THREE EXCHANGE RATE REGIMES AND A MONETARY UNION: DETERMINACY, CURRENCY CRISES, AND WELFARE

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Three Exchange Rate Regimes and a Monetary Union: Determinacy, Currency Crises, and Welfare*.

by

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Abstract. We study three different exchange rate regimes in a stochastic OLG model with free capital mobility and incomplete markets. The regimes are characterized by the type of coordinated seignorage financed transfer (or fiscal) policy in place. We are especially interested in how the different types of policies affect the possibility of sunspot equilibria with real and/or nominal effects. The first type of policy considered, where both governments are inactive does not lead to a Pareto optimal allocation (in the long run) and there is a continuum of equilibria as well as sunspot equilibria. The second type of policies are devaluation policies, where the country adversely affected by a shock devaluates. For most of these policies sunspots may have real affects, giving rise to suboptimal equilibria. If a central bank unilaterally tries to defend the exchange rate in order to avoid these sunspot equilibria, it may or may not, depending on its reserves, experience a currency crisis which forces it to give up intervention. Finally, we consider a type of policies that allow the exchange rate to be constant. For these policies sunspots cannot have real effects and all equilibria are Pareto optimal, but may affect the exchange rate. Coordinated intervention by both central banks rules out all nominal sunspot equilibria, but unilateral intervention does not. A final option, a monetary union also leads to a Pareto optimal allocation if the two countries agree on transfers to the country adversely hit by a shock, i.e. agree on a coordinated fiscal policy.

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1 Introduction

The foreign exchange markets seem to have a certain dichotomy to them, which makes it difficult to capture all their essential feature in one single model. On the one hand, fundamental economic activity arguably drives the exchange rate in the long run, while on the other hand, in the short run, the exchange rates seem to be highly sensitive to speculative forces and sentiments in the market. The dual nature of this market can also be exposed by pointing to the dual nature of the currencies being traded. On the one hand, each currency has a specific role in its country, being used as a primary means of payments, and does as such have an intrinsic not easily substitutable role[1]. On the other hand, the currencies do not, when traded on the international financial markets, have any intrinsic value and when there is free capital mobility currencies are close substitutes[2]; demand is only driven by their return structure. Many models, dealing with the markets for foreign exchange, emphasize the first aspect for instance by assuming a cash in advance constraint on what trade can take place in what currency or utility of the local currency only, i.e. assume no substitutability at all (see for instance Devereux and Engel, 2000 and Obstfeld and Rogoff,2000). In the model presented here another tradition is followed according to which currencies are perfect substitutes. We use a variant of an OLG model first introduced by Kareken and Wallace(1981). In this model agents care only about the return structure of the currencies and this makes the exchange rate sensitive to expectations. As such the model might be useful as a tool for thinking about the consequences of different exchange rate regimes for the "expectational stability" of the economy. One advantage of using the OLG structure for stochastic economies model is that it is tractable even when agents are heterogeneous. This is in contrast with the infinite horizon structure often being used. As it turns out, ex-ante heterogeneity among agents (countries) will be important for the existence of sunspot equilibria in the model studied here and may lead to ex-post heterogeneity too. We assume that markets are incomplete. Because the exchange rate may potentially be used to reallocate between countries hit by different shocks, i.e. provide insurance (an idea similar to many current studies among those Devereux and Engel, 2000 and Obstfeld and Rogoff, 2000) the choice between different exchange rate regimes is less trivial than in, for instance, the extension of Kareken and Wallace by Manuelli and Peck(1990), in some aspects the predecessor to the model studied here.

Three different fiscal policies/exchange rate regimes, as well as a monetary union, are then considered to see to what degree they may make up for the incompleteness being stipulated. The three regimes are refered to as respectively free float with non-intervention, floating exchange rates with devaluations, and policies set to fix the exchange rate. These regimes are studied mainly because, as their labels indicate, they have some resemblance to regimes that we actually do observe. Also, it is quite natural to study the first regime, to see if active governments are necessary for efficiency. The two other regimes also have the appealing feature that they lead to the symmetric Pareto optimal allocation, at least in the absence of sunspot influence.

Our results can be summarized as follows. Under a free float there is a continuum of equilibria for none of which the symmetric Pareto optimal allocation is achieved for more than a limited time. Furthermore, under this regime sunspots may have real effects (sunspot equilibria may dominate some non-sunspot equilibria and may be dominated by other non-sunspot equilibria)[3]. We next study a class of floating exchange rate regimes with devaluations. For all of them it is the case that sunspots may matter. The effect of these sunspots is real except for a single member of the said class. This single

[1] Absence of perfect currency substitutability can be phrased as follows: If the two currencies A and B happen to have the same return structure then a person in country A will strictly prefer currency A over currency B.

[2] McKinnon(1994) provides a criticism of models with perfect currency substitutability.

[3] The study of the (real) effects of sunspots originates with Cass and Shell(1983).

policy is characterized by leaving the agents ex-ante symmetric. With equal investment opportunities they will also be ex-post symmetric (as is the case in the model of Manuelli and Peck,1990) which, in turn, means that sunspots cannot affect the allocation of commodities. We also study the role of central bank intervention for these types of policies. If the two central banks coordinate, the exchange rate can be kept at the right levels for the policies to work as intended, but a single central bank may not be able to defend the exchange rate in the long run and currency crises may appear. Similar results are found for an exchange rate regime with devaluation, where the monetary policy of one ountry is passive, while that of the other country is active. Finally, when the policy is set to fix the exchange there may also be multible equilibria, but these only have nominal effects, affecting the exchange rate, but leaving consumption untouched at the symmetrical Pareto optimal allocation. The analysis thus shows that when there are two currencies the currency regimes can be chosen to "complete" otherwise incomplete markets. If the two countries form a monetary union direct subsidies have to replace the former arrangement. A monetary union removes endogenously created expectational instability but if the two countries are unable to create, in another way, the necessary insurance against country specific shocks, the cost of this expectational stability may be too high.

In studying the existence of sunspot equilibria under devaluation policies we rely on Chiappori,Geoffard, and R. Guesnerie (1992) who provide a very general link between indeterminacy and the existence of local sunspot equilibria. Some technical aspects of our results may be of independent interest. The indeterminacy exposed under the devaluation policy is different from what is ordinarily found in OLG models (or in finite horizon incomplete markets models); it is present due to the fact that the two currencies are subject to an inflation tax. Also, we show that when both governments are passive, only for sunspots that are correlated with fundamentals can sunspot equilibria exist. Finally, we provide examples of sunspots equilibria where, when the sunspots are correlated with fundamentals, the qualitative features of the equilibrium differs from the case where the process of sunspots is independent of the process of fundamentals.

The analysis that is probably most closely related to ours is found in King, Wallace, and Weber(1992) (KWW). Here an intermediate case between complete lack of and full substitutability of currencies is modeled by assuming that some traders have a cash in advance constraint in only one currency while others have it in any of the two currencies. Like in the present study, the main concern is with the influence of indeterminacy on welfare, however in their model there is no fundamental uncertainty and thus no "fundamental" need for insurance. Briefly put, the conclusion of KWW is that when markets are complete (i.e. agents can insure against sunspot related shocks) the choice between floating and fixed exchange rates is immaterial, while when markets are incomplete, inefficient sunspot equilibria may be present under floating exchange rates. The fact that the this kind of expectational instability may be present, even though only some agents ("a speculative fringe" in the words of KWW) regard the currencies as perfect substitutes lends some support to our concentrating on the case of perfect substitutability. The present study differs in at least two fundamental ways. Here, an exchange rate regime is to be chosen both with an eye to how it makes up for the incompleteness of markets and how vulnerable it is to expectational instability. Furthermore, the role of central bank intervention and the possibility and nature of currency crises is being studied. Other models have studied the issue of expectational stability without assuming perfect substitutability. In fact, the elaborate model of Woodford(1991) aims exactly at studying how increased substitutability may decrease the expectational stability of the economy[4]. Neumeier (1998) provides a general equilibrium study of the trade-off between a monetary union and multiple currencies in a finite horizon incomplete markets setting with cash in

[4] See also Weil(1991) for a simpler analysis along the same lines.

advance constraints. It is argued that a monetary union may be regarded as a commitment to reduce "non-fundamental" uncertainty injected into the system via the monetary policies of the different central banks. But a monetary union also reduces the span of the assets available, since there will no longer be bonds denominated in different currencies which can react differently to idiosyncratic fundamental shocks in different countries. This trade-off is then studied. The possible selffulfilling nature of currency crises is studied in, among others, Burnside, Eichenbaum, and Rebelo(2000) and Obstfeld(1996), both which are concerned with small open economies. In these studies, like in the one presented here, it is found that for some fundamentals (policy variables) there may be a positive probability both of a crisis and of no crisis while for others there is zero probability of a crisis. Burnside, Eichenbaum, and Rebelo(2000) emphasize the potential destabilizing effects of the financial sector caused by the governments' guarantees to bail out domestic banks. Such an arrangement, it is argued, may make the occurence of a currency crisis 'more likely' if the bailout is financed by seignorage. Thus like in the present study, the use of seignorage may create the instability that makes sunspots possible.

2 The model and equilibrium

The model[5] has two countries, A and B, each with an overlapping generation structure of representative agents. Thus at each date, t, the are in each country two agents, one old and one young. Since agents only consume when they are old, we can without confusion use the labels A and B for residents in countries A and B respectively. Whenever we in the following introduce stochastic variables these are supposed to be defined on some underlying probabilty space $(\Omega, \mathcal{S}, \mu)$. Also, distributions of stochastic variables are denoted by P. When agents take expectations they use this probability space, their (perfect) knowledge about the stochastic variables defined on it, and all information available up to and including the current date; we write E_t to emphasize the latter assumption. Thus in equilibrium agents have rational expectations. The endowment structure is as follows. When young, an agent in any of the two countries receives the amount e of the single commodity. When old, he receives a positive random amount of the same commodity; at date t, e_{Bt} and e_{Bt} for the two countries A and B respectively. We assume that the processes $\{e_{At}\}_t$ and $\{e_{Bt}\}_t$ are i.i.d. and mutually independent and also that: $e > |e_{At} - e_{Bt}|$ a.s. . This last assumption allows for full risk sharing to take place under the exchange regimes with active government policies.

Agents only receive utility of consumption, $C_{ct}, c = A, B$ when they are old[6], this utility being $u(C_c)$ in both countries, where u is defined on $\Re_+$, strictly increasing, strictly concave, and C^2. The agents of this economy want to transfer their initial real wealth, e, to the last period of their lifes. Since there are no restrictions on capital mobility in this model, this can be done by placing this wealth in a portfolio of one or two currencies (named after their countries). No other assets are available and markets are then incomplete (no short-selling can take place) although this incompleteness may not matter, depending on the particular exchange rate regime and equilibrium. Since the two countries are facing idiosyncratic risk but are ex-ante symmetric, it is of special interest to see, which institutional arrangements achieve the symmetric Pareto optimal allocation where $C_{At} = C_{Bt} = e + \frac{1}{2}(e_{At} + e_{Bt}) \equiv C_t^*$.

There is a government in each of the two countries, which each can issue its currency in negative or positive amounts and make transfers to or tax their own citizens (in fact the current old). If we denote the real transfer/tax in country $c = A, B$ at date t by S_{ct} and the outstanding money at the beginning of period t by M_{ct} we then have the budget constraint of the government: $\Delta M_{ct} = S_{ct} p_{ct}$ where p_{ct}

[5] The model and the polices defined below are taken from Nielsen(1998).

[6] Note that an immediate consequence of this assumptions on agents' preferences is that sunspots of the type studied in Azariadis(1981) - sunspots that amount to random redistributions between young and old - will never be present.

is the price of the commodity in terms of currency c and $\Delta M_{ct} = M_{ct+1} - M_{ct}$. Since governments' outlays are financed by seignorage the choice of S_A and S_B gives rise to particular exchange rate regimes, to be described below. When considering a particular exchange regime and its associated (stationary) equilibrium we could think of the horizon of the economy as stretching from $-\infty$ to $+\infty$. Under another interpretation the horizon is from 1 to $+\infty$ and in that case we have to specify what happens to the initial old under a specific new regime. This in turn depends on what kind of institution was in place before the regime started. Were the two economies for instance monetarized, but in autarky, or was there already some interaction between the two economies? We choose not to be too specific on this matter. It is here worth stressing one more time that this model is not concerned with the distribution between generations; In any reasonable equilibrium the current old will in each period consume all available commodities. The issue is rather insurance, the distribution of risk across agents in the same generation. Thus the scope for trade is not in physically differentiated commodities but in assets that deliver the commodity in specific states. This is clearly a very crude proxy for observed economic activity between countries. But it does allow us to capture the financial aspect of the markets for foreign exchange that we wish to emphasize while at the same time providing the model with a solid foundation for welfare analysis. At the bottomline then, at date 1 there is available for consumption by the old the stochastic quantity $2e + e_{A1} + e_{B1}$ of the commodity. We assume that this quantity is divided between the initially old in such a way that they are not worse off, than they would have been if the unspecified original institution would not have been displaced. With these remarks about the model in place, an equilibrium where both currencies are in circulation can be defined.

We let $\overline{M}_A$ and $\overline{M}_B$ denote the initial aggregate money holdings of the old of currencies A and B, respectively.

Definition 1: Equilibrium

A stochastic sequence $\{\pi_{At}, \pi_{Bt}, C_{At}, C_{Bt}, S_{At}, S_{Bt}, M_{At}, M_{Bt}, p_{At}, p_{Bt}\}_{t=1}^{\infty} \in (\Re_+^4 \times \Re^2 \times \Re_{++}^4)^{\infty}$ s.t. for all t, a.s. we have

(i) $M_{c1} = \overline{M}_{c1}, c = A, B$

(ii) π_{ct} solves the problem:

$$Max_{q\in[0,1]} E_t u \left[\left(q\frac{p_{At}}{p_{At+1}} + (1-q)\frac{p_{Bt}}{p_{Bt+1}} \right) e + e_{ct+1} + S_{ct+1} \right], c = A, B \qquad (1)$$

(iii) $(\pi_{At} + \pi_{Bt})p_{At}e = M_{At} + p_{At}S_{At}$

$(2 - \pi_{At} - \pi_{Bt})p_{Bt}e = M_{Bt} + p_{Bt}S_{Bt}$

$\frac{M_{At}}{p_{At}} + S_{At} + e_{At} + \frac{M_{Bt}}{p_{Bt}} + S_{Bt} + e_{Bt} = 2e + e_{At} + e_{Bt}$

(iv) $M_{ct+1} = M_{ct} + p_{ct}S_{at}, c = A, B$

and furthermore,

(v) $C_{c1} = \frac{\overline{M}}{p_{c1}} + S_{c1} + e_{c1}, c = A, B$

$C_{ct+1} = \left(\pi_{ct}\frac{p_{At}}{p_{At+1}} + (1-\pi_{ct})\frac{p_{Bt}}{p_{Bt+1}}\right) e + e_{ct+1} + S_{ct+1}, c = A, B, t \geq 1$ ■

The requirement that $M_{ct} > 0, c = A, B$ means that both currencies are in demand. The second condition states that both agents pick portfolios, in terms of fractions of real wealth invested, that maximize their expected utility. The third requirement states that total demand for currency A and B (on the left) is equal to total supply, namely what is sold by the old and what is sold (in negative or positive quantities) by the government, and furthermore that total demand of the commodity (on the left in the third equation) is equal to what is supplied. By Walras' law this third requirement is redundant. Finally, (iv) provides a dynamical relation between the stocks of the two currencies in two consecutive periods and (v) defines what consumption is in this equilibrium.

Let us briefly note that one equilibrium for this economy would give the same real outcome as autarky, where each country is isolated from the other. If we let $S_{ct} \equiv 0$ and $p_{ct} = \frac{\overline{M}_c}{e}, \forall t$ and $M_{ct} = \overline{M}_c, \forall t$ then with $\pi_{At} = 1$, $\pi_{Bt} = 0$, and $C_{ct} = e + e_{ct}, \forall t$ we have an equilibrium, where each country only consumes its own endowments. This would also, for any (feasible) transfer process, be the case if there were no free capital mobility. The outcome is clearly Pareto dominated by the consumption process $\{C_t^*, C_t^*\}_{t=1}^{\infty}$.

