>1.10</td>
<td>0.5</td>
<td>0.7</td>
<td>1.128, 1.338</td>
<td>0.117, 0.046</td>
</tr>
<tr>
<td>3</td>
<td>0.375</td>
<td>1.10</td>
<td>0.5</td>
<td>0.7</td>
<td>1.284, 1.292</td>
<td>0.063, 0.061</td>
</tr>
<tr>
<td>2</td>
<td>0.375</td>
<td>1.10</td>
<td>0.5</td>
<td>0.7</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3. Discount factor</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.375</td>
<td>1.10</td>
<td>0.5</td>
<td>0.7</td>
<td>1.251, 1.324</td>
<td>0.074, 0.050</td>
</tr>
<tr>
<td>5</td>
<td>0.4</td>
<td>1.10</td>
<td>0.5</td>
<td>0.7</td>
<td>1.208, 1.309</td>
<td>0.088, 0.055</td>
</tr>
<tr>
<td>4. Productivity of new technology</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.375</td>
<td>1.2</td>
<td>0.5</td>
<td>0.7</td>
<td>1.269, 1.307</td>
<td>0.068, 0.056</td>
</tr>
<tr>
<td>5</td>
<td>0.375</td>
<td>1.4</td>
<td>0.5</td>
<td>0.7</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4. Probability of North adopting</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.375</td>
<td>1.10</td>
<td>0.6</td>
<td>0.7</td>
<td>1.251, 1.327</td>
<td>0.071, 0.050</td>
</tr>
<tr>
<td>5</td>
<td>0.375</td>
<td>1.10</td>
<td>0.4</td>
<td>0.7</td>
<td>1.181, 1.299</td>
<td>0.097, 0.058</td>
</tr>
<tr>
<td>5</td>
<td>0.375</td>
<td>1.10</td>
<td>0.5</td>
<td>0.67</td>
<td>1.218, 1.411</td>
<td>0.085, 0.025</td>
</tr>
<tr>
<td>5</td>
<td>0.375</td>
<td>1.10</td>
<td>0.5</td>
<td>0.73</td>
<td>1.218, 1.224</td>
<td>0.085, 0.083</td>
</tr>
</tbody>
</table>

Column (6) in Table 1 gives the set of redistribution policies $Γ^{RU}$ consistent with rational underdevelopment. In the baseline case rational underdevelopment occurs for transfers that reduce the North’s relative income from 1.5 to a level ranging from 1.218 to 1.312. This corresponds to a 38% to 56% decrease in regional inequality. This reduction in inequality is substantial, though not unusual. Estimates for
Canadian provinces in the 1980s find a figure of 44% (Bayoumi and Masson, 1995). The two extremes of our range of redistribution policies have an easy interpretation: 1.218 corresponds to the maximum level of redistribution acceptable to the North; and 1.312 corresponds to the minimum level of redistribution acceptable to the South. Column (7) in Table 1 gives the tax rates in the North required to finance redistribution: to limit relative income to 1.218 we need a tax rate of 8.5% in the North; to limit relative income to 1.312 that tax rate drops to 5.4%.

As a robustness check, and to gain further insight into the role of the different parameters, we now look at some variations of the baseline case. As expected, higher risk aversion increases the range of redistribution policies consistent with rational underdevelopment. At greater levels of risk aversion the North is willing to pay higher transfers to remain in the lead, and the South is ready to accept lower transfers, since staying backward becomes more costly.

Changing the discount factor has contrary effects in North and South. A smaller discount rate increases the relative importance of the first period, so that the North wants less redistribution (since it is worried about insuring itself in the second period) and the South wants more redistribution (since it is the poor region in the first period).

Greater technological progress makes rational underdevelopment less likely. Rational underdevelopment occurs when the South foregoes the chance of adopting the new technology in exchange for transfers. This stance becomes more costly for the South if the new technology's productivity gain is large. In that case also the North becomes less likely to underwrite Southern backwardness: if productivity gains are substantial, the North actually gains from the South adopting the new technology.

