Finance Over the Life Cycle of Firms

Federico Kochen

CEMFI and Columbia University

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Motivation

- Large literature argues *finance* important for macro and economic development
- In theory, firms are more dependent on external financing early in their life
    - Have not had time to accumulate internal funds and grow out of borrowing constraints
    - Buera Kaboski Shin 2011; Guvenen et al. 2019; etc.
- Yet little is known about importance of financial frictions at different stages of firms' life cycles

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This Paper

- Present *new* evidence on the nature of external financing over firms' lifetimes
  - Use and cost of debt, frequency and size of equity injections
  - Contrasting high- and middle-income countries
- Develop a quantitative model of firm dynamics consistent with empirical patterns
  1. *Detailed capital structure*: internal funds, defaultable debt, and costly equity injections
  2. *Learning* about profitability and *age-specific volatility*
- Use model to quantify losses in output per worker from financial frictions
- *Main finding*: bulk of losses accounted by a *new* channel distorting firms' *exit* decisions
  - Young firms prematurely exit: costs of external financing higher than option value of learning

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▸ Literature

Outline

Life Cycle Facts

Model

Quantifying the Model

Aggregate Implications

Conclusions

Data and Empirical Specification

Orbis Global Database (Moody's Bureau van Dijk) ▸ Descrip Stats

- Firm-level harmonized data from national business registers
- Annual *balance sheet* and income statements for *privately-held* firms between 1996-2018
- Eleven high-income and seven middle-income European countries ▸ List ▸ Scatter Inc-Fin

Empirical Specification ▸ Details

- Non-parametric regression considering 9 age groups
- Middle-income country indicator interacting each age group
- Control for *sector* (NACE 4-digit), *cohort*, and *time* fixed effects

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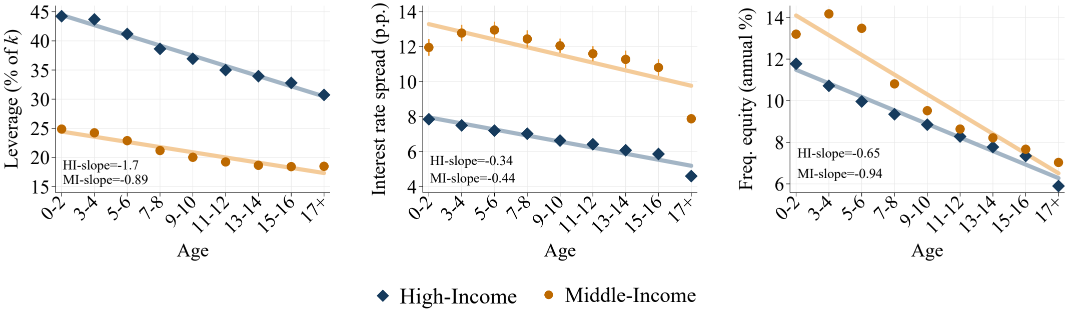
Finance Over the Life Cycle of Firms

Fact 1: Younger firms have higher leverage

- Firms in middle-income countries have lower leverage and flatter life cycle slope

1. Leverage 2. Interest rate spread 3. Equity financing



▸ Lev. a|s ▸ Spr. a|s ▸ Eq. a|s ▸ Eq. Size ▸ Measurement ▸ Firm FE ▸ BP

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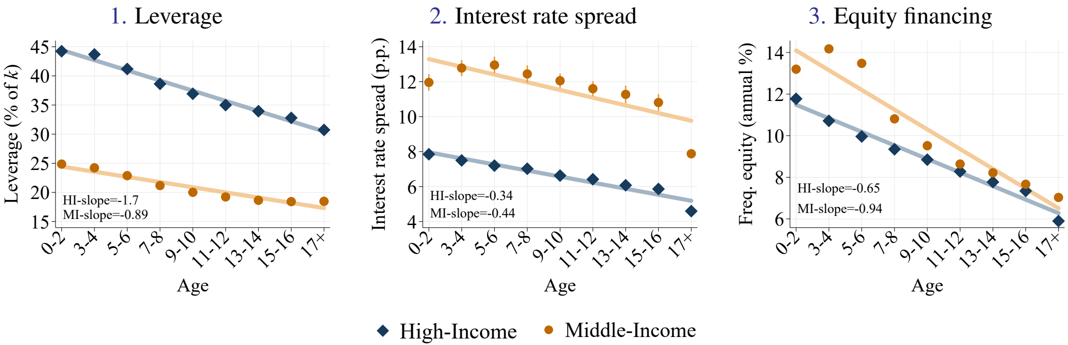
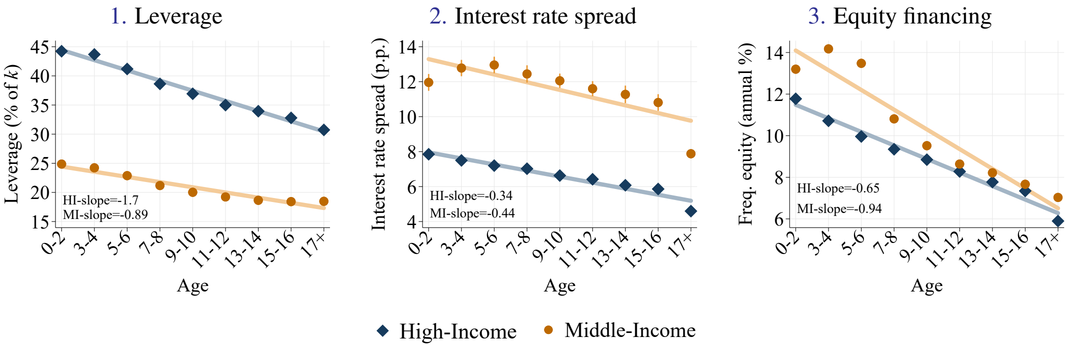


Lev. a|s · Spr. a|s · Eq. a|s · Eq. Size · Measurement · Firm FE · BP

Finance Over the Life Cycle of Firms

Fact 2: Younger firms pay higher interest rate spreads

- Firms in middle-income countries face higher spreads

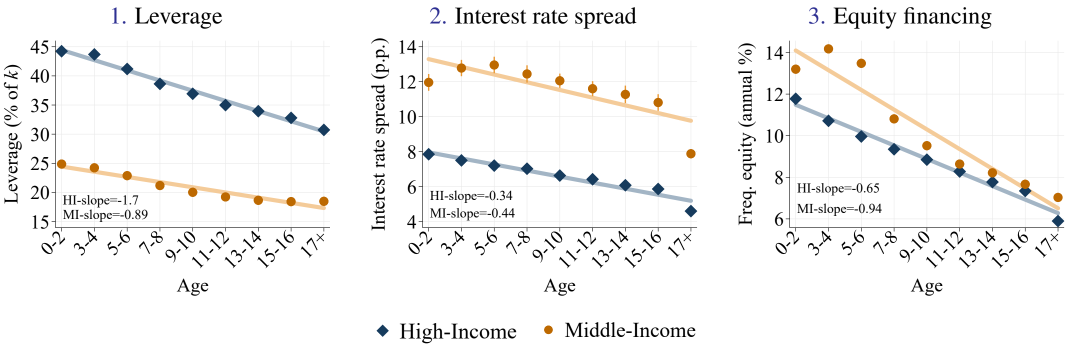


‣ Lev. a|s ‣ Spr. a|s ‣ Eq. a|s ‣ Eq. Size ‣ Measurement ‣ Firm FE ‣ BP

Finance Over the Life Cycle of Firms

Fact 3: Younger firms are more likely to receive equity injections from shareholders

- Equity financing is slightly more frequent in middle-income countries

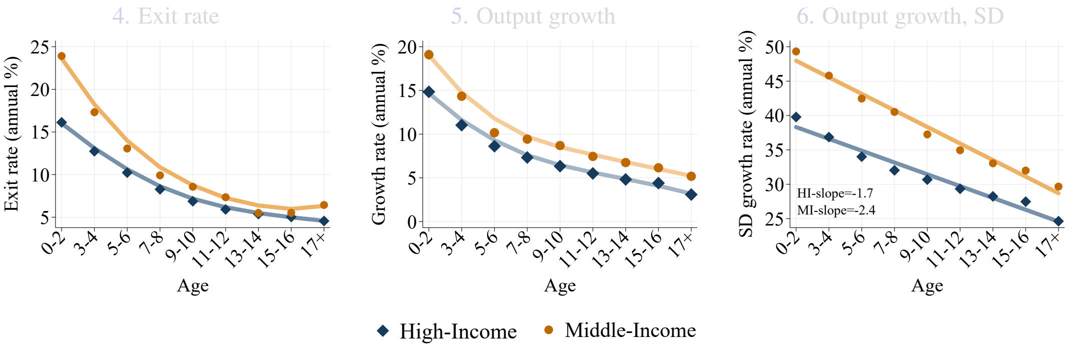


Lev. a|s   Spr. a|s   Eq. a|s   Eq. Size   Measurement   Firm FE   BP

Survival and Growth Over the Life Cycle of Firms

Fact 4: Younger firms have higher exit rates

- Firms in middle-income countries have higher exit rates, especially young firms