In order to study the equilibria arising from different policies it is convenient to formulate the equilibrium conditions in terms of fewer variables. Using the equilibrium condition $(\pi_{At} + \pi_{Bt})p_{At}e = M_{At} + p_{At}S_{At} = M_{At+1}$ we get that

$$\frac{p_{At}}{p_{At+1}} = \left(\frac{M_{At+1}}{(\pi_{At} + \pi_{Bt})e}\right)\left(\frac{M_{At+1}}{(\pi_{At+1} + \pi_{Bt+1})e - S_{At+1}}\right)^{-1} = \frac{(\pi_{At+1} + \pi_{Bt+1})e - S_{At+1}}{(\pi_{At} + \pi_{Bt})e} \tag{2}$$

and similarly,

$$\frac{p_{Bt}}{p_{Bt+1}} = \frac{(2 - \pi_{At+1} - \pi_{Bt+1})e - S_{bt+1}}{(2 - \pi_{At} - \pi_{Bt})e} \tag{3}$$

In the following, for notational convenience, subscript t is sometimes not used, when the designation of the period is irrelevant. Sometimes we will then use a prime on (portfolio) variables as in π_A' to differentiate between current (π_A) and future ones. Also, when we suppress reference to the period, C^* refers not to the sequence, but to $e + \frac{1}{2}(e_A + e_B)$. Now define for any π_A, π_B s.t $\pi_A + \pi_B \in (0, 2)$, any stochastic variables, π_A', π_B', S_A, and S_B, s.t. $(\pi_A' + \pi_B')e - S_A > 0$ and $(2 - \pi_A' - \pi_B')e - S_B > 0$, a.s., and any information $\mathcal{I}$[7]

$$Z_c(\pi_A, \pi_B, \pi_A', \pi_B', S_A, S_B, \mathcal{I}) = E\left[u'\left(\pi_c\frac{(\pi_A' + \pi_B')e - S_A}{\pi_A + \pi_B} + (1 - \pi_c)\frac{(2 - \pi_A' - \pi_B')e - S_B}{2 - \pi_A - \pi_B} + e_c + S_c\right) \cdot \left(\frac{(\pi_A' + \pi_B')e - S_A}{\pi_A + \pi_B} - \frac{(2 - \pi_A' - \pi_B')e - S_B}{2 - \pi_A - \pi_B}\right)|\mathcal{I}\right], c = A, B$$

Then consider the following definitions:

Definition 2: Temporary Equilibrium
$(\pi_A, \pi_B, S_A, S_B, \pi_A', \pi_B', S_A', S_B')$ and information $\mathcal{I}$ s.t.

(i) $\pi_c \in [0, 1], c = A, B,\ \ \pi_A + \pi_B \in (0, 2),\ \ \ (\pi_A + \pi_B)e - S_A > 0,\ \ \ (2 - \pi_A - \pi_B)e - S_B > 0$
(ii) a.s. $\pi_c' \in [0, 1], c = A, B,\ \ \pi_A' + \pi_B' \in (0, 2),\ \ \ (\pi_{A'} + \pi_B')e - S_A' > 0,\ \ \ (2 - \pi_A' - \pi_B')e - S_B' > 0$
(iii) $Z_c(\pi_A, \pi_B, \pi_A', \pi_B', S_A', S_B', \mathcal{I}) = 0, c = A, B$ a.s. ■

Definition 3: Equilibrium*
A stochastic sequence $\{\pi_{At}, \pi_{Bt}, S_{At}, S_{Bt}\}_{t=1}^{\infty}$ such that a.s.: $\forall t$, with $\mathcal{I}_t = \{\pi_{As}, \pi_{Bs}, S_{As}, S_{Bs}\}_{s=1}^{t}$, $(\pi_{At}, \pi_{Bt}, S_{At}, S_{Bt}, \pi_{At+1}, \pi_{Bt+1}, S_{At+1}, S_{Bt+1})$ and $\mathcal{I}_t$ form a temporary equilibrium■

It is then easy to see, that if we have an equilibrium* as defined in Definition 3 we also have an equilibrium as defined in Definition 1. In period 1, set $M_{c1} = \overline{M}_c, c = A, B$. Then let p_{c1} be s.t. $\frac{M_{A1}}{p_{A1}} = (\pi_{A1} + \pi_{B1})e - S_{A1}$ and $\frac{M_{B1}}{p_{B1}} = (2 - \pi_{A1} - \pi_{B1})e - S_{B1}$. Finally, let $M_{A2} = (\pi_{A1} + \pi_{B1})ep_{A1}$ and $M_{B2} = (2 - \pi_{A1} - \pi_{B1})ep_{B1}$. In any other period, t, M_{At} and M_{Bt} are given and given π_{At}, π_{Bt}, S_{At}, and S_{Bt}, we can then define p_{At}, p_{Bt}, M_{At+1}, and M_{Bt+1} in the same manner as for the first period.

[7] That is for any sub σ-algebra of $\mathcal{S}$.

3 Description and interpretation of the three main regimes

As earlier stated, the role of the governments' policy is to create options for insurance arrangements between the agents of the two countries. This captures to some extent what we see in reality: Individual agents cannot (in an efficient way) fully insure against country specific macroeconomic shocks. A negative shock in one country may then, for instance, be met with an expansionary fiscal policy and a devaluation, both measures to increase demand for domestic products and thus to increase domestic income to compensate for the negative shock. To the extend that the income in the foreign country decreases as a result of the revaluation of its currency, a transfer has, in effect, taken place.

We consider only coordinated policies[8], that is the two countries agree to pursuing a certain policy and stick to the rules stipulated by that policy[9]. Many policies result in the Pareto optimal allocation $C^* = \{C_t^*\}$. In particular, the governments could in principle agree to a redistribution, through taxes, that independently of the portfolio holdings of the the current old would result in the consumption C^*. Such a policy may have to depend not only on current and past shocks but also on current prices and portfolio holdings, implying that the informational requirements are strong. We concentrate on policies that arguably have some resemblance to observed policies, work through the "market", and for which the transfers, S_A and S_B, only depend on current and past (macroeconomic) shocks.

We then concider two active policies or regimes and, in addition, the consequences of a policy of no intervention. These policies are defined in terms of the taxes/subsidies, $S_c, c = A, B$, which by the governments' budget constraints also define their monetary policies[10]. The first policy, Policy 1s ('s' for 'symmetric'), has a straight forward interpretation as a devaluation policy. The country, c, that experiences a relatively negative shock, $e_{ct} < max\{e_{At}, e_{Bt}\}$, issues money using the proceeds to subsidize its own citizens, while the other country does not issue any money. As a result the currency of country c devaluates and the agents that hold this currency, presumably both foreigners and residents, pay a seignorage tax. The result will be a transfer from the foreign country to the home country, making up for the relatively negative shock. This policy resembles what Obstefeld and Rogoff(2000) call "an optimal float". Let $\Delta e_t = e_{Bt} - e_{At}$.

Definition 4: Policy 1s
$S_{At} = \tau max\{0, e_{Bt} - e_{At}\} = \tau max\{0, \Delta e_t\}$
$S_{Bt} = \tau max\{0, e_{At} - e_{Bt}\} = \tau max\{0, -\Delta e_t\}$
where $\tau \in (1/2, \infty]$ ■

The intention of this study is to focus on policies that can be interpreted as having some observed counterpart, and as being the outcome of an agreement between the two countries. We mostly concentrate on symmetric policies, but one might argue that in some of the exchange rate arrangement observed there is an implicitly agreed upon asymmetry. In such arrangements one country is "passive" (the phrase "anchor" country has been used in some contexts) , while the other is "active" in pursuing a policy of for instance devaluations or fixing the exchange rate. In the context of the simple model being studied here, such an asymmetric arrangement can, for the case of devaluations, be presented as follows:

[8] Obstfeld and Rogoff(2000) also focus on coordinated policies.

[9] As in Canzoneri(1989) transfers between countries, here for insurance purposes, are broght about by seignorage which hits both residents and foreigners. But while we assume cooperation between countries, Canzoneri assumes the opposite which leads to suboptimal inflation levels.

[10] In conjunction with these policies there may be further reallocations between the old in period 1 only. These reallocations would be chosen to guarantee that noone is worse off in the new regime and are left unspecified.

Definition 5: Policy 1a
$S_{At} \equiv 0$
$S_{Bt} = \tau(e_{At} - e_{Bt})$ for suitably chosen τ ■

As we shall see, Policy 2 results, in the absence of sunspots, in a constant exchange rate. In effect this policy designs a tightly coordinated monetary and fiscal policy in the two countries in order to keep the money stock increasing at the same rate in the two countries. Also this policy has a counterpart in Obstfeld and Rogoff under the title "an optimal fix".

Definition 6: Policy 2
$S_{At} = \frac{q}{2(q-1)}(e_{Bt} - e_{At}) = \frac{q}{2(q-1)}\Delta e_t$
$S_{Bt} = \frac{2-q}{2(q-1)}(e_{Bt} - e_{At}) = \frac{2-q}{2(q-1)}\Delta e_t$
where $q \in (0, 1 - \frac{sup(e_A - e_B)}{2e}] \cup [1 + \frac{sup(e_A - e_B)}{2e}, 2)$ ■

The assumption made earlier, guarantees that the two intervals above are non-empty. In a companion paper (Nielsen,1998) it is shown that Policy 2 is the only policy (chosen from a set of policies depending on current real shocks and past information) that results in the allocation C^* when agents have rational beliefs (as defined in Kurz, 1994). This is one reason to study this policy. Another is that it leads to the exchange rate, $x_t = \frac{p_{At}}{p_{Bt}}$, being constant while achieving the allocation C^*. However it is not the only such policy. Notice that when the exchange rate is constant, i.e. $x_{t-1} = x_t$ we have $R_{At} \equiv \frac{p_{At-1}}{p_{At}} = \frac{p_{Bt-1}}{p_{Bt}} \equiv R_{Bt}$, in other words the gross return on both currencies are the same. Suppose then that we have a particular equilibrium, where $C_{At} = C_{Bt} = C_t^*, \forall t$ and $x_t = x_{t+1}, \forall t$. Using (2) and (3) we would have

$$\pi_{ct}\frac{\pi_{t+1}e - S_{At+1}}{\pi_t} + (1 - \pi_{ct})\frac{(2 - \pi_{t+1})e - S_{Bt+1}}{2 - \pi_t} + e_{ct+1} + S_{ct+1} = C_{t+1}^*, c = A, B \tag{4}$$

$$\frac{\pi_{t+1}e - S_{At+1}}{\pi_t} = \frac{(2 - \pi_{t+1})e - S_{Bt+1}}{2 - \pi_t} \tag{5}$$

where $\pi_t \equiv \pi_{At} + \pi_{Bt}$. Combining (4) and (5) we get

$$e_{At} + S_{At} = e_{Bt} + S_{Bt} \tag{6}$$

Finally, using (6) in (5) we get

$$\pi_{t+1} = \pi_t + (1 - \pi_t)\frac{S_{Bt+1}}{2e} + (2 - \pi_t)\frac{e_{Bt+1} - e_{At+1}}{2e} \tag{7}$$

Clearly, there may be (f.i. for e large enough) many policies, S_A and S_B and portfolio sequences $\{\pi_t\}$ that fulfil (6) and (7) (which together with the requirement that $C_{At+1} + C_{Bt+1} = 2C_{t+1}^*$ imply (4) and (5)) such that $\pi_t e > S_{At}, \forall t$ and $(2 - \pi_t)e > S_{Bt}, \forall t$, i.e. such that the policies are feasible. Note that these policies require that at any date, $t + 1$, individual agents pick exactly the portfolios that make π_{t+1} fulfil (7) although, since both currencies have the same return, these individual agents are indifferent between all possible portfolios. Now, if we impose on the policy that π_t is a constant, $= \pi$, this means that $(1 - \pi)\frac{S_{Bt}}{e} + (2 - \pi)\frac{e_{Bt} - e_{At}}{2e} = 0, \forall t$ i.e. that $S_{Bt} = \frac{2-\pi}{2(\pi-1)}(e_{Bt} - e_{At})$ so that we have Policy 2. Thus Policy 2 is in this sense the "simplest" constant exchange rate policy that results in the Pareto optimal allocation C^*.

Note in passing, that any feasible policy S_A and S_B that fulfils (6) leads to the allocation C^*. When (6) holds the two agents have the same budget sets and hence, by strict concavity of u, their consumption will be identical: $C_{At} = C_{Bt}, \forall t$. But, since in equilibrium $C_{At} + C_{Bt} = 2C_t^*$, the result follows. In particular, for all policies that fulfil (6) sunspots cannot have any real effects.

4 The consequences of the three regimes and of a monetary union

In this section we ask the following questions for each of the three active/passive policies studied: (i) is there an equilibrium that achieves C^*, (ii) are there equilibria in which sunspots play a role, and (iii) if so, do sunspots have real effects. We will also consider the possibility and consequences of central bank intervention. Finally, we define what we mean by a monetary union for the model considered and consider what will be the consequences of such an arrangement.

4.1 No Intervention

As a beginning, it is natural to ask about the consequences if the governments do not pursue an active policy and in particular do not try to affect the movements of the exchange rate. Such a policy could in the context of the model at present be interpreted as a free float. Since the two governments keep the money supply constant, the policy could also be interpreted as the counterpart to what Obstfeld and Rogoff(2000) call "world monetarism a la McKinnon". However, under that interpretation we would expect that one or both central banks of the two countries would attempt to keep the exchange rate fixed. Let us at first ignore that possibilty. The first question then is, if the market can by itself establish an equilibrium in the currency market that compensates for the absense of a preset complete market structure.

When $S_{At} = S_{Bt} = 0$ we have

$$R_{At+1} = \frac{p_{At}}{p_{At+1}} = \frac{\pi_{t+1}}{\pi_t} \quad \text{and} \quad R_{Bt+1} = \frac{p_{At}}{p_{At+1}} = \frac{2-\pi_{t+1}}{2-\pi_t} \tag{8}$$

where, recall, $\pi_t = \pi_{At} + \pi_{Bt}$. Thus for any $\pi \in (0,2)$ and any $T \geq 1$, any $\pi_{ct}, c = A, B$, s.t. $\pi_t = \pi, \forall t \geq T$, is an equilibrium of this economy from T and onwards since in that case the exchange rate is constant implying that agents are indifferent between all possible portfolios (there is no risk sharing between agents).

C^* cannot be achieved

In the following we show that C^* cannot be achieved when governments are passive, but that for a limited (stochastic) number of periods C_t^* is possible. If C_t^* is the consumption in some state and at some t we have

$$[\pi_{ct-1} R_{At} + (1-\pi_{ct}) R_{Bt}]e + e_{ct} = e + \frac{1}{2}(e_{At} + e_{Bt}), c = A, B$$

implying that

$$R_{Bt} = \frac{e + \frac{1}{2}\Delta e_t}{e(1-\pi_{At-1})} - \frac{\pi_{At-1}}{1-\pi_{At-1}} R_{At} \quad \text{and} \quad R_{Bt} = \frac{e - \frac{1}{2}\Delta e_t}{e(1-\pi_{Bt-1})} - \frac{\pi_{Bt-1}}{1-\pi_{Bt-1}} R_{At}$$

These two equations can only hold if π_{At-1} is different from π_{Bt-1}. For that case, solving them gives

$$R_{At} = 1 + \frac{1}{2}\frac{2-\pi_{At-1}-\pi_{Bt-1}}{\pi_{At-1}-\pi_{Bt-1}}\frac{\Delta e_t}{e} = 1 + \frac{1}{2}\frac{2-\pi_{t-1}}{\pi_{At-1}-\pi_{Bt-1}}\frac{\Delta e_t}{e} \tag{9}$$

$$R_{Bt} = 1 - \frac{1}{2}\frac{\pi_{At-1}+\pi_{Bt-1}}{\pi_{At-1}-\pi_{Bt-1}}\frac{\Delta e_t}{e} = 1 - \frac{1}{2}\frac{\pi_{t-1}}{\pi_{At-1}-\pi_{Bt-1}}\frac{\Delta e_t}{e} \tag{10}$$

Note that it then follows that $R_{At} - R_{Bt} = \frac{1}{\pi_{At-1}-\pi_{Bt-1}}\frac{\Delta e_t}{e}$. So if the consumption, C_t^*, is achieved, we have that

$$E_{t-1}[u'(C_t^*)(R_{At} - R_{Bt})] = \frac{1}{e(\pi_{At-1}-\pi_{Bt-1})}E_{t-1}[u'(e + \frac{1}{2}(e_{At}+e_{Bt}))(e_{Bt}-e_{At})] = 0$$

where the last equality follows from the fact that e_{Bt} and e_{At} are identically and independently distributed. In other words, at any date $t-1$, if the consumption is C_t^* on the following date, the first order conditions, (iii) of Definition 2, hold for the two agents.

