Uncertainty and Economic Activity: A Multi-Country Perspective*

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Abstract

This paper develops a multi-country Lucas tree model to motivate identification of global growth and financial shocks in the context of a factor-augmented multi-country panel VAR model for the analysis of the relationship between output growth and realized stock market volatility. The proposed identification scheme is consistent with the cross-country correlations of volatility and output growth, and yields a number of interesting results regarding the importance of global growth shocks for the interpretation of the correlations between volatility and growth within countries, the extent to which growth shocks drive volatility, and the transmission of uncertainty shocks to output growth.

Keywords: Uncertainty, Business Cycle, Global Shocks, Multi-Country Asset Pricing Model, Panel VAR, Identification, Realized Volatility, Impulse Responses.
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1 Introduction

It is well-established that empirical measures of uncertainty behave countercyclically in the United States as well as in most other countries.[1] Interpreting correlations in economic terms is always problematic, as causation can run in both directions. From a theoretical standpoint, uncertainty can cause economic activity to slowdown and even contract through a variety of mechanisms, both on the household side *via* precautionary savings (Kimball (1990)) and on the firm side *via* investment delays or other frictions (see, for instance, Bernanke (1983), Dixit and Pindyck (1994) and, more recently, Bloom (2009), Christiano et al. (2014), Basu and Bundick (2017), among many others). But it is also possible that uncertainty responds to fluctuations in economic activity or other unobserved effects. Indeed, the theoretical literature highlights mechanisms through which adverse economic conditions can trigger spikes in uncertainty. Examples based on information and financial frictions include Van Nieuwerburgh and Veldkamp (2006), Fostel and Geanakoplos (2012), Kozlowski et al. (2015), and Ilut et al. (2017). Theory, therefore, does not provide a definite guidance on how to interpret the observed negative correlation between output growth and uncertainty measures.

In this paper we base our empirical analysis on realized stock market volatility as a measure of uncertainty and adopt a multi-country perspective.[2] To interpret the countercyclical nature of realized volatility, we consider a factor-augmented multi-country panel VAR model (PVAR) and develop explicit theoretical relationships for output growth and realized volatility over the business cycle, based on a multi-country Lucas (1978) tree model. We assume that country-specific output growth (i.e. the dividend or endowment growth process) is determined by a common persistent component, namely a global growth factor with time-varying volatility and heterogeneous loadings, and a country-specific business cycle component. We then show that, consistent with the econometric model we specify, country-specific equity returns and their realized volatility are driven by at least one more common shock than the common shock to the endowment process. Moreover, under the same identification assumptions made in the econometric model, the global growth factor is the only driver of the cross-section correlation of output growth, while realized volatility is driven by at least two common factors, the first being the level of global growth and the second a composite 'financial' factor of second and higher-order moment shocks. In other words, the global growth shock is shown to be sufficient to explain the cross-country correlations of the output growth series. The global financial shock is shown to be necessary to capture the remaining cross-country correlations of the volatility series.

In line with our theoretical derivations, we consider a multi-country econometric model in output growth and log volatility with two common shocks and two country-specific shocks. We assume that the first (which we refer to as 'global growth shock') is common to both GDP growth and volatility

[1] For the evidence on the United States see, for example, Schwert (1989) using the volatility of the aggregate stock market return; Campbell et al. (2001) and Bloom et al. (2007) using the volatility of firm-level stock returns; Bloom et al. (2012) and Bachmann and Bayer (2013) using the volatility of plant, firm, industry and aggregate output and productivity; Bachmann et al. (2013) using the behavior of expectations' disagreement. For evidence on other countries see Baker et al. (2018), Carriere-Swallow and Cespedes (2013), and Nakamura et al. (2017), among others.

[2] Realized volatility has been used extensively in the theoretical and empirical finance literature and implicitly assumes that uncertainty and risk can be characterized in terms of probability distributions.

in all countries, while the second (a broadly defined 'global financial shock') is only common to the volatility series after controlling for the global growth shock. To identify the two country-specific shocks we use auxiliary assumptions typically used in the literature. Identification of the common shocks, however, does not require any restriction on the within-country correlation between country-specific volatility and growth shocks. In the paper, we show that the identification restrictions that we impose are not only consistent with standard consumption-based asset pricing theory, but also fit well the stylized facts of the data. In fact, in addition to documenting that they are highly correlated *within* countries, we also show that realized volatility and output growth are closely correlated *across* countries, but this cross-country correlation is much stronger for volatility than for growth.

We establish that global growth shocks are consistently estimated as residuals from the regression of global output growth on its lagged values as well as the lagged values of global volatility, whilst global financial shocks are obtained as residuals from the regression of global volatility on the estimated global growth shocks and the lagged values of global output growth and global volatility. We note here that one cannot arrive at these estimates by principal component (PC) analysis, where the common factors are estimated as PCs of output growth and/or volatility series considered separately or together, since the PC analysis does not make use of the *a priori* identification of the shocks and, being static in nature, cannot cope with the heterogeneous dynamics of the interactions between volatility and growth across countries.

The empirical analysis yields three main results. First, we find that a large proportion of the observed negative country-specific correlations between volatility and output growth can be accounted for by shocks to the world growth factor. While unconditionally volatility behaves countercyclically for all but one of the 32 countries in our sample, when we condition on world growth shocks, the correlations between volatility and growth innovations become statistically and quantitatively much smaller in all countries. This implies that part of the explanatory power attributed to volatility shocks in empirical studies of individual countries (i.e. considered in isolation from the rest of the world economy) might be due to omitted common factors. Indeed, direct evidence on this hypothesis does not contradict it. In line with the insight of our theoretical model, we also show that shocks to the global growth factor correlate closely with proxies for global TFP growth and the world natural interest rate, and more weakly with a measure of global long-run risk.

Second, the paper finds that the time-variation of country-specific volatility is explained largely by shocks to the global financial factor (with a share of forecast error variance being larger than 60 percent) and innovations to country-specific volatility series themselves (with a share of forecast error variance of about 35 percent). Shocks to the global growth factor and to country-specific growth innovations jointly explain less than 5 percent of volatility forecast error variance. This result implies that the 'endogenous' component of country-specific volatility—namely, the component driven by common and country-specific growth shocks—is quantitatively small.

Third, we find that shocks to the global financial factor explain about 10 percent of the forecast error variance of country-specific output growth, and they have strong and persistent contractionary effects. In contrast, country-specific volatility shocks explain only $1-2$ percent of the country-specific forecast error growth variance. In our empirical model, the forecast error variance of output growth is explained mainly by innovations to country-specific growth rates themselves (with a share of at least 60 percent) and the world growth shock (with another 25 percent of the total). This third set of results illustrates that even though there might be a very high degree of international co-movement in equity prices, such a co-movement need not translate into a large explanatory power for real GDP growth. Nonetheless, when global financial shocks occur they can have deep and lasting negative effects, as we record during the recent global financial crisis. These results, therefore, bridge the debates between the proponents and the opponents of the of the Global Financial Cycle hypothesis (Rey (2013), Cerutti et al. (2017)).

Our paper is closely related to several contributions in the literature on uncertainty and the business cycle.[3] Ludvigson et al. (2015) and Berger et al. (2017) acknowledge that uncertainty has endogenous components and can be driven by the business cycle. We take a common factor approach to modeling the two-way relationship between volatility and growth in a multi-country setting as opposed to a single-country approach. The restrictions that we impose to identify the common factors apply to a cross-section of countries, as opposed to a single-country in isolation from the rest of the world, or the global economy as a single closed economy. The identification problem that we pose cannot be addressed in a single-country set up. Furthermore, we show that single-country approaches to the question we study could yield empirically biased estimates due to the omitted variables. Interestingly, despite the different approaches taken to proxy for uncertainty and to separate endogenous responses to the business cycle from exogenous changes in volatility, we reach very similar conclusions that the endogenous component of volatility is quantitatively small (Ludvigson et al. (2015)) and that first moment shocks account for the countercyclical nature of volatility (Berger et al. (2017)).

While other papers have highlighted similar patterns of cross-country correlations for equity returns and consumption growth (Tesar (1995), Colacito and Croce (2011), Lewis and Liu (2015)), as far as we are aware, this is the first paper that focusses on cross-country correlations of volatility and output growth. Consistent with Renault et al. (2018), who take an arbitrage asset pricing theory approach to the pricing of square returns, and Chamberlain and Rothschild (1982), where it is assumed that the idiosyncratic component of asset returns is weakly correlated, we develop an empirical methodology to determine the number of risk factors necessary to obtain weakly correlated idiosyncratic shocks and estimate them consistently from the data. As in Colacito et al. (2018), we focus on the persistent component of 'dividend' growth or the 'cash flow' risk, as proxied by real GDP growth. Unlike them, however, we take a multi-country perspective and consider the *interaction* among many economies, in addition to their heterogeneous loading on the common factor, thus coming to different conclusions on the importance of global growth for country-specific endowment risks.

Carriere-Swallow and Cespedes (2013) estimate a battery of 40 VARs for advanced and emerging economies in which the US VIX Index, assumed exogenous, is the only source of interdependence, and identification is achieved by imposing a recursive scheme, country-by-country. Baker et al. (2018)

[3]The literature is voluminous. See Bloom (2014) for a recent survey. Here we focus only on studies directly related to our paper.

study an unbalanced panel of 60 countries, documenting the counter-cyclicality of different proxies for uncertainty and use measures of natural disasters, terrorist attacks and political events as instruments to quantify the exogenous impact of uncertainty on GDP, without, however, quantifying the importance of activity measures for uncertainty. Hirata et al. (2012) estimate a factor-augmented VAR with factors computed based on data for 18 advanced economies, and use a recursive identification scheme in which the volatility variable is ordered first. Carriero et al. (2018) estimate a large Bayesian VAR with exogenously driven stochastic volatility to quantify the impact of macroeconomic uncertainty on OECD economies. All these papers, therefore, restrict the direction of economic causation from the outset of the analysis by assuming that the uncertainty proxy used is exogenous. In addition, in our framework, countries interact with each other not only *via* the common factors, but also *via* the covariance matrix of the country-specific volatility and growth innovations, which is an important source of additional spillover channels absent in other studies.

Notations: Generic positive finite constants are denoted by C_i for $i = 0, 1, 2, ...$ They can take different values at different instances. $\mathbf{w} = (w_1, w_2, ..., w_n)'$ is an $n \times 1$ vector, and $\mathbf{A} = (a_{ij})$ an $n \times n$ matrix, $\varrho_{\max}(\mathbf{A})$ denotes the largest eigenvalue of $\mathbf{A}$, $\|\mathbf{w}\| = \left(\Sigma_{i=1}^{n} w_i^2\right)^{1/2}$ and $\|\mathbf{A}\| = \left[\varrho_{\max}(\mathbf{A}'\mathbf{A})\right]^{1/2}$ are the Euclidean norm of $\mathbf{w}$, and the spectral norm of $\mathbf{A}$, respectively. $\boldsymbol{\tau}_T = (1, 1, ..., 1)'$ is a $T \times 1$ vector of ones. If $\{y_n\}_{n=1}^{\infty}$ is any real sequence and $\{x_n\}_{n=1}^{\infty}$ is a sequences of positive real numbers, then $y_n = O(x_n)$, if there exists a positive finite constant C_0 such that $|y_n| / x_n \leq C_0$ for all n. $y_n = o(x_n)$ if $f_n/g_n \to 0$ as $n \to \infty$. If $\{y_n\}_{n=1}^{\infty}$ and $\{x_n\}_{n=1}^{\infty}$ are both positive sequences of real numbers, then $y_n = \mathcal{O}\left(x_n\right)$ if there exists $n_0 \geq 1$ and positive finite constants C_0 and C_1, such that $\inf_{n \geq n_0} \left(y_n/x_n\right) \geq C_0$, and $\sup_{n \geq n_0} \left(y_n/x_n\right) \leq C_1$.

2 A Multi-country Model of Equity Market Volatility and the Business Cycle

In this section we set up a theoretical model to interpret the two common factors that we identify in the data. The framework is a multi-country version of the Lucas (1978) tree model with complete asset markets, persistent global endowment growth shocks, heterogeneous country loadings on this common factor, time-varying volatilities and cross-country spillovers.

2.1 Endowments and Volatilities

Consider a world consisting of N economies indexed by $i = 1, 2, ...N$, of similar but not necessarily identical size. Each country i is inhabited by an infinitely-lived representative agent endowed at time t with a stochastic stream of a single homogeneous good $\{Y_{i,t+s}, s = 0, 1, 2, ...,\}$ viewed as the economy's measure of real output or GDP, with $\Delta y_{it} = \ln\left(Y_{it}/Y_{i,t-1}\right)$. We make the following assumptions on the exogenous forces driving $\Delta y_{i,t+1}$:

$$\Delta y_{i,t+1} = a_i + \gamma_i f_{t+1} + \varepsilon_{i,t+1}, \qquad i = 1, 2, ..., N, \tag{1}$$

$$f_{t+1} = \phi_f f_t + \sigma_t \zeta_{t+1}, \tag{2}$$

$$\sigma^2_{t+1} = \sigma^2(1 - \phi_\sigma) + \phi_\sigma \sigma^2_t + \varphi_\chi \chi_{t+1}, \tag{3}$$

where $\sup_i |\gamma_i| < 1$, $|\phi_f| < 1$, $|\phi_\sigma| < 1$, $0 < \sigma^2 < C_0$, and a_i and φ_χ are bounded constants. These processes assume that a country's output growth fluctuates around a deterministic steady state, a_i, and is driven by a common factor, f_t, and country-specific shock, ε_{it}. The country-specific intercept, a_i, can be interpreted as a country's long-run mean growth rate. The common factor, f_t, which is assumed to follow a stationary first order auto-regressive process, can represent the stochastic trend growth of world productivity (e.g. Aguiar and Gopinath (2007)), and will be loosely referred to as the global growth factor, or simply 'growth factor' for brevity. As in Bansal and Yaron (2004), f_t is assumed to have mean zero, with conditionally heteroskedastic innovations, but unconditionally homoskedastic variance. Namely, $E(f_t) = 0$, $Var_t(f_{t+1}) = \sigma^2_t$, and $Var(f_{t+1}) = \sigma^2/(1 - \phi^2_f)$, where $Var_t(.) = Var(.|\Im_t)$ denotes the conditional variance operator with respect to the non-decreasing information set, $\Im_t$, which at least includes all risky asset returns, R_{it}, as well as Δy_{it} for all $i = 1, 2, ..., N$, and all their lagged values.

The term ε_{it} represents country-specific forces driving the country's business cycles, including technology shocks as well as other demand and supply shocks. Collecting the N country-specific shocks in the $N \times 1$ vector $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \varepsilon_{2t}, ..., \varepsilon_{Nt})'$ and denoting the conditional variance-covariance matrix of $\boldsymbol{\varepsilon}_{t+1}$ by $Cov_t(\boldsymbol{\varepsilon}_{t+1}) = \boldsymbol{\Sigma}_{t\varepsilon}$, where $\boldsymbol{\Sigma}_{t\varepsilon} = (\sigma_{t,ij})$ is an $N \times N$ matrix with time-varying elements assumed as $Var_t(\varepsilon_{i,t+1}) = \sigma_{t,ii} = \theta^2_t \sigma_{ii} > 0$, and $Cov_t(\varepsilon_{i,t+1}, \varepsilon_{j,t+1}) = \sigma_{t,ij} = \psi^2_t \sigma_{ji}$ for $i \neq j$.

In this specification, time-variation in variances of the country-specific shocks differs from the time-variation in covariances, while allowing also for the correlations of the shocks to differ across countries. This permits us to distinguish between the effects of the country-specific shocks, as defined by $\theta_t \sqrt{\sigma_{ii}}$, and the extent to which shocks get transmitted across countries, via $\psi^2_t \sigma_{ji}$, which in our application will play an important role. To keep the analysis manageable, however, we have assumed the patterns of time-variations to be the same across-country pairs. Similarly to the specification of σ^2_{t+1}, we assume that:

$$\theta^2_{t+1} = \theta^2(1 - \phi_\theta) + \phi_\theta \theta^2_t + \varphi_\varpi \varpi_{t+1}, \tag{4}$$

$$\psi^2_{t+1} = \psi^2(1 - \phi_\psi) + \phi_\psi \psi^2_t + \varphi_\eta \eta_{t+1}, \tag{5}$$

where $|\phi_\theta|, |\phi_\psi| < 1$, $\theta^2, \psi^2 > 0$, and φ_ϖ, and φ_η are bounded constants, implying the following unconditional moments $E(\sigma^2_{t+1}) = \sigma^2$, $E(\psi^2_t) = \psi^2$ and $E(\theta^2_t) = \theta^2$. Also, without loss of generality, we set $\psi^2 = \theta^2 = 1$, which ensures that $E(\boldsymbol{\Sigma}_{t\varepsilon}) = \boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}} = (\sigma_{ij})$.

For tractability, we also assume that the $N + 4$ shocks, $\boldsymbol{\varepsilon}_{t+1} = (\varepsilon_{1,t+1}, \varepsilon_{2,t+1}, ..., \varepsilon_{N,t+1})'$, ζ_{t+1}, χ_{t+1}, ϖ_{t+1}, and η_{t+1} are (a) serially independent (over t), (b) distributed independently of each other, and (c) ζ_{t+1}, χ_{t+1}, ϖ_{t+1}, and η_{t+1} are $IIDN(0,1)$ while $\boldsymbol{\varepsilon}_{t+1} | \mathcal{F}_t \sim N(\mathbf{0}, \boldsymbol{\Sigma}_{t\varepsilon})$. For future analysis we also introduce here the $N \times 1$ vector of weights, $\mathbf{w} = (w_1, w_2, ..., w_N)'$ associated with the $N \times 1$ vector of country-specific shocks, $\boldsymbol{\varepsilon}_{t+1}$, and define the following 'aggregate' output growth shock as $\varepsilon_{w,t+1} = \sum_{i=1}^{N} w_i \varepsilon_{i,t+1} = \mathbf{w}' \boldsymbol{\varepsilon}_{t+1}$. The Gaussianity assumption is only used to obtain analytically tractable expressions for returns (safe and risky), and is not required for the econometric analysis.

2.2 Asset Markets, Risk-free Rate, and Equity Returns

It will be assumed that the representative agent of country i can trade freely a globally available risk-free bond with known gross return, $R_{f,t+1}$, and N risky country-specific equity claims, defined on the country-specific entire endowment stream $\{Y_{i,t+s}\}$ with gross returns $R_{i,t+1} = (P_{i,t+1} + Y_{i,t+1}) / P_{it}$, where P_{it} is the t-dated market price of such claim for $i = 1, 2, ...N$. We further assume constant relative risk aversion (CRRA) preferences and complete international asset markets in the Arrow-Debreau sense, so that country-specific consumption growth is equalized across countries, and one can use the world endowment growth as the pricing kernel or stochastic discount factor (SDF) of country i, M_{t+1}, given by:

$$M_{t+1} = \left[\left(\beta \frac{C_{i,t+1}}{C_{it}} \right)^{-\varrho} \right] = \left[\left(\beta \frac{Y_{w,t+1}}{Y_{wt}} \right)^{-\varrho} \right] = \exp\left(\ln \beta - \varrho \Delta \ln Y_{w,t+1} \right), \tag{6}$$

where β $(0 < \beta < 1)$ is the discount rate, ϱ denotes the coefficient of relative risk aversion, and $Y_{w,t+1}$ is world output, defined by $Y_{w,t+1} = \sum_{i=1}^{N} Y_{i,t+1}$. Using the weights, $\mathbf{w}$, and (1), the world growth rate can be written as:

$$\Delta \ln Y_{w,t+1} \approx \sum_{i=1}^{N} w_i \Delta y_{i,t+1} = a_w + \gamma_w f_{t+1} + \varepsilon_{w,t+1}, \tag{7}$$

where $a_w = \sum_{i=1}^{N} w_i a_i$, $\gamma_w = \sum_{i=1}^{N} w_i \gamma_i$, and $\varepsilon_{w,t+1} = \sum_{i=1}^{N} w_i \varepsilon_{i,t+1}$. Hence, in view of (6), we have, $M_{t+1} = \exp\left(\ln \beta - \varrho a_w - \varrho \gamma_w f_{t+1} - \varrho \varepsilon_{w,t+1} \right)$, which implies the innovation in the (log) SDF is given by:

$$m_{t+1} - E_t(m_{t+1}) = -\varrho \gamma_w \sigma_t \zeta_{t+1} - \varrho \varepsilon_{w,t+1}. \tag{8}$$

By arbitrage, the following Euler equation must hold for any (riskless or risky) asset with gross return R_{t+1}:

$$E_t \left(M_{t+1} R_{t+1} \right) = 1.$$

For the risk free rate, $r_{t+1}^{f} = 1/E_t \left(M_{t+1} \right)$, since by assumption f_t and $\varepsilon_{w,t+1}$ are Gaussian, it follows that:

$$r_{t+1}^{f} \approx -\ln \beta + \varrho a_w + \varrho \gamma_w \phi_f f_t - \frac{1}{2} \varrho^2 \gamma_w^2 \sigma_t^2 - \frac{1}{2} \varrho^2 Var_t(\varepsilon_{w,t+1}), \tag{9}$$

where $Var_t(\varepsilon_{w,t+1}) = \mathbf{w}' \boldsymbol{\Sigma}_{t\varepsilon} \mathbf{w}$.

The derivation of country-specific equity returns, defined as $r_{i,t+1} = ln(R_{i,t+1})$, are much more involved and the details are provided in the Appendix. It is shown that

$$\begin{aligned} r_{i,t+1} &= b_{0i,N} + b_{1i} f_t + b_{2i}\sigma_t^2 + b_{3i,N}\psi_t^2 + b_{4i,N}\theta_t^2 \\ &\quad + c_{1i}\sigma_t\zeta_{t+1} + c_{2i}\chi_{t+1} + c_{3i,N}\eta_{t+1} + c_{4i,N}\varpi_{t+1} + \varepsilon_{i,t+1}, \end{aligned} \tag{10}$$

where $\{b_{0i}, b_{1i}, b_{2i}, b_{3i,N}, b_{4i,N}\}$ and $\{c_{1i}, c_{2i}, c_{3i,N}, c_{4i,N}\}$ are depend on the underlying risk and volatility parameters, with some coefficients varying with N (see (A13)). Comparing equation (1) and (10), it is clear that to explain the cross-section of equity returns requires at least one more common factor than the cross-section of output growth rates, even if ε_{it} were conditionally homoskedastic. Note also that, in our model, the price of global growth risk, $\varrho\gamma_w$, depends on the average exposure of countries to the global factor, γ_w, and not only on the degree of risk aversion, ϱ. Finally, it is of interest to note that, in our multi-country setting, equity returns are exposed to a second aggregate risk factor, $\varepsilon_{w,t+1}$ in equation (8), stemming from time-variations in cross-country correlations of idiosyncratic components ε_{it}, captured by θ_t^2 and the innovations to θ_{t+1}, namely ϖ_{t+1}, under the endowment and volatility assumptions above. As we shall see below, this additional risk factor and its effects vanish only when N is sufficiently large and country-specific shocks are weakly cross correlated, which is what we assume in our econometric model to identify ζ_t from the data.

More formally, suppose that $\mathbf{\Sigma}_{t\varepsilon}$ is such that $\sup_i \sigma_{ii} < C_0$ and $\sup_i \sum_{j=1}^N |\sigma_{ij}| < C_0$, and the weights $w_i > 0$, for $i = 1, 2, ..., N$ are granular, in the sense that they are of order N^{-1} such that $\mathbf{w}'\mathbf{w} = \sum_{i=1}^N w_i^2 = O(N^{-1})$. The first assumption is well understood in finance and allows the country-specific output shocks to be weakly cross-correlated, similar to the arbitrage asset pricing model of Ross (1976) and the approximate factor model of Chamberlain and Rothschild (1982), where it is assumed that the idiosyncratic component of asset returns is weakly correlated. Under this assumption, country-specific risk is fully diversified and will not be priced. The granularity condition rules out the presence of one or more dominant country sufficiently large that shocks to their outputs affect all other countries in the world. This condition could be relaxed by conditioning the analysis on the dominant unit(s), even though in our empirical analysis, as we shall argue, it will not be necessary to do so.

To investigate the implications of these two assumptions, we first note that

$$\begin{aligned} \mathbf{w}'\mathbf{\Sigma}_{t\varepsilon}\mathbf{w} &= \theta_t^2 \sum_{j=1}^N w_j^2 \sigma_{jj} + \psi_t^2 \sum_{j \neq i}^N w_j w_i \sigma_{ij} \\ &\leq \theta_t^2 \left(\sum_{j=1}^N w_j^2 \right) sup_i(\sigma_{ii}) + \psi_t^2 \left(\sum_{j=1}^N w_j^2 \right) \sup_i \sum_{j=1}^N |\sigma_{ij}| = O(N^{-1}). \end{aligned}$$

Hence, as we show in the Appendix, $A_{3i,N} = O\left(N^{-1}\right)$, $A_{4i,N} = \frac{1}{2}\left(\frac{\sigma_{ii}}{1-\kappa_{1i}\phi_\theta}\right) + O\left(N^{-1}\right)$, , and as a result the risk free rate simplifies to

$$r_{t+1}^f \approx -\ln\beta + \varrho a_w + \varrho\gamma_w\phi_f \; f_t - \frac{1}{2}\varrho^2\gamma_w^2\sigma_t^2 + O\left(N^{-1}\right), \tag{11}$$

which, as $N \to \infty$, converges to the standard results obtained in the literature for a single economy.[4] Similarly, the equity returns no longer depend on η_{t+1} and ψ_t^2 as $N \to \infty$ because $b_{3i,N} = O\left(N^{-1}\right)$, and the risk of time-variations in cross-country correlations is no longer priced, as they vanish with N, under the the assumptions of weak cross-country correlations and non-dominance.

To see this from a different perspective, consider the equity risk premium. From (9) and (10), it is easy, albeit tedious, to derive the risk premium for country $i = 1, 2, ..., N$ as (See Appendix):

$$E_t\left(r_{i,t+1} - r_{t+1}^f\right) = \varrho\gamma_w c_{1i}\sigma_t^2 - \frac{1}{2}c_{1i}^2\sigma_t^2 - \frac{1}{2}\sigma_{ii}\theta_t^2 + \varrho Cov_t\left(\varepsilon_{i,t+1}, \varepsilon_{w,t+1}\right) - \frac{1}{2}\kappa_{1i}^2 G_{iN}^2, \tag{12}$$

where, consistent with equation (10), $c_{1i} = \gamma_i + \kappa_{1i}A_{1i} = Cov_t(r_{i,t+1}, f_{t+1})/\sigma_t^2$ is the i^{th} equity exposure to σ_t^2, as defined in equation (11) of Bansal and Yaron (2004), for the special case of CRRA preferences, and $G_{iN}^2 = \varphi_\chi^2 A_{2i}^2 + \varphi_\eta^2 A_{3i,N}^2 + \varphi_\varpi^2 A_{4i,N}^2$. Once again, when N is sufficiently large, cross-country errors are weakly correlated, and in the absence of a dominant economy the risk premium simplifies to

$$E_t\left(r_{i,t+1} - r_{t+1}^f\right) = \varrho\gamma_w c_{1i}\sigma_t^2 - \frac{1}{2}c_{1i}^2\sigma_t^2 - \frac{1}{2}\sigma_{ii}\theta_t^2 - \frac{1}{2}\kappa_{1i}^2 G_i^2 + O(N^{-1}), \tag{13}$$

where $G_i^2 = \varphi_\chi^2 A_{2i}^2 + \frac{1}{2}\varphi_\varpi^2\sigma_{ii}/\left(1 - \kappa_{1i}\phi_\theta\right)$, and give the following result for the steady state (unconditional) risk premia:

$$E\left(r_{i,t+1} - r_{t+1}^f\right) = \varrho\gamma_w c_{1i}\sigma^2 - \frac{1}{2}c_{1i}^2\sigma^2 - \frac{1}{2}\sigma_{ii} - \frac{1}{2}\kappa_{1i}^2 G_i^2 + O(N^{-1}). \tag{14}$$

The assumption of weakly cross correlated country-specific shocks will play a crucial role in our econometric identification strategy to be discussed below. In the absence of such an assumption, country-specific output growths will be subject to another strong common factor, $\varepsilon_{w,t+1}$ in equation (8) that would be priced, besides the global growth factor f_t, and it will not be possible to identify f_t or its innovation, ζ_t, from world output growth as we propose to do in this paper.

2.3 Realized Volatility

In our empirical application, we consider the realized volatility of equity returns computed from squares of daily returns within a quarter (to match the available data on output growth). To link the above theoretical results to our empirical analysis, denote equity returns for a given day τ within a quarter t by $r_{it}(\tau)$, for $\tau = 1, 2, ..., D_t$, where D_t is the number of trading days within quarter t. The quarterly realized volatility associated to r_{it} is then given by:

$$RV_{it}^2 = \sum_{\tau=1}^{D_t}\left[r_{it}(\tau) - \bar{r}_{it}\right]^2, \tag{15}$$

where $\bar{r}_{it} = D_t^{-1}\sum_{\tau=1}^{D_t} r_{it}(\tau)$. By using the expression for (10) when N is large in (15), it is possible to derive the expression for RV_{it}^2 that depends on the first-moment innovation (ζ_t), second-moment

[4] An example is Bansal and Yaron (2004) in the special case of CRRA preferences.

innovations (χ_t and ϖ_t), as well as their many cross-products due to the square in (15). So, unlike country-specific returns (which depend linearly on ζ_t, χ_t, and ϖ_t), RV_{it}^2 is a non-linear function of all these innovations, and their impacts cannot be traced separately. Nonetheless, and crucially for our empirical analysis, as in the case of the equity returns, it continues to be the case that explaining the cross-section of realized volatilities requires at least one more common factor than the cross-section of output growth rates even if ε_{it} were conditionally homoskedastic.[5]

In the next section, therefore, we will use the model to interpret the difference in the degree of cross sectional dependence of the country output growth rates and realized volatilities documented in the introduction, after controlling for the effects of the common growth factor shock, ζ_t. All other common factors in (15) will be combined in a single common (or global) financial shock. This can also include any additional factors that may influence realized equity market volatilities, such as market imperfections, bubble effects, or time-varying risk preferences. In effect, our identification assumptions will distinguish between a first, level factor, ζ_t, common to both the growth and volatility series, and all other effects common only to the volatility series, lumped together in a second common financial factor.

3 A Multi-Country Econometric Framework

We now set up a factor-augmented, multi-country model in which country-specific output growth and realized stock market volatility can be driven by 2 common and $2 \times N$ country-specific shocks. We first discuss the identification of the common shocks in the context of a relatively simple static specification, and then consider model identification and estimation in the context of a more general dynamic setting. Identification of the country-specific shocks, which plays only an auxiliary role in our empirical analysis, will be addressed following conventional approaches.

Consider, without loss of generality, the following first-order panel vector autoregressive (PVAR) model in v_{it} and Δy_{it}, for $i = 1, 2, ..., N$ and $t = 1, 2, ..., T$, typically used in empirical work on volatility and the business cycle at the individual country level (assuming $N = 1$):[6]

$$v_{it} = a_{iv} + \phi_{i,11} v_{i,t-1} + \phi_{i,12} \Delta y_{i,t-1} + e_{iv,t}, \tag{16}$$

$$\Delta y_{it} = a_{iy} + \phi_{i,21} v_{i,t-1} + \phi_{i,22} \Delta y_{i,t-1} + e_{iy,t}. \tag{17}$$

where as before output growth (Δy_{it}) is measured as the log-difference of real GDP, $v_{it} = \ln(RV_{it})$ is the log of realized stock market volatility for country i during quarter t, $e_{iv,t}$ and $e_{iy,t}$ are country-specific reduced-form innovations assumed to be serially uncorrelated. A *single-country* approach to the identification of structural volatility and business cycle shocks in (16)-(17) must impose at least one *a priori* restriction on the covariance matrix of $e_{iv,t}$ and $e_{iy,t}$ or their long-run counterpart. Consistent with the theoretical model presented in the previous section, in this paper we posit the

[5]See Theorem 2 of Renault et al. (2018) for a general formulation of the APT approach applied to squared excess returns as payoffs on traded assets.