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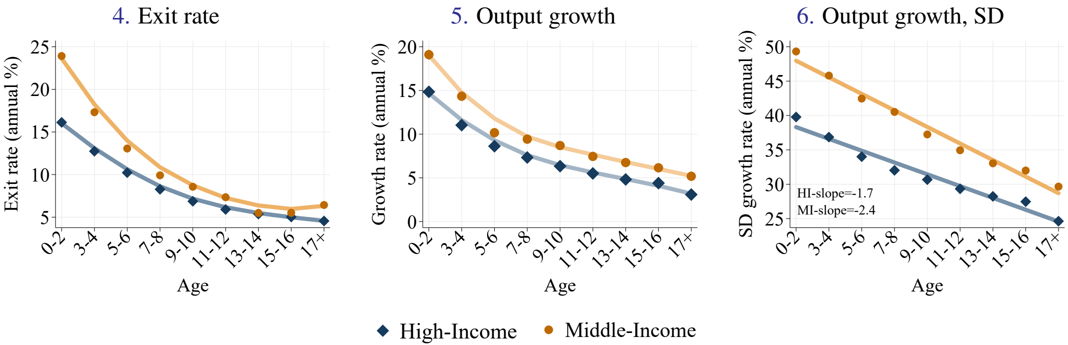

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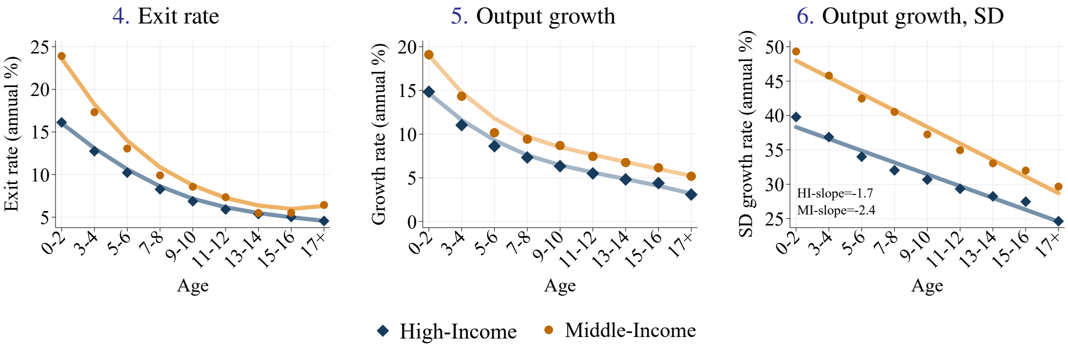
Survival and Growth Over the Life Cycle of Firms

Fact 5 and 6: Younger firms have higher and more volatile growth rates

- Firms in middle-income countries grow faster and have more dispersed growth



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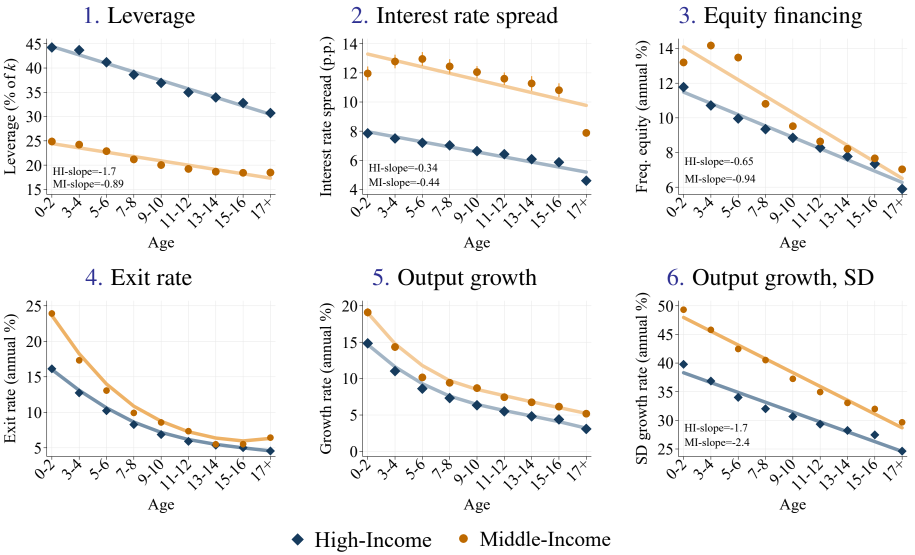
Finance, Survival, and Growth Over the Life Cycle of Firms



Outline

Environment

- Discrete time, infinite-horizon, small open economy
- Representative household preferences over final good and leisure
- Endogenously determined mass of risk neutral firms
  - CES demand (optimal scale), CRS technology, capital s.t. 1-period time-to-build ▸ Details

$$\pi(k_i, z_i) = \max_{l_i} A \exp(z_i)[k_i{}^{\alpha} l_i{}^{(1-\alpha)}]^{\frac{1}{\mu}} - wl_i$$

  - *Learn* about profitability and face age-specific volatility
  - *Finance* through: internal funds, defaultable long-term *debt*, and costly *equity* injections
  - Demographics: mass of prospective entrants, endogenous and exogenous exit
- Financial intermediaries price firms' debt competitively, symmetric information

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Learning About Profitability (1/2)

- Firm i at age t observe

$$z_{it} = s_{it} + \varepsilon_{it} \qquad \text{(not } s_{it} \text{ and } \varepsilon_{it} \text{ separately)}$$

where

$$s_{it} = \rho_s s_{it-1} + u_{it}, \quad s_{i0} \sim \mathcal{N}(\hat{s}_{i0}, \Sigma_0), \quad u_{it} \sim \mathcal{N}(0, \sigma_u^2), \quad \varepsilon_{it} \sim \mathcal{N}(0, \sigma_{\varepsilon t}^2), \quad \sigma_{\varepsilon t}^2 = (1 + \rho_\varepsilon^t C_\varepsilon)^2 \, \sigma_\varepsilon^2$$

- *Kalman filter* implies recursions for $\hat{s}_{it+1} = \mathbb{E}[s_{it+1}|z_i^t]$ and $\Sigma_{t+1} = \mathbb{V}(s_{it+1}|z_i^t)$

$$\hat{s}_{it+1} = \rho_s \hat{s}_{it} + K_t g_{it}$$
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KF Recursions

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Learning About Profitability (2/2)

Age-specific $\sigma_{\varepsilon t}$ slows down firms' learning (noisier signals)

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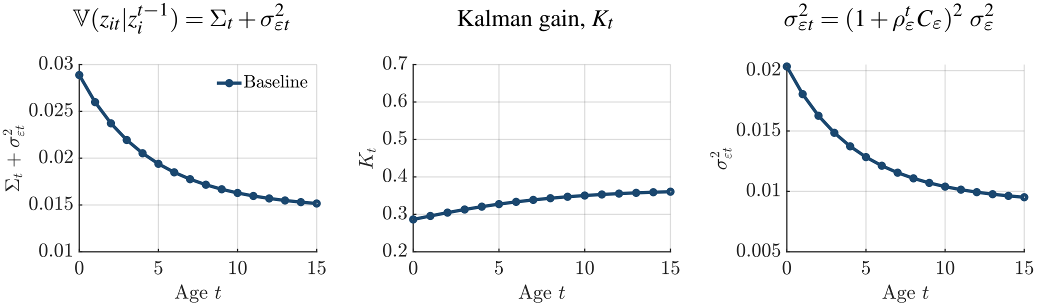

Notes: Baseline assumes $\Sigma_0/\Sigma_\infty = 1.211$, $\rho_s = 0.968$, $\sigma_u = 0.048$, $\sigma_\varepsilon/\sigma_u = 1.978$, $C_\varepsilon = 0.61$, and $\rho_\varepsilon = 0.827$. $C_\varepsilon = 0$ (no age-specific volatility) uses same parameters with the exception of $\Sigma_0/\Sigma_\infty = 1.816$. Perfect Info. assumes the firm perfectly observes s and ε, hence, $\mathbb{V}(z_{it}|z_i^{t-1}) = \sigma_u^2 + \sigma^2_{\varepsilon t}$.

SD growth

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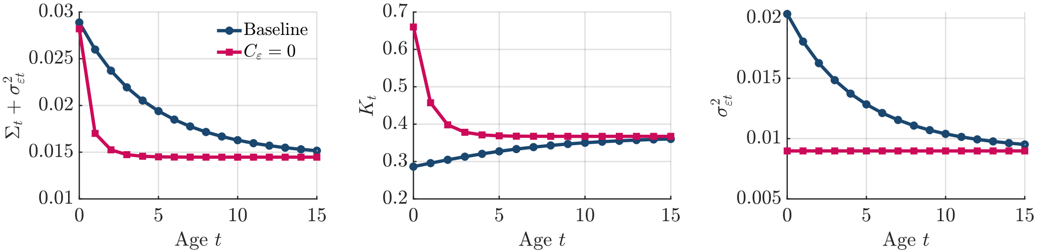
$\mathbb{V}(z_{it}|z_i^{t-1}) = \Sigma_t + \sigma^2_{\varepsilon t}$ | Kalman gain, K_t | $\sigma^2_{\varepsilon t} = \left(1 + \rho_\varepsilon^t C_\varepsilon\right)^2 \sigma^2_\varepsilon$



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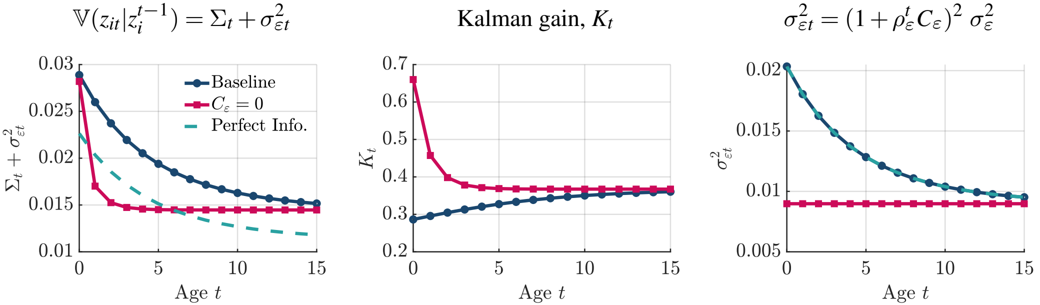