Combining (8) with (9) and (10) we then have to check if there is a stochastic sequence $\{\pi_{At}, \pi_{Bt}\}$ s.t. a.s.,$\forall t$

$$\pi_{At} \neq \pi_{Bt}, \quad \frac{\pi_t}{\pi_{t-1}} = 1 + \frac{1}{2}\frac{2-\pi_{t-1}}{\pi_{At-1}-\pi_{Bt-1}}\frac{\Delta e_t}{e} \text{ and } \frac{2-\pi_t}{2-\pi_{t-1}} = 1 - \frac{1}{2}\frac{\pi_{t-1}}{\pi_{At-1}-\pi_{Bt-1}}\frac{\Delta e_t}{e}$$

and s.t. $\pi_t \in (0,2), \forall t$. Each of these two equations is equivalent to

$$\pi_t = \pi_{t-1} + \frac{1}{2}\frac{(2-\pi_{t-1})\pi_{t-1}}{\pi_{At-1}-\pi_{Bt-1}}\frac{\Delta e_t}{e} \equiv H(\pi_{At-1}, \pi_{Bt-1}, \Delta e_t) \tag{11}$$

Note in passing that the stochastic relation described here is a martingale. So the question is, if there is a stochastic sequence $\{\pi_{At}, \pi_{Bt}\}$ with values in $[0,1]^2$ s.t. (11) holds and $\pi_t \in (0,2), \forall t$ a.s. The answer turns out to be negative. If $\pi_{t-1} \in (0,2)$ and $\pi_{ct-1} \in [0,1], c = A, B$, we have $\frac{\pi_{At-1}+\pi_{Bt-1}}{|\pi_{At-1}-\pi_{Bt-1}|} \geq 1$ and $\frac{2-\pi_{At-1}-\pi_{Bt-1}}{|\pi_{At-1}-\pi_{Bt-1}|} \geq 1$ in other words that

$$\frac{1}{2}\frac{(2-\pi_{t-1})\pi_{t-1}}{|\pi_{At-1}-\pi_{Bt-1}|}\frac{1}{e} \geq \frac{1}{2e}, \forall t, \text{ a.s.} \tag{12}$$

Suppose we were in an equilibrium where (11) held and $\pi_t \in (0,2), \forall t$ a.s. . In such an equilibrium Δe_t and (π_{At-1}, π_{Bt-1}) would be independent. This implies that there is some $k > 0$ s.t. for all $T > 1$ there is for a.a. $\omega \in \Omega$ some t' s.t for $t' < t \leq t' + T$, $\frac{\Delta e_t}{\pi_{At-1}-\pi_{Bt-1}} > k$. Then choose T such that $\frac{Tk}{2e} > 2$. We then have for a t' (depending on ω) $\pi_{t'+T} = \pi_{t'} + \sum_{t=t'+1}^{t'+T} \frac{1}{2}\frac{(2-\pi_t)\pi_t}{\pi_{At-1}-\pi_{Bt-1}}\frac{\Delta e_t}{e} \geq \pi_{t'} + \frac{Tk}{2e} > 2$, a contradiction. We conclude that in the non-intervention regime, with probability 1, C_t^* cannot be achieved indefinitely.

C_t^* can be achieved for a random number of periods

However, there is an equilibrium that achieves C_t^* for a positive (random) number of periods. The general principle is as follows. For any $t-1$, let π_{t-1} be given. If there is some (π_{At-1}, π_{Bt-1}) with $\pi_{At-1} + \pi_{Bt-1} = \pi_{t-1}$ and $\pi_{At} \neq \pi_{Bt}$ s.t. $H(\pi_{At-1}, \pi_{Bt-1}, \Delta e_t) \in (0,2)$ w. probability 1, let π_t be determined by (11), and the consumption in next period will be the random variable C_t^*. Consequently, with the gross returns on the two currencies defined by (9) and (10) the first order conditions will hold for the chosen (π_{At-1}, π_{Bt-1}), as was shown above. Else, if we cannot find (π_{At-1}, π_{Bt-1}) with the desired properties, let $\pi_s = \pi_{t-1}, \forall s \geq t$. Then the exchange rate is constant and the two representative agents are indifferent between any portfolio holdings, especially some (π_{At-1}, π_{Bt-1}) s.t. $\pi_{At-1} + \pi_{Bt-1} = \pi_{t-1}$. The two countries will then consume the autarky allocation from period t and onwards.

An example will demonstrate the possibility of such an equilibrium. Start with $\pi_1 = 1$ and let q be close to but below 1. For any t, given π_t define π_{At} and π_{Bt} as follows. If $\pi_t \leq 1$ let $\pi_{At} = q\pi_t$ and $\pi_{Bt} = (1-q)\pi_t$. Else let $1 - \pi_{At} = q(2-\pi_t)$ and $1 - \pi_{Bt} = (1-q)(2-\pi_t)$. Thus

$$\text{if } \pi_t \leq 1,\ \pi_{t+1} = \pi_t + \frac{2-\pi_t}{2(2q-1)}\frac{\Delta e_t}{e} \text{ and} \tag{13}$$

$$\text{if } \pi_t > 1,\ \pi_{t+1} = \pi_t + \frac{\pi_t}{2(1-2q)}\frac{\Delta e_t}{e} \tag{14}$$

When $\pi_t \approx 1$ we see that $\pi_{t+1} \in (0,2)$. At the first date t, at which with positive probability $\pi_{t+1} \notin (0,2)$ for the relevant of the two equations (13) and (14), let $\pi_{t+1} = \pi_t$ with probability one. In such an equilibrium it may seem for a long time that the Pareto optimal allocation, C^* is achieved and thus that the passive policy is achieving its goal. But eventually one of the currencies will be driven to near extinction and after that the exchange rate markets can no longer provide the role of insurance needed for Pareto optimality.

Sunspots
Clearly, there is a continuum of equilibria in the case of non-intervention and this should lead us to suspect that sunspots may matter. A very simple type of sunspot equilibria can be defined based on the equilibria just presented. Let $\{\sigma_t\}$ be a sequence of extrinsic stochastic variables taking values in $\{0, 1\}$. Consider the example given above and modify the equilibrium as follows. For any given date, t and given π_t define the continuation as follows. If both $\sigma_t = 1$ and $\pi_{t+1} \in (0, 2)$ with probability 1 (using the relevant of the two equations (13) and (14)) let π_{t+1} be defined accordingly. Else let $\pi_{t+1} = \pi_t$. A variant of this sunspot equilibrium would freeze the two countries at autarky, i.e. at the constant portfolio holding π_t the first time where $\sigma_t = 0$. Note, that the first of these sunspot equilibria are not Pareto dominated by the non-sunspot equilibrium it is derived from, since in the sunspot equilibrium, generations far into the future may achieve perfect risk sharing, where they would be in autarky in the non-sunspot equilibrium.

One may wonder if any other Pareto optimal allocations than C^* can be achieved under the non-intervention regime. In the appendix we show that this is not the case. Also to be found in the appendix is another example of a sunspot equilibrium. In this equilibrium, unlike in the one sketched above, with probability 1 sunspots will matter at all dates, although their influence will be diminishing. Finally, observe that the sunspot equilibria exhibited are characterized by (the effects of) the sunspots being correlated with fundamentals. We cannot have an equilibrium where sunspots have real effects and where $\{\pi_t\}$ is independent of fundamentals. In that case all agents would choose the same portfolios implying that their real return would be e, i.e. we would have the autarky consumption, $C_c = e + e_c, c = A, B$ for all sunspot states.

Remark: Central bank intervention
Suppose the purpose of the policy is to keep the exchange rate constant. Since sunspots may matter under this policy there is then a need for central bank intervention. If the two central banks coordinate they can keep the exchange rate at the desired level, but it will then only be the growth of world real money supply that is kept at 0 and not that of the individual countries. One individual central bank with limited reserves may not be able to keep the exchange rate constant, since when it is constant, agents are indifferent between all portfolios and currency crises may then appear in the same way as is treated in more detail below.

CONCLUSION: NON-INTERVENTION

When there is no intervention by governments (or central banks), i.e. no active monetary or fiscal policy, the returns on different currencies are entirely determined by demand conditions in the markets. It has been demonstrated that it is possible to design a random process of demands that achieves perfect risk sharing for a limited (random) time. However, in the long run such risk sharing is not achieved without active policies by the two governments. Moreover, when the governments are passive there is no guarantee that risk-sharing will be obtained even in the short run, since there is a continuum of equilibria for many of which (like the one in the appendix or the autarky equilibrium) risk sharing is less than perfect from date 1 onwards.

4.2 Devaluations

4.2.1 Symmetric Policies

We consider first a family of policies $(S_A, S_B) = (\tau_A max\{0, e_B - e_A\}, \tau_B max\{0, e_A - e_B\})$ (suppressing reference to time) where $\frac{sup\Delta e}{e} > \tau_c > 0$ and show that for the reduced family, where

$$\tau_A = \tau_B > 1/2 \qquad (15)$$

there is an equilibrium such that $\pi_{ct} = \pi_c, \forall t, c = A, B$ and the consumption is the Pareto optimal allocation, C^*. Else, if (15) does not hold, there is no such equilibrium for policies within the family considered. The interpretation of this family of policies is straigtforward. The government in country c makes a transfer to its own citizens if the shock they experience is relatively adverse and finance this transfer by issuing money. Since the other country does not issue money, this monetary expansion makes country c's currency depreciate. If we are in an equilibrium and C^* is achieved, for constant π_t we must have

$$E\left[u'(C^*)\left(\frac{\pi' e - \tau_A max\{0, e_B - e_A\}}{\pi} - \frac{(2-\pi')e - \tau_B max\{0, e_A - e_B\}}{2-\pi}\right)\right] = 0$$

This equality can only hold if $\frac{\tau_B}{2-\pi} = \frac{\tau_A}{\pi}$, i.e. if $\tau_B = \frac{2-\pi}{\pi}\tau_A$. Inserting this in the expression for the equilibrium consumption of agent A, equalizing to C^*, and rearranging we get:

$$-\pi_A \tau_A \frac{max\{0, e_B - e_A\}}{\pi} - (1-\pi_A)\frac{2-\pi}{\pi}\tau_A \frac{max\{0, e_A - e_B\}}{2-\pi} + \tau_A max\{0, e_B - e_A\} = \frac{1}{2}(e_B - e_A) \quad (16)$$

If $e_A > e_B$ (which happens with positive probability) this equality implies that $\frac{1-\pi_A}{\pi}\tau_A = 1/2$, while if $e_A < e_B$ it implies that $(\frac{-\pi_A}{\pi} + 1)\tau_A = 1/2$. Both these equalities can only hold if $\pi = 1$ which in turn implies $\tau_A = \tau_B = \tau$. It also follows that $\pi_A = \frac{\tau_A - 1/2}{\tau_A}$ (and consequently we require $\tau > 1/2$) and thus $\pi_B = \frac{1}{2\tau_B}$. Since (16) now holds, $C_A = C^*$, and consequently $C_B = C^*$. The first order conditions, (i) of definition 2, hold and with $\pi = 1$ and $\tau < \frac{sup\Delta e}{e}$ we have that also (ii) of the definition is fulfilled. Hence the desired equilibrium is achieved. Note that when e is large relatively to $sup\{e_A - e_B\}$, τ may be chosen to be very large implying that, in equilibrium, citizens of country c hold most of currency c and that the reactions, in terms of S_A and S_B, are large.

Sunspot equilibria

So far we have only studied the (in terms of π_{At} and π_{Bt}) determinstic and stationary equilibria associated with the (reduced) family of devaluation policies. Below we show that (a) associated with each member of this family there is a continuum of deterministic equilibria and a continuum of stationary Markov sunspot equilibria and (b) except when $\tau = 1$, the real part of these equilibria matters (i.e. sunspots matter). In doing this we use the results of Chiappori, Geoffard, and Guesnerie (1992), henceforth CG&G. Therefore, consider the system $Z = (Z_A, Z_B)^T$ (T for transpose). We then have $Z(1-\frac{1}{2\tau}, \frac{1}{2\tau}, 1-\frac{1}{2\tau}, \frac{1}{2\tau}) = (0,0)^T$ (where we now and below suppress reference to $S_A = \tau \max\{0, e_B - e_A\}$, $S_B = \tau \max\{0, e_A - e_B\}$, and $\mathcal{I}$). It is then straight forward to see that the technical Axiom A of CG&G holds[11]. Assuming that $\partial_0 Z$ is invertable, locally, the equilibrium is determined by

$$B = -(\partial_0 Z)^{-1}\partial_1 Z = \begin{pmatrix} \frac{\partial \pi_A}{\partial \pi'_A} & \frac{\partial \pi_A}{\partial \pi'_B} \\ \frac{\partial \pi_B}{\partial \pi'_A} & \frac{\partial \pi_B}{\partial \pi'_B} \end{pmatrix}$$

[11] This axiom states a relation between the derivatives of the deterministic and stochastic versions of the dynamical system.

where $\partial_0 Z$ is the Jacobean of Z with respect to the first two variables, π_A and π_B and $\partial_1 Z$ is the Jacobean of Z with respect to the last two variables, $\pi_A^{'}$ and $\pi_B^{'}$, both Jacobeans being evaluated in $(1-\frac{1}{2\tau},\frac{1}{2\tau},1-\frac{1}{2\tau},\frac{1}{2\tau})$. Below we show:
(i) $\partial_0 Z$ is actually invertible.
(ii) B has two real eigenvalues, one being 0 the other being > 1.
This means that the assumptions of CG&G's Theorem 3 holds, in particular Assumption(R). The conclusion of this theorem is that for any sufficiently small neighbourhood, N of $(1-\frac{1}{2\tau},\frac{1}{2\tau},1-\frac{1}{2\tau},\frac{1}{2\tau})$ and for any $k > 2$ there are k different points in N such that there is a stationary Markovian sunspot equilibrium with support being these k points. Note that we also need N to be so small that (i) and (ii) of Definition 2 holds.

Proving (i) and (ii)
Letting $R_A = e - \tau\max\{0, e_B^{'} - e_A^{'}\}$, $R_B = e - \tau\max\{0, e_A^{'} - e_B^{'}\}, \pi_A = 1-\frac{1}{2\tau}$, and $\pi_B = \frac{1}{2\tau}$ we have

$$\partial_0 Z = \begin{pmatrix} \frac{\partial \hat{Z}_A}{\partial \pi_A} + \frac{\partial \hat{Z}_A}{\partial \pi} & \frac{\partial \hat{Z}_A}{\partial \pi} \\ \frac{\partial \hat{Z}_B}{\partial \pi} & \frac{\partial \hat{Z}_B}{\partial \pi_B} + \frac{\partial \hat{Z}_B}{\partial \pi} \end{pmatrix}$$

Here, $\frac{\partial \hat{Z}_A}{\partial \pi_A} = \frac{\partial \hat{Z}_B}{\partial \pi_B} = E[u''(C^*)(R_A - R_B]^2$ while $\frac{\partial \hat{Z}_A}{\partial \pi} = E[u''(C^*)(R_A-R_B)(-\pi_A R_A + (1-\pi_A)R_B)] - E[u'(C^*)(R_A+R_B)]$ and $\frac{\partial \hat{Z}_B}{\partial \pi} = E[u''(C^*)(R_A-R_B)(-\pi_B R_A + (1-\pi_B)R_B)] - E[u'(C^*)(R_A+R_B) = E[u''(C^*)(R_A-R_B)(-(1-\pi_A)R_A + \pi_A R_B)] - E[u'(C^*)(R_A+R_B)]$ - the last equality following from the fact that $\pi_A = 1-\pi_B$.