[6]The analysis in this section applies to alternative business cycle indicators or other measures of volatility.

following unobservable common-factor representation for the reduced-form PVAR innovations:

$$e_{iv,t} = \lambda_i \zeta_t + \theta_i \xi_t + \eta_{it}, \tag{18}$$

$$e_{iy,t} = \gamma_i \zeta_t + \varepsilon_{it}, \tag{19}$$

where ζ_t and ξ_t are two common shocks, while η_{it} and ε_{it} are two country-specific shocks. Note that, by assumption, these shocks (or innovations) are serially uncorrelated. However, for the purpose of identifying the common shocks, they could be correlated both within each country and across countries.

Equations (16)-(19), taken together, specify a factor-augmented multi-country PVAR model that captures the main features of our theoretical model and could be formally shown to approximate its solution.[7] The model, in particular, posits that the volatility equations include at least one more common shock, ξ_t, than the output equations, capturing all common components not accounted for by ζ_t. The common shock ζ_t in (19) represents the same innovation as in the theoretical model, and therefore will continue to be labelled "global growth shock". The second common shock, ξ_t, instead, can be seen as a linear combination of a number of shocks that could reflect second and higher-order moment innovations (such as χ_{t+1} η_{t+1}, and ϖ_{t+1} introduced in the theoretical model and their squares and cross products) as well as changes in non-fundamental aspects of financial markets, such as over-reactions to news due to excessive optimism/pessimism or bubble components, ruled out by the theoretical model. For this reason, we refer to ξ_t as the "global financial shock" or "financial shock" for short. Similarly, while ε_{it} has a more direct mapping into the theoretical model, η_{it} is an all-encompassing country-specific financial shock, broadly defined.

The main idea of the paper is to achieve identification of ζ_t and ξ_t and their loadings, λ_i, γ_i, and θ_i (up to orthonormal transformations) by placing restrictions on the *cross-country* correlations of ε_{it} and η_{it}, while leaving their *within-country* correlation unrestricted. This is a problem that, obviously, cannot be addressed in a single-country framework, or in a model of the world economy viewed as a single entity. In a single-country model, the common shocks in (16)-(19) cannot be identified even if it is assumed that the idiosyncratic shocks η_{it} and ε_{it} are uncorrelated, or by adding more country-specific variables to the model. By adopting a multi-country perspective, we can pose this common factor identification problem and solve it. As an example, consider (16)-(19) with $N = 1$, and take this country to refer to the US economy. It is readily seen that the triangular factor representation in (18) and (19) does not impose any restriction on the observed covariance matrix of $(\Delta y_{US,t}, v_{US,t})$.

3.1 Identification of the Common Shocks in a Static Setting

To focus on our approach to the identification of ζ_t and ξ_t, let us drop deterministic components and lagged endogenous variables. Denote world GDP growth and world volatility by $\Delta \bar{y}_{\omega,t}$ and $\bar{v}_{\omega,t}$, respectively, and suppose that they are measured by the weighted cross-section averages of

[7] While other theoretical models are consistent with such a specification, one could also consider models for which the above triangular factor representation might not hold. As we shall see below, there is strong empirical support for the specification adopted in (16)-(19).

country-specific volatility and growth measures, namely:

$$\Delta\bar{y}_{\omega,t} = \sum_{i=1}^{N} w_i \Delta y_{it}, \qquad \text{and} \qquad \bar{v}_{\omega,t} = \sum_{i=1}^{N} \mathring{w}_i v_{it}, \tag{20}$$

where $\mathbf{w} = (w_1, w_2, ..., w_N)'$ and $\mathring{\mathbf{w}} = (\mathring{w}_1, \mathring{w}_2, ..., \mathring{w}_N)'$ are $N \times 1$ vectors of aggregation weights, which can be the same.[8] We make the following assumptions on the common shocks, ζ_t and ξ_t, their loadings, λ_i, γ_i, and θ_i, the weights, $\mathring{w}_i$ and w_i, and the country-specific innovations, ε_{it} and η_{it}:

Assumption 1 *(Common shocks and their loadings) The common unobservable shocks ζ_t and ξ_t have zero means and finite variances, and are serially uncorrelated. The loadings, λ_i, γ_i, and θ_i, are distributed independently across i and from the common shocks ζ_t and ξ_t for all i and t, with non-zero means λ, γ, and θ ($\lambda \neq 0$, $\gamma \neq 0$, and $\theta \neq 0$), and satisfy the following conditions:*

$$\lambda = \sum_{i=1}^{N} \mathring{w}_i \lambda_i \neq 0, \quad \gamma = \sum_{i=1}^{N} w_i \gamma_i \neq 0 \quad \textit{and} \quad \theta = \sum_{i=1}^{N} w_i \theta_i \neq 0, \tag{21}$$

$$\sum_{i=1}^{N} \lambda_i^2 = \mathcal{O}(N), \quad \sum_{i=1}^{N} \gamma_i^2 = \mathcal{O}(N), \quad \textit{and} \quad \sum_{i=1}^{N} \theta_i^2 = \mathcal{O}(N). \tag{22}$$

Assumption 2 *(Aggregation weights) The weights, w_i and $\mathring{w}_i$, for $i = 1, 2, ..., N$ are fixed non-zero constants such that $\sum_{i=1}^{N} w_i = 1$ and $\sum_{i=1}^{N} \mathring{w}_i = 1$, and satisfy the following "granularity" conditions:*

$$||\mathbf{w}|| = O(N^{-1}), \ \frac{w_i}{||\mathbf{w}||} = O(N^{-1/2}), \tag{23}$$

and

$$||\mathring{\mathbf{w}}|| = O(N^{-1}), \ \frac{\mathring{w}_i}{||\mathring{\mathbf{w}}||} = O(N^{-1/2}). \tag{24}$$

Assumption 3 *(Cross-country correlations) (a) The country-specific shocks, η_{it} and ε_{it}, have zero means and finite variances, and are serially uncorrelated, but can be correlated with each other both within and between countries. (b) Denoting the covariance matrices of the $N \times 1$ vectors $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \varepsilon_{2t}, ..., \varepsilon_{Nt})'$ and $\boldsymbol{\eta}_t = (\eta_{1t}, \eta_{2t}, ..., \eta_{Nt})'$ by $\mathbf{\Sigma}_{\varepsilon\varepsilon} = Var(\boldsymbol{\varepsilon}_t)$ and $\mathbf{\Sigma}_{\eta\eta} = Var(\boldsymbol{\eta}_t)$, respectively, we have:*

$$\varrho_{\max}(\mathbf{\Sigma}_{\varepsilon\varepsilon}) = O(1) \quad \textit{and} \quad \varrho_{\max}(\mathbf{\Sigma}_{\eta\eta}) = O(1). \tag{25}$$

Assumption 1 is standard in the factor literature (see, for instance, Assumption B in Bai and Ng (2002)). It ensures that ζ_t and ξ_t are strong (or pervasive) for both volatility and growth so that they can be estimated consistently either using principal components or by cross-section averages of country-specific observations (see Chudik et al. (2011)).[9] Assumption 2 requires that individual

[8] As we noted earlier, in practice, the weights in $\mathbf{w}$ and $\mathring{\mathbf{w}}$ need not be fixed and could be time-varying, but they must be predetermined.

[9] In the present context, the use of the cross-section-average (CSA) estimator of f_t has two advantages. First, it can be directly interpreted as world growth. Second, under Assumptions 1 and 3 the CSA estimator of f_t is consistent so long as N is large, whilst the principal component estimator requires both N and T to be large (See section 19.5.1 of

countries' contribution to world growth or world volatility is of order $1/N$. This is consistent with the notion that, since the 1990s, when our sample period starts, world growth and world capital markets have become progressively more diversified and integrated as a result of the globalization process. Part (a) of Assumption 3 is also standard and leaves the causal relation between the idiosyncratic components, ε_{it} and η_{it}, unrestricted. In our model, the correlation between ε_{it} and η_{it} captures any contemporaneous causal relation between volatility and growth at the country level, conditional on ζ_t and ξ_t, on which we do not impose any restrictions for the purpose of identifying the common shocks. The second part of Assumption 3 instead is the source of identification in our model and has been discussed at length in the theoretical model. It imposes that the country-specific shocks, ε_{it} and η_{it} are weakly cross correlated, which is in line with the approximate factor model of Chamberlain and Rothschild (1982), and ensures that country-specific shocks can be treated as idiosyncratic for asset pricing purposes.

Under Assumptions 1-3, for N sufficiently large, ζ_t can be identified (up to a scalar constant) by $\bar{y}_{\omega,t} = \sum_{i=1}^{N} w_i \Delta y_{it}$ as:

$$\zeta_t = \gamma^{-1} \Delta \bar{y}_{\omega,t} + O_p\left(N^{-1/2}\right), \tag{26}$$

This result follows noting that, using the definitions in (20), and dropping intercepts and dynamics, the following model for world GDP growth and volatility obtains:

$$\begin{aligned} \Delta \bar{y}_{\omega,t} &= \gamma \zeta_t + \bar{\varepsilon}_{\omega,t}, & (27)\\ \bar{v}_{\omega,t} &= \lambda \zeta_t + \theta \xi_t + \bar{\eta}_{\omega,t}, & (28) \end{aligned}$$

where $\bar{\varepsilon}_{\omega,t} = \mathbf{w}'\boldsymbol{\varepsilon}_t$, and $\bar{\eta}_{\omega,t} = \mathring{\mathbf{w}}'\boldsymbol{\eta}_t$. Furthermore, $Var\left(\bar{\varepsilon}_{\omega,t}\right) = \mathbf{w}'\boldsymbol{\Sigma}_{\varepsilon\varepsilon}\mathbf{w} \leq \left(\mathbf{w}'\mathbf{w}\right)\varrho_{\max}\left(\boldsymbol{\Sigma}_{\varepsilon\varepsilon}\right)$. Thus, under Assumptions 2 and 3, we have $Var\left(\bar{\varepsilon}_{\omega,t}\right) = O\left(\mathbf{w}'\mathbf{w}\right) = O\left(N^{-1}\right)$, and hence $\bar{\varepsilon}_{\omega,t} = O_p\left(N^{-1/2}\right)$, which allows us to recover ζ_t form $\Delta\bar{y}_{\omega,t}$ up to the scalar $1/\gamma$. Note that since volatility has at least one more factor than growth in the theoretical model, this model property can justify the econometric representation. But on its own it does not give us identification of the world growth factor. To get identification we need weak cross-country correlation and large N. This is because with small N we would not be able to disentangle ζ_t from $\bar{\varepsilon}_{\omega,t}$ (the average of ε_{it} in the equation for output growth above).

The first main empirical result of the paper does not require explicit identification of the second common shock (ξ_t) assumed to be exclusive to the volatility series, v_{it}. But doing so permits exploring other properties of the data that underpin the second and the third main empirical results of the paper summarized in the Introduction. Under our assumptions, ξ_t can be identified from the data as a linear combination of $\Delta\bar{y}_{\omega,t}$ and $\bar{v}_{\omega,t}$, up to an orthonormal transformation (as $N \to \infty$), given by:

$$\xi_t = \theta^{-1}\left(\bar{v}_{\omega,t} - \frac{\lambda}{\gamma}\Delta\bar{y}_{\omega,t}\right) + O_p\left(N^{-1/2}\right). \tag{29}$$

This result follows immediately from substituting (26) into (28) and applying the same reasoning as

Pesaran (2015)). A comparison of CSA and PC estimates of the common factors in the present static setting can be found in Cesa-Bianchi et al. (2018). But it is important to note that such a simple comparison is not possible once we allow for dynamics and heterogeneity in the country-specific VAR models to be discussed below in Section 3.2.

before.

3.2 Identification of the Common Shocks in a Dynamic Setting

Identifying the common shocks becomes considerably more complex in the heterogeneous PVAR specification given by (16) and (17). Let us first consider the much simpler case of a homogeneous model in which $\phi_{i,rs} = \phi_{rs}$ for all i and $r, s = 1, 2$. To simplify the exposition we continue to abstract from the intercepts. In this case the common shocks can be identified as before by averaging the country-specific VARs to obtain:

$$\begin{aligned}
\zeta_t &= \gamma^{-1}\left(\Delta\bar{y}_{\omega,t} - \phi_{21}\bar{v}_{\omega,t-1} - \phi_{22}\Delta\bar{y}_{\omega,t-1}\right) + O_p\left(N^{-1/2}\right), \\
\xi_t &= \frac{\left(\bar{v}_{\omega,t} - \frac{\lambda}{\gamma}\Delta\bar{y}_{\omega,t}\right)}{\theta} - \frac{\left(\phi_{11} - \frac{\lambda}{\gamma}\phi_{21}\right)\bar{v}_{\omega,t-1}}{\theta} - \frac{\left(\phi_{12} - \frac{\lambda}{\gamma}\phi_{22}\right)\Delta\bar{y}_{\omega,t-1}}{\theta} + O_p\left(N^{-1/2}\right),
\end{aligned}$$

which are obvious generalizations of (26) and (29), respectively. Allowing for heterogeneous dynamics, as needed for empirical implementation, presents a much bigger challenge because it could involve long memory processes as shown by Granger (1980).[10] Therefore, to tackle the general heterogeneous case requires stronger assumptions on PVAR coefficients, $\phi_{i,rs}$, and involves higher order lags of $(\bar{v}_{\omega,t}, \Delta\bar{y}_{\omega,t})$.

Specifically, consider the matrix version of equation (16) and (19):

$$\mathbf{z}_{it} = \mathbf{a}_i + \mathbf{\Phi}_i\mathbf{z}_{i,t-1} + \mathbf{\Gamma}_i\boldsymbol{\delta}_t + \boldsymbol{\vartheta}_{it}, \quad \text{for } i = 1, 2, ..., N;\ t = 1, 2, ..., T, \tag{30}$$

where $\mathbf{z}_{it} = (v_{it}, \Delta y_{it})'$ with

$$\mathbf{a}_i = \begin{pmatrix} a_{iv} \\ a_{iy} \end{pmatrix},\ \mathbf{\Phi}_i = \begin{pmatrix} \phi_{i,11} & \phi_{i,12} \\ \phi_{i,21} & \phi_{i,22} \end{pmatrix},\ \mathbf{\Gamma}_i = \begin{pmatrix} \lambda_i & \theta_i \\ \gamma_i & 0 \end{pmatrix},\ \boldsymbol{\delta}_t = \begin{pmatrix} \zeta_t \\ \xi_t \end{pmatrix},\ \boldsymbol{\vartheta}_{it} = \begin{pmatrix} \eta_{it} \\ \varepsilon_{it} \end{pmatrix},$$

and consider the following assumption that places additional restrictions on the PVAR coefficients and the factor loadings:

Assumption 4 *(Coefficients) The constants $\mathbf{a}_i$ are bounded, $\mathbf{\Phi}_i$ and $\mathbf{\Gamma}_i$ are independently distributed for all i, the support of $\varrho(\mathbf{\Phi}_i)$ lies strictly inside the unit circle, for $i = 1, 2, ..., N$, and the inverse of the polynomial $\mathbf{\Lambda}(L) = \sum_{\ell=0}^{\infty}\mathbf{\Lambda}_\ell L^\ell$, where $\mathbf{\Lambda}_\ell = \mathbb{E}\left(\mathbf{\Phi}_i^\ell\right)$ exists and has exponentially decaying coefficients, namely $\|\mathbf{\Lambda}_\ell\| \leq C_0\rho^\ell$, with $0 < \rho < 1$.*

We continue to maintain the earlier assumptions, 1, 2 and 3 which ensure the country-specific errors are weakly correlated and

$$\mathbf{\Gamma} = \mathbb{E}\left(\mathbf{\Gamma}_i\right) = \begin{pmatrix} \lambda & \theta \\ \gamma & 0 \end{pmatrix}, \tag{31}$$

where $\gamma\theta \neq 0$ which also ensures that $\mathbf{\Gamma}$ is invertible. The important additional condition in Assumption 4 is to control the effects of aggregation of dynamics across heterogenous units by requiring that

[10] For a review and more recent developments see, for example, Chapter 32 in Pesaran (2015).

$\Lambda_\ell = E\left(\mathbf{\Phi}_i^\ell\right)$ exists and has exponentially decaying coefficients. But it is easily seen that this latter condition holds if it is further assumed that $\sup_i E\left\|\mathbf{\Phi}_i\right\| < \rho < 1$. The latter ensures that the time series processes for the aggregates, $\Delta\bar{y}_{\omega,t}$ and $\bar{v}_{\omega,t}$, can be suitably truncated for empirical analyses and are devoid of long memory components.

The following proposition addresses the identification of the common shocks in the general dynamic heterogeneous setting.

Proposition 1 *(Identification of common shocks in the heterogeneous factor-augmented PVAR model) Consider the models given by (30) for country $i = 1, 2, ..., N$, and suppose that Assumptions 1, 2, 3 and 4 hold. Then:*

$$\zeta_t = b_\zeta + \gamma^{-1}\Delta\bar{y}_{\omega,t} + \sum_{\ell=1}^{\infty}\mathbf{c}'_{1,\ell}\bar{\mathbf{z}}_{\omega,t-\ell} + O_p\left(N^{-1/2}\right), \tag{32}$$

$$\xi_t = b_\xi + \theta^{-1}\left(\bar{v}_{\omega,t} - \frac{\lambda}{\gamma}\Delta\bar{y}_{\omega,t}\right) + \sum_{\ell=1}^{\infty}\mathbf{c}'_{2,\ell}\bar{\mathbf{z}}_{\omega,t-\ell} + O_p\left(N^{-1/2}\right), \tag{33}$$

where b_ζ and b_ξ are fixed constants, $\bar{\mathbf{z}}_{\omega,t} = (\bar{v}_{\omega,t}, \Delta\bar{y}_{\omega,t})$, $\{w_i,\ \text{for } i = 1, 2, ..., N\}$ are fixed weights that satisfy the granularity Assumption 2, $\mathbf{c}'_{1,\ell}$ and $\mathbf{c}'_{2,\ell}$ are the first and the second rows of $\mathbf{C}_\ell = \mathbf{\Gamma}^{-1}\mathbf{B}_\ell$, where $\mathbf{\Gamma} = \mathbb{E}\left(\mathbf{\Gamma}_i\right)$, $\mathbf{B}_\ell$ is defined by $\mathbf{\Lambda}^{-1}(L) = \mathbf{B}_0 + \mathbf{B}_1 L + \mathbf{B}_2 L^2 + ...$, $\mathbf{\Lambda}(L) = \sum_{\ell=0}^{\infty}\mathbf{\Lambda}_\ell L^\ell$, and $\mathbf{\Lambda}_\ell = \mathbb{E}\left(\mathbf{\Phi}_i^\ell\right)$, for all i.

Proof. See Appendix B. ■

Expressions (32) and (33) augment the corresponding results (30) and (30) obtained for the homogeneous case with higher order lags of $(\bar{v}_{\omega,t-\ell}, \Delta\bar{y}_{\omega,t-\ell})$, for $\ell > 1$, to take account of dynamic heterogeneity on the identification of common (or aggregate) shocks.

3.3 Consistent Estimation of the Factor-augmented, Heterogeneous PVAR Model

As they stand, the expressions given in Proposition 1 for ζ_t and ξ_t are formulated in terms of the observables $\{\bar{\mathbf{z}}_{\omega,t-\ell}, \text{ for } \ell \geq 0\}$, and cannot be used in empirical analysis as they depend on infinite order lags. But, as shown in Pesaran and Chudik (2014) and Chudik and Pesaran (2015), if slope heterogeneity is not extreme (i.e. if the coefficient matrices $\mathbf{\Phi}_i$ do not differ too much across i) and $\mathbf{C}_\ell$ decays exponentially in ℓ, the infinite order distributed lag functions in $\bar{\mathbf{z}}_{\omega,t}$ can be truncated. In practice, Pesaran and Chudik (2014) and Chudik and Pesaran (2015) recommend a lag length ℓ equal to $T^{1/3}$, where T is the time dimension of the panel. So considering a truncated approximation of the unobservable factors in equation (32) and (33) we can derive observable proxies for ζ_t and ξ_t, making sure that the resultant estimators are orthogonal to each other and with unit variance.[11] As the following proposition illustrates, the latter is achieved simply by choosing coefficients in the linear regression of $\bar{v}_{\omega,t}$ on $\Delta\bar{y}_{\omega,t}$ such that the observable proxy for the common shocks have zero-means and unit variances.

[11] Note that ζ_t and ξ_t can be identified only up to a non-singular transformation which we take to be orthonormal, as it simplifies the computation and interpretation of impulse responses and error variance decompositions that we conduct later on in the paper.

Proposition 2 *(Consistent estimation of observable orthonormalized common shocks in the heterogeneous factor-augmented PVAR model) Consider the p^{th} order truncated approximation of the unobservable factors in equation (32) and (33) above, and note that in matrix notations we have:*

$$\boldsymbol{\zeta} = \Delta\bar{\mathbf{y}}_\omega + \bar{\mathbf{Z}}_\omega \mathbf{C}_1 + O_p\left(N^{-1/2}\right), \tag{34}$$

$$\boldsymbol{\xi} = \bar{\mathbf{v}}_\omega - \lambda\Delta\bar{\mathbf{y}}_\omega + \bar{\mathbf{Z}}_\omega \mathbf{C}_2 + O_p\left(N^{-1/2}\right), \tag{35}$$

where $\boldsymbol{\zeta} = (\zeta_1, \zeta_2, ..., \zeta_T)'$, $\boldsymbol{\xi} = (\xi_1, \xi_2, ..., \xi_T)'$, $\bar{\mathbf{Z}}_\omega = (\boldsymbol{\tau}_T, \bar{\mathbf{z}}_{\omega,-1}, \bar{\mathbf{z}}_{\omega,-2}, ..., \bar{\mathbf{z}}_{\omega,-p})$, $\bar{\mathbf{z}}_{\omega,-l} = (\Delta\bar{\mathbf{y}}_{\omega,-l}\bar{v}_{\omega,-l})$, $\Delta\bar{\mathbf{y}}_{\omega,-l} = (\Delta\bar{y}_{\omega,1-l}, \Delta\bar{y}_{\omega,2-l}, ..., \Delta\bar{y}_{\omega,T-l})'$, $\Delta\bar{\mathbf{y}}_\omega = \Delta\bar{\mathbf{y}}_{\omega,0}$, $\bar{\mathbf{v}}_{\omega,-l} = (\bar{v}_{\omega,1-l}, \bar{v}_{\omega,2-l}, ..., \bar{v}_{\omega,T-l})'$, $\bar{\mathbf{v}}_\omega = \bar{\mathbf{v}}_{\omega,0}$, *and p denotes a suitable number of lags (or truncation order). Consistent estimators of the common shocks, denoted by $\hat{\boldsymbol{\zeta}}$ and $\hat{\boldsymbol{\xi}}$, can be obtained as residuals from the following OLS regressions:*

$$\hat{\boldsymbol{\zeta}} = \Delta\bar{\mathbf{y}}_\omega - \bar{\mathbf{Z}}_\omega \hat{\mathbf{C}}_1, \tag{36}$$

$$\hat{\boldsymbol{\xi}} = \bar{\mathbf{v}}_\omega - \hat{\lambda}\hat{\boldsymbol{\zeta}} - \bar{\mathbf{Z}}_\omega \hat{\mathbf{C}}_2, \tag{37}$$

where $\hat{\mathbf{C}}_1$ is the OLS estimator of the coefficients in the regression of $\Delta\bar{\mathbf{y}}_\omega$ on $\bar{\mathbf{Z}}_\omega$, and $\hat{\lambda}$ and $\hat{\mathbf{C}}_2$ are OLS estimators of the coefficients in the regression of $\bar{\mathbf{v}}_\omega$ on $\hat{\boldsymbol{\zeta}}$ and $\bar{\mathbf{Z}}_\omega$. The estimated shocks have zero means and are uncorrelated, namely $\boldsymbol{\tau}_T'\hat{\boldsymbol{\zeta}} = \boldsymbol{\tau}_T'\hat{\boldsymbol{\xi}} = \hat{\boldsymbol{\zeta}}'\hat{\boldsymbol{\xi}} = 0$.

Proof. See Appendix B. ■

Since $\hat{\zeta}_t$ and $\hat{\xi}_t$ are the residuals from regressions of $\Delta\bar{y}_{\omega,t}$ and $\bar{v}_{\omega,t}$ on an intercept and the lagged values $\bar{z}_{\omega,t-1}, ..., \bar{z}_{\omega,t-p}$, it follows that $\hat{\zeta}_t$ and $\hat{\xi}_t$ will have zero (in-sample) means and, for a sufficiently large value of p, will be serially uncorrelated. Therefore, $\hat{\zeta}_t$ and $\hat{\xi}_t$ can be viewed as estimators of the global shocks to the underlying factors, ζ_t and ξ_t. Note also that, in a dynamic setting, the orthogonalized components of $\Delta\bar{y}_{\omega,t}$ and $\bar{v}_{\omega,t}$, obtained by simply projecting $\bar{v}_{\omega,t}$ on $\Delta\bar{y}_{\omega,t}$, are not the same as our global shocks $\hat{\zeta}_t$ and $\hat{\xi}_t$, because this would ignore the contributions of $\bar{\mathbf{z}}_{\omega,t-\ell}$ for $\ell = 1, 2, ..., p$ to the estimation of ζ_t and ξ_t. As the common shocks depend on lagged variables, it is important to make sure that the past values of $\bar{\mathbf{z}}_{\omega,t}$ are filtered out.

The contemporaneous effects of the common shocks can now be estimated by substituting in (30) the orthogonal factor innovations, $\hat{\zeta}_t$ and $\hat{\xi}_t$, obtained from equations (36) and (37). We can then investigate their dynamic impact and relative importance for country-specific volatility and growth based on the following regressions:

$$v_{it} = a_{iv} + \phi_{i,11} v_{i,t-1} + \phi_{i,12}\Delta y_{i,t-1} + \sum_{\ell=1}^{p} \mathbf{d}'_{v,i\ell}\bar{\mathbf{z}}_{\omega,t-\ell} + \beta_{i,11}\hat{\zeta}_t + \beta_{i,12}\hat{\xi}_t + \eta_{it}, \tag{38}$$

$$\Delta y_{it} = a_{iy} + \phi_{i,21} v_{i,t-1} + \phi_{i,22}\Delta y_{i,t-1} + \sum_{\ell=1}^{p} \mathbf{d}'_{\Delta y,i\ell}\bar{\mathbf{z}}_{\omega,t-\ell} + \beta_{i,21}\hat{\zeta}_t + \varepsilon_{it}. \tag{39}$$

These country-specific equations can be estimated consistently by least squares so long as N and T are sufficiently large. Large N is required so that the probability order $O_p(N^{-1/2})$ in equations (34) and (35) becomes negligible. Large T is required to ensure that the dynamics are estimated accurately. We are now ready to present our empirical results; but before doing so we need to discuss how we measure volatility in our multi-country setting.

4 Volatility Measurement, Data and Summary Statistics

To proxy for volatility, we use quarterly (log) realized volatility based on the summation of squared daily equity returns as defined by (15). This is a natural application of within-day measures of volatility based on high frequency within-day price changes. See, for example, Andersen et al. (2001, 2003), and Barndorff-Nielsen and Shephard (2002, 2004).

There are, of course, a number of other measures that can be used. For example, using a panel data of equity returns on firms or sectors *within* a given country, volatility in that country can be measured as the cross-sectional dispersion of equity returns. In section S.4 in the online supplement we show that, under fairly general assumptions, realized volatility is closely correlated with this alternative measure and, given that it can be more readily constructed across many countries, it is preferable in our application. In the finance literature, the focus of the volatility measurement has now shifted to implied volatility measures obtained from option prices, like the US VIX Index. However, a key input for the implementation of our identification strategy is the availability of country-specific measures for a large number of countries over a long period of time, and implied volatility measures are not yet available for a meaningful number of countries. Moreover, in our robustness analysis, we will show that we find even stronger results by using the VIX Index rather than our RV measure of realized volatility. The literature has also used measures based on the dispersion of expectations such as, for instance, the one proposed by Bachmann et al. (2013) in the case of the United States, and by Rossi and Sekhposyan (2015) and Ozturk and Sheng (2018) in the international context. While the data set of Rossi and Sekhposyan (2015) covers a large number of countries, the time series dimension is unbalanced and often not long enough for our purposes. Finally, model based measures, such as those in Jurado et al. (2015) and Ludvigson et al. (2015) could in principle be computed for all countries in our sample, but the data requirements to construct such proxies for many countries over a sufficiently long time period are prohibitive.

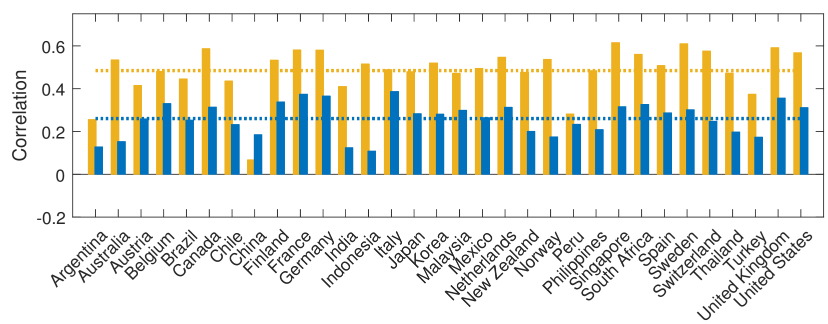
Data sources and their sampling information are reported in section S.1 in the online supplement. To construct a balanced panel for the largest number of countries for which we have sufficiently long time series, we first collect daily stock prices for 32 advanced and emerging economies from 1979 to 2016.[12] We then cut the beginning of the sample in 1993, as daily equity price data are not available earlier for two large emerging economies (Brazil and China) and for Peru. Better quality quarterly GDP data for China also became available from 1993. Our results are robust to excluding these three countries and/or starting the sample in 1988. Moreover, some steps of the empirical analysis, like the estimation of global shocks ($\hat{\zeta}_t$ and $\hat{\xi}_t$), can be easily implemented with the unbalanced panel from 1979 without any significant consequences for our main findings. In the online supplement to the paper, we report summary statistics, showing that realized volatility series are highly persistent

[12]Daily returns are computed abstracting from dividends, which are negligible by comparison to price changes at this frequency.

(more so than output growth series), but are nevertheless stationary, as required by our analysis.

The differential pattern of cross-country correlations of the growth and volatility *innovations* is crucial for our identification strategy. In order to gauge the extent to which volatility and growth series co-move across countries, one can use two techniques: standard principal component analysis and pair-wise correlation analysis across countries. The average pair-wise correlation of country i in the panel ($\bar{\rho}_i$) measures the average degree of co-movement of country i with all other countries j (i.e. for all $j \neq i$). The average pair-wise correlation *across* all countries, denoted by $\bar{\rho}_N$, is defined as the cross-country average of $\bar{\rho}_i$ over $i = 1, 2, ..., N$. This statistics relates to the degree of pervasiveness of the factors, as measured by the factor loadings. The attraction of the average pair-wise correlation, $\bar{\rho}_N$, lies in the fact that it applies to multi-factor processes, and unlike factor analysis does not require the factors to be strong. In fact, $\bar{\rho}_N$ tends to be a strictly positive number if the panel of series is driven by at least one strong factor, otherwise it must tend to zero as $N \to \infty$. Therefore, non-zero estimates of $\bar{\rho}_N$ are suggestive of strong cross-sectional dependence.[13] Figure 1 plots $\bar{\rho}_i$ for all $i = 1, 2, ...N$ and $\bar{\rho}_N$ for volatility and output growth series (light and dark bars, respectively). It can be seen that, on average across all countries, the average pair-wise correlation for the volatility series is more than twice the average for the growth series, at 0.58 and 0.27, respectively (the two dotted lines).

Figure 1 Average Pair-wise Correlations of Volatility and Growth



Note. For each country, the light (yellow) and the dark (blue) bar show the average pair-wise correlation with the remaining countries in the sample for volatility and GDP growth series, respectively. The dotted lines correspond to the overall average across all countries, equal to 0.55 and 0.27, respectively. Sample period: 1993:Q1-2016:Q4.

For completeness, we also use standard principal component analysis. Principal component analysis yields similar results. The first principal component in our panel of realized volatility series explains 65 percent of the total variation in the log-level of volatility, whilst the first principal component of the growth series accounts for only around 30 percent of total cross-country variations in these series. Thus, both in the case of the pair-wise correlation and principal component analysis, the results point to a much higher degree of cross-country co-movements for the volatility series than for the growth series.

