ANALYTICAL ARTICLES

Economic Bulletin

3/2019

BANCO DE ESPAÑA
Eurosistema

THE RELATIONSHIP BETWEEN AVERAGE ANNUAL AND QUARTER-ON-QUARTER GDP GROWTH RATES: IMPLICATIONS FOR PROJECTIONS AND MACROECONOMIC ANALYSIS

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ABSTRACT

The average annual growth rate of GDP can be formulated algebraically as a weighted average of the quarter-on-quarter growth rates of the preceding and the current year. Sometimes this can give rise to counterintuitive results and misinterpretations of how the economy is evolving. For example, a given sequence of GDP growth rates, in quarter-on-quarter terms, in the current year, may give rise to very different average annual rates depending on the trajectory of GDP in the preceding year. Also, with given quarter-on-quarter growth figures for the four quarters of a particular year, average GDP growth will be higher, the earlier in the year that the largest quarter-on-quarter increases occur.

In the context of macroeconomic projections, analysis tends to focus on average annual GDP growth rates, insofar as they offer a summarised version of the outlook. However, it should be noted that revisions to the current year's projections with a particular sign (for example, upwards) may reflect changes of two types: first, the publication of new, more favourable National Accounts data for past quarters; and second, a downward revision to growth prospects for the remaining quarters of the year. Therefore, it would be a mistake to conclude from the mere observation of an upward revision to average annual growth that the economic outlook has improved.

Keywords: growth rates, carry-over effect, GDP, macroeconomic forecasts, macroeconomic projections.

JEL codes: E01, E32, O40.

THE RELATIONSHIP BETWEEN AVERAGE ANNUAL AND QUARTER-ON-QUARTER GDP GROWTH RATES: IMPLICATIONS FOR PROJECTIONS AND MACROECONOMIC ANALYSIS

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Introduction

When an economic variable is monitored its growth rate is usually studied in the short term (in a particular month or quarter) and in average annual terms. In the case of GDP (the macro-aggregate used in this article to illustrate the analysis), its growth rate in the current quarter (the quarter-on-quarter rate) provides a contemporaneous signal of the trend in activity. The signal provided by average annual rates, in contrast, is less affected by temporary factors that may distort the short-term trend. Yet it has the disadvantage that it incorporates a greater lag than the signal provided by quarter-on-quarter rates. Despite this drawback, interest is often focused on the average change in output in a calendar year relative to the preceding year. In fact, this is the usual way in which the progress of a particular economy is assessed and future forecasts formulated.

A peculiarity of the average annual GDP growth rate in a particular year is that, contrary to what might be expected a priori, it is not only determined by the quarter-on-quarter rates observed during the current year, but also by the trajectory of GDP in the preceding year.

This has some important implications for economic analysis. First, weak (strong) growth on average in the current year may sometimes be more the result of the fragility (strength) of GDP in the preceding year than of its behaviour in the current year. Also, in the context of a revision to GDP growth forecasts, the relationship between average annual rates and quarter-on-quarter rates may be misinterpreted. For example, we cannot rule out that an improvement in the projected growth rate of the economy may be associated with an upward revision by the statistical authority of the quarter-on-quarter growth rates in previous quarters, which may even be consistent with a deterioration in the future outlook. Such a result is all the more likely the later it is in the current year, since that increases the weight of completed quarters in the annual average (known as the carry-over effect).[1]

1 As explained later in this article, the carry-over effect is often understood in a more restricted sense as the contribution to average GDP growth in year T of the trajectory of this variable in year T-1.

This article examines the link between quarterly and annual rates, which is not always completely intuitive, giving some specific hypothetical examples to illustrate this for the case of GDP.[2] This link is used below to explain how the average GDP growth rate is influenced by the carry-over effect, taking the recent performance of the Spanish economy as an example.