It follows that the determinant, $\det\partial_O Z = \frac{\partial \hat{Z}_A}{\partial \pi_A}[\frac{\partial \hat{Z}_A}{\partial \pi_A} + \frac{\partial \hat{Z}_B}{\partial \pi} + \frac{\partial \hat{Z}_A}{\partial \pi}] = -\frac{\partial \hat{Z}_A}{\partial \pi_A} 2E[u'(C^*)(R_A+R_B)] > 0$. So $\partial_0 Z$ is invertible. Furthermore,

$$\partial_1 Z = \begin{pmatrix} \frac{\partial Z_A}{\partial \pi_A^{'}} & \frac{\partial Z_A}{\partial \pi_B^{'}} \\ \frac{\partial Z_B}{\partial \pi_A^{'}} & \frac{\partial Z_B}{\partial \pi_B^{'}} \end{pmatrix} = \frac{\partial Z_A}{\partial \pi_A^{'}}\mathcal{E}$$

where $\mathcal{E}$ is the matrix with 1 in all positions. The equality follows since $\frac{\partial Z_c}{\partial \pi_k^{'}}$ is the same for $c = A, B, k = A, B$ (and equal to $E[u''(C^*)(R_A - R_B)(2\pi_A - 1)] + Eu'(C^*)2e = Eu'(C^*)2e)$. Consequently,

$$B = -\frac{\frac{\partial Z_A}{\partial \pi_A^{'}}}{\det\partial_0 Z}\begin{pmatrix} \frac{\partial \hat{Z}_A}{\partial \pi_A} + \frac{\partial \hat{Z}_B}{\partial \pi} - \frac{\partial \hat{Z}_A}{\partial \pi} & \frac{\partial \hat{Z}_A}{\partial \pi_A} + \frac{\partial \hat{Z}_A}{\partial \pi} - \frac{\partial \hat{Z}_B}{\partial \pi} \\ \frac{\partial \hat{Z}_A}{\partial \pi_A} - \frac{\partial \hat{Z}_B}{\partial \pi} + \frac{\partial \hat{Z}_A}{\partial \pi} & \frac{\partial \hat{Z}_A}{\partial \pi_A} - \frac{\partial \hat{Z}_B}{\partial \pi} + \frac{\partial \hat{Z}_A}{\partial \pi} \end{pmatrix}$$

Since this matrix is singular, one eigenvalue is 0. The other is trace$B$ which is

$$-2\frac{\frac{\partial Z_A}{\partial \pi_A^{'}}}{\det\partial_0 Z}\frac{\partial \hat{Z}_A}{\partial \pi_A} = \frac{E[u'(C^*)]2e}{E[u'(C^*)(R_A+R_B)]} = \frac{E[u'(C^*)]2e}{E[u'(C^*)(2e - \tau\max\{e_B^{'} - e_A^{'}, e_A^{'} - e_B^{'}\})]} > 1$$

This concludes the proofs of the two claims, (i) and (ii).

Why do we have sunspots influence in this model?
Agents are forced to invest in one of the two currencies even though, due to seignorage, the expected gross return on them, is less than one in the stationary equilibrium. This gives rise to an instability. Let us show this formally. Agent c solves the problem:

$$\max_q E\left\{u\left[q\frac{\pi^{'}e - S_A}{\pi} + (1-q)\frac{(2-\pi^{'})e - S_B}{2-\pi} + e_c^{'} + S_c\right]\right\}$$

with solution $\hat{\pi}_c(\pi,\pi')$. We also let $\hat{\hat{\pi}}_c(\pi) = \hat{\pi}_c(\pi,\pi)$ and study this function further. If the current aggregate holding, π, is equal to the future aggregate holding, π', the first order conditions for agent A reduce to

$$F(\pi,\pi_A) \equiv E\left[u'\left(e - \pi_A\frac{\tau\max\{0,e_B-e_A\}}{\pi} - (1-\pi_A)\frac{\tau\max\{0,e_A-e_B\}}{2-\pi} + e_A + \tau\max\{0,e_B-e_A\}\right)\cdot \left(\frac{\tau\max\{0,e_B-e_A\}}{\pi} - \frac{\tau\max\{0,e_A-e_B\}}{2-\pi}\right)\right] = 0$$

-the expectation is w.r.t. the stochastic variables, $e_c, S_c, c = A, B$. We differentiate π_A implicitly w.r.t. π and evaluate in $(1, 1-\frac{1}{2\tau})$.

$$\frac{\partial F}{\partial \pi} = (1-\frac{1}{2\tau})\tau^2 E\left[u''(C^*)[(\max\{0,e_A-e_B\})^2 - (\max\{0,e_B-e_A\})^2]\right] - \tau^2 E\left[u''(C^*)[\max\{0,e_A-e_B\} - \max\{0,e_B-e_A\}]\max\{0,e_A-e_B\}\right] + \tau E\left[u'(C^*)[\max\{0,e_A-e_B\} + \max\{0,e_B-e_A\}]\right]$$

The first element of this sum is equal to 0, the last is > 0 and the second is equal to

$$-\tau^2 E\left[u''(C^*)[e_A-e_B]\max\{0,e_A-e_B\}\right] > 0$$

Consequently, $\frac{\partial\hat{\hat{\pi}}_A}{\partial\pi} =$

$$\frac{-\tau^2 E\left[u''(C^*)[e_A-e_B]\max\{0,e_A-e_B\}\right] + \tau E\left[u'(C^*)[\max\{0,e_A-e_B\} + \max\{0,e_B-e_A\}]\right]}{-\tau^2 E\left[u''(C^*)[e_A-e_B]^2\right]}$$

We have $(e_A-e_B)^2 = (e_A-e_B)[\max\{0,e_A-e_B\} - \max\{0,e_B-e_A\}]$ and thus by symmetry and independence of the distributions of e_A and e_B that $E[u''(C^*)(e_A-e_B)^2] = 2E[u''(C^*)(e_A-e_B)\max\{0,e_A-e_B\}]$. In conclusion, $\frac{\partial\hat{\hat{\pi}}_A}{\partial\pi} > 1/2$ and since $\frac{\partial\hat{\hat{\pi}}_A}{\partial\pi} = \frac{\partial\hat{\hat{\pi}}_B}{\partial\pi}$ it follows that $\frac{\partial\hat{\hat{\pi}}_A}{\partial\pi} + \frac{\partial\hat{\hat{\pi}}_B}{\partial\pi} > 1$. In other words, when π and π' increase with the same amount (starting from 1 in the stationary deterministic equilibrium) then $\pi_A + \pi_B$ increases more. By continuity this continues to hold in some neighborhood $\hat{N}$ of 1 and this is, in the end, what makes the deterministic stationary equilibrium indeterminate.

The indeterminacy described in two dimensions

Notice that $\frac{\partial\pi_A}{\partial\pi'_A} + \frac{\partial\pi_B}{\partial\pi'_A} = \text{trace}B > 1$. Since in the temporary equilibrium, it is only the $\pi' = \pi'_A + \pi'_B$ that matters this shows the following, which we refer to as "two dimensional indeterminacy":

$$\exists\underline{\pi},\overline{\pi} \text{ with } \underline{\pi} < 1 < \overline{\pi} \text{ such that } \forall\pi \in (\underline{\pi},\overline{\pi}), \exists!\ \pi' \equiv H(\pi) \text{ and } (\pi_A,\pi_B,\pi'_A,\pi'_B) \text{ with } |\pi'-1| \leq |\pi-1|, \pi_A+\pi_B = \pi, \text{ and } \pi'_A+\pi'_B = \pi' \text{ such that} (\pi_A,\pi_B,\pi'_A,\pi'_B) \text{ form a temporary equilibrium}$$

What is proved in CG&G is essentially that this indeterminacy implies the existence of stationary local sunspot equilibria.

Revaluation policy

The two countries may also decide for a revaluation policy, according to which the country that experiences a relatively favourable shock revaluates, i.e its goverment buys money and levys a tax on its citizens. The policy would have the following form: $S_A = -\tau\max\{0,e_A-e_B\}$, $S_B = -\tau\max\{0,e_B-e_A\}$ and, with $\pi_A = 1-\frac{1}{2\tau}$ and $\pi_B = \frac{1}{2\tau}$, C^* is achieved. With this policy we do not observe said instability, i.e. the equilibrium is (locally) determinate. This policy does not seem to have a real counterpart. Furthermore, in the companion paper, Nielsen(1998) it is shown that this policy will not lead to the ex-post optimal allocation C^* when agents have (generic) rational beliefs, instead of rational expectations.

The effects of sunspots

When $\tau = 1$, $e_A + S_A = e_A + \max\{0, e_B - e_A\} = e_B + \max\{0, e_A - e_B\} = e_B + S_B$, i.e. (6) holds and as was noted before sunspots cannot have any real effects. For any other τ the sunspot will have real effects. To see this write out the consumption at date $t+1$ of agent A as a function of the aggregate portfolio holdings:

$$C_{At+1} = \left[\pi_{At}\frac{\pi_{t+1}}{\pi_t} + (1-\pi_{At})\frac{2-\pi_{t+1})}{2-\pi_t} \quad e + (1 - \frac{\pi_{At}}{\pi_t})S_{At+1} - \frac{1-\pi_{At}}{2-\pi_t}S_{Bt+1} + e_{At+1}\right]$$

Suppose that it were the case that for all t and all states, (π_{At}, π_{Bt}) that given that state, π_{t+1} were non-random. Consider then a t and a state (π_{At}, π_{Bt}) s.t. given that state (π_{At+1}, π_{Bt+1}) is random, taking each of the two values (π_A^1, π_B^1) and (π_A^2, π_B^2) different from each other, with positive probability. By assumption then

$$\pi_A^1 + \pi_B^1 = \pi_A^2 + \pi_B^2 = \pi_{t+1} \tag{17}$$

Furthermore, there would be states π^3 and π^4 s.t. $P(\pi_{t+2} = \pi^{j+1} | (\pi_{At+1}, \pi_{Bt+1}) = (\pi_A^j, \pi_B^j)) = 1, j = 1, 2$. Suppose without loss of generality that $\pi^3 \geq \pi^4$. This together with (17) would mean that $\hat{\pi}_c(\pi_A^1 + \pi_B^1, \pi^3) \geq \hat{\pi}_c(\pi_A^2 + \pi_B^2, \pi^4)$ (because $\frac{\partial \hat{\pi}_c}{\partial \pi'} > 0$) i.e. that $\pi_c^1 \geq \pi_c^2, c = A, B$ implying, in turn because of (17), that $(\pi_A^1, \pi_B^1) = (\pi_A^2, \pi_B^2)$, a contradiction. In conclusion, in a sunspot equilibrium, at some dates t π_{t+1} has to be random conditional on date t information.

Then consider a date, t and a state, (π_{At}, π_{Bt}) s.t. given that state, π_{t+1} is truely random. If it were the case that C_{At+1} were not effected by π_{t+1} this would, since $\{\pi_t\}$ is independent of $\{e_{At}, e_{Bt}\}$, mean that $\frac{\pi_{At}}{\pi_{t+1}} = \frac{1-\pi_{At}}{2-\pi_t}$ which is equivalent to

$$\pi_{At} = \frac{\pi_t}{2} \tag{18}$$

If (18) holds (which it does when $\tau = 1$) $\frac{1}{2}\tau \max\{0, e_{Bt} - e_{At}\} - \frac{1}{2}\tau \max\{0, e_{At} - e_{Bt}\} + e_{At}$ is not equal to $\frac{1}{2}(e_{At} + e_{Bt})$ (unless $\tau = 1$) with probability 1 and thus C_t^* is not achieved. The conclusion is that, locally, sunspots do matter.

Example of sunspot equilibrium

We provide an example of how to construct sunspot equilibria for the case $\tau = 1$[12]. When $\tau = 1$, in any equilibrium, $\pi_{At} = \pi_{Bt}$ and $C_{At} = C_{Bt} = C_t^*$. Suppose then that there are two sunspots, 1 and 2 with transition matrix

$$\begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix}$$

to be determined. In equilibrium, letting π_i be the aggregate portfolio associated with sunspot i,

$$E\left[\sum_{i=1}^{2} u'(C^*)\left(\frac{\pi_i e - S_A}{\pi_j} - \frac{(2-\pi_i)e - S_B}{2-\pi_j}\right)P_{ji}\right] = 0, j = 1, 2$$

Letting $X = E[u'(C^*)S_A] = E[u'(C^*)S_B] > 0$ and $Y = E[u'(C^*)e] > X$ this equality can be written as

$$X\left[\frac{1}{2-\pi_j} - \frac{1}{\pi_j}\right] + Y\sum_{i=1}^{2}\left[\frac{\pi_i}{\pi_j} - \frac{2-\pi_i}{2-\pi_j}\right]P_{ij} = 0$$

If different from 0, we can divide through by $(\frac{1}{2-\pi_j} - \frac{1}{\pi_j})$ to get

$$X + Y\frac{\sum_{i=1}^{2}\pi_i P_{ji} - \pi_j}{\pi_j - 1} = 0$$

[12] This example is taken from Nielsen (1998).

Letting $q \in (0,1)$ be the solution to $X - Y q = 0$ we then require that

$$P_{j1}\pi_1 + P_{j2}\pi_2 = \pi_j + (1-\pi_j)q, j = 1,2 \tag{19}$$

If we pick $\pi_1 < 1 < \pi_2$ s.t. (ii) of Definition 2 holds for π_1 and π_2, then since, for $j = 1,2$, $\pi_1 < \pi_j + (1-\pi_j)q = q + (1-q)\pi_j < \pi_2$ we can find a Markov matrix s.t. (19) holds. The result is a sunspot equilibrium where, as in Manuelli and Peck (1990), the sunspots have an influence on the exchange rate, but not on the real part of the economy, i.e. consumption. Note, that in contrast to Manuelli and Peck the sunspot equilibria considered here are stationary.

Central bank intervention

To avoid the type of sunspot equilibria described, the central banks of the two countries may consider to intervene on the markets to keep the exchange rate at the level found in the stationary Pareto optimal equilibrium without extrinsic uncertainty. Let us call this equilibrium $\mathcal{S}^*(\tau)$ and equip the associated variables with an $*$, including the exchange rate, $x_t^* = \frac{p_{At}^*}{p_{Bt}^*}$. In this equilibrium the aggregate real demand for each currency is $\pi_t^* e = e$ at all dates. The central banks would then have to make up for any differences between this "target" demand and the demand by private agents. If the two central bank coordinate to defend the exchange rate[13] in a devaluation regime, this is certainly possible. Central bank c can always meet any demand for currency c. But whenever the price levels are kept at (p_{At}^*, p_{Bt}^*) at all dates, the central banks do not have to be active, since aggregate real demand will then be e. The very credibility of the possible intervention makes intervention unnecesary. It is then when only one central bank with limited foreign reserves tries to defend an excange rate that currency crises might appear.

