The Network Origins of Bank Influence: Evidence from Bank–to–Firm and Firm–to–Firm Linkages*

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Abstract

Bank lending shocks affect firm behaviour and permeate, via firm interactions, throughout the real the economy. In this paper, we explore how the production architecture of the real economy determines the aggregate real impact of shocks that originate from individual banks. Confidential data on the universe of (*i*) firm–to–firm transactions and (*ii*) bank—firm borrowing relations of the Belgian economy allows us to reconstruct (*i*) the firm–level input—output production architecture of the Belgian economy and (*ii*) the bank credit network supporting it. We use these objects to structurally document the extent to which individual banks support value creation in the economy either directly or indirectly through endogenous network acceleration mechanisms. We then study how the current structure of the Belgian economy affects the size of aggregate real GDP fluctuations induced by shocks from individual banks. In that context, we revisit the Lucas (1976) argument – i.e. the rate at which this aggregate effect vanishes as the number of banks in the economy increases. We show that our analysis speaks to key research questions related to financial sector competition policy, strategic bank lending and selected macro prudential topics.

Keywords: Shock propagation · Input–output linkages · Bank–level influence
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1 Introduction

There is a long and well–established tradition in the macroeconomic literature of adding financial frictions to standard macroeconomic models in order to document the importance of the banking sector for real business cycle fluctuations (Quadrini, 2011; Bernanke and Gertler, 1989; Bernanke and Blinder, 1988). In most models, the banking sector acts either (*i*) as an amplification mechanism of shocks originating elsewhere in the economy or (*ii*) as an exogenous source of shocks that affect credit supply conditions to firms.[1] In the latter case, these models typically appeal to aggregate shocks to bank credit supply conditions, common across all banks, in order to explain aggregate real fluctuations (e.g. Gerali et al. (2010); Andrés et al. (2013); Andrés and Arce (2012)). Reversely, idiosyncratic, bank–specific, shocks to credit supply conditions are not taken into account as a material source of business cycles. Algebraically, this assumption enters these models through the inclusion of a representative bank (or atomistic banks) instead of a finite set of heterogeneous banks. The diversification argument by Lucas (1977)[2] justifies this approach: the real effects that stem from idiosyncratic variation in lending from individual banks cancels out across the real economy, implying that bank–specific lending shocks to firms are irrelevant for the study of fluctuations in real macroeconomic aggregates.

Despite this powerful dismissal of micro–level shocks on theoretical grounds, recent empirical contributions identify idiosyncratic shocks to bank credit–supply conditions as important drivers of macroeconomic variables, e.g. (*i*) aggregate investment (Amiti and Weinstein, 2018; Amador and Nagengast, 2016), (*ii*) gross domestic product (Buch et al., 2014; Buch and Neugebauer, 2011), (*iii*) international trade patterns (Tielens and Van Hove, 2019; Niepmann and Eisenlohr, 2014), (*iv*) aggregate employment (Greenstone et al., 2014), (*v*) economywide total factor productivity growth (Manaresi and Pierri, 2018), etc. Note that the evidence in this literature relies on idiosyncratic variation in bank lending to firms which is purged from general equilibrium mechanisms in the financial system (e.g. inter–bank claims, regulatory shocks, common exposures, etc.). These empirical results then suggest a role for important amplification mechanisms in the real economy instead of the financial system architecture.

In view of this disconnect between (*i*) traditional macro models and (*ii*) the emerging empirical evidence, we revisit the scale of aggregate real GDP fluctuations due to bank specific shocks to credit supply conditions in a New–Keynesian (NK) set–up. We focus on two types of shocks related to bank credit supply conditions: loan–to–value (LTV) requirements and risk

[1] When financial frictions act as amplification mechanisms, a shock originates in the non–financial sector (e.g. exogenous changes in productivity, preferences, etc.) and is consequently aggravated by financial frictions. Such models include Bernanke and Gertler (1989); Bernanke et al. (1999); Carlstrom and Fuerst (1997). Alternatively, the initial disruption arises in the financial sector without initial changes in the non–financial sector. Typically, financial frictions cause fewer funds to be channelled from banks to borrowers which in turn affects the real economy. Standard models include Kiyotaki and Moore (2012); Del Negro and Schorfheide (2008); Gertler and Karadi (2011); Christiano et al. (2010).

[2] Not to be confused with the well-known Lucas (1977) critique about behavioural changes in macroeconomic models.

premia shocks. Our model features three sources of heterogeneity that are shown to attenuate the extent to which real effects of bank–specific credit supply shocks balance out in the aggregate. (*i*) First, we embed a heterogeneous firm–level input–output production structure into the model (in the spirit of Long and Plosser (1983); Basu (1995)) where firms rely on each other for intermediate input requirements. (*ii*) Second, we allow for heterogeneity in the level of value added that individual firms produce. (*iii*) Finally, we introduce a monopolistic banking sector (a variant to Gerali et al. (2010); Cuciniello and Signoretti (2015)) featuring a firm–bank credit network and heterogeneous collateral constraints. The model, which combines a standard amplification mechanism brought about by financial frictions (such as analysed by Kiyotaki and Moore (1997)) with an amplification mechanism in the real economy caused by firm–to–firm interactions (such as analysed by e.g. Acemoglu et al. (2012)), is set up such that – under extreme parametrizations – the framework nests (or tightly tracks) the NK models of Iacoviello (2005, 2015); Pasten et al. (2018a,b); Carvalho and Lee (2011) and IO–like models of Bremus et al. (2018); Gabaix (2011); Acemoglu et al. (2012). Our model purposely leaves no role for an inter–bank market as a vehicle through which idiosyncratic bank–level events propagate.

The general equilibrium of the model is analytically tractable and delivers an influence measure for individual banks. The latter summarizes the extent to which credit of an individual bank supports value added creation in the economy. Key to our framework is that the effect of a shock to credit supply conditions from an individual bank does not remain confined to its borrowing firms, but permeates across the production architecture of the real economy and endogenously affects credit constraints elsewhere in the economy. The total influence of an individual bank on real GDP is then determined by the extent the bank, both directly and indirectly, affects added value creation by individual firms in the real economy.

We show that two interlocked networks (in casu individual firm credit portfolios and the production architecture of the real economy) potentially introduce large asymmetries in the influence of individual banks. Stated differently, some banks intensively support – directly and/or indirectly – value added of the non–financial sector, whereas other banks are only marginally connected to firms involved in value added creation. In contrast to the Lucas (1977) argument, such an asymmetric structure delivers banks whose shocks do not easily cancel out with that of other banks – even when the number of banks in the economy is large. As per Acemoglu et al. (2012), the law of large numbers on which the Lucas (1977) argument is built does no longer apply in the model set–up we propose. One contribution of this paper is to derive closed form expressions that characterize this relationship between (*i*) the asymmetry in bank influences, (*ii*) the number of banks in the economy and (*iii*) aggregate real GDP fluctuations.

The model is calibrated to the Belgian economy. The backbone of our calibration exercise relies on three confidential databases provided by the National Bank of Belgium; (*i*) the corporate credit register, (*ii*) the Business–to–Business database and (*iii*) value added tax declarations. The first datasource provides detailed information on the population of loans from banks to firms (as well as underlying collateral). It allows us to calibrate the credit network that ties banks to firms and LTV ratios that banks impose on firms. The second contains a quasi exhaustive list of all business–to–business transactions between VAT liable Belgian firms. We use it to calibrate an input–output matrix at the firm level.[3] Finally, the VAT declarations allow us to directly observe value added from individual firms. In sum, the three datasets reconstruct the production architecture of the Belgian economy as well as the credit network supporting it.

We find that the true structure of the Belgian economy, bank specific (LTV ratios and interest rate) shocks have material aggregate effects; Their aggregate effect is $40\% - 50\%$ as large as that of a common shock of similar size that affects all banks. This underscores that the widespread practice of discarding bank–specific shocks in macro models is highly restrictive. In a hypothetical full symmetric version of the Belgian economy, this multiplier is only 11% of what an aggregate bank shock would generate.

Asymmetries in the credit network and the production economy are shown to reinforce each other. E.g. banks that disproportionally lend to firms that create a lot of value added are also indirectly – through IO interactions – very supportive of value added creation of firms elsewhere in the economy to which they do not lend directly. In addition, from previous work it is well–documented that loose LTV requirements provide an endogenous financial propagation mechanism which increases macroeconomic volatility (Jensen et al., 2018; Walentin, 2014). We show that banks typically allow for loose LTV ratios when lending to key firms in the Belgian production network, which further enhances macroeconomic volatility originating from individual banks.

We distill three important policy implications from the analysis. First, the input–output structure of the real economy as a vehicle through which bank shocks propagate is an important feature in the context of skewed sector presence. Skewed sector presence implies that some individual banks are dominant credit providers to firms in specific (non–financial) sectors (e.g. Boeve et al. (2010); De Jonghe et al. (2019); Paravisini et al. (2016)).[4] An important takeaway from our framework is that specialization of banks can lead to increased macroeconomic volatility as bank shocks propagate, via their respective sector of specialization, to all other firms that rely on inputs from this sector. E.g. we show that, for Belgium, in the aftermath of the financial

[3]While there has been an increased interest in shock propagation across networks, the available firm–to–firm data is often restricted in terms of the (*i*) sectoral coverage (see e.g. Acemoglu et al. (2015a)), (*ii*) geographical coverage (see e.g. Carvalho et al. (2016)), (*iii*) reporting thresholds (see e.g. Atalay et al. (2011); Bernard et al. (2015)), (*iv*) nominal quantification of business relationships (see e.g. Kelly et al. (2014); Carvalho et al. (2016)) or (*v*) level of aggregation (see e.g. Acemoglu et al. (2012); Shea (2002)). The $B2B$ is unrestricted in aforementioned dimensions.

[4]Skewed sector presence, arising from bank specialization, is widespread. E.g. it is not uncommon for specific industries to be exclusively served by a single bank.

crisis, such shifts in strategic bank lending have caused an increase in macroeconomic volatility.

The Herfindahl–Hirschmann index (HHI) is a popular policy measure to quantify financial sector competition, concentration and stability. It is often used to identify potentially disruptive M&A's given that an increase in the HHI is often associated with increased macroeconomic volatility (Bremus, 2015; Bremus and Buch, 2017; Bremus et al., 2018). We show that the HHI is only marginally informative about macroeconomic volatility. The reason is that the HHI focuses exclusively on bank market shares. Our framework, instead, focuses on the (in)direct role that the borrowing non–financial corporations in the banks' portfolio play in the real economy. M&A activity in the banking sector might lead to an increase in the HHI, but might enhance symmetry among banks, which lowers volatility. We show that, in the face of modest M&A activity in the Belgian banking sector, the HHI has effectively remained relatively stable in Belgium over the last decade whereas we document macroeconomic volatility from bank–specific shocks to have slightly increased in the post–crisis period.[5]

Finally, our analysis also speaks to the identification of significant banks. In a European context, following SSM regulation, such identification is i.a. based on concepts such as size – "too big to fail" – and interconnections – "too interconnected to fail" (Freixas et al. (2000); Allen and Gale (2000); Acemoglu et al. (2015b)). The latter principle refers to connections arising from inter bank claims and connections to highly leveraged shadow financial institutions. Our view on interconnectivity complements this more traditional notion of financial connectedness and focuses on real connectedness, i.e. the way the production architecture of the economy is matched to individual banks.

Most directly, this paper contributes to the emerging literature on the micro origins of aggregate fluctuations. Although it has been a topic of interest in both the real business cycle literature and the financial economics literature, both research agendas developed largely in isolation from each other. Following Bigio and La'O (2019), our focus on the financial micro origins of aggregate real fluctuations aims to bridge the gap between these two research areas.[6]

[5]See Degryse et al. (2011).

[6]For the real economy, Gabaix (2011) was the first to develop the view that a large part of aggregate fluctuations arises from idiosyncratic productivity shocks to large individual firms. Empirical evidence strongly supports this view in the case of aggregate exports (Di Giovanni et al., 2014), GDP (Gabaix, 2011), the trade balance (Canals et al., 2007), etc. A related literature, originating from the seminal contribution of Long and Plosser (1983), explores the role of production networks in generating aggregate fluctuations in GDP. Idiosyncratic productivity shocks to individual sectors are found to propagate throughout the economy via input–output linkages, causing comovement across sectors and ultimately leading to aggregate fluctuations in GDP (see e.g. Shea (2002); Horvath (1998); Foerster et al. (2011); Dupor (1999); Conley and Dupor (2003); Acemoglu et al. (2012)). More recently, this network view has been taken to the firm level (e.g. Kelly et al. (2014), Carvalho et al. (2016), Carvalho (2014), Baqaee (2018), Stella (2015), Magerman et al. (2015), Tielens and Van Hove (2019), etc.). A parallel, but distinct, literature has focused on how the architecture of the financial system works as an amplification mechanism of shocks to individual banks. Early contributions include Freixas et al. (2000) and Allen and Gale (2000) who investigate how the nexus of inter–bank claims affect the resilience of the aggregate financial system to the insolvency of any individual bank. This network view has been central in subsequent contributions as well (e.g. Dasgupta (2004); Elliott et al. (2014); Allen et al. (2012); Acemoglu et al. (2015b)).

More generally, this paper is part of a larger agenda that investigates the impact of credit frictions on the real economy. Credit frictions are found to distort key economic activities, e.g. investment (Almeida and Campello, 2007), exports (Paravisini et al., 2015; Amiti and Weinstein, 2011; Strasser, 2013), pricing behaviour (Strasser, 2013), etc. While most of these studies offer compelling micro–level evidence that individual banks matter for individual firms, they have not addressed the question of how important individual banks are in determining the aggregate real economy. An emerging (but exclusively empirical) literature aims to fills this gap (e.g. Tielens and Van Hove (2019); Amiti and Weinstein (2018); Buch and Neugebauer (2011); Niepmann and Eisenlohr (2014)). This paper formally identifies the exact mechanisms and preconditions that drive these empirical results.

The remainder of the paper is organized as follows. Section 2 presents a basic economic environment. Section 3 analyses the log linearised model and relates the size of aggregate real volatility to the structure of the economy. Section 4 discusses the data sources and calibration exercise underlying the analysis of the model in section 5 and 6. Section 7 discusses policy implications. Section 8 concludes. All proofs and mathematical details are relegated to the appendix.

2 The model

The model is a variant of the standard NK model from which we make three key departures; (*i*) a heterogeneous firm–level input–output structure in the spirit of Long and Plosser (1983) where firms rely on each other for intermediate input requirements, (*ii*) heterogeneity in the level of value added individual firms produce, (*iii*) a monopolistic banking sector featuring a firm–bank credit network and heterogeneous collateral constraints.

2.1 Households

The representative household derives utility from non–durable consumption, land holdings and leisure. It maximizes the additive separable utility function

$$\mathbb{E}_0\Big[\sum_{t=0}^{\infty}\beta^t\big(\log(c_t)+\iota h_t-\sum_{j=1}^{J}g_j\frac{l_{jt}^{1+\varphi}}{1+\varphi}\big)\Big]$$

where c_t is an aggregate household consumption bundle, h_t denotes holding of land, and l_{jt} denotes the hours of labour supplied to firm j. The parameters β, φ, ι and $\{g_j\}_{j=1}^{J}$ denote the discount factor, the inverse of the (Frisch) elasticity of labor supply, relative land utility and the relative disutilities of supplying labour to individual firms, respectively. The aggregate consumption bundle is defined as

$$c_t = \Big(\sum_{j=1}^{J} \theta_j^{\frac{1}{\eta}} c_{jt}^{1-\frac{1}{\eta}}\Big)^{\frac{\eta}{\eta-1}} \qquad \sum_{j=1}^{J} \theta_j = 1 \wedge \theta_j \in [0,1]\ \forall j \tag{1}$$

where c_{jt} denotes consumption of goods/services produced by firm j and $\boldsymbol{\theta}$ collects the steady state share of good j in household consumption. If $\theta_j = 0$, firm j does not sell to households directly. The price index associated with aggregate consumption, P_t, is defined as

$$P_t = \Big(\sum_{j=1}^{J} \theta_j P_{jt}^{1-\eta}\Big)^{\frac{1}{1-\eta}}$$

where P_{jt} denotes the nominal price of goods from firm j. The households' demand schedule for goods/services from firm j is

$$c_{jt} = \theta_j \big(\frac{P_{jt}}{P_t}\big)^{-\eta} c_t$$

The budget constraint, in real terms, is given by

$$c_t + q_t(h_t - h_{t-1}) + \frac{R_{t-1} d_{t-1}}{\pi_t} = d_t + \sum_{j=1}^{J} w_{jt} l_{jt} + \sum_{j=1}^{J} \Delta_{jt} + \sum_{b=1}^{B} \Delta_{bt}$$

Where $\pi_t \equiv P_t/P_{t-1}$ denotes the gross inflation rate, $q_t \equiv \frac{Q_t}{P_t}$ is the real land price and $w_{jt} \equiv W_{jt}/P_t$ is the real wage paid by firm j. We assume that risk–free, one–period, bank deposits $d_t \equiv D_t/P_t$ are the only financial asset available to households. D_t pays an interest rate R_t set by the monetary authority. Δ_{jt} and Δ_{bt} are lump sum firm and bank profits, respectively, which are channelled to households.

The first order conditions w.r.t. $\{c_t, d_t, l_{jt}, h_t\}_{t=0}^{\infty}$ deliver firm–level labour supply schedules (2a), an Euler equation (2b) and demand for land (2c)

$$w_{jt} = g_j l_{jt}^{\varphi} c_t \tag{2a}$$

$$\frac{1}{c_t} = \beta \mathbb{E}_t \big(\frac{R_t}{\pi_{t+1} c_{t+1}}\big) \tag{2b}$$

$$\frac{q_t}{c_t} = \frac{\iota}{h_t} + \beta \mathbb{E}_t \Big(\frac{q_{t+1}}{c_{t+1}}\Big) \tag{2c}$$

2.2 Production

Production in the economy is shaped by J monopolistic competitive firms. Firm j produces quantity y_{jt} according to the Cobb–Douglas technology

$$y_{jt} = A_j \big(n_{jt}^{\phi_j} m_{jt}^{1-\phi_j} \big)^{\delta_j} k_{jt}^{1-\delta_j} - \Phi_j \quad s.t. \quad \phi_j, \delta_j \in (0,1)$$

where n_{jt}, m_{jt} and k_{jt} denote (i) hired household labour, (ii) an intermediate input bundle and (iii) capital services, respectively. The constraints impose constant returns to scale in variable inputs and ensure the existence of a stable equilibrium. Φ_j denotes a fixed cost and ensures zero steady state profit in order to rule out entry. A_j is an innocuous technology parameter.[7] The intermediate input bundle m_{jt}

$$m_{jt} = \left(\sum_{j'=1}^{J} \omega_{jj'}^{\frac{1}{\eta}} m_{jj't}^{1-\frac{1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \qquad \sum_{j'=1}^{J} \omega_{jj'} = 1 \wedge \omega_{jj'} \in [0,1] \ \forall j, j'$$

is a Dixit–Stiglitz aggregate over intermediate inputs sourced from other firms; $m_{jj't}$ denotes the amount of goods that firm j procures from firm j'. Optimal demand schedules for intermediate goods are given by

$$m_{jj't} = \omega_{jj'} \left(\frac{P_{j't}}{P_{jt}^{\omega}} \right)^{-\eta} m_{jt}$$

where $P_{jt}^{\omega} = \left(\sum_{j'=1}^{J} \omega_{jj'} P_{j't}^{1-\eta} \right)^{\frac{1}{1-\eta}}$ is the input–cost index of firm j. $\mathbf{\Omega} \in \mathbb{R}^{J \times J}$ with generic element $\omega_{jj'}$ denotes the input–output matrix of the economy at the firm level, describing the flow of intermediates across the economy. $\omega_{jj'} = 0$ if firm j does not directly buy from firm j' and the larger $\omega_{jj'}$, the more important firm j' is as an input supplier to firm j. This interconnected production architecture leaves firms exposed to distress originating with other firms.

Firms are further characterized by staggered price setting consistent with a Calvo (1983)–Yun (1996) framework. Firm j has a probability of $1 - \alpha_j$ to reset its price each period and is thus faced with the following dynamic price setting problem

$$\underset{P_{jt}^*}{\text{Max}} \ \mathbb{E}_t \sum_{s=0}^{\infty} \alpha_j^s \Lambda_{t,t+s} (P_{jt} y_{jt+s} - MC_{jt+s}(W_{jt+s}, P_{jt+s}^{\omega}, F_{jt+s}) y_{jt+s})$$

Where $\Lambda_{t,t+s} \equiv \frac{\beta^s c_t}{c_{t+s}} \frac{P_t}{P_{t+s}}$ is the discount kernel between period t and $t+s$. F_{jt} is the capital

[7] It is a normalization constant introduced for convenience when log–linearizing the model. Its value does not affect volatility of gross domestic product, the main quantity of interest in this paper.

rental rate and $MC_{jt}(\cdot)$ denotes nominal marginal costs and is defined as

$$MC_{jt} = \frac{1}{A_j}\Big(\frac{W_{jt}}{\delta_j\phi_j}\Big)^{\delta_j\phi_j}\Big(\frac{P^{\omega}_{jt}}{\delta_j(1-\phi_j)}\Big)^{\delta_j(1-\phi_j)}\Big(\frac{F_{jt}}{1-\delta_j}\Big)^{(1-\delta_j)}$$

after imposing the optimal mix of variable inputs.