The relationship between quarter-on-quarter rates and average annual rates

The average annual growth in the output of an economy is calculated as the percentage change in the level of GDP between two consecutive years, where the GDP for each of these two years is obtained as the sum of the GDP of the four quarters of the year. Quarter-on-quarter increases, meanwhile, reflect the percentage change in output between two consecutive quarters (whether or not they belong to the same year).

The average annual rates and the quarter-on-quarter rates are arithmetically related. Specifically, the average annual growth rate in a particular year can be expressed, approximately, as a weighted sum of a set of quarter-on-quarter increases. However, when calculating the average annual growth in year T, not only are the quarter-on-quarter rates recorded in each of the four quarters of that year used, but also the rates corresponding to the last three quarters of the preceding year, T-1).[3] Also, as seen in Chart 1, the weights assigned to the quarter-on-quarter rates of each of these seven quarters are not uniform. On the contrary, the weights increase linearly between the second quarter of T-1 and the first quarter of T (which is assigned the highest weight) and then decrease up to the fourth quarter of T. Chart 1.2 shows the cumulative value of these weights in each of the quarters of the period involved in the calculation of the annual rate. As can be seen, the average growth rate in a particular year is determined, to a large extent, by the quarter-on-quarter growth rates in the previous year. Specifically, the cumulative weight of the three quarters of T-1 that enter into the calculation amounts to almost 40% of the total and, if we also include the first quarter of T, their total weight exceeds 60%.

As already mentioned, the fact that, arithmetically, the average annual growth rate in year T reflects the quarter-on-quarter growth profile not only in that year but also in T-1, along with the weighting system described, means that the relationship between

2 This analysis may be applied in a similar way to examine the relationship between monthly and quarterly series.

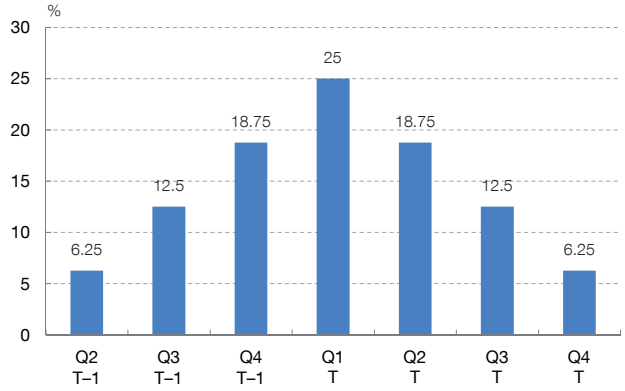
3 For an algebraic derivation, see Cross and Wyman (2011) or Appendix 1 in Tödter (2010).

Chart 1

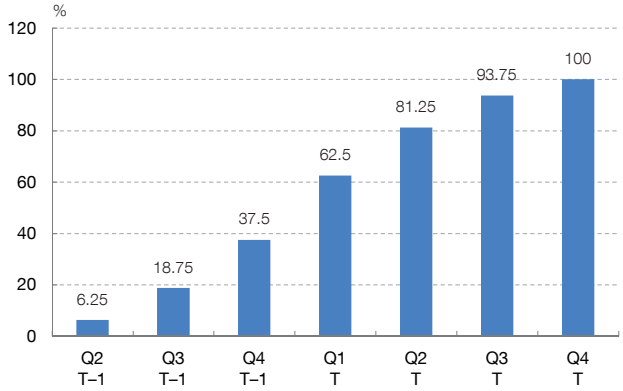
THE WEIGHTS OF THE QUARTER-ON-QUARTER GROWTH RATES IN THE AVERAGE ANNUAL GROWTH RATE (%) (a)

The average annual and quarter-on-quarter rates are algebraically related. Specifically, average annual growth in year T can be expressed approximately as a weighted sum of the quarter-on-quarter increases in the last three quarters of year T–1 and in the four quarters of year T, with the weights increasing up to the first quarter of year T and then decreasing.