A script for a currency crisis

In the context of our model a natural definition of a currency crisis is a situation where one central bank intervenes to defend the exchange rate, successfully or not. Consider the case where central bank A has foreign reserves in the amount of R units of currency B. We assume that this central bank is committed to intervention whenever called for and, in line with the rational expectations framework we are working within, that this is known by all agents. Many variations in terms of information and sequencing of events could be considered, but this is outside the scope of this contribution. We will merely provide an example, plus two variants, of how a currency crisis could be described in our model and show that they may appear. In the examples we consider there are three phases. In the pre-crisis phase the equilibrium values are like in $\mathcal{S}^*(\tau)$ and the central bank does not have to intervene. In the crisis phase the central bank is active on the markets for the two currencies. In the post-crisis phase the economy is either back at the stationary equilibrium, we could say the intervention was successful, or is behaving like one of the equilibria we have been studying, a sunspot equilibrium or an equilibrium in which the economy slowly moves back towards the stationary equilibrium. The main example considers the case where the central bank is unsuccessful with probability one and where the crisis only lasts one period. The two variants of this example consider respectively the case where the crisis lasts two periods and where intervention is successful with positive probability. Since we assume that only central bank A intervenes, a crisis can only occur in which it is forced to sell some or all of its foreign reserves. We now turn to describing the equilibrium, $\tilde{S}(\tau)$ with a currency crisis. Thereafter existence is considered.

The transfers are fixed at all dates, $S_{At} = \tau \max\{0, e_{Bt} - e_{At}\}$, $S_{Bt} = \tau \max\{0, e_{At} - e_{Bt}\}$. For $t < T$ the equilibrium is evolving exactly like in the stationary equilibrium $\mathcal{S}^*(\tau)$. At date T, after the realization of e_{AT} and e_{BT} the currency crisis occur, but central bank A is able to keep the prices at

[13] Something that does not seem to be observed in reality.

p^*_{AT} and p^*_{BT} (and thus the exchange rate at x^*_T) by selling all of its reserves. This justifies our assuming that all variables at $T-1$, in particular π_{AT-1} and π_{BT-1}, are the same as in $\mathcal{S}^*(\tau)$. The support of the aggregate portfolio at date $T+1$ in this equilibrium, $\tilde{\pi}_{T+1} = \tilde{\pi}_{AT+1} + \tilde{\pi}_{BT+1}$, is $\{\hat{\pi}^1, ..., \hat{\pi}^k\}$. The probabilities of these aggregate portfolios, known at date T, are $\tilde{P} = (\tilde{P}^1, \ldots, \tilde{P}^k\}$. The portfolio choices at date T are $\tilde{\pi}_{AT}$ and $\tilde{\pi}_{BT}$, with sum $\tilde{\pi}_T$, that solve:

$$max_q \sum_{i=1}^{k} \tilde{P}^i E\left[u(q\frac{\hat{\pi}^i e - S_{AT+1}}{\pi^*_T - \frac{R}{P^*_{BT}e}} + (1-q)\frac{(2-\hat{\pi}^i)e - S_{BT+1}}{2-\pi^*_T + \frac{R}{P^*_{BT}e}} + e_{cT+1} + S_{cT+1})\right] \tag{20}$$

Finally, $R = p^*_{BT}[\pi^*_T - \tilde{\pi}_T]e$ (in particular $\pi^*_T > \tilde{\pi}_T$). This means that by selling off all its foreign reserves the central bank can keep the prices at (p^*_{AT}, p^*_{BT}): Total demand (including that of central bank A) for currency A is

$$p^*_{AT}\tilde{\pi}_{AT} + R\frac{p^*_{AT}}{p^*_{BT}} = p^*_{AT}\tilde{\pi}_{AT} + p^*_{AT}[\pi^*_T - \tilde{\pi}_T]e = p^*_{AT}\pi^*_T e$$

and since supply, $M^*_{AT} + p^*_{AT}S_{AT}$, is unchanged, p^*_{AT} is indeed still the equilibrium price. The equilibrium on the market for currency B then follows.

We arrive at (20) as follows. $\tilde{p}_{cT} = p^*_{cT}, c = A, B$ and the portfolio holdings, $\tilde{\pi}_{cT}$ solve

$$\max_q E\left\{q\frac{p^*_{AT}}{\tilde{p}_{AT+1}}e + (1-q)\frac{p^*_{BT}}{\tilde{p}_{BT+1}}e + e_{cT+1} + S_{cT+1}\right\}$$

Furthermore, we have that

$$\tilde{M}_{AT+1} = \tilde{\pi}_T e p^*_{AT} = M^*_{AT} + p^*_{AT}S_{AT} - \frac{p^*_{AT}}{p^*_{BT}}R = \pi^* e p^*_{AT} - \frac{p^*_{AT}}{p^*_{BT}}R \tag{21}$$

$$\tilde{M}_{AT+1} = (2-\tilde{\pi}_T)e p^*_{BT} = M^*_{BT} + p^*_{BT}S_{BT} + R = (2-\pi^*)e p^*_{BT} + R \tag{22}$$

At date $T+s$, $\tilde{\pi}_{cT+s}$ solves

$$\max_q E\left\{q\frac{\tilde{p}_{AT+s}}{\tilde{p}_{AT+s+1}}e + (1-q)\frac{\tilde{p}_{BT+s}}{\tilde{p}_{BT+s+1}}e + e_{cT+s+1} + S_{cT+s+1}\right\}$$

and we have

$$\tilde{M}_{AT+s+1} = \tilde{\pi}_{T+s}e\tilde{p}_{AT+s} = \tilde{M}_{AT+s} + \tilde{p}_{AT+s}S_{AT+s} \tag{23}$$

$$\tilde{M}_{BT+s+1} = (2-\tilde{\pi}_{T+s})e\tilde{p}_{BT+s} = \tilde{M}_{BT+s} + \tilde{p}_{BT+s}S_{BT+s} \tag{24}$$

Combining (21) and (23) and (22) and (24) we arrive at (20).

(i) For the case where the continuation equilibrium is not affected by extrinsic uncertainty, we simply have $k=1$ and $\tilde{\pi}_{T+s+1} = H(\tilde{\pi}_{T+s})$ for $s \geq 1$.

(ii) For the case where the continuation equilibrium is a sunspot equilibrium, $\tilde{\pi}_{cT+s}$ is stochastic, in $\{\hat{\pi}^1, ..., \hat{\pi}^k\}$ with probability 1, and iff $\tilde{\pi}_{T+s} = \hat{\pi}^j$ there are $\tilde{\pi}_{AT+s}, \tilde{\pi}_{BT+s}$ with $\tilde{\pi}_{AT+s} + \tilde{\pi}_{BT+s} = \hat{\pi}^j$ s.t., for a transition probability $\{P^{ji}\}$, $\tilde{\pi}_{cT+s}$ solves

$$max_q \sum_{i=1}^{k} P^{ji} E\left[u(q\frac{\hat{\pi}^i e - S_{AT+s+1}}{\hat{\pi}^j} + (1-q)\frac{(2-\hat{\pi}^i)e - S_{BT+s+1}}{2-\hat{\pi}^j} + e_{cT+s+1} + S_{cT+s+1})\right]$$

Existence of a one-period currency crisis

Refer to the two dimensional indeterminacy described earlier. For case (i) suppose that $\pi^*_T - \frac{R}{P^*_{BT}e} \in (\underline{\pi}, \overline{\pi})$. This will be the case if R is sufficiently small or p^*_{BT} sufficiently large. Then let $\pi^1 = \pi^*_T - \frac{R}{P^*_{BT}e}$.

For case (ii) suppose that there is some sunspot equilibrium with support $\{\hat{\pi}^1, ..., \hat{\pi}^k\}$ s.t.

$$\pi^1 < H(\pi_T^* - \frac{R}{p_{BT}^* e}) \tag{25}$$

$$\pi^k > H(\pi_T^* - \frac{R}{p_{BT}^* e}) \tag{26}$$

Again, this is the case if R is sufficiently small. Let $\pi_c(P)$ be the solution to (20). Then for $P = (1, 0, .., 0)$, $\pi_A(P) + \pi_B(P) < \pi_T^* - \frac{R}{p_{BT}^* e}$, while for $P = (0, 0, .., 1)$, $\pi_A(P) + \pi_B(P) > \pi_T^* - \frac{R}{p_{BT}^* e}$. There is thus some $\tilde{P}$ s.t. $\pi_A(\tilde{P}) + \pi_B(\tilde{P}) = \pi_T^* - \frac{R}{p_{BT}^* e}$. For this $\tilde{P}$ the aggregate portfolio of the agents is such that with the prices p_{AT}^* and p_{BT}^* the reserves of the central bank A are exactly exhausted when it defends the prices/exchange rate. The fact that a currency crisis may happen at date T does not mean that it will actually happen. At date T agents have to come to believe that there will be a change of regime at date $T+1$ and the occurence of such a self-fulfilling belief is in itself extrinsic to the economy.

Other types of currency crises

Consider next a two-period crisis. In the first of the two crisis periods, T and $T+1$, the central bank uses some, but not all of its reserves to, successfully, defend the currency or exchange rate. In the second period, $T+1$, it uses the rest of its reserves but is not able to maintain the prices at p_{AT+1}^* and p_{BT+1}^*, the prices being $\tilde{p}_{AT+1}$ and $\tilde{p}_{BT+1}$ instead. This explains why agents, in period T, are not choosing the portfolios π_{AT}^* and π_{BT}^* and thus why the central bank has to intervene at that date. In period $T+2$ the equilibrium switches to some other continuation equilibrium and the prices in that equilibrium together with the current prices $\tilde{p}_{AT+1}$ and $\tilde{p}_{BT+1}$ determine, the portfolio choices, $\tilde{\pi}_{AT+1}$ and $\tilde{\pi}_{BT+1}$ with a sum different from 1. The details are found in the appendix.

Note in passing, that if the central bank is always able to successfully defend the exchange rate in two consecutive periods a currency crisis will never appear. For if it successfully defends the attack in the second of the two periods, the agents will hold the aggregate portfolio $\pi_T^* = 1$ in the first period, so there will be no need to defend in that period. A sufficient conditions for the impossibility of currency attacks at all dates is then $\frac{R^t}{p_{Bt}^*} > 2e$ for all t, where R^t is the reserves available at date t (since p_{Bt}^* is increasing over time, this inequality can only hold for R^t increasing).

Let us return to the one period crisis. If (25) and (26) hold, it is possible to construct an equilibrium where there are positive possibilities, P^a and P^b respectively, of the following two events: (a) at $T+1$ the equilibrium values continues to be the stationary Pareto optimal equilbrium, in particular the aggregate portfolio continues to be $\pi_{T+1}^* = 1$, (b) the equilibrium values switches to those of the sunspot equilibrium. So the distribution the agents are using, in making their portfolio choices at date T is now: Conditional on (a), $\pi_{T+1} = 1$ with probability 1 , and conditional on the event (b), $\pi_{T+1} = \hat{\pi}^i$ with probability $\tilde{P}^i$. Again we can choose suitable P^a,P^b and $\tilde{P}$ such that the reserves of the central bank are exactly exhausted. In this equilibrium, a currency crisis happens at date T, but with probability P^a the government is successful in keeping the exchange rate at the optimal level and hence consumption Pareto optimal. Of course, a new currency crisis may then arise at a later date.

4.2.2 Asymmetric Policies

The policies we consider here stipulate that $S_A \equiv 0$ (country A is passive), while $S_B = \tau(e_A - e_B)$. Note that these policies imply that both revaluations and devaluations take place and, what is more problematic (if we compare with reality), both inflation and deflation occur in country B. In order to achieve C^* in a stationary equilibrium where the aggregate portfolio is constant we need:

$$\pi_A e + (1-\pi_A)e - \frac{1-\pi_A}{2-\pi}\tau(e_A - e_B) + e_A = e + \frac{1}{2}(e_A + e_B) \equiv C^*$$

In other words that

$$\pi_A = 1 - \frac{2-\pi}{2\tau} \equiv \pi_A(\tau,\pi) \quad \text{and} \quad \pi_B = \pi - 1 + \tfrac{2-\pi}{2\tau} \equiv \pi_B(\tau,\pi) \;\; (= \pi - \pi_A) \tag{27}$$

Then $R_A - R_B = \frac{\tau}{2-\pi}(e_A - e_B)$ and since $E\{u'(C^*)\frac{\tau}{2-\pi}\frac{e_A-e_B}{e}\} = 0$ we have an equilibrium for any τ and $\pi \in (0,2)$ s.t.

$$(a)\;\; (2-\pi)e - \tau \sup(e_A - e_B) > 0 \qquad (b)\;\; 1 - \tfrac{2-\pi}{2\tau} \in (0,2) \qquad (c)\;\; \pi - 1 + \frac{2-\pi}{2\tau} \in (0,2)$$

When τ and π fulfil these requirements we say that they are admissible. Note that for $\tau \in (1/2, 1+\epsilon)$, for some positive ϵ, the three conditions hold for π in a neighborhood of 1. Note also, that if $\tau = 1$, $S_A + e_A = e_A = e_B + \tau(e_A - e_B) = e_B + S_B$, i.e. that (6) holds, so that sunspots can have no real effects.

We have, for given τ, that $\frac{\partial \pi_A}{\partial \pi} + \frac{\partial \pi_B}{\partial \pi} = 1$. The dynamical system thus has a large degree of nominal indeterminacy. In other words, for a given policy, τ, in the family considered, there are many stationary equilibria, each with different aggregate portfolios of the two currencies, but all of them achieving the consumption C^*. A counterpart to this observation is that the eigenvalues of the matrix associated with the dynamical system (defined is terms of π_A and π_B as in the analysis above) are 0 and 1. This means that we cannot apply Theorem 3 of CG&G, since they explicitly rule out that any eigenvalue is equal to 1 and require that some is greater than 1.

We show, for any admissible pair (τ,π), that there is a random variable $\epsilon(\tau,\pi)$ and an $\alpha(\tau,\pi) \in \Re$ such that if $\hat{\pi}_A(\tau,\pi) = \pi_A(\tau,\pi) + \alpha(\tau,\pi)$, $\hat{\pi}_B(\tau,\pi) = \pi_B(\tau,\pi) - \alpha(\tau,\pi)$, and $\pi'(\tau,\pi) = \pi + \epsilon(\tau,\pi)$, then
(i) With probability 1 $(\tau, \pi'(\tau,\pi))$ is admissible
(ii) For any random (π'_A, π'_B) s.t., with probability 1, $\pi'_A + \pi'_B = \pi'(\tau,\pi)$ and $\pi'_c \in [0,1], c = A, B$ $(\hat{\pi}_A(\tau,\pi)$,$\hat{\pi}_B(\tau,\pi)$,$\pi'_A, \pi'_B)$ form a temporary equilibrium (suppressing reference to policies and information) .

For given τ we can then construct a sunspot equilibrium as follows. Choose π_1 such that (τ,π_1) is admissible and $\pi_{1A} = \pi_A(\tau,\pi_1)$, and $\pi_{1B} = \pi_B(\tau,\pi_1)$. Then choose $\pi_2 = \pi'(\tau,\pi_1)$ and with probability one, (τ,π_2) is admissible and $(\pi_{1A}, \pi_{1B},\, \hat{\pi}_A(\tau,\pi_2),\, \hat{\pi}_B(\tau,\pi_2))$ form a temporary equilibrium. Then for any realization of π_2 we let $\pi_3 = \pi'(\tau,\pi_2) = \pi_2 + \epsilon(\tau,\pi_2)$ and continuing like this we get a sunspot equilibrium.