The production process of firm j is overseen by an entrepreneur (which we consider as integrally part of firm j and thus indexed as such). Entrepreneur j does not receive a wage, but receives income from combining its own land holdings h_{jt} with labour from the household, $\widetilde{n}_{jt}$, to produce capital services k_{jt} for firm j

$$k_{jt} = \widetilde{n}_{jt}^{1-\nu_j} h_{jt-1}^{\nu_j}$$

Following Iacoviello (2005), we assume entrepreneur j to consume and to maximize the utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \gamma^t \widetilde{c}_{jt}$$

subject the flow of funds (in real terms)

$$f_{jt}k_{jt} + s_{jt} = \widetilde{c}_{jt} + q_t(h_{jt} - h_{jt-1}) + \frac{R_{jt-1}s_{jt-1}}{\pi_t} + w_{jt}\widetilde{n}_{jt}$$

where for simplicity, but w.l.o.g., we assume that entrepreneur j has the same taste for varieties as households: $\widetilde{c}_{jt} = \Big(\sum_{j'=1}^{J} \theta_{j'}^{\frac{1}{\eta}}(\widetilde{c}_{jj't})^{1-\frac{1}{\eta}}\Big)^{\frac{\eta}{\eta-1}}$.

s_{jt} denotes the stock of credit borrowed from banks (infra) used to produce capital services for production of firm j. Along the lines of Iacoviello (2005) and Kiyotaki and Moore (1997), we cap this amount of bank debt one can obtain. More precisely, banks impose a collateral constraint: the nominal loan gross of interest payments cannot exceed a certain fraction (the pledgeability ratio) of the expected nominal resale value of the land holdings. The collateral constraint can be expressed in real terms as,

$$s_{jt} \leq \ell_{jt}\mathbb{E}_t \frac{q_{t+1}h_{jt}\pi_{t+1}}{R_{jt}} \tag{3}$$

Where ℓ_{jt} is the firm–level loan–to–value constraint.

Define λ_{jt} as the shadow value of the borrowing constraint. Optimizing w.r.t. $\{\widetilde{c}_{jt}, h_{jt}, s_{jt}, \widetilde{n}_{jt}\}_{t=0}^{\infty}$ then yields an (i) Euler equation (4a), (ii) real estate demand (4b) and (iii) labor demand schedule (4c)

$$\frac{1}{\widetilde{c}_{jt}} = \mathbb{E}_t\Big(\frac{\gamma R_{jt}}{\widetilde{c}_{jt+1}\pi_{t+1}}\Big) + \lambda_{jt}R_{jt} \tag{4a}$$

$$\frac{q_t}{\widetilde{c}_{jt}} = \mathbb{E}_t\Big(\frac{\gamma}{\widetilde{c}_{jt+1}}\big(\nu_j \frac{f_{jt}k_{jt}}{h_{jt}} + q_{t+1}\Big) + \lambda_{jt}\ell_{jt}\pi_{t+1}q_{t+1} \tag{4b}$$

$$w_{jt} = (1-\nu_j)k_{jt}f_{jt}/\widetilde{n}_{jt} \tag{4c}$$

Where the equations show that a tightening of the loan–to–value ratio (or a drop in the value of pledgeable collateral) curtails the amount of capital services and thus production capacities of firm j.

2.3 Banks

Banks are assumed to intermediate all credit flows between households (savers) and firms (borrowers). We assume that banks are perfectly competitive on the deposits market, and so they take the nominal deposit rate, R_t, which is set by the central bank, as given. However, competition in the loans market is imperfect, so each bank enjoys some monopolistic power when providing credit to firm j.

Following Gerali et al. (2010), we assume that units of loan contracts are a composite constant elasticity of substitution basket of slightly differentiated financial products – each supplied by a different bank

$$s_{jt} = \Big(\sum_{b=1}^{B} \psi_{jb}^{\frac{1}{\mu_{jt}}} s_{jbt}^{1-\frac{1}{\mu_{jt}}}\Big)^{\frac{\mu_{jt}}{\mu_{jt}-1}}$$

Where s_{jbt} denotes demand for credit of firm j from bank b. μ_{jt} are stochastic mark–ups, defined as a weighted average of bank–markup shocks with $\log(\frac{\mu_{jt}}{\mu_{jt}-1}) - \log(\frac{\mu_j}{\mu_j-1}) = \sum_{b=1}\psi_{jb}(\log(\frac{\mu_{bt}}{\mu_{bt}-1}) - \log(\frac{\mu_b}{\mu_b-1}))$. Optimal demand schedules of firm j are given by

$$s_{jbt} = \psi_{jb}\Big(\frac{R_{jbt}}{R_{jt}}\Big)^{-\mu_{jt}} s_{jt}$$

where $\mathbf{\Psi} \in \mathbb{R}^{J\times B}$, with generic element ψ_{jb} introduces a bank–firm credit network into the model. $\mathbf{\Psi}$ fixes the extensive margin; $\psi_{jb} > 0$ ($\psi_{jb} = 0$) if bank b lends (does not lend) to firm j. The CES specification allows for substitution in the intensive margin, i.e. across existing bank—firms relations. In the steady state, $\mathbf{\Psi}$ introduces the feature that some banks are large/small or intensively/marginally connected to firms in the real economy.

Bank b sets the interest rates on credit to firm j, R_{jbt}, in order to maximize a discounted stream of profits

$$\mathbb{E}_0 \sum_{s=0}^{\infty} \Lambda_{t,t+s} P_{t+s} \Delta_{bt+s}$$

subject to credit demand schedules and a flow of funds

$$\Delta_{bt} + \frac{R_{t-1}d_{bt-1}}{\pi_t} + \sum_{j=1}^{J} s_{jbt} = \sum_{j=1}^{J} \frac{R_{jbt-1}s_{jbt-1}}{\pi_t} + d_{bt}$$

where d_{bt} denotes bank b's deposits it takes from the household. In addition, bank b has to abide its balance sheet identity, $\sum_{j=1}^{J} s_{jbt} \equiv d_{bt}$, which implies that real profits simplify to $\Delta_{bt+1} = \sum_{j=1}^{J}(R_{jbt} - R_t)s_{jbt}\pi_{t+1}^{-1}$. The optimal interest rate set to firm j is then $R_{jbt} = \frac{\mu_{jt}}{\mu_{jt}-1}R_t$. By construction, the LTV ratio faced by firm j is $\ell_{jt} = \left(\sum_{b=1}^{B} \psi_{jb}^{\frac{1}{\mu_j}} \ell_{jbt}^{1-\frac{1}{\mu_j}}\right)^{\frac{\mu_j}{\mu_j - 1}}$.

2.4 Monetary authority

The monetary authority sets the gross short–term nominal interest rate, R_t, according to a Taylor rule

$$\frac{R_t}{R} = \left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi} \left(\frac{gdp_t}{gdp}\right)^{\phi_{gdp}}$$

where R and gdp are the steady state policy rate and real gross domestic product. Note that, in our model with intermediate inputs, gdp_t and economywide production $\sum_{j=1}^{J} y_{jt}$, do not collapse.

2.5 Market clearing

The model equilibrium is characterized by an allocation of quantities and prices that satisfy (i) the households/entrepreneurs optimality conditions and budget constraint, (ii) the firms optimality conditions, (iii) the monetary policy rule, (iv) the banks balance sheet identities and (v) market clearing conditions; $\{-d_t = \sum_{b=1}^{B} d_{bt}\}$, $\{l_t = \sum_{j=1}^{J}(n_{jt} + \widetilde{n}_{jt})\}$, $\{h = h_t + \sum_{j=1}^{J} h_{jt}\}$, $\{y_{jt} = c_{jt} + \sum_{j'=1}^{J} m_{j'jt} + \sum_{j'=1}^{J} \widetilde{c}_{j'jt}\}_{j=1}^{J}$. Where the expressions represent, respectively, the asset market clearing condition, the labour market–clearing condition, the fixed stock of land and Walras' law.

2.6 Exogenous processes

We assume the following processes on the shocks of the model

$$\begin{aligned} \log(\ell_{jbt}) - \log(\ell_{jb}) &= \epsilon_t^{(\ell)} + \varepsilon_{bt}^{(\ell)} \\ \log(\frac{\mu_{bt}}{\mu_{bt}-1}) - \log(\frac{\mu_b}{\mu_b - 1}) &= \epsilon_t^{(r)} + \varepsilon_{bt}^{(r)} \end{aligned}$$

where $\epsilon_t^{(\ell)}$ and $\epsilon_t^{(r)}$ represent an aggregate shock to LTV ratios and borrowing rates, respectively. $\varepsilon_{bt}^{(\ell)}$ and $\varepsilon_{bt}^{(r)}$ capture the bank–specific idiosyncratic counterparts. We make the following assumptions on the shock structure. (i) Zero mean: $\mathbb{E}_t(\epsilon_t^{(\ell)}) = \mathbb{E}_t(\epsilon_t^{(r)}) = \mathbb{E}_t(\varepsilon_{bt}^{(\ell)}) = \mathbb{E}_t(\varepsilon_{bt}^{(r)}) = 0$, (ii) orthogonality of all shocks and (iii) finite second moments, $\mathbb{V}(\epsilon_t^{(\ell)}) = \mathbb{V}(\epsilon_t^{(r)}) = \mathbb{V}(\varepsilon_{bt}^{(\ell)}) = \mathbb{V}(\varepsilon_{bt}^{(r)}) = \sigma^2 < \infty$.

2.7 Log–linearised model

We solve the model by log–linearizing the equilibrium conditions around the deterministic zero–inflation steady state. The appendix B provides a detailed derivation of the steady–state equilibrium as well as the full set of log–linearized equations.

2.8 Relation to the literature

Conceptually, our model combines a standard amplification mechanism brought about by financial frictions (such as analysed by Kiyotaki and Moore (1997)) with an amplification mechanism in the real economy caused by firm–to–firm interactions (such as analysed by e.g. Acemoglu et al. (2012)). Both amplification mechanisms are traditionally considered in isolation. E.g. as shown in appendix B, the single–firm/single–bank version of our model collapses to that analyzed in Iacoviello (2005). Alternatively, if we discard the role of entrepreneurs, land holdings and financial frictions, the model tightly tracks that elaborated by Pasten et al. (2018a,b); Carvalho and Lee (2011); Smets et al. (2018). Mappings to other models are further discussed in appendix B.5. We discuss the model mechanics in the following section.

3 Theoretical results

We now investigate how (i) the production architecture of the economy $(\mathbf{\Omega})$, the credit network supporting it $(\mathbf{\Psi})$, (iii) the concentration of value added in the economy $(\boldsymbol{\theta})$ and (iv) *LTV* requirements $(\boldsymbol{\ell})$, give rise to macroeconomic fluctuations from small idiosyncratic bank shocks. We also assess the rate at which this aggregate effect decays as the number of banks in the economy increases. The theoretical results in this section hold under a set of simplifying assumptions, which we gradually relax in the last subsection.

3.1 Simplifying assumptions

We make the following set of assumptions[8]:

Assumption 1. *Households have linear disutility of labor* $(\varphi = 0)$.

[8]Most assumptions parallel that in Pasten et al. (2018a).

Assumption 2. *Monetary policy targets steady state nominal gross domestic product*

$$P_t c_t = Pc$$

Assumption 3. *All firms are equally capital intensive ($\{\delta_j = \delta\}_{j=1}^J$).*

Assumption 4. *Entrepreneurs have zero consumption mass, $\{\widetilde{c}_{jt} = 0\}_{j=1}^J$ and the collateral constraint* (3) *does not bind.*

Assumption 5. *We replace the Calvo (1983)–Yun (1996) framework of staggerd price setting of firms by a simple rule. Prices are flexible, but with a fixed probability α, firm j has to set its price before observing banks shocks. Thus,*

$$P_{jt} = \begin{cases} \mathbb{E}_{t-1}[P_{jt}^*] & \text{with probability } \alpha \\ P_{jt}^* & \text{with probability } 1-\alpha \end{cases}$$

where $\mathbb{E}_{t-1}$ is the expectation operator conditional on the $t-1$ information set.

Assumptions 1–2 jointly fix labour supply and pin down wages. Assumption 3 implies that all firms are equally credit intensive. Assumption 4 shuts down an amplification mechanism via collateral constraints. Assumption 5 collapses the time dimension of the impulse response functions (irf).

3.2 Equilibrium and propagation mechanism

In our simplified model, aggregate real household consumption, c_t, equals total real value added (real gross domestic product). As shown in appendix B, under simplifying assumptions 1–5, the expression for log linearised real GDP is analytically tractable and given by (henceforth, hat notation signals log linearised variables)

$$\widehat{gdp}_{t|B} = \boldsymbol{\nu}'_B \boldsymbol{\iota}_B \epsilon_t^{(r)} + \boldsymbol{\nu}'_B \boldsymbol{\varepsilon}_{t|B}^{(r)} \tag{5}$$

where $\boldsymbol{\iota}_B$ is a unity vector of size B, $\epsilon_t^{(r)}$ is a common bank shock and $\boldsymbol{\varepsilon}_{t|B}^{(r)}$ is the vector capturing bank specific shocks $\varepsilon_{bt}^{(r)}$. By assumption 4, $\varepsilon_{bt}^{(\ell)}$, $\epsilon_t^{(\ell)}$ do not enter the simplified framework. Subscript B (or $|B$) is henceforth included to denote the number of banks in the economy. $\boldsymbol{\nu}_B$ is the influence vector defined as

$$\boldsymbol{\nu}_B \equiv -\kappa \boldsymbol{\Psi}'_B [\mathbb{I} - \widetilde{\boldsymbol{\Omega}}']^{-1} \boldsymbol{\theta} \quad ; \quad \widetilde{\boldsymbol{\Omega}} \equiv \delta(1-\alpha)(\mathbb{I} - \boldsymbol{\Phi})\boldsymbol{\Omega}, \kappa \equiv (1-\delta)(1-\alpha) \tag{6}$$

where $\boldsymbol{\Phi}$ is a diagonal matrix with ϕ_j on the diagonal. $\boldsymbol{\nu}_B \equiv [\nu_{1|B}, \ldots, \nu_{B|B}]' \in \mathbb{R}^{B\times 1}$ is a vector of elasticities that maps the vector of bank shocks, $\boldsymbol{\varepsilon}_{t|B}^{(r)}$, to aggregate GDP fluctuations. In that respect, $\boldsymbol{\nu}_B$ quantifies the aggregate real influence of the B individual banks in the economy.

To develop more intuition w.r.t. the structure of $\boldsymbol{\nu}_B$, it is useful to rewrite it as a converging Neumann series

$$\boldsymbol{\nu}_B = -\kappa \boldsymbol{\Psi}'_B \Big(\sum_{n=0}^{\infty} \widetilde{\boldsymbol{\Omega}}^n \Big)' \boldsymbol{\theta} = \sum_{n=0}^{\infty} \boldsymbol{\nu}_B^{(n)} \tag{7}$$

Given that the summands in the infinite sum capture the separate steps through which bank shocks propagate throughout the real economy, the expression identifies the channels through which individual banks affect the real economy. (6) is then interpreted as a collapsed irf whereas (7) disentangles the steps of this irf.

3.3 Aggregate volatility

From (5), it is possible to decompose the volatility of real gdp into an aggregate source and a bank–level source:

$$Var(\widehat{gdp}_{t|B}) = \underbrace{\|\boldsymbol{\nu}_B\|_2^2 \sigma^2}_{\substack{\text{Bank–specific} \\ \text{origin}}} + \underbrace{\|\boldsymbol{\nu}_B\|_1^2 \sigma^2}_{\substack{\text{Common bank} \\ \text{origin}}} \tag{8}$$

where $\|\boldsymbol{\nu}_B\|_2 = \sqrt{\sum_{b=1}^{B} \nu_{b|B}^2}$ is the Euclidean norm of $\boldsymbol{\nu}_B$. In addition, $\|\boldsymbol{\nu}_B\|_1 = \sum_{b=1}^{B} |\nu_{b|B}|$ is the Taxicab norm of $\boldsymbol{\nu}_B$.

The size of $\|\boldsymbol{\nu}_B\|_2$ depends on the number of banks in the economy. This means that, as the number of banks in the economy varies, the aggregate impact of bank–specific shocks varies. Moreover, $\|\boldsymbol{\nu}_B\|_2$ depends on the structure of the economy. This means that, as we consider alternative structures of the economy, the aggregate impact of bank–specific shocks varies. In contrast, $\|\boldsymbol{\nu}_B\|_1$ is a constant, the size of which depends only on the structure of the economy. It is invariant to the number of banks in the economy.[9]

In subsection 3.4 we study how the structure of the bank–to–firm network ($\boldsymbol{\Psi}$), firm–to–firm network ($\boldsymbol{\Omega}$) and heterogeneity in value added ($\boldsymbol{\theta}$) act as mechanisms through which individual bank shocks lead to aggregate fluctuations. To this end, we first investigate the size of aggregate volatility generated by micro level volatility σ in an economy with a given number of banks B and economic structure ($\boldsymbol{\Psi}$, $\boldsymbol{\Omega}$ and $\boldsymbol{\theta}$). Second, we assess the rate at which this aggregate effect decays as the number of banks increases. That is, we investigate how fast $\|\boldsymbol{\nu}_B\|_2$ vanishes as B increases.

3.4 Disentangling the network origins of bank influence

To achieve aforementioned two objectives, we do not analyse $\boldsymbol{\nu}_B$ directly. Recall from (7) that $\boldsymbol{\nu}_B$ can be rewritten as a Neumann series. The elements of this infinite sum are negative.

[9] More precisely, $\|\boldsymbol{\nu}_B\|_1 = \kappa \boldsymbol{\iota}' (\sum_{n=0}^{\infty} \widetilde{\boldsymbol{\Omega}^n})' \boldsymbol{\theta}$.

Hence, one can provide lower bounds on the size and decay of $\|\boldsymbol{\nu}_B\|_2$ by sequentially analysing $\boldsymbol{\nu}_B^{(n=0)}, \boldsymbol{\nu}_B^{(n=1)}, \ldots$. Not only is this analytically more tractable, it also delivers direct insight into the reasons why some banks are more influential than others.

3.4.1 First–order connections

This subsection analyses $\boldsymbol{\nu}_B^{(n=0)} \equiv [\boldsymbol{\nu}_{1|B}^{(n=0)}, \ldots, \boldsymbol{\nu}_{B|B}^{(n=0)}]'$

$$\boldsymbol{\nu}_B^{(n=0)} = -\kappa \boldsymbol{\Psi}'_B \boldsymbol{\theta} = -(1-\alpha) \begin{pmatrix} d_{1|B}^{(1)} \\ \vdots \\ d_{B|B}^{(1)} \end{pmatrix} \tag{9}$$

$\boldsymbol{\nu}_B^{(n=0)}$ does not depend on the production structure of the real economy since shock propagation through production chains in the real economy is muted in the first order approximation (i.e. $\widetilde{\boldsymbol{\Omega}}^0 = \mathbb{I}$). In $\boldsymbol{\nu}_{b|B}^{(n=0)}$, the influence of bank b on real gdp depends only on its connection with firms that produce value added. In order to quantify this notion, we define the *first–order outdegree of firm j*.

Definition 1. *The first–order outdegree of firm j is defined as $d_{j|J}^{(1)} \equiv \theta_j$.*

In $d_{j|J}^{(1)}$, the superscript (1) signals it is a *first–order* outdegree. The subscript $|J$ designates it is an outdegree for firms (not banks). The first–order outdegree of firm j trivially equals its share in aggregate real gdp (if $n = 0$). The motivation for including this trivial definition is to illustrate the analogy with higher order outdegrees to be defined below. The *first–order outdegree of bank b* is defined as,

Definition 2. *The first–order outdegree of bank b is defined as: $d_{b|B}^{(1)} \equiv \sum_{j=1}^{J}(1-\delta)\psi_{jb|B} d_{j|J}^{(1)}$. The set $d_B^{(1)} = \{d_{1|B}^{(1)}, \ldots., d_{B|B}^{(1)}\}$, is called the first–order outdegree sequence of the financial sector.*

Since $d_{j|J}^{(1)}$ quantifies the contribution of firm j to real gdp, $d_{b|B}^{(1)}$ quantifies the extent to which bank b lends to such firms. In general, here and below, the credit network $\boldsymbol{\Psi}$ maps the firm–level influence to individual banks. We next define the *coefficient of variation*, which measures the asymmetry in $d_B^{(1)}$

Definition 3. *The (population) coefficient of variation of $d_B^{(1)}$ is*

$$CV_{d_B^{(1)}} \equiv \frac{\sqrt{\mathbb{V}(d_B^{(1)})}}{\bar{d}_B^{(1)}} \tag{10}$$

where $\bar{d}_B^{(1)} \equiv \frac{1}{B}\sum_{b=1}^{B} d_{b|B}^{(1)}$ is the average bank outdegree and $\mathbb{V}(d_B^{(1)}) = \left(\frac{1}{B}\sum_{b=1}^{B}(d_{b|B}^{(1)} - \bar{d}_B^{(1)})^2\right)^{\frac{1}{2}}$ is the population variance of $d_B^{(1)}$.