1 WEIGHTS (APPROXIMATE VALUES)



2 CUMULATIVE WEIGHTS



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SOURCES: INE and Banco de España.

a Q1, Q2, Q3 and Q4 refer to the four quarters of the year. T refers to the current year and and T–1 to the preceding year.

the average annual growth rate and the quarter-on-quarter rates may not always be completely intuitive. Thus, for example, average growth in year T is influenced to a greater extent by the quarter-on-quarter growth rates recorded in the last two quarters of year T-1 than by the quarter-on-quarter growth rate in the fourth quarter of year T.

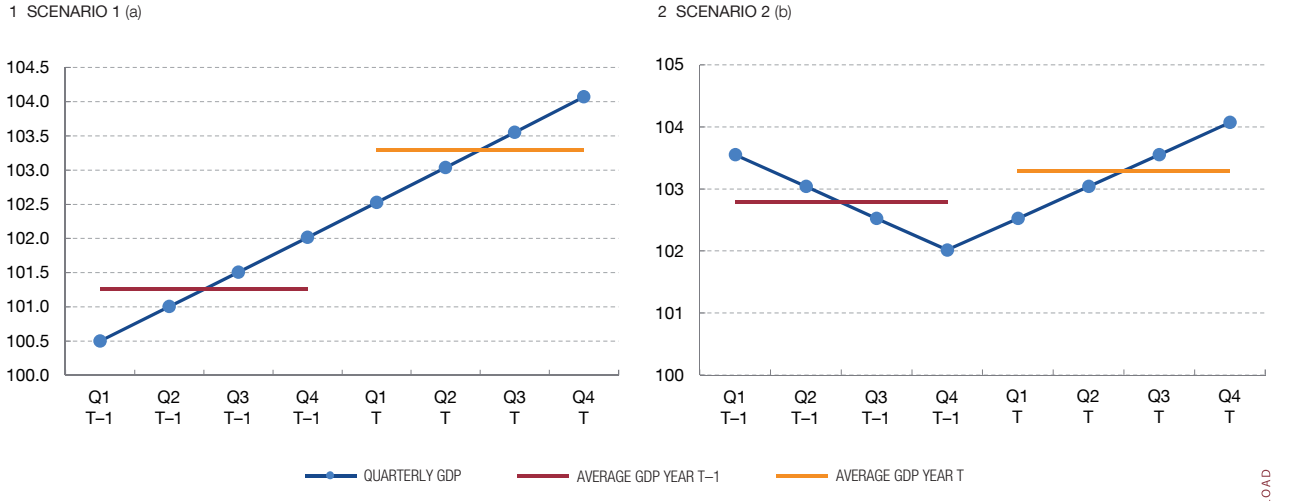
These results can be illustrated using some specific hypothetical scenarios. First, Chart 2 shows how the average growth in a particular year is affected by the profile of GDP in the preceding year. In the example, the hypothetical trajectories of GDP in year T are identical in both scenarios in terms of the quarter-on-quarter growth rates, and the level of GDP that year is also the same. However, the average rates in year T differ owing to the fact that the quarter-on-quarter rates differ in year T-1.

Specifically, the quarter-on-quarter GDP growth rate in each of the four quarters of year T is assumed to be a constant rate of 0.5% in both scenarios. However, while in scenario 1 (see Chart 2.1) the quarter-on-quarter GDP growth rate in Q2, Q3 and Q4 of year T-1 is also 0.5%, in scenario 2 (see Chart 2.2) this rate is -0.5%. As a result, even though GDP growth in each of the three quarters of year T is the same in both cases, annual GDP growth differs, being lower in scenario 2, in which the average level of GDP in year T-1, with respect to which the growth rate is calculated, is higher. Specifically, this example gives rise to GDP growth rates in year T of 2% (scenario 1) and 0.5% (scenario 2).

Chart 2

THE IMPORTANCE OF THE PREVIOUS YEAR'S GROWTH PROFILE

Average GDP growth in a particular year is affected by the profile of GDP in the previous year. With a given quarter-on-quarter growth path in year T, average annual GDP growth that year will be higher the greater the quarter-on-quarter increases in T–1 (especially, as that year progresses).