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▸ SD growth

Finance

- Firms can borrow using defaultable long-term debt b_{it+1}, random maturity ϕ, coupon r ▸ Details
  - Price of debt $\boldsymbol{q}_{t+1}(k_{it+1}, b_{it+1}, \hat{s}_{it+1})$ determined by intermediaries' ZEP condition ▸ Details
  - If the firm defaults, the intermediaries recover $\rho(1-\delta)k_{it+1}$

- Equity injections carry a fixed and convex cost (Hennessy Whited 2007)

$$\Lambda(x_{it}) = \begin{cases} \lambda_0 + \lambda_1 |x_{it}| + \lambda_2 |x_{it}|^2 & \text{if dividends } x_{it} < 0 \\ 0 & \text{eoc} \end{cases}$$

- Firms' budget constraint implies that capital investments equal

$$k_{it+1} - (1-\delta)k_{it} = \underbrace{\pi(k_{it}, z_{it}) - \exp(z_{it})c_{Fit} - (\phi + r)b_{it}}_{\text{internal funds}} \underbrace{- x_{it}}_{\text{equity/dividends}}$$

$$\underbrace{+ \boldsymbol{q}_{t+1}(k_{it+1}, b_{it+1}, \hat{s}_{it+1})[b_{it+1} - (1-\phi)b_{it}]}_{\text{new debt}}$$

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$$\underbrace{+ \boldsymbol{q}_{t+1}(k_{it+1}, b_{it+1}, \hat{s}_{it+1})[b_{it+1} - (1-\phi)b_{it}]}_{\text{new debt}}$$

Finance

- Firms can borrow using defaultable long-term debt b_{it+1}, random maturity ϕ, coupon r ▸ Details
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  - If the firm defaults, the intermediaries recover $\rho(1-\delta)k_{it+1}$

- Equity injections carry a fixed and convex cost (Hennessy Whited 2007)

$$\Lambda(x_{it}) = \begin{cases} \lambda_0 + \lambda_1 |x_{it}| + \lambda_2 |x_{it}|^2 & \text{if dividends } x_{it} < 0 \\ 0 & \text{eoc} \end{cases}$$

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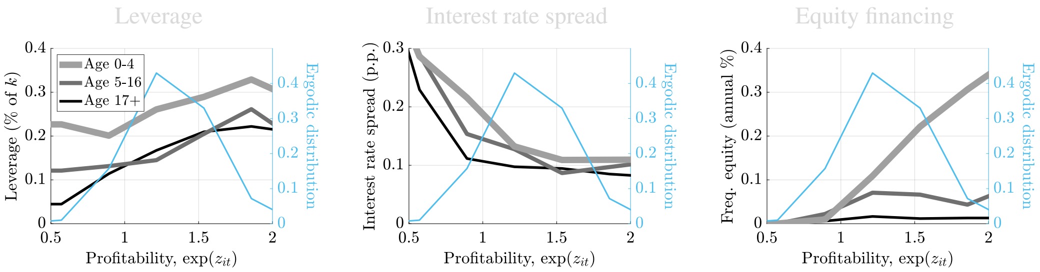
Learning and Financial Frictions

- Young firms have an *option value* of continue operating and learn about their profitability
- Signals (z) and degree of uncertainty (age) relevant for firms' capital structure
  - Young firms with *high* signals: more likely to use *equity* financing, pay lower spreads
  - Young firms with *low* signals: rely mostly on *debt*, pay high spreads



▸ Timing ▸ Entrants ▸ Incumbents' DP ▸ Equilibrium

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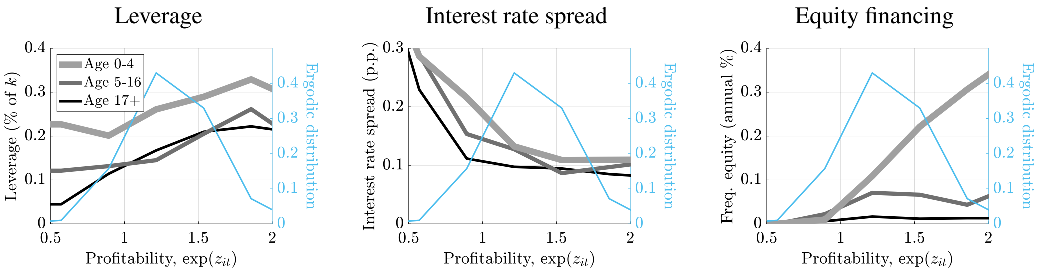

▸ Timing ▸ Entrants ▸ Incumbents' DP ▸ Equilibrium

Outline

Life Cycle Facts

Model

Quantifying the Model

Aggregate Implications

Conclusions

Calibration Strategy

- Annual frequency
- Assign parameters to standard values, same for both regions ▸ Assigned
- Parameters governing distribution of entrants, idiosyncratic shocks, and financial frictions ▸ Calibrated
  - *Separately* calibrated for high- and middle-income countries
  - Minimize distance between salient moments in data and model ▸ Model Fit
  - Severity of financing frictions chosen to match *micro facts* about leverage, spreads, equity

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Untargeted Moments and Validation

1. Life cycle patterns ▸ Figures
   - Model does a good job replicating *complete* life cycle patterns of the six facts
2. Output distribution by firms' age ▸ Figures
   - Model is consistent with the distribution of output by firms' age in the data
3. Forecast errors (FEs) ▸ Details
   - Model implied FEs decrease with firms' age, in line with evidence
   - Magnitude: model can account for financial and real facts with empirically plausible FEs
4. Capital investments and equity financing ▸ Table
   - Capital investments are twice as large during equity financing injections, as in data

▸ How Constrained Are Young Firms?

Outline

Aggregate Implications of Financial Frictions

- Steady state comparisons to perfect credit benchmark ($\{\lambda_j\} = \mathbf{0}$) show that financial frictions
  - Reduce output per worker in 15% and 24%, in high- and middle-income ▸ Table
  - TFP is 8% and 13% lower
- Decomposing the losses in three channels ▸ Details
  1. Capital deepening (lower K/Y) accounts for 25% of losses ▸ Table Y/L
  2. Capital misallocation accounts for 13% of the losses ▸ Table TFP
  3. Distortions in firms' exit margin account for bulk of the losses 62%

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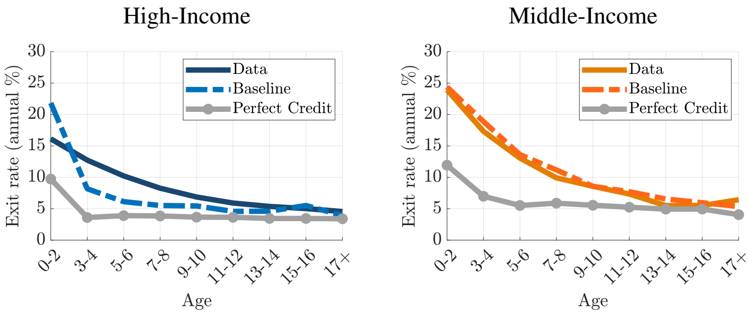
Financial Frictions and Firms' Exit

Distortions in exit margin driven by young firms, little effect on older firms

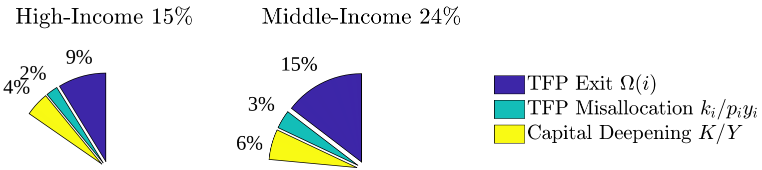


	High-Income		Middle-Income	
	Perfect Credit	Baseline	Perfect Credit	Baseline
Exit Rate	0.04	0.08	0.06	0.14
$\mathbb{E}$[lifespan]	25.3	12.5	17.9	7.1

Outline

Conclusions

- This paper presents new evidence about the nature of external financing over firms' lifetimes
  - Significant differences in firms' access to finance over their life cycles and across countries
- Develops a firm dynamics model with *rich capital structure* and *learning about profitability*
  - *Key*: younger firms rely more on external financing while facing higher uncertainty and risk
- Model calibrated to *micro facts* predicts that financial frictions generate
  - Losses in output per worker of 15% and 24%, in high- and middle-income countries
  - Bulk of losses explained by a *new* channel: young firms' premature exits



Appendix

Related Literature and Contribution

- Open debate: quantitative relevance of financial frictions as a source of *capital misallocation*
  - TFP differences across countries: Buera Kaboski Shin 2011; Midrigan Xu 2014; Moll 2014; Greenwood Sanchez Wang 2013; Gilchrist Sim Zakrajšek, 2013; Cavalcanti et al. 2021
  - Desirability of wealth taxation: Guvenen et al. 2019
- Empirical corporate finance (public firms)
  - Finance by firms' age: Rajan Zingales 1998; Hadlock Pierce 2010; Dinlersoz et al. 2019
- Age and firm dynamics
  - Empirical: Haltiwanger Jarmin Miranda 2013; Adelino Ma Robinson 2017; Dyrda 2019
  - Models w/ learning: Jovanovic 1982; Arkolakis Papageorgiou Timoshenko 2018; Chen et al. 2020

◂ Return

Descriptive Statistics

	High-Income		Middle-Income	
	Mean	SD	Mean	SD
T	10.1		8.2	
Age	12.4		9.1	
Employment	13.1	28.1	13.3	30.9
Sales (USD millions)	2.1	5.4	1.1	3.2
Sales Growth	0.06	0.29	0.08	0.37
Leverage	0.38	0.36	0.23	0.30
Interest Rate Spread	0.03	0.12	0.06	0.18
Fr. Equity-Fin.	0.06		0.08	
Size Equity-Fin.	0.14	0.23	0.16	0.27
Manufacturing	0.14		0.23	
Services	0.67		0.59	
Other	0.19		0.18	
N	30,056,311		6,324,422	