Not surprisingly, varying the probability of the new technology locating in

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26 This is an upper bound, since not all redistributive policies decrease the chance of poorer regions taking off.
the North has an asymmetric effect on both regions. As it becomes more likely for the North to adopt the new technology, the South wants more insurance, and the North wants less. Finally, increasing the initial productivity of the new technology in the South limits the possibility of rational underdevelopment; it now takes bigger transfers from the North to convince Southern workers not to switch to the new technology.

All of this suggests that rational underdevelopment is more probable if risk aversion is relatively high, and if the new technology's productivity gains are relatively limited. This does not imply choosing unreasonably high risk aversion or unrealistically low productivity gains though. As shown by the examples in Table 1, rational underdevelopment is consistent with a range of plausible parameter values.

5 Concluding remarks

In this paper we have proposed a model that explains rational underdevelopment. Fiscal transfers raise wages in the backward region, thus reducing its chances to attract new technologies and take off. With high enough transfers, the backward region is sure to remain backward. If this situation is stable, we have rational underdevelopment: the leading region pays transfers to make sure it remains in the lead, and the lagging region accepts those transfers because it is not sure to benefit from the new technology.

Our analysis rests on a number of crucial assumptions. First, wage subsidies affect technology adoption because workers are the ones to decide whether or not to adopt the new technology. If, instead, firms were to take the decision, the results would change. Second, we look at transfers that raise wages without raising productivity. Whereas wage subsidies, unemployment benefits and public employment would clearly fall under this category, the European Union's structural funds for backward regions would not: those funds subsidize infrastructure and human
capital, and therefore have a positive effect on productivity. Third, we assume that the adoption of new technologies by the backward region hurts the advanced region. This happens if productivity gains from new technologies are limited. If not, the advanced region's loss in industrial leadership would be more than compensated by a drop in prices.

As a final remark, note that we have not specified how regions agree on a particular level of redistribution $\alpha$. Our objective has been to delimit a set of Pareto superior redistribution policies consistent with rational underdevelopment, without saying how to pick a unique policy from that set $\Gamma^R_U$. This leaves room for political conflict about the size of inter-regional transfers: within $\Gamma^R_U$ the poor region will want high levels of redistribution, and the rich region will want low levels of redistribution. Whichever rule is applied to resolve this conflict though, it will not affect our result: any policy chosen from $\Gamma^R_U$ leads to rational underdevelopment.
Appendix

A Time consistency

In this section we show that our simple time-inconsistent two-period model can easily be thought of as a time-consistent infinite horizon model. To do so, we take a simple overlapping generations approach. Agents live for two periods: in the first period the agent is a “child”, and in the second period the agent becomes an “adult”. Adults in period \( t \) care about consumption in periods \( t \) and \( t + 1 \); in other words, they worry about their own and their children’s consumption. Only adults take part in the decision process determining the level of transfers.

In each period a new manufacturing technology is introduced (but not necessarily adopted). Starting at a level \( A_1 \) in period 1, the rate of technological progress in manufacturing is either \( \theta \) — if one of the regions adopts the new technology — or zero — if neither region adopts the new technology:

\[
A_t = \begin{cases} 
(1 + \theta)A_{t-1} & \text{if in } t \text{ the new technology is adopted} \\
A_{t-1} & \text{if in } t \text{ the new technology is not adopted}
\end{cases}
\]

(32)

where \( A_t \) is manufacturing productivity in \( t \). The poor region’s initial productivity in the new technology follows a similar time-path. Starting at a level \( a_t^* \) in period 1, we have:

\[
a_t^* = \begin{cases} 
(1 + \theta)a_{t-1}^* & \text{if in } t - 1 \text{ the new technology was adopted} \\
a_{t-1}^* & \text{if in } t - 1 \text{ the new technology was not adopted}
\end{cases}
\]

(33)

Assume \( v \) in (5) is a CRRA utility function. In period 1 the utility functions of the rich and the poor region are then:

\[
U_1(\alpha) = v\left(\frac{\alpha}{1 + \alpha} A_t^\mu\right) = \frac{(\frac{\alpha}{1 + \alpha} A_t^\mu)^{1-\rho}}{1 - \rho}
\]