To ground the intuition of $CV_{d_B^{(1)}}$, consider stylized economy A in figure 1. If $\{\theta_j = J^{-1}\}_{j=1}^{J}$, each bank supports an equal share of gdp, in which case $CV_{d_B^{(1)}} = 0$. To the extent that $\theta_1 \neq \theta_2 \neq ... \neq \theta_J$, some banks are better tied to value added production than others, $CV_{d_B^{(1)}}$ increases.

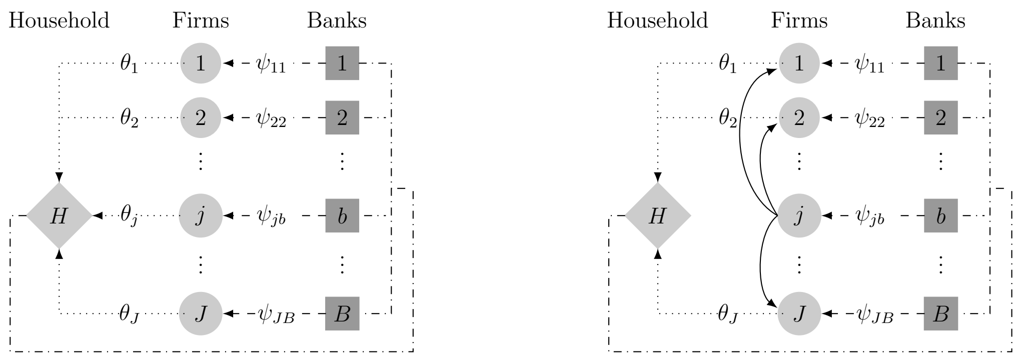


Figure 1: A graphical representation of two hypothetical economies.

The following proposition documents the impact of such an asymmetric structure on the size of aggregate volatility caused by bank–level shocks.

Proposition 1. *Provided an economy with a first–order outdegree sequence of the financial sector $d_B^{(1)}$, aggregate volatility satisfies*

$$\sqrt{Var\ (\widehat{gdp}_{t|B})} \geq (1-\alpha)\left(\sqrt{\sum_{b=1}^{B}(d_{b|B}^{(1)})^2}\right)\sigma$$

or, equivalently,

$$\sqrt{Var\ (\widehat{gdp}_{t|B})} \geq \frac{\kappa}{\sqrt{B}}\sqrt{1+CV^2_{d_B^{(1)}}}\sigma$$

Proof: See appendix.

Proposition 1 posits that, for a fixed number of banks B, a larger asymmetry in the first–order outdegree distribution of the financial sector implies a larger aggregate volatility that originates from bank–level shocks. Intuitively, for a fixed B, the shocks from the influential banks are not easily set off by shocks from other banks.

In addition, proposition 1 posits that, for a constant $CV_{d_B^{(1)}}$, a larger number of banks in the economy implies a lower aggregate volatility that originates from bank–level shocks. The reason is that, when B is small, bank–level idiosyncrasies generally have a smaller tendency to cancel out. When B increases, following a law of large numbers type of intuition, the real effects of idiosyncratic banks shocks tend to wash out, resulting in smaller aggregate fluctuations. The rate at which this happens depends on the distribution of the first–order outdegree of the financial sector. It is then interesting to verify what the squared expression in proposition 1 looks like for relevant distributions of the first–order outdegree of the financial sector.

In general, the first–order outdegree sequence of a network is often found to have power law tails (Clauset et al., 2009).[10] In keeping with Gabaix (2008), we define a power law as

Definition 4. *The random variable X follows a power law distribution with shape parameter ϱ when $Pr(X > x) = (\frac{x}{x_0})^{-\varrho}$ for $x \geq x_0$ and $\varrho > 0$.*

As shown in the appendix, the following proposition quantifies the rate of decay of aggregate volatility if the first–order outdegree of the financial sector follows a power law distribution

Corollary 1. *Consider an economy with a power law first–order outdegree sequence of the financial sector and shape parameter $\varrho \in (1, 2)$. Then aggregate volatility due to bank–specific shocks satisfies*

$$\sqrt{Var\ (\widehat{gdp}_{t|B})} = \Omega\Big(\frac{1}{B^{\frac{\varrho-1}{\varrho}}}\Big)$$

where the Landau notation $f(B) = \Omega(g(B))$ implies $f(B)$ is bounded below by $g(B)$ asymptotically.

The corollary establishes that, if the distribution of the first–order outdegree the financial sector is governed by a power law distribution, then aggregate volatility decays at a rate $\frac{1}{B^{\frac{\varrho-1}{\varrho}}}$. This rate becomes slower as $\varrho \to 1$, where ϱ close to 1 means that the distribution of the first–order outdegree is very asymmetric. Intuitively, when ϱ is close to 1, only a few banks have a large first–order outdegree (compared to the bulk of other banks, which have a small first–order outdegree). Then, even when the number of banks in the economy is very large, shocks to this minority of influential banks do not easily cancel out with the bulk of less influential banks. Note that the standard diversification argument of Lucas (1977), based on the central limit theorem, proposes a decay rate of $\sqrt{B}$, which is significantly slower than the one predicted by the corollary 1. Technically, the central limit theorem does not apply as it requires a finite variance whereas the power law distributed variable with shape parameter $\varrho \in (1, 2)$ does not have a finite variance.

The arguments in this subsection relied on a first–order approximation of the influence vector and ignored propagation of shocks through production linkages. We take this step in the next subsection.

[10] Stiglitz et al. (2011) find that the number of clients per bank follows a power law. Ennis (2001) finds that bank sizes can be characterized by a power law.

3.4.2 Second–order connections

This subsection analyses $\boldsymbol{\nu}_B^{(n=0)} + \boldsymbol{\nu}_B^{(n=1)}$

$$\boldsymbol{\nu}_B^{(n=0)} + \boldsymbol{\nu}_B^{(n=1)} = -\kappa \boldsymbol{\Psi}'_B \boldsymbol{\theta} - \kappa \boldsymbol{\Psi}'_B \widetilde{\boldsymbol{\Omega}}' \boldsymbol{\theta} = -(1-\alpha)(\begin{pmatrix} d^{(1)}_{1|B} \\ \vdots \\ d^{(1)}_{B|B} \end{pmatrix} + \begin{pmatrix} d^{(2)}_{1|B} \\ \vdots \\ d^{(2)}_{B|B} \end{pmatrix}) \tag{11}$$

$\boldsymbol{\nu}_B^{(n=0)} + \boldsymbol{\nu}_B^{(n=1)}$ not only takes into account that banks are influential because of their first–order interconnectedness. It also accounts for the fact that the impact of a shock from bank b to its borrowers does not remain confined with these borrowers. In a first step, the shock also propagates from these borrowing firms to firms with whom they interact in the production network. E.g. although an electricity distributor typically represents a material share in household consumption (large $d^{(1)}_{j|J}$), this direct share underestimates its importance for aggregate gdp given that a non–trivial share of other firms directly rely on this electricity distributor for their production (i.e. $d^{(2)}_{j|J}$ is large).

In order to formalize this notion, we define the second–order outdegree of firm j.

Definition 5. *The second–order outdegree of firm j is defined as $d^{(2)}_{j|J} \equiv \sum_{j'=1}^{J} \theta_{j'} [\widetilde{\Omega}]_{j'j}$.*

The *second–order outdegree of bank b* captures the extent to which bank b lends to firms with a high second–order outdegree

Definition 6. *The second–order outdegree of bank b is defined as: $d^{(2)}_{b|B} \equiv \sum_{j=1}^{J} (1-\delta) \psi_{jb|B} d^{(2)}_{j|J}$. The set $d^{(2)}_B = \{d^{(2)}_{1|B}, ..., d^{(2)}_{B|B}\}$, is called the second–order outdegree sequence of the financial sector.*

The next proposition refines proposition 1 using the additional information of the firm–to–firm production network:

Proposition 2. *Provided an economy with a first and second–order outdegree sequence of the financial sector $d^{(1)}_B$, $d^{(2)}_B$, aggregate volatility satisfies*

$$\sqrt{Var\ (\widehat{gdp}_{t|B})} \geq (1-\alpha) \sqrt{\sum_{b=1}^{B} (d^{(1)}_{b|B})^2 \sigma} + (1-\alpha) \sqrt{\sum_{b=1}^{B} (d^{(2)}_{b|B})^2 \sigma}$$

or, equivalently,

$$\sqrt{Var\ (\widehat{gdp}_{t|B})} \geq \Big(\frac{1}{\sqrt{B}} \sqrt{1 + CV_{d^{(1)}_{b|B}}} + \frac{\boldsymbol{\theta}' \widetilde{\boldsymbol{\Omega}} \boldsymbol{\iota}}{\sqrt{B}} \sqrt{1 + CV_{d^{(2)}_{b|B}}}\Big) \kappa \sigma$$

Proof: See appendix.

Proposition 2 is economically a more interesting result as it captures not only the fact that some banks intensively lend to important firms, but also the more subtle notion that some firms are key input suppliers to aforementioned firms, and banks gain indirect influence when lending to these firms. E.g. in economy B in figure 1, bank b has a zero first–order outdegree yet it is influential since its only borrower is a key input supplier to all other firms.

The scalar $\boldsymbol{\theta}'\widetilde{\boldsymbol{\Omega}}\boldsymbol{\iota}$ does not introduce asymmetry, but enters the expression as a level effect; if inter–firm trade is less important in an economy, the entries in $\widetilde{\boldsymbol{\Omega}}$ are smaller, which depresses the role of higher order interactions.

The input–output structure of the real economy as a vehicle through which bank shocks propagate is an interesting feature in the context of skewed sector presence. Skewed sector presence implies that some individual banks are dominant credit providers to firms in specific (non–financial) sectors (e.g. De Jonghe et al. (2019)). Following proposition 2, this can lead to increased macroeconomic volatility as bank shocks propagate, via their respective sector of specialization, to all other firms that rely on inputs from this sector. Moreover, proposition 2 allows us to better capture the role of "in–house banks", such as the "Volkswagen Bank", "Ford Credit NV", "Siemens Bank", etc. that are relatively small, but provide credit to few (mostly one) key firm(s). In addition, it also alludes to the identification of economic significance of banks (e.g. in the context of macroprudential policy, state aid, etc.) and competition policy concepts (such as the Herfindahl–Hirschmann Index). We come back to these policy related questions at the end of the paper.

We have the following counterpart to corollary 1

Corollary 2. *Consider an economy where (i) the first–order outdegree of the financial sector follows a power law outdegree sequence with shape parameter* $\varrho \in (1,2)$ *and (ii) the second–order outdegree of the financial sector follows a power law outdegree sequence with shape parameter* $\zeta \in (1,2)$*, then aggregate volatility due to bank–specific shocks satisfies*

$$\sqrt{Var\ (\widehat{gdp}_{t|B})} = \Omega\big(B^{-\frac{(\varpi-1)}{\varpi}}\big) \qquad where\ \varpi = Min\{\varrho, \zeta\}$$

As before, this corollary establishes that, if the distribution of the second–order outdegree of the financial sector has a power law tail, then aggregate volatility decays at a slower rate than predicted by the standard diversification argument.

The *dominance property* (see. Jessen and Mikosch (2006)) of the power law bridges corollary 1 and 2. For a sequence of economies in which the empirical distributions of the first and second order outdegrees of the financial sector have power law tails with shape parameters ϱ and ζ, the tighter bound for the decay rate of aggregate volatility is determined by $min\{\varrho, \zeta\}$.[11]

[11] In addition, if the first-order outdegree sequence of the financial sector does not have a power law tail but the second–order does (or vice versa), then the power law tail will dominate and the rate of decay is determined by the shape parameter of the power law tail.

3.4.3 Higher–order bank shocks

We now allow for shock propagation beyond the buyer–supplier relationship and account for higher order relationships (e.g. customers of customers). These interconnections in the real economy imply that bank shocks are not confined to its immediate borrowers (as in subsection 3.4.1) or corporate customers of these borrowers (as in subsection 3.4.2), but propagate further through the value chain.

To formally capture these complex patterns of interconnections, we define the p'th order outdegree of firm j.

Definition 7. *The p'th–order outdegree of firm j is defined as* $d^{(p)}_{j|J} \equiv \sum_{j'=1}^{J} \theta_{j'} \left[\widetilde{\mathbf{\Omega}}^{p-1}\right]_{j'j}$.

where $\left[\widetilde{\mathbf{\Omega}}^{p}\right]_{j'j}$ designates the (j', j)'th element of $\widetilde{\mathbf{\Omega}}^{p}$. Note that for $p = \{1, 2\}$, this definition collapses to $\{d^{(1)}_{j|J}, d^{(2)}_{j|J}\}$, respectively. Analogously, the p'th order outdegree of bank b is

Definition 8. *The p'th–order outdegree of bank b is defined as:* $d^{(p)}_{b|B} \equiv \sum_{j=1}^{J}(1-\delta)\psi_{jb} d^{(p)}_{j|J}$. *The set* $d^{(p)}_{B} = \{d^{(p)}_{1|B}, ..., d^{(p)}_{B|B}\}$, *is called the p'th–order outdegree sequence of the financial sector.*

The statistic captures the extent to which individual banks affect influential firms in the economy indirectly through input–output linkages in the real economy. The generalization of proposition 1 and proposition 2 is

Proposition 3. *Provided an economy with an outdegree sequences* $\{d^{(m)}_{B}\}_{m=1}^{M}$ *of the financial sector, aggregate volatility satisfies*

$$\sqrt{Var\ (\widehat{gdp}_{t|B})} \geq \Big(\sum_{m=1}^{M} \frac{\widetilde{\boldsymbol{\theta}}' (\widetilde{\mathbf{\Omega}})^{m-1} \boldsymbol{\iota}}{\sqrt{B}} \sqrt{1 + CV_{d^{(m)}_{b|B}}} \Big) \kappa\sigma$$

Proof: See appendix.

Proposition 3 captures the role of higher order interconnections. The proposition echoes the intuition of the *network–based financial accelerator* in Stiglitz et al. (2010) (although proposition 3 provides a closed form expression whereas Stiglitz et al. (2010) instead rely on a simulation exercise).

3.5 Relaxing assumptions

We now discuss the implications of relaxing the model assumptions 1–5. Throughout, we focus on the intuition.

3.5.1 Relaxing assumption 3

Allowing for heterogeneous capital intensity requires modifying definition 2

Definition 9. *The modified first–order outdegree of bank b is defined as:* $\widetilde{d}^{(1)}_{b|B} \equiv \sum_{j=1}^{J}(\delta - \delta_j)\psi_{jb|B}d^{(1)}_{j|J}$.

The modified first–order outdegree of bank b, $\widetilde{d}^{(1)}_{b|B}$ takes into account the fact that production of some of its borrowers is relatively more/less credit intensive. Proposition 1 then generalizes as follows

Proposition 4. *Provided an economy with a modified first–order outdegree sequence of the financial sector* $\widetilde{d}^{(1)}_{B}$, *aggregate volatility satisfies*

$$\sqrt{Var\ (\widehat{gdp}_{t|B})} \geq (1-\alpha)\sqrt{\sum_{b=1}^{B}(d^{(1)}_{b|B})^2 + \sum_{b=1}^{B}(\widetilde{d}^{(1)}_{b|B})^2 + \frac{1}{B}\big(Cov(d^{(1)}_{b|B}, \widetilde{d}^{(1)}_{b|B}) - \mathbb{E}\widetilde{d}^{(1)}_{b|B}\mathbb{E}d^{(1)}_{b|B}\big)\sigma}$$

where $Cov(\cdot,\cdot)$ is the population covariance operator.

The role of asymmetry is similar to before, but potentially exacerbated/moderated by a covariance term. If banks with a large first–order outdegree typically lend to firms that are relatively less credit intensive, the covariance term dampens aggregate volatility. A level effect (last term) does not affect the asymmetry of banks, but dampens aggregate volatility if less of the value added activities in the economy require bank debt. If $\{\delta_j = \delta\}_{j=1}^{J}$, $\widetilde{d}^{(1)}_{B} = 0$ and the proposition collapses to proposition 1.[12]

3.5.2 Relaxing assumption 4

Allowing for collateral constraints introduces an amplification mechanism into the model. To see this, consider the households' Euler equation, which can be solved forward to yield

$$\frac{q_t}{c_t} = \sum_{s=0}^{\infty}\beta^s \iota h^{-1}_{t+s} \equiv \mathcal{H}_t$$

As β is close to 1, any short term fluctuations in h_t affect $\mathcal{H}_t$ relatively little. The following approximation then holds

$$\frac{q_t}{c_t} \approx \mathcal{H}$$

This condition means that movements in land prices comove intimately with household consumption. Hence, shocks in the economy that stimulate household consumption will also drive up land prices (collateral values).

[12]Generalizing proposition 4 for higher order terms is straightforward, albeit the notation becomes very thorny and cumbersome. They are available upon request.

This brings forward a role for collateral requirements (and shocks to the latter). E.g., a shock to the required LTV ratio of bank b constrains production capacities of its borrowing firms. This is, in turn, reflected in these firms' output prices which, in the context of an interlinked production architecture, affects marginal costs and output prices of other firms in the economy. This cascades further through the IO structure of the economy. Higher prices in the economy then lower household consumption and depress the value of pledgeable collateral. The latter endogenously affects the borrowing capacity of all firms in the economy (even those not borrowing from bank b). This, in turn, affects their production capabilities, feeding again through the economy, etc.

Aforementioned mechanism introduces a level effect, i.e. it amplifies the size of individual bank shocks, but as such does not necessarily affect the incidence to which they cancel each other out. Heterogeneous bank–firm specific LTV ratios, however, do affect the tendency of averaging out. Banks that allow for mild collateral requirements with their borrowers install stronger financial accelerator mechanisms with these firms. Shocks from these banks get amplified more vis–à–vis banks that set lower LTV ratios (as per the discussion in Jensen et al. (2018); Walentin (2014)).

3.5.3 Relaxing assumption 5

Under assumption 5 the irf is collapsed to the single period in which the shock occurs. Relaxing assumption 5 introduces the Calvo (1983)–Yun (1996) framework which requires us to take into account the time dimensions. Following DeJong and Dave (2011), log–linearised gdp can be rewritten as

$$\widehat{gdp}_{t|B} = \sum_{b=1}^{B} \sum_{s=0}^{\infty} \nu_{b,t-s|B} \varepsilon_{bt-s|B} + \sum_{b=1}^{B} \sum_{s=0}^{\infty} \nu_{b,t-s|B} \epsilon_{t-s|B}$$

such that $Var(\widehat{gdp}_{t|B}) = \sigma^2 \sum_{s=0}^{\infty} \boldsymbol{\nu}'_{t-s|B} \boldsymbol{\nu}_{t-s|B} + \sigma^2 \sum_{s=0}^{\infty} (\boldsymbol{\iota}'_B \boldsymbol{\nu}_{t-s|B})(\boldsymbol{\iota}'_B \boldsymbol{\nu}_{t-s|B})$.

3.5.4 Relaxing assumption 1 and 2

As shown in our quantitative results below, a Taylor rule and non–linear labour supply only marginally affect the dynamics of the model variables. The role of relaxing these assumptions is thoroughly discussed in Pasten et al. (2018a).

4 Data sources and calibration

In this section we calibrate our model to the Belgian economy. We first elaborate the used data sources and subsequently discuss the details of our calibration.

4.1 Data sources

The calibration of our model relies on six confidential administrative data sources provided by the National Bank of Belgium (NBB): (*i*) the Business–to–Business transactions Database ($B2B$), (*ii*) the Corporate Credit Register (CCR), (*iii*) the International Trade database (ITD), (*iv*) Firm Annual Accounts (AA) (*v*) Value Added Tax declarations (VAT), and (*vi*) a panel of firm product–level price data underlying the official Belgian producer price index (PPI) statistics. All datasets are linked by VAT numbers and all minimally span the time frame $2002 - 2014$.

The backbone of our analysis constitutes the NBB B2B transactions database. This dataset documents both the extensive and the intensive margins of domestic buyer–supplier relationships in Belgium. In particular, an observation is the sum of all sales (in euros, excluding any value–added tax) from VAT–liable firm j to VAT–liable firm j' in a given calendar year. Coverage is quasi universal, as all relationships with annual sales of at least 250 euros must be reported.