[13] Formal tests of cross-sectional dependence based on estimates of $\bar{\rho}_N$ are discussed in Pesaran (2015) and reported, for our panel data sample, in the next section.

5 Empirical Results

5.1 Estimated Global Growth and Financial Shocks

While the theoretical model in section 2 provides one way to interpret of ζ_t and ξ_t, it is important to check the consistency of such interpretation against other observable measures.

Global growth shock. We first compare our global growth shock with the TFP measures from Huo et al. (2018). As the TFP data is annual, we take an average of our global growth shock within each year. We then compare our measure with the changes in global TFP, where global TFP is defined as the simple average of TFP for Canada, the United States, Germany, Japan, and the United Kingdom. The correlation between the global growth shock and global TFP growth is 0.64 for the sample period over which they overlap. Interestingly, when we use TFP estimates for the United States alone (as opposed to the average of the five countries), this correlation drops to 0.39. Huo et al. (2018) also provide a measure of capacity-adjusted TFP for these countries. We therefore repeat our exercise using the capacity adjusted TFP measure, and find qualitatively similar results (i.e. stronger correlation with the global measure and a weaker correlation with the US measure), even though magnitudes are smaller, with the correlation dropping to 0.33 for the global measure and to 0.16 for the US measure. This descriptive evidence is therefore supportive of our interpretation of ζ_t as a technology factor, even though the results based on the capacity-adjusted TFP suggest that our shock could also be capturing some 'demand' factors.

Consistent with this interpretation we then compare our measure with a quarterly global measure of the natural interest rate from Holston et al. (2017)—defined as the real short-term interest rate consistent with the economy operating at its full potential. Similarly to what is done above with TFP, we compute a global measure as the simple average of the natural rate for the United States, Canada, the euro area, and the United Kingdom. Consistent with our theoretical analysis, global growth shocks are closely associated with changes in this proxy for the global natural interest rate. The correlation between the global growth shock and changes in the global natural rate is 0.5 for the sample period over which they overlap. Again, when we use only the natural rate for the United States this correlation drops to 0.27.

Finally, since our theoretical model has a similar structure to the long-run risk model of Bansal and Yaron (2004), we also compare our estimated global growth shock to a global and US long-run risk measure used by Colacito et al. (2018)– calculated again as the average across the G10 economies of the projection of each country's GDP growth onto the lagged value of that country's price-dividend ratio (see equation 3 of their paper). We find that our estimated world growth shocks have a correlation of only 0.29 with this proxy of global long-run risk, dropping to 0.18 when we consider only the corresponding US-specific measure, at annual frequency.

Global financial shock. We now turn to the global financial shocks series. We start by comparing our global financial shocks with changes in the measure of US financial uncertainty of Ludvigson et al. (2015). The two measures co-move quite closely, with a correlation of 0.43. But this co-movement can vary substantially over time. For example, the two series differ quite substantially during periods in which there was no financial stress in the United States, like the 1993-1996 and 2003-2006 periods, and post 2011. However, they closely co-move during periods in which the US economy itself is under strain. For example, in 1998-2002, when we witnessed the rescue of Long Term Capital Management after the Russian default, or in 2001, when the dotcom bubble burst and the Twin Towers were struck, or in 2007-2009 when the United States was at the epicenter of the global financial crisis. Similarly, the correlation of $\hat{\xi}_t$ with changes in the US Baa-Treasury credit spread and Excess Bond Premium of Gilchrist and Zakrajsek (2012) are only 0.20 and 0.35, respectively, where Baa-Treasury credit spread is computed as the difference between yields on long-term Baa-rated industrial bonds and comparable maturity Treasury securities. This evidence suggests that our series of global financial shocks has a different information content than the US-specific measures of financial uncertainty.

5.2 Country-specific Correlations Between Volatility and Growth Innovations

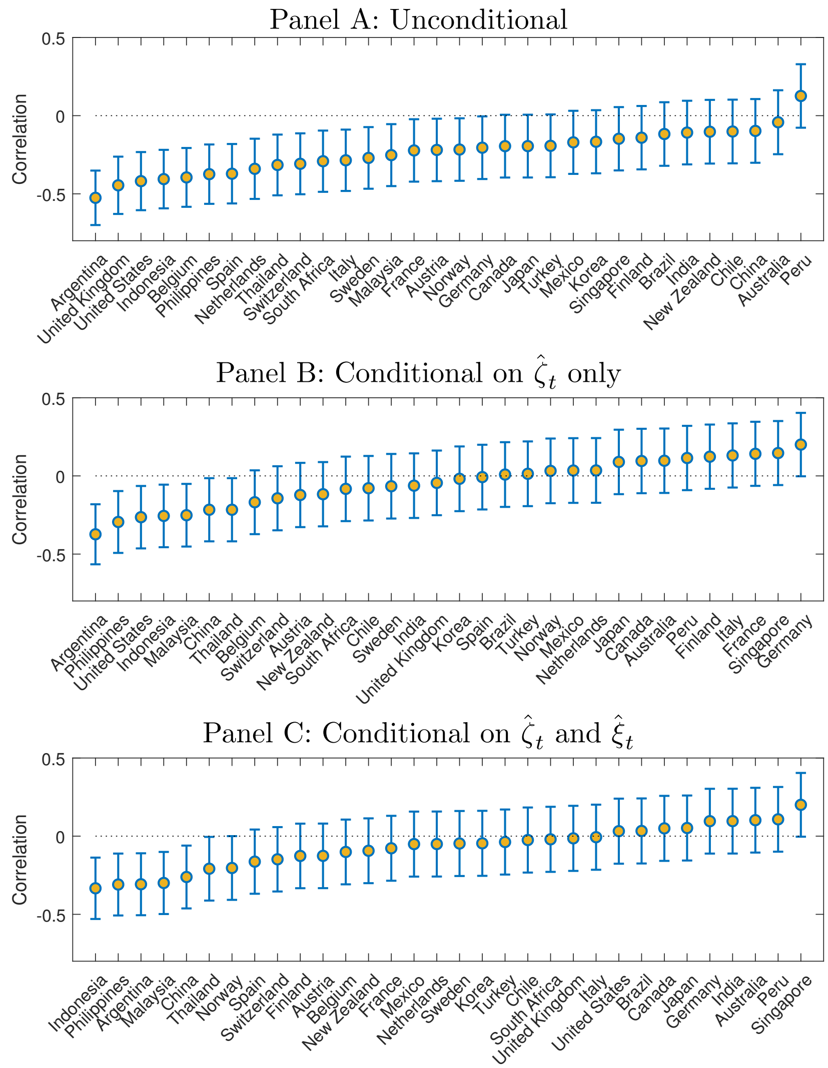
In this section, we report the within-country correlations between country-specific volatility and growth innovations, as a way of assessing the model's ability to capture the countercyclical nature of realized volatility. To compute these correlations, we first estimate the global factor innovations, $\hat{\zeta}_t$ and $\hat{\xi}_t$, using (36) and (37). Then we estimate country-specific VAR models conditional on these estimated global shocks and estimate the country-specific growth and volatility innovations, $\hat{\eta}_{it}$ and $\hat{\varepsilon}_{it}$, by running the OLS regressions in (38) and (39), respectively. To illustrate the strength of our main result, we will also report the same within-country correlations estimated conditional only on the global growth shock, $\hat{\zeta}_t$, in (38)-(39), rather than conditional on both global growth and financial shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$, and denote these estimated innovations by $\hat{u}_{it}$, where $u_{it} = \theta_i \xi_t + \eta_{it}$ and includes the global financial shock.

Figure 2 reports the main empirical result of the paper. It compares unconditional and conditional contemporaneous correlations between country-specific volatility and growth innovations, and suggests that this association is almost entirely driven by global output growth innovations, $\hat{\zeta}_t$. Panel A displays the unconditional correlations of volatility and growth series for all 32 countries in our panel, computed over the period 1993:Q1-2016:Q4, together with their 95-percent error band. Consistent with the large literature referred to in footnote 1, on average, this correlation is about -0.3, ranging from slightly more than -0.5 for Argentina to just above zero for Peru. With the exception of Austria, China, Indonesia, Peru, and South Africa, these correlations are statistically significant.

Panel B shows the correlation between country-specific volatility and growth innovations when we condition only on $\hat{\zeta}_t$ in model (38)-(39). The correlation between $\hat{u}_{it}$ and $\hat{\varepsilon}_{it}$ weakens substantially for all countries and it is no longer statistically significant, except in two cases. In the case of the United States, for instance, the conditional correlation does not vanish, but drops to about half its unconditional value and becomes borderline statistically insignificant when considered in isolation from the other correlations. Moreover, it is statistically not different from zero when estimated with the regularized multiple testing threshold estimator of the error covariance matrix proposed by Bailey et al. (2018b).[14]. Interestingly, this result is consistent with the main finding of Berger et al. (2017), who show that conditional on realized volatility, the VIX Index is not associated with indicators of

[14]See Section S.2.4 and Table S.6 of the online supplement

Figure 2 Country-specific Correlations Between Volatility and Growth Innovations



Note. Panel A displays the unconditional correlations between (log) realized stock market volatility and real GDP growth. Panel B plots the correlation between volatility and growth innovations when we condition only on $\hat{\zeta}_t$ in model (38)-(39). Panel C reports the same correlation when we condition on both $\hat{\zeta}_t$ and $\hat{\xi}_t$. The dots represent the contemporaneous correlations. The lines represent 95-percent confidence intervals. Sample period: 1993:Q1-2016:Q4.

economic activity like the unemployment rate or output growth for the United States.

Finally, Panel C reports the same correlation when we condition on both $\hat{\zeta}_t$ and $\hat{\xi}_t$. Remarkably, the results are very similar to those in Panel B, which is not surprising since $\hat{\xi}_t$ is common only to the volatility series. As we will see, these results are also robust to excluding from the analysis the sample period covering the global financial crisis (i.e. ending the sample period in 2006:Q4 or 2008:Q2) as well as dropping the United States, China, or both countries from the sample.

Overall, our findings suggest that volatility and growth share an important common component at quarterly frequency, and that conditioning on $\hat{\zeta}_t$ captures most of this dependence. As we will document by comparing impulse response functions to country-specific shocks with and without conditioning on our common shocks, this implies that some of the explanatory power attributed to uncertainty shocks in empirical studies of individual countries, considered in isolation from the rest of the world, is due to omitted common factors from the analysis.

Our estimates do not, however, imply that changes in volatility or growth over time are mostly driven by $\hat{\zeta}_t$. That is, while $\hat{\zeta}_t$ can account for most of the contemporaneous co-movement between country-specific volatility and growth, it does not necessarily explain a significant share of the observed time variations in volatility or growth. Indeed, as we will see in the next section, $\hat{\zeta}_t$ explains a relatively small share of the variation of volatility over time, with a much larger share explained by $\hat{\xi}_t$.

5.3 Volatility and Growth Forecast Error Variance Decompositions

Forecast error variance decompositions are routinely used to quantify the importance of a given shock for the time-variation of the endogenous variables at different time horizons, relative to other shocks in the model. Our factor augmented multi-country PVAR model can be readily used to decompose the forecast error variance of country volatility and growth in terms of the common shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$, as well as the 64×1 vector of country-specific shocks, $\hat{\eta}_{it}$ and $\hat{\varepsilon}_{it}$ for $i = 1, 2, ..., 32$. While the global growth and financial shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$, are orthogonal to the country-specific shocks and to each other by construction, the country-specific shocks $\hat{\eta}_{it}$ and $\hat{\varepsilon}_{it}$ are left unrestricted, and could be correlated, both within and between countries. In order to compute and interpret forecast error variance decompositions, we therefore have to deal with this second identification problem. To identify country-specific volatility and growth shocks, we exploit the empirical properties of their correlation matrix combined with alternative conventional assumptions regarding the causal relation between volatility and growth at the country-specific level. We will then show that the inference we draw is robust to alternative identification schemes used.

Consider the correlation between volatility and growth innovations within each country. We saw in Figure 2 that, once we condition on the global shocks ($\hat{\zeta}_t$ and $\hat{\xi}_t$), the contemporaneous within-country correlation between $\hat{\eta}_{it}$ and $\hat{\varepsilon}_{it}$ is very small and not statistically significant in most countries. Below, we will also show that conditional on both $\hat{\zeta}_t$ and $\hat{\xi}_t$, the country-specific shocks $\hat{\varepsilon}_{it}$ and $\hat{\eta}_{it}$ are weakly correlated across countries (Figure 6), with average pair-wise correlations below 0.05. Weak cross-sectional dependence means that, as N grows, the overall average pair-wise correlation must tend to zero; while some pairs of correlations can be different from zero, not all pairs can be so. In practice, this means that most correlation pairs will be very small and the covariance matrix, $\mathbf{\Sigma}_{(\varepsilon,\eta)}$, in the 64 shocks $\hat{\varepsilon}_{it}$ and $\hat{\eta}_{it}$, for $i = 1, 2, ..., N$, must be sparse. Indeed, when we apply the threshold estimation procedure of Bailey et al. (2018b) to the whole set of distinct off-diagonal elements of $\mathbf{\Sigma}_{(\varepsilon,\eta)}$ we find that only 57 out of 2016 off-diagonal elements are statistically different from zero.[15]

[15] Table S.6 in the online supplement shows that, of these 57, about half are positively correlated and the other half are negatively correlated, with an average value that is close to zero. Most notably, there is no surviving within-country contemporaneous correlation between volatility and growth. There are also very few significant GDP-GDP correlation pairs (i.e., $\hat{\varepsilon}_{it}$ with $\hat{\varepsilon}_{jt}$), with no obvious regional pattern of comovements. There are a few significant pairs of volatility-volatility correlations (i.e. $\hat{\eta}_{it}$ with $\hat{\eta}_{jt}$), but involving only a handful of countries, with no evidence of a dominant role for the United States. Finally, there are only two significant GDP-volatility correlation pairs (i.e. $\hat{\varepsilon}_{jt}$ with $\hat{\eta}_{it}$), again

Nonetheless, even a block diagonal estimated reduced form covariance matrix (that sets cross-country error correlations to zero), would not imply that innovations $\hat{\eta}_{it}$ and $\hat{\varepsilon}_{it}$ can be interpreted as 'structural' country-specific volatility and growth shocks. As is well known, there always exists an orthonormal transformation of η_{it} and ε_{it} that leads to the same forecast error variance decomposition. It is therefore important to complement this evidence with some explicit assumption about the 64×64 matrix of correlations. As a first approximation, we assume that the only source of interdependence among all growth and volatility series are the global real and financial shocks $\hat{\zeta}_t$ and $\hat{\xi}_t$ and that country-specific volatility and growth shocks have no contemporaneous impact on growth or volatility series within and across countries. In other words, we assume that the reduced form innovations are also structural. In the online supplement to the paper we report results imposing a recursive identification scheme, where we assume that within each country volatility shocks affect output growth contemporaneously, and also consider the case in which we refrain from interpreting these innovations structurally.

Figure 3 reports the forecast error variance decompositions (FEVDs) obtained assuming $\mathbf{\Sigma}_{(\varepsilon,\eta)}$ is diagonal.[16] Each figure reports the 'average' variance decomposition, weighting country-specific decompositions with PPP-GDP weights. All results are based on (38)-(39), which include both $\hat{\zeta}_t$ and $\hat{\xi}_t$. The left hand panel of Figure 3 plots the average forecast error variance decomposition of volatility across all countries in our sample. The right hand panel reports the FEVD of GDP growth. The figure shows that country-specific volatility is driven largely by common financial shocks (blue area with vertical lines), as suggested by the proponents of the Global Financial Cycle hypothesis (e.g. Rey (2013)), and country-specific volatility shocks (red area with crosses). Together, these two shocks explain about 95 percent of the total variance of realized volatility over time. World growth shocks (purple area with diagonal lines) explain less than 5 percent of the total volatility forecast error variance. Country-specific own growth shocks, as well as all other 31 country-specific foreign growth shocks in the full model, play essentially no role. These results imply that the endogenous component of country-specific volatility, namely the component driven by common and country-specific growth shocks, is quantitatively small.

It is worth noting that our estimated FEVD of country-specific realized volatility is similar to the central estimates of Ludvigson et al. (2015) for their US financial uncertainty measure. In that study, the share of the macroeconomic shock in the forecast error variance of financial uncertainty is estimated at just above 5 percent. However, while Ludvigson et al. (2015) attribute this to the US business cycle (as proxied by a shock to US industrial production), we attribute the outcome largely to the global growth shock, which can be interpreted as an international business cycle factor, as we find that country-specific growth shocks have little or no explanatory power for country-specific volatility.[17]

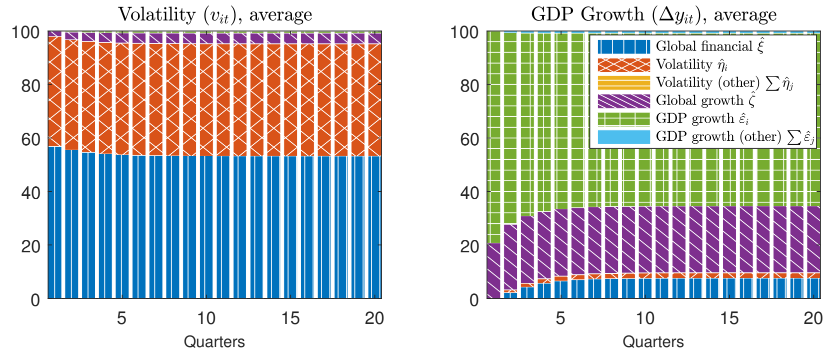
Consider now the FEVD of GDP growth reported on the right hand panel of Figure 3. The figure

revealing no specific patterns.

[16] The derivation of the FEVDs is reported in the online supplement to the paper. Robustness to the assumption of diagonal covariance matrix is reported below.

[17] Results for specific countries, including the United States, are reported in the online supplement. As we can see from Figures S.9 to S.12 in the online supplement, countries behave pretty similarly, with some but limited heterogeneity. The US results, in particular, are similar to those for the average economy reported here.

Figure 3 Forecast Error Variance Decomposition of Country-specific Shocks - Diagonal Error Covariance Matrix



Note. Average across countries with GDP-PPP weights. $\hat{\xi}$ is common financial shock (blue area with vertical lines); $\hat{\eta}_i$ is country-specific volatility shock (red area with crosses); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area with horizontal lines); $\hat{\zeta}$ is common growth shock (purple area with diagonal lines); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas with squares); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas with no pattern). The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

shows that, on average, the forecast error variance of country specific GDP growth is driven mostly by global and country-specific growth shocks, with a combined share approaching 90 percent of the total in the long run (green areas with squares and purple area with diagonal lines, respectively). The country-specific growth shock explains more than 60 percent of the total forecast error variance in the long-run, while the global growth shock on average explains around 30 percent of the total growth forecast error variance. This is in line with existing results in the international business cycle literature (see, for instance, Kose et al. (2003)).[18]

The global financial shock explains, on average across countries, $8-10$ percent of country-specific GDP growth forecast error variance. The importance of this shock picks up gradually over the forecast horizon and stabilizes within two years. In contrast, the own country-specific volatility shock explains $1-2$ percent of the total forecast error variance of GDP growth, while the combination of all other 31 country-specific volatility shocks in the model explains an even smaller share of country-specific growth variance. These results clearly show the quantitative importance of distinguishing between common and country-specific volatility shocks, but also illustrate that large explanatory power and predictability for financial variables does not necessarily mean explanatory power for real variables. These results therefore help to bridge the gap between proponents and opponents of the Global Financial Cycle hypothesis (e.g. Rey (2013) *versus* Cerutti et al. (2017)).

[18]Note that these results imply that countries' business cycles remain largely unexplained within our econometric model. Indeed, in the data, there are many shocks at work, and this is captured in our relatively simple empirical framework by the large share of growth forecast error variance accounted for by the own country-specific growth shocks.

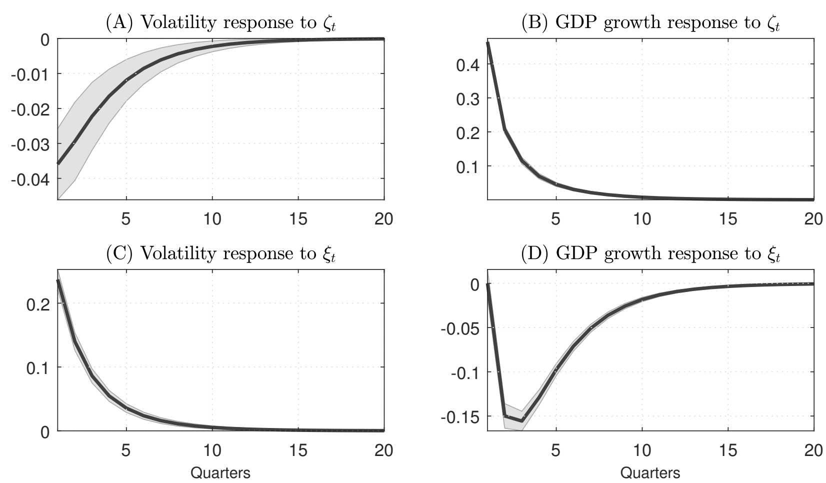
5.4 The Transmission of Common and Country-specific Growth and Volatility Shocks

The last step of our empirical analysis is the computation of impulse responses of country-specific volatility and growth to common and country-specific shocks. While forecast error variance decompositions speak to the importance of a particular shock for the time-variation of the endogenous variables relative to other shocks in the model, impulse responses provide information on the magnitude and the timing of the effects of the shocks across variables and countries.

5.5 Global Shocks

Figure 4 displays a weighted average of the country-specific impulse responses to the global growth and financial shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$, using PPP-GDP weights (solid line), together with two-standard deviation error bands (shaded areas). The error bands are computed based on the dispersion of the impulse responses across countries.[19]

Figure 4 Average Country Volatility and Growth Responses to Real and Financial Factor Shocks



Note. Average impulse responses to one-standard deviation real and financial shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$. The solid lines are the PPP-GDP weighted averages of the country-specific responses. The shaded areas are two standard deviations confidence intervals. The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

We focus on the effects of positive one-standard deviation shocks. Panels (A) and (B) of Figure 4 display the average across countries of the volatility and growth responses to a global growth shock. These figures show that a positive global growth shock increases country-specific output growth and lowers volatility. This reflects an endogenous volatility response to the fundamental improvements in the world economy. Note that the error bands around the average responses are very tight, reflecting relatively homogeneous country responses. In fact, as can be seen from Figure S.8 in the online

[19] The derivation of the average impulse response functions to common factor shocks and their error bands is provided in section S.5 in the online supplement, equations (S12) and (S13). The estimated country-specific responses to the two common shocks are reported in Figure S.8 in the online Supplement to the paper.

supplement, the impulse responses have a similar shape for most countries. The average impact of this shock on country-specific volatilities is one order of magnitude smaller than its impact on country-specific output growth, but it is quite persistent, taking more than three years for the effects of the shocks to vanish completely. Panel (B) also shows that, on average, country-specific growth loads positively on $\hat{\zeta}_t$, with persistent effects up to 8-10 quarters, as one would expect. Country-specific output growth increases by about half a percentage point following a one-standard error change in $\hat{\zeta}_t$. Again, this is consistent with the existing evidence on the international business cycle, which attributes an important role to a world factor, along with regional and country-specific factors, in driving the business cycle (e.g. Kose et al. (2003)).

Panels (C) and (D) of Figure 4 report the responses of volatility and growth to a positive global financial shock, $\hat{\xi}_t$. These average responses suggest that a positive shock to $\hat{\xi}_t$ is 'bad news' for the world economy, as volatility increases and growth declines. For a one-standard deviation shock to the common financial factor, volatility increases by 25 basis points, while growth declines by about 15 basis points within two quarters after the shock.[20] Although smaller than the growth response to the global growth shock in Panel B, the average growth response to the global financial shock in panel D is of the same order of magnitude, and hence quantitatively sizable. The average responses to the common financial shock are also very persistent, but there is much more heterogeneity in the country-specific growth responses, as can be seen from Figure S.8 in the online supplement. Therefore, these impulse responses suggest that, even though common financial shocks may not explain a very large share of the forecast error variance of GDP growth over time, they can cause large and persistent global recessions.

The pattern of shock transmissions in Figure 4 is consistent with country-specific volatility increasing in response to the large declines in world output in the second part of 2008, and the world recession being amplified by the exceptionally large common financial shock in the fourth quarter of 2008, and the first quarter of 2009. The transmission in Figure 4 can also help to explain the seemingly puzzling coexistence of high policy volatility (as in Baker et al. (2016)) and low equity market volatility after the beginning of the Trump administration with a combination of real and financial shocks partially offsetting each other.

5.6 Country-specific Shocks

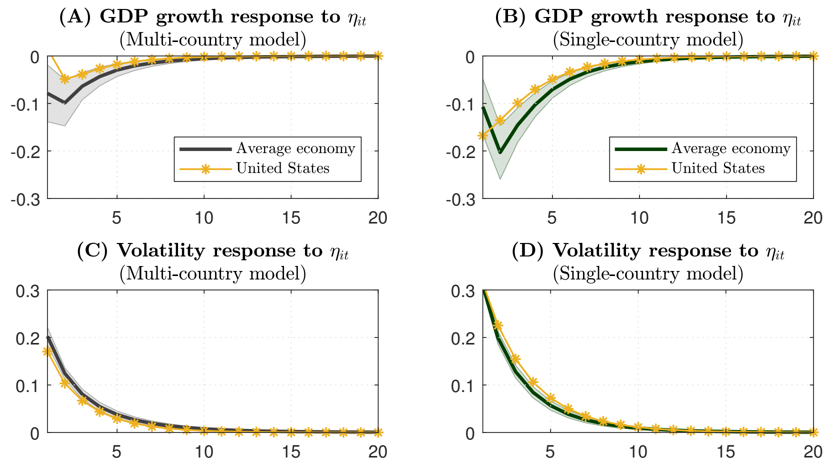
Impulse responses to country-specific shocks have qualitatively similar pattern of transmission, but are quantitatively much smaller than the responses to common shocks, as one would expect given the forecast error variance results.[21] It is therefore natural to ask whether impulse responses to country-specific volatility shocks computed from a single-country VAR estimated in isolation from the rest of the wold economy overestimate the impact on output growth if compared to the same shock computed with our multi-country model.

To address this question, we estimate single-country VARs in output growth and volatility without

[20]Note that the delayed growth response to the global financial shock follows from our identification assumptions, but it is not imposed directly on country-specific models.

[21]These results are not reported but are available from the authors on request.

Figure 5 MULTI-COUNTRY MODEL VS. SINGLE-COUNTRY BIVARIATE VARS



NOTE. Average (black solid line) and US (yellow line) impulse responses to one-standard deviation to the volatility innovations η_{it}. The average is a PPP-GDP weighted averages of the country-specific responses. The shaded areas are two standard deviations confidence intervals. The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

conditioning on our global shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$, and compute impulse responses to a volatility shock, identified recursively with volatility ordered first (as often assumed in the literature). We then compare the impulse responses from the single-country VARs with those obtained from our multi-country model (i.e. conditioning on $\hat{\zeta}_t$ and $\hat{\xi}_t$) imposing a block-diagonal covariance matrix with the same recursive identification within each country to be consistent with the single-country VARs.

The results are reported in Figure 5. The panels on the left hand side report the response to a country-specific volatility shock in our multi-country model. The panels on the right hand side report the response to a volatility shock in a single-country VAR. All panels report averages across countries as well as the US response for comparison. The output responses to a country-specific volatility shock obtained from single-country models, on average across countries and for the United States in particular, are much larger than those estimated from our multi-country model, thus providing strong evidence of omitted variable bias.

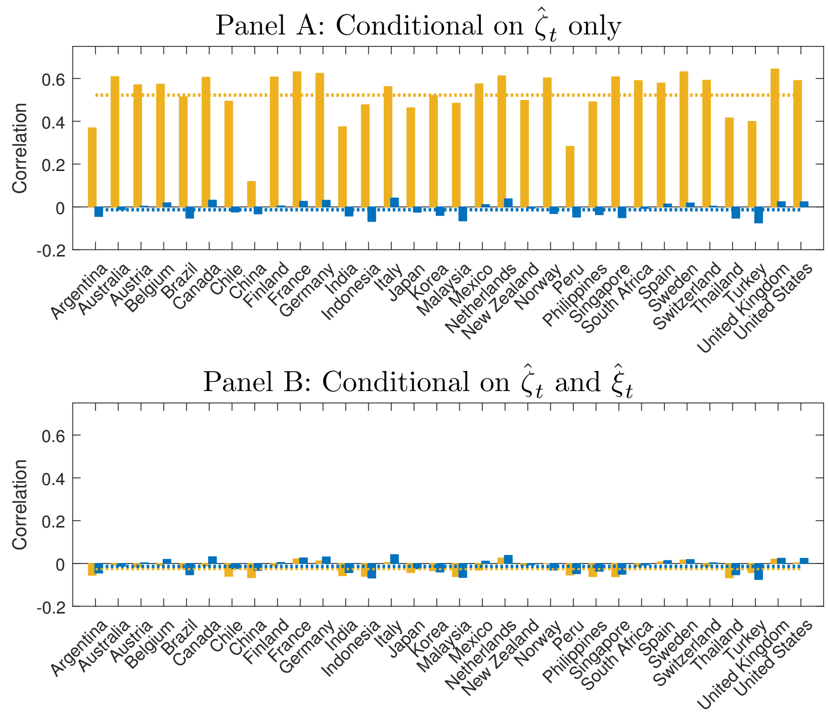
6 Cross-country Correlations of Volatility and Growth Innovations

Although the restrictions behind our identification assumptions for the common shocks cannot be formally tested, our multi-country approach permits us to investigate the extent to which the implications of the identified model are in line with the identification restrictions made. To this end, we explore the cross-country correlations of the estimated residuals from the dynamic regressions (38) and (39), with and without conditioning also on the global financial shock, $\hat{\xi}_t$.[22]

[22] Note that we can estimate ζ_t and ξ_t consistently by means of the OLS regressions (36) and (37) only under the identification assumptions made. As a result, whilst we can directly estimate pair-wise correlations of volatility and

Panel A of Figure 6 shows that, if we condition only on $\hat{\zeta}_t$ in (38)-(39), the volatility innovations display average pair-wise correlations comparable to those of the data reported for all countries in Figure 1. In contrast, the pair-wise correlations of the growth innovations are negligible, with an average across all countries of -0.01. Panel B of Figure 6 shows that, if we condition on both $\hat{\zeta}_t$ and $\hat{\xi}_t$, the cross-country correlations of the volatility innovations are now also negligible, as in the case of the growth innovations, with an average pair-wise correlation across all countries equal to -0.02. For instance, in the specific case of the United States, the average pair-wise correlation of the volatility innovations is equal to 0.6 conditioning on $\hat{\zeta}_t$ alone. But it drops to 0.00 if we condition on both factor innovations. By comparison, the US average pair-wise correlation of the growth innovations is 0.02.

Figure 6 CROSS-COUNTRY CORRELATION OF COUNTRY-SPECIFIC VOLATILITY AND GROWTH INNOVATIONS



NOTE. Country-specific average pair-wise correlation of volatility (yellow, lighter bars) and GDP growth (blue, darker bars) innovations conditional on $\hat{\zeta}_t$ only (Panel A) and on $\hat{\zeta}_t$ and $\hat{\xi}_t$ (Panel B). The volatility measures are based on (15). The dotted lines are the averages across all countries, equal to 0.52 and -0.01 for volatility and growth in Panel A; and equal to -0.02 and -0.01 for volatility and GDP growth in Panel B, respectively. Sample period: 1993:Q1-2016:Q4.

Figure 6, therefore, illustrates that, after conditioning on $\hat{\zeta}_t$—which is common to both growth and volatility series—not much commonality is left in the case of growth innovations, but the volatility innovations continue to share strong commonality. Moreover, after conditioning on both $\hat{\zeta}_t$ and $\hat{\xi}_t$, the volatility innovations also appear weakly correlated because of the near-zero average pair-wise correlation across all countries, thus suggesting that only two common shocks are necessary

growth series, we cannot examine cross-country pair-wise correlations of their innovations without imposing these identification conditions.

to span their correlations across-countries as we assumed in our theoretical model. It is, therefore, interesting to test whether the two sets of innovations also satisfy a formal definition of *weak* and *strong* dependence, as we assumed deriving them.