(34)

\[
U_1^*(\alpha) = v\left(\frac{1}{1 + \alpha} A_t^\mu\right) = \frac{(\frac{1}{1 + \alpha} A_t^\mu)^{1-\rho}}{1 - \rho}
\]

(35)

\(^{27}\)The model in this paper only studies two periods: manufacturing productivity in period 1 is \( A \), and manufacturing productivity in period 2 is either \( A \) (if the new technology is not adopted) or \( \hat{A} \) (if the new technology is adopted).

\(^{28}\)Because of leapfrogging the poor region in period \( t \) need not be the same region as the poor region in period \( t - 1 \). We therefore slightly change the notation: variables with an asterisk \( ^* \) now refer to the "poor" region, rather than the "South"; and variables without an asterisk refer to the "rich" region, rather than the "North".
For all subsequent periods, the utility functions are defined recursively. For instance, in period $t$, assuming the new technology is adopted, the utility function of the rich region can be written as:

$$U_t(\alpha) = v(\frac{\alpha}{1 + \alpha} A_t^\rho)$$

$$= \frac{(\frac{\alpha}{1 + \alpha} A_t^\rho)^{1-\rho}}{1 - \rho}$$

$$= (1 + \theta)^{\mu(1-\rho)} \frac{(\frac{\alpha}{1 + \alpha} A_{t-1}^\rho)^{1-\rho}}{1 - \rho}$$

$$= f(\theta)U'_{t-1}(\alpha)$$

But of course the new technology is not always adopted, so that the full expression of the rich region's utility function in $t$ is:

$$U_t(\alpha) = \begin{cases} f(\theta)U'_{t-1}(\alpha) & \text{if in } t \text{ the new technology is adopted} \\ U_{t-1}(\alpha) & \text{if in } t \text{ the new technology is not adopted} \end{cases}$$

(36)

In a similar way the utility function of the poor region in $t$ is:

$$U_t^*(\alpha) = \begin{cases} f(\theta)U'_{t-1}(\alpha) & \text{if in } t \text{ the new technology is adopted} \\ U_{t-1}^*(\alpha) & \text{if in } t \text{ the new technology is not adopted} \end{cases}$$

(37)

In period 1 the problem is identical to the two-period model described in the text. It suffices to realize that in period 1 the discounted sum of expected utility in the rich and the poor region are given by (26)-(31):

$$U(\alpha) = \begin{cases} U_1(\alpha) + \beta[pU_{2h}(\alpha) + (1 - p)U_{2l}(\alpha)] & \text{if } \alpha \in \Gamma_c \\ U_1(\alpha) + \beta[pU_{2h}(\alpha) + (1 - p)U_{2l}(\alpha)] & \text{if } \alpha \in \Gamma \end{cases}$$

(38)

$$U^*(\alpha) = \begin{cases} U_1^*(\alpha) + \beta[pU_{2h}^*(\alpha) + (1 - p)U_{2l}^*(\alpha)] & \text{if } \alpha \in \Gamma_c \\ U_1^*(\alpha) + \beta[pU_{2h}^*(\alpha) + (1 - p)U_{2l}^*(\alpha)] & \text{if } \alpha \in \Gamma \end{cases}$$

(39)

In period 1 we choose $\alpha$ for period 1 and period depending on a certain decision rule.\footnote{Note that in our discussion of rational underdevelopment we focused on the range of Pareto superior transfer policies, without specifying a decision rule for choosing one particular $\alpha$ from that range. One example of a possible decision rule would be Nash bargaining.} Assume this decision rule is invariant to affine transformations of the utility functions; i.e., if we multiply the utility functions of both regions by a constant, the $\alpha$ chosen does not change. In period 2 the new technology is or is not adopted, depending partly on the $\alpha$ chosen in period 1. The question is now whether our decision of period 1 is time-consistent: is the $\alpha$ chosen in period 1 still optimal in period 2? The
answer is yes. There is no reason to re-negotiate the level of redistribution in period 2. To see this, we distinguish between two cases, depending on whether or not the new technology has been adopted.