Note that, while the VAT ID is the legal entity of a firm in Belgium, some firms might be owned by other firms, generating intra–firm trade between parents and subsidiaries, which might not be subject to typical market forces. In this paper, we focus on propagation across firm boundaries. Given that the nature of propagation across firms, but within the confinements of a group structure, is likely to be different from across boundaries, we aggregate VAT IDs up to the group level using ownership filings in the annual accounts. We carry out this aggregation in all data sources.[13]

The CCR contains monthly updated information on the population of loans granted from banks to non–financial firms. The banks are established in Belgium and licensed by the NBB. This concerns both (*a*) branches incorporated under foreign law established in Belgium as well as (*b*) banks incorporated under Belgian law. An observation in the CCR documents: (*a*) the bank, (*b*) the firm, (*c*) the nominal value (denominated in euros) and (*d*) the collateral value of the granted loan at time t. The CCR is available at the monthly level but for our purpose converted to the annual level (i.e. the frequency of the other data sources). As of 2011, there is no reporting threshold (before, a threshold of $25,000$ euro was installed).

The AA report firm–level costs attributed to day–to–day business operations. Firm–level sales are also documented in the AA, but since their coverage is incomplete, we rely on VAT declarations to obtain firm–level sales. Given that both costs and sales also reflect cross–border trade we rely on the ITD to strip these numbers from imports and exports. This harmonizes our calibration with the closed economy nature of our model.

Finally, the Belgian National Institute of Statistics collects, on a monthly basis, prices of

[13]We aggregate variables across multiple VAT IDs owned by the same firm. Inputs (B2B), credit lines (CCR), exports and imports (ITD), labor costs (AA) are summed across subsidiaries to the group level. Intra–group transactions in the B2B database are dropped.

industrial products, which are used to compute the Belgian producer price index. The price information is collected through a monthly phone survey of around $1,500$ firms. The firms participate on a voluntary basis. We use this data source to back out price stickiness.

To significantly reduce the dimensions (and computational complexities) of our model, we impose a set of mild sample restrictions at the firm–level. We restrict our analysis to firms in the private and non–financial sector that report (i) fixed assets larger than 500 euro, (ii) at least one full time equivalent employee and (iii) non–negative wages. In addition, we require firms to participate in the Belgian production network (i.e. have at least one trade flow reported in the $B2B$ database) and borrow from at least one bank.

Applying these criteria reduces the number of firms significantly. Table 1 documents the dimensions of the used sample. In 2014, for example, only 22.10% of the then active firms in Belgium satisfy the above criteria. The large reduction in sample size is mostly driven by the exclusion of very small firms that do not submit annual accounts to the Central Balance Sheet Office. Although these firms account for a sizeable fraction of firms in the Belgian economy, they only represent a small fraction of Belgian economic activity. E.g. in 2002, our subset of firms represent 74.01% of aggregate value added and 90.01% of total private employment. Although it captures only 66.08% of all observed firm–to–firm linkages, it does captures 82.27% of its volume.

4.2 Calibration

In this subsection we discuss the calibration of our model. The calibration of parameters not specific to our model set–up $\{\beta, \gamma, \varphi, \iota, \phi_\pi, \phi_{gdp}\}$ are taken from the literature and their values are documented in table 2. The calibration of the parameters $\{\delta_j, \phi_j, \alpha_j, \ell_j, \omega_{jj'}, \theta_j, \psi_{jb}\}$ are elaborated upon below. Unless stated otherwise, parametrization relies on the 2014 vintage of the available datasets.

$\mathbf{\Omega}$ is constructed using the $B2B$ database. From the first–order conditions $\omega_{jj'} = P_{j'} M_{jj'}/(P_j^\omega M_j)$, i.e. $\widehat{\omega}_{jj'}$ is the share of firm j expenditures on j' inputs in total inputs sourced by firm j. Both $\widehat{P_{j'} M_{jj'}}$ and $\widehat{P_j^\omega M_j}$ are directly inferred from the $B2B$ database. Some firms do not buy intermediates, in which case $\widehat{\phi}_j = 1$ and $\sum_{j'=1}^J \widehat{\omega}_{jj'} = 0$. For all other firms, $\sum_{j'=1}^J \widehat{\omega}_{jj'} = 1$.

$\mathbf{\Psi}$ makes use of the CCR. In keeping with the model, $\widehat{\psi}_{jb}$ is calculated as the share of bank b credit to firm j in the total bank credit portfolio of firm j. The LTV ratio, $\widehat{\ell_{jb}}$, is inferred from the CCR as the total credit line from bank b to firm j over the amount of pledged collateral of firm j to bank b. Furthermore, following the FOCs, $\widehat{\ell}_j = \sum_{j=1}^J \widehat{\psi_{jb}} \widehat{\ell_{jb}}$.

$\boldsymbol{\theta}$ captures the steady state shares of firm sales to final demand. We obtain total firm–level sales from the VAT declarations, from which we subtract (i) intermediate sales to other firms (from the B2B) and (ii) exports (from the ITD). The remaining sum captures firm–level sales for final domestic consumption.

For simplicity, we assume that entrepreneurs do not hire labour, $\nu_j = 1$. Then, in order to arrive at a model consistent estimate for δ_j and ϕ_j, note that, under CRS Cobb–Douglas technologies, $\delta_j \phi_j$, $\delta_j(1-\phi_j)$ and $1-\delta_j$ reflect the share of wages, intermediates and capital services in total costs of firm j, respectively. Total costs, are observed from the AA. The wage bill, $\widehat{W_t N_{jt}}$ is directly reported in the AA and total intermediate inputs, $\widehat{P^{\omega}_{jt} M_{jt}}$, is observed from the $B2B$ database.

We use the confidential microdata underlying the producer price data (PPI) to calculate the frequency of price adjustments at the firm–level; α_j. Following Pasten et al. (2018a), we calculate the frequency of price changes at the firm level, as the ratio of the number of price changes to the total number of consecutive months available to us. For example, if an observed price trajectory is 1 euro for two months and then 1.5 euro for another four months, a single price change occurs during six months, and $\widehat{\alpha}_j$ is set to 1/6. For firms that are not in the micro price sample, we set $\widehat{\alpha}_j$ to the average $\widehat{\alpha}_j$ of the sector in which firm j is active.

Table 3 documents summary statistics related to our calibration. The average capital share of production is 0.22, which is close to macroeconomic estimates (Smets and Wouters (2007)). The average Calvo parameters is 0.88, which implies that the average firms keeps its prices fixed for 2.78 quarters. In addition, the average $\widehat{\omega_{jj'}}$ is small, meaning that most firms have a disaggregate input portfolio. The mean of LTV ratios, here directly calibrated from micro–level data, is close to that used in related models (e.g. 0.89 in Iacoviello (2015), 0.9 Iacoviello (2005)). Nonetheless, significant dispersion in LTV ratios is present.[14]

5 Size of aggregate fluctuations

From eq. (8), $\|\boldsymbol{\nu}_B\|_2$ maps micro–level volatility to gdp volatility. Similarly, $\|\boldsymbol{\nu}_B\|_1$ maps macro–level volatility to gdp volatility. In this section, we report both multipliers relative to one another. $(\|\boldsymbol{\nu}_B\|_2)(\|\boldsymbol{\nu}_B\|_1)^{-1}$ then quantifies the relative size of aggregate gdp volatility caused by bank specific shocks vis–à–vis common shocks.

We perform this exercise for the true economy, as calibrated in the previous section. In addition to the true (heterogeneous) scenario, we consider various hypothetical economies where we impose symmetry on $\widehat{\boldsymbol{\Psi}}, \widehat{\boldsymbol{\Omega}}, \widehat{\boldsymbol{\theta}}, \widehat{\boldsymbol{\alpha}}, \widehat{\boldsymbol{\phi}}$ and $\widehat{\boldsymbol{\delta}}$. More precisely, we define these as $\widehat{\boldsymbol{\Psi}}^S = B^{-1}\boldsymbol{\iota}_J \boldsymbol{\iota}'_B, \widehat{\boldsymbol{\Omega}}^S = J^{-1}\boldsymbol{\iota}_J\boldsymbol{\iota}'_J, \widehat{\boldsymbol{\theta}}^S = J^{-1}\boldsymbol{\iota}_J, \widehat{\boldsymbol{\alpha}}^S = \bar{\alpha}\boldsymbol{\iota}_J, \widehat{\boldsymbol{\phi}}^S = \bar{\phi}\boldsymbol{\iota}_J, \widehat{\boldsymbol{\delta}}^S = \bar{\delta}\boldsymbol{\iota}_J$. $\boldsymbol{\iota}$ are unit vectors and the superscript S is shorthand notation for symmetry. In words; $\widehat{\boldsymbol{\Psi}}^S$ shuts down all heterogeneity in the bank–firm network and considers an economy where all banks are equally tied to the real economy. $\widehat{\boldsymbol{\theta}}^S$ represents an economy where all firms are equally important contributors to gdp. $\widehat{\boldsymbol{\Omega}}^S$ captures a set–up where all firms take up an equal role in the production network.

[14] Note that our model assumes a fixed extensive margin of firm–firm and bank–firm relations. In appendix C, we provide evidence that such a simplified view on the economy provides a good approximation of the Belgian production architecture and credit network supporting it.

$\widehat{\boldsymbol{\alpha}}^S, \widehat{\boldsymbol{\phi}}^S, \widehat{\boldsymbol{\delta}}^S$ impose homogeneous price stickiness, homogeneous labour and capital shares for all firms in the economy, respectively.

5.1 Simplified framework

The calibration results from our simplified framework (abiding assumption 1–5) are documented in table 4, column (1). Across all rows, aggregate volatility from idiosyncratic bank shocks (vis–à–vis aggregate shocks) is the smallest in scenarios involving $\widehat{\boldsymbol{\Psi}}^S$ (scenario (2), (4), (6) and (8)). From the discussion in the theory section, each of these scenarios equalizes all $\{\nu_{b|B}\}_{b=1}^{B}$, implying that $(\|\boldsymbol{\nu}_B\|_2)(\|\boldsymbol{\nu}_B\|_1)^{-1} = \frac{1}{\sqrt{B}} = 13.4\%$.[15] Intuitively, if all banks are tied to the Belgian economy in the same way, their idiosyncratic shocks maximally cancel each other out relative to an economywide shock (which does not cancel out). This finding aligns with the Lucas (1977) argument and rationalizes the traditional NK focus on a representative bank: not much volatility is explained by including individual non–atomistic banks.

The role of micro–level shocks is the largest in the full heterogeneous scenario 1, in which the impact of bank–level shocks is 44.7% of what an aggregate shock of the same magnitude would cause. As per the discussion above, in this scenario the dominance of a minority of banks is most outspoken and their shocks do not easily cancel out with that of the other banks. Where does this dominance come from?

To answer that question, let us focus on the most relevant scenarios (1), (3), (5) and (7) which do not involve $\boldsymbol{\Psi}^S$. In scenario (7), we see that $\boldsymbol{\Psi}$ contributes the most to asymmetry in bank–level dominance. The multiplier rises from 13.4% to 31.8%. The reason is straightforward; irrespective of the heterogeneity among firms that are in the portfolios of individual banks, some banks simply have more firms in their portfolios.

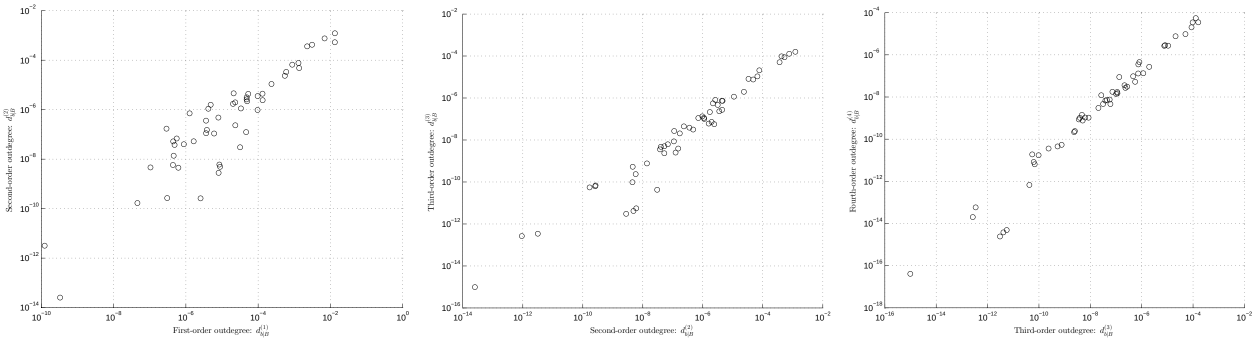
Allowing for a heterogeneous value added structure (scenario (3)) raises $(\|\boldsymbol{\nu}_B\|_2)(\|\boldsymbol{\nu}_B\|_1)^{-1}$ from 31.8% to 38.6% meaning that banks that are better tied to the real economy are also – on average – more tied to firms that create a lot of value added. A similar story is true for scenario (5) – an increase to 35.7%– where some banks are very intimately connected to firms that take up a central role in the production process. Both asymmetries brought about by $\boldsymbol{\Omega}$ and $\boldsymbol{\theta}$ reinforce each other in the full heterogeneous set up (scenario 1). Banks that disproportionally lend to firms that create a lot of value added are also indirectly – through IO interactions – very supportive to value added creation of firms to which they do not borrow directly. In the wording of the discussion in section 4, in figure 2, panel (a) we see that banks with a high (low) first–order outdegree typically also have a high (low) second–order outdegree, etc.

Across columns, we see that aggregate volatility slightly changes when allowing for heterogeneous Calvo probabilities and capital shares, respectively. The change however, is modest, suggesting that dominant banks are not systematically tied to flexible price firms/capital in-

[15] For 2002, in which $B = 78$, this number equaled 11%.

tensive firms.

In conclusion, throughout, the magnitude of aggregate volatility induced by individual bank shocks is sizeable compared to that of an aggregate shock common to all banks. Throughout scenarios (1), (3), (5) and (7), the multiplier associated with idiosyncratic bank shocks is almost half as large as the multiplier associated with one single aggregate (common) shock that affects all banks. This implies that the common practice of ignoring bank specific volatility is a strong restriction when explaining aggregate volatility.



(a) $d^{(1)}_{b|B}$ vs. $d^{(2)}_{b|B}$ (b) $d^{(2)}_{b|B}$ vs. $d^{(3)}_{b|B}$. (c) $d^{(3)}_{b|B}$ vs. $d^{(4)}_{b|B}$

Figure 2: The panels maps the $m'th$ order outdegree of bank b to the $m+1'th$ outdegree of bank b. Data are for 2014.

5.2 Full framework

Compared to the simplified framework, the full model allows for (i) the Calvo (1983)–Yun (1996) framework of staggered price setting, (ii) a collateral constraint amplification mechanism, (iii) a Taylor rule and (iv) non–linear labour disutility. The introduction of the Calvo (1983)–Yun (1996) framework gives rise to irfs whereas, in the previous subsection, all adjustments were instantaneous and the irfs were collapsed to the period of the shock.

Figures 6 and 7 plot irfs, relaxing assumptions $(i)-(v)$, and follow the (heterogeneous) calibration set out in subsection 4.2 – except for ℓ, which we assume to be homogeneous across firms and banks in the economy. We focus on a unit standard deviation in bank specific LTV requirements (figure 6) and borrowing rates (figure 7). We focus on two (randomly chosen) banks (left and right). One has an above average influence and one that is below average. On impact, an easing in LTV requirements from bank b increases the amount of debt its borrowers can take out. This unconstrains production of firms borrowing from bank b. An increase in output drives down prices, which – in the context of an interlinked production structure – spills over to other firms. The latter firms will pass this marginal cost decrease into their own prices, giving rise to a cascade effect on prices. This, in turn, affects consumption (real gdp). Moreover, an increase in the value of collateral q_t endogenously enhances the borrowing capacity of other firms in the economy, i.e. firms who do not borrow from the shocked bank b, which sets in motion a second round of amplification effects.

The irfs for the hypothetical scenarios (involving $\boldsymbol{\theta}$, $\boldsymbol{\Omega}$) illustrate that the main results documented from the simplified subsection still stand. That is, influential banks derive their influence from lending to important firms or firms that interact (indirectly) with important firms. Forcing symmetry ($\boldsymbol{\theta}^S$, $\boldsymbol{\Omega}^S$) then lowers these banks' irfs. The reverse is true for the non–influential banks.

In table 5, column (1), we document $(\|\boldsymbol{\nu}_B\|_2)(\|\boldsymbol{\nu}_B\|_1)^{-1}$ which largely confirm the results from the simplified framework. We focus on two additional scenarios; (i) one where individual banks set different LTV ratios – but homogeneous across firms in their portfolio. The LTV ratio set by the bank is an unweighted average across all LTVs it sets. (ii) Another is the full heterogeneous case, with bank–firm specific LTV ratios. In both scenarios, $(\|\boldsymbol{\nu}_B\|_2)(\|\boldsymbol{\nu}_B\|_1)^{-1}$ goes up vis–à–vis column (1). Why? Larger LTV ratios allow for larger endogenous amplification mechanisms from financial frictions. The slight increase from column (1) to column (2) implies that more influential banks set, on average, (marginally) looser LTV requirements than the average bank. The result in column (3) furthermore indicates that influential banks set, within their credit portfolio, these looser LTV requirements mainly with influential firms. This intuition is confirmed in figure 8, which plots irfs for LTV shocks of four (randomly chosen) influential banks.

6 Decay of aggregate fluctuations

In section 5 we quantified the aggregate effect of bank–level shocks for a fixed number of B banks. In this section we investigate the rate at which this multiplier would decay should the number of banks in the economy vary. Following proposition 1, 2 and 3 in our simplified framework this rate depends on the asymmetry of the constituent elements of the influence vector. We derived approximate (proposition 1, 2) and exact (proposition 3) rates of decay and focused on the particular case of power law distributions (corollary 1, 2). In view of these corollaries, table 6 fits a power law to the first four outdegrees of the banks (for 2002).

The first–order outdegree is found to follow a power law distribution with shape parameter 1.37, such that corollary 1 implies that $\|\boldsymbol{\nu}_B\|_2$ should not decay faster than the rate $B^{\frac{\varrho-1}{\varrho}} = B^{0.27}$. The law of large numbers argument, as posited by Lucas (1977), would suggest a rate of decay of $B^{0.5}$. The rate we identify is much slower.[16] Table 6 reveals that that the asymmetry in the higher–order effects is even larger.[17] Hence, by the dominance property (supra), these higher–order effects place an even tighter bound on the rate of decay. E.g. using corollary 2,

[16]Intuitively, when there is a large asymmetry, shocks from banks with a large first–order outdegree are not easily offset by shocks from other banks, even in the face of a large number of other banks.

[17]This can also be seen from figure 2.

the second–order outdegree suggests $\|\boldsymbol{\nu}_B\|_2$ to decay at the rate $B^{0.23}$.

In principle, it is possible to fit a power law tail to any distribution. Hence, using the goodness–of–fit test described in Clauset et al. (2009), we formally test whether the obtained power law is indeed a good characterization of the data. This test relies on a bootstrapping procedure and generates a p–value that can be used to quantify the plausibility of the hypothesis. If the p–value is large, then any difference between the empirical data and the power law can be explained with statistical fluctuations. If $p \simeq 0$, then the power law does not provide a plausible fit to the data. The goodness–of–fit tests reveal that the p–values for the first four outdegrees are well above the 0.1 threshold suggested by Clauset et al. (2009), confirming that the distributions indeed follow a power law.

7 Financial sector policy

In this section, we show that our framework provides insights into various policy topics that have recently been addressed in the literature.

7.1 Distortion of the Herfindahl–Hirschmann index

Gabaix (2011) develops a theory of granularity for non–financial corporations (NFCs). It states that the Herfindahl–Hirschmann index (HHI) of the non—financial sector is a sufficient statistic to map microlevel volatility of NFCs to economywide volatility.[18] Intuitively, a larger HHI reflects a larger market share of a subset of NFCs, in which case microeconomic shocks to these firms drive the macroeconomy.

Bremus et al. (2018) develop a parallel framework for the financial sector, which puts bank size (total credit supplied to NFCs) at the center of the analysis. Under extreme parametrizations of our model, i.e. no inter–firm trade $\{\phi_j = 1, \omega_{jj'} = 0\}_{j,j'=1}^{J}$, symmetric value added of firms $\{\theta_j = J^{-1}\}_{j=1}^{J}$, flexible prices $\{\alpha_j = 0\}_{j=1}^{J}$ and full labour intensity $\{\delta_j = 1\}_{j=1}^{J}$ on top of assumptions $1-5$, our framework collapses to theirs (see appendix B.5);

$$\begin{aligned}\sqrt{Var(\widehat{gdp}_{t|B})} &= \sqrt{\sum_{b=1}^{B}\Big(\frac{\sum_{j=1}^{J} s_{jb}}{\sum_{b=1}^{B}\sum_{j=1}^{J} s_{jb}}\Big)^2}\sigma \\ &= \|\boldsymbol{\nu}_B\|_2\sigma\end{aligned} \tag{12}$$

The top expression reflects the HHI, which, under aforementioned assumptions, collapses exactly to $\|\boldsymbol{\nu}_B\|_2$. The reason is that, under aforementioned assumptions, the market share of bank b (in terms of credit volume) equals the share of value added (in economywide value

[18]But see recent contributions of Pasten et al. (2018a) and Baqaee and Farhi (2017) who falsify this claim in the context of price rigidities and nonlinearities, respectively.