SOURCES: INE and Banco de España.

a GDP growth is 0.5% in each quarter from Q2 of year T–1 to Q4 of year T.
b GDP decreases by 0.5% in each of Q2, Q3 and Q4 of year T–1, and increases by 0.5% in each of the four quarters of year T.

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Second, the fact that the weights assigned to the quarter-on-quarter rates of the different quarters of year T are declining (as shown in Chart 1) means that the distribution of these rates over the year affects the annual average GDP growth rate. If in year T quarter-on-quarter growth rates of zero are recorded in three quarters and an increase of 1% is recorded in the other quarter, it is not irrelevant for the purposes of the average annual rate in which quarter this growth of 1% occurs. This is illustrated using the two scenarios of Chart 3 for the cases in which the quarter-on-quarter growth of 1% occurs in the first quarter (scenario 3) or in the fourth quarter (scenario 4) of the year.[4]

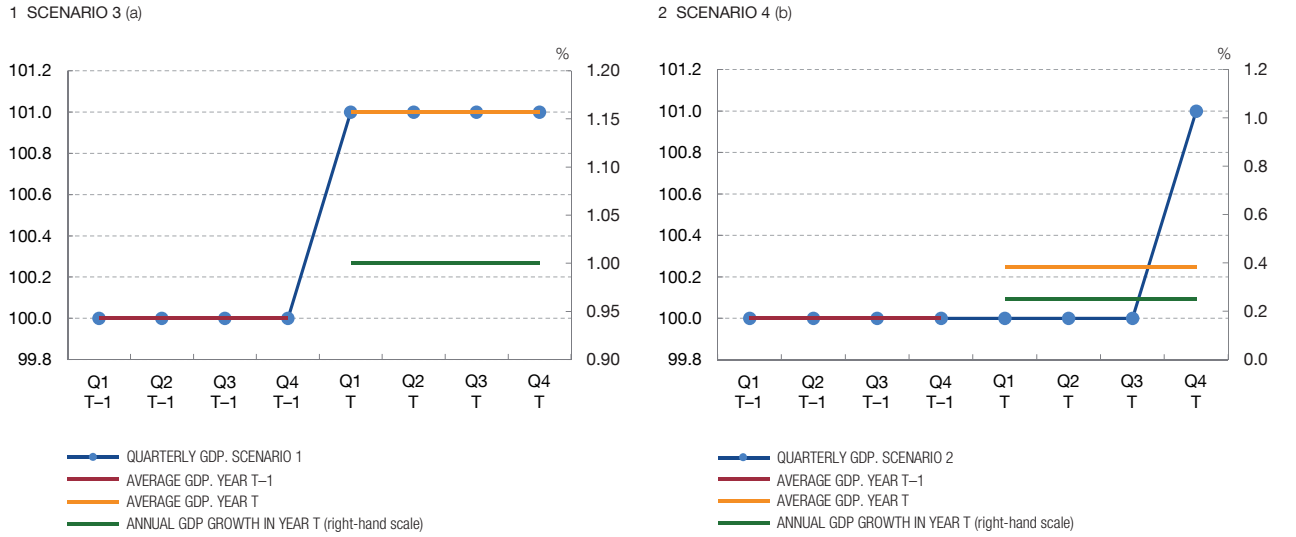
The intuitive reason why the changes recorded in the first quarter of the year analysed have a greater impact on the average annual rate is that these changes raise or reduce the level on which the rest of the growth recorded during the year is based (scenario 3). By contrast, the changes recorded in the fourth quarter of the year in question have no effect on the level of GDP in the other quarters, and therefore the changes recorded in this quarter have the smallest impact (i.e. a lower weight) on the average annual level (scenario 4). As a result, when the quarter-on-quarter GDP

4 In order to simplify matters it is also assumed that quarter-on-quarter GDP growth is zero in each of the three quarters of T-1 from Q2 to Q4.