Start off by supposing the new technology has been adopted. In that case utility in the rich region and utility in the poor region are given by \( f(\theta) \) times utility in (38) and (39). Note furthermore that, following (33), condition (20) is unchanged. This implies that the utility ordering is unchanged too, so that the decision problem in period 2 is identical to the one in period 1; the \( \alpha \) chosen in period 1 is therefore time-consistent. Now suppose, instead, that the new technology has not been adopted. In that case the conclusion is even more straightforward, since utility in the rich region and utility in the poor region coincide with (38) and (39). The decision problem is unchanged, so that the \( \alpha \) chosen in period 1 is time-consistent. It is clear that the same argument can be applied in all subsequent periods.

## B Proof of Theorem 1

Let \( \Omega \) be the set of possible redistribution schemes, i.e. \( \Omega = [1, \frac{\mu}{\mu - \mu'}] \). In the second period the South adopts the new technology if

\[
\alpha > \frac{A}{\mu a^*} - 1
\]

To simplify the notation we drop through this appendix "*" and write \( \alpha \) instead of \( \alpha^* \). Let \( \alpha(a) \equiv \frac{A}{\mu a} - 1 \). Thus the function \( \alpha(a) \) gives the level of redistribution that makes the South indifferent between adopting the new technology and continuing to produce exclusively food. The set of redistributive policies for which the South does not adopt the new technology is:

\[
\Gamma(a) \equiv \{ \alpha \in \Omega : \alpha \leq \alpha(a) \} = [1, \alpha(a)]
\]

Sometimes we just write \( \Gamma \). Let \( \Gamma^c(a) \equiv \Omega/\Gamma(a) \), i.e., \( \Gamma^c(a) \) is the set of \( \alpha \) for which the South adopts the technology. This set is also an interval of the form \([\alpha(a), 1/\mu]\)

Sometimes we write it as \( \Gamma^c \). Recall that the expressions for the expected utilities were given by

\[
U(\alpha) = \begin{cases} 
U_p(\alpha) & \text{if } \alpha \in \Gamma^c \\
U_n(\alpha) & \text{if } \alpha \in \Gamma
\end{cases}
\]
These functions are continuous everywhere in $\Omega$ except possibly at $\alpha(a)$ (see Figure 2). The strategy of the proof consists in showing that we can generate a situation similar to the one in Figure 2.

**Step 1.** $U_n(\alpha)$ is strictly increasing and $U^*_n(\alpha)$ is strictly decreasing.

$U_n(\alpha)$ is given by

$$v\left(\frac{\alpha}{1+\alpha} A^\mu + \beta \left[ p v\left(\frac{\alpha}{1+\alpha} \tilde{A}^\mu + (1-p) v\left(\frac{\alpha}{1+\alpha} A^\mu \right)\right)\right]\right)$$

which is, clearly, strictly increasing in $\alpha$. $U^*_n(\alpha)$ is given by

$$v\left(\frac{1}{1+\alpha} A^\mu + \beta \left[ p v\left(\frac{1}{1+\alpha} \tilde{A}^\mu + (1-p) v\left(\frac{1}{1+\alpha} A^\mu \right)\right)\right]\right)$$

which is, clearly, strictly decreasing in $\alpha$.