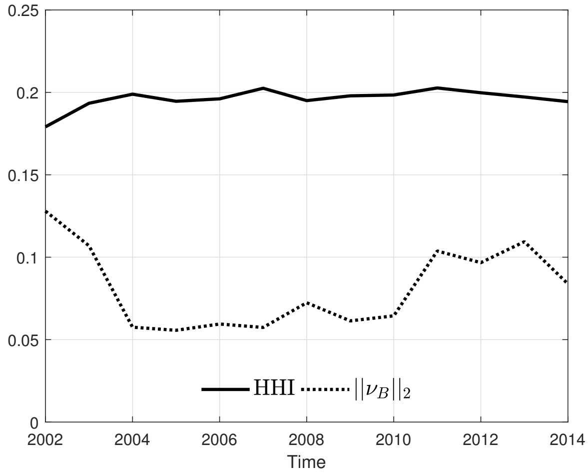


Figure 3: This figure plots the HHI and $\|\boldsymbol{\nu}_B\|_2$ in the simplified framework. All parameters $\widehat{\boldsymbol{\Psi}}, \widehat{\boldsymbol{\Omega}}, \widehat{\boldsymbol{\theta}}, \widehat{\boldsymbol{\phi}}$ and $\widehat{\boldsymbol{\delta}}$ vary over time ($\widehat{\boldsymbol{\alpha}}$ does not).

added) of the firms in the portfolio of bank b. In general $\|\boldsymbol{\nu}_B\|_2$ does not collapse to the HHI. Our framework, instead, focuses on the (in)direct role that the borrowing NFCs in the banks' portfolio play in the real economy. Providing a large volume of credit does not necessarily make a bank influential if this credit is only marginally used for value added activities.

Figure 3 plots the HHI of the Belgian banking sector for $2002 - 2014$. In addition, we graph $\|\boldsymbol{\nu}_B\|_2$, parametrized for the true, heterogeneous, Belgian economy. We document two interesting insights. First, $\|\boldsymbol{\nu}_B\|_2$ is smaller than the HHI. Using the HHI as a multiplier for microeconomic volatility overestimates aggregate volatility from bank–level shocks. In practice, sluggish firm behaviour (e.g. through positive Calvo frictions) and limited credit intensity of firms dampen the multiplier. Second, given the evolution of the HHI, equation (12) suggests that that macroeconomic volatility from micro–level shocks has remained stable over the last decade whereas our framework highlights that it has gone up in the post–crisis period.

7.2 Strategic bank lending

An emerging literature has documented the incidence of bank specialization. E.g. banks have been documented to specialize in particular domestic geographic regions (Boeve et al. (2010)), industrial sectors (De Jonghe et al. (2019)), export activities (Paravisini et al. (2016); Niepmann and Eisenlohr (2014)), etc. The benefits from such non–random matching of firms and banks include i.a. better screening abilities which reduce the problem of adverse selection and a better assessment of the collateral value (Acharya et al. (2006)). In addition, specialized banks are claimed to detect a deterioration of the borrower's business earlier and may react in a timely manner by risk mitigation, for example, by requesting additional collateral. The large concentration of the banks' lending portfolio and the increased sectoral HHI are often cited as the downsides of bank specialization.

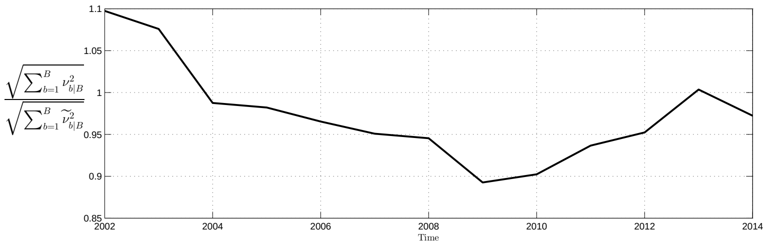


Figure 4: $\frac{\|\boldsymbol{\nu}_B\|_2}{\|\widetilde{\boldsymbol{\nu}}_B\|_2}$, i.e. the ratio of the norm of the influence vector with the actual credit network and the specialization–neutral credit network).

A relevant question is to ask whether such bank specialization is desirable from a macroeconomic viewpoint. That is, does sectoral specialization dampen or amplify macroeconomic volatility? In our framework, the question boils down to whether such specialization increases or decreases asymmetry in the economy. E.g. a non–influential bank that starts to specialize in the steel sector now indirectly impacts all sectors that (in)directly rely on steel. As specialization renders this bank more influential, its shocks offset those from other influential banks. On the other hand, if such specialization is initiated by an already influential bank, specialization enhances asymmetry and increases volatility.

In order to investigate the effect of bank specialization in the Belgian economy, we define a hypothetical "specialization–neutral" credit network, which mutes the role of specialization. Define $\widetilde{\boldsymbol{\Psi}}$ with $\widetilde{\psi}_{jb} = \frac{\sum_{j=1}^{J} \psi_{jb}}{\sum_{b=1}^{B} \sum_{j=1}^{J} \psi_{jb}}$. $\widetilde{\boldsymbol{\Psi}}$ homogenizes lending patterns of banks across sectors without compromising the intensity to which banks are tied to the real economy. Figure 4 plots $\frac{\|\boldsymbol{\nu}_B\|_2}{\|\widetilde{\boldsymbol{\nu}}_B\|_2}$, i.e. the ratio of the norm of the influence vector with the actual credit network and the specialization–neutral credit network, respectively. From the plot, sector specialization led to increased volatility before 2004, led to smaller aggregate volatility throughout $2002 - 2009$ (i.e. the ratio is lower than one). In the aftermath of the financial crisis, shifts in strategic bank lending have caused an increase in macroeconomic volatility.

7.3 LTV requirements

Caps on LTV ratios are part of the standard macroprudential toolkit. Our analysis nests the standard result that tighter LTV requirements lower macroeconomic volatility. We additionally document that tighter LTV ratios with particular key firms in the economy would also lower aggregate volatility. Such "Borrower–based" measures, tailored to the systemic nature of the borrowing NFCs in the bank's portfolio do, to the best of our knowledge, not exist. The exception are cases where lending to systemic NFCs is capped in order to prevent a too large exposure of the bank to a single entity.[19]

[19]E.g. following the ESRB recommendation (2018), French systemically important institutions can not incur an exposure that exceeds 5% of their eligible capital for NFCs (or group of connected NFCs) assessed to be highly indebted.

7.4 Big banks vs. influential banks

We next relate our influence measure with bank size. For that purpose, we calculate the rank correlation: $Kendalls\ \tau \in [-1, 1]$.[20]

$$\tau = \frac{2}{B(B-1)} \sum_{b<b'} sign(\nu_b - \nu_{b'}) sign(\sum_{j=1}^{J} s_{jb} - \sum_{j=1}^{J} s_{jb'})$$

τ measures the degree of similarity between two ordinal rankings. From table 7 we see that banks are ordered quite differently, according to both metrics. This is true, both for banks that are below and above medium size. Even among the top 10 banks, the ordering of banks can be very different depending on the measure used.

The imperfect ordinal correlation of bank size and influence is echoed in art. 4 of the SSM regulation which stipulates that some banks can be significant (and fall under supervision of the ECB) due to its economic importance to a member state – irrespective of its size. Our influence measure provides structural guidance on identifying such significant banks.

7.5 Why are some banks influential?

One interesting question is to investigate why individual banks are influential? Stated differently, what does the structure of $\nu_{b|B}$ for individual banks look like? In order to answer this question, recall, following 3, that individual bank influence $\nu_{b|B}$ can be additively decomposed into its outdegrees $d^{(1)}_{b|B}$, $d^{(2)}_{b|B}$, etc. In figure 5, we plot the relative share of each outdegree in total bank influence $\nu_{b|B}$. Along the horizontal axis, banks are ordered according to bank influence $\nu_{b|B}$ from left to right.[21]

A key insight from the emerging pattern is that, across banks, the origins of bank influence is very heterogeneous. Multiple banks derive their influence almost exclusively from lending to central firms in the production network. Other banks are influential because they directly support firms that create a lot of value added. Most banks derive their influence from the size of their immediate borrowers (first–order outdegree). Higher order outdegrees, which capture network propagation effects in the real economy, represent on average 32% of bank influence. Network effects tend to be more important for influential banks.

[20] Spearman's rank correlation coefficient yields similar results.
[21] To preserve confidentiality, the ordering of the banks in both tails of the distribution is random.

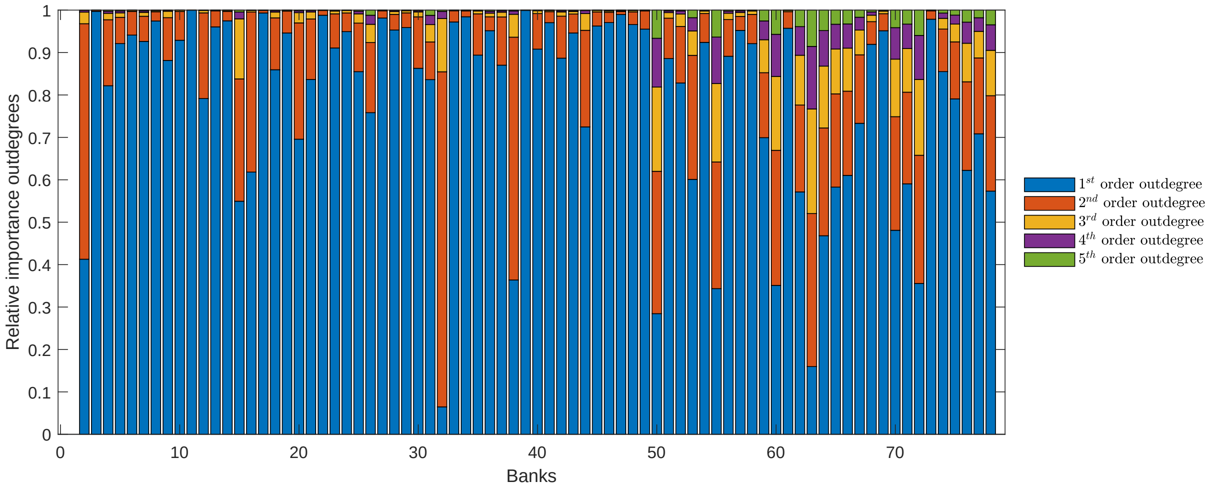


Figure 5: Decomposition of bank–specific influence (calibration for 2002, $B = 78$).

8 Conclusion

Any modern economy is characterized by an interlinked production architecture in which firms rely on each other for their input requirements. As credit is vital to support this production process, shocks to credit availability of individual firms propagate throughout this production network. In this paper, we study how the structure of the real economy determines the aggregate real impact of lending shocks from individual banks.

To that end, we develop a NK model which combines a standard amplification mechanism brought about by financial frictions with an amplification mechanism in the real economy caused by firm–to–firm interactions. The model provides a simple but powerful framework to identify how banks are both directly and indirectly supportive of value creation by non–financial firms in the real economy.

We show that two interlocked networks (in casu individual firm credit portfolios and the production architecture of the real economy) introduce large asymmetries in the influence of individual banks. Stated differently, some banks intensively support – directly and/or indirectly – value added of the non–financial sector, whereas other banks are only marginally connected to firms involved in value added creation. Consequently, when some banks are asymmetrically influential, their credit supply shocks do not easily cancel out with that of other banks. We show that this has policy implications related to the identification of significant banks, the usability the HHI and the role of sector specialization of individual banks.

9 Tables

Table 1: Dimensions used data

	B2B	CCR		Variables				
Year	Obs. (number)	Banks (number)	Obs. (number)	Value added (share total)	Employment (share total)	Trade transactions (share total)	Trade volume (share total)	Firms (share total)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
2002	4,091,210	78	165,081	74.01	90.01	66.08	82.27	15.69
2003	4,271,350	78	154,352	72.89	89.56	65.61	76.91	15.98
2004	4,318,907	72	153,872	70.26	88.12	66.05	76.44	16.08
2005	4,468,199	70	156,458	72.14	88.16	66.93	74.91	16.29
2006	4,677,163	67	161,938	72.22	88.07	67.07	74.38	16.42
2007	4,824,779	71	169,411	73.78	87.68	68.05	76.34	16.68
2008	4,982,887	68	173,061	62.35	86.51	67.44	75.26	16.91
2009	4,934,340	68	178,838	62.91	86.31	67.89	76.81	16.97
2010	5,089,192	63	179,208	60.03	86.41	38.36	74.56	16.82
2011	5,299,890	60	187,191	56.93	86.65	68.81	74.75	17.01
2012	6,697,987	62	289,286	54.22	89.94	74.34	73.33	22.86
2013	6,662,241	60	198,401	52.2	89.86	74.23	74.99	22.81
2014	6,708,970	55	194,046	50.24	88.71	72.86	75.41	22.1

This table reports the dimensions of the data sample used and reports the share of our sample in aggregate Belgian statistics. Value added (column (5)) is derived by summing firm–level value added (firm total sales minus imports and intermediate purchases). Employment (column (6)) is total full time equivalent employees obtained from the annual accounts. Total firms in the denominator of column (9) is the total number of distinct firms in the raw annual accounts, $B2B$ data and CCR prior to imposing the exclusion restrictions.

Table 2: Parameter values

Parameters	**Value**	**Explanation**
β	0.995	Discount factor households
η	2	Elasticity of substitution across firms
α	0.88	Price stickiness parameter
ℓ	0.91	LTV ratio
μ	3.5	Elasticity of substitution across banks
ϕ_π	1.25	Taylor rule parameter, inflation
ϕ_{gdp}	0.33/12	Taylor rule parameter, gdp
φ	1.5	Inverse of Frish labour supply

Calibration of model parameters. CES elasticity of substitution is taken from Kikkawa et al. (2018). Bank markups are taken from Gerali et al. (2010). The model is calibrated at the monthly frequency.

Table 3: DESCRIPTIVE STATISTICS STRUCTURAL PARAMETERS

Variable	**Mean**	**Percentile** 10	25	50	75	90
Calvo probability ($\widehat{\alpha}_j$)	0.88	0.78	0.83	0.93	0.96	0.98
Capital share ($1-\widehat{\delta}_j$)	0.22	0.01	0.053	0.17	0.31	0.52
Labour share ($\widehat{\delta}_j\widehat{\phi}_j$)	0.47	0.15	0.29	0.46	0.95	0.79
Intermediate input share ($\widehat{\delta}_j(1-\widehat{\phi}_j)$)	0.3	0.05	0.13	0.28	0.45	0.602
Loan–to–value ratio ($\widehat{\ell}_j$)	0.91	0.791	0.915	0.929	1	1
Intermediate input share ($\widehat{\omega}_{jj'} \times 100$)	9.28	1.25	3.15	7.82	18.45	24.11
Share credit portfolio ($\widehat{\psi}_{jb} \times 100$)	0.39	0.03	0.12	0.31	0.58	0.78

This table provides summary statistics on the structural parameters used in the model. Summary statistics are based on the pooled sample of firms throughout $2002-2014$. Descriptives of intermediate input shares and credit portfolio shares are based on the subsample $\widehat{\omega}_{jj'}, \widehat{\psi}_{jb} \in (0,1)$.

Table 4: RESULTS IN A SIMPLIFIED FRAMEWORK

	$\boldsymbol{\theta}$	$\boldsymbol{\Omega}$	$\boldsymbol{\Psi}$	Hom $\boldsymbol{\alpha}$ Het $\boldsymbol{\phi}$ Hom $\boldsymbol{\delta}$	Het $\boldsymbol{\alpha}$ Het $\boldsymbol{\phi}$ Het $\boldsymbol{\delta}$	Het $\boldsymbol{\alpha}$ Hom $\boldsymbol{\phi}$ Hom $\boldsymbol{\delta}$	Hom $\boldsymbol{\alpha}$ Hom $\boldsymbol{\phi}$ Het $\boldsymbol{\delta}$	Hom $\boldsymbol{\alpha}$ Hom $\boldsymbol{\phi}$ Hom $\boldsymbol{\delta}$
(1)	Het, $\widehat{\boldsymbol{\theta}}$	Het, $\widehat{\boldsymbol{\Omega}}$	Het, $\widehat{\boldsymbol{\Psi}}$	0.447	0.464	0.445	0.473	0.440
(2)	Het, $\widehat{\boldsymbol{\theta}}$	Het, $\widehat{\boldsymbol{\Omega}}$	Hom, $\widehat{\boldsymbol{\Psi}}^S$	0.134	0.134	0.134	0.134	0.134
(3)	Het, $\widehat{\boldsymbol{\theta}}$	Hom, $\widehat{\boldsymbol{\Omega}}^S$	Het, $\widehat{\boldsymbol{\Psi}}$	0.386	0.400	0.384	0.408	0.380
(4)	Het, $\widehat{\boldsymbol{\theta}}$	Hom, $\widehat{\boldsymbol{\Omega}}^S$	Hom, $\widehat{\boldsymbol{\Psi}}^S$	0.134	0.134	0.134	0.134	0.134
(5)	Hom, $\widehat{\boldsymbol{\theta}}^S$	Het, $\widehat{\boldsymbol{\Omega}}$	Het, $\widehat{\boldsymbol{\Psi}}$	0.357	0.370	0.355	0.378	0.352
(6)	Hom, $\widehat{\boldsymbol{\theta}}^S$	Het, $\widehat{\boldsymbol{\Omega}}$	Hom, $\widehat{\boldsymbol{\Psi}}^S$	0.134	0.134	0.134	0.134	0.134
(7)	Hom, $\widehat{\boldsymbol{\theta}}^S$	Hom, $\widehat{\boldsymbol{\Omega}}^S$	Het, $\widehat{\boldsymbol{\Psi}}$	0.318	0.330	0.317	0.336	0.313
(8)	Hom, $\widehat{\boldsymbol{\theta}}^S$	Hom, $\widehat{\boldsymbol{\Omega}}^S$	Hom, $\widehat{\boldsymbol{\Psi}}^S$	0.134	0.134	0.134	0.134	0.134

The calibrations are as follows: $\widehat{\boldsymbol{\Psi}}^S = B^{-1}\boldsymbol{\iota}_J\boldsymbol{\iota}_B', \widehat{\boldsymbol{\Omega}}^S = J^{-1}\boldsymbol{\iota}_J\boldsymbol{\iota}_J', \widehat{\boldsymbol{\theta}}^S = J^{-1}\boldsymbol{\iota}_J, \widehat{\boldsymbol{\alpha}}^S = \bar{\alpha}\boldsymbol{\iota}_J, \widehat{\boldsymbol{\phi}}^S = \bar{\phi}\boldsymbol{\iota}_J$. $\boldsymbol{\iota}$ are unit vectors and the superscript S is shorthand notation for symmetry.

Table 5: Results in the full framework

				Homogeneous LTV rates (Economywide)	Homogeneous LTV rates (Bank level)	Heterogeneous LTV rates
Panel A: Idiosyncratic LTV shocks						
(1)	Het, $\widehat{\boldsymbol{\theta}}$	Het, $\widehat{\boldsymbol{\Omega}}$	Het, $\widehat{\boldsymbol{\Psi}}$	0.465	0.473	0.492
(2)	Het, $\widehat{\boldsymbol{\theta}}$	Het, $\widehat{\boldsymbol{\Omega}}$	Hom, $\widehat{\boldsymbol{\Psi}}^S$	0.134	0.134	0.134
(3)	Het, $\widehat{\boldsymbol{\theta}}$	Hom, $\widehat{\boldsymbol{\Omega}}^S$	Het, $\widehat{\boldsymbol{\Psi}}$	0.386	0.392	0.408
(4)	Het, $\widehat{\boldsymbol{\theta}}$	Hom, $\widehat{\boldsymbol{\Omega}}^S$	Hom, $\widehat{\boldsymbol{\Psi}}^S$	0.134	0.134	0.134
(5)	Hom, $\widehat{\boldsymbol{\theta}}^S$	Het, $\widehat{\boldsymbol{\Omega}}$	Het, $\widehat{\boldsymbol{\Psi}}$	0.348	0.354	0.368
(6)	Hom, $\widehat{\boldsymbol{\theta}}^S$	Het, $\widehat{\boldsymbol{\Omega}}$	Hom, $\widehat{\boldsymbol{\Psi}}^S$	0.134	0.134	0.134
(7)	Hom, $\widehat{\boldsymbol{\theta}}^S$	Hom, $\widehat{\boldsymbol{\Omega}}^S$	Het, $\widehat{\boldsymbol{\Psi}}$	0.277	0.282	0.293
(8)	Hom, $\widehat{\boldsymbol{\theta}}^S$	Hom, $\widehat{\boldsymbol{\Omega}}^S$	Hom, $\widehat{\boldsymbol{\Psi}}^S$	0.134	0.134	0.134
Panel B: Idiosyncratic interest rate shocks						
(1)	Het, $\widehat{\boldsymbol{\theta}}$	Het, $\widehat{\boldsymbol{\Omega}}$	Het, $\widehat{\boldsymbol{\Psi}}$	0.474	0.490	0.499
(2)	Het, $\widehat{\boldsymbol{\theta}}$	Het, $\widehat{\boldsymbol{\Omega}}$	Hom, $\widehat{\boldsymbol{\Psi}}^S$	0.134	0.134	0.134
(3)	Het, $\widehat{\boldsymbol{\theta}}$	Hom, $\widehat{\boldsymbol{\Omega}}^S$	Het, $\widehat{\boldsymbol{\Psi}}$	0.398	0.404	0.420
(4)	Het, $\widehat{\boldsymbol{\theta}}$	Hom, $\widehat{\boldsymbol{\Omega}}^S$	Hom, $\widehat{\boldsymbol{\Psi}}^S$	0.134	0.134	0.134
(5)	Hom, $\widehat{\boldsymbol{\theta}}^S$	Het, $\widehat{\boldsymbol{\Omega}}$	Het, $\widehat{\boldsymbol{\Psi}}$	0.344	0.349	0.364
(6)	Hom, $\widehat{\boldsymbol{\theta}}^S$	Het, $\widehat{\boldsymbol{\Omega}}$	Hom, $\widehat{\boldsymbol{\Psi}}^S$	0.134	0.134	0.134
(7)	Hom, $\widehat{\boldsymbol{\theta}}^S$	Hom, $\widehat{\boldsymbol{\Omega}}^S$	Het, $\widehat{\boldsymbol{\Psi}}$	0.351	0.356	0.367
(8)	Hom, $\widehat{\boldsymbol{\theta}}^S$	Hom, $\widehat{\boldsymbol{\Omega}}^S$	Hom, $\widehat{\boldsymbol{\Psi}}^S$	0.134	0.134	0.134

The calibrations are as follows: $\widehat{\boldsymbol{\Psi}}^S = B^{-1}\boldsymbol{\iota}_J\boldsymbol{\iota}_B', \widehat{\boldsymbol{\Omega}}^S = J^{-1}\boldsymbol{\iota}_J\boldsymbol{\iota}_J', \widehat{\boldsymbol{\theta}}^S = J^{-1}\boldsymbol{\iota}_J, \widehat{\boldsymbol{\alpha}}^S = \bar{\alpha}\boldsymbol{\iota}_J, \widehat{\boldsymbol{\phi}}^S = \bar{\phi}\boldsymbol{\iota}_J$.