Perturbations that are permissible and form temporary equilibria
So let (τ,π) be admissible. Let $\pi_A^\alpha = 1 - \frac{2-\pi}{2\tau} + \alpha$, $\pi_B^\alpha = \pi - 1 + \frac{2-\pi}{2\tau} - \alpha$ and $\pi' = \pi + \epsilon$, where $\epsilon \in \{\epsilon_1, \epsilon_2\}$. For a given realization, ϵ_i the resulting consumptions are

$$C_{Ai} = C^* + \left[\frac{1}{\pi} + \frac{1}{2-\pi}\right] e\epsilon_i\alpha + \frac{1-\frac{1}{\tau}}{\pi}e\epsilon_i + \alpha\tau\frac{e_A - e_B}{2-\pi}$$

where we have used that $\frac{\pi_A(\tau,\pi)}{\pi} - \frac{1-\pi_A(\tau,\pi)}{2-\pi} = \frac{1-\frac{1}{\tau}}{\pi}$ and $C_{Bi} = 2C^* - C_{Ai}$. Finally, with the perturbed gross returns s.t. $R_A(\epsilon) - R_B(\epsilon) = (\frac{1}{\pi} + \frac{1}{2-\pi})e\epsilon + \tau\frac{e_A-e_B}{2-\pi}$ and letting the probability of ϵ_i be P_i we get the system: $F_A(\epsilon_1, \epsilon_2, \alpha) =$

$$\sum_{i=1}^{2} E\left[u'\left(C^* + \left[\frac{1}{\pi} + \frac{1}{2-\pi}\right] e\epsilon_i\alpha + \frac{1-\frac{1}{\tau}}{\pi}e\epsilon_i + \alpha\tau\frac{e_A - e_B}{2-\pi}\right)\left((\frac{1}{\pi} + \frac{1}{2-\pi})e\epsilon + \tau\frac{e_A - e_B}{2-\pi}\right)\right] P_i$$

and $F_B(\epsilon_1, \epsilon_2, \alpha) =$

$$\sum_{i=1}^{2} E\left[\left(C^* - \left[\frac{1}{\pi} + \frac{1}{2-\pi}\right] e\epsilon_i\alpha - \frac{1-\frac{1}{\tau}}{\pi} e\epsilon_i - \alpha\tau\frac{e_A - e_B}{2-\pi}\right)\left((\frac{1}{\pi} + \frac{1}{2-\pi})e\epsilon + \tau\frac{e_A - e_B}{2-\pi}\right)\right] P_i$$

We have $F_A(0,0,0) = F_B(0,0,0) = 0$ and taking derivatives w.r.t. ϵ_i and α and evaluating in $(0,0,0)$ we get:

$$\frac{\partial F_A}{\partial \epsilon_i} = \frac{\partial F_B}{\partial \epsilon_i} = Eu'(C^*)\left(\frac{1}{\pi} + \frac{1}{2-\pi}\right) eP_i > 0 \text{ and}$$

$$\frac{\partial F_A}{\partial \alpha} = -\frac{\partial F_B}{\partial \alpha} = Eu''(C^*)\left(\frac{\tau}{2-\pi}\right)^2 (e_A - e_B)^2 < 0, \text{ implying that}$$

$$\det\begin{pmatrix} \frac{\partial F_A}{\partial \epsilon_1} & \frac{\partial F_A}{\partial \alpha} \\ \frac{\partial F_B}{\partial \epsilon_1} & \frac{\partial F_B}{\partial \alpha} \end{pmatrix} = -2\frac{\partial F_A}{\partial \epsilon_1}\frac{\partial F_A}{\partial \alpha} > 0$$

So the implicit function theorem applies: We have implicitly defined ϵ_1 and α as functions of ϵ_2 in a neighborhood of $(0,0,0)$. By making ϵ_2 sufficiently small we can guarantee that $(\tau, \pi + \epsilon)$ is admissible with probability one. Note that, in general, C_{Ai} is not independent of i; sunspots have real effects. It is actually the case that $\frac{\partial \alpha}{\partial \epsilon_2} = 0$ for the implicit function, but one can also show that in a neighborhood of $(0,0,0)$ if, in equilibrium, ϵ_2 is different from 0 so is α.

CONCLUSION: DEVALUATIONS

We considered two types of devaluation policies, symmetric and asymmetric. For the first case we studied a continuum of policies that bear some resemblance to the devaluations policies seen, at least historically. According to these policies the government of the country that experiences a relatively negative shock pursues an expansionary monetary policy, i.e. issues money and transfers this money to the consumers, (or, in another interpretation, buys the commodity for the money and hands it over to the (old) consumers). As a consequence the currency of this country devaluates, something that causes all who holds this currency (including foreigners) to experience a negative net return. The end result is, in equilibrium, a net transfer from foreigners to the home country. It was shown that for most of these policies sunspots may have real effects, i.e. there are sunspot equilibria where the exchange rate as well as the consumption of agents fluctuate in response to the sunspot. The exception was the unique policy that makes the representative agents (two countries) ex-ante identical. Finally, we asked whether interventions by one or both central banks can prevent the countries from sliding into a suboptimal sunspot equilibrium. If the two central banks coordinate, the Pareto optimal allocation can indeed be defended, but if only one central bank defends the exchange rate this may not be possible. The conclusions for the asymmetric case are quite similar to those reached for the symmetric case. Only for one out of a continuum of policies can sunspots not have any real effects. It is obviously the case, also for this family of policies, that coordinated central bank intervention can make the exchange rate move in the desired way, while if only one central bank intervenes this may not prevent the allocation from being eventually suboptimal.

4.3 Policies for a Constant Exchange Rate

Let us first show that for each member of the family of fixed exchange rate policies there is an equilibrium, where C^* is achieved. We already, in the previous section, showed that if $\pi_{At} = \pi_{Bt} = {}^{q}\!/2, \forall t$, C^* is achieved. It is then clear that (iii) of Definition 2 is fulfilled. More generally, for $\pi^* \in (0,2)$ if

$\pi_A = \pi_B = \frac{1}{2}\pi^*$ $C_A = C_B = C^*$ and the first order condition holds:

$$E\left[u'(C^*_{t+1})\left(\frac{\pi^* e - \frac{q}{2(q-1)}(e_{Bt+1} - e_{At+1})}{\pi^*} - \frac{(2-\pi^*)e - \frac{2-q}{2(q-1)}(e_{Bt+1} - e_{At+1})}{2-\pi^*}\right)\right] =$$

$$E\left[u'(C^*_{t+1})\left(\frac{\frac{2-q}{2-\pi^*} - \frac{q}{\pi^*}}{2(q-1)}\frac{e_{Bt+1} - e_{At+1}}{e}\right)\right] = 0$$

On the other hand, the restrictions on q are chosen such that for π^* close to q if $\pi_{At} = \pi_{Bt} = \frac{1}{2}\pi^*, \forall t$ (i) (ii) of the definition holds. This establishes the counterpart to the Kareken and Wallace (1981) indeterminacy result. For Policy 2 there is a continuum of equilibria all with different exchange rates (to be studied shortly). We already observed, that with the family of policies considered, sunspots cannot have any real effects. In the following, we show that sunspots may have nominal effects. We also note that if both of the central banks of the two countries stand by the exchange rate, these nominal effects do not appear.

Since, with the family of policies considered here, the agents are identical, the equilibrium conditions of Definition 2 boil down to:

$$E_t\left[u'(C^*_{t+1})\left(\frac{\pi_{t+1}e}{\pi_t} - \frac{2-\pi_{t+1}}{2-\pi_t}\right)\right.= 0,\ \forall t \tag{28}$$

$$\left.\pi_{t+1}e - \frac{q}{2(q-1)}\Delta e_{t+1} > 0,\right]\forall t,\ \text{a.s.} \tag{29}$$

$$(2-\pi_{t+1})e - \frac{2-q}{2(q-1)}\Delta e_{t+1} > 0,\ \forall t,\ \text{a.s.} \tag{30}$$

To get (28) we used that $E_t[u'(C^*_{t+1})(\frac{q/\pi_t e}{2(q-1)}\Delta e_{t+1})] = 0$ and similarly, $E_t[u'(C^*_{t+1})(\frac{(2-q)/(2-\pi_t)e}{2(q-1)}\Delta e_{t+1})] = 0$, while (29) and (30) are simply restatements of (ii) of Definition 2. (28) is equivalent to:

$$E_t\left[u'(C^*_{t+1})\pi_{t+1}\right] = E_t\left[u'(C^*_{t+1})\pi_t\right] \tag{31}$$

If $\{\pi_t\}$ is a martingale independent of $\{e_{At}, e_{Bt}\}$, (31) holds. If, furthermore a.s.

$$\pi^d \equiv \frac{q}{2(q-1)}\frac{sup\Delta e}{e} < \pi_t < 2 - \frac{2-q}{2(q-1)}\frac{sup\Delta e}{e} \equiv \pi^u, \forall t$$

then (29) and (30) also hold and we have an equilibrium, where (if $\{\pi_t\}$ is truely random) the exchange rate will fluctuate in response to a sunspot, but where real variables are left unaffected. Note, that in this case, since $\{\pi_t\}$ is then a bounded martingale, by the martingale convergence theorem, it converges a.s. to some stochastic variable $\overline{\pi}$. Let us briefly look at the implications for the exchange rate.

The exchange rate in the long run

Write the exchange rate at date s, x_s, as: $x_s = x_1 \prod_{t=1}^{s-1} \frac{x_{t+1}}{x_t}$, i.e. $\log x_s = \log x_a + \sum_{t=1}^{s-1} \log \frac{x_{t+1}}{x_t}$

where $\frac{x_{t+1}}{x_t} = \frac{p_{At+1}/p_{Bt+1}}{p_{At}/p_{Bt}} = \frac{[2-\pi_{t+1}]e - \frac{2-q}{2(q-1)}\Delta e_t}{[2-\pi_t]e} \cdot \left[\frac{\pi_{t+1}e - \frac{q}{2(q-1)}\Delta e_t}{\pi_t e}\right]^{-1}$.

Suppose that $\pi_t = \pi^*, \forall t$. Then using the independence of $\{\Delta e_t\}$ and the strong law of large numbers we have

$$\frac{1}{s-1}\sum_{t=1}^{s-1}\log\frac{2(q-1)e-\frac{2-q}{2-\pi^*}\Delta e_{t+1}}{2(q-1)e-\frac{q}{\pi^*}\Delta e_{t+1}} \to E\left[\log\frac{2(q-1)e-\frac{2-q}{2-\pi^*}\Delta e}{2(q-1)e-\frac{q}{\pi^*}\Delta e}\right] \text{ a.s.}$$

This means that if $E\left[\log\frac{2(q-1)e-\frac{2-q}{2-\pi^*}\Delta e}{2(q-1)e-\frac{q}{\pi^*}\Delta e}\right] < 0(>0)$ then $\log x_s \to -\infty(\infty)$ i.e. $x_s \to 0(\infty)$. Letting P be the distribution of Δe we have

$$E\left[\log\frac{2(q-1)e-\frac{2-q}{2-\pi^*}\Delta e}{2(q-1)e-\frac{q}{\pi^*}\Delta e}\right] = \int_0^\infty \left[\log\frac{2(q-1)e-\frac{2-q}{2-\pi^*}y}{2(q-1)e-\frac{q}{\pi^*}y} + \log\frac{2(q-1)e+\frac{2-q}{2-\pi^*}y}{2(q-1)e+\frac{q}{\pi^*}y}\right]P(dy) =$$

$$\int_0^\infty \log\frac{4(q-1)^2e^2-\left(\frac{2-q}{2-\pi^*}\right)^2y^2}{4(q-1)^2e^2-\left(\frac{q}{\pi^*}\right)^2y^2}P(dy)$$

For $\pi^* = q$ this expression is 0. Furthermore, for any y the integrand is decreasing in π^*. Consequently, for $\pi^* < q$ the expectations is > 0 implying that $x_s \to \infty$ a.s. and for $\pi^* > q$ it is < 0, implying that $x_s \to 0$ a.s.. This is as we should expect: If there is less demand for currency A (and thus more for currency B) there is greater inflation in country A and currency A depreciates against currency B. In any of the two cases there is a serious inflation difference between the two countries. Since $\{\pi_t\}$ and $\{\Delta e_t\}$ are independent, the conclusion we found for constant π_t implies that if $\pi_t \to \overline{\pi} < q(> q)$ then $x_t \to \infty(0)$. If $\{\pi_t\}$ is a martingale, in general, a.s. either of the two will happen.

In the appendix it is shown by means of an example that such an extreme behavior may not be present in a sunspot equilibrium, i.e. x_t may fluctuate without ever tending to either 0 or ∞. Both types of sunspot equilibria may be thought of as depicting repeated speculative attacks on one or both of the two currencies. These attacks are rational in the sense that agents have correct expectations of what is going to happen in the future. The result could be, as we saw, that one of the currencies becomes almost valueless. For reasons strictly outside the model considered here this may not be deemed desirable. Our model is also too simple to capture the direct effects of wild fluctuations in the exchange rate[14], for instance on investments in export businesses. At any rate, if the two central banks are truely committed to a fixed exchange rate, no speculative attacks can effect this rate. It is only when a single centralbank is called to defend its own currency by selling its (limited) reserves that the fixed exchange rate arrangement may break down. We will briefly expose how this works in the present model. But first let us remark that not only the fear of speculative attacks may prompt the central banks to coordinate. When the exchange rate is fixed, agents are indifferent between any portfolio of currencies (this is what makes a fixed rate regime very vulnerable to attacks). However, for the exchange rate to stay fixed, the aggregate investment in currency A, πe has to be exactly equal to qe i.e. the agents have to coordinate, a questionable assumption in conjunction with the assumption that there are many small agents in the economy. When central banks coordinate, the individual agents need not do so.

Let $\{\pi^*_{At}, \pi^*_{Bt}, C^*_t, C^*_t, \frac{q}{2(q-1)}\Delta e_t, \frac{2-q}{2(q-1)}\Delta e_t, M^*_{At}, M^*_{Bt}, p^*_{At}, p^*_{Bt}\}_t$ be any Policy 2 equilibrium, where $\pi^*_{At} + \pi^*_{Bt} = q, \forall t$ and the exchange rate is fixed, $x_t = \frac{p_{At}}{p_{Bt}} = \overline{x}, \forall t$. Any sequence $\{\hat{\pi}_{At}, \hat{\pi}_{bt}, C^*_t, C^*_t, \frac{q}{2(q-1)}\Delta e_t, \frac{2-q}{2(q-1)}\Delta e_t, \hat{M}_{At}, \hat{M}_{Bt}, p_{At}, p_{Bt}\}_t$ where

$$\hat{M}_{At} = p_{At}(\hat{\pi}_{At} + \hat{\pi}_{Bt})e \text{ and } \hat{M}_{Bt} = p_{Bt}(2 - \hat{\pi}_{At} - \hat{\pi}_{Bt})e$$

is then an equilibrium where the two central banks coordinate by always offering their own currency at the rate $\overline{x}$. In this equilibrium, at any date t, $\hat{M}_{At+1}$ of currency A is demanded while $\hat{M}_{At} + S_{At}$ is

[14] Although, in the companion paper, Nielsen (1988) one possible consequence, that agents beliefs become more incorrect is considered.

supplied from private agents and the government in country A. If $\hat{M}_{At+1} > \hat{M}_{At} + S_{At}$ the difference is made up by central bank A buying ΔR_{At} in foreign reserves, where $\frac{p_{At}}{p_{Bt}}\Delta R_{At} = \hat{M}_{At+1} - \hat{M}_{At} + S_{At}$. If supply is bigger than demand $\hat{M}_{At+1} < \hat{M}_{At} + S_{At}$ then central bank B buys currency A to make up for the difference. The result will be a build-up of reserves over time in each central bank (unless they exchange them along the way).

Some Further Remarks

Clearly, if only one of the two central banks, say A, tries to defend the exchange rate, with limited foreign reserves it will most likely be unsuccesful. If $\hat{M}_{At} > \hat{M}_{At} + P_{At}S_{At} + \epsilon$, $\epsilon > 0$ for sufficiently many periods, which due to the agents' indifference between all portfolios under a fixed exchange rate can certainly happen, its reserves will be exhausted. A transition to a regime where the exchange rate is floating (but the consumption is still C_t^*) may then take place in the same way as described for the case of Policy 1s.

The counterpart to Policy 1a is the case where $S_{At} \equiv 0$. If C^* is achieved it follows from (6) and (7) that $S_{Bt} = e_{At} - e_{Bt}$ and $\pi_t = \pi_{t-1} + (1 - \pi_{t-1})\frac{e_{At}-e_{Bt}}{2e} + (2 - \pi_{t-1})\frac{e_{Bt}-e_{At}}{2e} = \pi_{t-1} - \frac{e_{At}-e_{Bt}}{2e}$ and then clearly, $\{\pi_t\}$ cannot be bounded with probability one so no equilibrium exists. Both countries need to be active in order for C^* to be achieved with a constant exchange rate.