To formally test for weak and strong cross-section dependence, we compute the cross-sectional dependence (CD) test statistic of Pesaran (2015) and the exponent of cross sectional dependence (α) proposed in Bailey et al. (2016). The CD statistic is normally distributed with zero-mean and unit-variance under the null of zero average pair-wise correlations. The critical value is around 2. When the null is rejected, Bailey et al. (2016) suggest estimating the strength of the cross-section dependence with an exponent, denoted α in the range $(1/2, 1]$, with unity giving the maximum degree of cross dependence. Any value above 1/2 and below 1, but significantly different from 1, suggests weak dependence.[23] So, in what follows, we present estimates of α for the volatility and the growth innovations, together with their confidence intervals. For comparison, we also report the same estimates for the (raw) growth and volatility series.

Table 1 TESTING FOR THE STRENGTH OF CROSS-SECTIONAL DEPENDENCE

	CD	Lower 5%	$\hat{\alpha}$	Upper 95%
Data				
v_{it}	104.57	0.94	0.99	1.05
Δy_{it}	55.73	0.87	1.00	1.14
Innovations (conditional on $\hat{\zeta}_t$)				
u_{it}	110.89	0.96	1.00	1.04
ε_{it}	-2.90	0.56	0.62	0.67
Innovations (conditional on $\hat{\zeta}_t$ and $\hat{\xi}_t$)				
η_{it}	-5.12	0.58	0.64	0.70

NOTE. CD is the cross-sectional dependence test statistic of Pesaran (2015). $\hat{\alpha}$ is the estimate of the exponent of cross-sectional dependence as in Bailey et al. (2016), together with its 90-percent confidence interval ('Lower 5%' and 'Upper95%').

The results are summarized in Table 1 and are in strong accordance with the identification assumptions made. The CD test statistic for the growth series is 55.73, with the associated α exponent estimated at 1.00. The CD statistic for the volatility series is even higher at 104.57 with an estimated α of 0.99. The CD statistics and the estimates of α confirm with a high degree of confidence that both series are cross-sectionally strongly correlated, containing *at least* one strong common factor. Conditional only on $\hat{\zeta}_t$, the CD statistic for the country-specific growth innovations ($\hat{\varepsilon}_{it}$) drops to -2.90, close to its critical value under the null of zero average pair-wise correlations, with its exponent of cross-sectional dependence estimated to be 0.62, significantly below 1. In sharp contrast, the CD statistic for the country-specific volatility innovations when we condition only on $\hat{\zeta}_t$ in model (38)-(39) (denoted by $\hat{u}_{it}$) remains close to that of the raw volatility series at 110.89 with an estimated α also not statistically different from unity. However, when we condition on both

[23]When estimating α one also needs to take into account the sampling uncertainty, which depends on the relative magnitude of N and T, and the null of weak cross dependence, which depends on the relative rates of increase of N and T.

$\hat{\zeta}_t$ and $\hat{\xi}_t$, the CD statistic for the volatility innovations ($\hat{\eta}_{it}$) also falls to -5.12, with an estimated α of 0.64 and a 95-percent confidence interval of $[0.58, 0.70]$, while the CD statistic and α are the same as before for the growth innovations ($\hat{\varepsilon}_{it}$). The test statistics in Table 1, therefore, accord very well with the assumptions made that the volatility innovations share at least one more, and only one more, strong common factor than the growth innovations.[24]

7 Robustness Analyses

In this section we report on a number of exercises we have conducted to check the robustness of our empirical findings. The results (reported in the online supplement to the paper, see section S.2) show that our main findings are not affected by the granularity assumption, the exclusion of the global financial crisis from the sample period, the volatility proxy used, and the assumptions on the error covariance matrix of the multi-country model.

Granularity assumption. One important question regarding our results is whether the estimated common shocks might in fact be idiosyncratic shocks to large countries such as the United States and China. To address this issue we re-estimate different versions of the model where we drop (i) the United States, (ii) China, and (iii) both the United States and China from the sample. We obtain essentially the same results. In contrast, when we replaced the estimated global growth and financial factors with US GDP growth and US realized volatility, we were able to control for only a fraction of the cross-country correlation in our data—thus violating our assumptions on weak cross-country correlation of the residuals conditional on the global factors. In this latter case, in particular, the estimated exponents of cross-sectional dependence (α) are not significantly different from one. Taken together, these two sets of results provide clear cut evidence against the United States (or China) driving our empirical results. See section S.2.1 in the online supplement.

Sample period. The results are robust to dropping the period of the global financial crisis from our sample. Specifically, we continue to obtain virtually the same results when we re-estimate our model on a sample that ends in 2006. Importantly, this is true for (i) the cross-sectional dependence of the raw data and the residuals conditional on the global shocks; (ii) the within-country correlation between volatility and GDP growth; and (iii) the role of the global growth and financial shocks in the FEVDs. Only the IRFs to the global growth and financial shocks display some small differences relative to our baseline, in that the size of the impact is slightly smaller, which is not surprising given the dramatic rise in volatility during 2008-2009. See section S.2.2 in the online supplement.

Realized *versus* Implied volatility. In the finance literature, the focus of the volatility measurement has recently shifted to implied volatility measures obtained from option prices, like the US VIX Index. At quarterly frequency, however, the realized volatility of US daily equity returns behaves very similarly to the VIX Index. For example, during the period over which they overlap, our US realized volatility measure and the VIX Index co-move very closely, with a correlation that exceeds 0.9. In addition, to more formally check the robustness of our results to the choice of the volatility measure, we re-estimated our model using the VIX Index as a measure of volatility for the United

[24] The estimates of α obtained with the residual-based approach of Bailey et al. (2018a) give very similar results.

States (instead of our realized volatility measure). We obtained even stronger results, in that the response of US GDP to a US volatility shock is less negative than in our baseline. This implies that the omitted variable bias from ignoring the global factors is even stronger when using the VIX Index as a proxy for US volatility. See section S.2.3 in the online supplement.

Variance Decompositions: Alternative Identification Assumptions for Country-Specific Shocks. While in our baseline estimates of the FEVDs we assume a diagonal covariance matrix for the residuals of the multi-country model (38)-(39), to check our results for robustness we re-estimate the FEVDs under other assumptions on the covariance matrix of country-specific shocks. We consider two alternatives. First, while maintaining the assumption of zero conditional correlations across countries, we assume that country-specific volatility shocks can have a contemporaneous causal impact on growth variables but not *vice-versa*, in line with much of the existing empirical literature as reviewed in the Introduction. This is done by allowing for a block-diagonal error covariance matrix in the multi-country model, in which the only non-zero off-diagonal elements are the estimated covariances between volatility and growth errors of each country-specific block. These within-country blocks are factorized with a Cholesky decomposition, ordering volatility before growth. Second, we refrain altogether from interpreting country-specific volatility and growth shocks as structural, and make use of a general unrestricted error covariance matrix (both within and across countries) and compute the generalized forecast error variance decompositions (GFEVD) of Pesaran and Shin (1998). However, before computing GFEVDs, we use the regularized multiple testing threshold estimator of the error covariance matrix proposed by Bailey et al. (2018b) to obtain a consistent estimator of the 64×64 error covariance matrix of the residuals of the multi-country model.[25] The FEVDs obtained for these alternative specifications of the error covariance matrix of country-specific shocks are very close to the ones reported in the paper. See section S.2.4 in the online supplement.

8 Conclusions

Volatility behaves counter-cyclically in most countries of the world, but economic theory suggests that causation can run both ways. In this paper, we take a common factor approach in a multi-country setting to study the interrelation between realized equity price volatility and GDP growth without imposing *a priori* restrictions on the direction of economic causation on country-specific volatility and growth shocks.

Based on new stylized facts of the data that we document in the paper, and consistent with a multi-country version of the Lucas tree model with time-varying volatility, a persistent common technology growth factor, heterogeneous exposure to this common factor, and cross country spillovers, we estimate a multi-country econometric model in output growth and realized volatilities for 32 countries over the period 1993:Q1-2016:Q2. Common growth and financial shocks are identified by assuming different patterns of correlation of volatility and output growth innovations across countries. Evidence based on the estimated innovations accords well with the identification assumptions made.

[25]This regularized estimator exploits the sparsity of the underlying error covariance matrix. More details are reported in the the online supplement.

Empirically, we report three main results. First, shocks to the world growth factor, which are closely associated with changes in proxy for the world TFP growth and the natural rate, account for most of the unconditional correlation between volatility and growth in all economies. Second, the share of forecast error variance of country-specific volatility explained by this global growth factor and country-specific growth shocks is less than 5 percent. Third, shocks to the financial common factor explain about 10 percent of the country-specific growth forecast error variance, while country-specific volatility shocks explain only about $1-2$ percent; but when a shock to the financial common factor is realized, its negative impact on country-specific growth is large and persistent as typically estimated in the existing literature.

The theoretical model that we set up and the econometric methodology that we propose in order to extract the risk factors from the data have a rich set of testable implications. An important question is the extent to which the risk factors we extracted from the data have pricing power over and above the traditional ones used in the literature. In our model, supply-side factors can potentially help predict cash flows and hence can help predict excess returns. Moreover, the identification strategy proposed provides a methodology for a model-free measure of long-run or disaster risk. We regard these as promising areas of future research.

Appendix A: Theoretical Model Details

Output Growth Equations Despite its simplicity, the production side of the economy summarized in equation (1) is consistent with multi-country versions of the international real business cycle models of Backus et al. (1992) and Baxter and Crucini (1995). To see this, assume a standard Cobb-Douglas production function in terms of output per worker denoted by $(Y_{it}/L_{it}) = exp(\tilde{y}_{it})$, $A_{it} = exp(a_{it})$ the country-specific level of technology, L_{it} the labor force, and K_{it} the capital stock in country i, so that we have:

$$\tilde{y}_{it} = \ln(Y_{it}/L_{it}) = a_{it} + \tilde{\alpha}_i \ln(K_{it}/L_{it}) = a_{it} + \tilde{\alpha}_i \log(k_{it}),$$

for $i = 1, 2, \ldots, N$. Further assume that the processes for L_{it} and a_{it} are exogenously given by

$$\ln(L_{it}) - \ln(L_{i,t-1}) = n_i, \text{ and } a_{it} = a_{0i} + \tilde{g}_i t + \gamma_i a_t + e_{it},$$

where the growth rate of the labour force, n_i, is assumed to be fixed, a_{0i} is an initial condition, $\tilde{g}_i$ is a deterministic growth component of a_{it}, a_t is the log-level of a stochastic common technology factor, and e_{it} is the country-specific technology shock, with γ_i measuring the extent to which country i is exposed to the global technology factor a_t. A key result from the stochastic growth literature is that, for all i, $\log(k_{it})$ is ergodic and stationary, in the sense that as t tends to infinity, $\log(k_{it})$ tends to a time-invariant random variable, $\log(k_{it}) = \log(k_i) + \tau_{it}$, where τ_{it} is a stationary process representing all country-specific forces driving the country's business cycles, possibly reflecting the effects of aggregate demand shocks, other supply shocks, as well as country-specific uncertainty shocks (see, for instance, Lee et al. (1997)).[26] So we have $\tilde{y}_{it} = a_{0i} + \tilde{\alpha}_i log(k_i) + \tilde{g}_i t + \gamma_i a_t + e_{it} + \tau_{it}$, and taking first differences we obtain $\Delta\tilde{y}_{it} = \tilde{g}_i + \gamma_i f_t + \varepsilon_{it}$, where $f_t = \Delta a_t = a_t - a_{t-1}$, and $\varepsilon_{it} = \Delta e_{it} + \Delta\tau_{it}$. In terms of log output, $y_{it} = \ln(Y_{it})$, therefore, we have $\Delta y_{it} = y_{it} - y_{i,t-1}$and $a_i = \tilde{g}_i + n_i$, which is equation (1) in the paper.

Equity Returns and Risk Premia To derive the equity return given by (10), where $r_{i,t+1} = \ln(R_{i,t+1})$, $R_{i,t+1} = (P_{i,t+1} + Y_{i,t+1})/P_{it}$, and P_{it} is the price of risky asset in country i at time t, we adopt the Campbell and Shiller (1988) approximation to the (log) one-period gross ex-post return given by:

$$r_{i,t+1} = \kappa_{0i} + \kappa_{1i} z_{i,t+1} - z_{it} + \Delta y_{i,t+1}, \tag{A1}$$

where κ_{0i} and κ_{1i} are fixed constants, and $z_{it} = \ln(P_{it}/Y_{i,t})$. It is possible to find an approximate closed-form solution for z_{it} and hence $r_{i,t+1}$. To accomplish this, similarly to Bansal and Yaron (2004) (BY hereafter) , we guess that the solution for z_{it} has the following linear form in our model's risk factors:

$$z_{i,t} = A_{0i,N} + A_{1i} f_t + A_{2i}\sigma_t^2 + A_{3i,N}\psi_t^2 + A_{4i,N}\theta_t^2, \tag{A2}$$

where $A_{0i,N}$, A_{1i}, A_{2i}, $A_{3i,N}$, and $A_{4i,N}$ are functions of all structural parameters of the model, some of which, under the assumptions made, will be shown to depend on the number of countries in the global economy, N. It is also worth noting that as compared to the single economy country model analyzed by BY, we have three additional risk factors due to the country-specific and cross-country time-varying volatilities.

The risky return $R_{i,t+1}$ satisfies the first order condition $E_t(R_{i,t+1}M_{t+1}) = 1$, where $E_t(\cdot) = E(\cdot|\Im_t)$, $\Im_t$ is the information set, and M_{t+1} is defined by (6) and (7). Using the expression for M_t, this first-order condition can be rewritten equivalently as:

$$1 = E_t\left[\exp\left[\ln\left(M_{t+1}R_{i,t+1}\right)\right]\right] = E_t\left[\exp\left(\ln\beta - \varrho a_w - \varrho\gamma_w f_{t+1} - \varrho\varepsilon_{w,t+1} + r_{i,t+1}\right)\right]. \tag{A3}$$

[26]For a discussion of volatility shocks interpreted as demand shocks see Leduc and Liu (2016) and Basu and Bundick (2017).

We can now derive the undetermined coefficients, $A_{0i,N}, A_{1i}, A_{2i}, A_{3i,N}, A_{4i,N}$, such that (A3) is satisfied. Substituting (A1) in (A3), using (1) and (A2), and using (2), (3), (4) and (5), we obtain:

$$E_t\left[\exp\left(q_{i,t+1}\right)\right] = \exp\left(-\ln\beta + \varrho a_w - \kappa_{0i} - a_i\right),$$

where:

$$\begin{aligned} q_{i,t+1} &= h_{it} + \left(\varepsilon_{i,t+1} - \varrho\varepsilon_{w,t+1}\right) + \left(\gamma_i - \varrho\gamma_w + \kappa_{1i}A_{1i}\right)\sigma_t\zeta_{t+1} \\ &\quad +\kappa_{1i}\varphi_\chi A_{2i}\chi_{t+1} + \kappa_{1i}\varphi_\eta A_{3i,N}\eta_{t+1} + \kappa_{1i}\varphi_u A_{4i,N}\varpi_{t+1}, \end{aligned}$$

and

$$\begin{aligned} h_{it} &= -\left(1-\kappa_{1i}\right)A_{0i,N} + \kappa_{1i}(1-\phi_\sigma)\sigma^2 A_{2i} + \kappa_{1i}(1-\phi_\psi)\psi^2 A_{3i,N} + \kappa_{1i}(1-\phi_\theta)\theta^2 A_{4i,N} \\ &\quad + \left[\gamma_i - \varrho\gamma_w - \left(1-\phi_f\kappa_{1i}\right)A_{1i}\right] f_t \\ &\quad - A_{2i}\left(1-\kappa_{1i}\phi_\sigma\right)\sigma_t^2 - A_{3i,N}\left(1-\kappa_{1i}\phi_\psi\right)\psi_t^2 - A_{4i,N}\left(1-\kappa_{1i}\phi_\theta\right)\theta_t^2. \end{aligned}$$

Under the assumption that the shocks are conditionally Gaussian (see the last paragraph of section 2.1), $q_{i,t+1}$ being a linear function of these shocks would also be Gaussian conditional on $\Im_t$, and we have:

$$E_t\left[\exp\left(q_{i,t+1}\right)\right] = \exp\left(h_{it}\right)\cdot\exp\left[\begin{array}{c} \frac{1}{2}Var_t\left(\varepsilon_{i,t+1} - \varrho\varepsilon_{w,t+1}\right) + \frac{1}{2}\left(\gamma_i - \varrho\gamma_w + \kappa_{1i}A_{1i}\right)^2\sigma_t^2 \\ +\frac{1}{2}\kappa_{1i}^2\varphi_\chi^2 A_{2i}^2 + \frac{1}{2}\kappa_{1i}^2\varphi_\eta^2 A_{3i,N}^2 + \frac{1}{2}\kappa_{1i}^2\varphi_\varpi^2 A_{4i,N}^2 \end{array}\right],$$

where

$$Var_t\left(\varepsilon_{i,t+1} - \varrho\varepsilon_{w,t+1}\right) = \theta_t^2 B_{iN} + \psi_t^2 C_{iN}, \tag{A4}$$

and

$$B_{iN} = \left(1 - 2\varrho w_i\right)\sigma_{ii} + \varrho^2\left(\sum_{j=1}^{N} w_j^2\sigma_{jj}\right), \tag{A5}$$

$$C_{iN} = \varrho^2\sum_{j\neq i}^{N} w_j w_i\sigma_{ij} - 2\varrho\sum_{j\neq i}^{N} w_j\sigma_{ji}. \tag{A6}$$

Thus:

$$\begin{aligned} -\ln\beta + \varrho a_w - \kappa_{0i} - a_i &= \ln\left[E_t\left[\exp\left(q_{i,t+1}\right)\right]\right], \\ &= h_{it} + \frac{1}{2}\left(\theta_t^2 B_{iN} + \psi_t^2 C_{iN}\right) + \frac{1}{2}\left(\gamma_i - \varrho\gamma_w + \kappa_{1i}A_{1i}\right)^2\sigma_t^2 \\ &\quad +\frac{1}{2}\kappa_{1i}^2\varphi_\chi^2 A_{2i}^2 + \frac{1}{2}\kappa_{1i}^2\varphi_\eta^2 A_{3i,N}^2 + \frac{1}{2}\kappa_{1i}^2\varphi_\varpi^2 A_{4i,N}^2. \end{aligned}$$

Using the expression for h_{it} and matching the terms on both sides of the above equation we have:

$$\begin{aligned} -\left(1-\kappa_{1i}\right)A_{0i,N}+\kappa_{1i}(1-\phi_{\sigma})\sigma^{2}A_{2i}+\kappa_{1i}(1-\phi_{\psi})\psi^{2}A_{3i,N}+\kappa_{1i}(1-\phi_{\theta})\theta^{2}A_{4i,N}+...& \\ \frac{1}{2}\kappa_{1i}^{2}\varphi_{\chi}^{2}A_{2i}^{2}+\frac{1}{2}\kappa_{1i}^{2}\varphi_{\eta}^{2}A_{3i,N}^{2}+\frac{1}{2}\kappa_{1i}^{2}\varphi_{\varpi}^{2}A_{4i,N}^{2}+\ln\beta-\varrho a_{w}+\kappa_{0i}+a_{i} &= 0, \\ -\left(1-\phi_{f}\kappa_{1i}\right)A_{1i}+\gamma_{i}-\varrho\gamma_{w} &= 0, \\ -\left(1-\kappa_{1i}\phi_{\sigma}\right)A_{2i}+\frac{1}{2}\left(\gamma_{i}-\varrho\gamma_{w}+\kappa_{1i}A_{1i}\right)^{2} &= 0, \\ -\left(1-\kappa_{1i}\phi_{\psi}\right)A_{3i}+\frac{1}{2}C_{iN} &= 0, \\ -\left(1-\kappa_{1i}\phi_{\theta}\right)A_{4i}+\frac{1}{2}B_{iN} &= 0. \end{aligned}$$

Using (A5) and (A6), the above expressions can now be used to solve for A_{1i}, $A_{2i}, A_{3i,N}$ and $A_{4i,N}$:

$$A_{1i} = \frac{\left(\gamma_{i}-\varrho\gamma_{w}\right)\phi_{f}}{1-\kappa_{1i}\phi_{f}}, \tag{A7}$$

$$A_{2i} = \frac{1}{2}\frac{\left(\gamma_{i}-\varrho\gamma_{w}+\kappa_{1i}A_{1i}\right)^{2}}{1-\kappa_{1i}\phi_{\sigma}}, \tag{A8}$$

$$A_{3i,N} = \frac{1}{2}\frac{\varrho^{2}\sum_{j\neq i}^{N}w_{j}w_{i}\sigma_{ij}-2\varrho\sum_{j\neq i}^{N}w_{j}\sigma_{ji}}{1-\kappa_{1i}\phi_{\psi}}, \tag{A9}$$

$$A_{4i,N} = \frac{1}{2}\frac{\left(1-2\varrho w_{i}\right)\sigma_{ii}+\varrho^{2}\sum_{j=1}^{N}w_{j}^{2}\sigma_{jj}}{1-\kappa_{1i}\phi_{\theta}}, \tag{A10}$$

which in turn can be used in (A11) to solve for $A_{0i,N}$:

$$A_{0i,N}=\frac{1}{\left(1-\kappa_{1i}\right)}\left(\begin{array}{c} \ln\beta+a_{i}-\varrho a_{w}+\kappa_{0i}+\kappa_{1i}\sigma^{2}(1-\phi_{\sigma})A_{2i}+\kappa_{1i}(1-\phi_{\psi})\psi^{2}A_{3iN} \\ +\kappa_{1i}(1-\phi_{\theta})\theta^{2}A_{4iN}+\frac{1}{2}\kappa_{1i}^{2}\varphi_{\chi}^{2}A_{2i}^{2}+\frac{1}{2}\kappa_{1i}^{2}\varphi_{\eta}^{2}A_{3i,N}^{2}+\frac{1}{2}\kappa_{1i}^{2}\varphi_{\varpi}^{2}A_{4i,N}^{2} \end{array}\right). \tag{A11}$$

Equipped with a solution for z_{it}, we can then easily derive $r_{i,t+1}$, its innovation and conditional variance, as well as its realized volatility. Using (A1), (A2) and the assumed processes (1)-(5) we have:

$$\begin{aligned} r_{i,t+1} &= \kappa_{0i}+a_{i}+\gamma_{i}f_{t+1}+\kappa_{1i}\left(A_{0i}+A_{1i}f_{t+1}+A_{2i}\sigma_{t+1}^{2}+A_{3i,N}\psi_{t+1}^{2}+A_{4i,N}\theta_{t+1}^{2}\right) \\ &\quad -\left(A_{0i,N}+A_{1i}f_{t}+A_{2i}\sigma_{t}^{2}+A_{3i,N}\psi_{t}^{2}+A_{4i,N}\theta_{t}^{2}\right)+\varepsilon_{i,t+1}, \end{aligned}$$

which can be written as equation (10) in the paper (which we reproduce here for convenience):

$$\begin{aligned} r_{it} &= b_{0i,N}+b_{1i}f_{t-1}+b_{2i}\sigma_{t-1}^{2}+b_{3i,N}\psi_{t-1}^{2}+b_{4i,N}\theta_{t-1}^{2} \\ &\quad +c_{1i}\sigma_{t-1}\zeta_{t}+c_{2i}\chi_{t}+c_{3i,N}\eta_{t}+c_{4i,N}\varpi_{t}+\varepsilon_{it}, \end{aligned} \tag{A12}$$

where

$$\left\{\begin{array}{c} b_{0i,N}=\left[\kappa_{0i}-\left(1-\kappa_{1i}\right)A_{0i,N}+a_{i}\right]+\kappa_{1i}A_{2i}\sigma^{2}(1-\phi_{\sigma})+\kappa_{1i}A_{3i,N}(1-\phi_{\psi})+\kappa_{1i}A_{4i,N}(1-\phi_{\theta}) \\ b_{1i}=\gamma_{i}\phi_{f}-A_{1i}(1-\kappa_{1i}\phi_{f}),\ b_{2i}=-A_{2i}(1-\kappa_{1i}\phi_{\sigma}), \\ b_{3i,N}=-A_{3i,N}(1-\kappa_{1i}\phi_{\psi}),\ b_{4i,N}=-A_{4i,N}(1-\kappa_{1i}\phi_{\theta}), \\ c_{1i}=\kappa_{1i}A_{1i}+\gamma_{i},\quad c_{2i}=\kappa_{1i}A_{2i}\varphi_{\chi},\ c_{3i,N}=\kappa_{1i}A_{3i,N}\varphi_{\eta},\ c_{4i,N}=\kappa_{1i}A_{4i}\varphi_{\varpi}. \end{array}\right. \tag{A13}$$

Hence, the innovation to the risky return is given by:

$$r_{i,t+1}-E_t\left(r_{i,t+1}\right)=\left(\gamma_i+\kappa_{1i}A_{1i}\right)\sigma_t\zeta_{t+1}+\kappa_{1i}A_{2i}\varphi_\chi\chi_{t+1}+\kappa_{1i}A_{3i,N}\varphi_\eta\eta_{t+1}+\kappa_{1i}A_{4i,N}\varphi_\varpi\varpi_{t+1}+\varepsilon_{i,t+1},$$

while the conditional variance of the return is:

$$Var_t\left(r_{i,t+1}\right)=E_t\Big[r_{i,t+1}-E_t\left(r_{i,t+1}\right)\Big]^2=\sigma_{ii}\theta_t^2+\left(\kappa_{1i}A_{1i}+\gamma_i\right)^2\sigma_t^2+\kappa_{1i}^2G_{iN}^2. \tag{A14}$$

where

$$G_{iN}^2=\varphi_\chi^2A_{2i}^2+\varphi_\eta^2A_{3i,N}^2+\varphi_\varpi^2A_{4i,N}^2. \tag{A15}$$

Risk Premium The risk premium can be derived by noting that the conditional mean of the equity return is given by:

$$E_t\left(r_{i,t+1}\right)=\pi_{iN}+\left[\gamma_i\phi_f-A_{1i}(1-\kappa_{1i}\phi_f)\right]f_t-A_{2i}(1-\kappa_{1i}\phi_\sigma)\sigma_t^2-A_{3i,N}(1-\kappa_{1i}\phi_\psi)\psi_t^2-A_{4i,N}(1-\kappa_{1i}\phi_\theta)\theta_t^2,$$

where

$$\pi_{iN}=a_i+\kappa_{0i}-(1-\kappa_{1i})A_{i0,N}+\kappa_{1i}H_{iN}, \tag{A16}$$

and

$$H_{iN}=\sigma^2(1-\phi_\sigma)A_{2i}+(1-\phi_\psi)A_{3i,N}+(1-\phi_\theta)A_{4i,N}. \tag{A17}$$

Subtracting the risk-free rate, Equation (9), we now have:

$$\begin{aligned} E_t\left(r_{i,t+1}-r_{t+1}^f\right) &= \pi_{iN}+\ln\beta-\varrho a_w+\left[\gamma_i\phi_f-A_{1i}(1-\kappa_{1i}\phi_f)-\varrho\gamma_w\phi_f\right]f_t \\ &\quad -\left[A_{2i}(1-\kappa_{1i}\phi_\sigma)-\frac{1}{2}\varrho^2\gamma_w^2\right]\sigma_t^2-A_{3i,N}(1-\kappa_{1i}\phi_\psi)\psi_t^2 \\ &\quad -A_{4i,N}(1-\kappa_{1i}\phi_\theta)\theta_t^2+\frac{1}{2}\varrho^2\mathbf{w}'\boldsymbol{\Sigma}_{t\varepsilon}\mathbf{w}. \end{aligned} \tag{A18}$$

Also since $\varepsilon_{w,t+1}=\sum_{i=1}^N w_i\varepsilon_{i,t+1}=w'\varepsilon_{t+1}$, then $Cov_t\left(\varepsilon_{i,t+1},\varepsilon_{w,t+1}\right)=w_i\theta_t^2\sigma_{ii}+\psi_t^2\sum_{j\neq i}^N w_j\sigma_{ji}$, and we have

$$\mathbf{w}'\boldsymbol{\Sigma}_{t\varepsilon}\mathbf{w}=\sum_{ij=1}^N w_iw_j\sigma_{t,ij}=\left(\sum_{i=1}^N w_i^2\sigma_{ii}\right)\theta_t^2+\left(\sum_{i\neq j}^N w_iw_j\sigma_{ij}\right)\psi_t^2.$$

The expression for $E_t\left(r_{i,t+1}-r_{t+1}^f\right)$ can now be simplified using (A7)-(A10). First note that

$$\left[\gamma_i\phi_f-A_{1i}(1-\kappa_{1i}\phi_f)-\varrho\gamma_w\phi_f\right]=\left(\gamma_i-\varrho\gamma_w\right)\phi_f-\left(\gamma_i-\varrho\gamma_w\right)\phi_f=0.$$

Also letting $c_{1i}=\gamma_i+\kappa_{1i}A_{1i}$, and using (A8) we obtain

$$-\left[A_{2i}(1-\kappa_{1i}\phi_\sigma)-\frac{1}{2}\varrho^2\gamma_w^2\right]=-\frac{1}{2}\left(\gamma_i+\kappa_{1i}A_{1i}-\varrho\gamma_w\right)^2+\frac{1}{2}\varrho^2\gamma_w^2=-\frac{1}{2}c_{1i}^2+\varrho\gamma_wc_{1i}.$$

Finally, using (A9) and (A10) we have

$$A_{3i,N}(1-\kappa_{1i}\phi_\psi)=\frac{1}{2}\left(\varrho^2\sum_{j\neq i}^N w_jw_i\sigma_{ij}-2\varrho\sum_{j\neq i}^N w_j\sigma_{ji}\right),\ A_{4i,N}(1-\kappa_{1i}\phi_\theta)=\frac{1}{2}\left(\sigma_{ii}+\varrho^2\sum_{j=1}^N w_j^2\sigma_{jj}-2\varrho w_i\sigma_{ii}\right).$$

Substituting the above result in (A18) we now obtain

$$\begin{aligned} E_t\left(r_{i,t+1}-r^f_{t+1}\right) = \;& \pi_{iN}+\ln\beta-\varrho a_w+\varrho\gamma_w c_{1i}\sigma_t^2-\tfrac{1}{2}c_{1i}^2\sigma_t^2 \\ & -\tfrac{1}{2}\left(\varrho^2\textstyle\sum_{j\neq i}^N w_j w_i\sigma_{ij}-2\varrho\sum_{j\neq i}^N w_j\sigma_{ji}\right)\psi_t^2 \\ & -\tfrac{1}{2}\left(\sigma_{ii}+\varrho^2\textstyle\sum_{j=1}^N w_j^2\sigma_{jj}-2\varrho w_i\sigma_{ii}\right)\theta_t^2 \\ & +\tfrac{1}{2}\varrho^2\left(\textstyle\sum_{i=1}^N w_i^2\sigma_{ii}\right)\theta_t^2+\tfrac{1}{2}\varrho^2\left(\textstyle\sum_{i\neq j}^N w_i w_j\sigma_{ij}\right)\psi_t^2, \end{aligned}$$

which after some algebra yields:

$$E_t\left(r_{i,t+1}-r^f_{t+1}\right) = \pi_{iN}+\ln\beta-\varrho a_w+\varrho\gamma_w c_{1i}\sigma_t^2-\tfrac{1}{2}c_{1i}^2\sigma_t^2-\tfrac{1}{2}\sigma_{ii}\theta_t^2+\varrho Cov_t\left(\varepsilon_{w,t+1},\varepsilon_{i,t+1}\right).$$

The intercept term of the above can also be simplified. Using (A11) we first note that A_{0i} can be written as using (A16) and (A17) as

$$(1-\kappa_{1i})A_{0i,N}=\ln(\beta)+a_i-\varrho a_w+\kappa_{0i}+\kappa_{1i}H_i+\frac{1}{2}\kappa_{1i}^2G_{iN}^2, \tag{A19}$$

where H_i is defined by (A17) and

$$G_{iN}^2=\varphi_\chi^2A_{2i}^2+\varphi_\eta^2A_{3i,N}^2+\varphi_\varpi^2A_{4i,N}^2. \tag{A20}$$

Hence using (A19) in (A16) we have

$$\pi_{iN}=-\ln(\beta)+\varrho a_w-\frac{1}{2}\kappa_{1i}^2G_{iN}^2. \tag{A21}$$

Using this expression in (A22) now provides the following result for the country-specific risk premia

$$E_t\left(r_{i,t+1}-r^f_{t+1}\right) = -\tfrac{1}{2}\kappa_{1i}^2G_{iN}^2+\varrho\gamma_w c_{1i}\sigma_t^2-\tfrac{1}{2}c_{1i}^2\sigma_t^2-\tfrac{1}{2}\sigma_{ii}\theta_t^2+\varrho Cov_t\left(\varepsilon_{w,t+1},\varepsilon_{i,t+1}\right). \tag{A22}$$

Finally, consider now the unconditional mean of the excess return, and taking the unconditional expectations of equation (A18) and using the above expression for π_i we have:

$$E\left(r_{i,t+1}-r^f_{t+1}\right)=\varrho\gamma_w c_{1i}\sigma^2-\frac{1}{2}c_{1i}^2\sigma^2-\frac{1}{2}\sigma_{ii}+\varrho\sum_{j=1}^N w_j\sigma_{ji}-\frac{1}{2}\kappa_{1i}^2G_{iN}^2. \tag{A23}$$

The equilibrium/steady state risk premium given by (A23) is a complicated function of ϱ and all the risk parameters, σ^2, σ_{ii}, φ_χ^2, φ_η^2, and φ_ϖ^2. It also depends on $\varrho\sum_{j=1}^N w_j\sigma_{ji}$, which vanishes (as we show below) only if the country-specific shocks are weakly correlated and N is sufficiently large.