Table 6: Power law fit to outdegrees

	First order outdegree $d_B^{(1)}$	Second order outdegree $d_B^{(2)}$	Third order outdegree $d_B^{(3)}$	Fourth order outdegree $d_B^{(4)}$
Shape parameter	1.37	1.31	1.23	1.21
Diversification $B^{\frac{\varrho-1}{\varrho}}$	$B^{0.27}$	$B^{0.23}$	$B^{0.18}$	$B^{0.17}$
Goodness-of-fit test	$p = 0.22$	$p = 0.14$	$p = 0.23$	$p = 0.12$

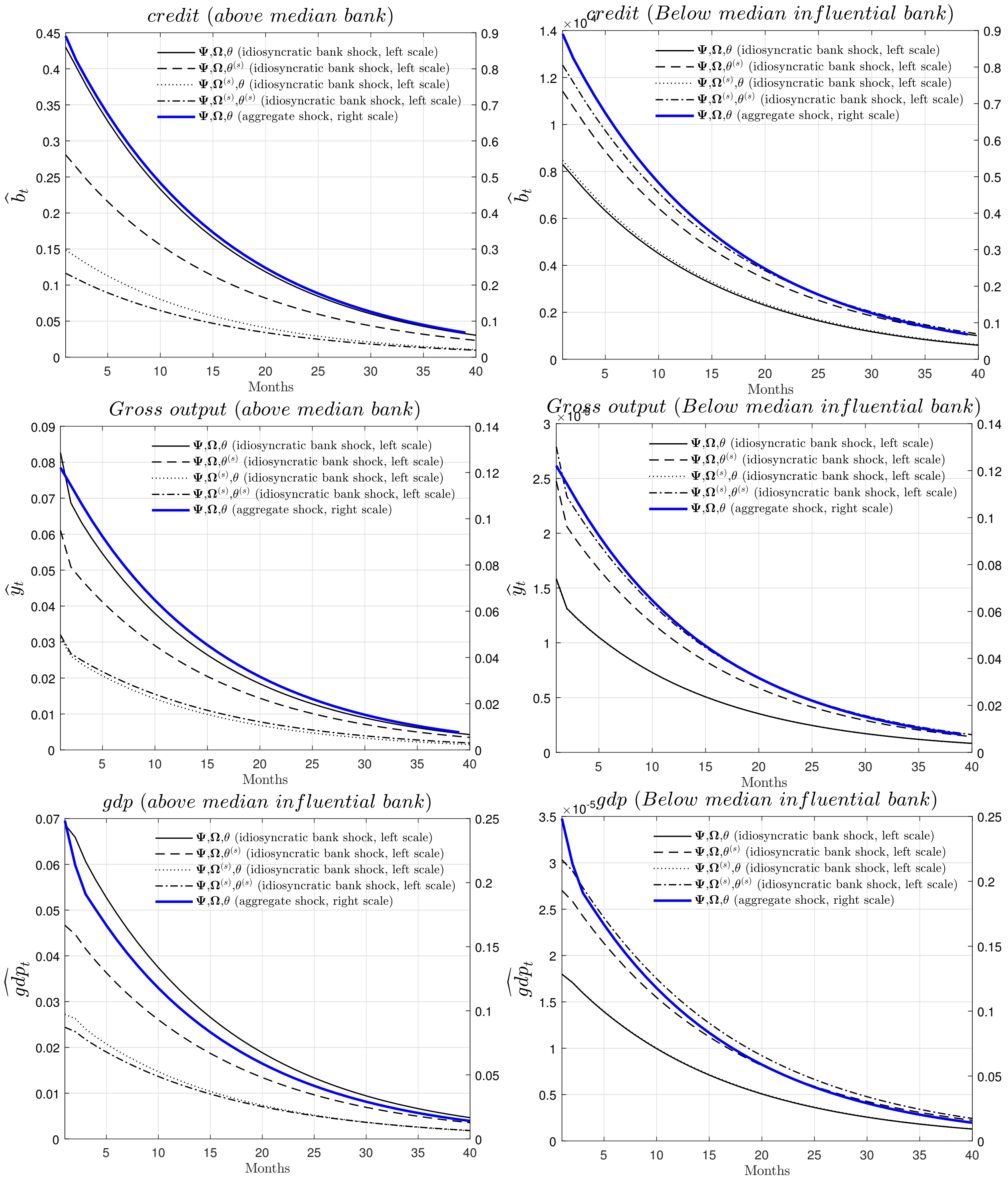
Power law fit using the methodology in Clauset et al. (2009). Power law fit using the rank correction methodology discussed in Gabaix and Ibragimov (2011) delivers similar results. Estimates are for 2002.

Table 7: Kendall rank correlation coefficient

Year	All banks	Below median size	Above median size	Top 10
2002	0.701	0.594	0.390	0.672
2003	0.665	0.651	0.346	0.745
2004	0.673	0.597	0.304	0.600
2005	0.666	0.558	0.350	0.709
2006	0.668	0.493	0.517	0.709
2007	0.678	0.622	0.412	0.709
2008	0.651	0.552	0.371	0.672
2009	0.684	0.640	0.453	0.709
2010	0.650	0.443	0.446	0.672
2011	0.636	0.384	0.516	0.709
2012	0.664	0.407	0.484	0.709
2013	0.638	0.522	0.398	0.709
2014	0.622	0.608	0.431	0.636

Kendall rank correlation coefficient between the size of individual banks and the influence of individual banks.

10 Figures



credit (above median bank)
credit (Below median influential bank)
Gross output (above median bank)
Gross output (Below median influential bank)
gdp (above median influential bank)
gdp (Below median influential bank)
Ψ,Ω,θ (idiosyncratic bank shock, left scale)
Ψ,Ω,$\theta^{(s)}$ (idiosyncratic bank shock, left scale)
Ψ,$\Omega^{(s)}$,θ (idiosyncratic bank shock, left scale)
Ψ,$\Omega^{(s)}$,$\theta^{(s)}$ (idiosyncratic bank shock, left scale)
Ψ,Ω,θ (aggregate shock, right scale)
$\widehat{b}_t$
$\widehat{y}_t$
$\widehat{gdp}_t$
Months
$\times 10^{-4}$
$\times 10^{-5}$

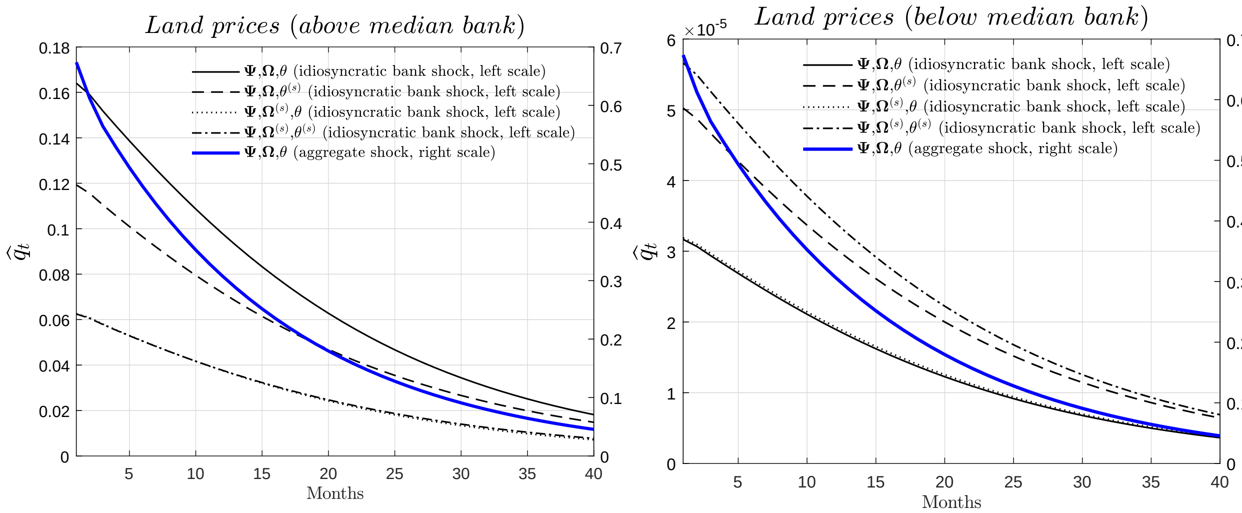
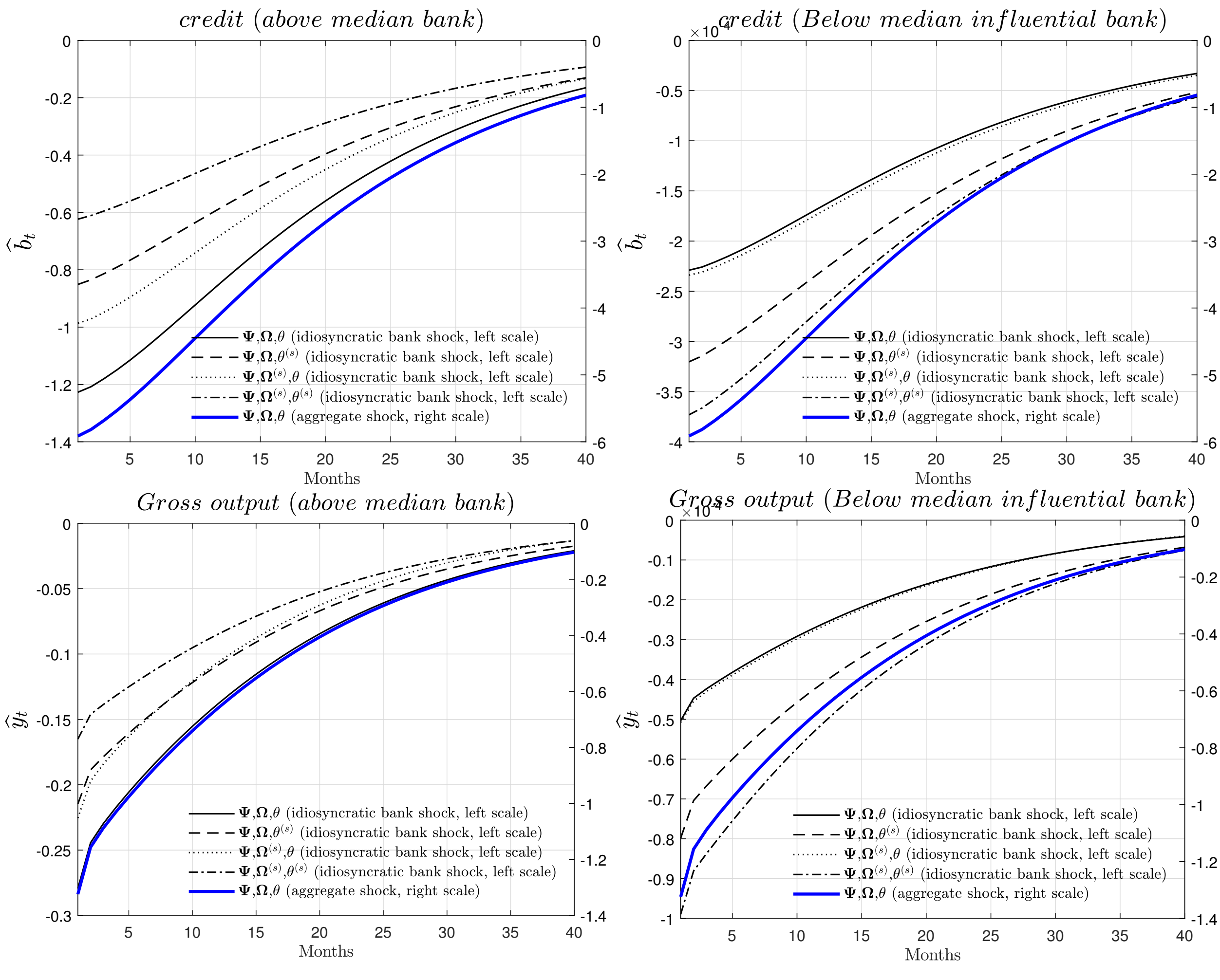


Figure 6: Impulse response function w.r.t. LTV shock.



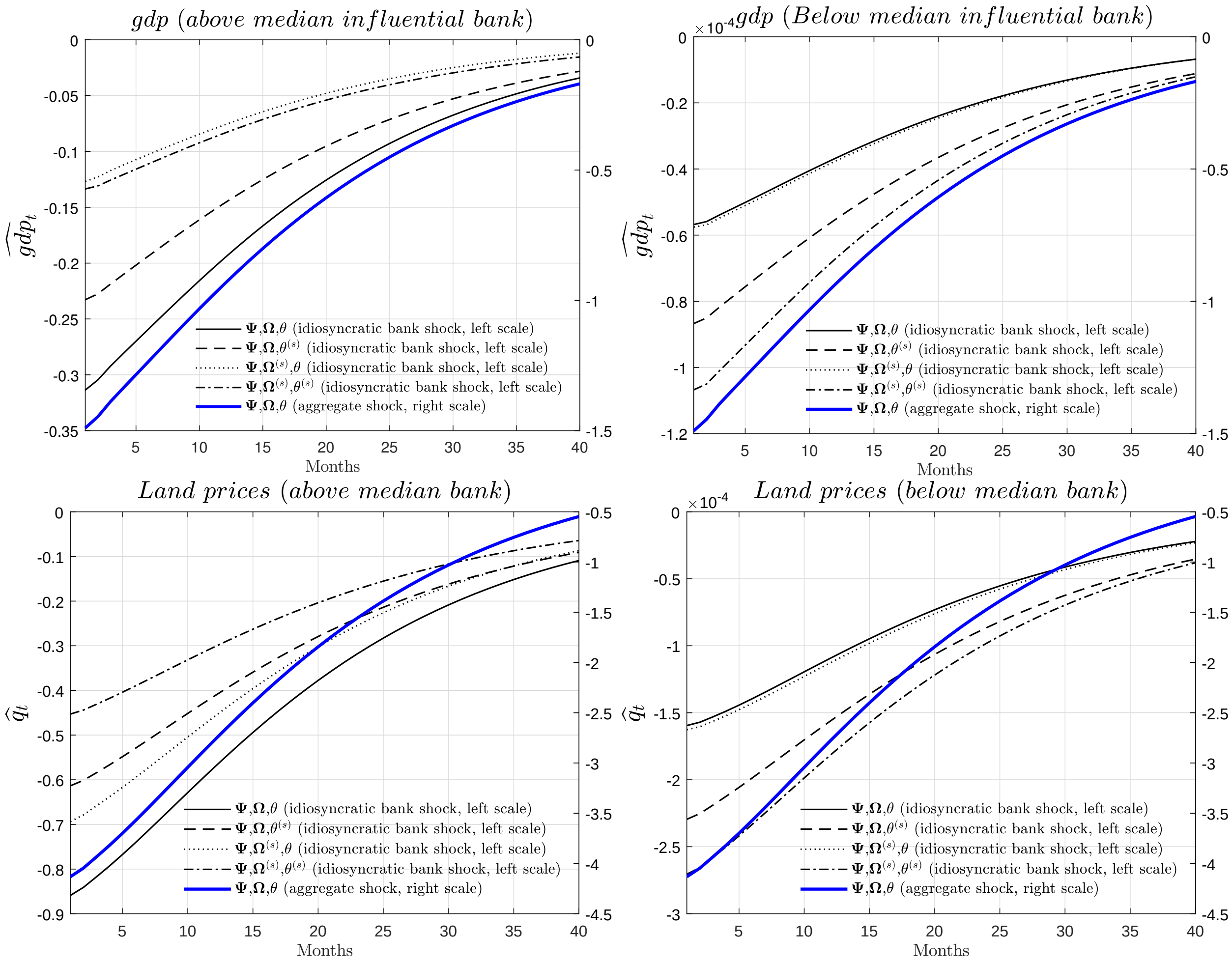


Figure 7: Impulse response functions w.r.t interest rate shock.

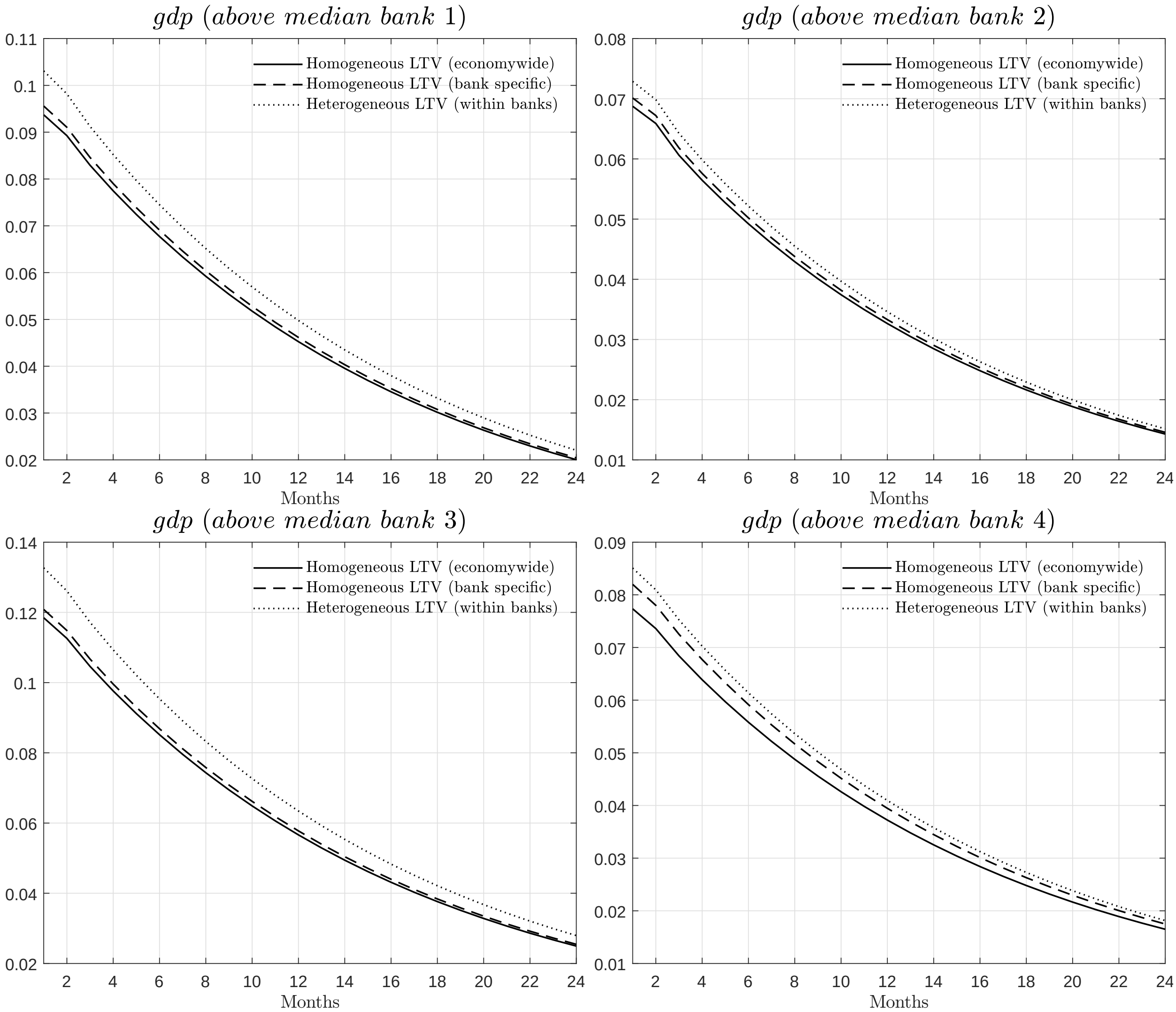


Figure 8: Impulse response function of individual banks (LTV shocks).