CONCLUSION: FIXED EXCHANGE RATES

Under policy 2 there is an equilibrium where the exchange rate is constant. However, there are also equilibria where the exchange rate reacts to non-fundamental shocks. Such sun-spot equilibria can be interpreted as regimes with repeated currency attacks. The real parts of all these equilibria are the same, namely the Pareto optimal allocation, (C^*, C^*). This should be contrasted with Obstfeld and Rogoff(2000) who found the optimal fix to be Pareto dominated by the optimal float[15]. The result on Pareto optimality, also for the sunspot equilibria, relies heavily on the assumption that agents hold rational expectations and the ex-ante symmetry between agents brought about by the policy. If the governments want to avoid exchange rate volatility they could do so by making their central banks coordinate to defend the fixed exchage rate. If on the other hand only one central bank intervenes it is doomed to fail under repeated currency attacks.

4.4 Monetary Union

In a monetary union there is only one currency and the exchange rate cannot provide for insurance. In our simple world the only alternative is, that governments agree to make direct transfers across borders. Consider transfer policy 1s, with $\tau = 1$. This policy would be funded by a seignorage tax on the commonly held currency.

For this regime there is (for a given initial money supply) a unique monetary equilibrium. The problem of the young agents is trivial, they invest all their real wealth in the single currency. So

$$M_{t+1} = 2eP_t = M_t + P_t(S_{At} + S_{Bt}) = M_t + P_t(\max\{e_{Bt} - e_{At}, e_{At} - e_{Bt}\})$$

The resulting consumption is, for agent A: $C_{At} = \frac{1}{2}\frac{M_t}{P_t} + e_{At} + S_{At} = \frac{1}{2}[2e - (S_{At} + S_{Bt})] + e_{At} + S_{At} = e + e_{At} + \max\{0, e_{Bt} - e_{At}\} - \frac{1}{2}\max\{e_{Bt} - e_{At}, e_{At} - e_{Bt}\} = C_t^*$. The interpretation is straight forward and in line with some arguments concerning the attractiveness of a monetary union. Only with a coordinated fiscal policy in place, according to which a country (or region) hit adversely by a shock is subsidized by the rest of the countries in the monetary union can a monetary union achieve Pareto optimality. If such a coordinated fiscal policy is in place, then the members of the union will reap the

[15] Our conclusion is almost the opposite, since most devaluation policies are expectationally unstable.

benefits of the absence of any extrinsic uncertainty affecting their economic interaction. On the other hand, if a coordinated fiscal policy can only be partially instituted, there is a real trade-off between, say, a devaluation policy and a monetary union.

5 Conclusion

The analysis presented here is focused on the possible role of expectation in the markets for foreign exchange. The model employed for this analysis allows us to provide a rationale for several exchange rate regimes some of which have rather straight forward interpretations in terms of what has, historically, been observed[16]. These government policies are neccessary remedies to make up for the lack of complete markets. What is demonstrated, within the context of the model, is that the possibility of sunspot equilibria should be taken into account when the choice of exchange rate regime is made. Our general conclusion for the model was that letting the exchange rate float without any government intervention will only achieve a Pareto optimal allocation for a limited time. If the governments pursue a coordinated devaluation policy a Pareto optimal allocation may be achieved, however for most of these polices there are many equilibria for which the exchange rate varies more than called for by fundamentals and which result in suboptimal consumptions. In the case of a policy aimed at keeping the exchange rate fixed, sunspots cannot affect fundamentels in our model, but may effect nominal variables. However, we pointed out that if central banks choose to cooperate a fixed exchange rate can be defended. Currency crises are always only possible if there is a lack of cooperation between monetary authorities.

Throughout the paper we assumed that agents have rational expectations. This is not a realistic assumption as argued in the companion paper (see also for instance Elliott and Ito, 1999 and Taylor,1995). There are also reasons to question whether it is an innocuous assumption: diversity of beliefs and in particular mistaken beliefs may have a considerable impact on the performance of the exchange rate markets . Seen in this light, the results of this paper can be seen as "lower bounds" on the differences between different regimes. If there is expectational instability under rational expectations there will certainly also be so under rational beliefs. In the companion paper where we assume that agents have, not rational expectations, but rational (and diverse) beliefs we consider all policies within a large family and including all policies considered here. We then show that only fixed exchange rates with central bank coordination or a monetary union with fiscal coordination will achieve C^*. In particular, the single devaluation policy, identified to achieve C^* under rational expectations, does not pass the test when we allow for diverse beliefs. The results in the companion paper could then be considered as "upper bounds" on to what degree we can distinguish between different exchange rate regimes. To judge whether the reality is closer to the upper bound one would then have to empirically assess, how important are the welfare losses that result from mistaken beliefs about the exchange rates.

Models like Obstfeld and Rogoff(2000) are specified to take into account the possible "insurance" role of flexible exchange rates in adjusting the terms of trade in reaction to asymmetric shocks[17], (something that is also allowed for in the current analysis, albeit in a much simpler and probably less realistic way). On the other hand the scope for expectational instability seems more limited in such models and in that sense the stakes are against a regime with fixed exchange rates. The model considered here assumes perfect substitutability between currencies, so the stakes are against a regime with floating exchange

[16] Not all combinations being studied can be argued to have real world counterparts. Thus one probably seldom sees full central bank coordination and maybe not even uncoordinated intervention under a devaluation policy - although this depends on whether we should think of, for instance, managed floats and pegs with infrequent realignments as being described by policy 1s or 1a .

[17] Devereux and Engel(2000) question this role.

rates. There seems still to be some way to go before arriving at a general equilibrium model where active policies are called for and which is more open to both aspects of the foreign exchange markets.

Appendix

1 *No Pareto optimal allocation can be achieved with non-intervention*

Any individually rational Pareto optimal allocation for date t, $(\hat{C}_{At}, \hat{C}_{Bt})$ has the form $(\gamma(C_t^*)2C_t^*, (1-\gamma(C_t^*)2C_t^*)$, a.s., where γ is a function defined on $\Re_+$, and fulfils for some $\mu \in (0,1)$

$$\mu u'[\gamma(C_t^*)C_t^*] = (1-\mu)u'[(1-\gamma(C_t^*))C_t^*] \text{ a.s.}$$

When $\mu = 1/2$, $\gamma \equiv 1/2$, and if $\mu > 1/2$ then $\gamma(C_t^*) > 1/2, \forall C_t^*$. Following the same procedure as in the main text, we ask what R_{At} and R_{Bt} must be for $(\hat{C}_{At}, \hat{C}_{Bt})$ to be achieved. We require

$$[\pi_{At-1}R_{At} + (1-\pi_{At-1})R_{Bt}]e + e_{At} = \gamma(C_t^*)2C_t^*$$

$$[\pi_{Bt-1}R_{At} + (1-\pi_{Bt-1})R_{Bt}]e + e_{Bt} = (1-\gamma(C_t^*))2C_t^*$$

It easy to see, that $\pi_{At-1} = \pi_{Bt-1}$ is not possible if these equations hold. They can then be solved to give: $R_{At} =$

$$\frac{(1-\pi_{Bt-1})\gamma(C_t^*) - (1-\pi_{At-1})(1-\gamma(C_t^*))}{\pi_{At-1} - \pi_{Bt-1}}2 + \frac{2-(\pi_{At-1}+\pi_{Bt-1})}{\pi_{At-1}-\pi_{Bt-1}}[\gamma(C_t^*)e_{Bt} - (1-\gamma(C_t^*))e_{At}]\frac{1}{e}$$

$$\text{and } R_{Bt} = \frac{\pi_{At-1}(1-\gamma(C_t^*)) - \pi_{Bt-1}\gamma(C_t^*)}{\pi_{At-1}-\pi_{Bt-1}}2 - \frac{\pi_{At-1}+\pi_{Bt-1}}{\pi_{At-1}-\pi_{Bt-1}}[\gamma(C_t^*)e_{Bt} - (1-\gamma(C_t^*))e_{At}]\frac{1}{e}$$

so that

$$R_{At} - R_{Bt} = \frac{2}{\pi_{At-1}-\pi_{Bt-1}}[2\gamma(C_t^*) - 1 + (\gamma(C_t^*)e_{Bt} - (1-\gamma(C_t^*))e_{At})\frac{1}{e}]$$

But then, if $\gamma(C_t^*) > (<)\ 1/2$ a.s.

$$\frac{2}{\pi_{At-1}-\pi_{Bt-1}}E\{u'(\hat{C}_{ct})\frac{2}{\pi_{At}-\pi_{Bt}}[2\gamma(C_t^*) - 1 + (\gamma(C_t^*)e_{Bt} - (1-\gamma(C_t^*))e_{At})\frac{1}{e}]\} > (<)\ 0, c = A, B$$

so the first order conditions of the two agents, required for equilibrium, cannot hold.

2 *Perpetual effects of sunspot under non-intervention*

Let $\{\pi_t\}_t$ be a martingale s.t. $\pi_t \in (0,2), \forall t$, a.s.. Furthermore, $\{\pi_t\}$ is picked such that for all t and all π_t there are $\overline{\pi}(\pi_t) > \pi_t > \underline{\pi}(\pi_t)$ such that $P(\pi_{t+1} = \overline{\pi}(\pi_t)|\pi_t) = P(\pi_{t+1} = \underline{\pi}(\pi_t)|\pi_t) = 1/2$. Finally, assume that this martingale is independent of $\{e_{At}, e_{Bt}\}_t$. Let for given π_t, R_{At} and R_{Bt} be the corresponding gross returns and define $\overline{\Delta}R_t = \frac{\overline{\pi}(\pi_t)}{\pi_t} - \frac{2-\overline{\pi}(\pi_t)}{2-\pi_t}$ and $\underline{\Delta}R_t = \frac{\underline{\pi}(\pi_t)}{\pi_t} - \frac{2-\underline{\pi}(\pi_t)}{2-\pi_t} = -\overline{\Delta}R_t$. So $R_{At} - R_{Bt} \in \{\overline{\Delta}R_t, \underline{\Delta}R_t\}$. We then have that

$$E\left[u'(e+e_{ct+1})(R_{At+1} - R_{Bt+1})|\pi_t\right] = 0, c = A, B$$

and letting $\pi_{ct} = \pi_t/2, \forall t$ we have an equilibrium.

Next we consider the following perturbations of the distribution of π_{t+1} given π_t. For ease of notation we suppress reference to π_t, taken to be given below. Let $\tilde{e}$ be chosen such that $P(e_{At} = \tilde{e}) = 0$ and $P(e_{At} < \tilde{e}) \in (0,1)$. Then define the perturbed distribution as follows:

$$P^\epsilon(\pi_{t+1} = \overline{\pi}|e_{At+1} > \tilde{e}, e_{Bt+1} > \tilde{e}) = P^\epsilon(\pi_{t+1} = \overline{\pi}|e_{At+1} < \tilde{e}, e_{Bt+1} < \tilde{e}) = \frac{1}{2}$$

$$P^\epsilon(\pi_{t+1} = \overline{\pi}|e_{At+1} > \tilde{e}, e_{Bt+1} < \tilde{e}) = \frac{1}{2} - \epsilon$$

$$P^\epsilon(\pi_{t+1} = \overline{\pi}|e_{At+1} < \tilde{e}, e_{Bt+1} > \tilde{e}) = \frac{1}{2} + \epsilon$$

This perturbation makes π_t and (e_{At}, e_{Bt}) dependent, but leaves the marginal distribution of π_t unchanged. We show that with this new distribution there is a $\delta > 0$ and a new equilibrium, where the optimal portfolios are $\hat{\pi}_{At} = \pi_t/2 + \delta$ and $\hat{\pi}_{Bt} = \pi_t/2 - \delta$. E^ϵ now refers to expectation under the perturbed distribution.

$$E^\epsilon \left[u'(e + e_{At+1})(R_{At+1} - R_{Bt+1})|\pi_t\right] =$$
$$E^\epsilon \left[u'(e + e_{At+1})(R_{At+1} - R_{Bt+1})|\pi_t, (e_{At+1}, e_{Bt+1}) > (\tilde{e}, \tilde{e}) \vee (e_{At+1}, e_{Bt+1}) < (\tilde{e}, \tilde{e})\right] \cdot$$
$$P((e_{At+1}, e_{Bt+1}) > (\tilde{e}, \tilde{e}) \vee (e_{At+1}, e_{Bt+1}) < (\tilde{e}, \tilde{e}))$$
$$+ \int_{\tilde{e}}^{\infty} \left[\int_0^{\tilde{e}} u'(e + e_{At+1})P(de_{At+1})\right] P(de_{Bt+1})[\overline{\Delta} R_t(\frac{1}{2} + \epsilon) + \underline{\Delta} R_t(\frac{1}{2} - \epsilon)]$$
$$+ \int_0^{\tilde{e}} \left[\int_{\tilde{e}}^{\infty} u'(e + e_{At+1})P(de_{At+1})\right] P(de_{Bt+1})[\overline{\Delta} R_t(\frac{1}{2} - \epsilon) + \underline{\Delta} R_t(\frac{1}{2} + \epsilon)]$$

The first element of this sum is equal to 0, while the second and third are equal to

$$\left(\int_{\tilde{e}}^{\infty} \left[\int_0^{\tilde{e}} u'(e + e_{At+1})dP(e_{At+1})\right] P(de_{Bt+1}) - \int_0^{\tilde{e}} \left[\int_{\tilde{e}}^{\infty} u'(e + e_{At+1})dP(e_{At+1})\right] P(de_{Bt+1})\right) \epsilon[\overline{\Delta} R_t - \underline{\Delta} R_t] \qquad (32)$$

Since, by the strict concavity of u, $u'(e_A) < u'(e_B)$ for $e_A \in (\tilde{e}, \infty), e_B \in (0, \tilde{e})$ we have

$$\int_0^{\tilde{e}} \left[\int_{\tilde{e}}^{\infty} u'(e + e_{At+1})P(de_{At+1})\right] P(de_{Bt+1}) < \int_0^{\tilde{e}} \left[\int_{\tilde{e}}^{\infty} u'(e + e_{Bt+1})P(de_{At+1})\right] P(de_{Bt+1}) =$$
$$\int_{\tilde{e}}^{\infty} \left[\int_0^{\tilde{e}} u'(e + e_{Bt+1})P(de_{Bt+1})\right] P(de_{At+1}) = \int_{\tilde{e}}^{\infty} \left[\int_0^{\tilde{e}} u'(e + e_{At+1})P(de_{At+1})\right] P(de_{Bt+1})$$

Consequently, (32) is positive. This means that agent A wants to invest more in currency A under the perturbed distribution. I.e. there is $\delta > 0$ such that under the perturbed distribution the optimal portfolio is $\hat{\pi}_{At} = \pi_t/2 + \delta$. We now only have to show that under the perturbed distribution the optimal portfolio for B is $\hat{\pi}_{Bt} = \pi_t/2 - \delta$, i.e. that

$$E^\epsilon[u'(e - \delta(R_{At+1} - R_{Bt+1}) + e_{Bt+1})(R_{At+1} - R_{Bt+1})|\pi_t] = 0$$

For any $\hat{e}$ then, $u'(e + \delta\overline{\Delta} R_{t+1} + \hat{e})\overline{\Delta} R_{t+1} = -u'(e - \delta\underline{\Delta} R_{t+1} + \hat{e})\underline{\Delta} R_{t+1}$

from which one can show that

$$E^\epsilon[u'(e - \delta(R_{At+1} - R_{Bt+1}) + e_{Bt+1})(R_{At+1} - R_{Bt+1})|\pi_t] =$$
$$-E^\epsilon[u'(e + \delta(R_{At+1} - R_{Bt+1}) + e_{Bt+1})(R_{At+1} - R_{Bt+1})|\pi_t] = 0$$

For instance, $E[u'(e+\delta\overline{\Delta} R_{t+1}+e_{At+1})\overline{\Delta} R_{t+1}|\pi_t, e_{At+1} > \tilde{e}, e_{Bt+1} < \tilde{e}](\frac{1}{2}-\epsilon)P(e_{At+1} > \hat{e}, e_{Bt+1} < \hat{e}) =$

$$-E[u'(e - \delta\underline{\Delta} R_{t+1} + e_{Bt+1})\underline{\Delta} R_{t+1}|\pi_t, e_{At+1} < \tilde{e}, e_{Bt+1} > \tilde{e}](\frac{1}{2} - \epsilon)P(e_{At+1} < \hat{e}, e_{Bt+1} > \hat{e})$$

and so on. This completes the example. Note, that when the distribution is unperturbated, there is autarky. Since, each agent c, $c = A, B$ can also under the perturbed distribution choose the portfolio $\pi/2$, resulting in the consumption $e + e_{ct}$, but does not, the consumption in the sunspot equilibrium strictly Pareto dominates $e + e_{ct}$. Finally, note that the influence of the sunspot will decrease over time, since the martingale process $\{\pi_t\}$, being bounded, converges.