Weakly Correlated Country-specific Shocks Consider first the aggregate risk factor $\varepsilon_{w,t+1}$ in the SDF innovation (8) and its conditional variance $Var_t(\varepsilon_{w,t+1})$:

$$Var_t(\varepsilon_{w,t+1})=\mathbf{w}'\mathbf{\Sigma}_{t\varepsilon}\mathbf{w}=\sum_{ij=1}^N w_iw_j\sigma_{t,ij}=\left(\sum_{i=1}^N w_i^2\sigma_{ii}\right)\theta_t^2+\left(\sum_{i\neq j}^N w_iw_j\sigma_{ij}\right)\psi_t^2.$$

Under our assumptions country-specific shocks, ε_{it}, are weakly cross correlated such that $sup_i\sum_{j\neq i}^N|\sigma_{ij}|<C_0<\infty$. Also by standard result on matrix norms we have $\lambda_{\max}(\mathbf{\Sigma}_{t\varepsilon})\leq sup_i\sum_{j\neq i}^N|\sigma_{t,ij}|$ (See, for example Theorem 5.6.9 of Horn and Johnson (1985)). Hence (noting that θ_t^2 and ψ_t^2 are given at time t, and σ_{ii} is bounded)

$$\lambda_{\max}\left(\mathbf{\Sigma}_{t\varepsilon}\right) \leq \sup_i \sum_{j=1}^{N} |\sigma_{t,ij}| = \theta_t^2 \sup_i \sigma_{ii} + \psi_t^2 \sup_i \sum_{j\neq i}^{N} |\sigma_{ij}| < C_1, \tag{A24}$$

and it immediately follows that:

$$Var_t(\varepsilon_{w,t+1}) = \mathbf{w}'\mathbf{\Sigma}_{t\varepsilon}\mathbf{w} \leq \left(\mathbf{w}'\mathbf{w}\right)\lambda_{\max}\left(\mathbf{\Sigma}_{t\varepsilon}\right) \leq C_1\left(\mathbf{w}'\mathbf{w}\right), \tag{A25}$$

which establishes that $Var_t(\varepsilon_{w,t+1}) = O\left(N^{-1}\right)$, given the granularity of the weight vector $\mathbf{w}$. Moreover, if N is sufficiently large (assuming that $1-\kappa_{1i}\phi_\theta \neq 0$ and $1-\kappa_{1i}\phi_\psi \neq 0$) we also have:

$$A_{0i,N} = A_{0i} + O\left(N^{-1}\right),\ A_{3i,N} = O\left(N^{-1}\right),\ \text{and } A_{4i,N} = A_{4i} + O(N^{-1}), \tag{A26}$$

where

$$A_{4i} = \frac{1}{2}\left(\frac{\sigma_{ii}}{1-\kappa_{1i}\phi_\theta}\right),\ A_{0i} = \frac{1}{(1-\kappa_{1i})}\left(\begin{array}{c} \ln\beta + a_i + \kappa_{0i} - \varrho a_w + \kappa_{1i}\sigma^2(1-\phi_\sigma)A_{2i} \\ +\kappa_{1i}(1-\phi_\theta)\theta^2 A_{4i} + \frac{1}{2}\kappa_{1i}^2\varphi_\chi^2 A_{2i}^2 + \frac{1}{2}\kappa_{1i}^2\varphi_\varpi^2 A_{4i,\infty}^2 \end{array}\right). \tag{A27}$$

To establish the result for $A_{3i,N}$ in (A26), using (A9) note that

$$\begin{aligned} 2\left(1-\kappa_{1i}\phi_\psi\right)A_{3i,N} &= \varrho^2\sum_{j\neq i}^{N} w_j w_i \sigma_{ij} - 2\varrho\sum_{j\neq i}^{N} w_j\sigma_{ji} \\ &= \varrho^2\mathbf{w}'\mathbf{\Sigma}_\varepsilon\mathbf{w} - \varrho^2\sum_{i=1}^{N} w_i^2\sigma_{ii} - 2\varrho\sum_{j\neq i}^{N} w_j\sigma_{ji}, \end{aligned}$$

where $\Sigma_\varepsilon = (\sigma_{ij})$. Then:

$$|A_{3i,N}| \leq \frac{1}{2}\left|\frac{1}{1-\kappa_{1i}\phi_\psi}\right|\left[\varrho^2\left(\mathbf{w}'\mathbf{w}\right)\lambda_{max}\left(\mathbf{\Sigma}_\varepsilon\right) + \varrho^2\left(\sup_i \sigma_{ii}\right)\left(\mathbf{w}'\mathbf{w}\right) + 2|\varrho|\sup_j |w_j|\sup_i \sum_{j=1}^{N}|\sigma_{ji}|\right].$$

But under the assumptions made $\lambda_{max}\left(\mathbf{\Sigma}_\varepsilon\right) < C_0$, $(\mathbf{w}'\mathbf{w}) = O(N^{-1})$, $\sup_j |w_j| = O(N^{-1})$ and $\sup_i \sum_{j=1}^{N}|\sigma_{ji}| < C_1$, which establishes that $|A_{3i,N}| = O(N^{-1})$.

The results for $A_{0i,N}$ and $A_{4i,N}$ follow similarly. Using the above results it is also easily established that

$$\begin{aligned} b_{0i,N} &= \left[\kappa_{0i} - (1-\kappa_{1i})A_{0i} + a_i\right] + \kappa_{1i}A_{2i}(1-\phi_\sigma)\sigma^2 + \kappa_{1i}A_{4i}(1-\phi_\theta) + O(N^{-1}), \\ b_{3i,N} &= O\left(N^{-1}\right),\ b_{4i,N} = -\frac{1}{2}\left(\frac{\sigma_{ii}}{1-\kappa_{1i}\phi_\theta}\right) + O(N^{-1}), \\ c_{3i,N} &= O\left(N^{-1}\right),\ c_{4i,N} = \frac{1}{2}\left(\frac{\kappa_{1i}\sigma_{ii}\varphi_\varpi}{1-\kappa_{1i}\phi_\theta}\right) + O(N^{-1}), \end{aligned}$$

where A_{0i} and A_{4i} are given by (A27). Also note that b_{1i}, b_{2i}, c_{1i} and c_{2i} do not depend on N, and are given as before (see (A13)).

Finally, under weakly cross correlated country-specific shocks we have (using (A22) and (A23))

$$E_t\left(r_{i,t+1} - r_{t+1}^f\right) = -\tfrac{1}{2}\kappa_{1i}^2 G_i^2 + \varrho\gamma_w c_{1i}\sigma_t^2 - \tfrac{1}{2}c_{1i}^2\sigma_t^2 - \tfrac{1}{2}\sigma_{ii}\theta_t^2 + O(N^{-1}),$$

and

$$E\left(r_{i,t+1}-r_{t+1}^{f}\right)=\varrho\gamma_w c_{1i}\sigma^2-\frac{1}{2}c_{1i}^2\sigma^2-\frac{1}{2}\sigma_{ii}-\frac{1}{2}\kappa_{1i}^2G_i^2+O(N^{-1}),$$

where

$$G_i^2=\varphi_\chi^2A_{2i}^2+\frac{1}{2}\left(\frac{\sigma_{ii}\varphi_\varpi^2}{1-\kappa_{1i}\phi_\theta}\right). \tag{A28}$$

Realized Volatility At daily frequency within a given quarter t, the daily returns can be written as:

$$\begin{aligned} r_{it}(\tau) \;=\;& b_{0i,N}+b_{1i}f_{t-1}(\tau)+b_{2i}\sigma_{t-1}^2(\tau)+b_{3i,N}\psi_{t-1}^2(\tau)+b_{4i,N}\theta_{t-1}^2(\tau) \\ &+c_{1i}\sigma_{t-1}(\tau)\zeta_t(\tau)+c_{2i}\chi_t(\tau)+c_{3i,N}\eta_t(\tau)+c_{4i,N}\varpi_t(\tau)+\varepsilon_{it}(\tau), \end{aligned} \tag{A29}$$

where $r_{it}(\tau)$ is the return on country i^{th} return on risky asset for day τ in quarter t. Similarly, $f_{t-1}(\tau)$ stands for the realization of factor f_t for day τ in quarter $t-1$, and etc. Therefore, quarterly realized volatility associated to r_{it} is given by (assuming there are D_t days in quarter t):

$$\sigma_{it}^2=\sum_{\tau=1}^{D_t}\left[r_{it}(\tau)-\bar{r}_{it}\right]^2,$$

where $\bar{r}_{it}=D_t^{-1}\sum_{\tau=1}^{D_t}r_{it}(\tau)$. It is now easily seen that realized return volatilities are complicated functions of lagged realized volatilities of the growth factor, f_t, realized volatility of the innovations to the growth factor, the realized volatilities of the country-specific shocks and their correlations (when N is finite), and their many interactions. Specifically, $r_{it}(\tau)-\bar{r}_{it}$ is given by:

$$\begin{aligned} & b_{1i}\left[f_{t-1}(\tau)-\bar{f}_{t-1}\right]+b_{2i}\left[\sigma_{t-1}^2(\tau)-\bar{\sigma}_{t-1}^2\right]+b_{3i,N}\left[\psi_{t-1}^2(\tau)-\bar{\psi}_{t-1}^2\right]+b_{4i,N}\left[\theta_{t-1}^2(\tau)-\bar{\theta}_{t-1}^2\right]\ldots \\ & +c_{1i}\left[\sigma_{t-1}(\tau)v_t(\tau)-\overline{\sigma_{t-1}v_t}\right]+c_{2i}\left[\chi_t(\tau)-\bar{\chi}_t\right]+c_{3i,N}\left[\eta_t(\tau)-\bar{\eta}_t\right]+c_{4i,N}\left[\varpi_t(\tau)-\bar{\varpi}_t\right]+\left[\varepsilon_{it}(\tau)-\bar{\varepsilon}_{it}\right], \end{aligned}$$

where the bar on the variables denotes the sample mean of daily values in a given quarter. To simplify notation, we set the sample means of the shocks to zero (namely $\overline{\sigma_{t-1}v_t}=0$, $\bar{\chi}_t=0$, etc.), and note that the realized volatility of r_{it}, denoted by σ_{it}^2, is given by the following long expression:

$$\begin{aligned} \sigma_{it}^2 \;=\;& b_{1i}^2\textstyle\sum_{\tau=1}^{D_t}\left[f_{t-1}(\tau)-\bar{f}_{t-1}\right]^2+b_{2i}^2\sum_{\tau=1}^{D_t}\left[\sigma_{t-1}^2(\tau)-\bar{\sigma}_{t-1}^2\right]^2+b_{3i,N}^2\sum_{\tau=1}^{D_t}\left[\psi_{t-1}^2(\tau)-\bar{\psi}_{t-1}^2\right]^2 \\ &+b_{4i,N}^2\textstyle\sum_{\tau=1}^{D_t}\left[\theta_{t-1}^2(\tau)-\bar{\theta}_{t-1}^2\right]^2+c_{1i}^2\sum_{\tau=1}^{D_t}\sigma_{t-1}^2(\tau)\zeta_t^2(\tau)+c_{2i}^2\sum_{\tau=1}^{D_t}\chi_t^2(\tau)+c_{3i,N}^2\sum_{\tau=1}^{D_t}\eta_t^2(\tau) \\ &+c_{4i,N}^2\textstyle\sum_{\tau=1}^{D_t}\varpi_t^2(\tau)+\sum_{\tau=1}^{D_t}\varepsilon_{it}^2(\tau)+2b_{1i}b_{2i}\sum_{\tau=1}^{D_t}\left[f_{t-1}(\tau)-\bar{f}_{t-1}\right]\left[\sigma_{t-1}^2(\tau)-\bar{\sigma}_{t-1}^2\right] \\ &+\text{other cross terms.} \end{aligned}$$

Appendix B: Proofs

Proof of Proposition 1. Using the country-specific models given by (30), and solving for $\mathbf{z}_{it}$ in terms of current and past values of factors and shocks we have:

$$\mathbf{z}_{it}=\boldsymbol{\mu}_i+\sum_{\ell=0}^{\infty}\boldsymbol{\Phi}_i^{\ell}\boldsymbol{\Gamma}_i\boldsymbol{\delta}_{t-\ell}+\varkappa_{it}, \tag{A30}$$

where

$$\boldsymbol{\mu}_i=(\mathbf{I}_2-\boldsymbol{\Phi}_i)^{-1}\mathbf{a}_i,\ \boldsymbol{\delta}_t=(\zeta_t,\xi_t)',\ \varkappa_{it}=\sum_{\ell=0}^{\infty}\boldsymbol{\Phi}_i^{\ell}\boldsymbol{\vartheta}_{i,t-\ell},\text{ and }\boldsymbol{\vartheta}_{it}=(\eta_{it},\varepsilon_{it})'. \tag{A31}$$

Assumption 4, ensures that the infinite sums are convergent. Pre-multiplying both sides of (A30) by (w_i) and summing over i yields:

$$\bar{\mathbf{z}}_{\omega t}=\bar{\boldsymbol{\mu}}_{\omega}+\sum_{\ell=0}^{\infty}\mathbf{A}_{\ell,N}\boldsymbol{\delta}_{t-\ell}+\bar{\varkappa}_{\omega t}, \tag{A32}$$

where

$$\begin{array}{l}\bar{\mathbf{z}}_{\omega t}=\sum_{i=1}^{N}w_i\mathbf{z}_{it},\quad \bar{\boldsymbol{\mu}}_{\omega}=\sum_{i=1}^{N}w_i\boldsymbol{\mu}_i,\\ \mathbf{A}_{\ell,N}=\sum_{i=1}^{N}w_i\boldsymbol{\Phi}_i^{\ell}\boldsymbol{\Gamma}_i,\ \text{ and } \bar{\varkappa}_{\omega t}=\sum_{i=1}^{N}w_i\varkappa_{it}.\end{array} \tag{A33}$$

Under Assumption 3, $\varkappa_{it}$ are cross-sectionally weakly correlated and the weights $w=(w_1,w_2,...,w_N)'$ are granular. Using the results in Pesaran and Chudik (2014), it readily follows that:

$$\bar{\varkappa}_{\omega t}=O\left(\|\mathbf{w}\|\right)=O\left(N^{-1/2}\right),\ \text{ for each } t. \tag{A34}$$

Under Assumptions 3 and 4, we also have

$$\mathbb{E}\left(\boldsymbol{\Phi}_i^{\ell}\boldsymbol{\Gamma}_i\right)=\mathbb{E}\left(\boldsymbol{\Phi}_i^{\ell}\right)\mathbb{E}\left(\boldsymbol{\Gamma}_i\right)=\boldsymbol{\Lambda}_{\ell}\boldsymbol{\Gamma},$$

and since $\boldsymbol{\Phi}_i$ and $\boldsymbol{\Gamma}_i$ are distributed independently across i, using again results in Pesaran and Chudik (2014) we have:

$$\mathbf{A}_{\ell,N}-\mathbb{E}\left(\mathbf{A}_{\ell,N}\right)=\sum_{i=1}^{N}w_i\left[\boldsymbol{\Phi}_i^{\ell}\boldsymbol{\Gamma}_i-\mathbb{E}\left(\boldsymbol{\Phi}_i^{\ell}\boldsymbol{\Gamma}_i\right)\right]=O\left(\|\mathbf{w}\|\right)=O\left(N^{-1/2}\right). \tag{A35}$$

Using (A34) and (A35) in (A32), and setting $\boldsymbol{\Lambda}(L)=\sum_{\ell=0}^{\infty}\boldsymbol{\Lambda}_{\ell}L^{\ell}$ we now have:

$$\bar{\mathbf{z}}_{\omega t}=\bar{\boldsymbol{\mu}}_{\omega}+\boldsymbol{\Lambda}(L)\boldsymbol{\Gamma}\mathbf{f}_t+O_p\left(N^{-1/2}\right).$$

But under Assumptions 3 and 4, $\boldsymbol{\Gamma}$ and $\boldsymbol{\Lambda}(L)$ are both invertible and:

$$\boldsymbol{\delta}_t=\boldsymbol{\Gamma}^{-1}\boldsymbol{\Lambda}^{-1}(L)\left(\bar{\mathbf{z}}_{\omega t}-\bar{\boldsymbol{\mu}}_{\omega}\right)+O_p\left(N^{-1/2}\right),$$

where $\boldsymbol{\Lambda}^{-1}(L)=\sum_{\ell=0}^{\infty}\mathbf{B}_{\ell}L^{\ell}$, with $\mathbf{B}_0=\boldsymbol{\Lambda}_0=\mathbf{I}_2$, and

$$\boldsymbol{\Gamma}^{-1}=\begin{pmatrix}0 & \gamma^{-1}\\ \theta^{-1} & -\frac{\lambda}{\theta\gamma}\end{pmatrix}.$$

Hence,

$$\begin{aligned}\boldsymbol{\delta}_t &= \boldsymbol{\Gamma}^{-1}\left(\bar{\mathbf{z}}_{\omega t}-\bar{\boldsymbol{\mu}}_{\omega}\right)+\left(\mathbf{C}_1+\mathbf{C}_2L+\mathbf{C}_3L^2+....\right)\left(\bar{\mathbf{z}}_{\omega,t-1}-\bar{\boldsymbol{\mu}}_{\omega}\right)+O_p\left(N^{-1/2}\right)\\ &= \mathbf{b}+\left(\sum_{\ell=0}^{\infty}\mathbf{C}_{\ell}L^{\ell}\right)\bar{\mathbf{z}}_{\omega,t}+O_p\left(N^{-1/2}\right),\end{aligned}$$

where $\mathbf{C}_\ell = \mathbf{\Gamma}^{-1}\mathbf{B}_\ell$, for $\ell = 0, 1, 2, ...$, and $\mathbf{b} = -\mathbf{\Gamma}^{-1}\mathbf{\Lambda}^{-1}(1)\,\bar{\boldsymbol{\mu}}_\omega$. But given the lower triangular form of $\mathbf{\Gamma}^{-1}$, we have

$$\zeta_t = \gamma^{-1}\Delta\bar{y}_{\omega t} + \sum_{\ell=1}^{\infty}\mathbf{c}'_{1,\ell}\bar{\mathbf{z}}_{\omega,t-\ell} + O_p\left(N^{-1/2}\right), \tag{A36}$$

$$\xi_t = \theta^{-1}\bar{v}_{\omega t} - \left(\frac{\lambda}{\theta\gamma}\right)\Delta\bar{y}_{\omega t} + \sum_{\ell=1}^{\infty}\mathbf{c}'_{2,\ell}\bar{\mathbf{z}}_{\omega,t-\ell} + O_p\left(N^{-1/2}\right), \tag{A37}$$

where $\mathbf{c}'_{1,\ell}$ and $\mathbf{c}'_{2,\ell}$ are the first and the second rows of $\mathbf{C}_\ell$, respectively, and $\bar{v}_{\omega t}$, $\Delta\bar{y}_{\omega t}$, $\bar{\mathbf{z}}_{\omega t}$ are defined as above. Consider now $\mathbf{C}_\ell$ and note that $\|\mathbf{C}_\ell\| \leq \left\|\mathbf{\Gamma}^{-1}\right\|\|\mathbf{B}_\ell\|$, where $\left\|\mathbf{\Gamma}^{-1}\right\|$ is bounded for fixed non-zero values of γ and θ. Further, $\mathbf{B}_\ell$ is given by the recursions $\mathbf{B}_\ell = -\sum_{i=1}^{\ell}\mathbf{\Lambda}_i\mathbf{B}_{\ell-i}$, for $\ell = 0, 1, ...$, with $\mathbf{B}_0 = \mathbf{I}_2$, and $\mathbf{B}_\ell = 0$, for $\ell < 0$. Hence, $\|\mathbf{B}_\ell\| \leq \sum_{i=1}^{\ell}\|\mathbf{\Lambda}_i\|\,\|\mathbf{B}_{\ell-i}\|$, where $\|\mathbf{B}_0\| = 1$. However,

$$\|\mathbf{\Lambda}_\ell\| = \left\|\mathbb{E}\left(\mathbf{\Phi}_i^\ell\right)\right\| \leq \mathbb{E}\left\|\mathbf{\Phi}_i^\ell\right\| \leq \left(\mathbb{E}\,\|\mathbf{\Phi}_i\|\right)^\ell \leq \rho^\ell.$$

Hence, $\|\mathbf{B}_1\| \leq \rho$, $\|\mathbf{B}_2\| \leq \rho^2$, and so on, and as required $\|\mathbf{C}_\ell\| \leq \ell\left\|\mathbf{\Gamma}^{-1}\right\|\rho^\ell$.[27] ■

Proof of Proposition 2. Consider equations (34) and (35) in the paper. Let $\mathbf{M}_{\bar{Z}_\omega} = \mathbf{I}_T - \bar{\mathbf{Z}}_\omega\left(\bar{\mathbf{Z}}'_\omega\bar{\mathbf{Z}}_\omega\right)^{-1}\bar{\mathbf{Z}}'_\omega$, and note that:

$$\mathbf{M}_{\bar{Z}_\omega}\boldsymbol{\zeta} = \mathbf{M}_{\bar{Z}_\omega}\Delta\bar{\mathbf{y}}_\omega, \qquad \mathbf{M}_{\bar{Z}_\omega}\boldsymbol{\xi} = \mathbf{M}_{\bar{Z}_\omega}\bar{\mathbf{v}}_\omega - \lambda\mathbf{M}_{\bar{Z}_\omega}\Delta\bar{\mathbf{y}}_\omega,$$

since $\mathbf{M}_{\bar{Z}_\omega}\bar{\mathbf{Z}}_\omega = 0$. We set the first normalized vector of innovations, denoted by $\hat{\boldsymbol{\zeta}}$, to $\mathbf{M}_{\bar{Z}_\omega}\boldsymbol{\zeta}$, namely $\hat{\boldsymbol{\zeta}} = \mathbf{M}_{\bar{Z}_\omega}\Delta\bar{\mathbf{y}}_\omega$, and set the second factor, that we label $\hat{\boldsymbol{\xi}}$, as the linear combination of $\mathbf{M}_{\bar{Z}_\omega}\boldsymbol{\zeta}$ and $\mathbf{M}_{\bar{Z}_\omega}\boldsymbol{\xi}$, such that $\hat{\boldsymbol{\zeta}}'\hat{\boldsymbol{\xi}} = 0$. This can be achieved by selecting λ so that:

$$\hat{\boldsymbol{\zeta}}'\hat{\boldsymbol{\xi}} = \Delta\bar{\mathbf{y}}'_\omega\mathbf{M}_{\bar{Z}_\omega}\left(\mathbf{M}_{\bar{Z}_\omega}\bar{\mathbf{v}}_\omega - \lambda\mathbf{M}_{\bar{Z}_\omega}\Delta\bar{\mathbf{y}}_\omega\right) = 0.$$

The value of λ that solves this equation is given by, $\hat{\lambda} = \left(\Delta\bar{\mathbf{y}}'_\omega\mathbf{M}_{\bar{Z}_\omega}\bar{\mathbf{v}}_\omega\right)/\left(\Delta\bar{\mathbf{y}}'_\omega\mathbf{M}_{\bar{Z}_\omega}\Delta\bar{\mathbf{y}}_\omega\right)$, where $\hat{\lambda}$ is the OLS estimator of the coefficient of the regression of $\mathbf{M}_{\bar{Z}_\omega}\bar{\mathbf{v}}_\omega$ on $\mathbf{M}_{\bar{Z}_\omega}\boldsymbol{\Delta}\bar{\mathbf{y}}_\omega$. Hence, the orthogonalized factors are

$$\hat{\boldsymbol{\zeta}} = \mathbf{M}_{\bar{Z}_\omega}\Delta\bar{\mathbf{y}}_\omega, \quad \text{and} \quad \hat{\boldsymbol{\xi}} = \mathbf{M}_{\bar{Z}_\omega}\bar{\mathbf{v}}_\omega - \hat{\lambda}\mathbf{M}_{\bar{Z}_\omega}\Delta\bar{\mathbf{y}}_\omega.$$

In practice, this implies that $\hat{\boldsymbol{\zeta}}$ can be recovered as residuals from the OLS regression of $\boldsymbol{\Delta}\bar{\mathbf{y}}_\omega$ on an intercept and $\bar{\mathbf{z}}_{\omega,t-\ell}$, for $\ell = 1, 2, ..., p$:

$$\Delta\bar{\mathbf{y}}_\omega = \bar{\mathbf{Z}}_\omega\hat{\mathbf{c}}_1 + \hat{\boldsymbol{\zeta}}. \tag{A38}$$

While $\hat{\boldsymbol{\xi}}$ can be recovered as residuals from the OLS regression of $\bar{v}_\omega$ on $\hat{\boldsymbol{\zeta}}$, an intercept, and $\bar{\mathbf{z}}_{\omega,t-\ell}$, for $\ell = 1, 2, ..., p$:

$$\bar{\mathbf{v}}_\omega = \hat{\lambda}\hat{\boldsymbol{\zeta}} + \bar{\mathbf{Z}}_\omega\hat{\mathbf{c}}_2 + \hat{\boldsymbol{\xi}}. \tag{A39}$$

■

[27] Note that for any matrix $\mathbf{A}$, $\|\mathbf{A}^p\| \leq \|\mathbf{A}\|^p$, and for any random variable X, $\|E(X)\| \leq E\,\|X\|$.

References

Aguiar, M. and G. Gopinath (2007): "Emerging Market Business Cycles: The Cycle Is the Trend," *Journal of Political Economy*, 115, 69–102.

Andersen, T. G., T. Bollerslev, F. X. Diebold, and P. Labys (2001): "The Distribution of Realized Exchange Rate Volatility," *Journal of the American Statistical Association*, 96, 42–55.

——— (2003): "Modeling and Forecasting Realized Volatility," *Econometrica*, 71, 579–625.

Bachmann, R. and C. Bayer (2013): "'Wait-and-See' business cycles?" *Journal of Monetary Economics*, 60, 704–719.

Bachmann, R., S. Elstner, and E. R. Sims (2013): "Uncertainty and Economic Activity: Evidence from Business Survey Data," *American Economic Journal: Macroeconomics*, 5, 217–249.

Backus, D. K., P. J. Kehoe, and F. E. Kydland (1992): "International Real Business Cycles," *Journal of Political Economy*, 100, 745–775.

Bai, J. and S. Ng (2002): "Determining the Number of Factors in Approximate Factor Models," *Econometrica*, 70, 191–221.

Bailey, N., G. Kapetanios, and M. H. Pesaran (2016): "Exponent of Cross-Sectional Dependence: Estimation and Inference," *Journal of Applied Econometrics*, 31, 929–960.

——— (2018a): "Exponent of Cross-sectional Dependence for Residuals," CESifo Working Paper Series 7223, CESifo Group Munich.

Bailey, N., M. H. Pesaran, and L. V. Smith (2018b): "A multiple testing approach to the regularisation of large sample correlation matrices," Forthcoming in the *Journal of Econometrics*.

Baker, S. R., N. Bloom, and S. J. Davis (2016): "Measuring Economic Policy Uncertainty," *The Quarterly Journal of Economics*, 131, 1593–1636.

——— (2018): "Does Uncertainty Reduce Growth? Using Disasters as Natural Experiments," Unpublished manuscript, https://sites.google.com/site/srbaker/academic-work.

Bansal, R. and A. Yaron (2004): "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles," *Journal of Finance*, 59, 1481–1509.

Barndorff-Nielsen, O. E. and N. Shephard (2002): "Estimating quadratic variation using realized variance," *Journal of Applied Econometrics*, 17, 457–477.

——— (2004): "Econometric Analysis of Realized Covariation: High Frequency Based Covariance, Regression, and Correlation in Financial Economics," *Econometrica*, 72, 885–925.

Basu, S. and B. Bundick (2017): "Uncertainty Shocks in a Model of Effective Demand," *Econometrica*, 85, 937–958.

Baxter, M. and M. J. Crucini (1995): "Business Cycles and the Asset Structure of Foreign Trade," *International Economic Review*, 36, 821–854.

Berger, D., I. Dew-Becker, and S. Giglio (2017): "Uncertainty Shocks as Second-Moment News Shocks," NBER Working Papers 23796, National Bureau of Economic Research, Inc.

Bernanke, B. S. (1983): "Irreversibility, Uncertainty, and Cyclical Investment," *The Quarterly Journal of Economics*, 98, 85–106.

Bloom, N. (2009): "The Impact of Uncertainty Shocks," *Econometrica*, 77, 623–685.

——— (2014): "Fluctuations in Uncertainty," *Journal of Economic Perspectives*, 28, 153–176.

Bloom, N., S. Bond, and J. V. Reenen (2007): "Uncertainty and Investment Dynamics," *Review of Economic Studies*, 74, 391–415.

Bloom, N., M. Floetotto, N. Jaimovich, I. Saporta-Eksten, and S. J. Terry (2012): "Really Uncertain Business Cycles," NBER Working Papers 18245, National Bureau of Economic Research, Inc.

Campbell, J. Y., M. Lettau, B. G. Malkiel, and Y. Xu (2001): "Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk," *Journal of Finance*, 56,

1–43.
Campbell, J. Y. and R. J. Shiller (1988): "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors," *Review of Financial Studies*, 1, 195–228.
Carriere-Swallow, Y. and L. F. Cespedes (2013): "The impact of uncertainty shocks in emerging economies," *Journal of International Economics*, 90, 316–325.
Carriero, A., T. E. Clark, and M. Marcellino (2018): "Assessing International Commonality in Macroeconomic Uncertainty and Its Effects," Working Paper 1803, Federal Reserve Bank of Cleveland.
Cerutti, E., S. Claessens, and A. K. Rose (2017): "How Important is the Global Financial Cycle? Evidence from Capital Flows," NBER Working Papers 23699, National Bureau of Economic Research, Inc.
Cesa-Bianchi, A., M. H. Pesaran, and A. Rebucci (2018): "Uncertainty and Economic Activity: A Multi-Country Perspective," NBER Working Papers 24325, National Bureau of Economic Research, Inc.
Chamberlain, G. and M. Rothschild (1982): "Arbitrage, Factor Structure, and Mean-Variance Analysis on Large Asset Markets," Scholarly Articles 3230355, Harvard University Department of Economics.
Christiano, L., R. Motto, and M. Rostagno (2014): "Risk Shocks," *American Economic Review*, 104, 27–65.
Chudik, A. and M. H. Pesaran (2015): "Common correlated effects estimation of heterogeneous dynamic panel data models with weakly exogenous regressors," *Journal of Econometrics*, 188, 393–420.
Chudik, A., M. H. Pesaran, and E. Tosetti (2011): "Weak and strong cross-section dependence and estimation of large panels," *Econometrics Journal*, 14, C45–C90.
Colacito, R. and M. M. Croce (2011): "Risks for the Long Run and the Real Exchange Rate," *Journal of Political Economy*, 119, 153–181.
Colacito, R., M. M. Croce, F. Gavazzoni, and R. Ready (2018): "Currency Risk Factors in a Recursive Multicountry Economy," Forthcoming in the *The Journal of Finance*.
Dixit, A. K. and R. S. Pindyck (1994): *Investment under Uncertainty*, Princeton University Press.
Fostel, A. and J. Geanakoplos (2012): "Why Does Bad News Increase Volatility and Decrease Leverage?" *Journal of Economic Theory*, 147, 501–525.
Gilchrist, S. and E. Zakrajsek (2012): "Credit Spreads and Business Cycle Fluctuations," *American Economic Review*, 102, 1692–1720.
Granger, C. (1980): "Long memory relationships and the aggregation of dynamic models," *Journal of Econometrics*, 14, 227 – 238.
Hirata, H., M. A. Kose, C. Otrok, and M. E. Terrones (2012): "Global House Price Fluctuations: Synchronization and Determinants," in *NBER International Seminar on Macroeconomics 2012*, National Bureau of Economic Research.
Holston, K., T. Laubach, and J. C. Williams (2017): "Measuring the natural rate of interest: International trends and determinants," *Journal of International Economics*, 108, 59–75.
Horn, R. A. and C. R. Johnson, eds. (1985): *Matrix Analysis*, Cambridge: Cambridge University Press.
Huo, Z., A. Levchenko, and N. Pandalai-Nayar (2018): "Technology and Non-Technology Shocks: Measurement and Implications for International Comovement," Unpublished manuscript.
Ilut, C., M. Kehrig, and M. Schneider (2017): "Slow to Hire, Quick to Fire: Employment Dynamics with Asymmetric Responses to News," Forthcoming in the *Journal of Political Economy*.
Jurado, K., S. C. Ludvigson, and S. Ng (2015): "Measuring Uncertainty," *American Economic Review*, 105, 1177–1216.