A Proofs

Proof of Proposition 1. The first part of the proposition readily follows without proof. For the second part, it is verifiable that

$$|\boldsymbol{\nu}_B| \geq |\boldsymbol{\nu}_B^{(n=0)}|$$
$$\boldsymbol{\nu}_B^{(n=0)} = -\kappa \boldsymbol{\Psi'}_B \boldsymbol{\theta}$$

where $|\cdot|$, $\geq$ and $=$ hold element–wise. Hence, it must be true that

$$\|\boldsymbol{\nu'}_B\|_2^2 \geq \|\kappa \boldsymbol{\theta'} \boldsymbol{\Psi}_B\|_2^2 = (1-\alpha)^2 \sum_{b=1}^{B} (d_{b|B}^{(1)})^2$$

Recall from definition 3 that $CV_{d_B^{(1)}} \equiv \frac{1}{\bar{d}_B^{(1)}}\Big(\frac{1}{B}\sum_{b=1}^{B}(d_{b|B}^{(1)} - \bar{d}_B^{(1)})^2\Big)^{\frac{1}{2}}$. Rewrite the r.h.s. using $\bar{d}_B^{(1)} = (1-\delta)B^{-1}$, $\sum_{b=1}^{B}\bar{d}_B^{(1)} = 1-\delta$ and $\sum_{b=1}^{B} d_{b|B}^{(1)} = 1-\delta$ in order to obtain

$$CV_{d_B^{(1)}} = \frac{B}{(1-\delta)}\sqrt{\frac{1}{B}\Big(\sum_{b=1}^{B}(d_{b|B}^{(1)})^2 - \frac{(1-\delta)^2}{B}\Big)}$$

$$\sum_{b=1}^{B}(d_{b|B}^{(1)})^2 = \frac{(1-\delta)^2 B}{B^2} CV^2_{d_B^{(1)}} + \frac{(1-\delta)^2}{B}$$

Use this expression to rewrite the first part in the proposition

$$\begin{aligned}
\sqrt{Var\ (\widehat{gdp}_{t|B})} &\geq (1-\alpha)\sqrt{\Big(\sum_{b=1}^{B}(d_{b|B}^{(1)})^2\Big)}\sigma \\
&\geq \kappa\Big(\sqrt{\frac{1}{B}CV^2_{d_B^{(1)}} + \frac{1}{B}}\Big)\sigma \\
&\geq \frac{\kappa}{\sqrt{B}}(\sqrt{1 + CV^2_{d_B^{(1)}}})\sigma
\end{aligned}$$

which delivers the result.

Q.E.D.

Proof of Corollary 1. The proof is a mixture of the proofs included in Gabaix (2011) and Acemoglu et al. (2012). By definition, the CDF of a random variable is uniformly distributed. Hence, the first–order outdegree of bank b out of B banks follows $\mathbb{E}[(d_{b|B}^{(1)})^{-\varrho}] = b/B$, such that – in a heuristic sense, $d_{b|B}^{(1)} \approx (b/B)^{\frac{-1}{\varrho}}$. This, then, allows us to write

$$\begin{aligned}
\sqrt{\sum_{b=1}^{B}(d_{b|B}^{(1)})^2} &\sim \frac{1}{1-\delta}\sqrt{\sum_{b=1}^{B}(d_{b|B}^{(1)})^2} \\
&\sim \frac{1}{B}\frac{\sqrt{\sum_{b=1}^{B}(d_{b|B}^{(1)})^2}}{\mathbb{E}d_{b|B}^{(1)}} \\
&\sim \frac{B^{-1+\frac{1}{\varrho}}\sqrt{\sum_{b=1}^{B} b^{-\frac{2}{\varrho}}}}{\mathbb{E}[d_{b|B}^{(1)}]}
\end{aligned}$$

In the case $\varrho < 2$, the series $\sum_{b=1}^{B} b^{-\frac{2}{\varrho}}$ converges such that

$$\sqrt{\sum_{b=1}^{B}(d_{b|B}^{(1)})^2} \sim \frac{C}{B^{\frac{\varrho-1}{\varrho}}}$$

where C is a constant. In contrast; if $\varrho \geq 2$, the series $\sum_{b=1}^{B} b^{-\frac{2}{\varrho}}$ diverges, such that $\sum_{b=1}^{B} b^{-\frac{2}{\varrho}} \sim \int_{1}^{B} b^{-\frac{2}{\varrho}} \mathrm{d}b \sim B^{1-\frac{2}{\varrho}}/(1-2/\varrho)$, which delivers

$$\sqrt{\sum_{b=1}^{B}(d_{b|B}^{(1)})^2} \sim B^{-1+\frac{1}{\varrho}}(B^{1-\frac{2}{\varrho}}/(1-2/\varrho))^{\frac{1}{2}} = \frac{C'}{\sqrt{B}}$$

and volatility scales with $\sqrt{B}$. Q.E.D.

Proof of Proposition 2. It is readily verifiable that

$$|\boldsymbol{\nu}_B| \geq |\sum_{n=0}^{1} \boldsymbol{\nu}_B^{(n)}|$$
$$\sum_{n=0}^{1} \boldsymbol{\nu}_B^{(n)} = -\kappa \boldsymbol{\Psi}_B'(\mathbb{I}+\widetilde{\boldsymbol{\Omega}}')\boldsymbol{\theta}$$

where $\geq$ and $=$ hold element–wise. It must be true that

$$\begin{aligned}\|\sum_{n=0}^{1} \boldsymbol{\nu}_B^{(n)}\|_2^2 &= \big(\kappa\boldsymbol{\theta}'(\mathbb{I}+\widetilde{\boldsymbol{\Omega}})\boldsymbol{\Psi}_B\big)\big(\kappa\boldsymbol{\Psi}_B'(\mathbb{I}+\widetilde{\boldsymbol{\Omega}}')\boldsymbol{\theta}\big)\\ &= \underbrace{\|\kappa\boldsymbol{\theta}'\boldsymbol{\Psi}_B\|_2^2}_{(1)} + \underbrace{\|\kappa\boldsymbol{\theta}'\widetilde{\boldsymbol{\Omega}}\boldsymbol{\Psi}_B\|_2^2}_{(2)} + \underbrace{2\kappa^2\boldsymbol{\theta}'\boldsymbol{\Psi}_B\boldsymbol{\Psi}_B'\widetilde{\boldsymbol{\Omega}}'\boldsymbol{\theta}}_{(3)}\end{aligned}$$

Or, after rewriting

$$(1): \|\kappa\boldsymbol{\theta}'\boldsymbol{\Psi}_B\|_2^2 = (1-\alpha)^2\sum_{b=1}^{B}(d_{b|B}^{(1)})^2$$
$$(2): \|\kappa\boldsymbol{\theta}'\widetilde{\boldsymbol{\Omega}}\boldsymbol{\Psi}_B\|_2^2 = (1-\alpha)^2\sum_{b=1}^{B}(d_{b|B}^{(2)})^2$$
$$(3): 2\kappa^2\boldsymbol{\theta}'\boldsymbol{\Psi}_B\boldsymbol{\Psi}_B'\boldsymbol{\Omega}'\boldsymbol{\theta} = 2(1-\alpha)^2\sum_{b=1}^{B} d_{b|B}^{(2)}d_{b|B}^{(1)}$$

Note that the following expression holds:

$$\sum_{b=1}^{B}((1-\alpha)d_{b|B}^{(1)} - (1-\alpha)d_{b|B}^{(2)})^2 \geq 0$$
$$\underbrace{(1-\alpha)^2\sum_{b=1}^{B}(d_{b|B}^{(1)})^2 + (1-\alpha)^2\sum_{b=1}^{B}(d_{b|B}^{(2)})^2}_{(1)} \geq \underbrace{2(1-\alpha)^2\sum_{b=1}^{B} d_{b|B}^{(2)}d_{b|B}^{(1)}}_{(2)}$$

such that (1) always dominates (2). As a result:

$$\|\boldsymbol{\nu}_B\|_2 \geq (1-\alpha)^2 \sqrt{\sum_{b=1}^{B} (d_{b|B}^{(1)})^2} + (1-\alpha)^2 \sqrt{\sum_{b=1}^{B} (d_{b|B}^{(2)})^2}$$

From proof A, $\sum_{b=1}^{B} (d_{b|B}^{(1)})^2 = \frac{(1-\delta)^2}{B} CV^2_{d_B^{(1)}} + \frac{(1-\delta)^2}{B}$. For the second term, using the same strategy as before, it is readily verifiable that $\sum_{b=1}^{B} (d_{b|B}^{(2)})^2 = (1-\delta)^2 (\boldsymbol{\theta}' \widetilde{\boldsymbol{\Omega}} \boldsymbol{\iota})^2 (\frac{1}{B} CV^2_{d_B^{(2)}} + \frac{1}{B})$. Hence,

$$\|\boldsymbol{\nu}_B\|_2 \geq \Big(\frac{1}{\sqrt{B}} \sqrt{1 + CV^2_{d_{b|B}^{(1)}}} + \frac{\boldsymbol{\theta}' \widetilde{\boldsymbol{\Omega}} \boldsymbol{\iota}}{\sqrt{B}} \sqrt{1 + CV^2_{d_{b|B}^{(2)}}} \Big) \kappa$$

Q.E.D.

Proof of Corollary 2. The rest of the proof is similar to the proof of corollary 1. Q.E.D.

Proof of Proposition 3. The proof follows the same logical flow as the previous proofs.

1. Rewrite the approximation of the influence vector $\boldsymbol{\nu}_B^{(n)}$ as a finite power series.
2. Show that the squared norm of this finite sum, when the order of approximation is N, equals $\|\boldsymbol{\nu}_B^{(n=N)}\|_2^2 = (1-\alpha)^2 \sum\limits_{1 \leq i,j \leq N} \sum_{b=1}^{B} (d_{b|B}^{(i)})(d_{b|B}^{(j)})$.
3. The terms for which $i = j$ dominate all terms for which $i \neq j$ since expansion of $\sum_{b=1}^{B} (d_{b|B}^{(i)} - d_{b|B}^{(j)})^2$ delivers $\sum_{b=1}^{B} (d_{b|B}^{(i)})^2 + \sum_{b=1}^{B} (d_{b|B}^{(j)})^2 \geq 2 \sum_{b=1}^{B} d_{b|B}^{(i)} d_{b|B}^{(j)}$ for all $i \neq j$.

Q.E.D.

B Steady state solution and log–linear system

This section elaborates the first–order conditions and other relevant equations used to log linearize the model.

B.1 First–order conditions

B.1.1 Household

$$w_{jt} = g_j l_{jt}^{\varphi} c_t \tag{B-1}$$

$$\frac{1}{c_t} = \beta \mathbb{E}_t \Big(\frac{R_t}{\pi_{t+1} c_{t+1}} \Big) \tag{B-2}$$

$$\frac{q_t}{c_t} = \frac{\iota}{h_t} + \beta \mathbb{E}_t \Big(\frac{q_{t+1}}{c_{t+1}} \Big) \tag{B-3}$$

B.1.2 Firms

Let $\Lambda_{t,t+s} \equiv \frac{\beta^s c_t}{c_{t+s}} \frac{P_t}{P_{t+s}}$ denote the discount factor of nominal profits of firm j with $\Lambda_{t,t} = 1$ and $\Lambda_{t+s,t+s+1} = R^{-1}_{t+s}$. Firm j maximizes the present value of profits

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}(P_{jt+s}y_{jt+s} - (W_{jt+s}n_{jt+s} + P^{\omega}_{jt+s}m_{jt+s} + F_{jt+s}k_{jt+s}))$$

s.t.

$$\begin{cases} y_{jt+s} & = \sum_{j'=1}^{J} m_{j'jt+s} + c_{jt+s} + \sum_{j'=1}^{J} \widetilde{c}_{j'jt+s} \\ y_{jt+s} & = A_j \big(n^{\phi_j}_{jt+s} m^{1-\phi_j}_{jt+s}\big)^{\delta_j} k^{1-\delta_j}_{jt+s} - \Phi_j \\ P_{jt+s} & = \begin{cases} P^*_{jt+s} & \text{with probability } 1-\alpha_j \\ P_{jt+s-1} & \text{with probability } \alpha_j \end{cases} \end{cases}$$

The first order conditions w.r.t. n_{jt}, m_{jt} and k_{jt} deliver

$$\begin{aligned} n_{jt} : &\qquad W_{jt} = MC_{jt}(\partial y_{jt}/\partial n_{jt}) \\ m_{jt} : &\qquad P^{\omega}_{jt} = MC_{jt}(\partial y_{jt}/\partial m_{jt}) \\ k_{jt} : &\qquad F_{jt} = MC_{jt}(\partial y_{jt}/\partial k_{jt}) \end{aligned}$$

Where MC_{jt} denotes nominal marginal costs. Furthermore

$$\begin{cases} \frac{m_{jt}}{n_{jt}} & = \frac{\delta_j(1-\phi_j)}{\delta_j\phi_j} \frac{W_{jt}}{P^{\omega}_{jt}} \\ \frac{n_{jt}}{k_{jt}} & = \frac{\delta_j\phi_j}{(1-\delta_j)} \frac{F_{jt}}{W_{jt}} \end{cases}$$

Nominal marginal costs are obtained using

$$y_{jt} = A_j \Big[\Big(\frac{\delta_j(1-\phi_j)}{\delta_j\phi_j}\Big)\Big(\frac{W_{jt}}{P^{\omega}_{jt}}\Big)\Big]^{\delta_j(1-\phi_j)} \Big[\Big(\frac{(1-\delta_j)}{\delta_j\phi_j}\Big)\Big(\frac{W_{jt}}{F_{jt}}\Big)\Big]^{1-\delta_j} n_{jt} - \Phi_j \tag{B-4}$$

$$y_{jt} = A_j \Big[\Big(\frac{\delta_j\phi_j}{\delta_j(1-\phi_j)}\Big)\Big(\frac{P^{\omega}_{jt}}{W_{jt}}\Big)\Big]^{\delta_j\phi_j} \Big[\Big(\frac{(1-\delta_j)}{\delta_j(1-\phi_j)}\Big)\Big(\frac{P^{\omega}_{jt}}{F_{jt}}\Big)\Big]^{(1-\delta_j)} m_{jt} - \Phi_j \tag{B-5}$$

$$y_{jt} = A_j \Big[\Big(\frac{\delta_j\phi_j}{(1-\delta_j)}\Big)\Big(\frac{F_{jt}}{W_{jt}}\Big)\Big]^{\delta_j\phi_j} \Big[\Big(\frac{\delta_j(1-\phi_j)}{(1-\delta_j)}\Big)\Big(\frac{F_{jt}}{P^{\omega}_{jt}}\Big)\Big]^{\delta_j(1-\phi_j)} k_{jt} - \Phi_j \tag{B-6}$$

such that

$$MC_{jt} = \frac{1}{A_j}\Big(\frac{W_{jt}}{\delta_j\phi_j}\Big)^{\delta_j\phi_j}\Big(\frac{P^{\omega}_{jt}}{\delta_j(1-\phi_j)}\Big)^{\delta_j(1-\phi_j)}\Big(\frac{F_{jt}}{1-\delta_j}\Big)^{1-\delta_j}$$

or in real terms $mc_{jt} \equiv \frac{MC_{jt}}{P_t}$

$$mc_{jt} = \frac{1}{A_j}\Big(\frac{w_{jt}}{\delta_j\phi_j}\Big)^{\delta_j\phi_j}\Big(\frac{p^{\omega}_{jt}}{\delta_j(1-\phi_j)}\Big)^{\delta_j(1-\phi_j)}\Big(\frac{f_{jt}}{1-\delta_j}\Big)^{1-\delta_j}$$

with $w_{jt} \equiv \frac{W_{jt}}{P_t}$, $p^{\omega}_{jt} \equiv \frac{P^{\omega}_{jt}}{P_t}$ and $f_{jt} \equiv \frac{F_{jt}}{P_t}$.

B.1.3 Entrepreneurs

$$\{\frac{1}{\widetilde{c}_{jt}} = \mathbb{E}_t\Big(\frac{\gamma R_{jt}}{\widetilde{c}_{jt+1}\pi_{t+1}} + \lambda_{jt}R_{jt}\Big)\}_{j=1}^{J} \tag{B-7}$$

$$\{\frac{q_t}{\widetilde{c}_{jt}} = \mathbb{E}_t\Big(\frac{\gamma}{\widetilde{c}_{jt+1}}\big(\nu_j\frac{f_{jt+1}k_{jt+1}}{\widetilde{h}_{jt}} + q_{t+1}\big) + \lambda_{jt}\ell_{jt}\pi_{t+1}q_{t+1}\}_{j=1}^{J} \tag{B-8}$$

$$\{w_{jt}\widetilde{n}_{jt} = (1-\nu_j)k_{jt}f_{jt}\}_{j=1}^{J} \tag{B-9}$$

$$\{f_{jt}k_{jt} + s_{jt} = \widetilde{c}_{jt} + q_t(h_{jt} - h_{jt-1}) + \frac{R_{jt-1}s_{jt-1}}{\pi_t} + w_{jt}\widetilde{n}_{jt}\}_{j=1}^{J} \tag{B-10}$$

$$\{k_{jt} = \widetilde{n}_{jt}^{1-\nu_j}h_{jt-1}^{\nu_j}\}_{j=1}^{J} \tag{B-11}$$

$$\{s_{jt} = \ell_{jt}\mathbb{E}_t\frac{q_{t+1}h_{jt}\pi_{t+1}}{R_{jt}}\}_{j=1}^{J} \tag{B-12}$$

B.1.4 Market clearing

$$\{D_{bt} = \sum_{j=1}^{J} S_{jbt}\}_{b=1}^{B}$$

$$D_t = \sum_{b=1}^{B} D_{bt}$$

$$l_t = \sum_{j=1}^{J}(n_{jt} + \widetilde{n}_{jt})$$

$$h = h_t + \sum_{j=1}^{J} h_{jt}$$

$$\{y_{jt} = c_{jt} + \sum_{j'=1}^{J}\widetilde{c}_{j'jt} + \sum_{j'=1}^{J} m_{j'jt}\}_{j=1}^{J}$$

B.2 Steady state

For simplicity, we make three assumptions.

Assumption A-1. *$gdp \equiv c + \widetilde{c} = 1$.*

Assumption A-2. *As in Carvalho and Lee (2011); Pasten et al. (2018a,b) we take $g_j = (\frac{l_j}{l})^{-\varphi}$ where $\frac{l_j}{l}$ denotes the steady state share of household labour employed by firm j.*

Assumption A-3. *We normalize* $\{A_j\}_{j=1}^J$ *such that steady state prices of goods/capital services equal the aggregate price level; i.e.* $\frac{P_j}{P} = p_j = \frac{P_j^\omega}{P} = p_j^\omega = \frac{F_j}{P} = f_j = 1$.

Assumption A-1, comes w.l.o.g. and merely pins down the size of the economy. Assumption A-2, equalizes steady state wages across firms. Assumption A-3 allows for a symmetric steady state.

From the household Euler equation, $R = \frac{1}{\beta}$. From the bank first order conditions, $R_{jb} = \frac{\mu}{\mu-1}R$. From the firm first–order conditions,

$$
\begin{aligned}
n_j w_j &= mc_j \delta_j \phi_j (y_j + \Phi_j) \\
m_j p_j^\omega &= mc_j \delta_j (1 - \phi_j)(y_j + \Phi_j) \\
k_j f_j &= mc_j (1 - \delta_j)(y_j + \Phi_j)
\end{aligned}
$$

Real firm profit is then defined as

$$
\begin{aligned}
\Delta_j &= y_j - \big(n_j w_j + m_j p_j^\omega + k_j f_j\big) \\
&= y_j - mc_j (y_j + \Phi_j)
\end{aligned}
$$

In order to rule out entry, $\Delta_j = 0$, we pin down the fixed costs

$$
\Phi_j = \frac{1 - mc_j}{mc_j} y_j
$$

and since $mc_j = \frac{\eta-1}{\eta}$, we have that $\Phi_j = (\eta - 1)^{-1} y_j$. Consequently, from the first–order conditions of intermediate goods producers, we have that

$$
\begin{aligned}
n_j w_j &= \frac{\eta - 1}{\eta} \delta_j \phi_j (y_j + \Phi_j) = \delta_j \phi_j y_j \\
m_j p_j^\omega &= \frac{\eta - 1}{\eta} \delta_j (1 - \phi_j)(y_j + \Phi_j) = \delta_j (1 - \phi_j) y_j \\
k_j f_j &= \frac{\eta - 1}{\eta} (1 - \delta_j)(y_j + \Phi_j) = (1 - \delta_j) y_j
\end{aligned}
$$

Under assumption A-3, it is true that

$$
\begin{aligned}
c_j &= \theta_j c \\
\sum_{j'=1}^J \widetilde{c}_{j'j} &= \theta_j (1 - c) \\
m_{jj'} &= \omega_{jj'} m_j
\end{aligned}
$$

Furthermore, from assumption A-2,

$$
\begin{aligned}
y_j &= c_j + \theta_j(1-c) + \sum_{j'=1}^{J} m_{j'j} \\
&= \theta_j + \sum_{j'=1}^{J} \delta_{j'}(1-\phi_{j'})\omega_{j'j} y_{j'}
\end{aligned}
$$

implying that $\boldsymbol{y} = (\mathbb{I} - (\boldsymbol{\Delta}(\mathbb{I} - \boldsymbol{\Phi})\boldsymbol{\Omega})')^{-1}\boldsymbol{\theta}$, where $\boldsymbol{y} = [y_1, ..., y_j]$.

