Cournot Fire Sales

Unexpected Consequences of Internalizing Price Impact

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Motivation

- Theory: canonical macro-finance models with fire sales
  - Liquidity holdings inefficiently low (Allen and Gale, 2004)
  - Levered investment inefficiently high (Lorenzoni, 2008)
  - → Pecuniary externalities (Dávila and Korinek, 2017)

→ Implicit intuition: "if only agents internalized price impacts"

- Data: increasing concentration...
  - in financial sector (Corbae and Levine, 2018)
  - in real sector (Gutierrez and Philippon, 2017)

→ Worry less about the externalities?

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This paper — check intuition

Cournot fire sales

- Finite number of agents internalizing price impacts
  - Banks choosing portfolio liquidity
  - Firms choosing levered investment
- Internalizing price impact can. . .
  - exacerbate inefficiently low liquidity
  - overcorrect inefficiently high investment

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Portfolio tradeoff: liquidity vs. return

Standard: pecuniary externality ➔ inefficiently low liquidity

New: internalizing price effect can exacerbate inefficiency

Bank portfolio model

Assets and preferences á la Diamond-Dybvig

- $t = 0, 1, 2$
- Assets: trade-off liquidity vs. return
    1. Liquid asset: 1 at $t = 0 \longrightarrow 1$ at $t = 1$ or at $t = 2$
    2. Illiquid asset: 1 at $t = 0 \longrightarrow R > 1$ only at $t = 2$
- Preferences: liquidity shocks
    1. Early consumer $u(c_1)$
    2. Late consumer $u(c_1) + \beta u(c_2)$
- $\beta < 1$, $\beta R \geq 1$ and RRA > 1 ➔ liquidity insurance

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Structure of Uncertainty

Agg. state	Prob.	Liquidity shock	Cons.
Good	α	Nobody hit	$\overline{c}$
Bad	$1-\alpha$	Hit with $\Pr = \frac{1}{2}$	c_L
		Not hit with $\Pr = \frac{1}{2}$	c_H

Trade in financial assets

Banks and trade á la Allen-Gale

- $2N$ banks
  - Liquidity shocks perfectly correlated within bank
  - Portfolio $(\ell_i, 1-\ell_i)$ at $t=0$
- Cash-in-the-market pricing at $t=1$:

$$\overbrace{\sum_{i \in \text{buy}} \ell_i}^{\text{total cash}} = p \times \overbrace{\sum_{j \in \text{sell}} (1-\ell_j)}^{\text{total assets}}$$

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First-order condition

- Consumption:

$$\overline{c} = \ell_i + (1 - \ell_i)\, R \qquad \begin{aligned} c_L &= \ell_i + (1 - \ell_i)\, p \\ c_H &= \ell_i \tfrac{R}{p} + (1 - \ell_i)\, R \end{aligned}$$

- Expected utility:

$$\alpha\beta u(\overline{c}) + (1 - \alpha)\left(\tfrac{1}{2} u(c_L) + \tfrac{1}{2}\beta u(c_H)\right)$$

- First-order condition for ℓ_i — Walrasian equilibrium:

$$\overbrace{\alpha\beta\,(R - 1)\, u'(\overline{c})}^{\text{cost in good state}}$$

$$= \underbrace{(1 - \alpha)\left(\frac{1}{2}(1 - p)\, u'(c_L) + \frac{1}{2}\beta\left(\frac{R}{p} - R\right) u'(c_H)\right)}_{\text{benefit in bad state } (p < 1)}$$

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First-order condition — Social planner

- Extra term for social planner:

$$\underbrace{\frac{dp}{d\ell}}_{>0} \times \underbrace{\left(u'(c_L) - \beta \frac{R}{p} u'(c_H)\right)}_{>0} > 0$$

- Extra liquidity increases price
  - Benefits sellers: $u'(c_L)$
  - Hurts buyers: $-\beta \frac{R}{p} u'(c_H)$
  - ➔ Net effect positive (liquidity insurance)

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$$\frac{dp_L}{d\ell_i} \times u'(c_L) - \frac{dp_H}{d\ell_i} \times \beta \frac{R}{p} u'(c_H)$$

- Price impacts weight benefit and cost of liquidity
  - High $\frac{dp_L}{d\ell_i}$ (seller) ➔ more liquidity
  - High $\frac{dp_H}{d\ell_i}$ (buyer) ➔ less liquidity

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Price impacts

$$p = \frac{\sum_{i \in \text{buy}} \ell_i}{\sum_{j \in \text{sell}} (1 - \ell_j)} = \frac{\ell}{1 - \ell}$$

i as buyer	i as seller
$p_H = \frac{\ell_i + (N-1)\ell}{N(1-\ell)}$	$p_L = \frac{N\ell}{(1-\ell_i)+(N-1)(1-\ell)}$
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- Low price (bad state unlikely) ➔ low weight on liqu. benefit