◂ Return

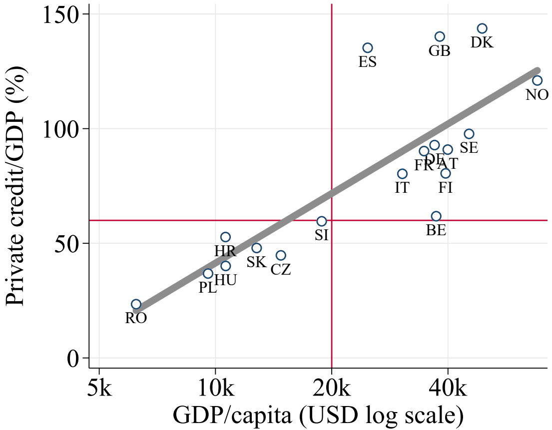
List of Countries

High-Income			Middle-Income		
Austria	EU-€	39.5	Croatia	EU	10.4
Belgium	EU-€	36.9	Czechia	EU	14.4
Denmark	EU	36.7	Hungary	EU	10.4
Finland	EU-€	38.9	Poland	EU	9.3
France	EU-€	34.4	Romania	EU	6.0
Germany	EU-€	36.7	Slovakia	EU-€	12.5
Italy	EU-€	30.1	Slovenia	EU-€	18.5
Norway	EEA	66.8			
Spain	EU-€	24.4			
Sweden	EU	44.7			
UK	EU	37.5			
Average		38.8			11.6

Notes: $ amounts correspond to average GDP/capita between 1994-2018 in '000 2015 USD. EU denotes European Union membership (as of 2018). EEA denotes membership to the European Economic Area. € denotes the Euro as currency.

◂ Return

Income and Financial Development in Europe



Notes: Country-level averages for the sample period 1996-2018.

◂ Return

Empirical Specification

- For each variable y, run the non-parametric regression

$$y_{it} = \sum_{a \in \mathcal{A}} (\gamma_a + \gamma_a^{\text{MI}} \text{MI}_i) \text{D}_{it}^a + \alpha_n + \alpha_c + \alpha_t + \epsilon_{it}$$

  where

  - $\mathcal{A}$ includes 9 age groups: 0-2, 3-4, 5-6, ..., 13-14, 15-16, and 17+
  - D_{it}^a equals 1 if firm i belongs to the age group a in period t
  - MI_i is equal to one if firm i is located in one of the middle-income countries
  - α_n denotes NACE 4-digit industry fixed effects
  - α_c and α_t correspond to cohort and time fixed effects, respectively*

Return

* The *Deaton-Hall* normalization is used on the time dummies to simultaneously control for age, cohort, and year effects

Measurement: Mapping Between Data and Model

- In data

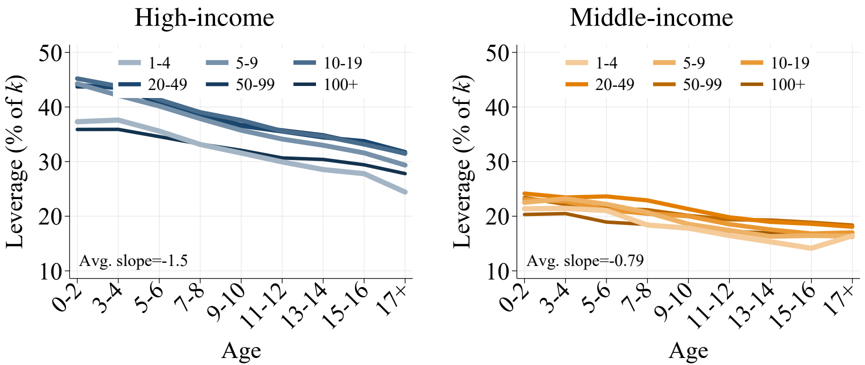
$$
\begin{aligned}
(n_{it} - x_{it}) &= \texttt{TOAS}_{it} - \texttt{CULI}_{it} - \texttt{NCLI}_{it} = \texttt{CAPI}_{it} + \texttt{OSFD}_{it} \\
\mathrm{b}_{it+1} &= \texttt{LOAN}_{it} + \texttt{LTDB}_{it} - \texttt{CASH}_{it} \\
k_{it+1} &= n_{it} - x_{it} + b_{it+1} \\
&= \underbrace{k^{\text{tan}}}_{0.76} + \underbrace{k^{\text{int}}}_{0.11} + \underbrace{k^{\text{wk}}}_{0.13}
\end{aligned}
$$

- k is known as *financial capital* in the corporate finance literate (Welch IRF 2011)
- Leverage is measured as $\ell_{it} = \max\{\mathrm{b}_{it}, 0\}/k_{it}$
- The spread is the average interest on outstanding debt relative to the country risk-free rate, $\frac{(rb)_{it}}{b_{it}} - r_{ft}$, where *rb* is measured by firm financial expenses ($\texttt{FIEX}_{it}$)
- Equity injections are measured by the variable $x_{it} = -\Delta\texttt{CAPI}_{it}$, where $\texttt{CAPI}$ is issued share capital

◂ Return

Leverage: By Age Conditional on Size

$$y_{it} = \sum_{a \in \mathcal{A}} \sum_{s \in \mathcal{S}} (\gamma_{as} + \gamma_{as}^{\text{MI}} \text{MI}_i) D_{it}^a D_{it}^s + \alpha_n + \alpha_c + \alpha_t + \epsilon_{it}$$

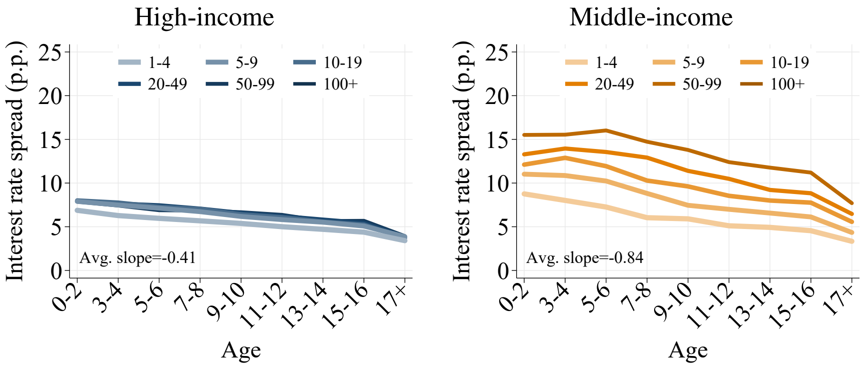


Notes: Leverage is net financial debt over capital, $\max\{b_{it}, 0\}/k_{it}$. Predicted values are scaled using the unconditional mean of the omitted group (oldest and biggest firms in HI). The regression considers firms' capital as weights.

Return

Spreads: By Age Conditional on Size

$$y_{it} = \sum_{a \in \mathcal{A}} \sum_{s \in \mathcal{S}} (\gamma_{as} + \gamma_{as}^{\text{MI}} \text{MI}_i) D_{it}^a D_{it}^s + \alpha_n + \alpha_c + \alpha_t + \epsilon_{it}$$

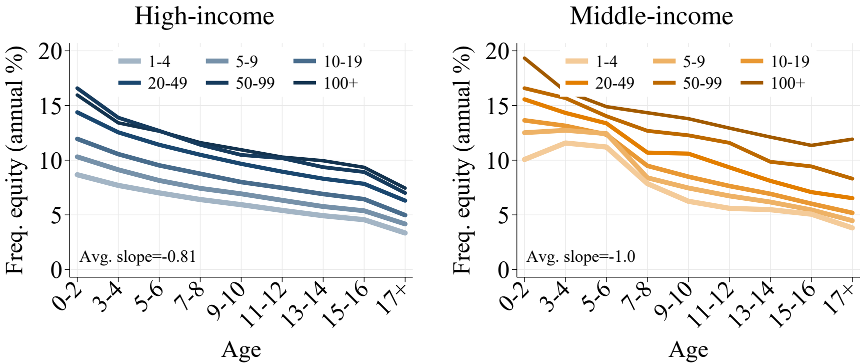


Notes: The spread is the average interest rate relative to the country risk-free rate, $\frac{(rB)_{it}}{B_{it}} - r_{ft}$. Predicted values are scaled using the unconditional mean of the omitted group (oldest and biggest firms in HI). The regression considers firms' debt as weights.

◂ Return

Fr. Equity Financing: By Age Conditional on Size

$$y_{it} = \sum_{a \in \mathcal{A}} \sum_{s \in \mathcal{S}} (\gamma_{as} + \gamma_{as}^{\text{MI}} \text{MI}_i) D_{it}^{a} D_{it}^{s} + \alpha_n + \alpha_c + \alpha_t + \epsilon_{it}$$

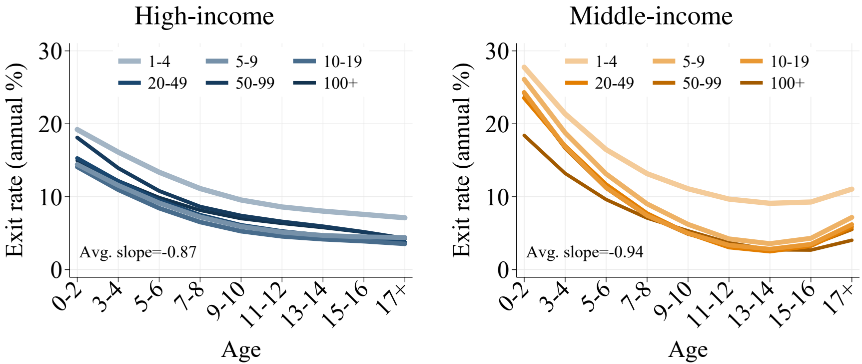


Notes: The annual frequency of equity financing is the average of $\mathbb{1}_{\{x_{it}<0\}}$. Predicted values are scaled using the unconditional mean of the omitted group (oldest and biggest firms in HI). The regression considers firms' sales as weights.