**Step 2.** We will construct a set $P \subset \Gamma$ of redistribution policies that are preferred by the North to any policy in $\Gamma^c$. (See Figure 3).
i) For $\bar{\alpha} := \frac{A(1-\mu)}{\mu}$ we have $\Gamma(\bar{\alpha}) = [1, \frac{1}{1-\mu}]$, i.e., the new technology is never adopted.

ii) We have

$$U_n \left( \frac{\mu}{1-\mu} \right) = u(\mu A^\mu) + \beta \left[ p u\left( \mu \hat{A}^\mu \right) + (1-p) u\left( \mu A^\mu \right) \right]$$

and

$$U_y \left( \frac{\mu}{1-\mu} \right) = u(\mu A^\mu) + \beta \left[ p u\left( \mu \hat{A}^\mu \right) + (1-p) u\left( (1-\mu) \hat{A}^\mu \right) \right]$$

so that $U_n \left( \frac{\mu}{1-\mu} \right) > U_y \left( \frac{\mu}{1-\mu} \right)$ if $p < 1$, $\beta > 0$ and $\frac{\mu}{1-\mu} > \left( \frac{\hat{A}}{\hat{A}} \right)^\mu$. This corresponds to Condition 1 and Condition 2, so that $U_n (\frac{\mu}{1-\mu}) > U_y (\frac{\mu}{1-\mu})$.

iii) By the previous two points and by continuity of $U_n$, $U_y$, and $\alpha(a)$, there exists $\bar{a} > \bar{\alpha}$ close enough to $\bar{\alpha}$ such that $U_n(\alpha(\bar{\alpha})) > U_y(\bar{\alpha})$ for all $\alpha \in [\alpha(\bar{\alpha}), \frac{\mu}{1-\mu}]$.

iv) Let $z = \max_{\alpha \in [\alpha(\bar{\alpha}), \frac{\mu}{1-\mu}]} U_y(\bar{\alpha})$. By continuity of $U_y(\alpha)$ the value $z$ is well defined. Let $\bar{\alpha}(\bar{\alpha})$ be the solution to

$$z = U_n(\alpha)$$

and if $z < U_n(\alpha)$ for all $\alpha \in \Omega$ we set $\bar{\alpha}(\bar{\alpha}) = 1$. Since $U_n(\alpha)$ is strictly increasing $\bar{\alpha}(\bar{\alpha})$ is well defined.

Let $P(\bar{\alpha}) \equiv \{ \alpha : \bar{\alpha}(\bar{\alpha}) \leq \alpha \leq \alpha(\bar{\alpha}) \}$. By construction $P(\bar{\alpha}) \subset \Gamma(\bar{\alpha})$. Moreover, $P(\bar{\alpha})$ is non-empty; to see this, it suffices to show that $\bar{\alpha}(\bar{\alpha}) < \alpha(\bar{\alpha})$; this is immediate, given that $U_n(\alpha(a))$ is strictly increasing and given that $U_n(\alpha(\bar{\alpha})) > U_y(\bar{\alpha})$ for all $\alpha \in [\alpha(\bar{\alpha}), \frac{\mu}{1-\mu}]$. It is also clear that for any $\alpha \in P(\bar{\alpha})$ we have $U_n(\alpha(a)) \geq U_y(\alpha'(a))$ for all $\alpha' \in \Gamma(\bar{\alpha})$. Thus, the transfer schemes in $P(\alpha)$ are Pareto superior for the North to the transfers schemes for which the South adopts the new technology. Notice that the North is (weakly) better off under $\alpha \in P(\bar{\alpha})$ than under zero transfers, i.e. all the policies in $P(\bar{\alpha})$ are also individually rational for the North.

Recall that $\tilde{\alpha}$ satisfies $U_n(\tilde{\alpha}) = U_y(\frac{\mu}{1-\mu})$. It is not difficult to see that for a small enough the value $\alpha(a)$ approaches to $\frac{\mu}{1-\mu}$ and $\alpha(a)$ approaches to $\tilde{\alpha}$ (see Figure 3), i.e.:

$$\lim_{a \to (\frac{A(1-\mu)}{\mu})^+} \alpha(a) = \tilde{\alpha}$$

and

$$\lim_{a \to (\frac{A(1-\mu)}{\mu})^+} \alpha(a) = \frac{\mu}{1-\mu}$$
The idea is simple, when $a$ approaches (from right) its lowest possible value, the South almost never adopts the new technology and the set $P(a)$ approaches the set of all $\alpha$'s such that $U_n(\alpha) > U_y(\frac{\mu}{1-\mu})$, and the lower bound of this set is $\alpha$ and the upper bound is $\frac{\mu}{1-\mu}$. Given that $\alpha < \alpha(\bar{a})$ and $\frac{\mu}{1-\mu} > \alpha(\bar{a})$, and given that $P(\bar{a})$ is non empty, it is clear that for $a$ small enough, $P(a)$ is non empty.