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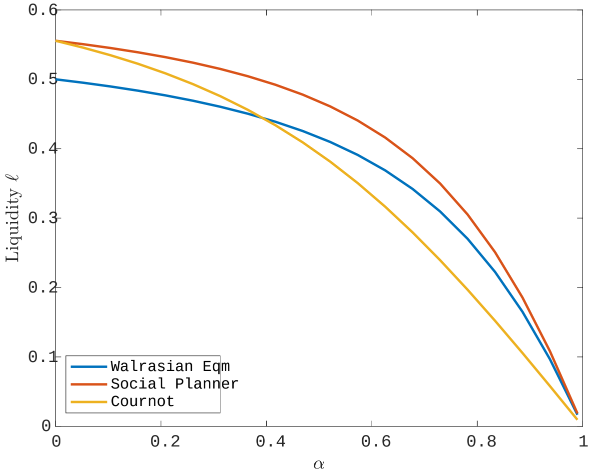
Equilibrium liquidity



- Bad state likely ➔ Cournot mitigates externality ✓
- Bad state unlikely ➔ Cournot exacerbates externality ↯

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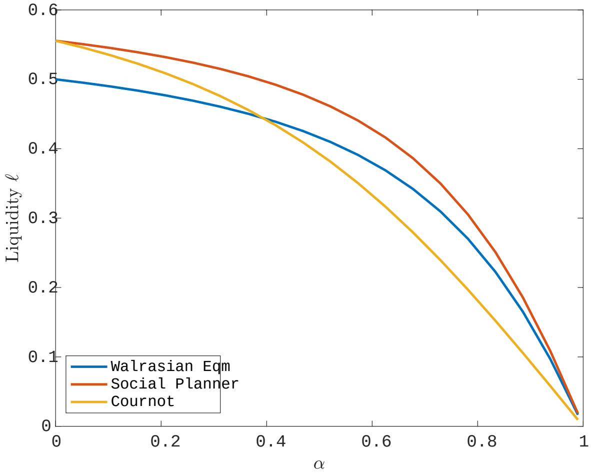

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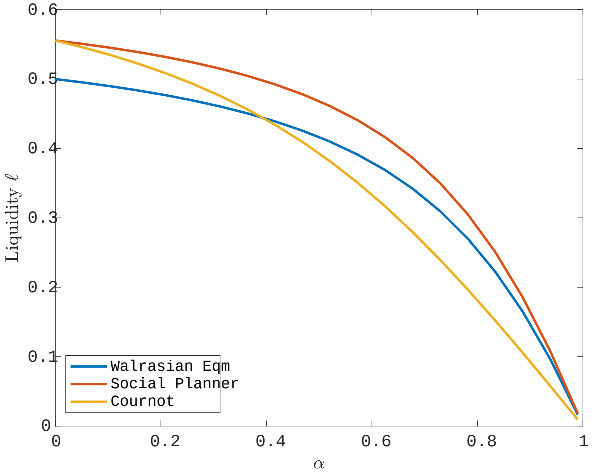

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Intuition

- Portfolio allocation liquid vs. illiquid asset $(\ell, 1-\ell)$
  - Higher $\ell \Leftrightarrow$ lower $1-\ell$
  - More liquidity: bad if buyer (cost), good if seller (benefit)
  - ➔ Cost–benefit tradeoff weighted by price impacts
- CITM pricing $\sum_{i \in \text{buy}} \ell_i = p \times \sum_{j \in \text{sell}} (1-\ell_j)$
  - Buyer ℓ enters with factor 1, seller ℓ with factor p
  - $\underbrace{\text{seller price impact}}_{\text{weight on liqu. benefit}} = p \times \underbrace{\text{buyer price impact}}_{\text{weight on liqu. cost}}$
  - ➔ relative weight on benefit shrinks with p

➔ Internalizing price effect can exacerbate low liquidity

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Levered investment in productive assets

Standard: pecuniary externality ➔ inefficiently high investment

New: internalizing price effect can overcorrect inefficiency

Firm investment model

Model of firm borrowing á la Lorenzoni

- $t = 0, 1, 2$
- $2N$ Firms
  - Production: capital k at $t = 0$ $\longrightarrow$ output Ak at $t = 1$
  - Borrowing: net worth & risk-free debt $\longrightarrow$ $k = n + d$
- Productivity shocks:
  - High productivity: $A_H k > Rd$ $\longrightarrow$ surplus $A_H k - Rd$
  - Low productivity: $A_L k < Rd$ $\longrightarrow$ shortfall $Rd - A_L k$
- $2N$ Households — inefficient users: $F(k) = a \log(1 + k)$