Kimball, M. S. (1990): "Precautionary Saving in the Small and in the Large," *Econometrica*, 58, 53–73.

Kose, M. A., C. Otrok, and C. H. Whiteman (2003): "International Business Cycles: World, Region, and Country-Specific Factors," *American Economic Review*, 93, 1216–1239.

Kozlowski, J., L. Veldkamp, and V. Venkateswaran (2015): "The Tail that Wags the Economy: Beliefs and Persistent Stagnation," NBER Working Papers 21719, National Bureau of Economic Research, Inc.

Leduc, S. and Z. Liu (2016): "Uncertainty shocks are aggregate demand shocks," *Journal of Monetary Economics*, 82, 20–35.

Lee, K., M. H. Pesaran, and R. Smith (1997): "Growth and Convergence in Multi-country Empirical Stochastic Solow Model," *Journal of Applied Econometrics*, 12, 357–392.

Lewis, K. K. and E. X. Liu (2015): "Evaluating international consumption risk sharing gains: An asset return view," *Journal of Monetary Economics*, 71, 84–98.

Lucas, Robert E, J. (1978): "Asset Prices in an Exchange Economy," *Econometrica*, 46, 1429–1445.

Ludvigson, S. C., S. Ma, and S. Ng (2015): "Uncertainty and Business Cycles: Exogenous Impulse or Endogenous Response?" NBER Working Papers 21803, National Bureau of Economic Research, Inc.

Nakamura, E., D. Sergeyev, and J. Steinsson (2017): "Growth-Rate and Uncertainty Shocks in Consumption: Cross-Country Evidence," *American Economic Journal: Macroeconomics*, 9, 1–39.

Ozturk, E. and X. Sheng (2018): "Measuring Global and Country-specific Uncertainty," Forthcoming in the *Journal of International Money and Finance*.

Pesaran, H. H. and Y. Shin (1998): "Generalized impulse response analysis in linear multivariate models," *Economics Letters*, 58, 17–29.

Pesaran, M. H. (2015): *Time Series and Panel Data Econometrics*, Oxford University Press, Oxford.

Pesaran, M. H. and A. Chudik (2014): "Aggregation in large dynamic panels," *Journal of Econometrics*, 178, 273–285.

Renault, E., T. van der Heijden, and B. J. Werker (2018): "Arbitrage Pricing Theory for Idiosyncratic Variance Factors," Unpublished manuscript, http://dx.doi.org/10.2139/ssrn.3065854.

Rey, H. (2013): "Dilemma not trilemma: the global cycle and monetary policy independence," *Proceedings - Economic Policy Symposium - Jackson Hole*, 1–2.

Ross, S. A. (1976): "The arbitrage theory of capital asset pricing," *Journal of Economic Theory*, 13, 341–360.

Rossi, B. and T. Sekhposyan (2015): "Macroeconomic Uncertainty Indices Based on Nowcast and Forecast Error Distributions," *American Economic Review*, 105, 650–655.

Schwert, G. W. (1989): "Why Does Stock Market Volatility Change over Time?" *Journal of Finance*, 44, 1115–53.

Tesar, L. L. (1995): "Evaluating the gains from international risk sharing," *Carnegie-Rochester Conference Series on Public Policy*, 42, 95–143.

Van Nieuwerburgh, S. and L. Veldkamp (2006): "Learning asymmetries in real business cycles," *Journal of Monetary Economics*, 53, 753–772.

Online Supplement to

Uncertainty and Economic Activity: A Multi-Country Perspective

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Introduction

This supplement gives data sources and some summary statistics, and provides details of robustness analysis, country-specific results, and the derivation of impulse responses and error variance decompositions for global and country-specific shocks used in the paper.

S.1 Data Sources and Summary Statistics

Data Sources To construct a balanced panel for the largest number of countries for which we have sufficiently long time series, we first collect daily stock prices (excluding dividends) for 32 advanced and emerging economies from 1979 to 2016. We then cut the beginning of the sample in 1993, as daily equity price data are not available earlier for two large emerging economies (Brazil and China) and for Peru. Better quality quarterly GDP data for China also became available from 1993.[S1]

For equity prices we use the MSCI Index in local currency. We collected daily observations from January 1993 to December 2016. The data source for the daily equity price indices is Datastream. The countries included in the sample are the following: Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, China, Finland, France, Germany, India, Indonesia, Italy, Japan, Korea, Malaysia, Mexico, Netherlands, Norway, New Zealand, Peru, Philippines, South Africa, Singapore, Spain, Sweden, Switzerland, Thailand, Turkey, United Kingdom, and United States. The list of Bloomberg tickers is as follows: TOTMKAR, TOTMKAU, TOTMKOE, TOTMKBG, TOTMKBR, TOTMKCN, TOTMKCL, TOTMKCA, TOTMKFN, TOTMKFR, TOTMKBD, TOTMKIN, TOTMKID, TOTMKIT, TOTMKJP, TOTMKKO, TOTMKMY, TOTMKMX, TOTMKNL, TOTMKNZ, TOTMKNW, TOTMKPE, TOTMKPH, TOTMKSG, TOTMKSA, TOTMKES, TOTMKSD, TOTMKSW, TOTMKTH, TOTMKTK, TOTMKUK, TOTMKU.

Real GDP data come from the latest update of the GVAR data set. The data set is balanced and good quality quarterly data are available for all countries in our sample from 1993:Q1 to 2016:Q4. For more details see: https://sites.google.com/site/gvarmodelling/.

Cross-country Correlations The differential pattern of cross-country correlations of the growth and volatility innovations is crucial for our identification strategy. Here we consider the properties of the observed time series as displayed in Figure 1 in the paper. In order to gauge the extent to which volatility and growth series co-move across countries, we use two techniques: standard principal component analysis and pair-wise correlation analysis across countries.

[S1] Note that some steps of the empirical analysis can be easily implemented with the unbalanced panel from 1979. This is the case, for example, for the estimates of factor innovations ($\hat{\zeta}_t$ and $\hat{\xi}_t$), which we report in Section S.2 below.

In a panel of countries indexed by $i = 1, 2, ..., N$, the average pair-wise correlation of country i in the panel ($\bar{\rho}_i$) measures the average degree of co-movement of country i with all other countries j (i.e. for all $j \neq i$). The average pair-wise correlation across all countries, denoted by $\bar{\rho}_N$, is defined as the cross-country average of $\bar{\rho}_i$ over $i = 1, 2, ..., N$. This statistic relates to the degree of pervasiveness of the factors, as measured by the factor loadings. To see this, consider equation (1) of our model, $\Delta y_{it} = \gamma_i f_t + \varepsilon_{it}$, where $Var(f_t) = 1$, and $Var(\varepsilon_{it}) = \sigma_{ii}$.[S2] The average pair-wise correlation across all countries is given by:

$$\bar{\rho}_N = \frac{2}{N(N-1)} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \rho_{ij} = \frac{1}{N(N-1)} \left(\sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{ij} - N \right), \tag{S1}$$

where

$$\rho_{ij} = \begin{cases} \frac{\tilde{\gamma}_i}{\sqrt{1+\tilde{\gamma}_i^2}} \frac{\tilde{\gamma}_j}{\sqrt{1+\tilde{\gamma}_j^2}} & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$$

and $\tilde{\gamma}_i = \gamma_i / \sqrt{\sigma_{ii}}$. Hence

$$\bar{\rho}_N = O\left(\bar{\gamma}_N^2\right), \tag{S2}$$

where $\bar{\gamma}_N = N^{-1} \sum_{i=1}^{N} \tilde{\gamma}_i$ measures the degree of pervasiveness of the factor.

The attraction of the average pair-wise correlation, $\bar{\rho}_N$, lies in the fact that it applies to multi-factor processes, and unlike factor analysis does not require the factors to be strong. In fact, the average pair-wise correlation, $\bar{\rho}_N$, tends to be a strictly positive number if Δy_{it} contains at least one strong factor, otherwise it tends to zero as $N \to \infty$. Therefore, non-zero estimates of $\bar{\rho}_N$ are suggestive of strong cross-sectional dependence.[S3] For completeness, and to show that our analysis is robust to using an alternative methodology, in what follows, we also use standard principal component analysis. (See also Chapter 29 in Pesaran (2015) for more details).

The average pair-wise correlation across all countries for the realized volatility series in Figure 1 is 0.56. In contrast, the average pair-wise correlation across all countries for the growth series at 0.27 is much smaller. Principal component analysis yields similar results. The first principal component in our panel of realized volatility series explains 65 percent of the total variation in the log-level of volatility, whilst the first principal component of the growth series accounts for only around 30 percent of total cross-country variations in these series. Thus, both in the case of the pair-wise correlation and principal component analysis, the results point to a much higher degree of cross-country co-movements for the volatility series than for the growth series. As we will see, these differences are even more pronounced in the case of the estimated shocks obtained using equations (38) and (39).

Summary Statistics Table S.1 reports the summary statistics for the realized volatility series for each country in our sample. These results support the use of the log-level of realized volatilities as stationary series in our empirical analysis. Tables S.2 and S.3 give similar summary statistics for log of real GDP and its growth rate, and justifies using the latter as a stationary variable along with the log of realized volatility.

[S2] Under our assumptions $Var_t(\varepsilon_{i,t+1}) = \theta_t^2 \sigma_{ii}$, which gives $Var(\varepsilon_{i,t+1}) = \sigma_{ii}$.

[S3] Formal tests of cross-sectional dependence based on estimates of $\bar{\rho}_N$ are discussed in Pesaran (2015) and reported, for our panel of countries, in the next section.

Table S.1 SUMMARY STATISTICS FOR COUNTRY-SPECIFIC REALIZED VOLATILITY (LOG-LEVEL)

	ARG	AUS	AUT	BEL	BRA	CAN	CHL	CHN	FIN	FRA	DEU
Obs. in quarters	96	96	96	96	96	96	96	96	96	96	96
Mean	-2.12	-2.74	-2.70	-2.68	-2.20	-2.78	-2.85	-2.09	-2.17	-2.48	-2.53
Max	-1.23	-1.42	-1.17	-1.41	-1.12	-1.16	-1.77	-0.60	-1.09	-1.27	-1.23
Min	-2.86	-3.52	-3.38	-3.55	-2.84	-3.53	-3.56	-3.01	-2.87	-3.22	-3.44
St. Dev.	0.36	0.36	0.43	0.45	0.42	0.44	0.38	0.48	0.46	0.38	0.41
Auto Corr.	0.37	0.64	0.66	0.70	0.64	0.70	0.46	0.66	0.77	0.61	0.64
ADF	-3.12^b	-2.37	-2.17	-2.69^c	-3.55^a	-2.64^c	-3.19^b	-2.69^c	-2.31	-2.91^c	-2.79^c

	IND	IDN	ITA	JPN	KOR	MYS	MEX	NLD	NZL	NOR	PER
Obs. in quarters	96	96	96	96	96	96	96	96	96	96	96
Mean	-2.31	-2.25	-2.36	-2.40	-2.21	-2.76	-2.52	-2.56	-2.98	-2.42	-2.72
Max	-1.27	-1.07	-1.27	-1.13	-1.03	-0.82	-1.46	-1.21	-1.68	-0.93	-1.39
Min	-3.06	-3.19	-3.32	-3.17	-3.06	-3.87	-3.40	-3.33	-3.75	-3.09	-3.79
St. Dev.	0.40	0.43	0.40	0.34	0.48	0.59	0.43	0.46	0.38	0.39	0.49
Auto Corr.	0.58	0.60	0.58	0.32	0.79	0.72	0.66	0.69	0.67	0.61	0.67
ADF	-2.73^c	-2.46	-2.52	-3.6^a	-1.50	-2.24	-2.36	-2.79^c	-3.09^b	-2.81^c	-2.56

	PHL	SGP	ZAF	ESP	SWE	CHE	THA	TUR	GBR	USA
Obs. in quarters	96	96	96	96	96	96	96	96	96	96
Mean	-2.46	-2.65	-2.53	-2.42	-2.35	-2.67	-2.20	-1.83	-2.67	-2.64
Max	-1.54	-1.44	-1.45	-1.35	-1.21	-1.41	-1.20	-0.90	-1.28	-1.10
Min	-3.22	-3.47	-3.23	-3.18	-3.02	-3.39	-3.12	-2.81	-3.35	-3.40
St. Dev.	0.37	0.45	0.37	0.39	0.40	0.41	0.42	0.45	0.42	0.44
Auto Corr.	0.46	0.68	0.53	0.58	0.66	0.54	0.64	0.73	0.66	0.69
ADF	-2.97^b	-2.06	-3.28^b	-2.56	-2.59^c	-3.23^b	-2.25	-1.43	-2.56	-2.27

NOTE. Summary statistics of the log-level of volatility (v_{it}). *ADF* is the Augmented Dickey-Fuller t-statistic computed with 4 lags and a constant, where a, b, and c denote associated p-values at 1-percent, 5-percent, and 10-percent. Sample period 1993:Q1-2016:Q4.

Table S.2 Summary Statistics for Country-specific Real GDP (Log-Level)

	ARG	AUS	AUT	BEL	BRA	CAN	CHL	CHN	FIN	FRA	DEU
Obs. in quarters	96	96	96	96	96	96	96	96	96	96	96
Mean	4.78	4.73	4.66	4.65	4.75	4.67	4.79	5.14	4.65	4.64	4.64
Max	5.20	5.08	4.84	4.82	5.05	4.92	5.24	6.26	4.84	4.77	4.79
Min	4.42	4.30	4.40	4.40	4.39	4.32	4.23	3.99	4.29	4.43	4.47
St. Dev.	0.27	0.23	0.13	0.12	0.20	0.18	0.30	0.72	0.16	0.11	0.09
Auto Corr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
ADF	-0.30	-3.57^{a}	-1.86	-1.76	-1.83	-2.12	-1.62	-0.24	-2.20	-2.00	-0.72

	IND	IDN	ITA	JPN	KOR	MYS	MEX	NLD	NZL	NOR	PER
Obs. in quarters	96	96	96	96	96	96	96	96	96	96	96
Mean	4.91	4.88	4.60	4.65	4.77	4.79	4.67	4.62	4.98	4.59	4.86
Max	5.73	5.46	4.69	4.75	5.19	5.35	4.96	4.78	5.26	4.80	5.44
Min	4.08	4.37	4.46	4.55	4.19	4.15	4.31	4.36	4.63	4.23	4.24
St. Dev.	0.48	0.31	0.06	0.06	0.28	0.32	0.19	0.12	0.17	0.15	0.35
Auto Corr.	1.00	1.00	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
ADF	-0.37	0.64	-2.44	-0.78	-1.63	-0.59	-0.31	-1.97	-0.98	-2.8^{c}	0.21

	PHL	SGP	ZAF	ESP	SWE	CHE	THA	TUR	GBR	USA
Obs. in quarters	96	96	96	96	96	96	96	96	96	96
Mean	4.84	4.78	4.77	4.66	4.68	4.67	4.82	4.78	4.66	4.66
Max	5.45	5.31	5.07	4.84	4.97	4.85	5.18	5.28	4.86	4.89
Min	4.33	4.06	4.39	4.37	4.36	4.48	4.37	4.27	4.37	4.33
St. Dev.	0.32	0.36	0.22	0.15	0.17	0.12	0.23	0.29	0.14	0.16
Auto Corr.	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	1.00	1.00
ADF	1.96	-1.37	-1.36	-1.65	-0.54	-0.61	-0.68	-0.59	-1.77	-1.65

Note. Summary statistics for the log-level of real GDP (y_{it}). *ADF* is the Augmented Dickey-Fuller t-statistic computed with 4 lags and a constant, where a, b, and c denote associated p-values at 1-percent, 5-percent, and 10-percent. Sample period 1993:Q1-2016:Q4.

Table S.3 Summary Statistics for Country-specific Real GDP (Log-Difference)

	ARG	AUS	AUT	BEL	BRA	CAN	CHL	CHN	FIN	FRA	DEU
Obs. in quarters	95	95	95	95	95	95	95	95	95	95	95
Mean	0.78	0.81	0.46	0.43	0.60	0.63	1.06	2.40	0.55	0.36	0.34
Max	4.04	2.46	2.48	2.23	4.83	1.64	5.91	5.91	4.41	1.56	2.19
Min	-6.35	-0.99	-2.61	-2.12	-5.19	-2.27	-3.38	-1.33	-6.01	-1.70	-4.16
St. Dev.	1.92	0.56	0.92	0.69	1.44	0.61	1.28	1.24	1.32	0.50	0.74
Auto Corr.	0.59	-0.04	0.00	0.24	0.25	0.54	0.12	0.08	0.12	0.47	0.33
ADF	-3.49^a	-3.75^a	-3.68^a	-4.64^a	-3.63^a	-3.78^a	-3.18^b	-2.45	-3.88^a	-3.19^b	-4.65^a
	IND	IDN	ITA	JPN	KOR	MYS	MEX	NLD	NZL	NOR	PER
Obs. in quarters	95	95	95	95	95	95	95	95	95	95	95
Mean	1.73	1.14	0.18	0.18	1.05	1.26	0.63	0.44	0.67	0.60	1.27
Max	5.41	5.11	1.79	2.53	3.81	4.65	3.77	1.70	2.54	4.48	4.44
Min	-1.43	-8.17	-3.70	-4.09	-8.94	-7.10	-6.07	-2.10	-1.67	-1.74	-2.52
St. Dev.	1.28	1.76	0.69	1.00	1.48	1.59	1.43	0.65	0.71	1.15	1.36
Auto Corr.	-0.06	0.34	0.42	0.29	0.27	0.33	0.33	0.58	0.24	-0.20	0.28
ADF	-4.66^a	-3.47^a	-3.3^b	-5.08^a	-4.74^a	-5.34^a	-4.1^a	-3.04^b	-3.94^a	-3.46^a	-4.2^a
	PHL	SGP	ZAF	ESP	SWE	CHE	THA	TUR	GBR	USA	
Obs. in quarters	95	95	95	95	95	95	95	95	95	95	
Mean	1.17	1.32	0.71	0.50	0.64	0.39	0.86	0.99	0.52	0.59	
Max	3.21	6.77	1.86	2.49	2.94	1.98	10.79	6.57	1.41	1.81	
Min	-2.44	-3.77	-1.63	-1.57	-3.71	-3.50	-11.97	-11.93	-2.11	-2.18	
St. Dev.	0.87	2.03	0.60	0.60	1.12	0.78	2.47	2.91	0.58	0.59	
Auto Corr.	0.11	0.23	0.60	0.80	-0.01	0.16	-0.02	-0.01	0.69	0.41	
ADF	-4.13^a	-5.28^a	-2.7^c	-2.47	-4.9^a	-4.15^a	-4.11^a	-5.21^a	-4^a	-3.34^b	

Note. Summary statistics for the log-difference of real GDP (Δy_{it}). *ADF* is the Augmented Dickey-Fuller t-statistic computed with 4 lags and a constant, where a, b, and c denote associated p-values at 1-percent, 5-percent, and 10-percent. Sample period 1993:Q1-2016:Q4.

S.2 Robustness Analysis

We report here the results from a few exercises showing robustness of our results.

S.2.1 Robustness to Choice of Countries (Granularity Assumptions)

This section compares the results from four robustness exercises with respect to the choice of the countries in our sample with the estimates reported in the paper that are based on all countries. In particular, we consider the following cases: (1) exclude the United States from the sample; (2) exclude China from the sample; (3) exclude the United States and China from the sample; and (4) we treat the United States as the global factor, namely we substitute $\hat{\zeta}_t$ and $\hat{\xi}_t$ with $\Delta y_{US,t}$ and $v_{US,t}$ respectively.

Table S.4 shows that in cases (1), (2), and (3)—i.e. when we exclude the United States, China, or both—the cross-sectional dependence of the country-specific innovations is very similar to the baseline. So, our common factors cannot be driven by shocks to these large economies. This is not true for case (4), i.e. when we treat the US economy as the common factor. In this case, Table S.4 shows that the country-specific GDP growth and volatility innovations display a significant degree of cross-sectional dependence even after conditioning on US GDP growth and US (log) volatility. Consistently with that, the CD test rejects the null of zero average pair-wise correlation of the innovations. In other words, when replacing the common factors $\hat{\zeta}_t$ and $\hat{\xi}_t$ with US GDP growth and US volatility, we can control for some, but not all, the cross-country correlation of the GDP growth and volatility series. Table S.5 reports similar evidence based on 'long-run' (i.e. 12 quarters ahead) forecast error variance decompositions (FEVD). The Table shows that the FEVDs in cases (1), (2), and (3) are very similar to our baseline, while this is not true for case (4).

Table S.4 Cross-Sectional Dependence of the Innovations

	Pairwise Correlation			Exponent of cross-sectional dependence		
	$\hat{\varepsilon}_i$	$\hat{u}_i$	$\hat{\eta}_i$	$\hat{\varepsilon}_i$	$\hat{u}_i$	$\hat{\eta}_i$
Baseline (All countries)	-0.01	0.52	-0.02	0.56 [0.62,0.67]	0.96 [1.00,1.04]	0.58 [0.64,0.70]
Excluding US	-0.02	0.52	-0.02	0.54 [0.60,0.65]	0.96 [1.00,1.04]	0.52 [0.59,0.66]
Excluding China	-0.01	0.55	-0.03	0.57 [0.62,0.68]	0.96 [1.00,1.04]	0.61 [0.68,0.74]
Excluding US & China	-0.01	0.54	-0.03	0.56 [0.61,0.66]	0.96 [1.00,1.04]	0.57 [0.63,0.69]
US as global factor	0.15	0.49	0.27	0.91 [0.99,1.06]	0.96 [0.99,1.03]	0.96 [0.99,1.03]

Note. Pair-wise correlations and exponent of cross-sectional dependence ($\hat{\alpha}$) as in Bailey et al. (2016), together with the associated 90-percent confidence interval in square brackets. Sample period 1993:Q1-2016:Q4.

Table S.5 Forecast Error Variance Decomposition (Long-run)

	FEVD of GDP growth					
	$\hat{\xi}$	$\hat{\eta}_i$	$\sum \hat{\eta}_j$	$\hat{\zeta}$	$\hat{\varepsilon}_i$	$\sum \hat{\varepsilon}_j$
Baseline	7.6	1.9	0.1	24.9	64.6	0.9
Excluding US	7.8	2.0	0.1	24.8	64.5	0.8
Excluding China	7.5	2.0	0.1	25.6	63.9	0.9
Excluding US & China	7.7	2.1	0.1	25.4	63.8	0.9
US as global factor	5.7	2.9	0.2	6.5	83.9	0.9
	FEVD of Volatility					
	$\hat{\xi}$	$\hat{\eta}_i$	$\sum \hat{\eta}_j$	$\hat{\zeta}$	$\hat{\varepsilon}_i$	$\sum \hat{\varepsilon}_j$
Baseline	53.2	41.9	0.1	3.9	0.6	0.2
Excluding US	53.0	42.4	0.1	3.7	0.6	0.2
Excluding China	54.5	40.3	0.1	4.3	0.6	0.2
Excluding US & China	54.3	40.8	0.1	4.0	0.6	0.2
US as global factor	32.8	57.8	0.2	8.5	0.6	0.2

Note. Average across countries with GDP-PPP weights at horizon $h = 12$ quarters. $\hat{\xi}$ is common financial shock; $\hat{\eta}_i$ is country i's volatility shock; $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in country j, for all $j \neq i$; $\hat{\zeta}$ is common growth shock; $\hat{\varepsilon}_i$ is country i's GDP growth shock; $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in country j, for all $j \neq i$. Sample period: 1993:Q1-2016:Q4.

S.2.2 Robustness to the Choice of Sample Periods

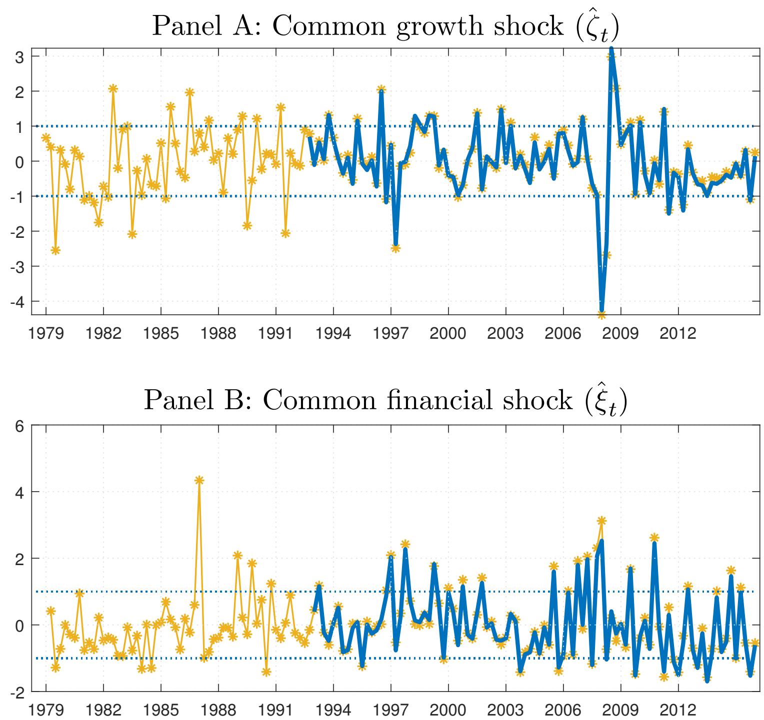
We report here results from a longer unbalanced sample period and when we exclude the global financial crisis period from the sample.

Unbalanced Panel Estimates of the Common Shocks. We consider in this section a longer sample period starting from 1979. While for a few emerging economies quarterly GDP data is not available from this starting date, it is possible to interpolate annual series to obtain a balanced sample of GDP growth series at quarterly frequency for all countries considered in our study. For more details see: https://sites.google.com/site/gvarmodelling/. We then collected daily equity prices from January 1979 to December 2016. Note that, over this sample, it is possible to obtain a balanced panel only for 16 economies.

Estimates of the global shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$, recovered from the OLS estimation of (36) and (37) are reported in Figure S.1 when estimated using the unbalanced panel from 1979 (thin lines with asterisks), and when we use the balanced panel from 1993 (thick solid lines), so as to better illustrate their time profiles. The figure also reports one-standard deviation bands for the shocks. Note that the shocks are standardized and have zero means and unit in-sample variances. They are also serially uncorrelated and orthogonal to each other by construction. Interestingly, the Jarque-Bera test strongly rejects normality in the case of the growth shocks, with strong evidence of left skewness and kurtosis, and only marginally rejects in the case of the financial shock with mild evidence of right skewness. The figure shows that the largest negative realization of the real common shock was after the second oil shock in 1979, and during the fourth quarter of 2008 after the Lehman Brother's collapse, consistent with prevailing narratives on the characterization of world recessions. Figure S.1 illustrates that the largest realizations of the common financial shock, $\hat{\xi}_t$, coincide with the 1987 stock market crash and the 2008 Lehman Brother's collapse.

Excluding the global financial crisis. The results are robust to dropping the period of the global financial crisis from our sample. For example, we report in Figures S.2 and S.3 the FEVDs and IRFs that we obtained when re-estimating the sample from 1993 to 2006.

Figure S.1 ESTIMATED COMMON GROWTH ($\hat{\zeta}_t$) AND FINANCIAL ($\hat{\xi}_t$) SHOCKS



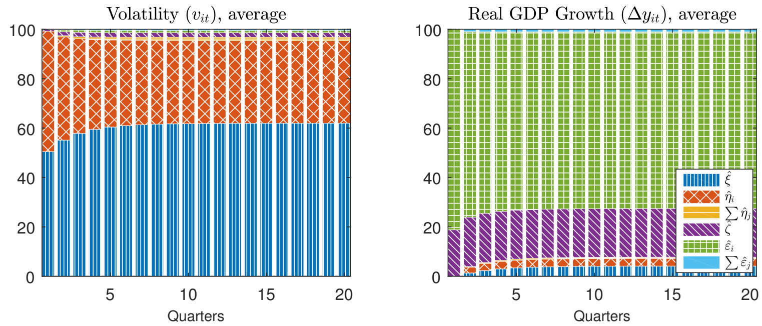
NOTE. The common shocks $\hat{\zeta}_t$ and $\hat{\xi}_t$ are computed using (36) and (37), with one lag of $\mathbf{z}_{it}$, using an unbalanced sample 1979:Q2-2016:Q4 (thin lines with asterisks) and the shorter balanced sample 1993:Q1-2016:Q4 (thick solid lines). The shocks are standardized and the dotted lines are the one-standard deviation bands around the zero mean.

S.2.3 Robustness to Choice of Uncertainty Measures: Realized *versus* Implied volatility

At quarterly frequency, the realized volatility of US daily equity returns behaves very similarly to the VIX Index. For example, during the period over which they overlap, our realized volatility measure and the VIX Index co-move very closely, with a correlation that exceeds 0.9. See Figure S.4. In addition, to check more formally the robustness of our results, we re-estimated our model using the VIX Index as a measure of volatility for the United States (instead of our realized volatility measure) and obtain virtually identical results.

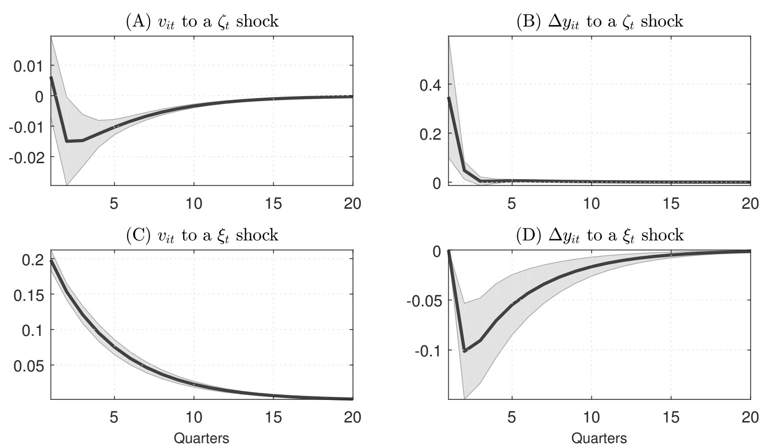
Figure S.5 compares our baseline IRFs of US volatility and US GDP growth to a US country-specific volatility shock (solid blue line) with those obtained from a specification where we used the VIX Index as a measure of US volatility instead of our realized volatility measure (yellow line with asterisks). The comparison shows that, in the robustness exercise, the correlation between US GDP and volatility residuals is even more positive than in our baseline scenario; thus reinforcing our main result.

Figure S.2 FEVD - Diagonal covariance matrix - Sample period : 1993-2006



Note. We impose a diagonal covariance matrix. Average across countries with GDP-PPP weights. $\hat{\xi}$ is common financial shock (blue area with vertical lines); $\hat{\eta}_i$ is country-specific volatility shock (red area with crosses); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area with horizontal lines); $\hat{\zeta}$ is common growth shock (purple area with diagonal lines); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas with squares); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas with no pattern). The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2006:Q2.