3 *A two-period currency crisis*

The crisis starts in period T. Before then, the equilibrium values, identified by a $\tilde{}$, are equal to those prevailing in the stationary Pareto optimal equilibrium with no extrinsic uncertainty, identified by an $*$. During the crisis the central bank in country A uses up all its reserves. From period $T+2$ and onwards the equilibrium values are like in a sunspot equilibrium. In period T we then have:

$$\tilde{M}_{AT+1} = \tilde{\pi}_T e P^*_{AT} = M^*_{AT} + P^*_{AT} S_{AT} - \frac{P^*_{AT}}{P^*_{BT}} R_T = \pi^*_T e P^*_{AT} - \frac{P^*_{AT}}{P^*_{BT}} R_T$$

$$\tilde{M}_{BT+1} = (2 - \tilde{\pi}_T) e P^*_{BT} = M^*_{BT} + P^*_{BT} S_{BT} + R_T = (2 - \pi^*_T) e P^*_{BT} + R_T$$

where R_t is the amount of reserves that central bank A supplies to the market in period $t = T, T+1$. Furthermore, $\tilde{\pi}_{cT}$ solves for $c = A, B$:

$$\max_{q \in [0,1]} E \left\{ u \left[q \frac{P^*_{AT}}{\tilde{P}_{AT+1}} e + (1-q) \frac{P^*_{BT}}{\tilde{P}_{BT+1}} e + e_{cT+1} + S_{cT+1} \right] \right\}$$

In period $T+1$ we have

$$\tilde{M}_{AT+2} = \tilde{\pi}_{T+1} e \tilde{P}_{AT+1} = \tilde{M}_{AT+1} + \tilde{P}_{AT+1} S_{AT+1} - \frac{\tilde{P}_{AT+1}}{\tilde{P}_{BT+1}} R_{T+1}$$

$$\tilde{M}_{BT+2} = (2 - \tilde{\pi}_{T+1}) e \tilde{P}_{BT+1} = \tilde{M}_{BT+1} + \tilde{P}_{BT+1} S_{BT+1} + R_{T+1}$$

and $\tilde{\pi}_{cT+2}$ solves, for $c = A, B$,

$$\max_{q \in [0,1]} E \left\{ u \left[q \frac{\tilde{P}_{AT+1}}{\tilde{P}_{AT+2}} e + (1-q) \frac{\tilde{P}_{BT+1}}{\tilde{P}_{BT+2}} e + e_{cT+2} + S_{cT+2} \right] \right\}$$

From $T+2$ and onwards the usual equilibrium conditions apply.
We can then, in the usual way, rewrite the equilibrium returns as follows:

$$\frac{P^*_{AT}}{\tilde{P}_{AT+1}} e = \frac{\tilde{\pi}_{T+1} e - S_{AT+1} + \frac{R_{T+1}}{\tilde{P}_{BT+1}}}{\pi^*_T - \frac{R_T}{P^*_{BT} e}} \quad \text{and} \quad \frac{P^*_{BT}}{\tilde{P}_{BT+1}} e = \frac{(2 - \tilde{\pi}_{T+1}) e - S_{BT+1} - \frac{R_{T+1}}{\tilde{P}_{BT+1}}}{2 - \pi^*_T + \frac{R_T}{P^*_{BT} e}}$$

Then using that $R_{T+1} = R - R_T$ and that $\tilde{M}_{BT+2} = \tilde{M}_{BT+1} + \tilde{P}_{BT+1} S_{BT+1} + R_{T+1} = M^*_{BT} + P^*_{BT} S_{BT} + R_T + \tilde{P}_{BT+1} S_{BT+1} + R_{T+1} = M^*_{BT+1} + R + \tilde{P}_{BT+1} S_{BT+1}$ so that $(2 - \tilde{\pi}_{T+1}) e \tilde{P}_{BT+1} = M^*_{BT+1} + R + \tilde{P}_{BT+1} S_{BT+1}$, i.e.

$$\tilde{P}_{BT+1} = \frac{M^*_{BT+1} + R}{(2 - \tilde{\pi}_{T+1}) e - S_{BT+1}}$$

we get a description of the relevant part of the equlibrium as follows.
$\tilde{\pi}_{AT}, \tilde{\pi}_{BT}, \tilde{\pi}_{AT+1}, \tilde{\pi}_{BT+1}, \tilde{R}_T, (\hat{\pi}^1, \ldots, \hat{\pi}^k), (\tilde{P}^1, \ldots, \tilde{P}^k)$ such that
(1) $\tilde{R}_T < R$
(2) $\tilde{\pi}_{AT} + \tilde{\pi}_{BT} = \pi^*_T - \frac{\tilde{R}_T}{P^*_{BT} e} \in (0, 2)$

(3) $\tilde{\pi}_{cT} = \tilde{\pi}_{cT}(\tilde{R}_T, \tilde{\pi}_{T+1})$ where $\tilde{\pi}_{cT}(R_T, \pi^{'})$ solves:

$$\max_{q\in[0,1]} E\left\{ u\left[q\frac{\pi^{'}e - S_{AT+1} + (R - R_T)\frac{(2-\pi^{'})e - S_{BT+1}}{M^{*}_{BT+1}+R}}{\pi^{*}_{T} - \frac{R_T}{P^{*}_{BT}e}} + \right.\right.$$

$$\left.\left.(1-q)\frac{(2-\pi^{'})e - S_{BT+1} - (R - R_T)\frac{(2-\pi^{'})e - S_{BT+1}}{M^{*}_{BT+1}+R}}{(2-\pi^{*}_{T}) + \frac{R_T}{P^{*}_{BT}e}} + e_{cT+1} + S_{cT+1}\right]\right\}$$

(4) $\tilde{\pi}_{cT+1} = \tilde{\pi}_{cT+1}(\{\tilde{P}^i\}, \tilde{\pi}_{T+1})$ where for any probability vector, P^i and $\pi^{'} \in (0,2)$, $\tilde{\pi}_{cT+1}(\{P^i\}, \pi^{'})$ solves

$$\max_{q\in[0,1]} \sum_{i=1}^{k} E\left\{ u\left[q\frac{\hat{\pi}^i e - S_{AT+2}}{\pi^{'}} + (1-q)\frac{(2-\hat{\pi}^i)e - S_{BT+2}}{2-\pi^{'}} + e_{cT+2} + S_{cT+2}\right]\right\} P^i$$

and where there are for $i = 1, 2, .., k$ $\hat{\pi}^i_A$ and $\hat{\pi}^i_B$ summing to $\hat{\pi}^i$ where $\hat{\pi}^i_c$ solves

$$max_q \sum_{j=1}^{k} E\left\{ u\left[q\frac{\hat{\pi}^j e - S_A}{\hat{\pi}^i} + (1-q)\frac{(2-\hat{\pi}^j)e - S_B}{2-\hat{\pi}^j} + e_c + S_c\right]\right\} P^{ji} \tag{33}$$

for some transition matrix $[P^{ij}]$.
(5) Finally, we need that both $\tilde{\pi}_{T+1}$ and the $\hat{\pi}^i$'s fulfill the feasibility requirements that $\pi e - S_A > 0$ and $(2-\pi)e - S_B > 0$, something which holds in a neigborhood of $\pi^*_t \equiv 1$.
We use the characterization of a two dimensional indeterminacy provided in 4.2.1 . Suppose then that
(a) $\pi^*_T - \frac{R}{P^*_{BT}} \in (\underline{\pi}, \overline{\pi})$
(b) There is some sunspot equilibrium $[P^{ji}]$, $(\hat{\pi}^1, \ldots, \hat{\pi}^k)$ (i.e. (33) holds) such that $\hat{\pi}^1 < H(\pi^*_T - \frac{R}{P^*_{BT}}) < \hat{\pi}^k$.
These two requirements hold if $\frac{R}{P^*_{BT}}$ is sufficiently small. Starting by setting $R_T = R$ we get a one period currency crisis, with $\tilde{\pi}_{T+1} = H(\pi^*_T - \frac{R}{P^*_{BT}})$, $\tilde{\pi}_{T+2} = H(\tilde{\pi}_{T+1}) = H(H(\pi^*_T - \frac{R}{P^*_{BT}}))$ and so on. Since $\frac{\partial \tilde{\pi}_{cT}}{\partial \pi^{'}} > 0$ (for R_T close to R), for R_T in a neighborhood of R there is $\pi^{'}(R_T)$ such that $\tilde{\pi}_{AT}(R_T, \pi^{'}(R_T)) + \tilde{\pi}_{BT}(R_T, \pi^{'}(R_T)) = \pi^*_T - \frac{R_T}{P^*_{BT}e}$. When R_T is sufficiently close to R, $\pi^{'}(R_T) \in (\hat{\pi}^1, \hat{\pi}^k)$ continues to hold so there is some $(\tilde{P}^1, \ldots, \tilde{P}^k)$ such that

$$\tilde{\pi}_{AT+1}(\{\tilde{P}^i\}, \pi^{'}(R_T)) + \tilde{\pi}_{BT+1}(\{\tilde{P}^i\}, \pi^{'}(R_T)) = \pi^{'}(R_T)$$

This concludes the proof. The two-period currency crisis depicted here is "close" to a one-period currency crisis. One would expect that there would also be two-period currency crises where the government would spend substantial amounts of reserves in the second period.

4 *Perpetual effects of sunspots under Policy 2*
As previously stated the equilibrium conditions are

$$E_t\left[u'(C^*_{t+1})\pi_{t+1}\right] = E\left[u'(C^*_{t+1})\right]\pi_t$$

together with the boundary conditions on π_t. Let $\pi_{t+1} = \pi_t + \delta_{t+1}$, where δ_{t+1} is random, and the requirement is

$$E_t\left[u'(C^*_{t+1})\delta_{t+1}\right] = 0$$

Find $\hat{e}$ such that $\mu(e_A + e_B > \hat{e}) \in (0,1)$ and $\mu(e_A + e_B = \hat{e}) = 0$. We then have $E[u'(e + (C^*)/2)|e_A + e_B < \hat{e}] > E[u'(e + (C^*)/2)|e_A + e_B > \hat{e}]$. We now define three distributions for δ. Let $\underline{\delta}^u = -1$ and

$$\overline{\delta}^u = \frac{\int_0^{\hat{e}} u'(C^*)/2)P(d(e_A + e_B))}{\int_{\hat{e}}^{\infty} u'(e + (C^*)/2)P(d(e_A + e_B))}$$

where P is the distribution of $e_{At} + e_{Bt}$. Then for each t define two random variables, δ_t^u and δ_t^d, as follows:

$$\text{if } e_{At} + e_{Bt} > \hat{e}, \quad \delta_t^u = \frac{\overline{\delta}^u}{\max\{\overline{\delta}^u, 1\}} \quad \text{else,} \quad \delta_t^u = \frac{\underline{\delta}^u}{\max\{\overline{\delta}^u, 1\}}$$

and let $\delta_t^d = -\delta_t^u$. Both are assumed to be independent of all variables but $e_{At} + e_{Bt}$.
We then have $E[u'(C_t^*)\delta_t^u)] = E[u'(C_t^*)\delta_t^u|e_{At}+e_{Bt} > \hat{e}]P(e_{At}+e_{Bt} > \hat{e}) + E[u'(C_t^*)\delta_t^u|e_{At}+e_{Bt} < \hat{e}]P(e_{At}+e_{Bt} < \hat{e}) = \frac{\int_0^{\hat{e}} u'(C_t^*)P(d(e_{At}+e_{Bt}))}{\int_{\hat{e}}^{\infty} u'(C_t^*)P(d(e_{At}+e_{Bt}))} \int_{\hat{e}}^{\infty} u'(C_t^*)P(d(e_{At} + e_{Bt})) - \int_0^{\hat{e}} u'(C_t^*)P(d(e_{At} + e_{Bt})) = 0$. Consequently, $E[u'(C_t^*)\delta_t^d)] = 0$, also.
Furthermore,

$$E\delta_t^u = -P(e_{At} + e_{Bt} < \hat{e}) + P(e_{At} + e_{Bt} > \hat{e})\frac{\int_0^{\hat{e}} u'(C_t^*)P(d(e_{At} + e_{Bt}))}{\int_{\hat{e}}^{\infty} u'(C_t^*)P(d(e_{At} + e_{Bt}))} =$$

$$-P(e_{At} + e_{Bt} < \hat{e}) + P(e_{At} + e_{Bt} < \hat{e})\frac{\int_0^{\hat{e}} u'(C_t^*)P(d(e_{At} + e_{Bt}))/P(e_{At} + e_{Bt} < \hat{e})}{\int_{\hat{e}}^{\infty} u'(C_t^*)P(d(e_{At} + e_{Bt}))/P(e_{At} + e_{Bt} > \hat{e})}$$

$$= P(e_{At} + e_{Bt} < \hat{e})\left[-1 + \frac{E[u'(C_t^*)|e_{At} + e_{Bt} < \hat{e}]}{E[u'(C_t^*)|e_{At} + e_{Bt} > \hat{e}]}\right] > 0$$

Then $E\delta^d < 0$. Finally, let $\delta^n(\pi_t) = -1$ with probability $1/2$ and $\delta^n(\pi_t) = 1$ with probability $1/2$, independently of $e_{At} + e_{Bt}$ and let $\epsilon > 0$. Let $0 < \underline{x} < x^* < \overline{x}$, where x^* is the constant equilibrium exchange rate. Then define $\{\pi_t\}_t$ as follows. $\pi_1 = q$ and $\pi_2 = \pi_1 + \delta_2^n min\{|\pi_t - \pi^d|, |\pi_t - \pi^u\}/(1+\epsilon)$

$$\text{if } \underline{x} < x_t < \overline{x} \text{ let } \pi_{t+1} = \pi_t + \delta^n min\{|\pi_t - \pi^d|, |\pi_t - \pi^u\}/(1+\epsilon)$$

$$\text{if } x_t \leq \underline{x} \text{ let } \pi_{t+1} = \pi_t + \delta^d min\{|\pi_t - \pi^d|, |\pi_t - \pi^u\}/(1+\epsilon) \text{ if } \pi_t \geq q, \quad \pi_{t+1} = \pi_t, \text{ else}$$

$$\text{if } x_t \geq \overline{x} \text{ let } \pi_{t+1} = \pi_t + \delta^u min\{|\pi_t - \pi^d|, |\pi_t - \pi^u\}/(1+\epsilon) \text{ if } \pi_t < q, \quad \pi_{t+1} = \pi_t, \text{ else .}$$

This process has the desired properties: x_t neither tends to 0 nor to ∞. The idea of the construction is as follows. As long as, say, $x_t \leq \underline{x}$, we try to make $\{x_s\}$ increase by making π decrease if $\pi_t \geq q$, since we know that if $\pi_t < q$ then $\{x_s\}$ (informally speaking) will (tend to) increase. The desired decrease is brought about by adding δ^d, which has a negative expected value to π_t. As soon as $\pi_t < q$ we do not need to change it since the increase in $\{x_s\}$ then takes place. As $\{x_s\}$ increases it eventually crosses into being $> \underline{x}$ and the process is now governed by one of the other two cases.

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