Chart 3

THE IMPORTANCE OF THE GROWTH PROFILE IN THE YEAR IN QUESTION

The fact that the weights assigned to the quarter-on-quarter rates in the different quarters of year T are declining means that the distribution of these rates within the year affects the annual GDP growth rate. In particular, the latter will be greater the earlier in the year that the largest quarter-on-quarter increases occur.



SOURCES: INE and Banco de España.

a The level of GDP remains unchanged between the first quarter of year T–1 and the last quarter of year T–1. It increases by 1% in the first quarter of year T and thereafter remains unchanged.
b The level of GDP remains unchanged between the first quarter of year T–1 and the third quarter of year T. It increases by 1% in the fourth quarter of year T.

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growth of 1% takes place in the first quarter of the year (scenario 3) the average GDP growth rate is also 1%, but when the 1% increase occurs in the fourth quarter the average GDP growth rate is only 0.25% (scenario 4).[5]

The cumulative weight of the information used to calculate the average growth rate in T exceeds 60% by the end of the first quarter of that year, when the average level of GDP in the previous year and the value to which the increases recorded between Q2 and Q4 of the current year will be anchored are already known. Likewise, more than 80% of the annual growth will be determined by the information known as at the end of the first half of year T and, therefore, any deviations in GDP growth from the

5 This line of argument gives rise to the profile of weights shown in Chart 1.1. On one hand, given zero growth in the rest of the quarters of year T, the sooner a quarterly increase of 1% occurs that year the higher the average level of GDP that year. On the other hand, given zero growth likewise in the other quarters of year T-1, the later a quarterly increase of 1% occurs that year the lower the average level of GDP that year. It can be shown algebraically that the resulting system of weights is that shown in Chart 1.1, so that in a hypothetical scenario in which zero growth is observed in six of the seven quarters used to calculate the average annual rate and 1% growth in the other, the impact on the average rate for T is greatest when the quarter in which the rate of 1% is observed is the first quarter of T, and smallest when the increase of 1% takes place either in the second quarter of T-1 or in the fourth quarter of T.

path expected in the final two quarters of the year will have a relatively modest impact on the annual growth rate eventually recorded.

The carry-over effect

The calculation of the average annual growth rate of GDP for a specific year is determined, as explained above, by the seven quarter-on-quarter growth rates observed in years T-1 and T. Let us assume that we group together, first the contributions of the three quarter-on-quarter growth rates for T-1, and second the contributions of the four quarter-on-quarter growth rates for T, or in other words, that we decompose the average annual growth for the year into the previous year's and current year's contributions.[6] In its narrowest definition, the carry-over effect is the previous year's contribution to average GDP growth in year T. Equally, in this sense, it may be defined as the average annual growth rate of year T assuming that the level of GDP remains the same throughout the four quarters of year T as in the fourth quarter of year T-1 (or, in other words, that the quarter-on-quarter growth rates in the four quarters of year T are zero).

In consequence, the carry-over effect is the part of GDP growth in year T that depends on GDP growth in year T-1. For this reason, not only is the average annual level important but also the time profile whereby that level is reached. Specifically it is possible to show algebraically that the carry-over effect may be calculated as the percentage difference between the level of GDP in the fourth quarter of year T-1 and its average level in that year.[7] Accordingly, the steeper the upward profile of GDP over year T-1, the greater the carry-over effect will tend to be. Thus, for example, in the two scenarios in Chart 4, the average level of GDP for year T-1 is the same. However, the intra-year profiles are different. In scenario 5, quarter-on-quarter GDP growth is 0.5% in each of the last three quarters of year T-1, whereas in scenario 6 it is 0.8% in Q2 and Q3 and -1% in Q4. In consequence, in scenario 6, the level of GDP at the end of year T-1 is lower than the average for the year and gives rise to a negative carry-over effect of -0.15 pp, compared with a positive carry-over effect of 0.75 pp in scenario 5.