**Step 3.** We will construct, in a parallel way to what we did in Step 2 for the North, a set $P^* \subset \Gamma$ of redistribution policies that are preferred by the South to any policy in $\Gamma^c$. (See Figure 4).

i) Condition 3 implies that $U_n^*(1) > U_y^*(\frac{\mu}{1-\mu})$. Step 1 showed that $U_n^*(\alpha)$ is strictly decreasing in $\alpha$. Hence, and by continuity of $U_n^*$, $U_y^*$, and $\alpha(a)$, there exists $a' > \bar{a} \equiv \frac{\mu}{1-\mu}$ close enough to $\bar{a}$ such that $U_n^*(1) > U_y^*(\alpha)$ for all $\alpha \in [\alpha(a'), \frac{\mu}{1-\mu}]$.

ii) Let $z = \max_{\alpha \in [\alpha(a'), \frac{\mu}{1-\mu}]} U_y^*(\alpha)$. By continuity of $U_y^*(\alpha)$ the value $z$ is well defined. Let $\alpha(a')$ be the solution to

$$z = U_n^*(\alpha)$$

Since $U_n^*(\alpha(a')) < U_y^*(\alpha(a'))$, since $U_n(\alpha)$ is strictly decreasing, and since $U_n^*(1) > z$, we can conclude that $\alpha(\alpha')$ is well defined.
Let $P^*(a') \equiv \{\alpha : 1 \leq \alpha \leq \alpha(a')\}$. By construction $P^*(a') \subseteq \Gamma(a')$. Moreover, $P^*(a')$ is non empty; to see this, it suffices to show that $\alpha(a') < 1$; this is immediate, given that $z < U_n^*(1)$. It is also clear that for any $\alpha \in P^*(a')$ we have $U_n^*(\alpha) \geq U_p^*(a')$ for all $a' \in \Gamma(a')$. Thus, the transfer schemes in $P^*(a)$ are Pareto superior for the South to the transfers schemes for which the South adopts the new technology. Notice that the South is (weakly) better off under $\alpha \in P^*(a')$ than under zero transfers, i.e. all the policies in $P(a')$ are also individually rational for the South. Recall that $\alpha^*$ satisfies $U_n(\alpha^*) = U_p^*(1)$. It is not difficult to see that for a small enough value $\alpha(a)$ approaches $\frac{\mu}{\mu - \mu}$, and $\alpha(a)$ approaches to $\alpha^*$ (see Figure 4), i.e.:

$$\lim_{\alpha \rightarrow \left(\frac{\mu}{\mu - \mu}\right)^+} \alpha(a) = \alpha^*$$

Given that $\alpha^* > \alpha(a')$ and given that $P^*(a')$ is non empty, it is clear that for a small enough the set $P^*(a)$ is non empty.

**Step 4** We want to show that for a small enough the set $R(a) \equiv P(a) \cap P^*(a)$ is not empty. Remember that $P(a)$ is given by the interval $[\alpha(a), \alpha(a)]$ and $P^*(a)$ is given by the interval $[1, \alpha(a)]$, and for a small enough those intervals are not empty: We showed that $\lim_{a \rightarrow \left(\frac{\mu-\mu}{\mu}\right)^+} \alpha(a) = \alpha^*$ and $\lim_{a \rightarrow \left(\frac{\mu-\mu}{\mu}\right)^+} \alpha(a) = \alpha^*$. Thus, for a small enough, $R(a)$ is not empty if $\alpha < \alpha^*$, and this inequality is just Condition 3. Since $R(a) \subseteq \Gamma^{RU}(a)$ we have that, for a small enough, $\Gamma^{RU}(a)$ is not empty, and this concludes the proof.
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