Figure S.3 Average Country Volatility and Growth Responses to Real and Financial Factor Shocks (In Percent) - Sample period: 1993-2006



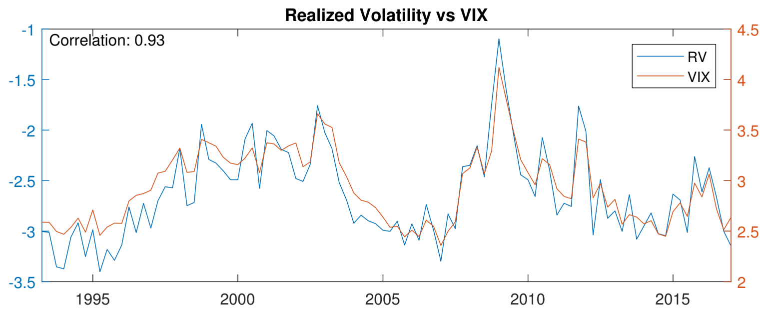
Note. Average impulse responses to one-standard deviation real and financial shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$. The solid lines are the PPP-GDP weighted averages of the country-specific responses. The shaded areas are the two standard deviations confidence intervals. See equations (S12) and (S13) for the derivations and Figure S.8 for the country-specific responses. The horizontal axis is in quarters. Sample period: 1993:Q1-2006:Q2.

S.2.4 Variance Decompositions: Alternative Identification Assumptions for Country-Specific Shocks

While in our baseline estimates of the FEVDs we assume a diagonal covariance matrix for the residuals of the multi-country model (38)-(39), to check the robustness of our results we re-estimate the FEVDs with two alternative specifications of the covariance matrix of country-specific shocks.

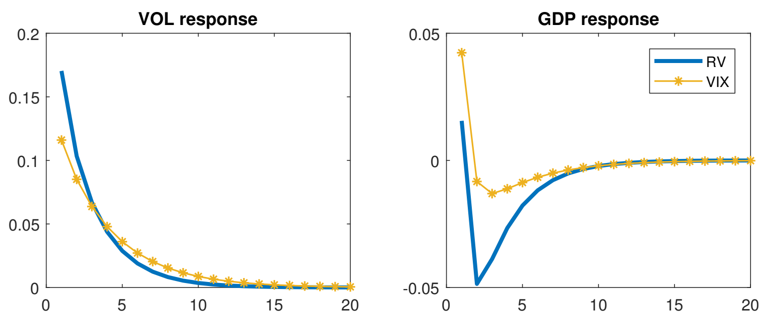
Block-Diagonal Covariance Matrix and Orthogonal Decomposition. We now maintain the assumption of zero correlations of country-specific shocks (after conditioning on the common shocks) across countries, but allow for a possibly non-zero correlation between country-specific volatility and growth shocks within each country. Specifically, we assume that, at the country level, a country-specific volatility shock can affect growth contemporaneously but not *vice versa*. This is the as-

Figure S.4 United States: VIX Index *versus* RV



Note. Blue line is the (log) realized volatility of equity prices for the United States, as in our baseline model (RV). The red line is the (log) VIX Index (average across days within the quarter). Sample period: 1993:Q1-2016:Q4.

Figure S.5 US Response to US Volatility Shock ($\hat{\eta}_{US,t}$)



Note. US impulse responses to a one-standard deviation shock to US volatility, $\hat{\eta}_{US,t}$. The blue lines are our baseline; the yellow lines with asterisks are obtained from a specification where we used the VIX Index as a measure of US volatility instead of our realized volatility measure. The horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

sumption typically made in the empirical and theoretical literature on volatility and the business cycle. So, here, we are 'identifying' exogenous country-specific volatility changes with a Cholesky decomposition of the within-country covariance matrix. We do so by ordering volatility first in the model (38)-(39).

The results for this specification are given in Figure S.6 and can be seen to be virtually identical to the estimates obtained for the diagonal error covariance matrix reported in Figure 3 in the paper. This is perhaps not surprising given that the correlations between the country-specific innovations, once the effects of the common shocks are removed, are very small as in Figure 2 in the paper.

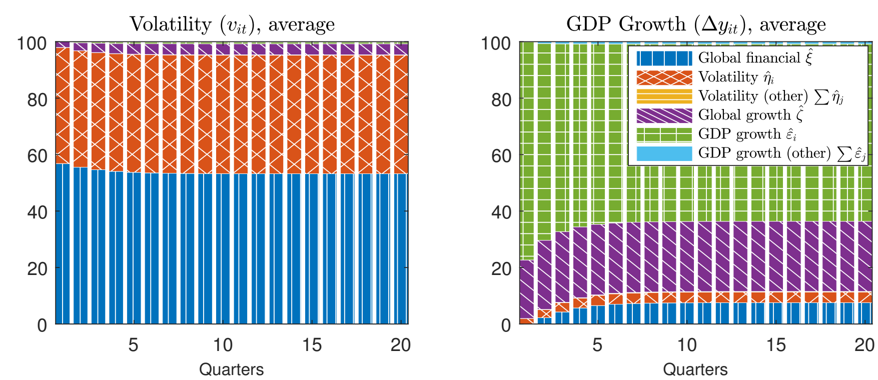
Thresholding the Country-specific Error Covariance Matrix and Generalized Error Variance Decomposition. We finally allow for a fully estimated (64×64) correlation matrix or country-specific errors, both within and across countries, and compute the GFEVDs. However, given the large size of this matrix, we regularize it by computing a threshold estimator following Bailey et al. (2018b), who developed a procedure based on results from the multiple testing literature. Specifically, we first test for the statistical significance of each of the 2016 distinct off-diagonal elements of the (64×64) matrix. We then set to zero all those elements that are not statistically significant, using suitably adjusted critical values to allow for the large number of tests that are being carried out. We then finally compute the GVEDs by using the regularized estimates as set out in Section S.5 below.

Table S.6 below lists all the non-zero correlation pairs. As can be seen, only 57 out of 2016 total off-diagonal elements are statistically different from zero. Of these, about half are positively correlated and the other half are negatively correlated, with an average value that is close to zero. Most notably, there is no surviving within-country contemporaneous correlations between volatility and growth. There are also very few significant GDP-GDP correlation pairs (i.e. $\hat{\varepsilon}_{it}$ with $\hat{\varepsilon}_{jt}$), with no obvious regional pattern of co-movements. There are a few significant pairs of volatility-volatility correlations (i.e. $\hat{\eta}_{it}$ with $\hat{\eta}_{jt}$), but involving only a handful of countries, with no evidence of a dominant role for the United States. Finally, there are only two significant GDP-volatility correlation pairs (i.e. $\hat{\varepsilon}_{jt}$ with $\hat{\eta}_{it}$), again revealing no specific patterns.

The estimated generalized forecast error variance decompositions (GFEVDs), reported in Figure S.7, are consistent with those obtained assuming a diagonal or block-diagonal error covariance matrix.[S4] Relative to the results with diagonal or block-diagonal covariance matrix in Figures 3 and S.6, the contribution of foreign country-specific volatility (growth) shocks, $\sum \hat{\eta}_j$ ($\sum \hat{\varepsilon}_j$), to domestic volatility (growth) is now larger, but the spillover effects of foreign volatility shocks to growth (and foreign growth shocks to volatility) remain negligible. Moreover, global financial shocks and domestic country-specific volatility shocks continue to explain the bulk of the forecast error variance of volatility. Similarly, global growth shocks and the country-specific growth shocks remain the main drivers of the forecast error variance of growth.

We interpret the above results as strong evidence of robustness of our conclusions reached by assuming a diagonal or block-diagonal error covariance matrix. In particular, it remains the case that common or country-specific output growth shocks have a small quantitative importance for volatility, and home and foreign country-specific volatility shocks have little or no quantitative consequence for output growth.

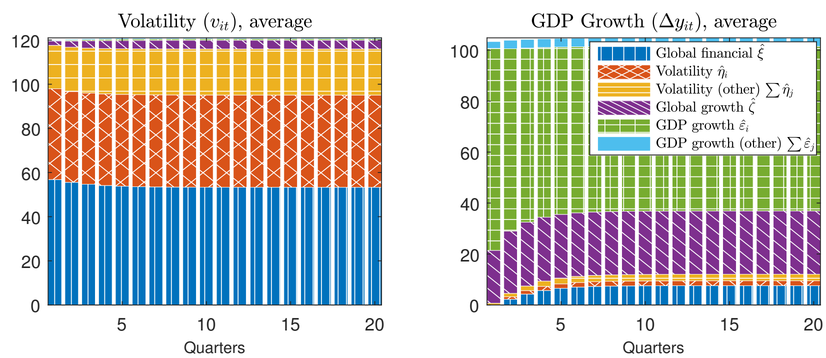
Figure S.6 Forecast Error Variance Decomposition of Country-specific Shocks - Block Diagonal Error Covariance Matrix (In Percent)



Note. Block-diagonal covariance matrix, with Cholesky decomposition of within-country covariance. Average across countries with GDP-PPP weights. $\hat{\xi}$ is common financial shock (blue area with vertical lines); $\hat{\eta}_i$ is country-specific volatility shock (red area with crosses); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area with horizontal lines); $\hat{\zeta}$ is common growth shock (purple area with diagonal lines); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas with squares); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas with no pattern). The horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

[S4] Notice here that the GFEVDs need not sum to 100 as the underlying shocks are not orthogonal.

Figure S.7 Generalized Forecast Error Variance Decomposition of Country-specific Shocks - Estimation of Regularized Full Error Covariance Matrix (In Percent)



Note. Threshold estimator of the population covariance matrix. Average across countries with GDP-PPP weights. $\hat{\xi}$ is common financial shock (blue area with vertical lines); $\hat{\eta}_i$ is country-specific volatility shock (red area with crosses); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area with horizontal lines); $\hat{\zeta}$ is common growth shock (purple area with diagonal lines); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas with squares); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas with no pattern). The horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

Table S.6 Non-zero Elements of the Regularized Error Covariance Matrix Estimate

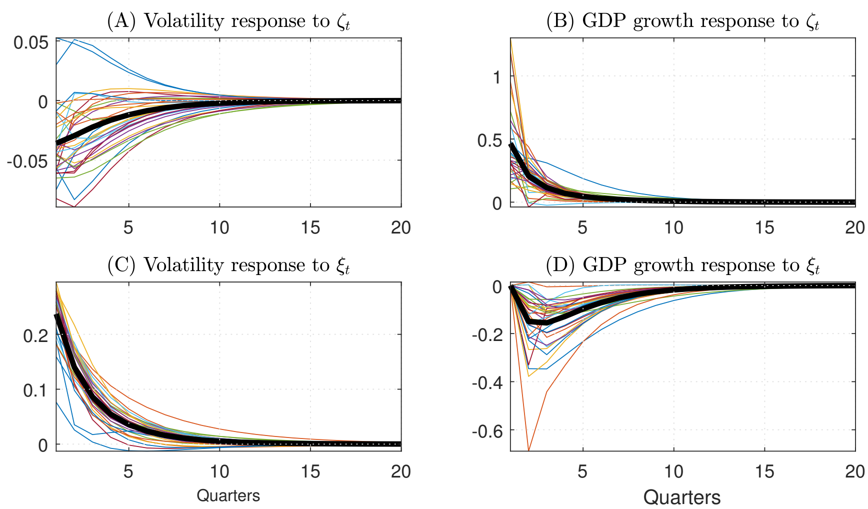
All Significant			Between-county correlations		
Country - Variable Pairs		Corr	$\hat{\varepsilon}_{it}, \hat{\varepsilon}_{jt}$	$\hat{\eta}_{it}, \hat{\eta}_{jt}$	$\hat{\varepsilon}_{it}, \hat{\eta}_{jt}$
ARG VOL	NLD VOL	-0.34		ARG,NLD	
AUS VOL	NZL VOL	0.37		AUS,NZL	
AUT GDP	PHL GDP	-0.36	AUT,PHL		
BEL VOL	ITA VOL	0.48		BEL,ITA	
BEL VOL	NLD VOL	0.56		BEL,NLD	
BEL VOL	CHE VOL	0.38		BEL,CHE	
BEL VOL	GBR VOL	0.50		BEL,GBR	
BRA VOL	MEX VOL	0.45		BRA,MEX	
CAN VOL	NOR VOL	0.44		CAN,NOR	
CHL VOL	FRA VOL	-0.35		CHL,FRA	
CHL VOL	NLD VOL	-0.35		CHL,NLD	
CHL VOL	ESP VOL	-0.36		CHL,ESP	
FIN VOL	SWE VOL	0.51		FIN,SWE	
FIN GDP	ITA GDP	0.37	FIN,ITA		
FRA VOL	DEU VOL	0.62		FRA,DEU	
FRA VOL	IND VOL	-0.38		FRA,IND	
FRA VOL	IDN VOL	-0.43		FRA,IDN	
FRA VOL	ITA VOL	0.44		FRA,ITA	
FRA VOL	MEX VOL	-0.43		FRA,MEX	
FRA VOL	NLD VOL	0.67		FRA,NLD	
FRA VOL	PHL VOL	-0.35		FRA,PHL	
FRA VOL	SGP VOL	-0.44		FRA,SGP	
FRA VOL	ESP VOL	0.58		FRA,ESP	
FRA VOL	SWE VOL	0.48		FRA,SWE	
FRA VOL	THA VOL	-0.37		FRA,THA	
FRA VOL	GBR VOL	0.60		FRA,GBR	
DEU VOL	ITA VOL	0.37		DEU,ITA	
DEU VOL	MEX VOL	-0.38		DEU,MEX	
DEU VOL	NLD VOL	0.62		DEU,NLD	
DEU VOL	ESP VOL	0.48		DEU,ESP	
DEU VOL	GBR VOL	0.37		DEU,GBR	
IDN VOL	PER GDP	-0.36			IDN,PER
IDN VOL	PHL VOL	0.38		IDN,PHL	
IDN VOL	SGP VOL	0.35		IDN,SGP	
IDN VOL	THA VOL	0.39		IDN,THA	
IDN VOL	GBR VOL	-0.36		IDN,GBR	
IDN GDP	KOR GDP	0.35	IDN,KOR		
ITA VOL	MYS VOL	-0.38		ITA,MYS	
ITA VOL	NLD VOL	0.53		ITA,NLD	
ITA VOL	PHL VOL	-0.34		ITA,PHL	
ITA VOL	SGP VOL	-0.36		ITA,SGP	
ITA VOL	ESP VOL	0.56		ITA,ESP	
ITA VOL	GBR VOL	0.39		ITA,GBR	
KOR GDP	MYS GDP	0.47	KOR,MYS		
MYS VOL	SGP VOL	0.43		MYS,SGP	
MYS VOL	SWE VOL	-0.48		MYS,SWE	
MEX VOL	NLD VOL	-0.36		MEX,NLD	
NLD VOL	PER VOL	-0.35		NLD,PER	
NLD VOL	ESP VOL	0.46		NLD,ESP	
NLD VOL	CHE VOL	0.50		NLD,CHE	
NLD VOL	GBR VOL	0.70		NLD,GBR	
NOR VOL	PER GDP	0.36			NOR,PER
PHL VOL	SGP VOL	0.48		PHL,SGP	
SGP VOL	ESP VOL	-0.39		SGP,ESP	
SWE VOL	GBR VOL	0.37		SWE,GBR	
CHE VOL	GBR VOL	0.46		CHE,GBR	
THA VOL	GBR VOL	-0.38		THA,GBR	

Note. Non-zero elements of the regularized error covariance matrix estimate proposed by Bailey et al. (2018b). Sample period: 1993:Q1-2016:Q4.

S.3 Country-specific Results

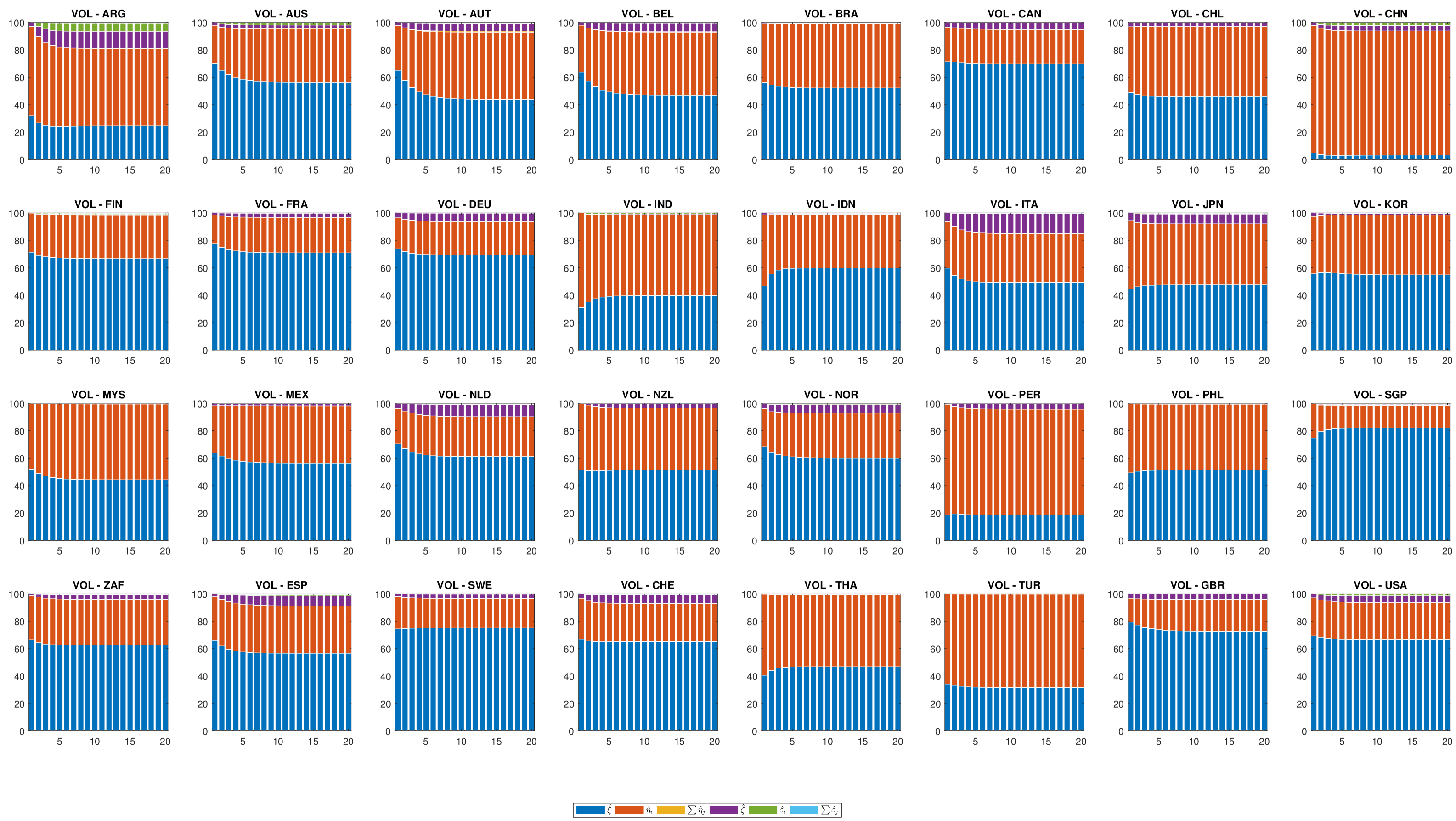
In this section we report selected country-specific results, namely the individual country impulse response functions and forecast error variance decompositions. Figure S.8 plots the country-specific impulse response of volatility and growth to a positive, one-standard-deviation shock to the global shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$. We can see from Figure S.8 that for most countries the impulse responses have a very similar profile. Figures S.9 to S.14 report forecast error variance decompositions for each country, for both volatility and growth, computed with different assumptions on the covariance matrix of the volatility and growth innovations. As can be seen the estimates are very similar across countries and for all the three schemes assumed for the error covariances.

Figure S.8 COUNTRY-SPECIFIC VOLATILITY AND GROWTH IMPULSE RESPONSES TO COMMON REAL AND FINANCIAL SHOCKS



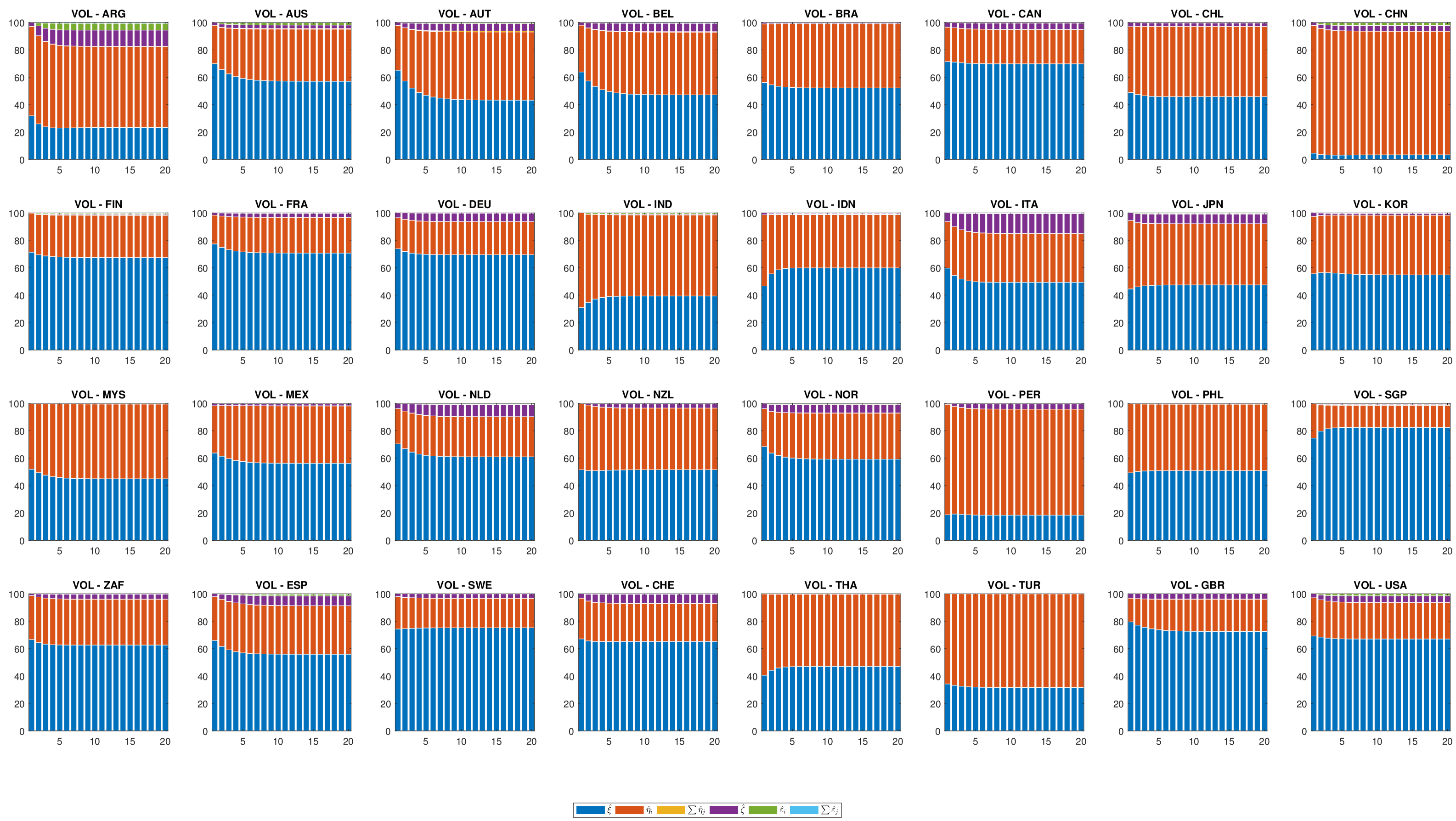
NOTE. One standard deviation shocks to $\hat{\zeta}_t$ and $\hat{\xi}_t$. Thin lines are individual country responses. The solid lines are the PPP-GDP weighted averages, as the ones reported in the main text. The horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

Figure S.9 Forecast Error Variance Decomposition of Country-Specific Volatility Shocks - Diagonal Error Covariance Matrix



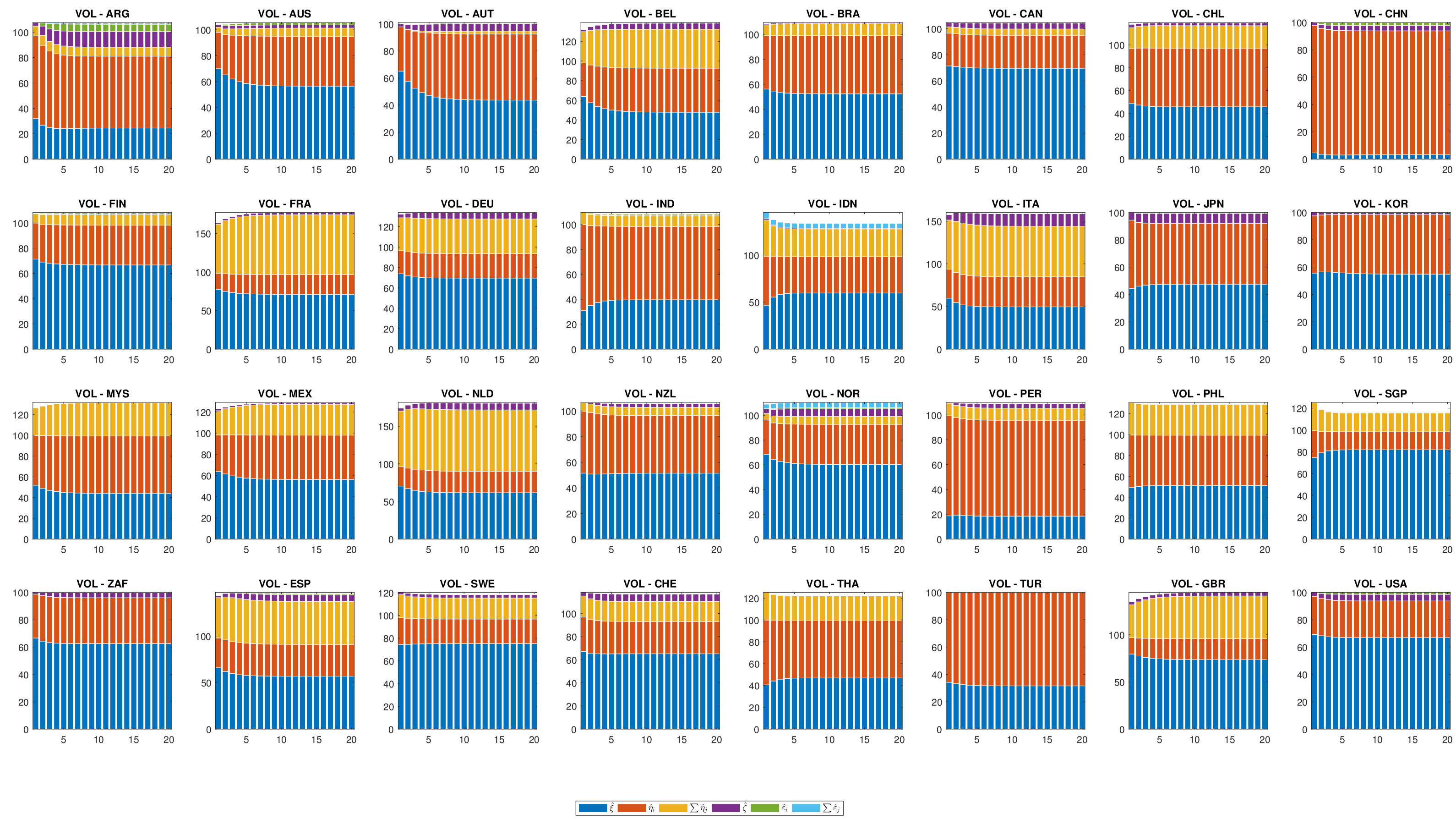
Note. $\hat{\xi}$ is common financial shock (blue area); $\hat{\eta}_i$ is country-specific volatility shock (red area); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area); $\hat{\zeta}$ is common growth shock (purple area); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas). The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

Figure S.10 Forecast Error Variance Decomposition of Country-Specific Volatility Shocks - Block Diagonal Error Covariance Matrix



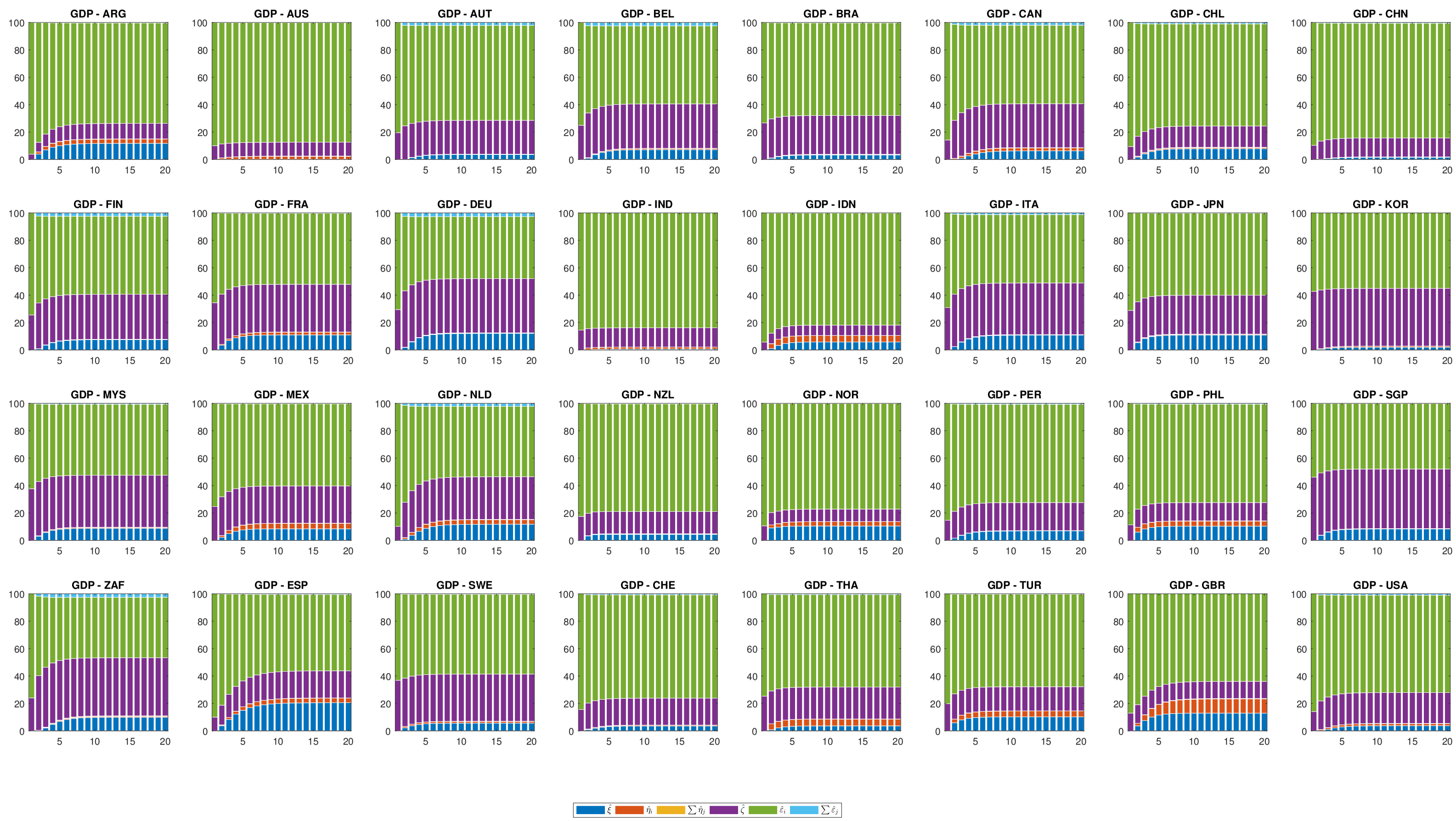
Note. $\hat{\xi}$ is common financial shock (blue area); $\hat{\eta}_i$ is country-specific volatility shock (red area); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area); $\hat{\zeta}$ is common growth shock (purple area); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas). The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