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Trade in real capital

- At $t = 1$ capital price $q < 1$
  - Low productivity: sell z_L such that

$$qz_L = Rd - A_L k = \underbrace{(R - A_L)\,k - Rn}_{\text{cash shortfall}}$$

  - High productivity: buy x_L such that

$$qx_H = A_H k - Rd = \underbrace{Rn + (A_H - R)\,k}_{\text{cash surplus}}$$

  ➔ shortfall and surplus increasing in k

- Households: residual demand $qx_{hh} = a - q$

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Trade in real assets

- Market clearing: $Nx_H + 2Nx_{hh} = Nz_L$

$$\sum_{i \in \text{buy}} \left(Rn + (A_H - R)\, k_i\right) + 2N\,(a - q)$$
$$= \sum_{j \in \text{sell}} \left((R - A_L)\, k_j - Rn\right)$$

- Equilibrium price

$$q = a + Rn + \sum_{i \in \text{buy}} \frac{(A_H - R)\, k_i}{2N} - \sum_{j \in \text{sell}} \frac{(R - A_L)\, k_j}{2N}$$
$$= a + Rn - \underbrace{\left(R - \frac{A_L + A_H}{2}\right)}_{>0} \times k$$

Trade in real assets

- Market clearing: $Nx_H + 2Nx_{hh} = Nz_L$

$$\sum_{i \in \text{buy}} \left(Rn + (A_H - R)\, k_i\right) + 2N\,(a - q) = \sum_{j \in \text{sell}} \left((R - A_L)\, k_j - Rn\right)$$

- Equilibrium price

$$q = a + Rn + \sum_{i \in \text{buy}} \frac{(A_H - R)\, k_i}{2N} - \sum_{j \in \text{sell}} \frac{(R - A_L)\, k_j}{2N} = a + Rn - \underbrace{\left(R - \frac{A_L + A_H}{2}\right)}_{>0} \times k$$

Equilibrium investment

- q decreasing in overall investment (social planner perspective)

→ Walrasian investment inefficiently high

- q increasing in buyers' investment
  - marginal investment → greater cash surplus → higher price

  → bad for buyers
- q decreasing in sellers' investment
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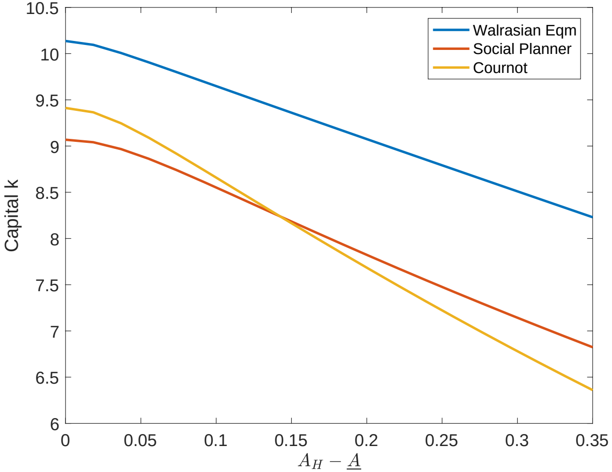

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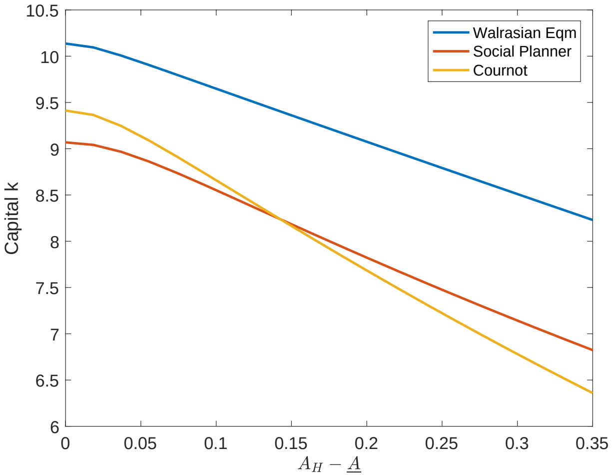

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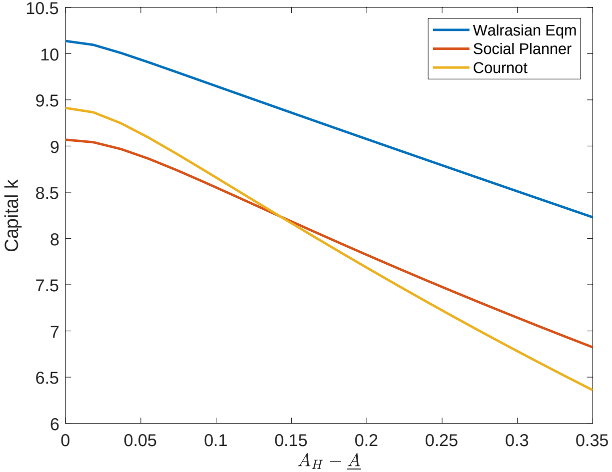

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  - Uncertain payoff scales with investment
  - Fixed debt repayment also scales with investment

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- More investment. . .
  - drives up price when buying
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➔ Internalizing price effect can overcorrect high investment

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