Discussion
Cournot Fire Sales

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Summary

- Starting point for this paper
  - Pecuniary/Fire-Sale externalities as rationale for regulation
  - Root of externalities: price-taking behavior
    - In addition to incomplete markets and/or binding constraints

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- This paper
  - Explores the role of non-price taking behavior (oligopoly)
- Interesting question
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  - Explores the role of non-price taking behavior (oligopoly)
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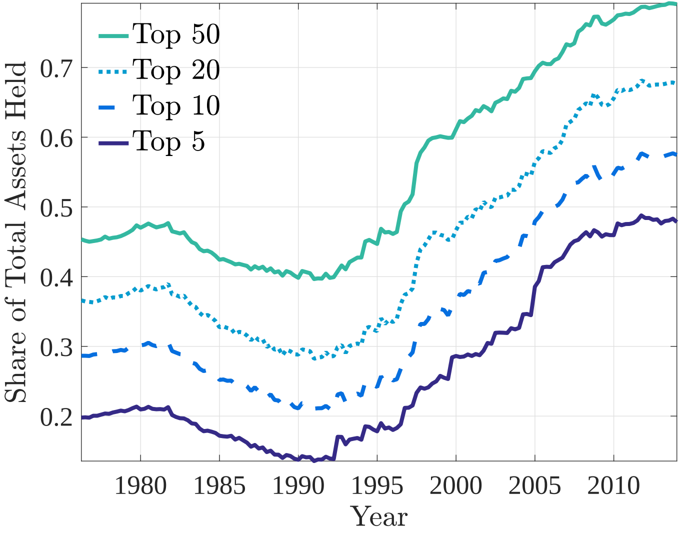
  - Practically: increased concentration in banking/intermediation
- Main takeaways
  - Cournot solution is different from planning solution
    - Different price impact
  - Cournot solution can *reverse* normative prescriptions
    - Move further away from planning solution (worsens lack of liquidity provision)
    - Under-investment (Cournot) instead of over-investment (CE) relative to planning solution

Increasing Concentration



- See Corbae-Levine 19

Roadmap

1. Abstract framework
2. Liquidity model
3. Final comments

Abstract Framework: Competitive Equilibrium

- General framework (incomplete markets)
  - $i \in I$ agents, single asset, many states, single good economy

$$\max_{x_t^i} \mathbb{E}_0 \left[\sum_t \beta^t u_i \left(c_t^i \right) \right]$$

$$c_t^i = e_t^i + d_t x_{t-1}^i - p_t \Delta x_t^i$$

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- **Benchmark 1: Competitive Equilibrium**

$$p_t = \mathbb{E}_t \left[\frac{\beta u_i' \left(c_{t+1}^i \right)}{u_i' \left(c_t^i \right)} \left(d_{t+1} + p_{t+1} \right) \right], \forall i, t$$

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- **Remark:** MRS generically not equalized, $\frac{\beta u_i'\left(c_{t+1}^i\right)}{u_i'\left(c_t^i\right)}$ vary across i

Abstract Framework: Distributive Externalities

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  - Consider perturbation: $\tilde{x}_t^i = x_t^i + \varepsilon h_t^i$ (e.g., $h_t^i = 1$, $\forall i$)

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- Incomplete markets: scope for Pareto Improvements (**distributive externalities**, see Davila/Korinek 18)
  1. Differences in MRS
  2. Net trading positions
  3. Price impact

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- Computing $\frac{dp_t}{d\varepsilon}$? Implicit Function Thm on $\int_i \Delta\tilde{x}_t^i\left(p,\varepsilon\right) = 0$, $\forall t$

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$$\int_i \frac{\partial\tilde{x}_t^i\left(p,\varepsilon\right)}{\partial\varepsilon} + \int_i \frac{\partial\tilde{x}_t^i\left(p,\varepsilon\right)}{\partial\varepsilon}\frac{dp}{d\varepsilon} = 0 \Rightarrow \frac{dp}{d\varepsilon} = -\left(\int_i \frac{\partial\tilde{x}_t^i\left(p,\varepsilon\right)}{\partial p}\right)^{-1}\int_i \underbrace{\frac{\partial\tilde{x}_t^i\left(p,\varepsilon\right)}{\partial\varepsilon}}_{=h_t^i}$$

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- Key difference: *Price impacts* are perceived differently
  - Formally, $\frac{dp_t^i}{d\varepsilon}$ instead of $\frac{dp_t}{d\varepsilon}$
  - Computing $\frac{dp_t^i}{d\varepsilon}$? Residual demands are agent specific

$$\Delta\tilde{x}_t^i(\varepsilon) + \int_{-i} \Delta\tilde{x}_t^{-i}(p) = 0 \Rightarrow \frac{dp_t^i}{d\varepsilon} = -\left(\int_{-i} \frac{\partial\tilde{x}_t^i(p,\varepsilon)}{\partial p}\right)^{-1} \underbrace{\frac{\partial\tilde{x}_t^i(p,\varepsilon)}{\partial\varepsilon}}_{=h_t^i}$$

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- Cournot solution must be bad under complete markets

$$\int_i \Delta \tilde{x}_t^i \frac{dp_t^i}{d\varepsilon} \neq 0$$

Liquidity Provision Model

- Elegant model
  - Ex-ante identical agents simplifies welfare comparisons
- Too much or too little liquidity depends on

$$\underbrace{\frac{dp_L}{d\ell}u'\left(c_L\right) - \frac{dp_H}{d\ell}\frac{1}{p}\beta R u'\left(c_H\right)}_{\text{cournot}} \gtreqless \underbrace{\left(u'\left(c_L\right) - \frac{1}{p}\beta R u'\left(c_H\right)\right)\frac{dp}{d\ell}}_{\text{constrained planner}}$$

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- Key intuition:
  - If bad state unlikely ($\alpha \to 1$)
  - Agents hold little liquidity ($\ell \to 0$)
  - And $\frac{dp_L}{d\ell} \to 0$ (but $\frac{dp_L}{d\ell} \to \frac{1}{N}$): small amount of liquidity, minimal price impact
- **Comment**: How robust are $\frac{dp_L}{d\ell}$ and $\frac{dp_H}{d\ell}$ results? Ideally empirically disciplined

Comments/Thoughts

1. Include welfare rankings
   - It is not obvious whether Cournot $\succ$ Competitive or vice versa
   - Paper focuses on ℓ (allocations)
2. Explore joint antitrust and insurance policies
   - Benchmark with imperfect competition and *complete* markets
3. Single agent case (full monopolist with RoW/fringe pricing)
   - Converges to constrained efficient benchmark
   - Worth discussing
4. Both models would benefit from sensible numerical illustrations
   - Sense of magnitudes
   - Calibration?