In turn, the current year's contribution would be the difference between the average annual growth rate in year T and the carry-over effect. In the examples in Charts 4.1 and 4.2 it has been assumed that quarter-on-quarter GDP growth in each of the four quarters of year T is equal to zero. Hence their contribution to average annual growth is also zero.[8]

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6 See, for example, Box 6, "The carry-over effect on average annual real GDP growth", in ECB (2010).

7 See National Bank of Romania (2013).

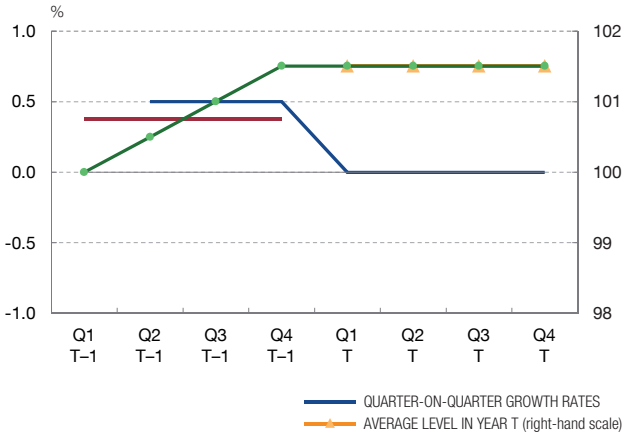
8 It is possible to show that the contribution of these quarters to average annual growth in year T is equal to the percentage difference between the average level of GDP and its level in the fourth quarter of T-1.

Chart 4

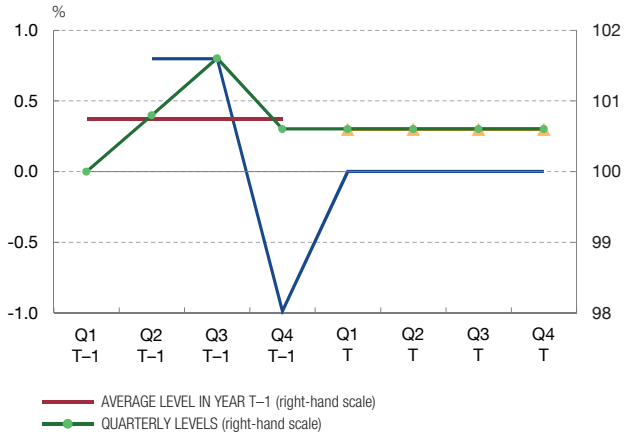
THE CARRY-OVER EFFECT

The carry-over effect, defined as the contribution of GDP growth in year T–1 to average GDP growth in year T, will tend to be higher, the steeper the upward profile of GDP over year T–1. Sometimes, the carry-over effect can give rise to counterintuitive results: for example, even if there is a downturn in year T, average annual growth may be positive, as a result of the positive contribution of the carry-over effect.