◂ Return

Exit Rates: By Age Conditional on Size

$$y_{it} = \sum_{a \in \mathcal{A}} \sum_{s \in \mathcal{S}} (\gamma_{as} + \gamma_{as}^{\text{MI}} \text{MI}_i) D_{it}^a D_{it}^s + \alpha_n + \alpha_c + \alpha_t + \epsilon_{it}$$

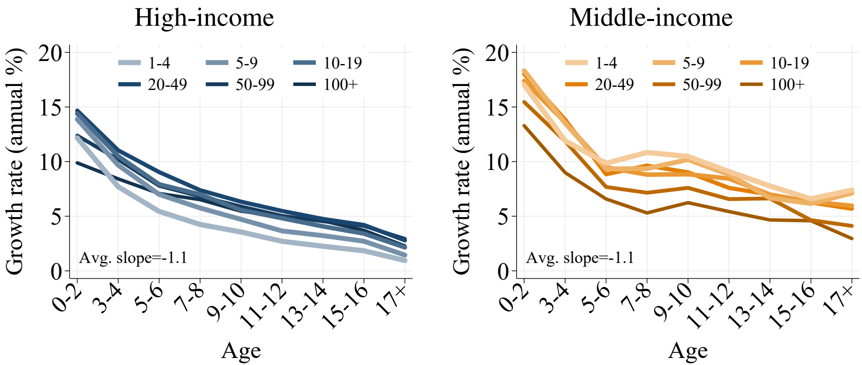


Notes: Exit are measured using Orbis' status identifiers.

◂ Return

Output Growth: By Age Conditional on Size

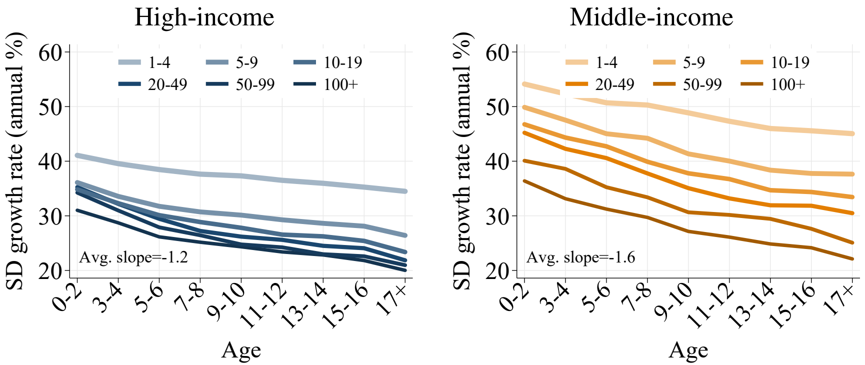
$$y_{it} = \sum_{a \in \mathcal{A}} \sum_{s \in \mathcal{S}} (\gamma_{as} + \gamma_{as}^{\text{MI}} \text{MI}_i) D_{it}^a D_{it}^s + \alpha_n + \alpha_c + \alpha_t + \epsilon_{it}$$



Notes: Annual growth rates are measured as $100(y_{it} - y_{it-1})/(0.5y_{it} + 0.5y_{it-1})$. Output is measured by firms' value added. The regression considers firms' current output as weights.

◂ Return

Output Growth SD: By Age Conditional on Size

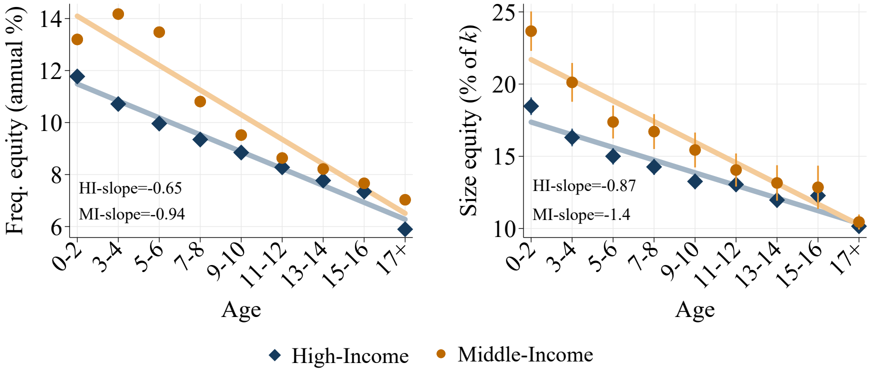


Notes: Annual growth rates are measured as $100(y_{it} - y_{it-1})/(0.5y_{it} + 0.5y_{it-1})$. Output is measured by firms' value added. The dispersion measure is the standard deviation of residuals after controlling for sector and year fixed-effects.

◂ Return

Equity injections are more frequent and larger for younger firms

1. Frequency of equity financing   2. Size of equity financing

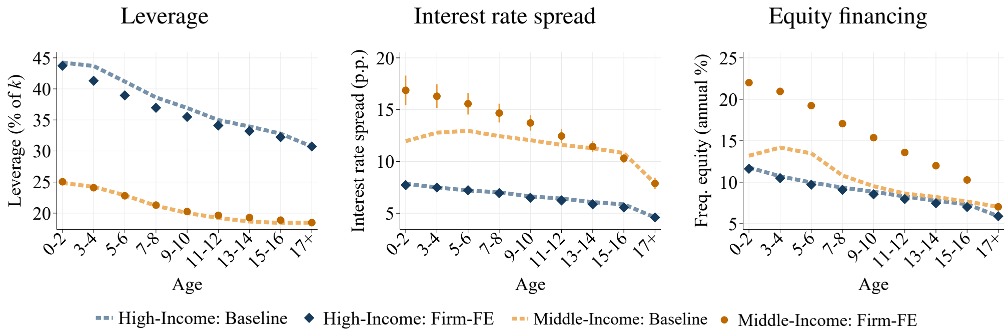


Notes: Equity injections refers to the resources put by shareholders into the firm, after the first year of operation (negative dividends), and can be financed by the founder of the firm or by new shareholders. The frequency is reported at an annual basis. The size of equity injections, conditional on adjustment, is measured with respect to next period capital $|x_{it}|/k_{it+1}$, where $x_{it} < 0$. The regression in panel (b) uses next period capital k_{it+1} as weights.

◂ Return

Finance Over the Life Cycle of Firms: Firm Fixed Effects

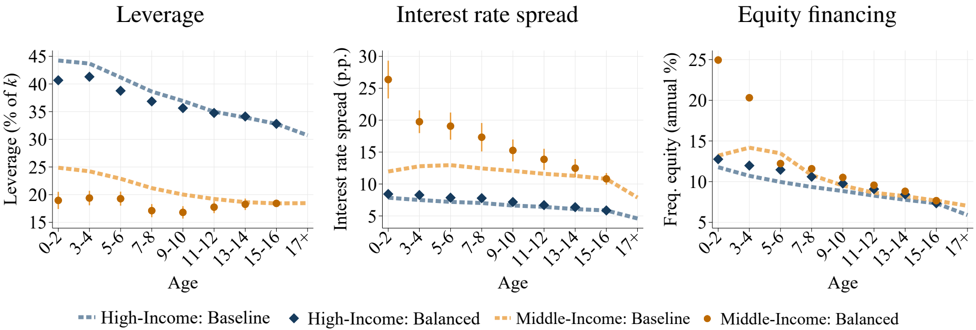
$$y_{it} = \sum_{a \in \mathcal{A}} \gamma_a \mathrm{D}^a_{it} + \alpha_i + \alpha_t + \epsilon_{it}$$



Notes: For presentation purposes the numbers are scaled using the unconditional mean of the omitted group. The vertical lines correspond to 95% confidence intervals considering robust standard errors. Leverage is net financial debt over capital. The spread is the average interest rate relative to the country risk-free rate. Equity financing measures the share of firms that receive an equity injection. Leverage is weighted by capital, and spreads are credit-weighted.

◂ Return

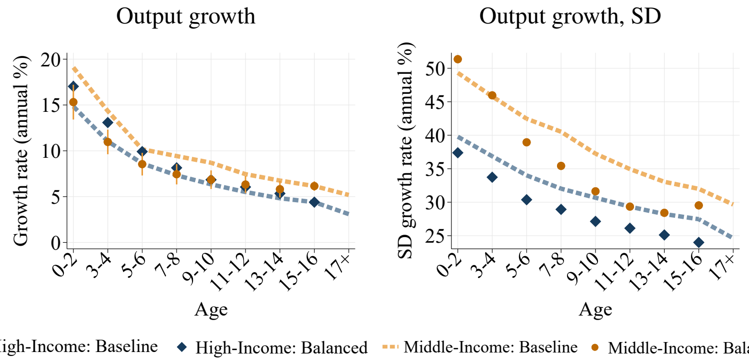
Finance Over the Life Cycle of Firms: Balanced Panel



Notes: Predicted values from baseline regression considering alternative samples. For presentation purposes the numbers are scaled using the unconditional mean of the omitted group. The vertical lines correspond to 95% confidence intervals considering robust standard errors. Leverage is net financial debt over capital. The spread is the average interest rate relative to the country risk-free rate. Equity financing measures the share of firms that receive an equity injection. Leverage is weighted by capital, and spreads are credit-weighted.