Next, from the entrepreneur Euler equation (B-7)

$$
\lambda_j = \frac{1}{\widetilde{c}_j}\Big(\beta(\frac{\mu-1}{\mu}) - \gamma\Big)
$$

where, as is common, we assume that $\beta(\frac{\mu-1}{\mu}) > \gamma$. From the entrepreneur first–order condition (B-8)

$$
\frac{qh_j}{k_j} = \frac{\gamma\nu_j}{1-\gamma-\ell_j(\beta(\frac{\mu-1}{\mu})-\gamma)} = \frac{\gamma\nu_j}{1-\widetilde{\gamma}_j}
$$

with $\widetilde{\gamma}_j \equiv (1-\ell_j)\gamma + \ell_j\beta(\frac{\mu-1}{\mu})$. From the entrepreneur budget constraint, we have that

$$
\frac{\widetilde{b}_j}{\widetilde{k}_j} = \frac{\mu-1}{\mu}\frac{\beta\ell_j\gamma\nu_j}{1-\widetilde{\gamma}_j}
$$

From the budget constraint of entrepreneur j

$$
\widetilde{c}_j = y_j\Big(1 + \big(1 - (\frac{\mu-1}{\mu})(\frac{1}{\beta})\big)\big(\frac{\beta\frac{\mu-1}{\mu}\ell_j\gamma\nu_j}{1-\widetilde{\gamma}_j}\big) - (1-\nu_j)\Big)
$$

which can be used to determine $c = 1 - \sum_{j=1}^{J}\widetilde{c}_j$ and $c_j = \theta_j c$. From the labour supply schedule and assumption $A-2$

$$
w_j = l^{\varphi}c
$$

Note that $q\sum_{j=1}^{J} h_j = \sum_{j=1}^{J}\frac{\gamma\nu_j\tau_j(1-\delta_j)}{1-\widetilde{\gamma}_j}$, such that

$$
\frac{\sum_{j=1}^{J} h_j}{h - \sum_{j=1}^{J} h_j} = \sum_{j=1}^{J}\frac{\gamma\nu_j\tau_j(1-\delta_j)(1-\beta)}{(1-\widetilde{\gamma}_j)\iota c}
$$

B.3 Log linearization

We next summarize the set of log–linearized equations. Hats denote deviations from steady state.

Household

$$\begin{aligned}
\widehat{c}_t &= \widehat{c}_{t+1} - \widehat{r}r_t \\
\widehat{c}_t &= \sum_{j=1}^{J} \theta_j \widehat{c}_{jt} \\
\{\widehat{c}_{jt} &= \widehat{c}_t - \eta \widehat{p}_{jt}\}_{j=1}^J \\
\{\widehat{w}_{jt} &= \varphi \widehat{l}_{jt} + \widehat{c}_t\}_{j=1}^J \\
\{\widehat{l}_{jt} &= \zeta_j^n \widehat{n}_{jt} + (1-\zeta_j^n)\widehat{\widetilde{n}}_{jt}\}_{j=1}^J \\
\widehat{q}_t &= \beta \widehat{q}_{t+1} + (1-\beta)\widehat{h}_t + \widehat{c}_t - \beta \widehat{c}_{t+1}
\end{aligned}$$

Firms & entrepreneurs

$$\begin{aligned}
\{\widehat{p}^{\omega}_{jt} &= \sum_{j'=1}^{J} \omega_{jj'} \widehat{p}_{j't}\}_{j=1}^J \\
\{\widehat{y}_{jt} &= \zeta_j^c \widehat{c}_{jt} + \sum_{j'=1}^{J} \zeta_{j'j}^{\widetilde{c}} \widehat{\widetilde{c}}_{j'jt} + \sum_{j'=1}^{J} \zeta_{j'j}^{m} \widehat{m}_{j'jt}\}_{j=1}^J \\
\{\widehat{\pi}_{jt} &= \widehat{p}_{jt} - \widehat{p}_{jt-1} + \widehat{\pi}_t\}_{j=1}^J \\
\{\widehat{\pi}_{jt} &= \beta \widehat{\pi}_{jt+1} + \zeta^{\pi}(\widehat{mc}_{jt} - \widehat{p}_{jt})\}_{j=1}^J \\
\{\widehat{mc}_{jt} &= \delta_j \phi_j \widehat{w}_{jt} + \delta_j(1-\phi_j)\widehat{p}^{\omega}_{jt} + (1-\delta_j)\widehat{f}_{jt}\}_{j=1}^J \\
\{\widehat{m}_{jt} &= \sum_{j'=1}^{J} \omega_{jj'} \widehat{m}_{jj't}\}_{j=1}^J \\
\{\widehat{m}_{jj't} &= \widehat{m}_{jt} - \eta(\widehat{p}_{j't} - \widehat{p}^{\omega}_{jt})\}_{j=1}^J \\
\{\widehat{y}_{jt} &= \frac{\eta}{\eta-1}(\delta_j \phi_j \widehat{l}_{jt} + \delta_j(1-\phi_j)\widehat{m}_{jt} + (1-\delta_j)\widehat{k}_{jt})\}_{j=1}^J \\
\{\widehat{w}_{jt} - \widehat{p}^{\omega}_t &= \widehat{m}_{jt} - \widehat{n}_{jt}\}_{j=1}^J \\
\{\widehat{f}_{jt} - \widehat{p}^{\omega}_t &= \widehat{m}_{jt} - \widehat{k}_{jt}\}_{j=1}^J \\
\{\widehat{\widetilde{c}}_{jj't} &= \widehat{\widetilde{c}}_{jt} - \eta \widehat{p}_{j't}\}_{j=1}^J \\
\{\widehat{k}_{jt} &= (1-\nu_j)\widehat{\widetilde{n}}_{jt} + \nu_j \widehat{h}_{jt-1}\}_{j=1}^J \\
\{\widehat{s}_{jt} &= \widehat{h}_{jt} + \widehat{\ell}_{jt} + \widehat{q}_{t+1} - (\widehat{r}_{jt} - \pi_t)\}_{j=1}^J \\
\{\widehat{q}_t &= \widetilde{\gamma}_j \widehat{q}_{t+1} + (1-\widetilde{\gamma}_j)(\widehat{k}_{jt+1} + \widehat{f}_{jt+1} - \widehat{h}_{jt}) - \widehat{\ell}_j \beta \frac{\mu-1}{\mu}(\widehat{R}_{jt} - \pi_t) \\
&\quad - (1-\beta\frac{\mu-1}{\mu})(\widehat{\widetilde{c}}_{jt+1} - \widehat{\widetilde{c}}_{jt}) + (\beta\frac{\mu-1}{\mu} - \gamma)\ell_j \widehat{\ell}_{jt}\}_{j=1}^J
\end{aligned}$$

Monetary policy

$$\widehat{R}_t = \phi_\pi \widehat{\pi}_t + \phi_{gdp}\widehat{gdp}_t$$
$$\widehat{gdp}_t = c\widehat{c}_t + (1-c)\widehat{\widetilde{c}}_t$$
$$\widehat{rr}_t = \widehat{R}_t - \widehat{\pi}_{t+1}$$

Exogenous processes

$$\{\widehat{\ell}_{jt} = \sum_{b=1}^{B} \psi_{jb}(\epsilon_t^{(\ell)} + \varepsilon_{bt}^{(\ell)})\}_{j=1}^{J}$$
$$\{\widehat{r}_{jt} = \widehat{R}_t + \sum_{b=1}^{B} \psi_{jb}(\varepsilon_{bt}^{(r)} + \varepsilon_t^{(r)})\}_{j=1}^{J}$$

Structural composite parameters

$$\zeta_j^n = \frac{\delta_j\phi_j}{\delta_j\phi_j + (1-\nu_j)(1-\delta_j)}$$
$$\zeta_{j'j}^{\widetilde{c}} = c_{j'j}/\tau_j$$
$$\zeta_j^c = \widetilde{c}_j/y_j$$
$$\zeta_{j'j}^m = \delta_{j'}(1-\phi_{j'})\frac{y_j}{y_{j'}}$$
$$\zeta^\pi = \frac{(1-\beta\alpha_j)(1-\alpha_j)}{\alpha_j}$$

B.4 Equilibrium in a simplified framework

First, under assumptions 1–2, household labour supply to firm j yields

$$\frac{W_{jt}}{P_t} = g_j C_t = 0$$

such that $\widehat{W}_{jt} = 0$. Second, by assumptions 3 and 4, normal marginal cost collapses to

$$MC_{jt} = \frac{1}{A_j}\Big(\frac{W_{jt}}{\delta\phi_j}\Big)^{\delta\phi_j}\Big(\frac{P_{jt}^\omega}{\delta(1-\phi_j)}\Big)^{\delta(1-\phi_j)}\Big(\frac{R_{jt}}{1-\delta}\Big)^{(1-\delta)}$$

or in log linear form $\widehat{MC}_{jt} = \delta(1-\phi_j)\widehat{P}_{jt}^\omega + (1-\delta)(\epsilon_t^{(r)} + \sum_{b=1}^{B}\psi_{jb}\varepsilon_{bt}^{(r)})$. Third, under assumption 3,

$$\widehat{P}_{jt} = (1-\alpha)\widehat{MC}_{jt}$$

This implies that

$$\widehat{P}_{jt} = (1-\alpha)\big(\delta(1-\phi_j)\widehat{P}_{jt}^\omega + (1-\delta)(\epsilon_t^{(r)} + \sum_{b=1}^{B}\psi_{jb}\varepsilon_{bt}^{(r)})\big)$$

Or in matrix form with $\boldsymbol{P}_t = [P_{1t}, ..., P_{jt}]'$, $\boldsymbol{P}_t = \kappa[\mathbb{I} - \widetilde{\boldsymbol{\Omega}}]^{-1}\boldsymbol{\Psi}(\boldsymbol{\varepsilon}^{(r)}_{t|B} + \boldsymbol{\iota}_B \epsilon^{(r)}_t)$. Given that $\widehat{P}_t = \boldsymbol{\theta'}\boldsymbol{P}_t$ and $\widehat{GDP}_t = \widehat{C}_t = -\widehat{P}_t$,

$$\widehat{gdp}_t = -\kappa\boldsymbol{\theta'}[\mathbb{I} - \widetilde{\boldsymbol{\Omega}}]^{-1}\boldsymbol{\Psi}(\boldsymbol{\varepsilon}^{(r)}_{t|B} + \boldsymbol{\iota}_B \epsilon^{(r)}_t)$$

B.5 Nesting the literature

B.5.1 Iacoviello (2005)

If we take (1) $J = 1$, (2) $\mu \to \infty$, (3) $\delta_j = 0$, (4) $\phi_j = 1$, (5) $\ell_{jbt} = \ell\ \forall t$. Then,

$$\begin{aligned}
\widehat{k}_t &= \frac{c}{k}\widehat{c}_t + \frac{\widetilde{c}}{k}\widehat{\widetilde{c}}_t \\
\widehat{c}_t &= \mathbb{E}_t\widehat{c}_{t+1} - \widehat{rr}_t \\
\frac{\widetilde{c}}{k}\widehat{\widetilde{c}}_t &= \frac{\widetilde{b}}{k}\widehat{\widetilde{b}}_t + \frac{\widetilde{b}}{\beta k}(\widehat{\pi}_t - \widehat{R}_{t-1} - \widehat{\widetilde{b}}_{t-1}) + \nu k(\widehat{k}_t + \widehat{f}_t) - qh\Delta\widehat{h}_t \\
\widehat{q}_t &= \widetilde{\gamma}\mathbb{E}_t\widehat{q}_{t+1} + (1-\widetilde{\gamma})(\widehat{k}_{t+1} - \widehat{h}_t + \widehat{f}_{t+1}) - \ell\beta\widehat{rr}_t - (1-\ell\beta)\mathbb{E}_t(\widehat{\widetilde{c}}_{t+1} - \widehat{\widetilde{c}}_t) \\
\widehat{q}_t &= \beta\widehat{q}_{t+1} + (1-\beta)\frac{\widetilde{h}}{h}\widehat{\widetilde{h}}_t + \widehat{c}_t - \beta\widehat{c}_{t+1} \\
\widehat{\widetilde{b}}_t &= \widehat{q}_{t+1} + \widehat{\widetilde{h}}_t - \widehat{rr}_t \\
\widehat{k}_t &= \frac{\eta\nu}{\eta-(1-\nu)}\widehat{\widetilde{h}}_{t-1} - \frac{1-\nu}{\eta-(1-\nu)}(\widehat{\widetilde{c}}_t - \widehat{f}_t) \\
\widehat{\pi}_t &= \beta\widehat{\pi}_{t+1} + \frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\widehat{f}_t \\
\widehat{R}_t &= \phi_\pi\widehat{\pi}_t + \phi_{gdp}\widehat{c}_t
\end{aligned}$$

with $\widetilde{\gamma} \equiv (1-\ell)\gamma + \ell\beta$ which is the simplified model analyzed by Iacoviello (2005).

B.5.2 Pasten et al. (2018a)

If we take (1) $\delta_j = 1$, (2) $A_j = A_{jt}$, (3) zero consumption mass on entrepreneurs, $c = 1$, (4) $h = \iota = 0$. Then,

Household

$$\begin{aligned}
\widehat{c}_t &= \widehat{c}_{t+1} - \widehat{rr}_t \\
\widehat{c}_t &= \sum_{j=1}^{J}\theta_j\widehat{c}_{jt} \\
\{\widehat{c}_{jt} &= \widehat{c}_t - \eta\widehat{p}_{jt}\}_{j=1}^{J} \\
\{\widehat{w}_{jt} &= \varphi\widehat{l}_{jt} + \widehat{c}_t\}_{j=1}^{J}
\end{aligned}$$

Firms

$$\{\widehat{p}^{\omega}_{jt} = \sum_{j'=1}^{J} \omega_{jj'}\widehat{p}_{j't}\}_{j=1}^{J}$$

$$\{\widehat{y}_{jt} = \zeta^{c}_{j}\widehat{c}_{jt} + \sum_{j'=1}^{J} \zeta^{m}_{j'j}\widehat{m}_{j'jt}\}_{j=1}^{J}$$

$$\{\widehat{\pi}_{jt} = \widehat{p}_{jt} - \widehat{p}_{jt-1} + \widehat{\pi}_{t}\}_{j=1}^{J}$$

$$\{\widehat{\pi}_{jt} = \beta\widehat{\pi}_{jt+1} + \zeta^{\pi}(\widehat{mc}_{jt} - \widehat{p}_{jt})\}_{j=1}^{J}$$

$$\{\widehat{mc}_{jt} = \phi\widehat{w}_{jt} + (1-\phi)\widehat{p}^{\omega}_{jt}$$

$$\{\widehat{m}_{jt} = \sum_{j'=1}^{J} \omega_{jj'}\widehat{m}_{jj't}\}_{j=1}^{J}$$

$$\{\widehat{y}_{jt} = \frac{\eta}{\eta-1}(\phi\widehat{l}_{jt} + (1-\phi)\widehat{m}_{jt})\}_{j=1}^{J}$$

$$\{\widehat{w}_{jt} - \widehat{p}^{\omega}_{t} = \widehat{m}_{jt} - \widehat{l}_{jt}\}_{j=1}^{J}$$

$$\{\widehat{m}_{jj't} = m_{jt} - \eta(\widehat{p}_{jt} - \widehat{p}^{\omega}_{jt}\}_{j=1}^{J}$$

Monetary policy

$$\widehat{R}_t = \phi_{\pi}\widehat{\pi}_t + \phi_{gdp}\widehat{c}_t$$

$$\widehat{rr}_t = \widehat{R}_t - \widehat{\pi}_{t+1}$$

B.5.3 Acemoglu et al. (2012)

If we take (1) $\delta_j = 1$, (2) $A_j = A_{jt}$, (3) zero consumption mass on entrepreneurs, $c = 1$, (4) $h = \iota = 0$, (5) $\{\alpha_j\}_{j=1}^{J} = 0$, (6) $\{\phi_j = \phi\}_{j=0}^{J}$, (7) $\{\theta_j = 1/J\}_{j=0}^{J}$. (8) $\eta = \infty$ Then, the model collapses to Acemoglu et al. (2012).

B.5.4 Gabaix (2011)

The framework of Acemoglu et al. (2012) can easily be reformulated to the Gabaix (2011) framework (Acemoglu et al., 2012, p. 1983).

B.5.5 Bremus et al. (2018)

If we take (1) Assumptions 1–5 body text. (2) $\{\phi_j\}_{j=1}^{J} = 1$, (3) $\{\omega_{jj'}\}_{j,j'}^{J} = 0$, (4) $\{\theta_j\}_{j=1}^{J} = J^{-1}$. Then, $\sqrt{Var(\widehat{gdp}_t)} = \sqrt{\sum_{b=1}^{B}\left(\frac{\sum_{j=1}^{J} s_{jb}}{\sum_{b=1}^{B}\sum_{j=1}^{J} s_{jb}}\right)^2}\sigma$.

C Additional results

C.1 Model assumptions: supplier switching

Our model does not allow firms to adjust their supplier portfolio in the extensive margin (adding and dropping suppliers) in order to offset shocks from suppliers. In order to gauge the role of the extensive margin, we probe the stability of the real economy network for the period $2002-2014$.

In table C.1 we show that, for the average firm, 44.57% of its suppliers at time t were not its suppliers in the previous period $t-1$. Reversely, for the average firm, 42.89% of the current supplier relationships at time t are discontinued in the following period $t+1$. These high levels of variation in the extensive margin conceal that added (discontinued) supplier relations only represent small shares in the overall input portfolio of firms. At the firm level, new and discontinued purchases from input suppliers in total intermediate purchases have an average input weight (in total intermediates) of 8.45% and 8.81%, respectively. In addition to genuine supplier switching, these numbers also reflect (*i*) firm entry, (*ii*) firm exit, (*iii*) one–time capital purchases, (*iv*) transactions crossing the reporting threshold. In sum, firms actively discontinue supplier relationships and establish new ones but the size of these relationships are found to be small.

Table C.1: Firm switching

		Percentiles				
	Average	$p=90$	$p=75$	$p=50$	$p=25$	$p=10$
New firm–to–firm contracts at t (% of total firm–to–firm contracts active at t)	44.57	62.19	46.77	37.5	31.25	26.26
Number of firm-to-firm contracts discontinued at $t+1$ (% of total firm-to-firm contracts active at t)	42.89	64.12	44.89	36.04	30.00	25.27
Total weight of new supplier contracts in firm portfolio	8.45	17.69	10.85	6.75	3.40	0.00
Total weight of discontinued supplier contracts in firm portfolio	8.81	18.22	11.28	7.15	3.86	0.00

t refers to years. The reported statistics are averages across $2002-2014$. Numbers are denoted in percentages.

C.2 Modelling assumptions: bank switching

We study the impact of bank shocks taking the credit network $\mathbf{\Psi}$ as given. Our model does not allow firms to establish new banking relationships in order to offset adverse credit–supply shocks. In order to gauge the restrictiveness of this assumption, we exploit the high frequency panel structure of the CCR (monthly periods from $2002m1-2011m3$).

In table C.2 we observe that the extensive margin of the credit network is very stable. On average, only $1-3\%$ of the firm–bank relationships are discontinued in the next month. Similarly, on average about $1-3\%$ of firms do not add a bank to their portfolio. These numbers are corrected for changes of the bank identity due to M&A by creating hypothetical pre and post merger banks.

This evidence suggests that active bank switching is not a feature of the Belgian credit network. Various theoretical arguments explain why we observe this high stability in the bank–to–firm network; the existence of fixed costs Detragiache et al. (2000), relationship lending (Degryse and Van Cayseele, 2000), bank specialization (Paravisini et al., 2016), etc.

Table C.2: Bank switching of firms

	2004	2006	2008	2010	2012	2014
Firm–to–Firm level						
Firms that add a bank to their portfolio at time t	1.54	1.56	1.57	1.36	2.57	2.68
Firms that drop a bank to their portfolio at time t	1.91	1.80	1.71	1.65	2.62	2.70

t refers to years and are 12 month averages. Numbers are denoted in percentages.

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