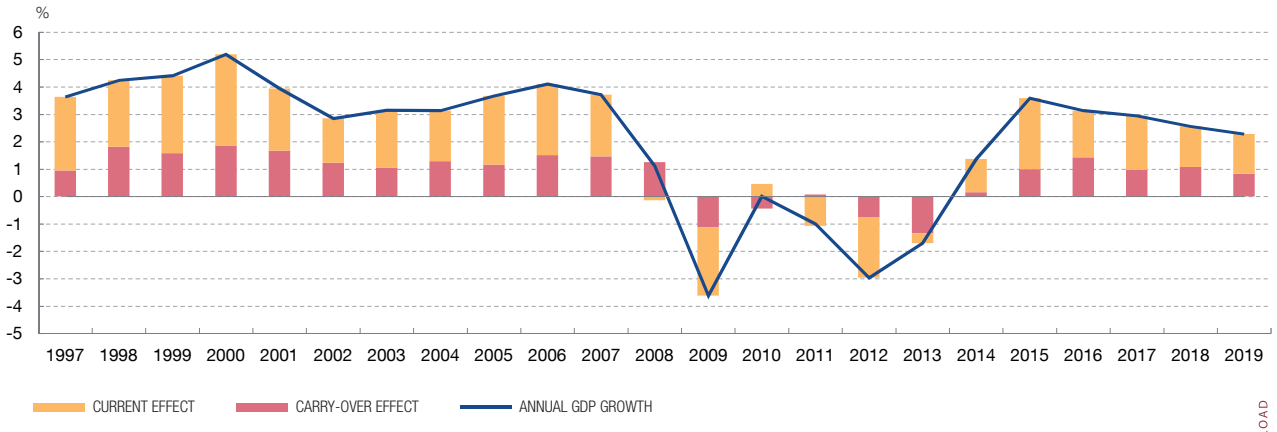
1 CARRY-OVER EFFECT. SCENARIO 5



2 CARRY-OVER EFFECT. SCENARIO 6



3 ANNUAL GDP GROWTH AND CARRY-OVER EFFECT



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SOURCES: INE and Banco de España.
Note: The article draws on the information available up to 12 September. Accordingly, the chart does not take into account the National Accounts update published on 16 September.

If we assume that the quarterly rate of GDP growth is uniform over time, the previous year's contribution to average GDP growth in year T would be 37.5%, as that is the sum of the weights of the three quarter-on-quarter growth rates in year T-1 that give rise to the carry-over effect (see Chart 1). Accordingly, the share of the current year's contribution would be 62.5%.

In the case of the Spanish economy, in the period 1996 to 2018 the contributions to GDP growth of the carry-over effect and the current effect were 32% and 68% on average; these figures are, therefore, very similar to what might be expected with a uniform growth rate. Thus, for an average GDP growth rate of 2.2% in the period, the

contributions of the carry-over effect and the current effect were 0.8 pp and 1.4 pp, respectively.

Obviously, in practice, the quarter-on-quarter GDP growth rate does not tend to be uniform, which means that in certain years there may be significant departures from this pattern. Thus, for example, in 2008, when GDP grew by 1.1%, the contribution of the current effect was actually negative (-0.1 pp), compared with the positive carry-over effect of 1.3 pp, which was above the average of 0.8 pp for the period considered (see Chart 4.3). In other words, despite the downturn in the second half of 2008, which gave rise to a slightly negative current effect, average GDP growth was positive thanks to the upward path of GDP throughout 2007.

Conversely, GDP recorded zero growth in 2010, despite its rising path in that year (reflected in the positive contribution of 0.5 pp of the current effect), given the downward GDP growth path that prevailed in 2009 (which gave rise to a negative carry-over effect of -0.4 pp).

More broadly speaking, the carry-over effect may refer to the contribution to average GDP growth in year T made by the quarter-on-quarter data up to the last known quarter, whether or not that quarter is the last quarter of a calendar year. In consequence, when previously published National Accounts figures are revised, changes to quarter-on-quarter increases in past quarters will imply changes to average annual growth rates, and the more the revisions affect the quarters that have a higher weighting in the calculation of average annual rates, the greater those changes will be.

Thus, for example, in its macroeconomic projections published in June 2019, the Banco de España made an upward revision to its GDP growth forecast for the year of 0.2 pp.[9] However, as explained in the corresponding projections report, the revision was not due to an improvement in the outlook for the coming quarters, as "for the second half of 2019, the GDP growth path is held unchanged", but rather to an upward revision of the National Accounts figures for the first half of the year. This specific case illustrates the importance, when interpreting a revision made to the average annual growth rate, of analysing any change in the carry-over effect associated with the quarter-on-quarter path in the known period.

18.9.2019.

9 The cut-off date for these projections was 15 May. In consequence, they do not take into account the National Accounts update published on 16 September. See Banco de España (2019).

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