Return

Output Growth Over the Life Cycle of Firms: Balanced Panel



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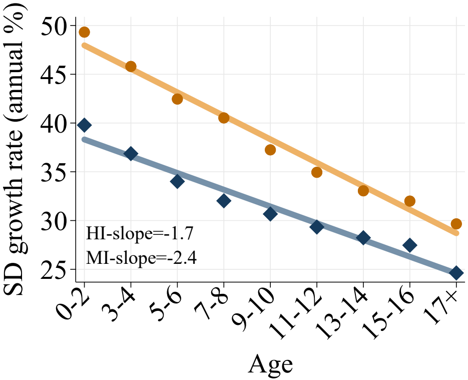
◂ Return

Output growth dispersion is larger for younger firms

Dispersion in middle-income countries is higher and slope is steeper



Notes: Notes: Annual growth rates are measured as $100(y_{it} - y_{it-1})/(0.5y_{it} + 0.5y_{it-1})$. Output is measured by firms' value added. Results controls for sector and year fixed-effects. The regression considers firms' current output as weights.$N =$25,042,906

◂ Return

Kalman Filter

- State space representation (s hidden state, z signal)

$$s_{it+1} = \rho_s s_{it} + u_{it+1}$$
$$z_{it} = s_{it} + \varepsilon_{it}$$
$$s_{i0} \sim \mathcal{N}(\hat{s_{i0}}, \Sigma_0), \quad \varepsilon_t \sim \mathcal{N}(0, \sigma_{\varepsilon t}^2), \quad u_t \sim \mathcal{N}(0, \sigma_u^2), \quad \sigma_{\varepsilon t} = (1 + \rho_\varepsilon^t C_\varepsilon)\, \sigma_\varepsilon$$

- Kalman filter recursions for $\hat{s}_{it+1} = \mathbb{E}[s_{it+1} | z_i^t]$ and $\Sigma_{t+1} = \mathbb{V}(s_{it+1} | z_i^t)$ are

$$g_{it} = z_{it} - \hat{s}_{it}$$
$$K_t = \rho_s \frac{\Sigma_t}{\Sigma_t + \sigma_{\varepsilon t}^2}$$
$$\hat{s}_{it+1} = \rho_s \hat{s}_{it} + K_t g_{it}$$
$$\Sigma_{t+1} = (\rho_s - K_t)^2 \Sigma_t + K_t^2 \sigma_{\varepsilon t}^2 + \sigma_u^2$$

◂ Return

Long-Term Debt

- Every period, a fraction $\phi(b_{it}) = \begin{cases} \phi & \text{if } b_{it} > 0 \\ 1 & \text{if } b_{it} \leq 0 \end{cases}$ of debt mature (average maturity equals $1/\phi$)

- Each bond pays a coupon rate r, the debt and interest payments at t equal

$$\underbrace{\phi(b_{it})b_{it}(1+r)}_{\text{Matures: principal+coupon}} + \underbrace{(1-\phi(b_{it}))b_{it}r}_{\text{Does not mature: coupon}} = (\phi(b_{it})+r)b_{it}$$

- If the firm acquires new debt between age t and $t+1$ it receives

$$\underbrace{\boldsymbol{q}_{t+1}(k_{it+1}, b_{it+1}, \hat{s}_{it+1})}_{\text{Price of debt}}\underbrace{[b_{it+1} - (1-\phi(b_{it}))b_{it}]}_{\text{New debt}}$$

◂ Return

Price of Debt

- The price of debt $\boldsymbol{q}_{t+1}$ faced by a firm of age t, when choosing k_{it+1} and b_{it+1}, is defined by

$$\boldsymbol{q}_{t+1}(k_{it+1}, b_{it+1}, \hat{s}_{it+1}) b_{it+1} = \underbrace{R^{-1}\mathbb{E}_t\left[\, d_{it+1}\ \min\{b_{it+1}(1+r),\ \rho(1-\delta)k_{it+1}\}\,\right]}_{\text{Recovery under default}}$$

$$+ \underbrace{R^{-1}\mathbb{E}_t\left[(1-d_{it+1})\, b_{it+1}\left(\phi + r + (1-\phi)\boldsymbol{q}_{t+2}(k_{it+2}, b_{it+2}, \hat{s}_{it+2})\right)\right]}_{\text{Repayment no default}}$$

where $d_{it+1} = 1$ if the firm defaults, $R = (1+r)$, and ρ is the recovery rate

Return

Unconstrained Allocation

- The unconstrained capital of a firm of age t, denoted by $k^*_{t+1}(\hat{s}_{t+1})$, solves the FOC

$$[k^*_{t+1}]: \quad -1+\beta\mathbb{E}_t\left[\text{MRPK}(k^*_{t+1},z_{t+1})+(1-\delta)\right]=0$$

with $\text{MRPK} = \partial\pi/\partial k$

- It can be showed that

$$k^*_{t+1}(\hat{s}_{t+1}) = \left(\frac{\hat{\alpha}\; G\; \mathbb{E}_t[f(z_{t+1})]}{(\beta^{-1}-1)+\delta}\right)^{\frac{1}{1-\hat{\alpha}}}$$

where $\alpha_1 = \frac{\alpha\eta}{\mu}$, $\alpha_2 = \frac{(1-\alpha)\eta}{\mu}$, $\hat{\alpha} = \frac{\alpha_1}{1-\alpha_2}$, $f(z) = \exp(z)^{\frac{1}{1-\alpha_2}}$, and

$$G = (1-\alpha_2)\left(\frac{\alpha_2}{w}\right)^{\frac{\alpha_2}{1-\alpha_2}}\left(PY^{\frac{1}{\sigma}}\right)^{\frac{1}{1-\alpha_2}}$$

captures the effect of aggregate variables

Return

Law of Motion for the Mass of Active Firms

- Law of motion for the mass of operating firms Ω over $(k_t, b_t, \hat{s}_{t+1}, g_t, t)$ is given by

$$\Omega' = \mathcal{C}[\Omega] + \mathcal{E}$$

  where $\mathcal{C}$ is a function mapping current to next period states for continuing firms

- The mass of entrants is equal to

$$\mathcal{E} = M \int_{\mathbb{1}\{\mathcal{V}_e(k_0,\hat{s}_0) \geq \boldsymbol{n}_e(k_0,\hat{s}_0)\}} H(k_0, \hat{s}_0)\, \mathrm{d}G(k_0, \hat{s}_0).$$

  where H is a function mapping entrants states $(k_0, \hat{s}_0)$ to $(k_0, b_0, \hat{s}_1, g_0, 0)$ and $M = 1$

◂ Return

Market Structure, Technology, and Earnings

- Aggregate output $$Y = \left[\int \exp(z_i)\, y_i^{\frac{\sigma-1}{\sigma}} \mathrm{d}i \right]^{\frac{\sigma}{\sigma-1}}$$

  implies demand curve $y_i = \left[\frac{p_i}{P}\right]^{-\sigma} \exp(z_i)^{\sigma} Y$

- Technology $y_i = k_i{}^{\alpha} l_i{}^{(1-\alpha)}$

- Earnings $$\pi(k_i, z_i) = \max_{l_i} A \exp(z_i) \left(k_i{}^{\alpha} l_i{}^{(1-\alpha)} \right)^{\frac{1}{\mu}} - w l_i$$

  where $A = PY^{\frac{1}{\sigma}}$, $\mu = \frac{\sigma}{\sigma-1}$, $\sigma \in (1, \infty)$, $\alpha \in (0, 1)$

◂ Return

Entrants

- Every period there is a constant mass of prospective entrants $M > 0$

- Prospective entrants are heterogeneous on their signal $\hat{s}_{i0}$ and initial capital k_{i0}
  - $(k_{i0}, \hat{s}_{i0}) \sim G(\alpha_\kappa, \alpha_0)$ ▸ Details
  - For each $(k_{i0}, \hat{s}_{i0})$, b_{i0} chosen to match leverage of entrants in data
  - Initial equity injection $\qquad \boldsymbol{n}_e(k_{i0}, \hat{s}_{i0}) = k_{i0} - q_e(k_{i0}, b_{i0}, \hat{s}_{i0}) b_{i0}$

- New firms of type $(k_{i0}, \hat{s}_{i0})$ will enter the market if and only if

$$\mathcal{V}_e(k_{i0}, \hat{s}_{i0}) - \boldsymbol{n}_e(k_{i0}, \hat{s}_{i0}) \geq 0$$

where $\quad \mathcal{V}_e(k_{i0}, \hat{s}_{i0}) = \beta \mathbb{E}[\mathcal{V}_0(n_{i0}, b_{i0}, \hat{s}_{i1})], \quad n_{i0} = \boldsymbol{n}(k_{i0}, z_{i0}, c_{Fi0}), \quad z_{i0} = \hat{s}_{i0} + g_{i0}$