Figure S.11 GENERALIZED FORECAST ERROR VARIANCE DECOMPOSITION OF COUNTRY-SPECIFIC VOLATILITY SHOCKS - REGULARIZED ESTIMATION OF FULL ERROR COVARIANCE MATRIX



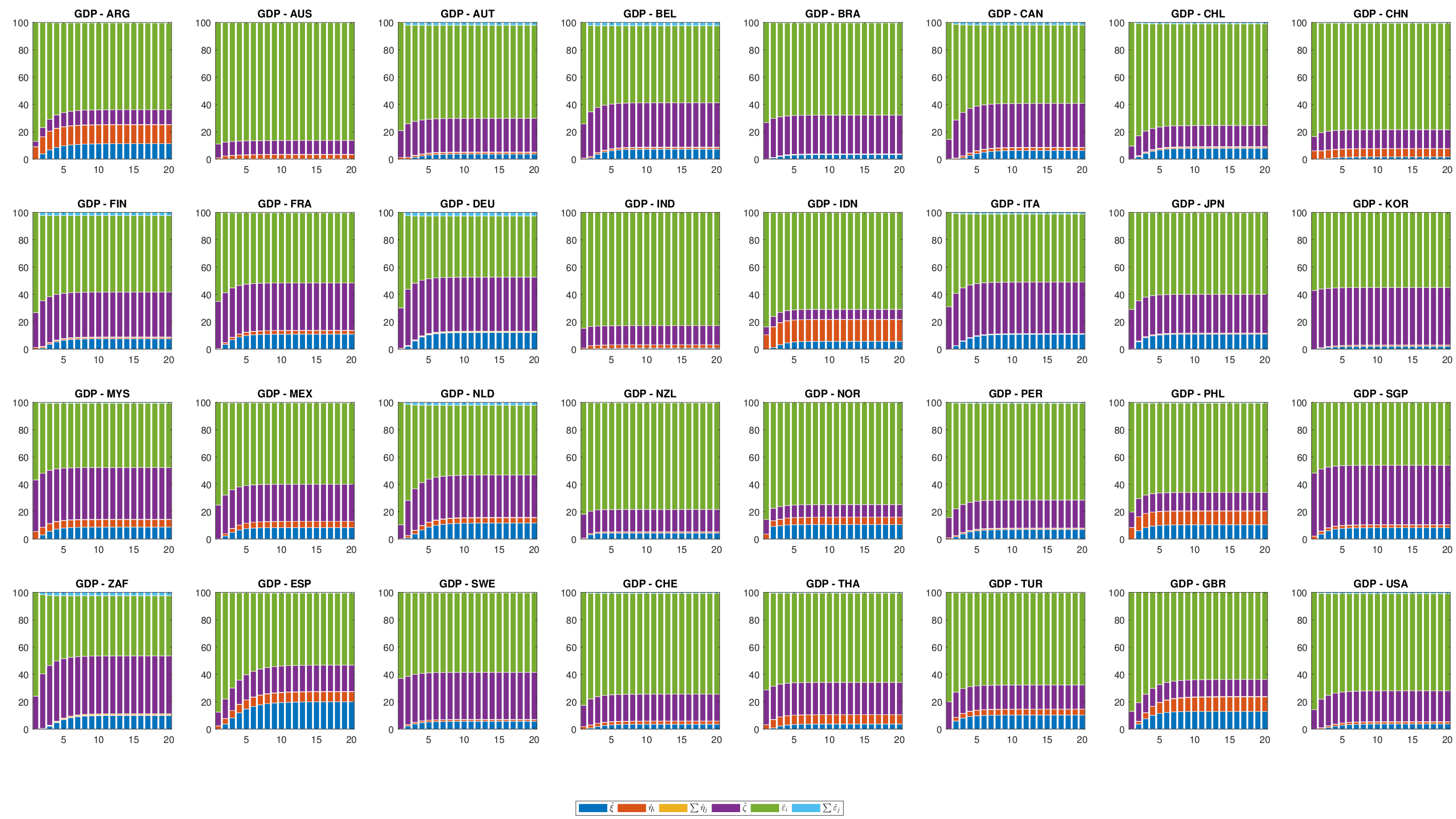
NOTE. $\hat{\xi}$ is common financial shock (blue area); $\hat{\eta}_i$ is country-specific volatility shock (red area); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area); $\hat{\zeta}$ is common growth shock (purple area); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas). The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

Figure S.12 Forecast Error Variance Decomposition of Country-Specific Growth Shocks - Diagonal Error Covariance Matrix



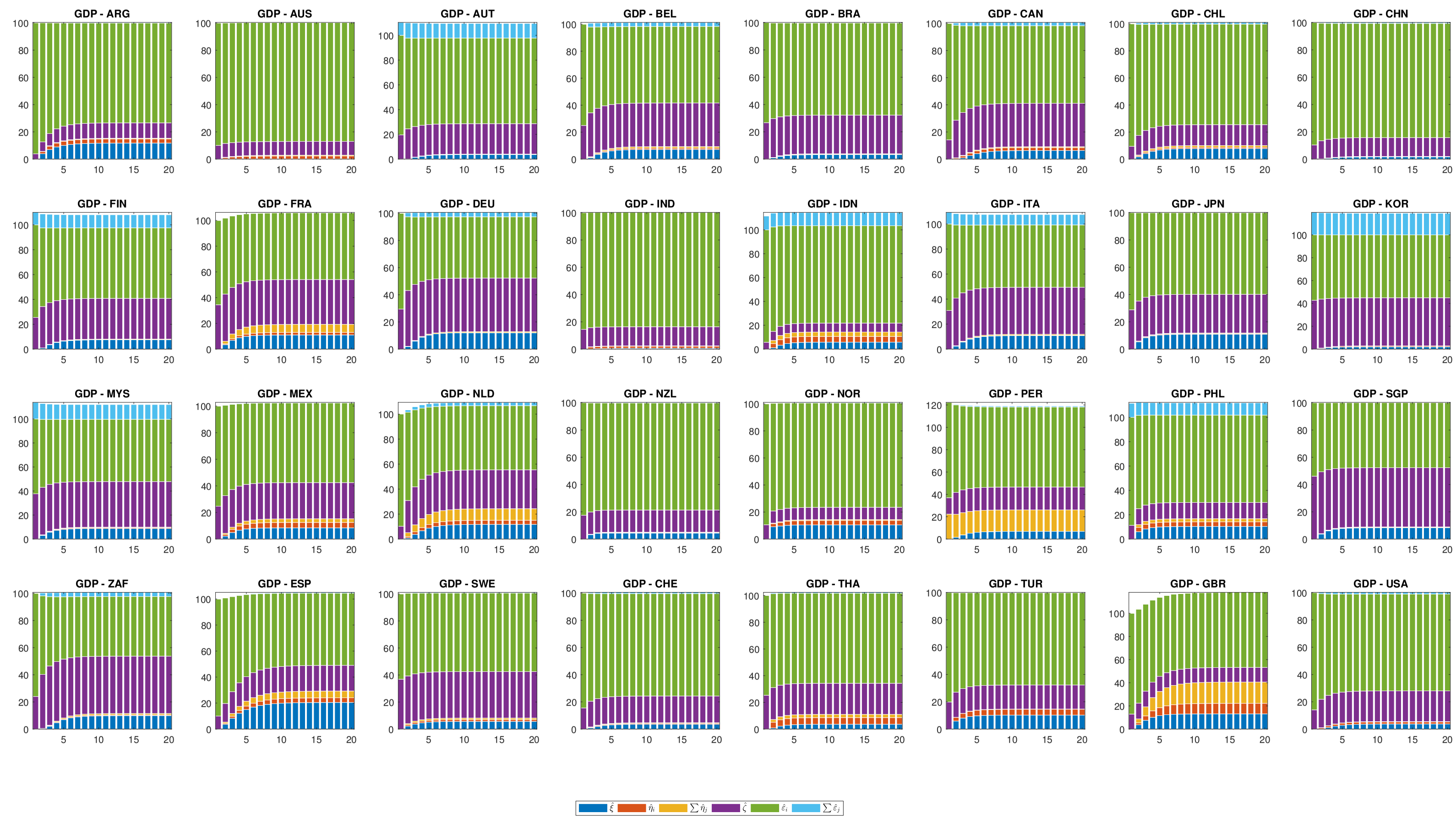
Note. $\hat{\xi}$ is common financial shock (blue area); $\hat{\eta}_i$ is country-specific volatility shock (red area); $\sum\hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area); $\hat{\zeta}$ is common growth shock (purple area); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas); $\sum\hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas). The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

Figure S.13 FORECAST ERROR VARIANCE DECOMPOSITION OF COUNTRY-SPECIFIC GROWTH SHOCKS - BLOCK DIAGONAL ERROR COVARIANCE MATRIX



NOTE. $\hat{\xi}$ is common financial shock (blue area); $\hat{\eta}_i$ is country-specific volatility shock (red area); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area); $\hat{\zeta}$ is common growth shock (purple area); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas). The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

Figure S.14 Generalized Forecast Error Variance Decomposition of Country-Specific Growth Shocks - Regularized Estimation of Full Error Covariance Matrix



Note. $\hat{\xi}$ is common financial shock (blue area); $\hat{\eta}_i$ is country-specific volatility shock (red area); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area); $\hat{\zeta}$ is common growth shock (purple area); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas). The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

S.4 Realized Volatility versus Cross-sectional Dispersion

As noted in the paper, if we consider a panel of country-specific equities (e.g. of firms or sectors within a country), a different measure of uncertainty can be computed as the cross-sectional dispersion of equity prices. In this section we show that this concept is closely related to the realized volatility measure we consider. To illustrate the point with the data that we use in our application, we derive results at the 'country-specific versus world level' rather than 'firm-specific versus country level'.[S5] Specifically, we compare the cross-sectional dispersion of equity returns across countries with the realized volatility of 'world' equity returns.

Define the daily cross-country dispersion of equity returns as:

$$\sigma_{cdt} = \sqrt{D_t^{-1} \sum_{\tau=1}^{D_t} \sum_{i=1}^{N} w_i \left[r_{it}(\tau) - \bar{r}_t(\tau) \right]^2}, \tag{S1}$$

and the daily realized volatility of world equity returns as:

$$\sigma_{rvt} = \sqrt{D_t^{-1} \sum_{i=1}^{N} \sum_{\tau=1}^{D_t} w_i \left[r_{it}(\tau) - \bar{r}_{it} \right]^2}, \tag{S2}$$

where $r_{it}(\tau) = \Delta \ln P_{it}(\tau)$ and $\bar{r}_{it} = D_t^{-1} \sum_{\tau=1}^{D_t} r_{it}(\tau)$ is the average daily price change over the quarter t, and D_t is the number of trading days in quarter t; and w_i is the weight attached to country i. To establish the relation between these two measures it is easier to work with their squares:

$$\sigma_{rvt}^2 = D_t^{-1} \sum_{i=1}^{N} \sum_{\tau=1}^{D_t} w_i \left[r_{it}(\tau) - \bar{r}_{it} \right]^2, \quad \sigma_{cdt}^2 = D_t^{-1} \sum_{\tau=1}^{D_t} \sum_{i=1}^{N} w_i \left[r_{it}(\tau) - \bar{r}_t(\tau) \right]^2.$$

Note also that

$$\sigma_{rvt}^2 = D_t^{-1} \sum_{i=1}^{N} \sum_{\tau=1}^{D_t} w_i r_{it}^2(\tau) - \sum_{i=1}^{N} w_i \bar{r}_{it}^2,$$

and

$$\sigma_{cdt}^2 = D_t^{-1} \sum_{\tau=1}^{D_t} \sum_{i=1}^{N} w_i r_{it}^2(\tau) - \sum_{i=1}^{N} w_i \left(D_t^{-1} \sum_{\tau=1}^{D_t} \bar{r}_t^2(\tau) \right).$$

Hence, since $\sum_{i=1}^{N} w_i = 1$, it follows that

$$\sigma_{cdt}^2 - \sigma_{rvt}^2 = \sum_{i=1}^{N} w_i \bar{r}_{it}^2 - D_t^{-1} \sum_{\tau=1}^{D_t} \bar{r}_t^2(\tau),$$

where as before $\bar{r}_{it} = D_t^{-1} \sum_{\tau=1}^{D_t} r_{it}(\tau)$, and $\bar{r}_t(\tau) = \sum_{i=1}^{N} w_i r_{it}(\tau)$.

Suppose now that daily returns have the following single-factor structure:[S6]

$$r_{it}(\tau) = \beta_i f_t(\tau) + \varepsilon_{it}(\tau),$$

[S5] Our analysis holds at the firm-specific versus country level as well.

[S6] The analysis readily extends to more general multiple factor settings.

where the factor is strong in the sense that (Bailey et al. (2016))

$$\lim_{N\to\infty}\sum_{i=1}^{N} w_i\beta_i = \bar{\beta} \neq 0, \text{ and } \lim_{N\to\infty}\sum_{i=1}^{N} w_i\beta_i^2 = \sigma_\beta^2 + \bar{\beta}^2 > 0.$$

The idiosyncratic components, $\varepsilon_{it}(\tau)$, are assumed to be independently distributed from $\beta_i f_t(\tau)$, cross-sectionally weakly correlated, and serially uncorrelated with zero means and finite variances. Also let:

$$\lim_{D_t\to\infty} D_t^{-1}\sum_{\tau=1}^{D_t} f_t^2(\tau) = h_{f_t}^2.$$

We now note that

$$\begin{aligned}\sum_{i=1}^{N} w_i\bar{r}_{it}^2 &= \left(\sum_{i=1}^{N} w_i\beta_i^2\right)\bar{f}_t^2 + \left(\sum_{i=1}^{N} w_i\bar{\varepsilon}_{it}^2\right) + 2\left(\sum_{i=1}^{N} w_i\beta_i\bar{\varepsilon}_{it}\right)\bar{f}_t \\ &= \left(\sigma_\beta^2 + \bar{\beta}^2\right)\bar{f}_t^2 + O_p\left(D_t^{-1/2}\right) + O_p\left(N^{-1/2}\right),\end{aligned}$$

where $\bar{f}_t = D_t^{-1}\sum_{\tau=1}^{D_t} f_t(\tau)$, and $\bar{\varepsilon}_{it} = D_t^{-1}\sum_{\tau=1}^{D_t}\varepsilon_{it}(\tau)$. Also

$$\begin{aligned}D_t^{-1}\sum_{\tau=1}^{D_t}\bar{r}_t^2(\tau) &= D_t^{-1}\sum_{\tau=1}^{D_t}\left[\bar{\beta}f_t(\tau) + \bar{\varepsilon}_t(\tau)\right]^2 \\ &= \bar{\beta}^2\left[D_t^{-1}\sum_{\tau=1}^{D_t} f_t^2(\tau)\right] + D_t^{-1}\sum_{\tau=1}^{D_t}\bar{\varepsilon}_t^2(\tau) + 2D_t^{-1}\sum_{\tau=1}^{D_t}\bar{\beta}\bar{\varepsilon}_t(\tau)f_t(\tau) \\ &= \bar{\beta}^2 h_{f_t}^2 + O_p\left(N^{-1/2}\right) + O_p\left(D_t^{-1/2}\right).\end{aligned}$$

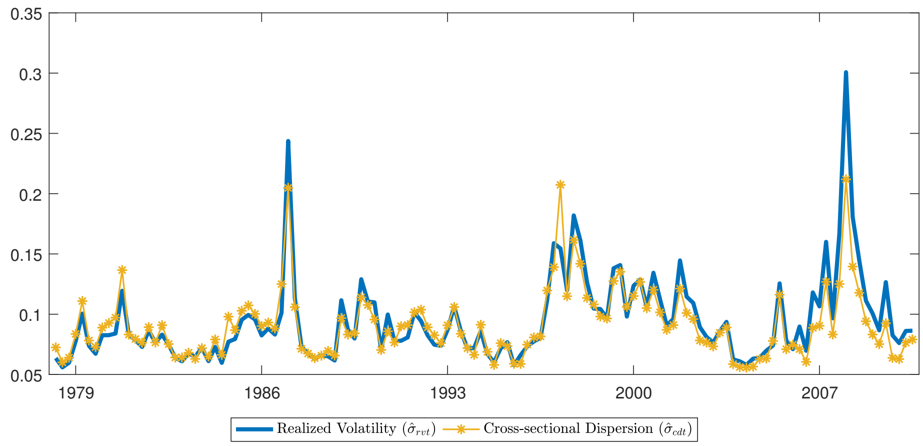
Hence

$$\begin{aligned}\sigma_{cdt}^2 - \sigma_{rvt}^2 &= \left(\sigma_\beta^2 + \bar{\beta}^2\right)\bar{f}_t^2 - \bar{\beta}^2 h_{f_t}^2 + O_p\left(N^{-1/2}\right) + O_p\left(D_t^{-1/2}\right) \\ &= \sigma_\beta^2\bar{f}_t^2 - \bar{\beta}^2\sigma_{f_t}^2 + O_p\left(N^{-1/2}\right) + O_p\left(D_t^{-1/2}\right).\end{aligned}$$

where $\sigma_{f_t}^2 = \left(h_{f_t}^2 - \bar{f}_t^2\right) \geq 0$, is the variance of the common factor. This expression shows that, under fairly general assumptions (and for N and D_t sufficiently large) we would expect the cross-sectional dispersion measure to be closely related to asset-specific measures of realized volatility when the factor loadings, β_i, are not too dispersed across countries. The results also show that the relative magnitudes of the cross section dispersion and realized volatility measures depend on the relative values of $\sigma_\beta^2\bar{f}_t^2$ and $\bar{\beta}^2\sigma_{f_t}^2$.

Figure S.15 compares world realized volatility (σ_{rvt}, light thick line) and cross-sectional dispersion (σ_{cdt}, dark thin line), computed as in equations (S2) and (S1), respectively, with equal weights. Their sample correlation over the 1979:Q1 to 2016:Q4 period is 0.92. Figure S.15 suggests that the two measures are very closely related, which is in line with the evidence provided by Bloom et al. (2012).

Figure S.15 Realized Volatility and Cross-sectional Dispersion



Note. World realized volatility of equity returns ($\hat{\sigma}_{rvt}$) and cross-sectional dispersion of equity returns across countries ($\hat{\sigma}_{cdt}$), computed as in equations (S2) and (S1), respectively. Both measures are expressed at quarterly rates and computed over the 1979:Q2-2016:Q4 period.

S.5 Computing Impulse Responses and Error Variance Decompositions

Consider the factor-augmented country-specific VAR models augmented with lagged cross section averages, $\bar{\mathbf{z}}_{\omega,t-\ell}$, for $\ell = 1, 2, ..., p$ as in equations (38)-(39) in the main text:

$$\mathbf{z}_{it} = \mathbf{\Phi}_i \mathbf{z}_{i,t-1} + \sum_{\ell=1}^{p} \mathbf{d}_{i\ell} \bar{\mathbf{z}}_{\omega,t-\ell} + \boldsymbol{\beta}_i \boldsymbol{\delta}_t + \boldsymbol{\vartheta}_{it}, \text{ for } i = 1, 2, ..., N, \tag{S1}$$

where:

$$\mathbf{d}_{i\ell} = \begin{pmatrix} d_{1v,i\ell} & d_{2v,i\ell} \\ d_{1\Delta y,i\ell} & d_{2\Delta y,i\ell} \end{pmatrix}, \boldsymbol{\beta}_i = \begin{pmatrix} \beta_{i,11} & \beta_{i,12} \\ \beta_{i,21} & 0 \end{pmatrix}, \boldsymbol{\upsilon}_t = \begin{pmatrix} \zeta_t \\ \xi_t \end{pmatrix}.$$

Intercepts are omitted to simplify the exposition. Note also that $\bar{\mathbf{z}}_{\omega,t} = \sum_{i=1}^{N} w_i \Delta \mathbf{z}_{it} = \mathbf{W}\mathbf{z}_t$, where $\mathbf{z}_t = (\mathbf{z}'_{1t}, \mathbf{z}'_{2t}, ..., \mathbf{z}'_{Nt})'$, and $\mathbf{W}$ is a $2 \times 2N$ matrix of weights. Stacking the VARs in (S1) over i we obtain:

$$\mathbf{z}_t = \mathbf{\Phi}\mathbf{z}_{t-1} + \sum_{\ell=1}^{p} \mathbf{d}_\ell \mathbf{W} \mathbf{z}_{t-\ell} + \boldsymbol{\beta}\boldsymbol{\delta}_t + \boldsymbol{\vartheta}_t, \tag{S2}$$

where $\boldsymbol{\vartheta}_t = (\boldsymbol{\vartheta}'_{1t}, \boldsymbol{\vartheta}'_{2t}, ..., \boldsymbol{\vartheta}'_{Nt})'$ and:

$$\mathbf{\Phi} = \begin{pmatrix} \mathbf{\Phi}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \cdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{\Phi}_N \end{pmatrix}, \quad \mathbf{d}_\ell = \begin{pmatrix} \mathbf{d}_{1,\ell} \\ \mathbf{d}_{2,\ell} \\ \vdots \\ \mathbf{d}_{N,\ell} \end{pmatrix}, \quad \boldsymbol{\beta} = \begin{pmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \\ \vdots \\ \boldsymbol{\beta}_N \end{pmatrix}.$$

The high-dimensional VAR in (S2) can now be written as a standard FAVAR(p) model in $2N$ variables:

$$\mathbf{z}_t = (\mathbf{\Phi} + \mathbf{d}_1 \mathbf{W})\, \mathbf{z}_{t-1} + \sum_{\ell=2}^{p} \mathbf{d}_\ell \mathbf{W} \mathbf{z}_{t-\ell} + \boldsymbol{\beta}\boldsymbol{\delta}_t + \boldsymbol{\vartheta}_t. \tag{S3}$$

For example, when $p = 1$ we have the FAVAR(1):

$$\mathbf{z}_t = (\mathbf{I}_{2N} - \mathbf{\Psi}_1\mathbf{L})^{-1}(\boldsymbol{\beta}\boldsymbol{\delta}_t + \boldsymbol{\vartheta}_t),$$

where $\mathbf{\Psi}_1 = \mathbf{\Phi} + \mathbf{d}_1\mathbf{W}$ and

$$\mathbf{z}_t = (\mathbf{I}_{2N} - \mathbf{\Psi}_1\mathbf{L})^{-1}\boldsymbol{\beta}\boldsymbol{\delta}_t + (\mathbf{I} - \mathbf{\Psi}_1\mathbf{L})^{-1}\boldsymbol{\vartheta}_t.$$

Note that by construction $\boldsymbol{\delta}_t$ and $\boldsymbol{\vartheta}_t$ are orthogonal, and for sufficiently large p, they are serially uncorrelated. Hence, the impulse response of shocks to elements of $\boldsymbol{\delta}_t$ and $\boldsymbol{\vartheta}_t$ can be computed using the following moving average representation:

$$\mathbf{z}_t = \sum_{n=0}^{\infty} \mathbf{A}_n \boldsymbol{\delta}_{t-n} + \sum_{n=0}^{\infty} \mathbf{C}_n \boldsymbol{\vartheta}_{t-n}, \tag{S4}$$

where $\mathbf{A}_n = \mathbf{\Psi}_1^n \boldsymbol{\beta}$, and $\mathbf{C}_n = \mathbf{\Psi}_1^n$, for $n = 0, 1, 2, ...$

S.5.1 Responses to Common and Country-specific Shocks

Let $\mathfrak{e}_i$ be a selection vector such that $\mathfrak{e}_i'\mathbf{z}_t$ picks the i^{th} element of $\mathbf{z}_t$. Also let $\mathbf{s}_f = (1, 0)'$ and $\mathbf{s}_g = (0, 1)'$, the vectors that select ζ_t and ξ_t from $\boldsymbol{\delta}_t$, namely:

$$\mathbf{s}_f'\boldsymbol{\delta}_t \equiv \zeta_t, \quad \mathbf{s}_g'\boldsymbol{\delta}_t \equiv \xi_t. \tag{S5}$$

Recall now that ζ_t and ξ_t have zero means, unit variances and are orthogonal to each other. Then the impulse responses to a positive unit shock to ζ_t or ξ_t are given by:

$$IR_{i,\zeta,n} = \mathfrak{e}_i'\mathbf{A}_n\mathbf{s}_f \quad \text{and} \quad IR_{i,\xi,n} = \mathfrak{e}_i'\mathbf{A}_n\mathbf{s}_g \quad \text{for } n = 0, 1, 2, ..., \tag{S6}$$

where $\mathbf{A}_n$ is given by the moving average representation, (S4)

To derive impulse response functions for country-specific shocks, namely the j^{th} element of $\boldsymbol{\vartheta}_t$, we need to make assumptions about the correlation between volatility and growth innovations within each country and across countries. Since the elements of $\boldsymbol{\vartheta}_t$ are weakly correlated across countries, they have some, but limited correlations across countries (see Figure 6). We also documented that, conditional on the common factors ζ_t and ξ_t, the country-specific correlation of volatility and growth innovations are statistically insignificant for all except for four countries.

As a first order approximation, therefore, we will assume that the covariance matrix of $\boldsymbol{\vartheta}_t$ in (S3) is diagonal. Under this assumption, the impulse response function of a positive, unit shock to the j^{th} element of $\boldsymbol{\vartheta}_t$ on the the i^{th} element of $\mathbf{z}_t$ is given by:

$$IR_{i,\vartheta_j,n} = \sqrt{\hat{\omega}_{jj}}\,\mathfrak{e}_i'\mathbf{C}_n\mathfrak{e}_j \ , \tag{S7}$$

where $\mathbf{C}_n$ is given by the moving average representation, (S4), $\hat{\omega}_{jj}$ is the (estimate) of the variance of the j^{th} country-specific shock and $\mathfrak{e}_j$ is a selection vector such that $\mathfrak{e}_j'\mathbf{z}_t$ picks the j^{th} element of $\mathbf{z}_t$.

The above impulse responses can be compared to the generalized impulse responses of Pesaran and Shin (1998). The latter are given by:

$$GIR_{i,\vartheta_j,n} = \frac{\mathfrak{e}_i'\mathbf{C}_n\hat{\Omega}\mathfrak{e}_j}{\sqrt{\hat{\omega}_{jj}}}, \tag{S8}$$

where $\hat{\mathbf{\Omega}} = (\hat{\omega}_{ij})$ is the estimate of the covariance of $\boldsymbol{\vartheta}_t$. The generalized impulse responses allow for non-zero correlations across the idiosyncratic errors. The two sets of impulse responses coincide if the covariance matrix of $\boldsymbol{\vartheta}_t$ is diagonal.

S.5.2 Forecast Error Variance Decompositions

Traditionally, the forecast error variance decomposition of a VAR model is performed on a set of orthogonalized shocks, whereby the contribution of the j^{th} orthogonalized innovation to the mean square error of the n-step ahead forecast of the model is calculated. In our empirical application this is not the case as —even if the country-specific volatility and growth innovations η_{it} and ε_{it} are weakly correlated across countries— some pairs of innovations can still display some non-zero correlation. An alternative approach is to compute Generalized Forecast Error Variance Decompositions (GVD) of Pesaran and Shin (1998). The Generalized Forecast Error Variance Decompositions consider the proportion of the variance of the n-step forecast errors of the endogenous variables that is explained by conditioning on the non-orthogonalized shocks, while explicitly allowing for the contemporaneous correlations between these shocks and the shocks to the other equations in the system.

Let $GVD_{i,\zeta,n}$ and $GVD_{i,\xi,n}$ be the share of the n-step ahead forecast error variance of the i^{th} variable in $\mathbf{z}_t$ that is accounted for by ζ_t and ξ_t, respectively, and $GVD_{i,j}$ the variance share of a generic country-specific shock, then:

$$GVD_{i,\zeta,n} = \frac{\sum_{\ell=0}^{n} (\mathfrak{e}_i'\mathbf{A}_\ell \mathbf{s}_f)^2}{\sum_{\ell=0}^{n} \mathfrak{e}_i'\mathbf{A}_\ell\mathbf{A}_\ell'\mathfrak{e}_i + \sum_{\ell=0}^{n} \mathfrak{e}_i'\mathbf{C}_\ell\hat{\mathbf{\Omega}}\mathbf{C}_\ell'\mathfrak{e}_i}, \quad n = 1,2,....,H, \tag{S9}$$

$$GVD_{i,\xi,n} = \frac{\sum_{\ell=0}^{n} (\mathfrak{e}_i'\mathbf{A}_\ell \mathbf{s}_g)^2}{\sum_{\ell=0}^{n} \mathfrak{e}_i'\mathbf{A}_\ell\mathbf{A}_\ell'\mathfrak{e}_i + \sum_{\ell=0}^{n} \mathfrak{e}_i'\mathbf{C}_\ell\hat{\mathbf{\Omega}}\mathbf{C}_\ell'\mathfrak{e}_i}, \quad n = 1,2,....,H, \tag{S10}$$

$$GVD_{i,j,n} = \frac{\hat{\omega}_{jj}^{-1}\sum_{\ell=0}^{n} \left(\mathfrak{e}_i'\mathbf{C}_\ell \hat{\mathbf{\Omega}}\mathfrak{e}_j\right)^2}{\sum_{\ell=0}^{n} \mathfrak{e}_i'\mathbf{A}_\ell\mathbf{A}_\ell'\mathfrak{e}_i + \sum_{\ell=0}^{n} \mathfrak{e}_i'\mathbf{C}_\ell\hat{\mathbf{\Omega}}\mathbf{C}_\ell'\mathfrak{e}_i}, \quad j = 1,2,....,2N, \quad n = 1,2,....,H; \tag{S11}$$

Note that the different assumptions we make on the covariance matrix of all country-specific shocks, $\hat{\mathbf{\Omega}}$, have implications for the error variance decompositions. Specifically, when we assume that (i) $\hat{\mathbf{\Omega}}$ is diagonal or (ii) $\hat{\mathbf{\Omega}}$ is block-diagonal with Cholesky-orthogonalized blocks, the relative importance of shocks to country-specific volatility and growth for all countries (η_{it} and ε_{it}, for $j = 1,2,....,2N$) and shocks to the two common factors ζ_t and ξ_t, is easily characterized as $VD_{i,\zeta,n} + VD_{i,\xi,n} + \sum_{j=1}^{2N} VD_{i,j,n} = 1$. That is the GVD formula coincides with the standard VD formula. In contrast, when we consider an unrestricted covariance matrix $\hat{\mathbf{\Omega}}$, the sum of the variance shares does not necessarily add up to 1.

S.5.3 Average Impulse Responses and Forecast Error Variance Decompositions

As a summary measure of the effects of shocks to the common factors we report the following average measures. Denote the impulse response (or forecast error variance decomposition) of a particular shock on the j^{th} variable in country i at horizon n by $X_{i,j,n}$. Let $\mathbf{w} = (w_1, w_2, ..., w_N)'$ be a vector of fixed weights such that $\Sigma_{i=1}^{N} w_i = 1$. Then the average impulse response (or forecast error variance decomposition) of the shock to variable j, at horizon n, is computed as:

$$X_{\omega,j,n} = \sum_{i=1}^{N} w_i X_{i,j,n}. \tag{S12}$$

and its dispersion is computed by:

$$\sigma_{X_{\omega,j,n}} = \left[\sum_{i=1}^{N} w_i^2 \left(X_{i,j,n} - X_{\omega,j,n} \right)^2 \right]^{1/2}, \tag{S13}$$

assuming country-specific impulse responses or forecast error variance decompositions are approximately uncorrelated.

Supplement References

Bailey, N., G. Kapetanios, and M. H. Pesaran (2016): Exponent of Cross-Sectional Dependence: Estimation and Inference," *Journal of Applied Econometrics*, 31, 929-960.

— (2018a): "Exponent of Cross-sectional Dependence for Residuals," CESifo Working Paper Series 7223, CESifo Group Munich.

Bailey, N., M. H. Pesaran, and L. V. Smith (2018b): "A multiple testing approach to the regularisation of large sample correlation matrices," Forthcoming in the *Journal of Econometrics*.

Bloom, N., M. Floetotto, N. Jaimovich, I. Saporta-Eksten, and S. J. Terry (2012): "Really Uncertain Business Cycles," NBER Working Papers 18245, National Bureau of Economic Research, Inc.

Pesaran, M. H. (2015): *Time Series and Panel Data Econometrics*, Oxford University Press, Oxford.

Pesaran, H. H. and Y. Shin (1998): "Generalized impulse response analysis in linear multivariate models," *Economics Letters*, 58, 17-29.