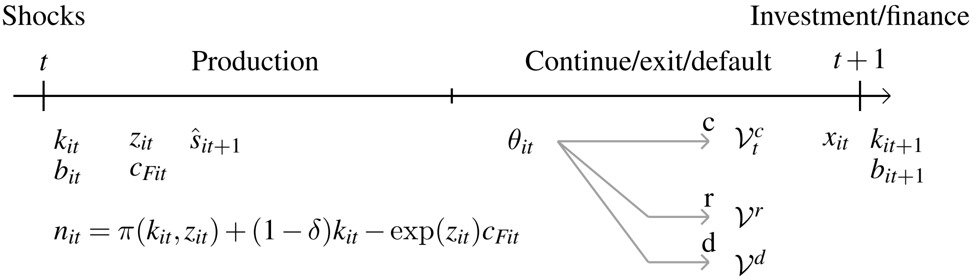
◂ Return

Entrants' Initial States

- Entrants are heterogeneous on their signal $\hat{s}_{i0} \sim G(\alpha_0)$ and initial scale $\kappa_{i0} \sim \text{Beta}(\alpha_\kappa, 1)$
  - Initial capital $k_{i0} = \kappa_{i0}\, k_0^*(\hat{s}_{i0}), \quad \text{where } \kappa_{i0} \in (0,1)$
  - As $(k_{i0}, \hat{s}_{i0}) = (\kappa_{i0}\, k_0^*(\hat{s}_{i0}), \hat{s}_{i0})$, on average, firms with higher $\hat{s}_{i0}$ have higher k_{i0}
  - Entrants debt b_{i0} chosen to match leverage of entrants in data
  - Initial equity injection $n_e(\kappa_{i0}, \hat{s}_{i0}) = k_{i0} - q_e(k_{i0}, b_{i0}, \hat{s}_{i0}) b_{i0}$

◂ Return

Timing



1. The firm observes z_{it}, produces, and draw an operating cost $c_{Fit} \sim \log \mathcal{N}(\mu_{cF}, \sigma_{cF}^2)$
2. Updates $\hat{s}_{it+1}$, and observe its cash on hand n_{it}, before debt expenses
3. Draws exit shock $\theta_{it} \sim \text{Ber}(\theta)$, and decides to continue (c), exit and repay (r), or exit and default (d)
4. If the firm continues, it chooses next period capital k_{it+1} and how to finance it

◂ Return

Incumbents' Problem

- The value of a firm with age $t \geq 0$, cash on hand n_{it}, debt b_{it}, and belief $\hat{s}_{it+1}$ is

$$\mathcal{V}_t(n_{it}, b_{it}, \hat{s}_{it+1}) = \mathbb{E}_{\theta_{it}} \left[\theta_{it} \max_{r,d} \left\{ \mathcal{V}^r(n_{it}, b_{it}),\ \mathcal{V}^d \right\} \ +\ (1-\theta_{it}) \max_{c,r,d} \left\{ \mathcal{V}^c_t(n_{it}, b_{it}, \hat{s}_{it+1}),\ \mathcal{V}^r(n_{it}, b_{it}),\ \mathcal{V}^d \right\} \right]$$

- If the firm exits and repays $\mathcal{V}^r(n_{it}, b_{it}) = n_{it} - (1+r)b_{it}$, if defaults $\mathcal{V}^d = 0$

- If the firm continues, it solves

$$\mathcal{V}^c_t(n_{it}, b_{it}, \hat{s}_{it+1}) = \max_{k_{it+1}, b_{it+1}} \ x_{it} - \Lambda(x_{it}) + \beta \mathbb{E}_t \left[\mathcal{V}_{t+1}\left(n_{it+1}, b_{it+1}, \hat{s}_{it+2}\right) \right]$$

$$\text{s.t.} \qquad k_{it+1} = n_{it} - (\phi + r)b_{it} - x_{it} + \boldsymbol{q}_{t+1}(k_{it+1}, b_{it+1}, \hat{s}_{it+1})[b_{it+1} - (1-\phi)b_{it}]$$

Return

Equilibrium

A stationary competitive equilibrium consists of: (i) aggregate wage w; (ii) value functions $\{\mathcal{V}_t\}$ and $\{\mathcal{V}_t^c\}$; (iii) policies $\{k_{t+1}\}$, $\{b_{t+1}\}$, and $\{x_t\}$; (iv) debt schedules $\{\boldsymbol{q}_t\}$; (v) a measure of incumbent firms Ω over $(k_t, b_t, \hat{s}_{t+1}, g_t, t)$; and ($vi$) a measure of entrants $\mathcal{E}$, such that

1. $\mathcal{V}_t$ solves the Bellman equation of age t incumbents, with associated extensive margin decision rules
2. For age t continuing firms, $\mathcal{V}_t^c$ solves the Bellman equation with policies for k_{t+1}, b_{t+1}, and x_t
3. The debt schedule $\{\boldsymbol{q}_t\}$ solves the financial intermediaries' zero expected profit condition
4. Labor market clears
$$\int l_i \, \mathrm{d}\Omega(i) = \bar{L}w^{\gamma}$$
5. The mass of operating firms Ω solves the law of motion
$$\Omega' = \mathcal{C}[\Omega] + \mathcal{E}$$
where $\mathcal{C}$ is a function mapping current to next period states for continuing firms, and
$$\mathcal{E} = M \int_{\mathbb{1}\{\mathcal{V}_e(k_0,\hat{s}_0) \geq \boldsymbol{n}_e(k_0,\hat{s}_0)\}} H(k_0, \hat{s}_0) \, \mathrm{d}G(k_0, \hat{s}_0)$$

Return

Assigned Parameters

		Description
r	0.03	Gross risk-free rate
$\beta^{-1} - 1$	0.06	Discount factor
γ	2	Labor supply elasticity
σ	10	CES
α	1/3	Capital elasticity
δ	0.1	Capital depreciation rate
θ	0.025	Exogenous exit rate
ϕ^{-1}	4.5	Debt expected duration (Hernandez Koeter, 2008)

◂ Return

Calibrated Parameters

	High	Middle	Description
α_κ	0.448	0.205	Entrants' capital, shape
α_0	2.86	2.30	Entrants' $\hat{s}_0$ signal, shape
Σ_0/Σ_∞	1.29	1.21	Entrants' uncertainty
ρ_s	0.980	0.968	Persistent shock, AR
σ_u	0.042	0.048	Persistent shock, SD
σ_ε	0.069	0.095	Transitory shock, SD
ρ_ε	0.803	0.827	Transitory shock, SD persistence
C_ε	0.517	0.610	Transitory shock, SD initial
μ_{c_F}	-0.14	-0.81	Operation cost, mean
σ_{c_F}	1.98	2.58	Operation cost, SD
ρ	0.34	0.29	Lenders' recovery rate
λ_0	10.196	7.202	Equity cost, fixed
λ_1	0.382	0.390	Equity cost, linear
λ_2	0.011	0.070	Equity cost, quadratic

◂ Return

Model Fit

Entrants and Real Variables

	High-Income		Middle-Income	
	Data	Model	Data	Model
Entrants (age 0-2)				
Output growth	0.15	0.17	0.19	0.22
Exit rate	0.16	0.21	0.24	0.24
Real Variables				
Exit rate	0.08	0.08	0.12	0.14
log Output, SD	1.71	2.13	2.09	2.17
Output growth				
Mean	0.06	0.07	0.08	0.10
SD	0.29	0.32	0.37	0.40
SD age-slope	-0.017	-0.023	-0.024	-0.022
Profits/k	0.08	0.11	0.12	0.12
Profits/k, SD	0.18	0.08	0.20	0.16

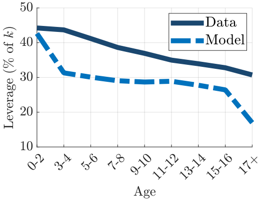
Financial Variables

	High-Income		Middle-Income	
	Data	Model	Data	Model
Leverage				
Age-slope	-0.017	-0.020	-0.009	-0.009
Mean age 9-10	0.37	0.29	0.20	0.18
SD	0.35	0.14	0.28	0.13
Interest Rate Spread				
Age-slope	-0.003	-0.003	-0.004	-0.005
Mean age 9-10	0.066	0.074	0.121	0.096
SD	0.119	0.103	0.178	0.117
Equity Financing				
Fr., age-slope	-0.007	-0.006	-0.009	-0.014
Fr., age 9-10	0.09	0.09	0.10	0.06
Size, mean	0.14	0.15	0.16	0.13
Size, SD	0.23	0.17	0.27	0.18

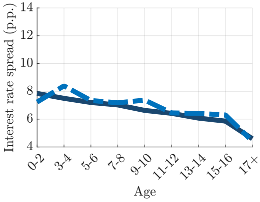
◂ Return

Life Cycle of Firms in Data and Model: High-Income

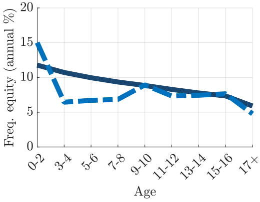
1. Leverage



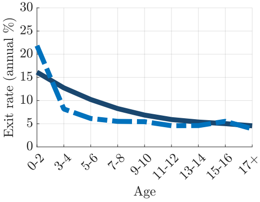
2. Interest rate spread



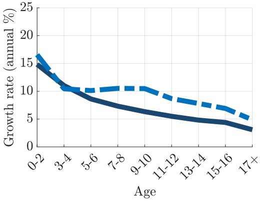
3. Equity financing



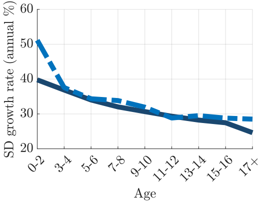
4. Exit rate



5. Output growth



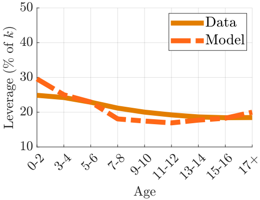
6. Output growth, SD



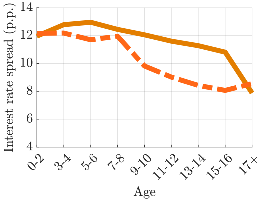
◂ Return

Life Cycle of Firms in Data and Model: Middle-Income

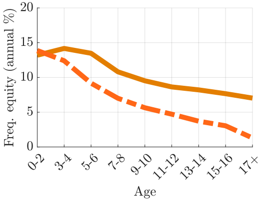
1. Leverage



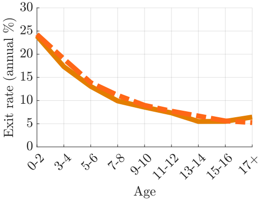
2. Interest rate spread



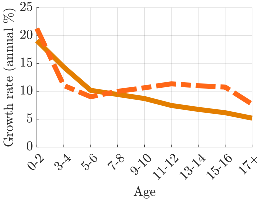
3. Equity financing



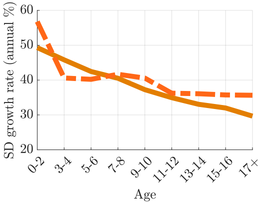
4. Exit rate



5. Output growth

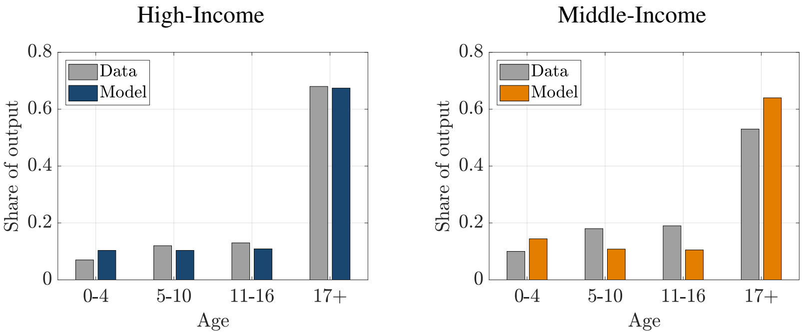


6. Output growth, SD



◂ Return

Output Distribution by Firms' Age in Data and Model



Notes: Data numbers corresponds to the cross-sectional distribution of value added in the year 2018. Model moments were computed using simulated data from the stationary distribution Ω.

◂ Return

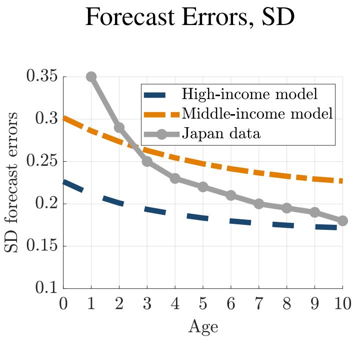
Forecast Errors

- Conditional on z_i^t and k_{it+1}, age t+1 log earnings' forecast error equals

$$FE_{it+1|t} \equiv \log \pi(z_{it+1}, k_{it+1}) - \mathbb{E}_t \left[\log \pi(z_{it+1}, k_{it+1})\right]$$
$$= \frac{\mu}{\mu - (1-\alpha)} \left(g_{it+1} - \mathbb{E}_t[g_{it+1}]\right)$$

where

$$\mathbb{E}[g_{it+1}] = 0$$
$$\mathbb{V}(g_{it+1}) = \mathbb{V}(z_{it+1}|z_i^t) = \Sigma_{t+1} + \sigma^2_{\varepsilon t+1}$$



Notes: Japan-data numbers are from Chen Senga Sun Zhang 2020, who document log sales' forecast errors using panel and survey data from Japanese firms.

◂ Return

Capital Investments and Equity Financing

Dependent Variable: Investment Rate $(k_{it+1} - (1-\delta)k_{it})/k_{it}$

	High-Income				Middle-Income			
	Data		Model		Data		Model	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Constant	0.113	0.104	0.102	0.101	0.136	0.125	0.123	0.121
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$\mathbb{1}\{x_{it} < 0\}$		0.147		0.139		0.150		0.166
		(0.001)		(0.005)		(0.003)		(0.004)
α_n, α_t	No	Yes			No	Yes		
N	19,904,118		500,000		3,778,009		500,000	

Notes: Robust standard errors are presented in parentheses. α_n and α_t denote industry (NACE 4-digit) and time fixed effects, respectively. Model regressions were computed using simulated data from the stationary distribution Ω.

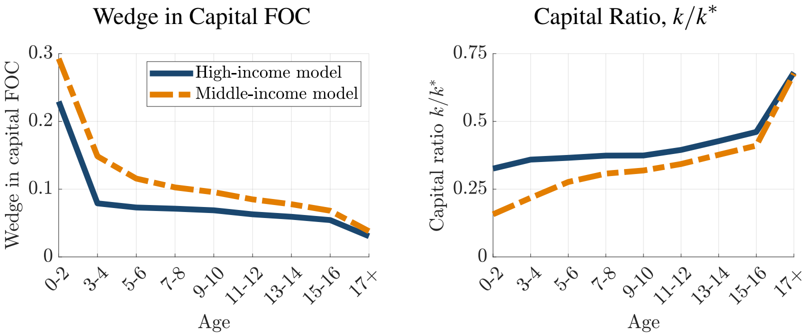
◂ Return

How Constrained Are Young Firms?

Entrants start at 0.31 and 0.17 k/k^* in high- and middle-income countries

The typical firm in high- and middle-income exit at 0.42 and 0.3 k/k^* ($\mathbb{E}[dur]$ 12.5 and 7.1)

Capital Wedge and Ratio, Relative to Unconstrained Allocation



◂ Return

Notes: Panel (a) reports the average wedge in the first order condition (FOC) of capital, relative to the unconstrained level, $\mathbb{E}_{z_{it+1}|\hat{s}_{it+1}}\left[\text{MRPK}(k_{it+1}, z_{it+1})\right] - (\beta^{-1} - 1 + \delta)$.

Aggregate Implications of Financial Frictions

Financial frictions reduce aggregate output in 15% and 24%, TFP is 8% and 13% lower

Lower mass of firms Ω explained by exit margin: lower share of continuing firms $m(\mathcal{C}[\Omega])$

	High-Income		Middle-Income	
	Perfect Credit	Baseline	Perfect Credit	Baseline
(a) *Relative to Perfect Credit*				
Y/L	1.00	0.85	1.00	0.76
TFP	1.00	0.92	1.00	0.87
K/Y	1.00	0.91	1.00	0.88
$m(\Omega)$	1.00	0.48	1.00	0.41
$m(\mathcal{C}[\Omega])$	1.00	0.46	1.00	0.37
$m(\mathcal{E})$	1.00	0.97	1.00	1.09
(b) *Levels*				
Exit Rate	0.04	0.08	0.06	0.14
$\mathbb{E}$[lifespan]	25.3	12.5	17.9	7.1

▸ LofM
◂ Return

Decomposing Losses in Output Per Worker Y/L

TFP explains 3/4 of losses in Y/L, lower K/Y accounts for 1/4

$$\Delta^* \log(Y/L) = \frac{1}{1-\alpha}\Delta^* \log(\text{TFP}) + \frac{\alpha}{1-\alpha}\Delta^* \log(K/Y).$$

Losses in Output Per Worker, Decomposition

	High-Income	Middle-Income
$\Delta^* \log(Y/L)$	0.17	0.27
$\frac{1}{1-\alpha}\Delta^* \log(\text{TFP})$	0.12	0.21
$\frac{\alpha}{1-\alpha}\Delta^* \log(K/Y)$	0.05	0.06

Notes: Δ^* denotes the difference between the perfect credit allocation ($\{\lambda_j\} = \mathbf{0}$) and the baseline.

◂ Return

Unpacking TFP Losses

Extensive margin account for 6.1 and 10.4% of the TFP losses

Capital misallocation (intensive margin) explains 1.5 and 2.4% of the TFP losses

$$\text{TFP} = \left(\frac{\int \left(f(z_i)^{\frac{1}{1-\hat{\alpha}}} (k_i/p_i y_i)^{\frac{\hat{\alpha}}{1-\hat{\alpha}}} \right) d\Omega(i)}{\left[\int \left(f(z_i)^{\frac{1}{1-\hat{\alpha}}} (k_i/p_i y_i)^{\frac{1}{1-\hat{\alpha}}} \right) d\Omega(i) \right]^{\hat{\alpha}}} \right)^{\mu-(1-\alpha)}$$

TFP Losses, Extensive (Ω) *and Intensive Margins* (k/py)

		High-Income	Middle-Income
Ω	(k/py)	7.6%	12.8%
Ω	$(k/py)^*$	6.1%	10.4%
Ω^*	(k/py)	1.5%	2.4%
Ω^*	$(k/py)^*$	0.0%	0.0%

◂ Return

Aggregation, Output Per Worker, and TFP

- Aggregate output $Y = \text{TFP}\, K^{\alpha}\, L^{(1-\alpha)}$, output per worker $Y/L = \text{TFP}^{\frac{1}{1-\alpha}}\, (K/Y)^{\frac{\alpha}{1-\alpha}}$

$$\text{TFP} = \left(\frac{\int \left(\varphi(z_i)^{\frac{1}{1-\hat{\alpha}}} (k_i/p_i y_i)^{\frac{\hat{\alpha}}{1-\hat{\alpha}}} \right) d\Omega(i)}{\left[\int \left(\varphi(z_i)^{\frac{1}{1-\hat{\alpha}}} (k_i/p_i y_i)^{\frac{1}{1-\hat{\alpha}}} \right) d\Omega(i) \right]^{\hat{\alpha}}} \right)^{\mu-(1-\alpha)}, \quad K = \int k_i \, d\Omega(i), \quad L = \int l_i \, d\Omega(i)$$

where $\varphi(z_i) = \exp(z_i)^{\frac{\mu}{\mu-(1-\alpha)}}$ and $\hat{\alpha} = \frac{\alpha}{\mu-(1-\alpha)}$

- Financial frictions can reduce output per worker through three channels:
  1. Capital deepening: lower aggregate capital-output ratio K/Y
  2. TFP Capital misallocation: dispersion in active firms' capital-output ratios $k_i/p_i y_i$
  3. TFP Extensive margin: distortions in firms' entry/exit decisions $d\Omega(i)$

Return